Vorticities in relativistic plasmas: from waves to reconnection

Felipe A. Asenjo[1]
Universidad Adolfo Ibáñez, Chile

- *Part I: Waves in relativistic plasmas*
- *Part II: Electro–Vortical formulation*
- *Part III: Generalized Connetion and Reconnection*

[1] felipe.asenjo@uai.cl; felipe.asenjo@gmail.com

Part III:
GENERALIZED CONNECTION AND RECONNECTION

Newcomb's Connection Theorem

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Motion of Magnetic Lines of Force*

WILLIAM A. NEWCOMB†

Project Matterhorn, Princeton University, Princeton, New Jersey

In 1958, Newcomb showed that in a plasma that satisfies the ideal Ohms law, two plasma elements connected by a magnetic field line at a given time will remain connected by a field line for all subsequent times. This occurs because the plasma moves with a transport velocity that preserves the magnetic connections between plasma elements. This is one of the most fundamental and relevant ideas in plasma physics.

Proof: d/dt is the convective derivative

Ohm's law $\vec{E} + \vec{v} \times \vec{B} = 0$ implies

$$\partial_t \vec{B} = \nabla \times (\vec{v} \times \vec{B}) = \frac{d\vec{B}}{dt} - (\vec{v} \cdot \nabla)\vec{B}$$

Proof: d/dt is the convective derivative

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$$\partial_t \vec{B} = \nabla \times (\vec{v} \times \vec{B}) = \frac{d\vec{B}}{dt} - (\vec{v} \cdot \nabla)\vec{B}$$

Be $d\vec{l} = \vec{x}' - \vec{x}$ the 3D vector connecting two infinitesimally close fluid elements.

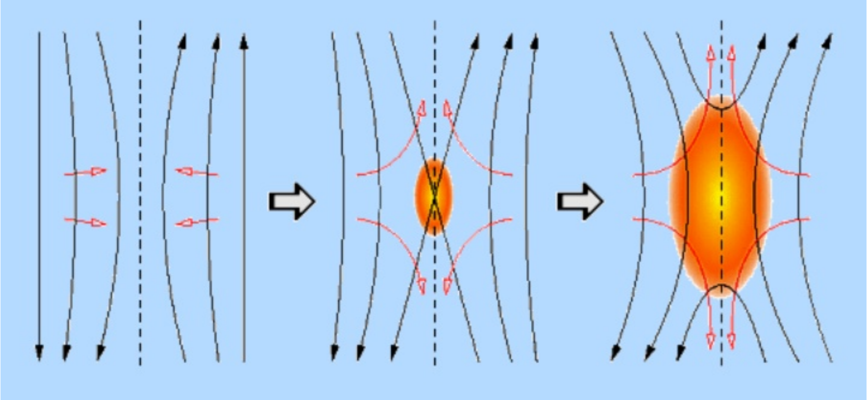
$$\frac{d}{dt} d\vec{l} = \vec{v}(\vec{x}') - \vec{v}(\vec{x}) = \vec{v}(\vec{x} + d\vec{l}) - \vec{v}(\vec{x}) = (d\vec{l} \cdot \nabla)\vec{v}$$

Then

$$\frac{d}{dt}(d\vec{l} \times \vec{B}) = -(d\vec{l} \times \vec{B})(\nabla \cdot \vec{v}) - \left[(d\vec{l} \times \vec{B}) \times \nabla\right] \times \vec{v}$$

Wich means that if $d\vec{l} \times \vec{B} = 0$, it always remains null

Why is important? Because when the Connetion Theorem is violated, then it can occur reconnection



Pegoraro's covariant generalization for ideal MHD in flat–spacetime

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Covariant form of the ideal magnetohydrodynamic "connection theorem" in a relativistic plasma

F. PEGORARO[a]

Dipartimento di Fisica "Enrico Fermi", Università di Pisa - Largo B. Pontecorvo 3, I-56127 Pisa, Italy, EU

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Abstract – The magnetic connection theorem of ideal magnetohydrodynamics by Newcomb (NEWCOMB W. A., *Ann. Phys. (N.Y.)*, **3** (1958) 347) and its covariant formulation are rederived and reinterpreted in terms of a "time resetting" projection that accounts for the loss of simultaneity in different reference frames between spatially separated events.

Ideal MHD in flat–spacetime

Ohm's law

$$u^\mu = \frac{dx^\mu}{d\tau}$$

$$F^{\mu\nu} u_\nu = 0$$

$$\frac{dF_{\mu\nu}}{d\tau} = (\partial_\mu u^\alpha) F_{\nu\alpha} - (\partial_\nu u^\alpha) F_{\mu\alpha}$$

$$d/d\tau = u^\mu \partial_\mu$$

$$\frac{d}{d\tau} dl^\mu = dl^\alpha \partial_\alpha u_\mu$$

where dl^μ is the 4D displacement of a plasma fluid element.

$$\frac{d}{d\tau}(dl^\mu F_{\mu\nu}) = -(\partial_\nu u^\beta) dl^\alpha F_{\alpha\beta}$$

This means that if $dl^\mu F_{\mu\nu} = 0$, it always remains null

What about ideal MHD in curved spacetimes?

PHYSICAL REVIEW D **96,** 123004 (2017)

Magnetic connections in curved spacetime

Felipe A. Asenjo[1,*] and Luca Comisso[2,†]

[1]*Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez, Santiago 7941169, Chile*

[2]*Department of Astrophysical Sciences and Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08544, USA*

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The ideal magnetohydrodynamic theorem on the conservation of the magnetic connections between plasma elements is generalized to relativistic plasmas in curved spacetime. The connections between plasma elements, which are established by a covariant connection equation, display a particularly complex structure in curved spacetime. Nevertheless, it is shown that these connections can be interpreted in terms of magnetic field lines alone by adopting a $3+1$ foliation of spacetime.

Ideal MHD in curved–spacetime

$$u_\mu u^\mu = g_{\mu\nu} u^\mu u^\nu = -1\,; \qquad F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu\,; \qquad d/d\tau = u^\mu \nabla_\mu$$

Ideal MHD in curved–spacetime

$$u_\mu u^\mu = g_{\mu\nu} u^\mu u^\nu = -1\,; \qquad F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu\,; \qquad d/d\tau = u^\mu \nabla_\mu$$

Ohm's law

$$F^{\mu\nu} u_\nu = 0 = \frac{dA_\mu}{d\tau} - u^\nu \nabla_\mu A_\nu$$

Ideal MHD in curved–spacetime

$$u_\mu u^\mu = g_{\mu\nu}u^\mu u^\nu = -1\,; \qquad F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu\,; \qquad d/d\tau = u^\mu \nabla_\mu$$

Ohm's law

$$F^{\mu\nu}u_\nu = 0 = \frac{dA_\mu}{d\tau} - u^\nu \nabla_\mu A_\nu$$

$$\begin{aligned} \frac{dF_{\mu\nu}}{d\tau} &= u^\alpha \nabla_\mu \nabla_\alpha A_\nu - u^\alpha \nabla_\nu \nabla_\alpha A_\mu + u^\alpha R_{\beta\nu\mu\alpha} A^\beta - u^\alpha R_{\beta\mu\nu\alpha} A^\beta \\ &= (\nabla_\mu u^\alpha) F_{\nu\alpha} - (\nabla_\nu u^\alpha) F_{\mu\alpha} \end{aligned}$$

$$\frac{d}{d\tau} dl^\mu = dl^\alpha \nabla_\alpha u_\mu$$

$$\frac{d}{d\tau}(dl^\mu F_{\mu\nu}) = -(\nabla_\nu u^\beta) dl^\alpha F_{\alpha\beta}$$

This means that if $dl^\mu F_{\mu\nu} = 0$, it always remains null

3+1 foliation of spacetime

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

where α is the lapse function, β^i is the shift vector (rotation), and γ_{ij} is the 3-metric of spacelike hypersurfaces

$$n_\mu = (-\alpha, 0, 0, 0)\,; \quad n^\mu = (1/\alpha, -\beta^i/\alpha)\,; \quad \gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$

$$\gamma_{\mu\nu}n^\mu = 0\,; \qquad n_\mu n^\mu = -1$$

$$E^\mu = n_\nu F^{\mu\nu}\,; \quad B^\mu = \frac{1}{2}n_\rho \epsilon^{\rho\mu\sigma\tau}F_{\sigma\tau}\,; \quad F^{\mu\nu} = E^\mu n^\nu - E^\nu n^\mu - \epsilon^{\mu\nu\rho\sigma}B_\rho n_\sigma$$

$$u^\mu = \alpha\Gamma n^\mu + \Gamma\gamma^\mu{}_\nu v^\nu\,; \qquad \Gamma = \left[\alpha^2 - \gamma_{ij}(\beta^i\beta^j + \beta^i v^j + \beta^j v^i + v^i v^j)\right]^{-1/2}$$

The connected field in ideal MHD in curved spacetimes for simultaneous events $n_\mu dl^\mu = 0$

$$dl^\mu F_{\mu\nu} = 0$$

$$n_\mu (dl^\alpha E_\alpha) - \epsilon_{\mu\nu\rho\sigma} dl^\nu B^\rho n^\sigma = 0$$

The connected field in ideal MHD in curved spacetimes for simultaneous events $n_\mu dl^\mu = 0$

$$dl^\mu F_{\mu\nu} = 0$$

$$n_\mu (dl^\alpha E_\alpha) - \epsilon_{\mu\nu\rho\sigma} dl^\nu B^\rho n^\sigma = 0$$

$$dl^i E_i = 0$$

$$\epsilon_{0ijk} dl^j B^k = 0$$

The connected magnetic field is

$$B^i = \frac{-\alpha}{2} \epsilon^{0ijk} F_{jk}$$

with gravitational corrections

What happens if we include resistivity in MHD?

- The Connection Theorem does not hold
- Reconnection in flat or curved spacetimes!

Reconnection around rotating black holes

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Relativistic Magnetic Reconnection in Kerr Spacetime

Felipe A. Asenjo[1,*] and Luca Comisso[2,†]
[1]*Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez, Santiago 7941169, Chile*
[2]*Department of Astrophysical Sciences and Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08544, USA*
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$$\nabla_\nu (H u^\mu u^\nu) = -\nabla^\mu p + J^\nu F^\mu{}_\nu$$
$$u^\nu F^\mu{}_\nu = \eta (J^\mu + u^\alpha J_\alpha u^\mu)$$
$$\nabla_\nu F^{\mu\nu} = J^\nu$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \sum_{i=1}^{3} (h_i dx^i - \alpha \beta^i dt)^2$$

$$\alpha^2 = h_0^2 + \sum_{i=1}^{3} (h_i \omega_i)^2 \, ; \quad \beta^i = h_i \omega_i / \alpha \, ; \quad h_0 = \sqrt{-g_{00}}$$

$$h_0 = (1 - 2 r_g r/\Sigma)^{1/2} \, ; \quad h_1 = (\Sigma/\Delta)^{1/2} \, ; \quad h_2 = \Sigma^{1/2} \, ; \quad h_3 = (A/\Sigma)^{1/2} \sin\theta$$
$$\omega_1 = 0 = \omega_2 \, ; \quad \omega_3 = 2 r g^2 a r/\Sigma \, ; \quad A = (r^2 + a^2 r_g^2)^2 - \Delta a^2 r_g^2 \sin^2\theta$$
$$\Sigma = r^2 + a^2 r_g^2 \cos^2\theta \, ; \quad \Delta = r^2 - 2 r_g r + a^2 r_g^2 \, ; \quad r_g = GM \, ; \quad a = J/GM^2 \le 1$$

Reconnection around rotating black holes

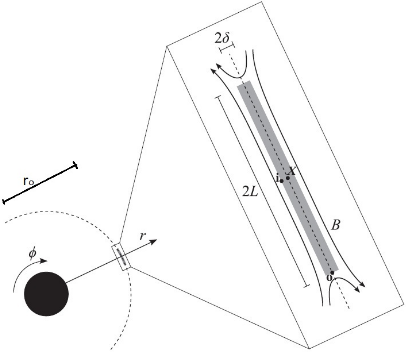

FIG. 1. Sketch of a reconnection layer in the azimuthal direction. The rotating black hole is represented by the black circle, while the magnetic diffusion region is marked by the shaded area.

The inflow velocity (reconnection rate)

$$v_i \approx S^{-1/2}\left(1 - \frac{a^2 r_g^2}{4 r_o^2}\right) \; ; \qquad S = Lc/\eta$$

Let's go further: Electro-Vortical unification

Is a Connetion Theorem for a general one–fluid plasma?

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General connected and reconnected fields in plasmas

Swadesh M. Mahajan[1,2,a)] and Felipe A. Asenjo[3,b)]
[1]*Institute for Fusion Studies, The University of Texas at Austin, Austin, Texas 78712, USA*
[2]*Department of Physics, Shiv Nadar University, Lucknow, UP 201314, India*
[3]*Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez, Santiago 7941169, Chile*

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Consider two oppositely charged plasma fluids under the Electro-Vortical unification

$$en_+ u_{\nu+} \mathcal{M}_+^{\mu\nu} = 0$$

$$en_- u_{\nu-} \mathcal{M}_-^{\mu\nu} = 0$$

The general one–fluid plasma under the Electro-Vortical unification

momentum equation

$$J_\nu D^{\mu\nu} + 2ne\mathcal{U}_\nu Z^{\mu\nu} = 0$$

Ohm's law

$$J_\nu Z^{\mu\nu} + 2ne\mathcal{U}_\nu D^{\mu\nu} = 0$$

$$\mathcal{U}^\mu = \mathbf{U}^\mu - \frac{\Delta\mu}{2ne} J^\mu\,; \qquad n = \frac{m_+ n_+ + m_- n_-}{m_+ + m_-}\,; \quad \Delta\mu = \frac{m_+ - m_-}{m_+ + m_-}$$

$$\mathbf{U}^\mu = \frac{m_+ n_+ u_+^\mu + m_- n_- u_-^\mu}{m_+ n_+ + m_- n_-}\,; \quad J^\mu = e n_+ u_+^\mu - e n_- u_-^\mu\,; \quad \partial_\mu (n\mathcal{U}^\mu) = 0$$

$$Z^{\mu\nu} = \frac{1}{2}(\mathcal{M}_+^{\mu\nu} - \mathcal{M}_-^{\mu\nu})$$

$$D^{\mu\nu} = \frac{1}{2}(\mathcal{M}_+^{\mu\nu} + \mathcal{M}_-^{\mu\nu})$$

Connection Theorem?

Ohm's law

$$\mathcal{U}_\nu D^{\mu\nu} = \Gamma^\mu = \frac{J_\nu Z^{\nu\mu}}{2ne}$$

$$\frac{d}{d\tau} = \mathcal{U}^\mu \partial_\mu$$

$$\frac{d}{d\tau} dl^\mu = dl^\nu \partial_\nu \mathcal{U}^\mu$$

$$\frac{d}{d\tau}(dl_\alpha D^{\alpha\mu}) = -(dl_\alpha D^{\alpha\nu})\partial^\mu \mathcal{U}_\nu + dl_\alpha(\partial^\alpha \Gamma^\mu - \partial^\mu \Gamma^\alpha)$$

- If initially $dl_\alpha D^{\alpha\mu} = 0$, then later $dl_\alpha D^{\alpha\mu} \neq 0$.
- THE CONNECTION THEOREM DOES NOT HOLD IN GENERAL.
- This implies general reconnection.
- For ideal MHD, $\Gamma^\mu \to 0$

Condition for Connection Theorem

If

$$\Gamma^{\mu} = \frac{J_{\nu} Z^{\nu\mu}}{2ne} = \Lambda_{\mu} D^{\mu\nu}$$

for some Λ_{μ}, then

$$\frac{d}{d\tau} = (\mathcal{U}^{\mu} + \Lambda^{\mu})\partial_{\mu}$$

$$\frac{d}{d\tau}(dl_{\alpha} D^{\alpha\mu}) = -(dl_{\alpha} D^{\alpha\nu})\partial^{\mu}(\mathcal{U}_{\nu} + \Lambda_{\nu})$$

However, it is very difficult to justify physically Λ_{μ}, as it depends on the electromagnetic properties of the fluid.

Thanks!