Singularities of mean curvature flow and surgery

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Let $\gamma : M \times [0.T) \to \mathbb{R}^{n+1}$ be a family of smooth immersions of an $n-$dimensional manifold M evolving by mean curvature flow. When M is compact, it is well known that the mean curvature flow is defined up to a finite singular time T at which the curvature of the hypersurface becomes unbounded. For instance, if the initial surface is convex, the evolving surfaces become spherical and contract to a point. On the other hand, if the initial surface is not convex, the evolution may become singular without shrinking.

The study of singularities for the mean curvature flow is of great interest but, also in the last decades several methods have been developed to continue the flow beyond the first singular time. Huisken and Sinestrari considered a new approach based on a surgery procedure to extend the mean curvature flow after singularities. This construction was inspired by a procedure originally introduced by Hamilton for the Ricci flow which played a fundamental role in the work by Perelman to prove the Poincarè and Geometrization conjectures.

The results about singularities and surgery for the mean curvature flow were obtained for manifolds of codimension 1. In this case the comparison principle is fundamental which, among others things, ensures that the embeddedness is preserved. In contrast, the comparison principle cannot be applied for higher codimension manifolds making more difficult the study of such evolution.

In this talk I will introduce the types of singularities as well as the procedure of surgery for mean curvature flow. We are going to discuss some new results concerning the simplest case in higher codimension, space curves.