Part II

In one year, the LHC provides ~10^{14} *pp* collisions

An observation of ~ 10 events could be a discovery of new physics.

Searching for a needle in a haystack?

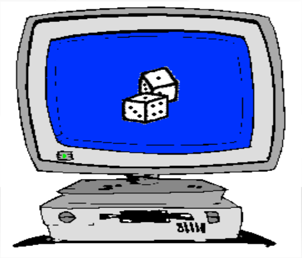
- **typical needle: 5 mm^3**
- **typical haystack: 50 m^3**

→ **needle : haystack = 1 : 10^{10}**

→ **Looking for new physics at the LHC is like looking for a needle in thousands of haystacks … meaning probability!!**

The Concept of Probability

- Many processes in nature have uncertain outcomes.
- A *random process is a process* that can be reproduced, to some extent, within some given boundary and initial conditions, but whose outcome is uncertain.
- For example, quantum mechanics phenomena have intrinsic randomness.
- *Probability* is a measurement of how *favored one of the possible outcomes of such a random* process is compared with any of the other possible outcomes.



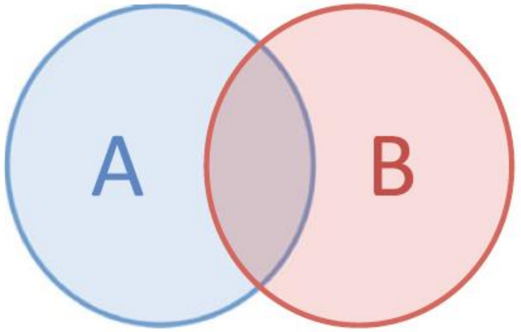
The Meaning of Probability: 2 approaches

- *Frequentist probability is defined as the fraction of the number of occurrences* of an event of interest over the total number of possible events in a repeatable experiment, in the *limit of very large number of experiments.*

- *Bayesian probability measures someone's degree of belief that a statement, and it makes u*se of an extension of the Bayes theorem: the probability of an event *A given the condition that the event B has occurred is given by:*

$$P(A|B) = \frac{P(A \cap B)}{P(B)} .$$

- The conditional probability is equal to the area of the intersection divided by the area if B



A Word on Simulation

- What a (computer) simulation does:
  - Applies mathematical methods to the analysis of complex, real-world problems
  - Predicts what might happen depending on various actions/scenarios

- Use simulations when
  - Doing the actual experiments is not possible
  - The cost in money, time, or danger of the actual experiment is prohibitive (e.g. nuclear reactors)
  - The system does not exist yet (e.g. an airplane)
  - Various alternatives are examined (e.g. hurricane predictions)

An example for illustration

Correct dice

every number has probability 1/6

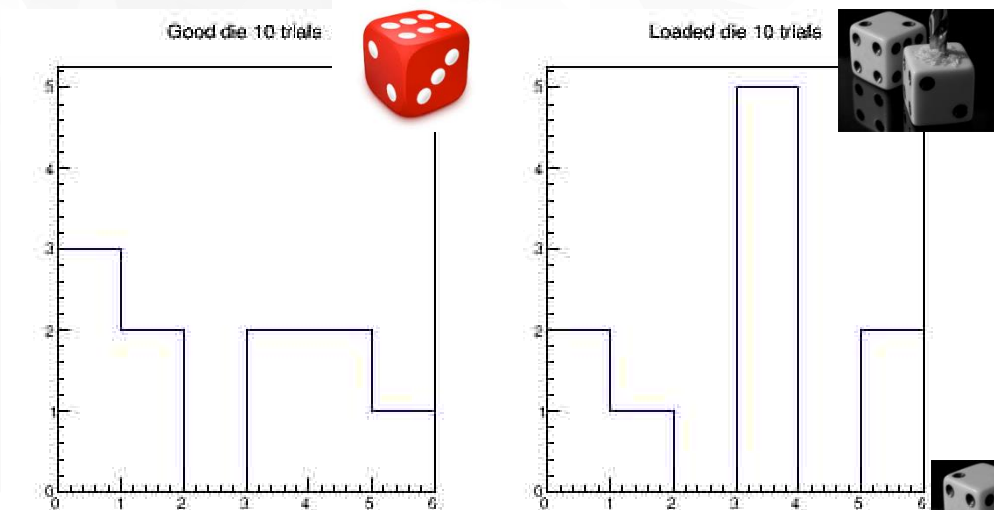
Manipulated dice

numbers 1..5 probability <1/6

number 6 probability > 1/6

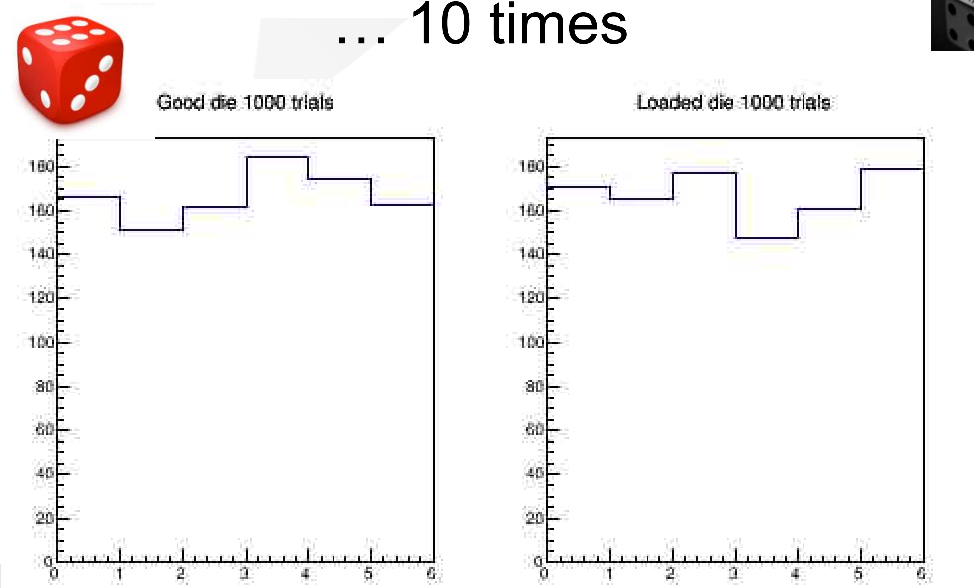
An example for illustration

Role the dice and record the number in a bar chart



Just random fluctuations

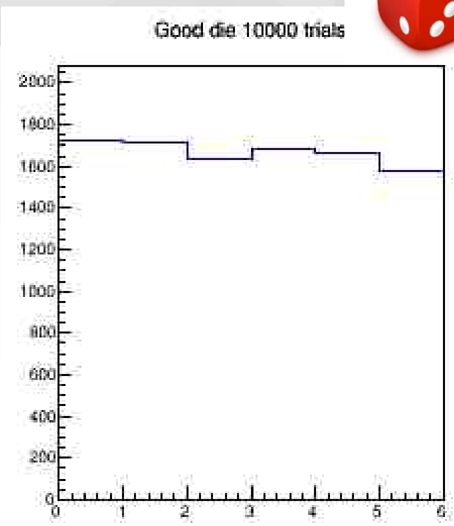
… 10 times

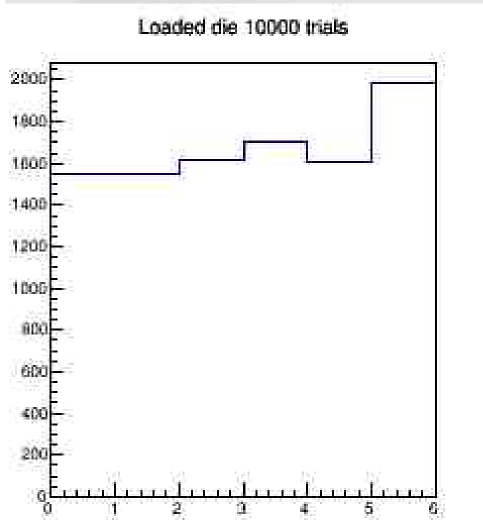


Still nothing can be concluded

… 1000 times

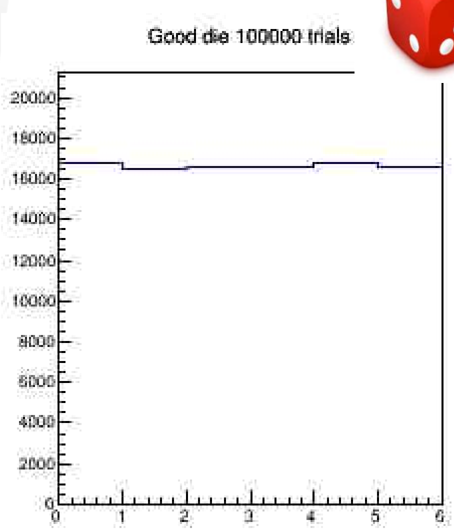
An example for illustration

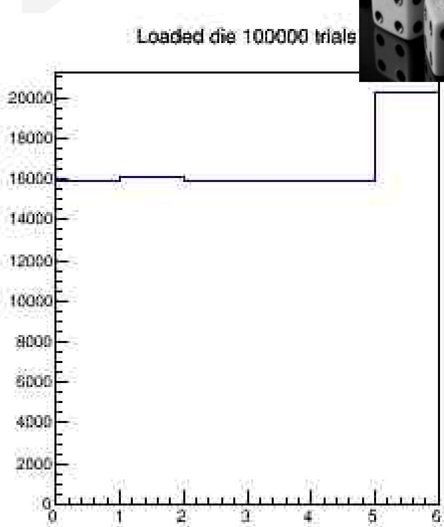




… 10000 times

Evidence is rising …

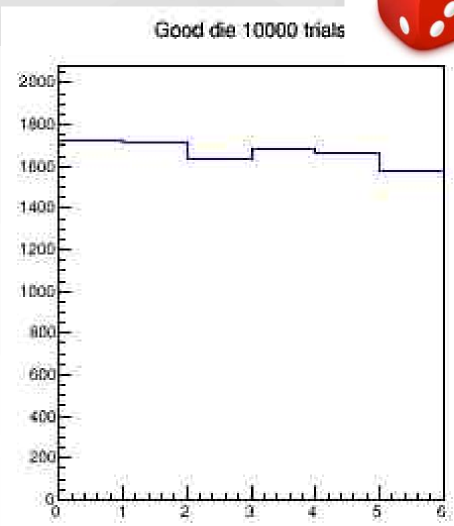


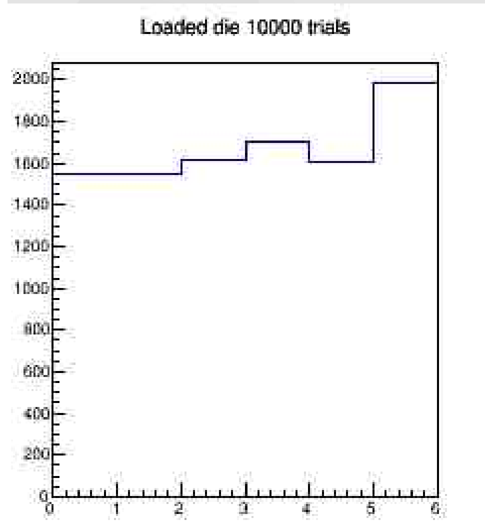


… 100000 times

For sure there is something wrong with the dice

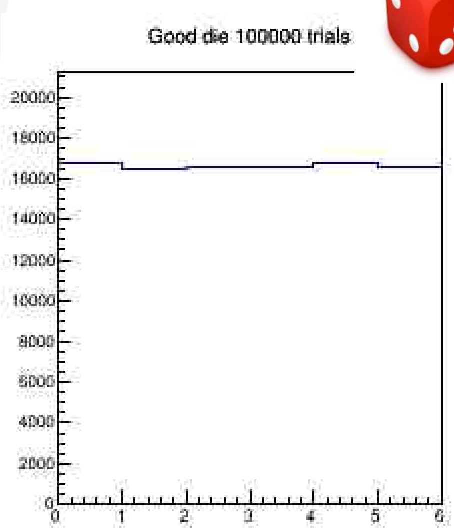
An example for illustration

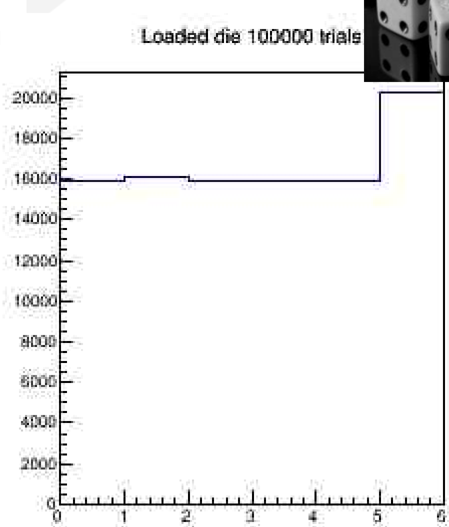




… 10000 times

Evidence is rising …



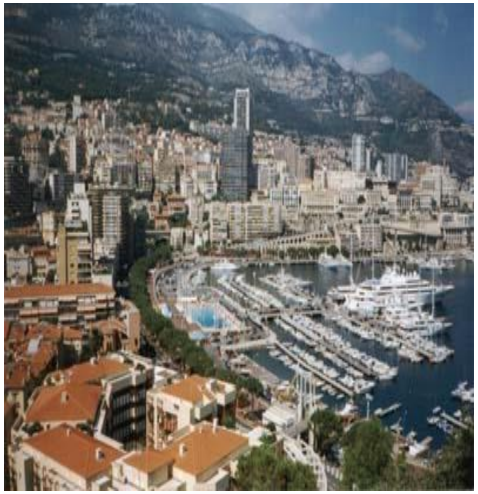


… 100000 times

For sure there is something wrong with the dice

The more data you take the smaller your error gets (Gauss)

Monte Carlo Method



- A numerical simulation method which uses sequences of random numbers to solve complex problems
- MC assumes the system is described by probability density functions (PDF) which can be modeled with no need to write down equations
- These PDF are sampled randomly, many simulations are performed and the result is the average over the number of observations

Practical example: estimating the value of π using the Monte Carlo method

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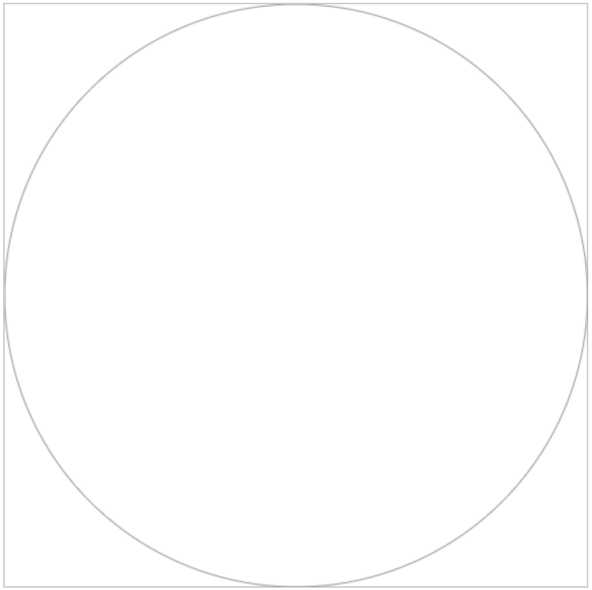
- A: Generate a large number of random points and see how many fall in the circle enclosed by the unit square.

Practical example: estimating the value of π using the Monte Carlo method

- A: Generate a large number of random points and see how many fall in the circle enclosed by the unit square

- Build a circle of radius 0.5, enclosed by a 1 × 1 square. The area of the circle is: $\pi R^2 = \pi/4$

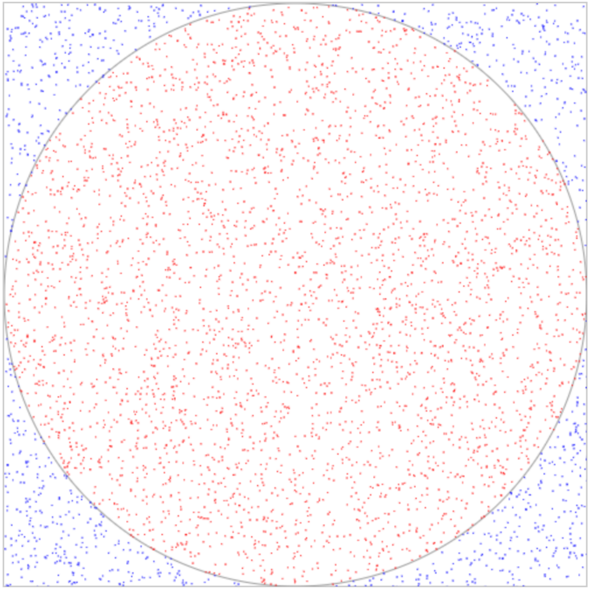


- The area of the square is 1.

- If we divide the area of the circle, by the area of the square we get: $\pi/4$

Practical example: estimating the value of π using the Monte Carlo method

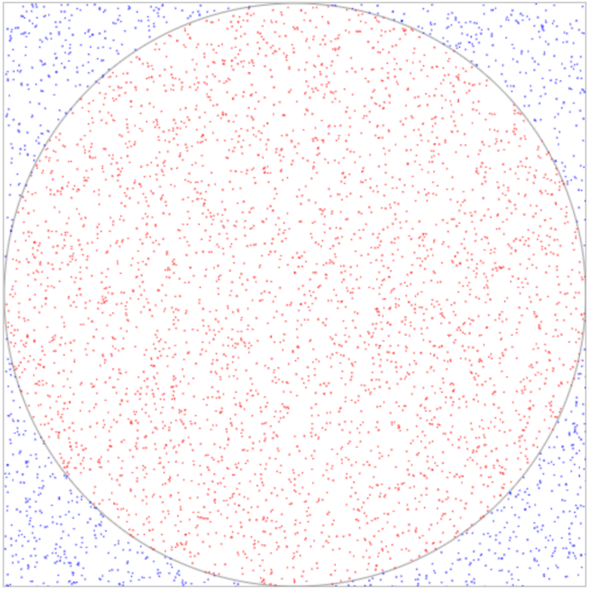
- Generate a large number of uniformly distributed random points and plot them on the graph. These points can be in any position within the square i.e. between (0,0) and (1,1).
- If they fall within the circle, they are coloured red, otherwise they are coloured blue.



Practical example: estimating the value of π using the Monte Carlo method

- We keep track of the total number of points, and the number of points that are inside the circle.
- If we divide the number of points within the circle, Ninner, by the total number of points, Ntotal, we should get a value that is an approximation of the ratio of the areas we calculated above, π/4

$$\pi \approx 4\frac{N_{inner}}{N_{total}}$$



- With a small number of points, the estimation is not very accurate, but with thousands of points, we get closer to the actual value

Practical example: estimating the value of π using the Monte Carlo method

- Play here:

https://academo.org/demos/estimating-pi-monte-carlo/