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Electromagnetic Waves in Vacuum.

⇒ In region of free space (i,e the vacuum)

→ where no electric charges, no electric currents and no matter of any are present

→ Maxwell's equations are.

1, $\vec{\nabla} \cdot \vec{E}(\bar{r},t) = 0$

2, $\bar{\nabla} \cdot \bar{B}(\bar{r},t) = 0$

3, $\bar{\nabla} \times \bar{E}(\bar{r},t) = \frac{-\partial \bar{B}(\bar{r},t)}{\partial t}$

4, $\bar{\nabla} \times \bar{B}(\bar{r},t) = \mu_0 \epsilon_0 \frac{\partial \bar{E}(\bar{r},t)}{\partial t}$

$$= \frac{1}{c^2} \frac{\partial \bar{E}}{\partial t} \qquad \text{where } c^2 = \frac{1}{\mu_0 \epsilon_0}$$

These eqns. are set of coupled first-order partial equations

→ Can be decoupled by applying curl operator to eqns 3 & 4

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$$\nabla\times(\nabla\times E) = \nabla\times\left(-\frac{\partial B}{\partial t}\right) \qquad \nabla\times(\nabla\times B) = \nabla\times\left(\frac{1}{c^2}\frac{\partial \bar{E}}{\partial t}\right)$$

using vector identity

$$\nabla\times(\nabla\times A) = \nabla(\bar{\nabla}\cdot\bar{A}) - \bar{\nabla}^2\bar{A}$$

$$\Rightarrow \bar{\nabla}(\cancel{\bar{\nabla}\cdot E}^{\,0}) - \bar{\nabla}^2 E = -\frac{\partial}{\partial t}(\nabla\times B)$$

$$-\nabla^2 \bar{E} = -\frac{\partial}{\partial t}\left(\frac{1}{c^2}\frac{\partial \bar{E}}{\partial t}\right)$$

$$\Rightarrow \quad \left.\begin{aligned} \nabla^2 E &= \frac{1}{c^2}\frac{\partial^2 \bar{E}}{\partial t^2} \\ \text{Similarly } \nabla^2 B &= \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} \end{aligned}\right\} \text{3D - de-coupled wave equations}$$

→ Have exactly the same structure

→ Both are linear, homogenous, 2nd order differential equations.

Both eqn's have explicit dependent on space and time

$$\nabla^2\bar{E}(\bar{r},t) - \frac{1}{c^2}\frac{\partial^2\bar{E}(\bar{r},t)}{\partial t} = 0$$

$$\nabla^2\bar{B}(\bar{r},t) - \frac{1}{c^2}\frac{\partial^2\bar{B}(\bar{r},t)}{\partial t^2} = 0$$

⇒ Maxwell's equations implies that empty space – the vacuum – support the propagation of electromagnetic waves – at the speed of light

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/sec}$$

Monochromatic EM Plane waves

→ A plane wave is a constant freq. $\nu(\lambda)$ wave whose

→ wavefronts are infinite parallel planes.

→ Have constant amplitude normal ($\perp$) to the phase velocity vector.

→ Propagates with speed of light in vacuum.

$$c = \nu\lambda = \frac{\omega}{k}$$

Mathematical form is

$$\vec{F}(\bar{r}, t) = \vec{F_0}\, e^{i(\bar{k}\cdot\bar{r} - \omega t)}$$

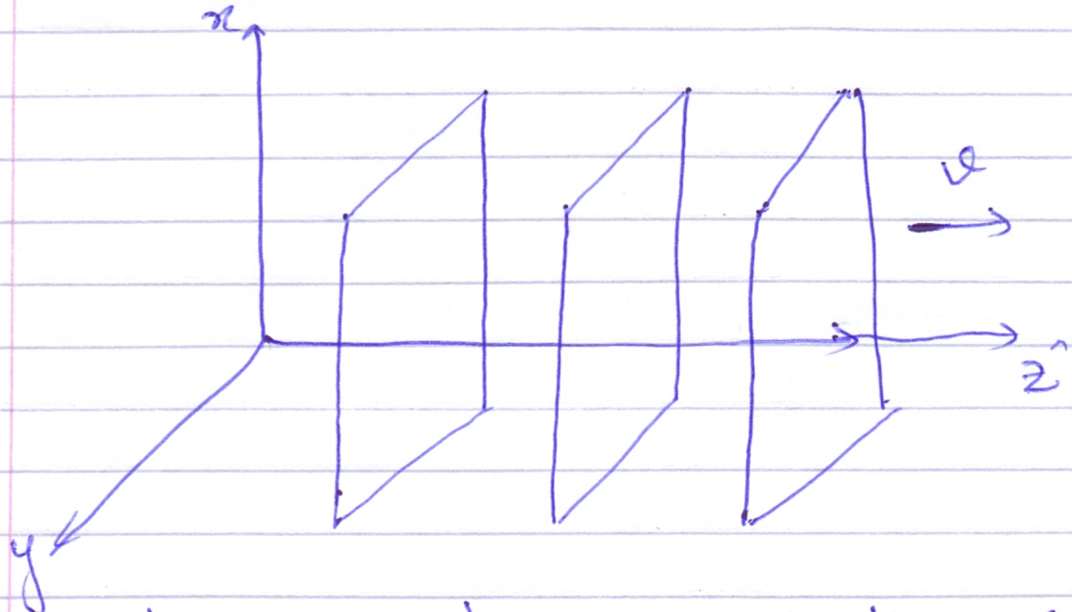
Uniform plane wave

Generally have uniform or constant properties in plane $\perp$ to their direction of propagation.

⇒ The magnitude of the electric and magnetic fields are the same at all points in the direction of propagation

⇒ The Electric & Magnetic fields are orthogonal to the direction of propagation.

⇒ EM wave that propagates in z-direction



Lie in plane $\perp$ to the $\hat{z}$-axis.

$\vec{E}$ & $\vec{B}$ are function of (z,t)

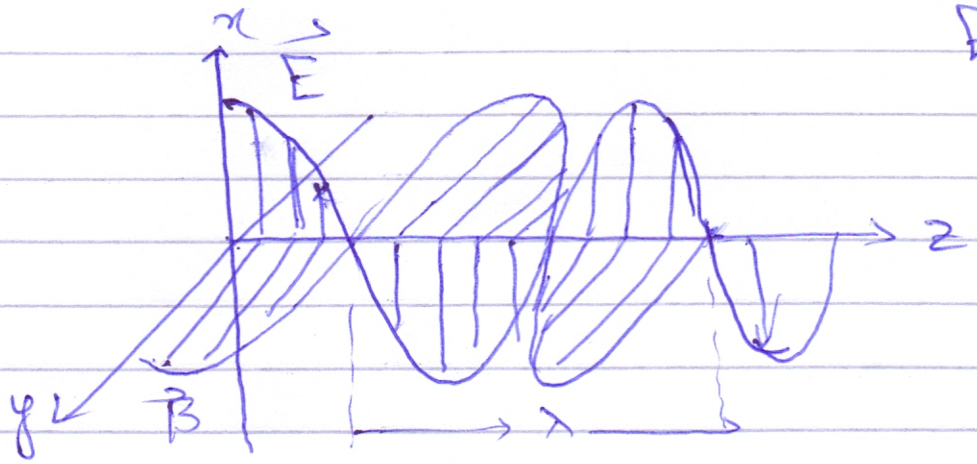
⇒ The direction of propagation is taken to be along z-axis.

⇒ The direction of propagation is normal to the plane formed by the electric & magnetic field vectors.

⇒ The phase of these fields is indep-endent of x & y.

⇒ no phase variation exist over the planer surfaces orthogonal to the direction of the propagation.

$\vec{E} = E_0 \hat{x}$



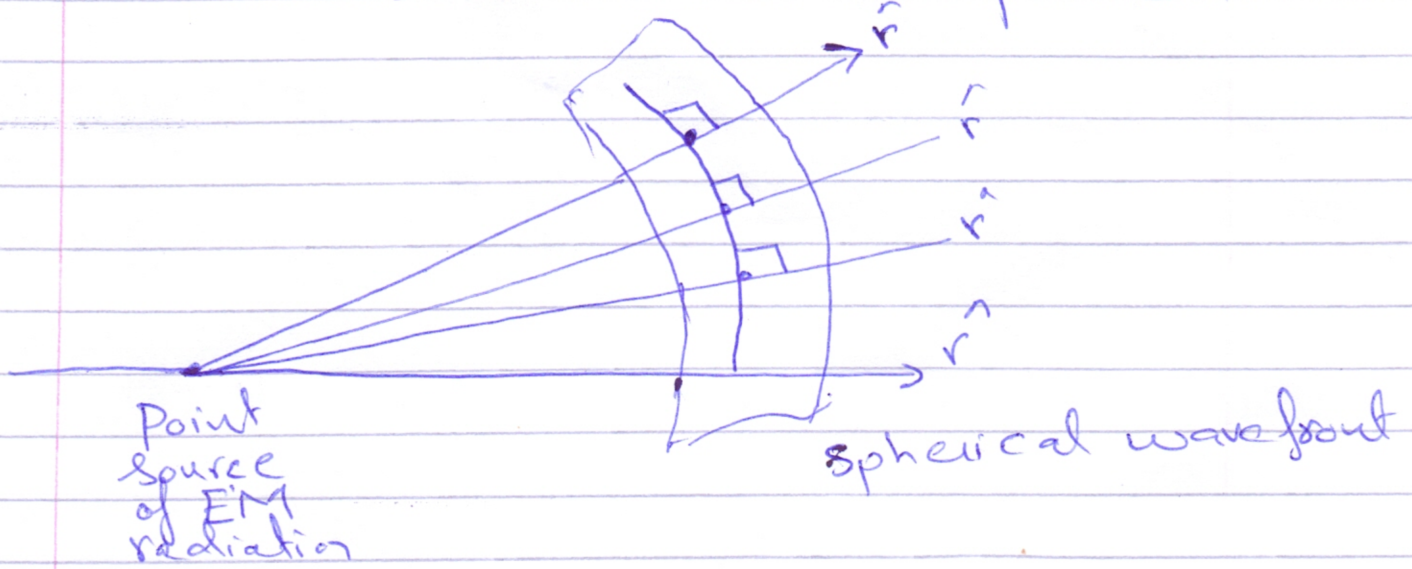
Important properties of waves are Amp, phase or frequency which allows the waves to carry information from source to destination.

$\vec{E}$ is function of $(\bar{x}, t)$ and independent of y & z.

Electromagnetic Spherical waves.

→ Another possible solution of wave equation can be spherical EM waves — emitted from a point source

→ Wave-fronts associated with these EM waves are spherical.



Mathematical form

$$\vec{F}(r,t) = \frac{F_0}{r} e^{i(k \cdot r - \omega t)}$$

r – radial distance from the point source to a given pt on wave front.

~~A~~ $\frac{F_0}{r}$ — amplitude

→ If point source is infinitely far

away from field point (observer)

→ A spherical wave → plane wave in this limit ($R_c \to \infty$)

Criterion for a plance wave

$$\lambda \ll R_c$$

Monochromatic Plane waves associated with $\vec{E}$ & $\vec{B}$

Using complex notation i,e

$$e^{i\omega t} = \cos\omega t + i\sin\omega t \quad \text{Euler's eqn.}$$

$$\Rightarrow \quad \vec{\tilde{E}}(\bar{r},t) = \vec{\tilde{E}}_0 \left(e^{i(k \cdot r - \omega t)}\right)$$

$$\& \quad \vec{\tilde{B}}(\bar{r},t) = \vec{\tilde{B}}_0 \left(e^{i(k \cdot r - \omega t)}\right)$$

For wave propagating in z-directi

$$\vec{K} \cdot \bar{r} = (k_x \hat{x} + k_y \hat{y} + k_z \hat{z}) \cdot (x\hat{x} + y\hat{y} + z\hat{z})$$

$$= kz$$

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Monochromatic EM Plane Waves

$$\tilde{\vec{E}}(z,t) = \tilde{\vec{E}}_0 \underbrace{e^{i(kz-\omega t)}}_{\text{propagating in } +z \text{ direc.}}$$

complex vectors

Similarly for M. field

$$\tilde{\vec{B}}(z,t) = \tilde{\vec{B}}_0 e^{i(kz-\omega t)}$$

with $\tilde{\vec{E}}_0 = \vec{E}_0 e^{i\delta_x} = E_0 e^{i\delta_x} \hat{x}$

& $\tilde{\vec{B}}_0 = \bar{B}_0 e^{i\delta_y} \hat{y}$

$\Rightarrow$ The real, physical (instantaneous) fields are:

$$\vec{E}(\bar{r},t) \equiv Re\left(\tilde{\vec{E}}(\bar{r},t)\right)$$

$$\& \; \vec{B}(\bar{r},t) \equiv Re\left(\tilde{\vec{B}}(\bar{r},t)\right)$$

$\Rightarrow$ Maxwell's equations impose additional constraints on $\tilde{\vec{E}}_0$ & $\tilde{\vec{B}}_0$

As $\bar{\nabla}\cdot\bar{E} = 0$ & $\bar{\nabla}\cdot\bar{B} = 0$

$Re(\bar{\nabla}\cdot\tilde{\vec{E}}) = 0$ $\quad$ $Re(\bar{\nabla}\cdot\tilde{\vec{B}}) = 0$

only satisfied if.

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if $\bar{\nabla} \cdot \tilde{\vec{E}} = 0$ for all $\bar{r}, t$

and $\bar{\nabla} \cdot \tilde{\vec{B}} = 0 \quad \forall (\bar{r}, t)$

In cartesian Co-ordinates.

$\nabla \cancel{\cdot E} = \quad \vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$

$\bar{\nabla} \cdot \tilde{\vec{E}} = 0 \quad ; \quad \bar{\nabla} \cdot \tilde{\vec{B}} = 0$

$$\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) \cdot \left(\tilde{\vec{E}}_0 e^{i(kz - \omega t)}\right) = 0$$

If we allow all polarization direction

$$\Rightarrow \tilde{\vec{E}}_0 = \left(E_{0x}\hat{x} + E_{0y}\hat{y} + E_{0z}\hat{z}\right) e^{i\delta} \equiv \vec{E}_0 e^{i\delta}$$

$$\bar{\nabla} \cdot \tilde{\vec{E}} = 0$$

$$\Rightarrow \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) \cdot \left(E_{0x}\hat{x} + E_{0y}\hat{y} + E_{0z}\hat{z}\right) e^{i\delta} e^{i(kz - \omega t)} = 0$$

E_{0x}, E_{0y} & E_{0z} $\Rightarrow$ Amplitudes of the E.F components in x, y, z directions

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$$\Rightarrow \frac{\partial}{\partial x}\hat{x}\cdot E_{0x}\hat{x}\,e^{i(kz-\omega t)}e^{i\delta} = 0$$

$$\frac{\partial}{\partial y}\hat{y}\cdot E_{0y}\hat{y}\,e^{i(kz-\omega t)}e^{i\delta} = 0$$

$$\frac{\partial}{\partial z}\hat{z}\cdot E_{0z}\hat{z}\,e^{i(kz-\omega t)}e^{i\delta} = ikE_{0z}e^{i(kz-\omega t)}e^{i\delta}$$

This will = zero if and only if

$$E_{0z} \equiv 0$$

Similarly $B_{0z} \equiv 0$ iff ~~ff~~

$\Rightarrow$ Maxwell's eqns impose the restriction that an electromagnetic plane wave cannot have any component of $\vec{E}$ or $\vec{B}$ ~~parallel~~ parallel ~~and~~ or anti-parallel to the propagation direction.

$\Rightarrow$ EM wave is a transverse wave (at least for propagation in free space)

→ Maxwell's eqns. impose another restriction on the allowed form of $\vec{E}$ and $\vec{B}$ for an EM wave.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{or} \quad \vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

$$= Re(\vec{\nabla} \times \vec{\tilde{E}}) = Re\left(-\frac{\partial \vec{\tilde{B}}}{\partial t}\right)$$

It can only be satisfied if and only if.

$$\vec{\nabla} \times \vec{\tilde{E}} = -\frac{\partial \vec{\tilde{B}}}{\partial t} \quad \text{or} \quad \vec{\nabla} \times \vec{\tilde{B}} = \frac{1}{c^2}\frac{\partial \vec{\tilde{E}}}{\partial t}$$

$$\vec{\nabla} \times \vec{\tilde{E}} = \left(\frac{\partial \tilde{E}_z}{\partial y} - \frac{\partial \tilde{E}_y}{\partial z}\right)\hat{x} + \left(\frac{\partial \tilde{E}_x}{\partial z} - \frac{\partial \tilde{E}_z}{\partial x}\right)\hat{y} + \underbrace{\left(\frac{\partial \tilde{E}_y}{\partial x} - \frac{\partial \tilde{E}_x}{\partial y}\right)}_{\text{zero } \because \text{ no } x \,\&\, y \text{ dependence in } \tilde{E}_x \,\&\, \tilde{E}_y}$$

$$= -\frac{\partial \tilde{B}_x}{\partial t}\hat{x} - \frac{\partial \tilde{B}_y}{\partial t}\hat{y} - \frac{\partial \tilde{B}_z}{\partial t}\hat{z}$$

As E_z & $B_z = 0$

$$\vec{\nabla} \times \vec{\tilde{E}} = -\frac{\partial \tilde{E}_y}{\partial z} + \frac{\partial \tilde{E}_x}{\partial z}\hat{y} = -\frac{\partial \tilde{B}_x}{\partial t}\hat{x} - \frac{\partial \tilde{B}_y}{\partial t}\hat{y} \quad \Big\{ \text{hold it till next page}$$

Complex Electric field vector $\vec{\tilde{E}}$ is given by

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$$\tilde{\vec{E}} = \tilde{E}_x \hat{x} + \tilde{E}_y \hat{y} + \cancel{\tilde{E}_z \hat{z}}^{\,0}$$

$$= \left(E_{0x}\hat{x} + E_{0y}\hat{y}\right) e^{i(kz-\omega t)}\, e^{i\delta} \quad \Bigg| \quad \tilde{E}_x = E_{0x} e^{i\delta} \times e^{i(kz-\omega t)}$$

Similarly

$$\tilde{\vec{B}} = \tilde{B}_x \hat{x} + \tilde{B}_y \hat{y} = \left(B_{0x}\hat{x} + B_{0y}\hat{y}\right) e^{i(kz-\omega t)}\, e^{i\delta}$$

$$\vec{\nabla}\times\tilde{\vec{E}} = -\frac{\partial \tilde{E}_y}{\partial z}\hat{x} + \frac{\partial \tilde{E}_x}{\partial z}\hat{y} = -\frac{\partial \tilde{B}_x}{\partial t}\hat{x} - \frac{\partial \tilde{B}_y}{\partial t}\hat{y} \quad (1)$$

$$\vec{\nabla}\times\tilde{\vec{B}} = -\frac{\partial \tilde{B}_y}{\partial t}\hat{x} + \frac{\partial \tilde{B}_x}{\partial t}\hat{y} = \frac{1}{c^2}\frac{\partial \tilde{E}_x}{\partial t}\hat{x} + \frac{1}{c^2}\frac{\partial \tilde{E}_y}{\partial t}\hat{y} \quad (2)$$

Comparing same components

x-component

$$\vec{\nabla}\times\tilde{\vec{E}} : \boxed{-\frac{\partial \tilde{E}_y}{\partial z}\hat{x} = -\frac{\partial \vec{B}_x}{\partial t}\hat{x}}$$

$$\Rightarrow \quad \frac{\partial \tilde{E}_y}{\partial z} = \frac{\partial \tilde{B}_x}{\partial t} \Rightarrow ikE_{0y} e^{i(kz-\omega t)} e^{i\delta} = -i\omega B_{0x} e^{i(kz-\omega t)} e^{i\delta}$$

$$\Rightarrow \quad ikE_{0y} = -i\omega B_{0x} \quad (3)$$

For y-compt.

$$\frac{\partial E_x}{\partial z}\hat{y} = -\frac{\partial \tilde{B}_y}{\partial t}\hat{y} \Rightarrow ikE_{0x} = i\omega B_{0y} \quad (4)$$

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For $\bar{\nabla} \times \bar{B} = \mu_0 \epsilon_0 \frac{\partial \bar{E}}{\partial t}$

$\nabla \times B$: $\boxed{-\frac{\partial \tilde{B}_y}{\partial z} \hat{x} = \frac{1}{c^2} \frac{\partial E_x}{\partial t} \hat{x}} \Rightarrow -ikB_{oy} = \frac{-1}{c^2} i\omega E_{ox}$ (5)

&

$+\frac{\partial \tilde{B}_x}{\partial z} \hat{y} = \frac{1}{c^2} \frac{\partial \tilde{E}_y}{\partial t} \hat{y} \Rightarrow ikB_{ox} = -\frac{1}{c^2} i\omega E_{oy}$ (6)

From eqn 3, 4, 5 & 6 we have

$ikE_{oy} = -i\omega B_{ox} \Rightarrow E_{oy} = -\left(\frac{\omega}{k}\right) B_{ox}$ (7)

$ikE_{ox} = i\omega B_{oy} \Rightarrow E_{ox} = \left(\frac{\omega}{k}\right) B_{oy}$ - (8)

$-ikB_{oy} = -\frac{1}{c^2} i\omega E_{ox} \Rightarrow B_{oy} = \frac{1}{c^2}\left(\frac{\omega}{k}\right) E_{ox}$ - 9

$ikB_{ox} = -\frac{1}{c^2} i\omega E_{oy} \Rightarrow B_{ox} = -\frac{1}{c^2}\left(\frac{\omega}{k}\right) E_{oy}$ - (10)

As $c = f\lambda$ or $c = \nu\lambda = (2\pi f)\left(\frac{\lambda}{2\pi}\right) = \frac{\omega}{k}$

$\frac{1}{c} = \frac{k}{\omega}$ and $k = \frac{2\pi}{\lambda}$

eqn (7) in terms of $B_{ox} = -\left(\frac{k}{\omega}\right) E_{oy}$

" (8) " " of $B_{oy} = \left(\frac{\omega}{k}\right) E_{ox}$

$\vec{\nabla} \times \vec{\tilde{E}}$: $B_{ox} = -\frac{1}{c} E_{oy}$

$B_{oy} = \frac{1}{c} E_{ox}$

$\vec{\nabla} \times \vec{B}$: $B_{oy} = \frac{1}{c} E_{ox}$

$B_{ox} = -\frac{1}{c} E_{oy}$

Redundancy of relations.

Two independent relations are

$B_{ox} = -\frac{1}{c} E_{oy} \Rightarrow -\hat{x} = \hat{z} \times \hat{y}$

$B_{oy} = \frac{1}{c} E_{ox} \Rightarrow \hat{y} = \hat{z} \times \hat{x}$

In compact form.

$$\vec{\tilde{B}}_o = \frac{1}{c}\left(\hat{z} \times \vec{\tilde{E}}_o\right)$$

Physically this relation states that $\vec{E}$ & $\vec{B}$ are

$\Rightarrow$ In phase with each other.

$\Rightarrow$ Mutually $\perp$ to each other $-(E \perp B) \perp \hat{z}$

The real amplitudes of E and B are related to each othe.

$$B_o = \frac{1}{c} E_o$$

where $B_o = \sqrt{B_{ox}^2 + B_{oy}^2}$

& $E_o = \sqrt{E_{ox}^2 + E_{oy}^2}$

Instantaneous Poynting's Vector associated with an EM Wave

$$\vec{S}(\cancel{\bar{r}}, t) = \frac{1}{\mu_o} \vec{E}(\bar{r},t) \times \vec{B}(\bar{r},t) \qquad \bar{r} = z$$

$$= \frac{1}{\mu_o} Re\{\vec{\tilde{E}}(z,t)\} \times Re\{\vec{\tilde{B}}(z,t)\} \quad \frac{\text{watts}}{m^2}$$

For linearly polarized plane wave propagating in z-direction

$$\vec{S}(z,t) = c\,\varepsilon_o E_o^2 \cos^2(kz - \omega t + \delta)\,\hat{z}$$

$$u_{EM}(z,t) = \epsilon_o E_o^2 \cos^2(kz - \omega t + \delta)$$

as $u_E = u_{BM}$ in free space.

$$I(\bar{r}) \equiv \langle S(\bar{r},t) \rangle = c \langle u_{EM} \rangle = \frac{1}{2} c \epsilon_o E_o^2$$

Electromagnetic Wave Propagation in Linear Media

→ EM wave is propagating inside matter.

→ There are no free charges and no free currents

→ The medium is an insulator / non conductor

Maxwell's equations become

1, $\vec{\nabla}\cdot\vec{D}(\bar{r},t) = 0$ $\qquad \vec{D} = \epsilon_0\bar{E}+\bar{P} = \epsilon\bar{E}$

2, $\vec{\nabla}\cdot\vec{B}(\bar{r},t) = 0$ $\qquad \vec{B} = \mu_0(\bar{H}+\bar{M}) = \mu\vec{H}$

3, $\vec{\nabla}\times\vec{E}(\bar{r},t) = -\dfrac{\partial B(\bar{r},t)}{\partial t}$ $\qquad P = \epsilon_0\chi_e E$

$M = \chi_e H$

4, $\vec{\nabla}\times\bar{H}(\bar{r},t) = \dfrac{\partial\vec{D}(\bar{r},t)}{\partial t}$ $\qquad \dfrac{\epsilon}{\epsilon_0} = 1+\chi_e$

$\dfrac{\mu}{\mu_0} = (1+\chi_m)$

Medium is assumed to be linear, homogeneous and isotropic

$\Rightarrow \vec{D} = \epsilon\vec{E}(\bar{r},t)$ and $\vec{H}(\bar{r},t) = \dfrac{1}{\mu}\vec{B}(\bar{r},t)$

and $P = \epsilon_0\chi_e E$ and $M = \chi_e H$

(4)

Maxwell's equation interms of $\vec{E}$ & $\vec{B}$

1, $\bar{\nabla} \cdot \vec{E}(\bar{r},t) = 0$

2, $\bar{\nabla} \cdot \vec{B}(\bar{r},t) = 0$

3, $\bar{\nabla} \times \vec{E}(\bar{r},t) = \frac{-\partial B(\bar{r},t)}{\partial t}$

4, $\bar{\nabla} \times \bar{B}(\bar{r},t) = \mu\epsilon \frac{\partial \vec{E}(\bar{r},t)}{\partial t}$

The $\vec{E}$ and $\vec{B}$ fields in medium obey the following wave function.

$$\nabla^2 \bar{E}(\bar{r},t) = \epsilon\mu \frac{\partial^2 \bar{E}(\bar{r},t)}{\partial t^2} = \frac{1}{V_{prop}^2} \frac{\partial^2 E(\bar{r},t)}{\partial t^2}$$

$$\nabla^2 \bar{B}(\bar{r},t) = \epsilon\mu \frac{\partial^2 \bar{B}(\bar{r},t)}{\partial t^2} = \frac{1}{V_{prop}^2} \frac{\partial^2 B(\bar{r},t)}{\partial t^2}$$

where $V_{prop} = \frac{1}{\sqrt{\epsilon\mu}}$

is ~~Vprop~~ Speed of propagation of EM wave in linear, homogeneous isotropic medium.

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For linear, homogeneous and isotropic media.

$$\epsilon = k_e \epsilon_0 = (1+\chi_e)\epsilon_0 \Rightarrow k_e = \frac{\epsilon}{\epsilon_0} = (1+\chi_e)$$

relative permittivity or dielectric constant

$$\mu = k_m \mu_0 = (1+\chi_m)\mu_0$$

$$\Rightarrow k_m = \frac{\mu}{\mu_0} = (1+\chi_m)$$ relative magnetic permeability.

as $$V_{prop} = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{k_e \epsilon_0 k_m \mu_0}} = \frac{1}{\sqrt{k_e k_m}} \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$= \frac{1}{\sqrt{k_e k_m}} c$$ where $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

If $k_e k_m \geq 1$

$$\Rightarrow \frac{1}{\sqrt{k_e k_m}} \leq 1$$

$$\Rightarrow V_{prop} = \frac{1}{\sqrt{k_e k_m}} c \leq c$$

as $k_e = \frac{\epsilon}{\epsilon_0}$ and $k_m = \frac{\mu}{\mu_0}$ are dimensionless.

$$\Rightarrow \frac{1}{\sqrt{k_e k_m}} \text{ is also dimensionless}$$

Define the index of refraction of Linear, Homogeneous and isotropic medium as:

$$n \equiv \sqrt{k_e k_m} = \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}}$$

$$\Rightarrow \quad \cancel{c} = \frac{\cancel{v_{prop}}}{n} \qquad v_p = \frac{c}{n} \leq c$$

OR

$$C = n\, v'_{prop}$$

For many paramagnetic and dia-magnetic-type materials.

$$\mu = \mu_0 (1 + \chi_m) \simeq \mu_0$$

$$\text{As } |\chi_m| \sim O(10^{-8}) \sim 0$$

$$\Rightarrow \quad k_m = \frac{\mu}{\mu_0} = (1+\chi_m) = 1$$

$$\Rightarrow \quad n = \sqrt{k_e} \quad \& \quad v' = \frac{c}{n} = \frac{c}{\sqrt{k_e}}$$

Maxwell's Equations for Linear dielectric

i) $\bar{\nabla}\cdot\bar{D}(r,t) = 0$

ii) $\bar{\nabla}\cdot B(\bar{r},t) = 0$

iii) $\nabla\times E(\bar{r},t) = -\frac{\partial B(\bar{r},t)}{\partial t}$

iv) $\nabla\times H(\bar{r},t) = \frac{\partial D}{\partial t}$

where $D = \epsilon_0 E + P$ & $B = \mu_0 H$

P — is the macroscopic Polarization of the medium.

ϵ_0 — permittivity of free space
μ_0 — permeability of " "

Applying curl operator to both sides of eqn iii we get.

$$\bar{\nabla}\times\bar{\nabla}\times\bar{E} = \bar{\nabla}(\bar{\nabla}\cdot\bar{E}) - \nabla^2 E$$

$$\Rightarrow \bar{\nabla}(\bar{\nabla}\cdot\bar{E}) - \nabla^2 E = -\frac{\partial}{\partial t}(\nabla\times\mu_0 H)$$

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$$\Rightarrow \nabla(\nabla\cdot E) - \nabla^2 E = -\frac{\partial}{\partial t}\left(\mu_0 \frac{\partial D}{\partial t}\right)$$

As $D = \epsilon_0 E + P$

$$\nabla(\nabla\cdot E) - \nabla^2 E = -\epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} - \mu_0 \frac{\partial^2 P}{\partial t^2}$$

For transverse fields $\bar{\nabla}\cdot\bar{E} = 0$

$$\Rightarrow \vec{\nabla}^2 E - \frac{1}{c^2}\frac{\partial^2 \bar{E}}{\partial t^2} = +\mu_0 \frac{\partial^2 P}{\partial t^2}$$