Review of Quantum Mechanics

Classical Mechanics

Classical mechanics is based on

on the assumption that any physically interesting variables connected with a system/particle, such as its position, velocity or its energy can be measured with arbitrary precisions and without mutual interference for any other such measurement.

LAWs of Classical mechanics can be expressed in various mathematical forms.

1, Newtonian Mechanics

2, Hamilltonian Mechanics

⇒ Quantum Mechanics is based on the realization that the measuring process may affect the physical system.

⇒ It is therefore impossible to measure simultaneously certain pair of variables with precision.

⇒ Quantum mechanics can be expressed by:

1, Wave Mechanics

2, Dirac's Notion.

Wave Mechanics

A quantum system, such as atoms, molecules, ion etc, are given by its wave function. $\psi(\bar{r},t)$

⇒ Itself $\psi(\bar{r},t)$ has no physical meaning but it allows to calculate the expectation values of all observables of interest.

Observables.

⇒ Measurable quantities are called observable and are represented by Hermitien operators $\hat{O}$

Expectation values:

$$\langle\hat{O}\rangle = \int \psi^*(r,t)\,\hat{O}\,\psi(r,t)\,d\vec{r}$$

Probability

⇒ As the system exist, its probability of being somewhere has to equal 1.

$$\int \psi^*(\bar{r},t)\,\psi(\bar{r},t)\,d^3\bar{r} = 1$$

$$\int \psi_n^*(r,t)\,\psi_m(r,t)\,d^3\bar{r} = \delta_{nm} \begin{cases} 1 & \text{for } n=m \\ 0 & \text{for } n\neq m \end{cases}$$

The time development of system

Schrodinger equation

$$i\hbar \frac{\partial}{\partial t}\psi(r,t) = H\psi(r,t)$$

⇒ H → Hamiltonian of the system.
→ Energy of the system.

→ For unperturbed system for instance an atom not interacting with light (EM-field) is the sum of its potential and kinetic energies

$$H = \frac{p^2}{2m} + V(\bar{r})$$

Stationar States

States for which space and time dependence are separated.

$$\Psi_n(\bar{r},t) = U_n(\bar{r})\, A(t) = U_n(\bar{r})\, e^{-i\omega_n t}$$

Time independent equation.

$$H U_n(\bar{r}) = E_n U_n(\bar{r}) = \hbar\omega_n U_n(r)$$

will prove it.

$U_n(\bar{r})$ — called eigen function of H
E_n — called eigen value.

⇒ The eigen functions of Hermitian operators belonging to different eigen values are orthogonal

⇒ Eigen functions having same eigen values are normal.

$$\int U_n^*(\bar{r})\, U_m(\bar{r})\, d\bar{r} = \delta_{nm}$$

and Complete.

$$\sum_n U_n^*(r)\, U_m(r) = 1$$

⇒ The completeness relation means that any function can be written as a linear combination of the $U_n(\bar{r})$

The wave function

$$\psi(\bar{r},t) = \sum_n \psi_n(\bar{r},t) = \sum_n C_n(t) U(\bar{r}) e^{-i\omega_n t}$$

$C_n(t)$ — expansion co-efficients.

$C_n(t)$ — constant for problems related to free part of Hamiltonian.

$C_n(t)$ — change with time for interaction Hamiltonian.

Putting value of $\psi(\bar{r},t)$ in normalization condition.

$$\psi_n(\bar{r},t) = \sum_n C_n(t) U_n(\bar{r}) e^{-i\omega_n t}$$

$$\psi_m^*(\bar{r},t) = \sum_m C_m^*(t) U_m^*(r) e^{+i\omega_m t}$$

$$\int \psi_m^*(\bar{r},t) \psi_n(\bar{r},t) \, d\bar{r} = \sum_{n,m} \int C_n(t) C_m^*(t) U_n(\bar{r}) U_m^*(\bar{r}) e^{-i(\omega_n - \omega_m)t} \, d\bar{r}$$

Using

$$\int u_m^*(\bar{r})\, u_n(\bar{r})\, d\bar{r} = \delta_{nm}$$

$$\int \psi_m^*(\bar{r},t)\, \psi_n(\bar{r},t)\, d\bar{r} = \sum_{n,m} C_n(t)\, C_m^*(t)\, \delta_{nm}\, e^{-i(\omega_n - \omega_m)t}$$

$$= \sum_n |C_n|^2 = 1$$

$\Rightarrow$ gives the probability of finding the system in state n.

Expectation Value

$$\langle \hat{O} \rangle = \int \sum_{n,m} C_n(t)\, C_m^*(t)\, U_m^*(r)\, \hat{O}\, U_m(r) \times e^{-i(\omega_n - \omega_m)t}\, d\bar{r}$$

$$= \sum_{n,m} C_n(t)\, C_m^*(t)\, O_{nm}\, e^{-i\omega_{nm}}$$

where $\omega_n - \omega_m = \omega_{nm}$

and

$$O_{nm} = \int U_m^*(\vec{r})\, \hat{O}\, U_n(r)\, d\vec{r}$$

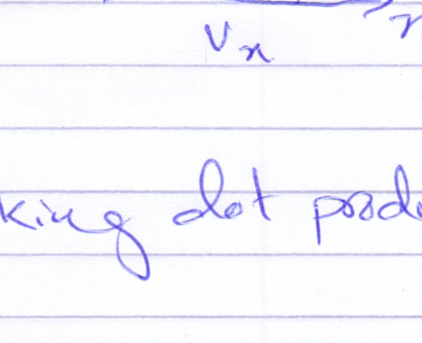
Matrix element of operator.

DIRAC NOTATION

⇒ The wave function of wave mechanics corresponds to the stat vector in Dirac's formulation of quantum Mechanics.

⇒ The relation between state vector and wave function is analogous to using vectors instead of co-ordinates.

A vector $\vec{V}$ is 2-D space can be expanded as.



$$\vec{V} = V_x \hat{x} + V_y \hat{y}$$

$\hat{x}$ - compt of vector V is

$$\left.\begin{array}{l} \vec{V} \cdot \hat{x} = V_x \\ \vec{V} \cdot \hat{y} = V_y \end{array}\right\} \text{By taking dot product.}$$

In Diracs notation

$$|V\rangle = V_x|x\rangle + V_y|y\rangle$$

Taking inner product with

$$\langle x|V\rangle = V_x$$

& $\langle y|V\rangle = V_y$.

putting in above eqn we get.

$$|V\rangle = |x\rangle\langle x|V\rangle + |y\rangle\langle y|V\rangle$$

$$= \left[|x\rangle\langle x| + |y\rangle\langle y|\right]|V\rangle$$

The identity diadic (outer product of two vectors)

$$|x\rangle\langle x| + |y\rangle\langle y| = I$$

For n-dimensional space

$$|V\rangle = \sum_n |n\rangle\langle n|V\rangle$$

$$\Rightarrow \quad \sum_n |n\rangle\langle n| = I$$

$\{|n\rangle\} \longrightarrow$ complete set of vectors

$\rightarrow$ a basis.

The inner products $\langle n|v\rangle$ are the expansion co-efficients of the vector V in this basis.

Expansion co-effs are in general Complex.

$$\langle k|v\rangle = \langle v|k\rangle^*$$

For continuous basis $\{|r\rangle\}$

$$I = \int d\bar{r}\ |r\rangle\langle r|$$

$\Rightarrow$ The wave vector

$$|\psi(\bar{r},t)\rangle = \int d\bar{r}\ |r\rangle\langle r|\psi(\bar{r},t)\rangle$$

where $\psi(\bar{r}) = \langle r|\psi\rangle$ are the wave functions of wave mechanics.

$\Rightarrow$ Hermitian

$$\langle \psi(\bar{r},t)|\hat{O}|\psi(\bar{r},t)\rangle = [\langle\psi(t)|O^{+}|\psi(t)\rangle]^*$$

$$= \langle\psi(t)|\hat{O}|\psi\rangle^*$$

$$\hat{O} = O^{+}$$

⇒ The set of eigen vectors of a Hermitian operator is complete.

⇒ Any arbitrary vector $|\psi(t)\rangle$ can be expressed as a sum of orthogonal eigen vectors.

$$|\psi(r)\rangle = \sum_{n=0}^{\infty} C_n |X_n\rangle e^{-i\omega_n t}$$

Eigen vectors are orthogonal

$$\langle X_n | X_m \rangle = \delta_{nm} \qquad \delta_{nm} = \begin{matrix} 0 & n \neq m \\ 1 & n = m \end{matrix}$$

Completeness relation

$$\sum |X_n\rangle\langle X_n| = I$$

$$|\psi(r)\rangle = \int d\bar{r}\, |\vec{r}\rangle\langle \bar{r} | \psi\rangle$$

$$\Rightarrow \int d\bar{r}\, |\bar{r}\rangle\langle \bar{r}| = I$$

State vector obeys the Schrodinger's equation.

$$i\hbar|\dot{\psi}\rangle = H|\psi\rangle$$

$$|\psi\rangle = \sum_n C_n e^{-i\omega_n t}|n\rangle$$

Expectation value can be written as.

$$\langle\psi|\hat{O}|\psi\rangle = \sum_{n,m} C_n C_m^* e^{-i(\omega_n - \omega_m)t}\,\hat{O}_{mn}$$

$$\hat{O}_{mn} = \langle m|\hat{O}|n\rangle = \hat{O}_{nm}$$

Matrix element of operator $\hat{O}$.

<u>Two-level System</u>

Wave function for two-level system is

$$\psi(\bar{r},t) = C_a U_a(\bar{r})e^{-i\omega_a t} + C_b U_b(\bar{r})e^{-i\omega_b t}$$

State-vector.

$$|\psi(\bar{r},t)\rangle = C_a e^{-i\omega_a t}|a\rangle + C_b e^{-i\omega_b t}|b\rangle$$

Schrodinger, Heisenberg and Interaction Pictures:

Schrodinger Picture:

→ The interaction of radiation with matter involves a hamiltonian.

$$H = H_0 + V$$

H_0 — unperturbed energy

V — Interaction energy

The corresponding Schrodinger eqn

$$|\dot{\psi}(\bar{r},t)\rangle = \frac{-i}{\hbar} H |\psi(\bar{r},t)\rangle$$

$$\Rightarrow |\dot{\psi}(\bar{r},t)\rangle = \frac{-i}{\hbar}(H_0 + V)|\psi(\bar{r},t)\rangle$$

Integrating

$$\Rightarrow |\psi(\bar{r},t)\rangle = e^{-iHt/\hbar} |\psi(\bar{r},0)\rangle$$

$$\dot{x} = -iAx$$

$$\frac{dx}{x} = -iA\,dt$$

$$\left.\ln(x)\right|_0^t = \left.-iAt\right|_0^t$$

$$x(t) = e^{-iAt} x(0)$$

The expectation of an operator $\hat{O}$

$$\langle \hat{O} \rangle = \langle \psi(t) | \hat{O}(0) | \psi(t) \rangle$$

Operator $\hat{O}$ is independent of time, but $|\psi(t)\rangle$ is a function of time.

$\Rightarrow$ Schrodinger picture way of writing the expectation value of an operator.

Heisenberg Picture

$\rightarrow$ Total time dependence goes into operator $\rightarrow$ State vector is independent of time.

Expectation value of $\hat{O}(0)$ in Sch. pic

$$\Rightarrow \langle \hat{O} \rangle = \langle \psi(t) | \hat{O}(0) | \psi(t) \rangle$$

Can be written as

$$\langle \hat{O}(t) \rangle = \langle \psi(t) | e^{-iHt/\hbar} e^{+iHt/\hbar} \hat{O} e^{-iHt/\hbar} e^{+iHt/\hbar} | \psi(t) \rangle$$

where $H = H_0 + V$ — Total hamiltonian

As $|\psi(t)\rangle = e^{-iHt/\hbar} |\psi(0)\rangle$

$$\Rightarrow e^{iHt/\hbar} |\psi(t)\rangle = |\psi(0)\rangle$$

As

$$e^{iHt/\hbar}|\psi(t)\rangle = |\psi(0)\rangle$$

Taking complex conjugate of

$$|\psi(t)\rangle = e^{-iHt/\hbar}|\psi(0)\rangle$$

$$\langle\psi(t)| = \langle\psi(0)|e^{iHt/\hbar}$$

$$\langle\psi(t)|e^{-iHt/\hbar} = \langle\psi(0)|e^{iHt/\hbar}e^{-iHt/\hbar}$$

$$\Rightarrow \langle\hat{O}(t)\rangle = \langle\psi(0)|e^{iHt/\hbar}\hat{O}(0)e^{-iHt/\hbar}|\psi(0)\rangle$$

Define

$$\hat{O}(t) = e^{iHt/\hbar}\hat{O}(0)e^{-iHt/\hbar}$$

Then

$$\langle O(t)\rangle = \langle\psi(0)|\hat{O}(t)|\psi(0)\rangle$$

Total time dependence is with operator.

$\Rightarrow$ State vector is time independent

Heisenberg Picture method to calculate expectation value.

Why called Heisenberg Picture?

$$\hat{O}(t) = e^{+iHt/\hbar}\, O(0)\, e^{-iHt/\hbar}$$

$$\dot{\hat{O}}(t) = \frac{i}{\hbar} H\hat{O} + \frac{-i}{\hbar}\hat{O}H$$

$$= \frac{i}{\hbar}[H, \hat{O}]$$

is Heisenberg equation of motion.

INTERACTION PICTURE

$$\langle \hat{O}(t) \rangle = \langle \psi(0) | e^{iHt/\hbar} O(0) e^{-iHt/\hbar} | \psi(0) \rangle$$

As $H = H_0 + V$

$\Rightarrow$ If the time dependence created by the interaction energy is only assigned to the state vector and rest of time dependence goes to the operator, the the expectation value is written as.

$$\langle \hat{O}(t) \rangle = \langle \psi(0) e^{iVt/\hbar} | e^{iH_0 t/\hbar} \hat{O}(0) e^{-iH_0 t/\hbar} | e^{-iVt/\hbar} \psi(0) \rangle$$

$$\Rightarrow \langle \hat{O}(t) \rangle = \langle \psi_I(t) | \hat{O}_I(t) | \psi_I(t) \rangle$$

$\Rightarrow$ The Interaction picture state vector

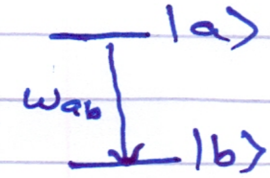
$$|\psi_I(t)\rangle = e^{-iVt/\hbar} |\psi(0)\rangle$$

Pauli Spin Matrix:

→ Another way to write two-level atom is matrix notation

$$|a\rangle \longleftrightarrow U_a \longleftrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$|b\rangle \longleftrightarrow U_b \longleftrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

} Column matrix



$$\psi(\vec{r},t) = \begin{bmatrix} C_a e^{-i\omega_a t} \\ C_b e^{-i\omega_b t} \end{bmatrix} = \begin{bmatrix} C_a(t) \\ C_b(t) \end{bmatrix}$$

⇒ Two-level atom is analogus to spin up & down states

The spin-flip operators:

$$\sigma_+ = \frac{1}{2}(\sigma_x + i\sigma_y) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$
$$\sigma_- = \frac{1}{2}(\sigma_x - i\sigma_y) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

where

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli Spin matrices

$$\sigma_- |a\rangle = \sigma_- \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |b\rangle$$

$\Rightarrow$ σ_- flips the system from upper-level to a lower-level

while σ_+

$$\sigma_+ |b\rangle = \sigma_+ \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |a\rangle$$

flips the system from lower-level to the upper-level.

<u>Dipole operator for two-level atom</u>

The expectation value of any operator is given by.

$$\langle \psi | \hat{O} | \psi \rangle = C_a C_a^* \hat{O}_{aa} + C_b C_b^* \hat{O}_{bb} + \left\{ C_a C_b^* \hat{O}_{ab} \times e^{-i\omega_{ab} t} + c.c \right\}$$

The expectation value of er is

$$\langle \psi | er | \psi \rangle = e C_a C_a^* \langle a | r | a \rangle + e C_b C_b^* \langle b | r | b \rangle + e \left\{ C_a C_b^* e^{-i(\omega_a - \omega_b)t} \langle b | r | a \rangle + c.c \right\}$$

As the diagonal matrix element of "er" between eigen states of the Hamiltonian generally vanishes:

$$e r_{aa} = \langle a|er|a\rangle = e\int U_a^*(r)\,\hat{r}\,U_a(\hat{r})\,d\hat{r} = 0$$

$$e r_{bb} = \langle b|er|b\rangle = e\int U_b^*(r)\,r\,U_b(r)\,d\hat{r} = 0$$

$$e r_{ab} = \langle a|er|b\rangle = e\int U_a^*(r)\,r\,U_b(r)\,d\hat{r} \neq 0$$

$$\Rightarrow \quad \langle er\rangle = e\,C_a C_b^*\,e^{-i(\omega_a-\omega_b)t}\,r_{ba} + c.c$$

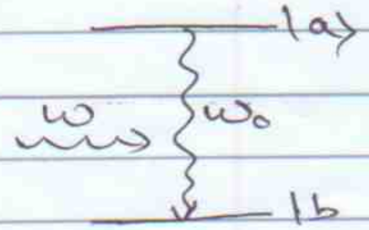
$$= e\begin{bmatrix} 0 & r_{ab} \\ r_{ba} & 0 \end{bmatrix}$$

ATOM - FIELD INTERACTION SEMICLASSICAL THEORY

$\Rightarrow$ Atom $\longrightarrow$ Quantum Mechanical

Field $\longrightarrow$ Classical

Two-Level atomic system



$$|\psi(t)\rangle = c_a(t)|a\rangle + c_b(t)|b\rangle$$

Hamiltonian of the system

$$H = H_0 + H'$$

H_0 — free part of Hamiltonian

H' — perturbed part of Hamiltonian

Completeness relation for atomic system

$$|a\rangle\langle a| + |b\rangle\langle b| = 1$$

$$\Rightarrow \quad H_0 = \underbrace{\left[|a\rangle\langle a| + |b\rangle\langle b|\right]}_{1} H_0 \left[|a\rangle\langle a| + |b\rangle\langle b|\right]$$

As $H_0|a\rangle = E_a|a\rangle$; $H_0|b\rangle = E_b|b\rangle$

$\Rightarrow$ $H_0 = E_a|a\rangle\langle a| + E_b|b\rangle\langle b|$

$\Rightarrow$ $H_0 = \hbar\omega_a|a\rangle\langle a| + \hbar\omega_b|b\rangle\langle b|$

Interaction part of Hamiltonian is written as.

$$H' = -er \cdot E(r,t)$$

under dipole approximation

$$E(R,t) = E(r,t) \cong E(t) \simeq \mathcal{E}_0 \cos\omega t$$

$$H' = -er \cdot \mathcal{E}_0 \cos\omega t$$

$$H' = \left[|a\rangle\langle a| + |b\rangle\langle b|\right] H' \left[|a\rangle\langle a| + |b\rangle\langle b|\right]$$

$$= \cancel{|a\rangle\langle a|H'|a\rangle}^{\,0} \langle a| + |b\rangle\langle b|H'|a\rangle\langle a|$$

$$+ |a\rangle\langle a|H'|b\rangle\langle b| + |b\rangle\cancel{\langle b|H'|b\rangle}^{\,0}\langle b|$$

As $H'_{aa} = 0 = H'_{bb}$

$$H' = |a\rangle\langle b| H'_{ab} + |b\rangle\langle a| H'_{ba}$$

Where $H'_{ab} = \langle a|H'|b\rangle$

$$= -\langle a|er|b\rangle \mathcal{E}_0 \cos\omega t$$

$$= -p_{ab} \mathcal{E}_0 \cos\omega t$$

and

$$H'_{ba} = -p_{ba} \mathcal{E}_0 \cos\omega t$$

$$\Rightarrow H' = -\left(p_{ab}|a\rangle\langle b| + p_{ba}|b\rangle\langle a|\right) E(t)$$

where

$p_{ab} = p_{ba}^* = e\langle a|r|b\rangle$ – matrix element of the electric dipole moment and $E(t)$ is the field at the atom.

The time-development of the system is given by Schrodinger equation.

$$|\dot{\psi}(t)\rangle = \frac{1}{i\hbar} H|\psi(t)\rangle$$

Substitute values of state-vector and Hamiltonian we get.

$$\dot{C}_a|a\rangle + \dot{C}_b|b\rangle = \frac{-i}{\hbar}\left[C_a\hbar\omega_a|a\rangle + C_b\hbar\omega_b|b\rangle + H'_{ab}C_b|a\rangle + H'_{ba}C_a|b\rangle\right]$$

Multipling with $\langle a|$ and using

$\langle a|a\rangle = 1$; $\langle a|b\rangle = 0$
$\langle b|b\rangle = 1$; $\langle b|a\rangle = 0$

$$\dot{C}_a = \frac{-i}{\hbar}\left[\hbar\omega_a C_a + H'_{ab}C_b\right]$$

and

$$\dot{C}_b = -\frac{i}{\hbar}\left[\hbar\omega_b C_b + H'_{ba}C_a\right]$$

Putting values of interaction Hamiltonian

$$\dot{C}_a = -i\omega_a C_a + \frac{i\,p_{ab}}{\hbar}\mathcal{E}_0 \cos\omega t\, C_b$$

and

$$\dot{C}_b = -i\omega_b C_b + \frac{i\,p_{ba}}{\hbar}\mathcal{E}_0 \cos\omega t\, C_a$$

Define

$$\Omega_R = \frac{|p_{ba}|}{\hbar}\mathcal{E}$$ — Rabi frequency

$p_{ba} = |p_{ba}| e^{i\phi}$

$\Rightarrow p_{ab} = p_{ba}^* = |p_{ba}| e^{-i\phi}$

where ϕ is the phase of the dipole matrix element

$\Rightarrow \dot{c}_a = -i\omega_a c_a + i\Omega_R e^{-i\phi} \cos\omega t \, c_b$

$\dot{c}_b = -i\omega_b c_b + i\Omega_R e^{i\phi} \cos\omega t \, c_a$

Transform c_a & c_b (sch. picture amplitudes) into slowly varying interaction picture amplitudes

$C_a = c_a e^{i\omega_a t}$

$C_b = c_b e^{i\omega_b t}$

Differentiating above eqn's

$\dot{C}_a = \dot{c}_a e^{i\omega_a t} + i\omega_a c_a e^{i\omega_a t}$

$\dot{C}_a = (\cancel{-i\omega_a c_a} + i\Omega_R e^{-i\phi} \cos\omega t \, c_b) e^{i\omega_a t} + \cancel{i\omega_a c_a e^{i\omega_a t}}$

$= i\Omega_R e^{-i\phi} \cos\omega t \, c_b e^{i\omega_a t}$

$= i\Omega_R e^{-i\phi} \cos\omega t \, e^{i\omega_{ab} t} C_b \qquad \omega_{ab} = \omega_a - \omega_b$

Similarly.

$$\dot{C}_b = i\Omega_R e^{i\phi} \cos\omega t \, e^{-i\omega_{ab}t} C_a$$

$\omega_{ab} = \omega_0$ — transition frequency.

Expanding $\cos\omega t$.

$$\dot{C}_a = \frac{i\Omega_R}{2} e^{-i\phi} \left[e^{i\omega t + i\omega_0 t} + e^{-i\omega t + i\omega_0 t} \right] C_b$$

Neglecting rapidly oscillating terms like $e^{i(\omega+\omega_0)t}$

$$\dot{C}_a = \frac{i\Omega_R}{2} C_b e^{i(\omega_0-\omega)t} e^{-i\phi}$$

similarly.

$$\dot{C}_b = \frac{i\Omega_R}{2} C_a e^{-i(\omega_0-\omega)t} e^{+i\phi}$$

where $\Delta = \omega_0 - \omega$. — detuning.

⇒ Consider atom initially in the excited state $C_a(0) = 1$

$$C_b(0) = 0$$

Assume resonance $\Delta = \omega_0 - \omega = 0$

$$C_a(t) = \cos\frac{\Omega_R t}{2}$$

$$C_b(t) = i e^{i\phi} \sin\frac{\Omega_R t}{2}$$

Probability of atom in state $|a\rangle$ at time t

$$P_a(t) = |\langle\psi|a\rangle|^2 = |C_a(t)|^2$$

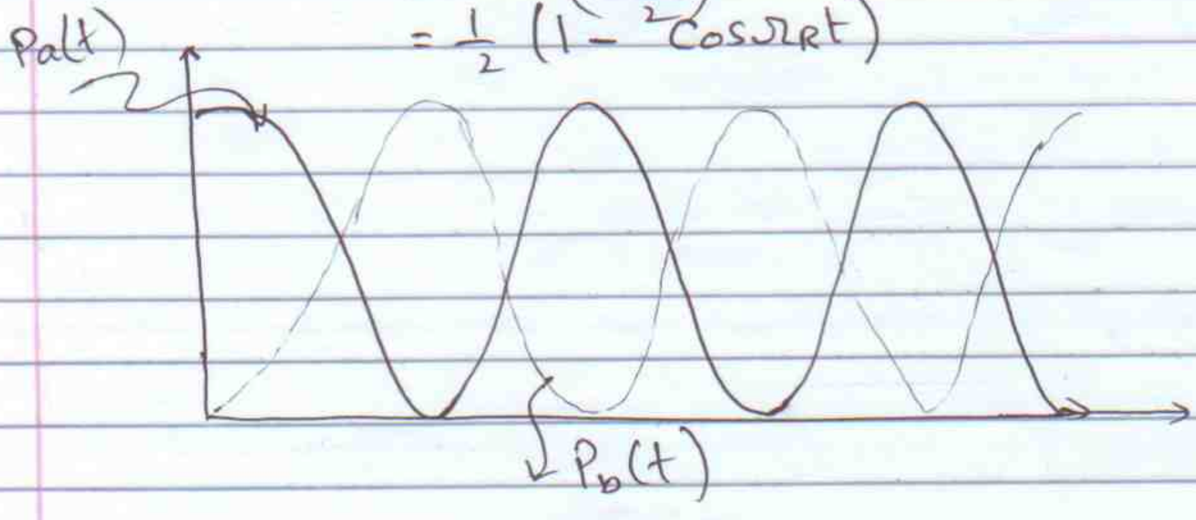
$$= \cos^2\left(\frac{\Omega_R t}{2}\right) = \frac{1}{2}\left(1 + \cos\Omega_R t\right)$$

and

$$P_b(t) = |C_b(t)|^2$$ — Probability of atom in state $|b\rangle$

$$= \sin^2\left(\frac{\Omega_R t}{2}\right)$$

$$= \frac{1}{2}\left(1 - \cos\Omega_R t\right)$$



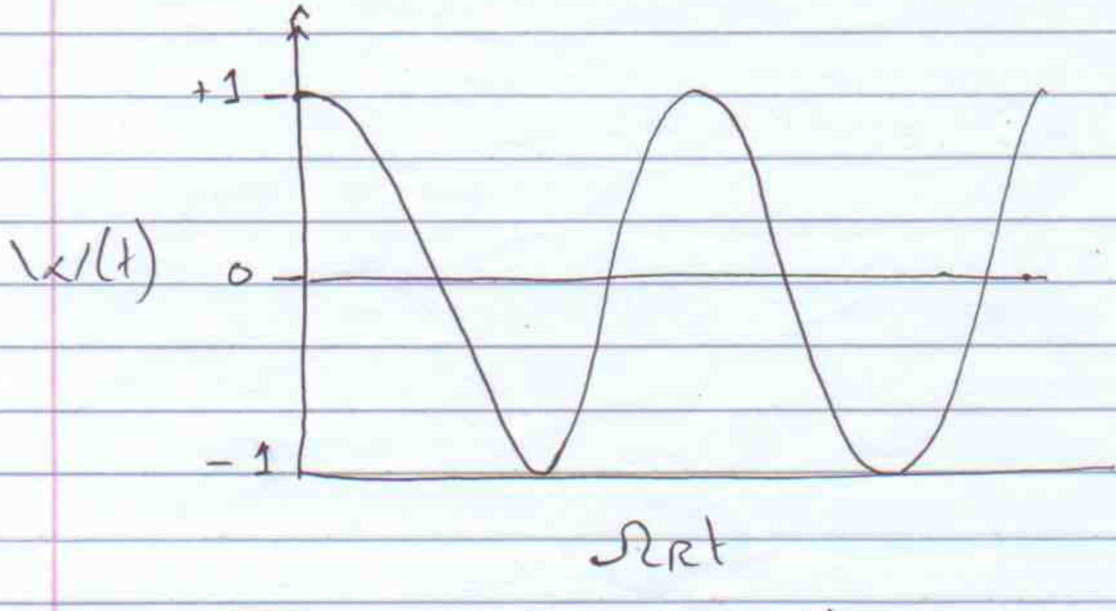
Atom oscillates with Rabi-freq.

Population inversion

$$W(t) = P_a(t) - P_b(t) \quad \text{at } \Delta = 0$$

$$W(t) = \cos^2\left(\frac{\Omega_R t}{2}\right) - \sin^2\left(\frac{\Omega_R t}{2}\right) = \cos \Omega_R t$$

It oscillates between -1 and $+1$



There are three frequency involved

1, $\omega_0 = \omega_a - \omega_b = \frac{E_a - E_b}{\hbar}$ — Transition frequency

2, ω frequency of the field

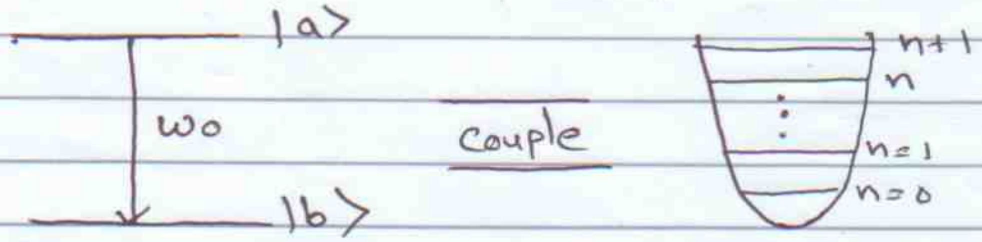
3, Rabi frequency $\Omega_R = \frac{|P_{ab}| \mathcal{E}}{\hbar}$

Atom-Field Interaction
Quantum Theory.

⇒ Atom ⟶ Quantum Mechanically

⇒ Field ⟶ Quantum Mechanically.

Interaction between a single-mode radiation field and a two-level atom inside a cavity



Atomic state-vector

$$|\psi_{atom}\rangle = c_a|a\rangle + c_b|b\rangle$$

Field state-vector

$$|\psi_{field}\rangle = \sum_n C_n|n\rangle$$

Atom-field state-vector

$$|\psi_{a-f}\rangle = \sum_n \left[C_{a,n}|a\rangle|n\rangle + C_{b,n}|b\rangle|n\rangle \right]$$

$C_{a,n}$ ⟶ probability amplitude

If at time $t=0$

$$|\psi(0)_{a-f}\rangle = |a\rangle|n\rangle$$

Then at later time t.

$$|\psi(t)_{a-f}\rangle = c_{a,n}|a\rangle|n\rangle + c_{b,n+1}|b\rangle|n+1\rangle$$

$\Rightarrow$ a finite probability that atom has made a transition to the lower level and emitted a photon.

Total Hamiltonian of the system

$$H = H_A + H_F + H_I$$

H_A — Energy of free-atom

$$= \sum_i E_i |i\rangle\langle i|$$

H_F = Energy of free-field

$$= \sum_k \hbar\omega_k \left(a_k^{+} a_k + \frac{1}{2}\right)$$

H_I = Interaction energy under dipole-approximation

$$H_I = -e\vec{r}\cdot\vec{E}$$

Define – atom transition operators

$\sigma_{ij} = |i\rangle\langle j|$ $\qquad$ i,j – atomic level

$\Rightarrow$

$$H_A = \sum_i E_i \sigma_{ii}$$

and

$$e\vec{r} = \sum_i \sum_j |i\rangle\langle i|e\vec{r}|j\rangle\langle j|$$
$$= \sum_{i,j} p_{ij}\sigma_{ij}$$

with

$p_{ij} = e\langle i|\vec{r}|j\rangle$ – electric-dipole transition matrix element.

Electric field operator

$$E = \sum_k \hat{\epsilon}_k \mathcal{E}_k (\hat{a}_k + \hat{a}_k^\dagger)$$

with

$$\mathcal{E}_k = \left(\hbar\omega_k / 2\epsilon_0 V\right)$$

$\hat{\epsilon}_k$ — represents polarization

$\Rightarrow$ Complete Hamiltonian

$$H = \sum_k \hbar\omega_k \left(\hat{a}_k^\dagger \hat{a}_k + \frac{1}{2}\right) + \sum_i E_i \sigma_{ii} +$$

$$+ \hbar \sum_{i,j} \sum_k g_k^{ij} \sigma_{ij} \left(\hat{a}_k + \hat{a}_k^\dagger\right)$$

Here

$$g_k^{ij} = -\frac{p_{ij} \cdot \hat{e}_k \mathcal{E}_k}{\hbar}$$ — coupling constant
— dimensions of frequency
— similar to Rabi-frequency

For a two-level atom and single-mode field

$$k = 1$$
$$\left.\begin{array}{l} i = a, b \\ j = a, b \end{array}\right\}$$ for two-level atom.

$$\Rightarrow \quad H = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) + E_a \sigma_{aa} + E_b \sigma_{bb} +$$

$$+ \hbar\left(g^{aa}\sigma_{aa} + g^{ab}\sigma_{ab} + g^{ba}\sigma_{ba} + g^{bb}\sigma_{bb}\right)\left(a + a^\dagger\right)$$

$$g^{aa} = g^{bb} = 0 \qquad \because p_{aa} = \langle a | er | a \rangle = 0$$
dipole-transitions

and

$$g^{ab} = g^{ba} = g \qquad g \text{ is real.}$$

As $\sigma_{aa} + \sigma_{bb} = |a\rangle\langle a| + |b\rangle\langle b| = 1$

Hamiltonian reduces to

$$H = E_a\sigma_{aa} + E_b\sigma_{bb} + \hbar\omega\left(a^{+}a + \tfrac{1}{2}\right) +$$

$$+ \hbar g\left(\sigma_{ab} + \sigma_{ba}\right)\left(a + a^{+}\right)$$

<u>First term</u>

$$E_a\sigma_{aa} + E_b\sigma_{bb} = \frac{1}{2}\left(E_a - E_b\right)\left(\sigma_{aa} - \sigma_{bb}\right)$$

$$+ \frac{1}{2}\left(E_a + E_b\right)\left(\sigma_{aa} + \sigma_{bb}\right)$$

As $E_a - E_b = \hbar\omega_0$

$$\Rightarrow E_a\sigma_{aa} + E_b\sigma_{bb} = \frac{1}{2}\hbar\omega_0\left(\sigma_{aa} - \sigma_{bb}\right) + \frac{1}{2}\left(E_a + E_b\right)$$

The total Hamiltonian

$$H = \hbar\omega\left(a^{+}a + \frac{1}{2}\right) + \frac{1}{2}\hbar\omega_0\left(\sigma_{aa} - \sigma_{bb}\right) + \frac{1}{2}\left(E_a + E_b\right)$$

$$+ \hbar g\left(\sigma_{ab} + \sigma_{ba}\right)\left(a + a^{+}\right)$$

$$\sigma_{aa} - \sigma_{bb} = \sigma_z$$

$$\sigma_{ab} = |a\rangle\langle b| = \sigma_+$$

$$\sigma_{ba} = |b\rangle\langle a| = \sigma_-$$

Ignoring

$\frac{1}{2}(E_a + E_b)$ & $\frac{1}{2}\hbar\omega$ } constant enegy terms

$$H = \hbar\omega a^{+}a + \frac{1}{2}\hbar\omega_0\sigma_z + \hbar g(\sigma_{+} + \sigma_{-})(a + a^{+})$$

Here

$$\sigma_{-} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \text{ — lowering operator}$$

$$\sigma_{+} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \text{ — raising operator}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Rightarrow H = \hbar\omega a^{+}a + \frac{1}{2}\hbar\omega_0\sigma_z + \hbar g(\sigma_{+}\hat{a} + \sigma_{+}a^{+} + \sigma_{-}\hat{a} + \sigma_{-}\hat{a}^{+})$$

$\left.\begin{matrix}\sigma_{+}a^{+} \\ \sigma_{-}a\end{matrix}\right\}$ enegy non-conservingterms Dropping unda RWA

$$\Rightarrow$$

$$H = \hbar\omega a^{+}a + \frac{1}{2}\hbar\omega_0\sigma_z + \hbar g(\sigma_{+}a + a^{+}\sigma_{-})$$

$$H_0 = \hbar\omega a^{+}a + \frac{1}{2}\hbar\omega_0\sigma_z$$

$$H_I = \hbar g(\sigma_{+}a + a^{+}\sigma_{-})$$

For multimode field

$$H = \sum_k \hbar\omega_k a_k^\dagger a_k + \frac{1}{2}\hbar\omega_0\sigma_z + \hbar\sum_k g_k(\sigma_+ a_k + a_k^\dagger \sigma_-)$$

Hamiltonian

$$H = \hbar\omega a^\dagger a + \frac{1}{2}\hbar\omega_0\sigma_z + \hbar g(\sigma_+ a + a^\dagger \sigma_-)$$

Describes the atom-field interaction under dipole and rotating-wave-approximation

This Hamiltonian is exactly solvable is called Jaynes-Cumming Model (JCM)

INTERACTION PICTURE

Operator in Interaction Picture

$$\hat{O}_I = e^{iH_0t/\hbar}\,\hat{O}(0)\,e^{-iH_0t/\hbar}$$

Hamiltonian in Interaction picture

$$V = e^{iH_0t/\hbar}\,H_I\,e^{-iH_0t/\hbar}$$

with

$$H_0 = \frac{1}{2}\hbar\omega_0\sigma_z + \hbar\omega a^+a$$

$$\Rightarrow V = \hbar g\left\{e^{i(\omega a^+a + \frac{\omega_0\sigma_z}{2})t}(\sigma_+\hat{a} + a^+\sigma_-)e^{-i(\omega a^+a + \frac{\omega_0\sigma_z}{2})t}\right\}$$

Atomic and field operators comute

$$\Rightarrow V = \hbar g\left\{\left(e^{\frac{i\omega_0\sigma_z t}{2}}\sigma_+ e^{-\frac{i\omega_0\sigma_z t}{2}}\right)\left(e^{i\omega a^+at}\hat{a}\,e^{-i\omega a^+at}\right)\right.$$
$$\left. + \left(e^{i\omega a^+at}a^+e^{-i\omega a^+at}\right)\left(e^{\frac{i\omega_0\sigma_z t}{2}}\sigma_- e^{-\frac{i\omega_0\sigma_z t}{2}}\right)\right\}$$

Using

$$e^{\alpha A}Be^{-\alpha A} = B + \alpha[A,B] + \frac{\alpha^2}{2!}[A[A,B]] + \cdots$$

First consider

$$e^{i\omega a^\dagger a t}\, \hat{a}\, e^{-i\omega a^\dagger a t} = ?$$

here $\alpha = i\omega t$; $A = \hat{a}^\dagger \hat{a}$ and $B = \hat{a}$

$$e^{i\omega a^\dagger a t}\, \hat{a}\, e^{-i\omega a^\dagger a t} = \hat{a} + i\omega t\,[\hat{a}^\dagger \hat{a}, \hat{a}] + \frac{(i\omega t)^2}{2!}[a^\dagger a, [a^\dagger a, a]] + \ldots$$

As $[\hat{a}, \hat{a}^\dagger] = 1$ & $[\hat{a}^\dagger, \hat{a}] = -1$

$$\therefore [a^\dagger a, a] = \hat{a}^\dagger \hat{a} \hat{a} - \hat{a} \hat{a}^\dagger \hat{a} = [\hat{a}^\dagger \hat{a} - \hat{a}\hat{a}^\dagger]\hat{a}$$

$$= [\hat{a}^\dagger, \hat{a}]\hat{a} = -\hat{a}$$

And

$$[a^\dagger a, [a^\dagger a, a]] = [\hat{a}^\dagger \hat{a}, -\hat{a}] = \hat{a}$$

Putting in above eqn we get.

$$e^{i\omega a^\dagger a t}\, \hat{a}\, e^{-i\omega a^\dagger a t} = \hat{a} - i\omega t \hat{a} + \frac{(i\omega t)^2}{2!} a - \frac{(i\omega t)^3}{3!} a + \ldots$$

$$= a\left[1 - i\omega t + \frac{(i\omega t)^2}{2!} - \frac{(i\omega t)^3}{3!} + \ldots\right]$$

$$= a\, e^{-i\omega t} = \hat{a}_I$$ – destruction in I.P

Similarly,

$$e^{i\omega a^\dagger a t}\,\hat{a}\,e^{-i\omega a^\dagger a t} = a^+ e^{i\omega t} = \hat{a}^+_I$$

$$e^{i\frac{\omega_0 t}{2}\sigma_z}\,\sigma_+\,e^{-i\frac{\omega_0 t}{2}\sigma_z} = \sigma_+ e^{i\omega_0 t} = \sigma_{+I}$$

And

$$e^{i\frac{\omega_0 t}{2}\sigma_z}\,\sigma_-\,e^{-i\frac{\omega_0 t}{2}\sigma_z} = \sigma_- e^{-i\omega_0 t} = \sigma_{-I}$$

Putting in V we get.

$$V = \hbar g\left[\sigma_+ a e^{i(\omega_0-\omega)t} + a^\dagger\sigma_- e^{-i(\omega_0-\omega)t}\right]$$

$$= \hbar g\left(\sigma_+\hat{a}e^{i\Delta t} + a^\dagger\sigma_- e^{-i\Delta t}\right)$$

where $\Delta = \omega_0 - \omega$ — detuning.

The non-conservative terms

$$\left.\begin{array}{l}\sigma_+\hat{a}^+ \sim e^{i(\omega_0+\omega)t} \\ \sigma_-\hat{a} \sim e^{-i(\omega_0+\omega)t}\end{array}\right\}$$ Rapidly oscillating neglected in R.W.A.

$\Rightarrow$ In I. P

$$\hat{a}^+_I = a^+ e^{i\omega t} \quad ; \quad a_I = a e^{-i\omega t}$$

and

$$\sigma_{+I} = \sigma_+ e^{i\omega_0 t} \quad ; \quad \sigma_{-I} = \sigma_- e^{-i\omega_0 t}.$$

At exact resonance

$$\Delta = \omega_0 - \omega = 0 \quad \Rightarrow \omega = \omega_0$$

$$\Rightarrow V = \hbar g \left(a\sigma_+ + a^\dagger \sigma \right)$$

Is the interaction part of Hamiltonian in Interaction picture in RWA approximationa and at exact resonance.

Equation of motion in Interaction picture is written as.

$$|\dot{\psi}_I\rangle = \frac{-i}{\hbar} V |\psi\rangle$$

The state-vector in I.P

$$|\psi(t)\rangle_I = \sum_n \left[C_{an}(t) |a,n\rangle + C_{bn}(t) |b,n\rangle \right]$$

C_{an} & C_{bn} — slowly varying probability amplitudes.

The interaction energy can only cause transitions b/w $|a,n\rangle$ & $|b,n+1\rangle$

$\Rightarrow$

$$|\psi_I(t)\rangle = C_{a,n}|a,n\rangle + C_{b,n+1}|b,n+1\rangle$$

Putting in eqn. of motion and multipling the result by $\langle a,n|$

$$\dot{C}_{an} = -ig\langle a,n|(a\sigma_+ + a^\dagger\sigma_-)|a,n\rangle C_{a,n}$$
$$-ig\langle a,n| a\sigma_+ + a^\dagger\sigma_-)|b,n+1\rangle C_{b,n+1}$$

Using

$$a|n\rangle = \sqrt{n}|n-1\rangle \quad : \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\sigma_+|b\rangle = |a\rangle \quad ; \quad \sigma_-|a\rangle = |b\rangle$$

and $\langle \alpha,n|\alpha,n\rangle = 1$

$\Rightarrow$
$$\dot{C}_{an}(t) = -ig\sqrt{n+1}\,C_{b,n+1}(t)$$
$$\dot{C}_{b,n+1}(t) = -ig\sqrt{n+1}\,C_{a,n}(t)$$

Coupled differential equations.

For atom initially in level $|a\rangle$

$C_{an}(0) = C_n(0)$ where $C_a(0) = 1$

$C_{bn}(0) = 0$ " $C_b(0) = 0$

$\Rightarrow C_{an}(t) = C_{an}(0)\cos(g\sqrt{n+1}\,t) = C_n(0)\cos g\sqrt{n+1}\,t$

and

$C_{b,n+1}(t) = -i\,C_{an}(0)\sin(g\sqrt{n+1}\,t) = -i\,C_n(0)\sin(g\sqrt{n+1}\,t)$

These are the conditions for emission

For stimulated absorption

$|\psi_I(0)\rangle = |b\rangle|n+1\rangle$

Solutions are

$C_{a,n}(t) = -i\sin(g\sqrt{n+1}\,t)\,C_{b,n+1}(0)$

$C_{b,n+1}(t) = \cos(g\sqrt{n+1}\,t)\,C$

The factor $g\sqrt{n+1}$ is the Rabi flopping frequency.

Population Inversion

Probability of finding atom in state $|a\rangle$

- taking trace over field variables

$$P(a) = Tr_f |C_{an}|^2 = Tr_f \sum_n |n\rangle\langle n| |C_{an}|^2$$

$$= \sum_n \langle n|n\rangle |C_{an}|^2 = \sum_n |C_{an}|^2$$

Probability of finding the atom in state $|b\rangle$

$$P(b) = \sum_n |C_{bn}|^2$$

$\Rightarrow$ Population Inversion

$$W = P(a) - P(b) = \sum_n \left[|C_{an}|^2 - |C_{bn}|^2 \right]$$

Probability of finding n-photons in the field at time t is

$$P(n,t) = Tr_\alpha |C_{\alpha,n}(t)|^2 = \sum_{\alpha=a,b} |C_{\alpha,n}(t)|^2$$

$$= |C_{an}(t)|^2 + |C_{bn}(t)|^2$$

$$\Rightarrow \quad P(n,t) = \rho_{nn}(t)$$

$$\Rightarrow \quad \rho_{nn}(0) = |C_{an}(0)|^2 + |C_{bn}(0)|^2$$

For initial condition

$$C_{an}(0) = C_n(0) \;\&\; C_{b,n+1}(0) = 0$$

$$\Rightarrow \rho_{nn}(0) = |C_{an}(0)|^2 = |C_n(0)|^2 \quad \because C_a(0) = 1$$

It gives the probability that there are n-photons in the field at time $t=0$

$$\Rightarrow P(n,t) = |C_{an}(0)|^2 \cos^2(g\sqrt{n+1}\,t) + |C_{a,n-1}(0)|^2 \sin^2 g\sqrt{n}\,t$$

$$= \rho_{nn}(0) \cos^2(g\sqrt{n+1}\,t) + \rho_{n-1,n-1}(0) \sin^2 g\sqrt{n}\,t$$

Using $\rho_{nn}(0) = |C_{an}(0)|^2$ Population inversion can be written as.

$$W = P(a) - P(b) = \sum_n \left[|C_{an}|^2 - |C_{bn}|^2 \right]$$

$$= \sum_n \left[\rho_{nn}(0) \cos^2 g\sqrt{n+1}\,t - \rho_{n-1,n-1}(0) \sin^2 g\sqrt{n}\,t \right]$$

We need $\rho_{nn}(0) = ?$ for W

In semiclassical theory probability amplitudes are

$$C_a(t) = \cos\left(\frac{\Omega_R t}{2}\right)$$

$$C_b(t) = i\sin\left(\frac{\Omega_R t}{2}\right)$$

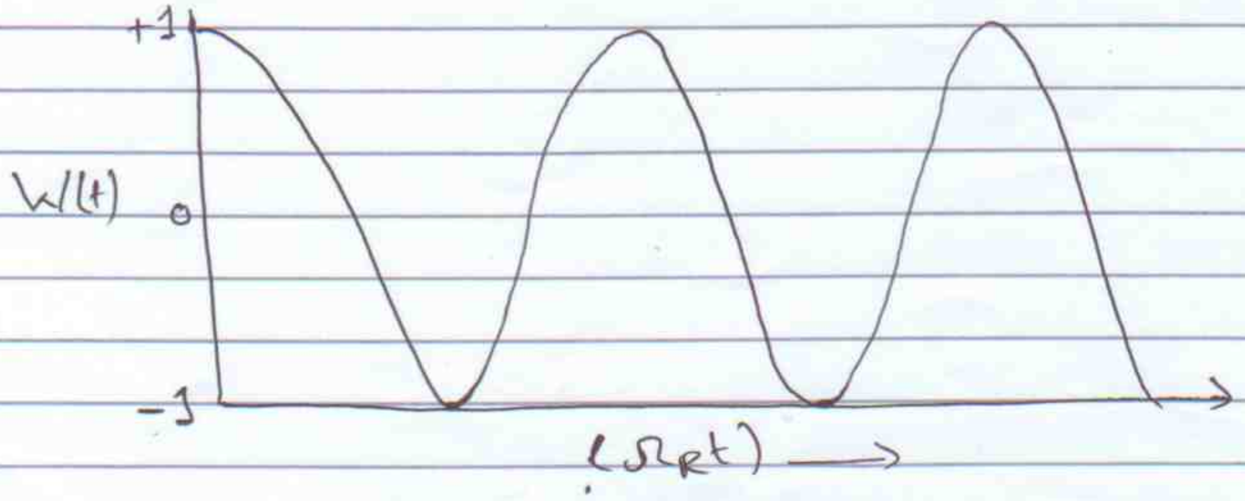
$\Omega_R = \frac{p \cdot \mathcal{E}}{\hbar}$ = Rabi-freq

$\Rightarrow$

Population inversion in semiclassical theory

$$W(t) = P_a(t) - P_b(t) = \cos^2\left(\frac{\Omega_R t}{2}\right) - \sin^2\left(\frac{\Omega_R t}{2}\right)$$

$$= \cos(\Omega_R t)$$



Population inversion oscillates b/w −1 & +1 at at freq. Ω_R. Atom undergoes a Rabi flopping b/w the upper and lower level under the action of field.

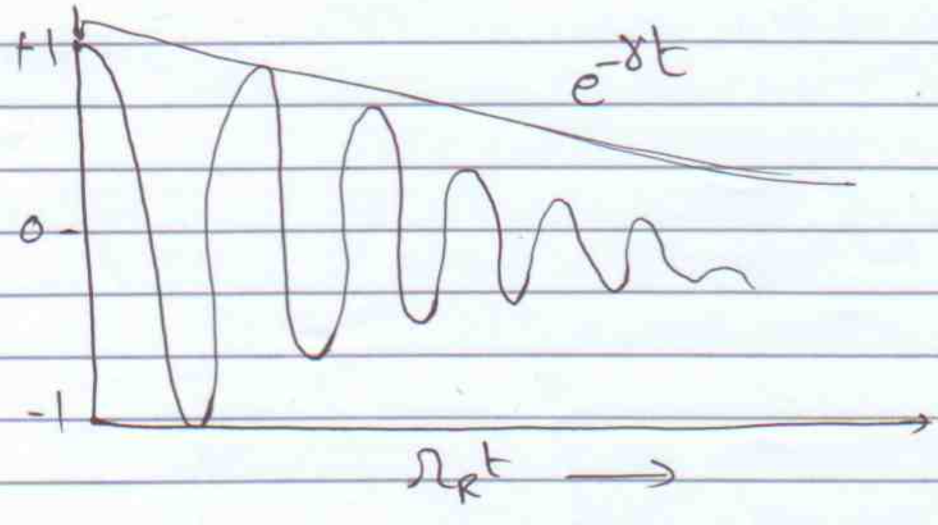
If atomic is included the probability amplitudes are written as.

$$C_a(t) = e^{-\gamma/2\, t} \cos\left(\frac{\Omega_R t}{2}\right)$$

$$C_b(t) = i\, e^{-\gamma/2\, t} \sin\left(\frac{\Omega_R t}{2}\right)$$

$\Rightarrow$ Population inversion at time t is

$$W(t) = e^{-\gamma t} \cos(\Omega_R t)$$



Rabi-oscillations are damped due to atomic decay.

In quantum theory of atom-field interaction

For atom initially in excited state we have

$$W(t) = \sum_n \left[\rho_{nn}(0) \cos^2 g\sqrt{n+1}\, t - \underbrace{\rho_{n-1,n-1}(0)}_{\text{shifting by one}} \sin^2 g\sqrt{n}\, t \right]$$

$\rho_{nn}(0) = |c_n(0)|^2$ — probability that there are n-photons at time $t=0$

Field can be

(i) Vaccum
(ii) Fock state
(iii) Coherent state.

(i) For a vacuum state

$$\rho = |0\rangle\langle 0|$$

$$\Rightarrow \rho_{nn}(0) = \langle n|0\rangle\langle 0|n\rangle = \delta_{n0} = \underset{\text{for } n=0 \text{ only}}{1}$$

As

$$W(t) = \sum_{n=0}^{\infty} \left[\rho_{nn}(0) \cos^2 g\sqrt{n+1}\, t - \rho_{nn}(0) \sin^2 g\sqrt{n+1}\, t \right]$$

$$= \sum_{n=0}^{\infty} \rho_{nn}(0) \left[\cos 2g\sqrt{n+1}\, t \right]$$

$$\Rightarrow W(t) = \sum_{n=0}^{\infty} \delta_{n0} \cos(2gt\sqrt{n+1})$$

$$= \cos 2gt\sqrt{1} \qquad \text{for } n=0$$

$\Rightarrow$ The Rabi-oscillations take place even when there is no field.

$\Rightarrow$ The square root of 1 i.e. $\sqrt{1}$ corresponds to spontaneous emission.

This is drastically different from the predictions of semi-classical theory.

$\Rightarrow$ In Semi-classical theory the probability of finding the atom in state $|b\rangle$ is

$$P_b(t) = \sin^2(\Omega_R t) \qquad \text{where } \Omega_R = \frac{\mathbf{P} \cdot \mathbf{E}}{\hbar}$$

If there is no field.

$$P_b(t) = \sin^2(\Omega_R t) = 0$$

$\Rightarrow$ No transition in the absences of field.

In quantum theory. the probability of finding atom in state $|b\rangle$ is

$$P_b(t) = \sum_{n=0}^{\infty} |C_{bn}(t)|^2$$

$$= \sum_{n=0}^{\infty} \rho_{nn}(0)\ \sin^2\left(g\sqrt{n+1}\,t\right)$$

For ~~vacuum~~ vacuum = no field

$$\rho_{nn}(0) = \delta_{n0}\ \Big\}= 1 \text{ for } n=0$$

$$\Rightarrow P_b(t) = \sum_{n=0}^{\infty} \delta_{n0} \sin^2\left(g\sqrt{n+1}\,t\right) = \sin^2 gt$$

vacuum Rabi-freq.

$\Rightarrow$ In semi-classical theory - atom in excited state cannot make a transition to lower-level in the absence of field.

In quantum treatment transition from $|a\rangle \to |b\rangle$ in vacuum becomes possible due to spontaneous emission.

For field initially in number state

$\rho = |n_0\rangle\langle n_0|$

$\Rightarrow \rho_{nn}(0) = \langle n|n_0\rangle\langle n_0|n\rangle = \delta_{nn_0}$

$$\Rightarrow W(t) = \sum_{n=0}^{\infty} \delta_{nn_0} \cos(2g\sqrt{n+1}\,t)$$

$$= \cos 2gt\sqrt{n_0+1}$$

for $n_0 \gg 1$

$$W(t) \simeq \cos(2gt\sqrt{n_0})$$

$\Rightarrow$ This is like semi-classical result

For $n_0 \gg 1$

$$W(t) \simeq \cos(2gt\sqrt{n_0})$$

here for $n_0 = 0$

$W(t) = 0$ like classical treatment.

For the field to be initially in the coherent state.

$$\rho_{nn}(0) = \frac{|\alpha|^{2n} e^{-|\alpha|^2}}{n!}$$

$\Rightarrow$

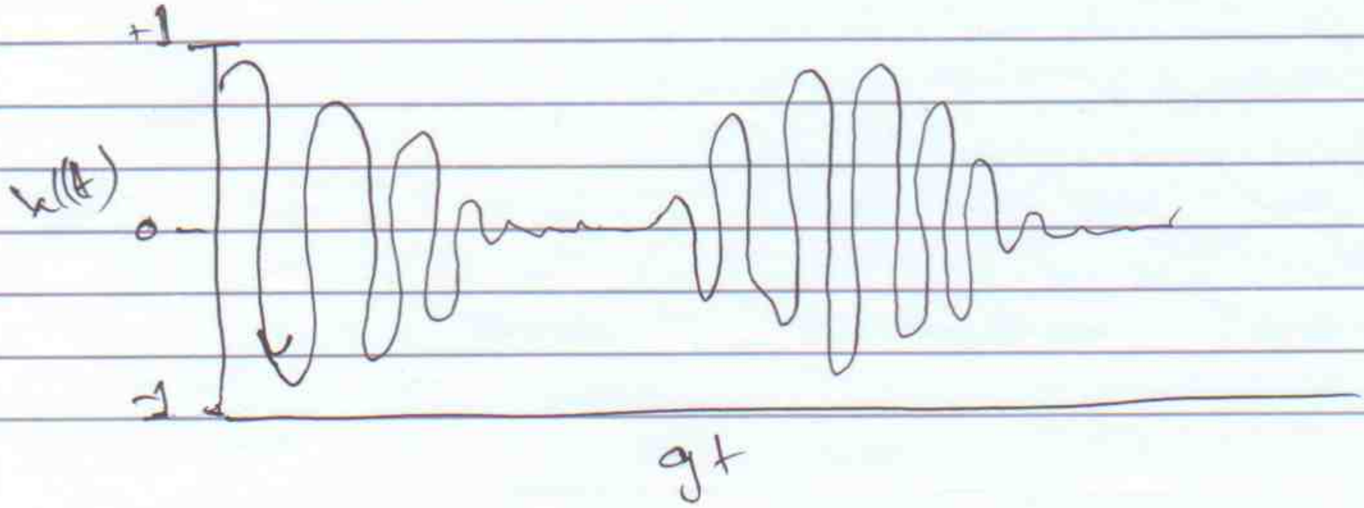
$$W(t) = \sum_{n=0}^{\infty} \left[\frac{e^{-|\alpha|^2} |\alpha|^{2n}}{n!} \cos^2 g\sqrt{n+1}\,t - \frac{e^{-|\alpha|^2} |\alpha|^{2n-1}}{(n-1)!} \sin^2 g\sqrt{n}\,t \right]$$

here $|\alpha|^2 = \bar{n}$

$$W(t) = \sum_{n=0}^{\infty} \left(\frac{e^{-\bar{n}} \bar{n}^{n}}{n} \cos^2 g\sqrt{n+1}\,t - \underbrace{\frac{e^{-\bar{n}} \bar{n}^{(n-1)}}{(n-1)!}}_{\text{shifting by one}} \sin^2 g\sqrt{n}\,t \right)$$

$$= \sum_{n=0}^{\infty} \frac{e^{-\langle n \rangle} \langle n \rangle^{n}}{n!} \left[\cos^2 g\sqrt{n+1}\,t - \sin^2 g\sqrt{n+1}\,t \right]$$

$$= \sum_{n=0}^{\infty} \rho_{nn}(0) \left[\cos 2g\sqrt{n+1}\,t \right]$$



The phenomena of collapse and revival can be understood from

$$W(t) = \sum_{n=0}^{\infty} \rho_{nn}(0) \left[\cos 2g\sqrt{n+1}\, t \right]$$

⇒ Each term in the summation represents Rabi-oscillations, for a definite value of n. At time $t=0$ all terms are correlated. As time increases the Rabi-oscillations associated with different excitations have different frequenies and therefore becomes un-correlated leading to a collapse of inversion. As time further increased the correlation is restored and revival occurs.

⇒ Revival is pure Q.M phenomenon and accurss due to the discrete value of n.

The number state behaves like semi-classical state because both have definite Intensity, needed to avoid the interference leading to a collapse. The random phase associated with number state (but not with the classical field) is not important for Rabi-flopping since the atom and field maintain a precise relative phase in the absence of decay process.

While ~~coher~~ minimum uncertainty ~~in the~~ intensity in coherent state, causes the atom-field relative phase to diffuse away i,e any spread in field strength will dephase Rabi-oscillations.