PREPARATORY SCHOOL TO THE

Winter College on Optics 2019:

Application of Optics and Photonics in Food Science

EXPERIMENTS in the DIFFRACTION LABORATORY

ANNA CONSORTINI

UNIVERSITA` DEGLI STUDI DI FIRENZE

anna.consortini@unifi.it

Three sets of experiments will be presented, namely on:

1 - MICHELSON INTERFEROMETER, WITH AN INTRODUCTION ON INTERFERENCE

2 - BASIC OF SPECTROSCOPY: DECOMPOSITION OF LIGTH BY DIFFRACTION GRATINGS

3 - DIFFRACTION and FOURIER TRANSFORM

----

1 - INTERFERENCE AND MICHELSON INTERFEROMETER

1.1 Simple formulas on interference

Let us start by considering the superposition of two spherical waves, of a given wavelength, λ, like those used in the theory of the Young interferometer. Remember that the interferometer was developed by Young to show the wave nature of light. It consisted of two very small holes on an opaque screen, at a small separation with respect to the other dimensions. A plane wave, impinging on one side, gives rise to two spherical waves on the other side, one from each hole, according to Huygens-Fresnel principle.

Let us consider two spherical waves v_1 and v_2 from two source points, e.g. the two holes. As usual in optics, we refer to one component of the electromagnetic field, that in our case is enough to describe the entire phenomenon.

With reference to an orthogonal system of coordinates, Fig 1, be z_o and $-z_o$ the locations of the two sources. Let r and r' be their distances from a point P(x,z), respectively. We assume that the distance of P from the two sources be large with respect to distance $2z_o$ of the two sources.

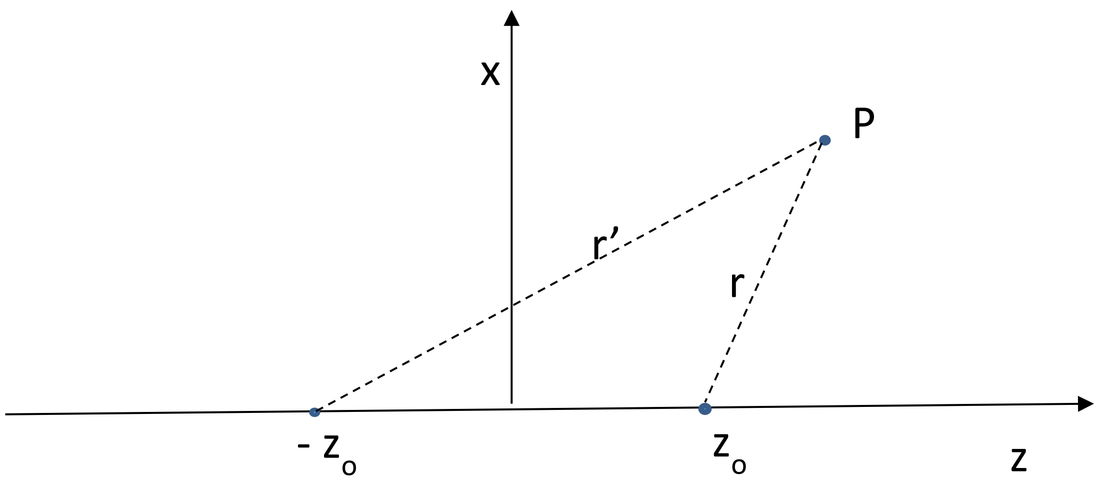

Fig 1. Scheme to evaluate interference at point P(x, z) of two spherical waves, v_1 and v_2, originating at points z_o and $-z_o$ respectively.

We assume, as usual, time dependence $e^{-i\omega t}$. The complex amplitudes of the two waves reaching P are given by:

1) $$v_1 = \frac{a}{r} e^{ikr} \quad \text{and} \quad v_2 = \frac{a}{r'} e^{ikr'}$$

where a denotes amplitude, $k=2\pi/\lambda$, λ wavelength, and

2) $$r = \sqrt{(z - z_o)^2 + x^2} \quad \text{and} \quad r' = \sqrt{(z + z_o)^2 + x^2}$$

The two distances r and r' can be assumed to be equal

in the amplitudes of the two waves, of course not in the phase. Eq.s 1) become

3) $$v_1 = A\, e^{ikr} \quad \text{and} \quad v_2 = A\, e^{ikr'}$$

Where A = a/r. The total field v at P is therefore

4) $$v = v_1 + v_2 = A\, e^{ikr} + A\, e^{ikr'}$$

With a simple mathematical trick v can be written as:

5) $$v = A\, e^{ik\frac{r+r'}{2}} \left[e^{ik\left(\frac{r-r'}{2}\right)} + e^{-ik\left(\frac{r-r'}{2}\right)}\right] = 2A\, e^{ik\frac{r+r'}{2}} \cos\left[\frac{k(r-r')}{2}\right]$$

We now evaluate the intensity, I, that is the modulus square of v. As is well known our eyes are sensitive to the intensity.

6) $$I = 4\,A^2\ cos^2\left[\frac{k(r-r\prime)}{2}\right]$$

Eq.6) represent regions of the plane of variable intensity. The equation of the lines of maximum intensity is

7) $$\frac{k\,(r - r')}{2} = \mathbf{n}\,\pi$$

where **n** denotes an integer number positive or negative. The lines of maximum intensity (fringes) are therefore hyperbolas with focus in the sources. If we extend the formulas to the space, by noting that there is rotation symmetry with respect to the z axis, we reach the final results that: in the space, the maxima are hyperboloids of rotation, with respect to the z axis, with focus on the sources. Their equation is:

8) $$r - r' = \mathbf{n}\,\lambda$$

On a **far plane, located perpendicularly to the x axis, the interference fringes are hyperbolas**. They become parallel lines near the x axis, as are the fringes of the Young interferometer in a first approximation.
On a **far plane, located perpendicularly to the z axis, the interference fringes are circles.**
The above simple formulas allow one to explain many interference phenomena. For instance, when one of the two sources go to infinity, Newton rings are obtained. Here they are useful to understand the Michelson interferometer.

1.2 - MICHELSON INTERFEROMETER

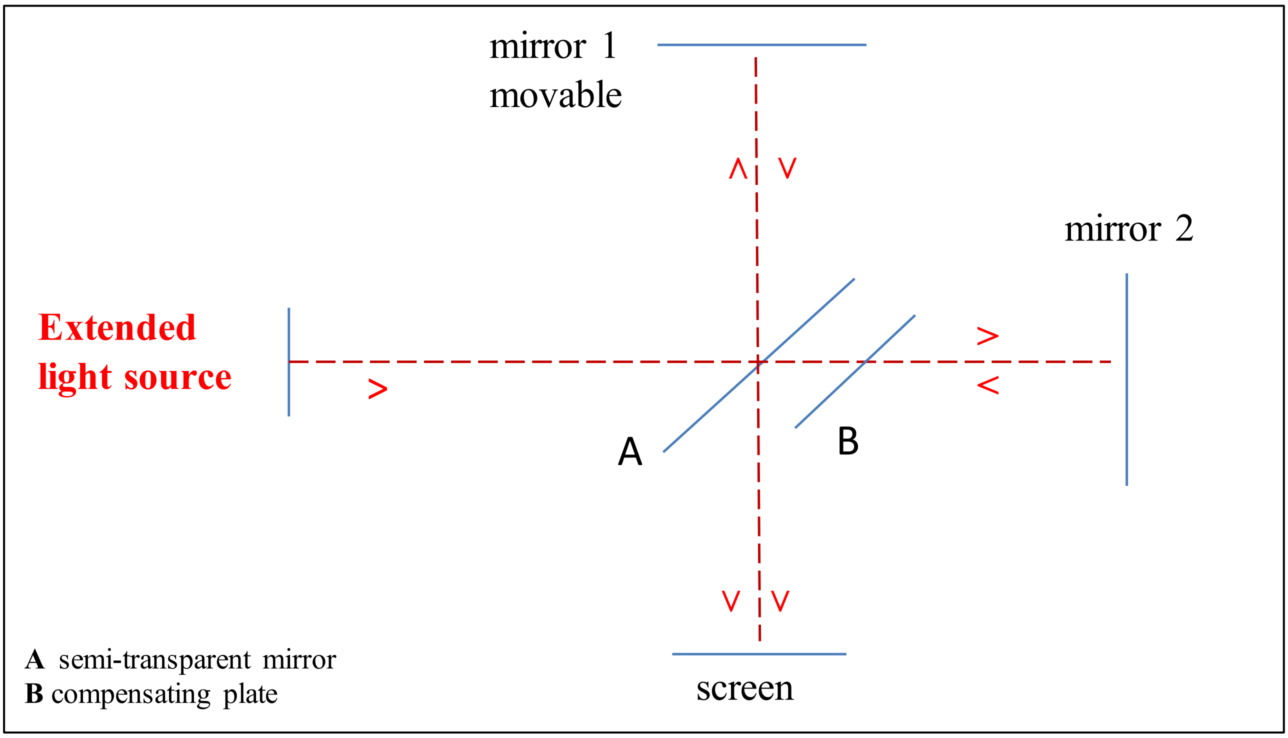

Fig. 2 - Simple scheme of Michelson interferometer

An extended beam of light, from a coherent source (e.g. a laser beam) on the left side of Fig.2, travels toward a semi-transparent mirror, A (beamsplitter), where it is split in two beams, one reflected towards mirror 1 and another transmitted towards mirror 2. The beam reflected from mirror 1 crosses the beamsplitter and a part is transmitted towards the screen/receiver. The beam reflected from mirror 2 crosses a compensating plate and part of it is reflected from the semi-transparent mirror towards the receiver. Of course, the part going back to the source is not of interest here. The two superposed beams interfere and give rise to a pattern in the receiver screen/plane. One of the two mirrors, here mirror 1, can be moved back and forth or rotated.
If the two mirrors are perfectly parallel, and so are the beams, the two beams reach the screen with a phase difference, for instance, they can be in phase or out of phase and so on. The phase difference depends on the different optical paths of the two beams in the arms. Circular fringes are produced on the screen. In particular, if the optical path difference is zero, the screen is completely illuminated, if the two beams are out of phase the screen is black. In the general case, there are a number of circular fringes depending on the difference in the optical path. To understand this behaviour, one can refer to the previous section 1.1 by considering a plane normal to the z axis at a very far positive distance. In the far region near the z axis, the spherical wavefronts are approximately plane surfaces and the above results can be applied here. In this comparison, the difference of optical path of the interferometer corresponds to the difference $2z_0$ of the two sources.
If the two mirrors are tilted, and therefore **the beams are not parallel** the comparison with the results of Section 1.1 requires to consider a **far plane** perpendicular to x axis. Here the fringes are hyperbolas and become straight lines near the x axis.
The shapes of the fringes can help aligning the interferometer. In the Laboratory, we will learn how to align a Michelson interferometer by utilizing the shapes of the fringes.
The previous analysis was made for a perfectly coherent source, that is a precise wavelength. In the **case of a partially coherent source** the visibility of the fringes decreases from the maximum. One can utilize the decrease of visibility to measure the coherence of a source.

1.3 - Michelson interferometer: experiment procedure

by Dr Miltcho Danailov

The **Laboratory experiment** can include: setting up and alignment of a Michelson interferometer, fringe observation, and change of the relative phase of the two beams by a wedge pair in one arm. Wedge pairs are often used in ultrashort pulse setups for controlling and stabilizing the Carrier-to-Envelope phase of ultrafast lasers.

Main components of the set up: **He Ne laser, Telescope, Negative lens, Beamsplitter, HR mirrors, Fused Silica wedges**

1. Interferometer alignment and fringe observation in slightly tilted wavefront geometry
   a. Insert the telescope in-front of the He-Ne laser and achieve a good beam collimation
   b. Setup the Michelson interferometer by inserting the beamsplitter and placing the HR mirrors appropriately
   c. Position the white screen in a position where fringes can be observed and align the beams reflected by the mirrors to coincide on the screen
   d. Find out how different fringe spacing can be obtained and explain it
2. Interferometer alignment in parallel wavefront geometry
   a. Insert the negative lens for easy fringe observation
   b. Align the interferometer for observing circular fringes
   c. How the number of fringes in this geometry can be reduced? Tray to get few fringes or even a single fringe

3. Interferometer measurement of phase changes by a wedge pair
    a. Insert and align the two wedges in one of the interferometer arms in antiparallel configuration
    b. Using the micrometer change the insertion of the wedge and measure the relation between wedge movement and fringe changes,
    c. on the basis of the above measurement estimate the wedge angle, given that the wedge material is fused silica with refractive index of 1.46

2 - BASIC of SPECTROSCOPY: DECOMPOSITION of LIGHT by DIFFRACTION GRATINGS

Spectroscopy is separation of light from a source in its (time) frequency components. The basic element allowing it is the diffraction grating.

A linear diffraction grating is a transparent, or reflecting, element where a number of parallel lines are drawn. The elements can be reflecting or transmitting.
A beam impinging onto the grating is "diffracted" by each element, a system of diffracted waves propagates in reflection or transmission. Let us now refer to a transmitting grating and a beam impinging normally. The diffracted waves at each point in the space give rise to interference; there are directions where they interfere constructively and other directions where the interference is destructive. Let d denote the period of the grating and θ denote the angle between the normal to the surface and an arbitrary direction. The interference is constructive in those directions θ where one has, Fig.2,

$$d \sin \theta = m \lambda,$$

where m is an entire number denoting the order of the diffracted field, spectrum. It is clear that in the main direction, where m=0, there is no frequency separation. From the equation it is also clear that, apart from m=0, for any other value of m, the diffraction angle depends on the wavelength.

If θ is small $$\theta = m \lambda/d$$

Therefore a radiation constituted by different wavelengths can be decomposed in its basic components. By measuring angle θ it is possible to measure the wavelength of a given radiation, spectroscopy. A large number of applications are based on spectroscopy.

In addition to linear gratings there are also two-dimensional gratings, where two sets of perpendicular lines are present. In this case one has a spectrum in the space where the directions of constructive interference are given by a double set of integer numbers.

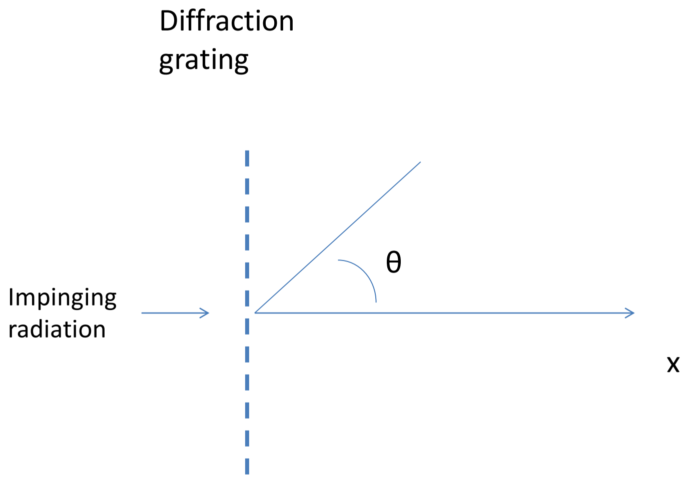

Fig. 2

Scheme for description of the spectrum from a diffraction grating

EXPERIMENTS: by use of gratings of different periods, decomposition will be shown of radiation from different sources.
By use of linear gratings of different periods (100, 300, and 600 and 1000 lines/mm) decomposition will be shown of radiation from different sources including lasers and leds. A number of orders will be seen. In particular cases the relationship between the wavelength of two lasers, namely green and red, will be found by measuring the diffraction angles. Examples will be also seen of spectra produced by a two-dimensional grating.

3 - EXPERIMENTS on DIFFRACTION and FOURIER TRANSFORM

It is well known that diffraction from an aperture in a screen gives rise to a system of waves propagating from the aperture. In addition, there are evanescent waves that do not propagate but "flow" along the surface. For simplicity let us refer to a plane monochromatic wave impinging normally on the screen.
At each point behind the screen the field is the result of the interference of the diffracted waves and, depending on the distance, in wavelengths, from the screen, two different regions are considered, namely **Fresnel region** (far but not too much from the aperture) and **Fraunhofer region** (very far, practically at infinity). Depending on the shape of the aperture there are different shapes and formulas for the diffracted field.

Here we will experience diffraction of laser radiation (HeNe, wavelength 632,8nm) by
1- wires, by
2- slits of different width and by

3- circular apertures of different radius.

Of course, as our eyes see the **energy**, we will always "see" the **square of the field**. By using a moving screen, we will follow the development of the field (intensity) from the diffracting screen through the Fresnel region up to the Fraunhofer region. We will check the angular dependence on the aperture width by measuring the width of the intensity patterns in the Fraunhofer region in different cases.

- Note that, mathematically, the diffracted field at infinity is the **Spatial Fourier Transform** of the field on the diffracting aperture. Commonly **diffraction is said to operate a Fourier transform**, the transform is also called **spectrum**. Each spatial frequency corresponds to a plane wave propagating in a suitable direction. As mentioned, interference of all waves gives rise to the field at any point in the space. Evanescent waves do not propagate, as they flow along the surface, and information carried by them is lost.

From the point of view of the transform, we will also check the different shapes of the patterns according to the different apertures. The transform of a Rect (slit uniformly illuminated) is Sinc = (sin(x))/x and the transform of a Circ (diffraction from a circular aperture) is a Bessel function divided its argument.

Of course, we "see" the modulus square of each transforms, precisely:

FUNCTION	TRANSFORM	WE SEE
1 - Rect	Sinc = [sin(arg)] /arg	**Sinc^2**
2 - Circ	Airy Function = [Bessel J_1(arg)] /arg	**Airy Function^2**

Note that in the case of a slit the Fourier transform is a transform in one variable, while in the case the circular hole we are faced to a transform in two variables.

If the aperture is the border of a converging lens, the **lens "carries" the field from infinity to the focal plane**. This property, commonly referred to as the property of a lens of "making Fourier transforms", is the **basis of image elaboration**. However, the transform is performed by the border of the lens, by diffraction, not by the lens as such. When a lens forms an image, the propagating part of the diffracted field is collected, while the evanescent waves are lost. Information carried by the evanescent waves is also lost and this explains why the image of a source point is not a point but a diffraction figure, and is the cause of the "resolution" problem.

For more information on Diffraction and Evanescent waves see Lectures of Winter College 1993:
`http://indico.ictp.it/event/a02251/speakers`

Acknowledgements

Thanks are due to:
- the Elettra Sinchrotrone for supplying a laser and some optical components, and help of Miltcho Danailov.

- Personal material for diffraction is carried by Anna Consortini and Imrana Ashraf.

The laboratory experiments are organized by Miltcho Danailov, Imrana Asharaf and Anna Consortini.