Exercises for Statistical Mechanics of Two Dimensional Critical Curves

1. Consider infinitesimal coordinate transformations $x_\mu \to x_\mu + w_\mu(x)$, and derive the conditions satisfied by $w_\mu(x)$ so that the metric only changes by a factor, $g_{\mu\nu} \to \rho(x)\, g_{\mu\nu}$. Rewrite your results for 2d in terms of a complex transformation parameter.
2. Show that the trace of energy-momentum tensor vanishes if you have scale invariance.
3. Calculate the fractal dimension of Sierpinski Carpet.
4. Numerically estimate the fractal dimension of an Object by the correlation method and compare the result with the usual Hausdorff method. (Sierpinski carpet is good for this)
5. Explain why we need $f' \neq 0\ on\ whole\ of\ D$ For Riemann's theorem.
6. Estimate the Percolation threshold for a bond percolation on square lattice (20x20) and (100x100) by Monte Carlo simulation.
7. By explicit calculation show that $\frac{-dz+b}{cz-a}$, inverts $\frac{az+b}{cz+d}$.
8. Numerically solve Loewner's equation (derive γ(t))for a couple of driving functions of your own choice. For example a) a(t)=t b)a(t)=t^2 or you can choose your pet function.
9. The renormalization group equations for a system are given by:

$$\lambda \frac{dc_1}{d\lambda} = -3c_1 + c_1 {c_2}^2$$
$$\lambda \frac{dc_2}{d\lambda} = -4c_2 + c_2 {c_1}^2$$

   a) Find all the fixed points
   b) Among the fixed points choose a saddle point.
   c) Find all the eigenvalues of the stability matrix around this point
   d) Identify the relevant and irrelevant directions in the RG flow

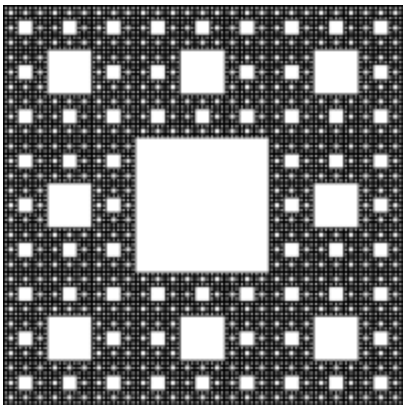
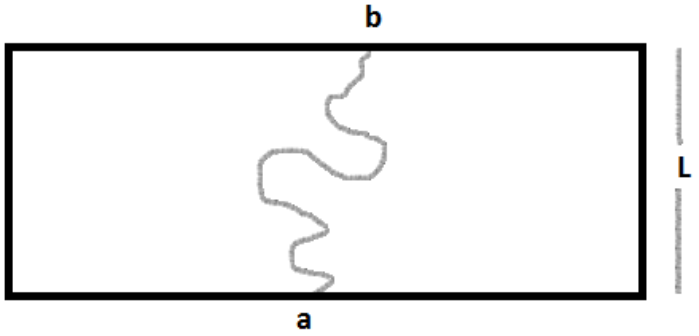


Figure 1: Sierpinski carpet.

10. Numerically evaluate fractal dimension. Write a computer code to construct a percolation cluster boundary on a rectangular domain. Fix the boundary conditions such that it starts at “a” and exits at “b”. Do site percolation on a hexagonal lattice and assign nodes with the critical probability. Calculate the length of the path , plot its variation with the size L , calculate the fractal dimension



You can also find the distribution of the winding angle on the paths you generate this way. Check that k=6.

The hull of percolation boundaries is supposed to be a saw. Find the hull of these paths, calculate the boundary fractal dimension, check that k=8/3.

You can repeat the above for the Ising model, choosing the correct boundary conditions, a little more work for generating the configurations.

11. Write a computer code for simulating ASM on a square lattice no bigger than 100 by 100. Count the number of avalanches of a given size estimate the exponent of pdf of avalanche size.

12. Consider the dimer covering of the 2 X n ladder. Show that the number of coverings is given by the Fibonacci numbers. If m=n , even, then the number of coverings is given by the Pfaffian of n^2 anti symmetric matrix. Write this result as an integral over Grassmann variables.