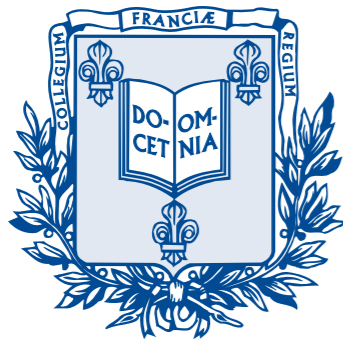


Thermoelectrical response beyond the quantum limit of 3D electron gas systems

Benoît Fauqué['4K']

Quantum Matter Group

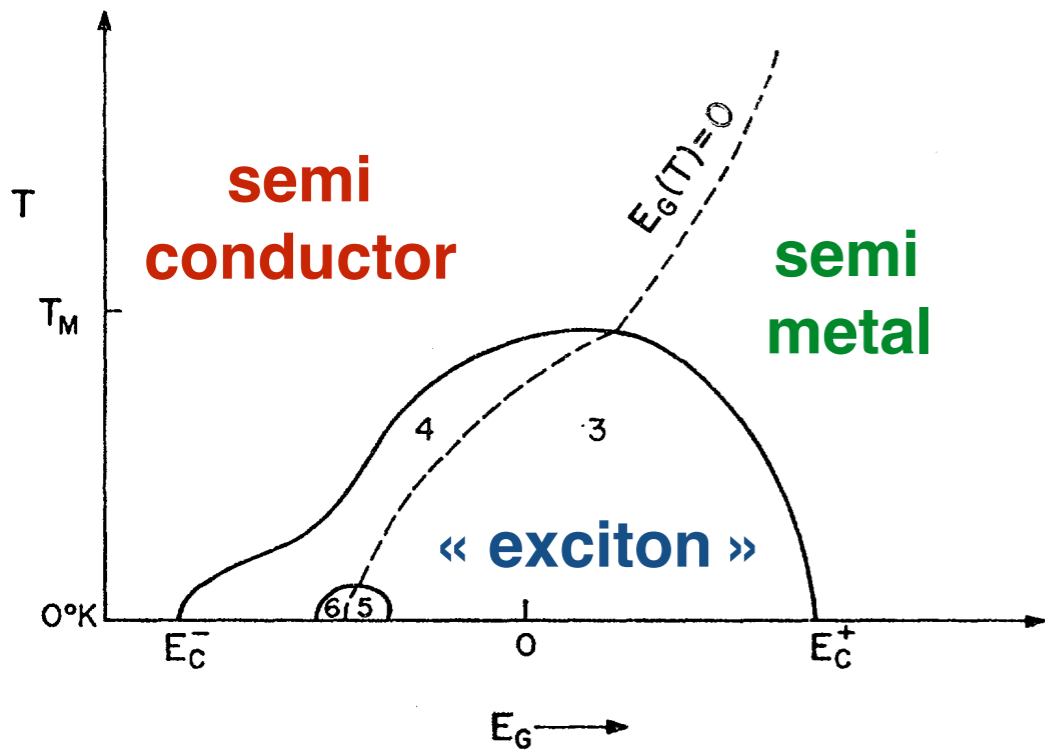
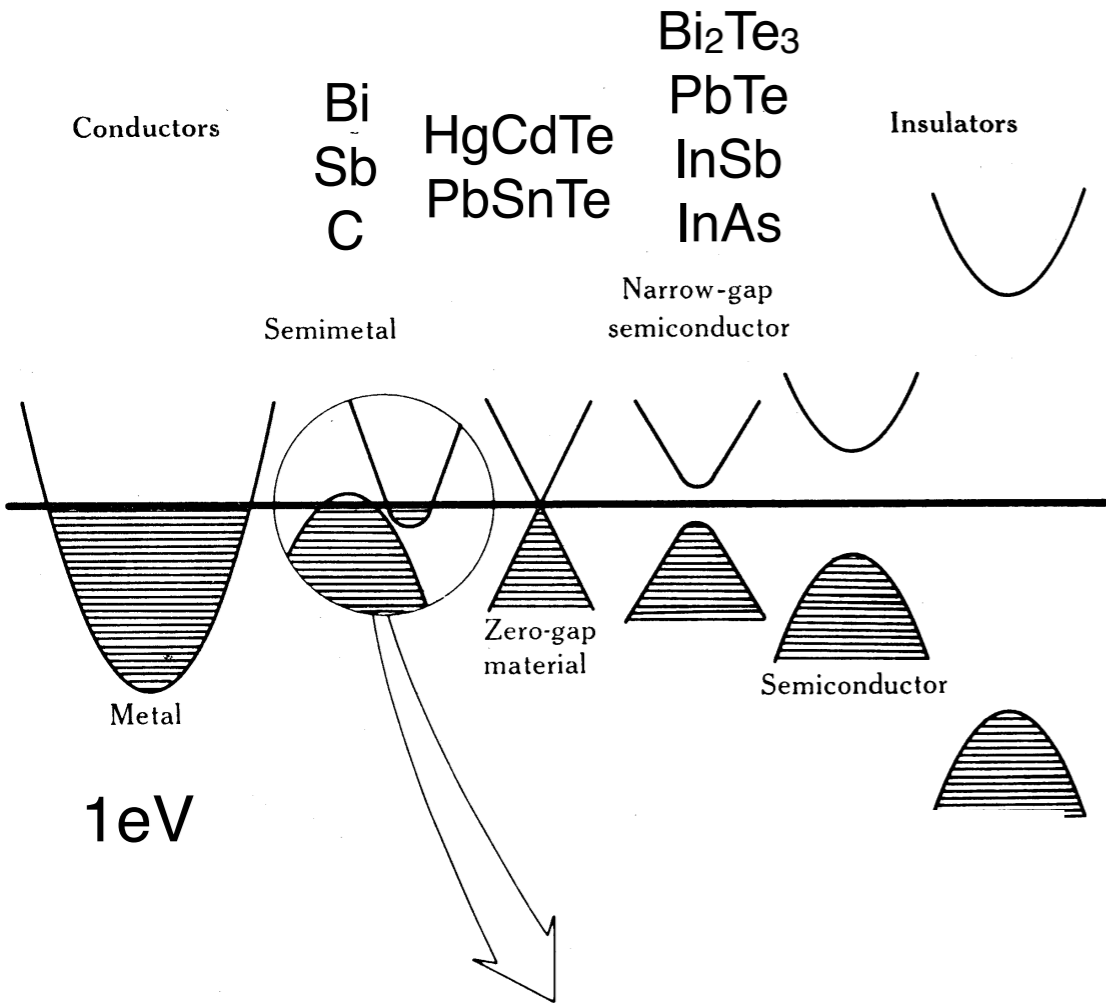
<http://qm.ipem.espci.fr/>



COLLÈGE
DE FRANCE
— 1530 —



Introduction : dilute metals



Dilute metals:
 low carrier conductors
 ($n=1e18cm^{-3} \Rightarrow$ small $E_F \approx 10meV$)

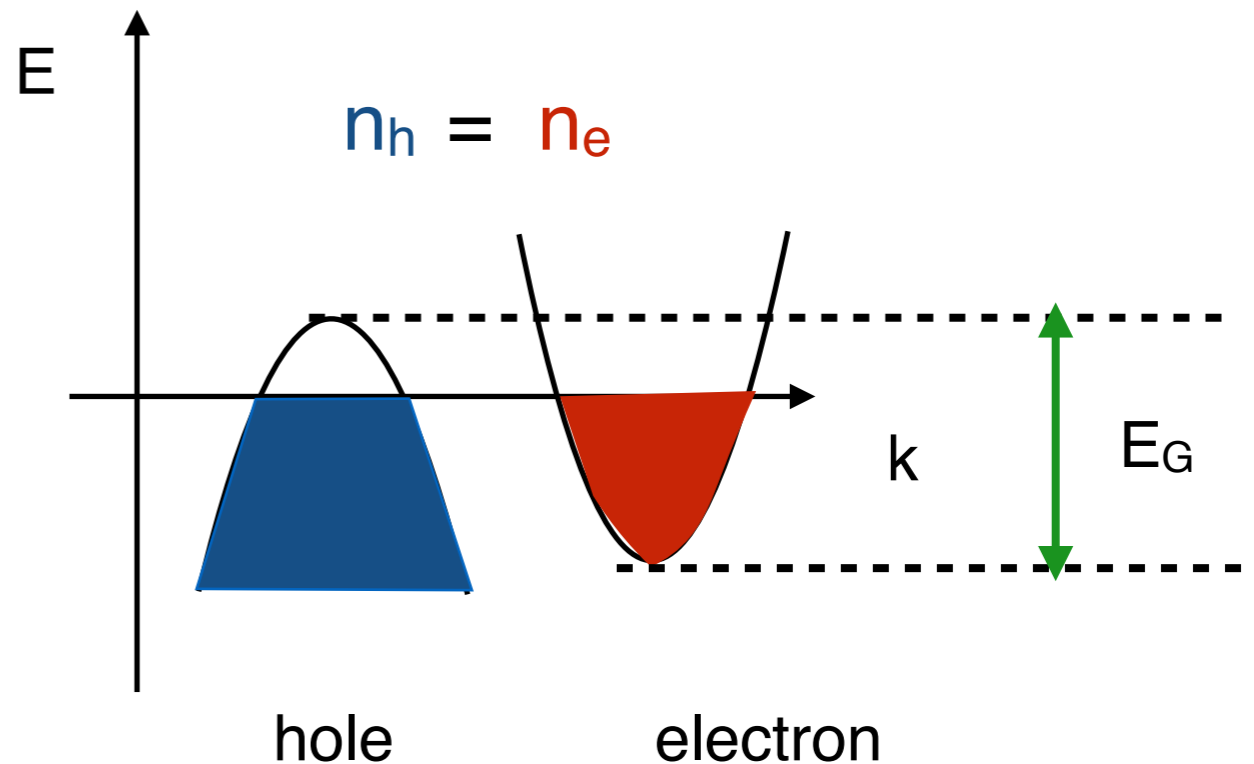
+
 well defined Fermi Surface resolved by QO

Enhancement of the effect of the interaction:

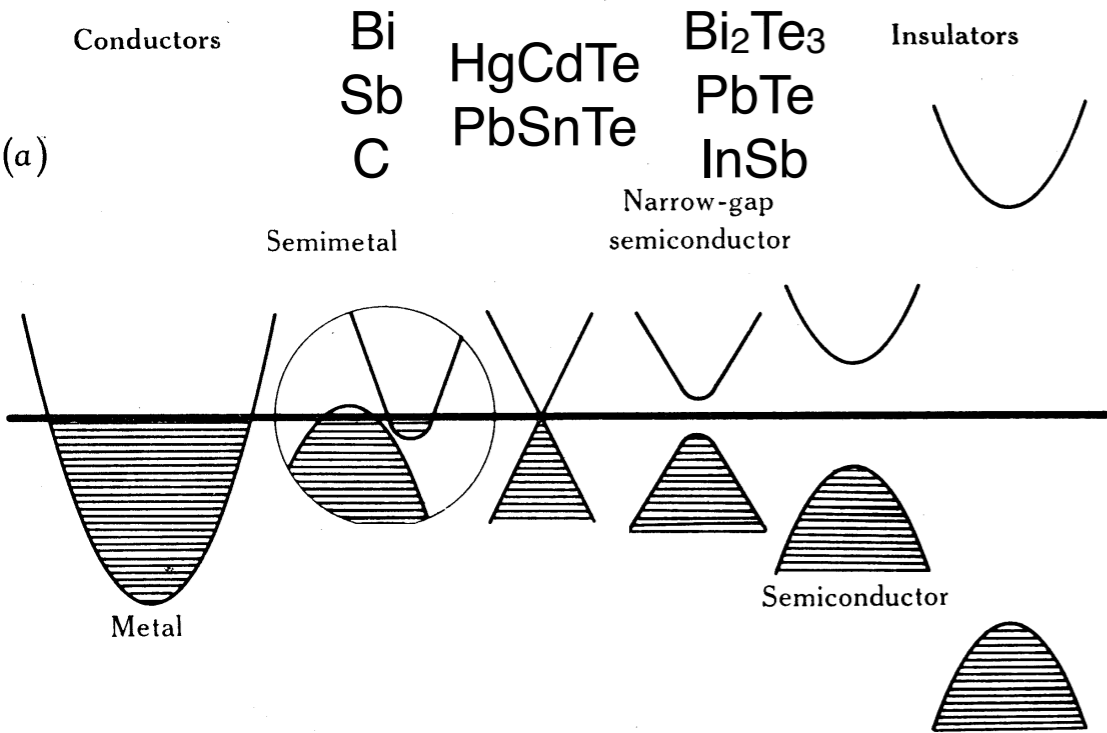
$$E_F \sim E_C$$

N.F Mott, Phil. Mag **6**, 287, (1961)

BCS like instability when $n \rightarrow 0$:
 electrons-holes form bound pairs -
 « an excitonic insulator »



Introduction : dilute metals



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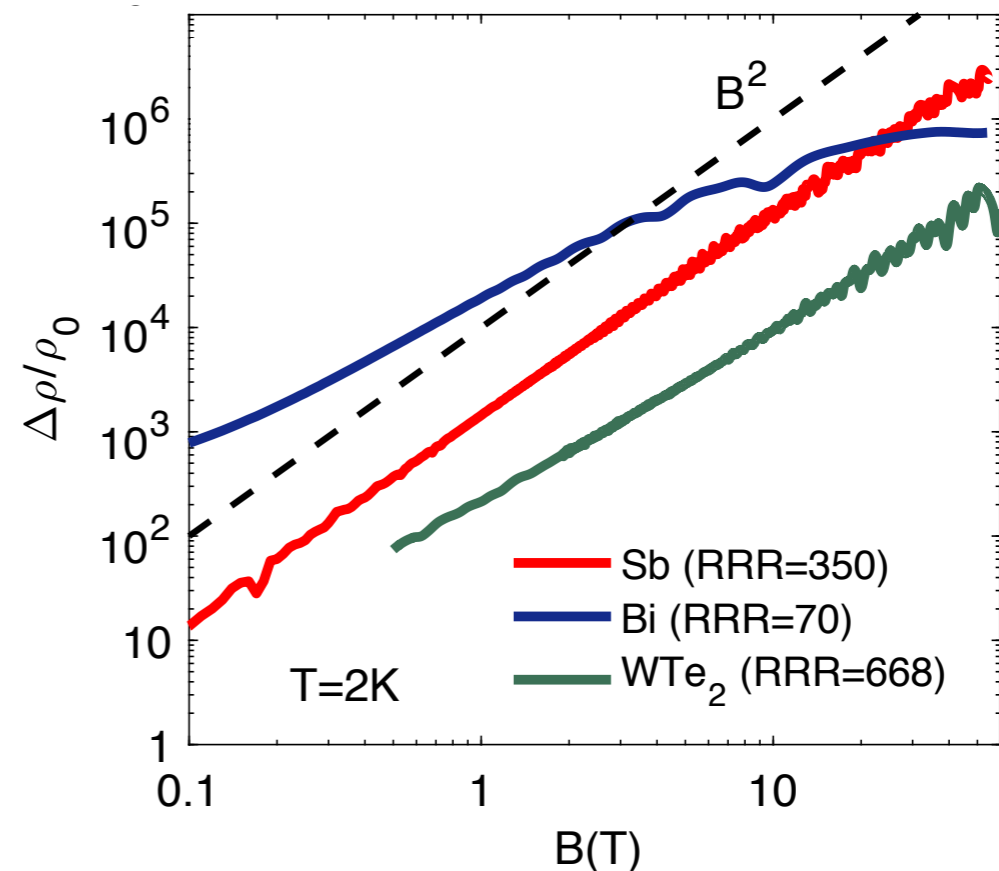
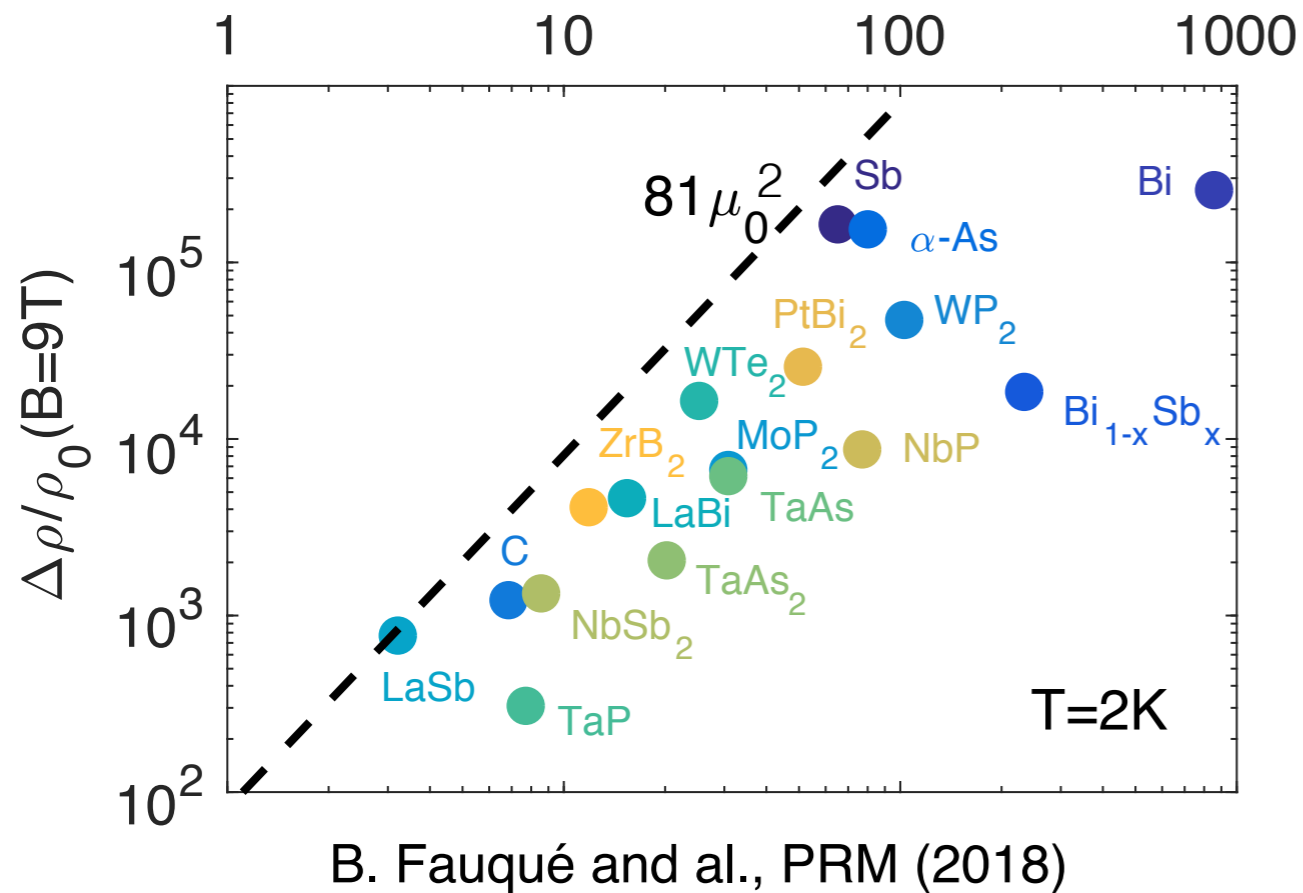
Dirac like dispersion in 3D :

3D « Dirac » dispersion Na₃Bi, Cd₃As₂
Type-II Weyl semi-metals WTe₂, WP₂ ...

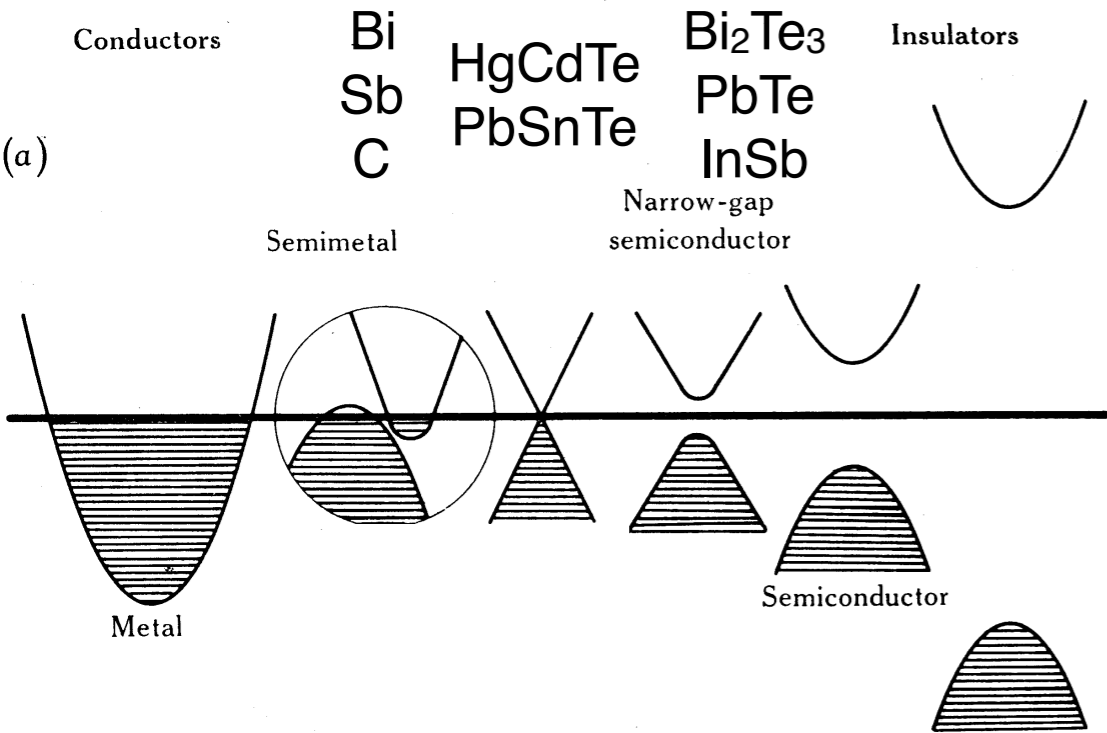
$$\mu_0 = \frac{1}{\rho_0(n+p)e}$$

$$\mu_0 \text{ (T}^{-1}\text{)}$$

Perfect semi-metal : $\Delta\rho/\rho_0 = \mu_0 B^2$



Introduction : dilute metals



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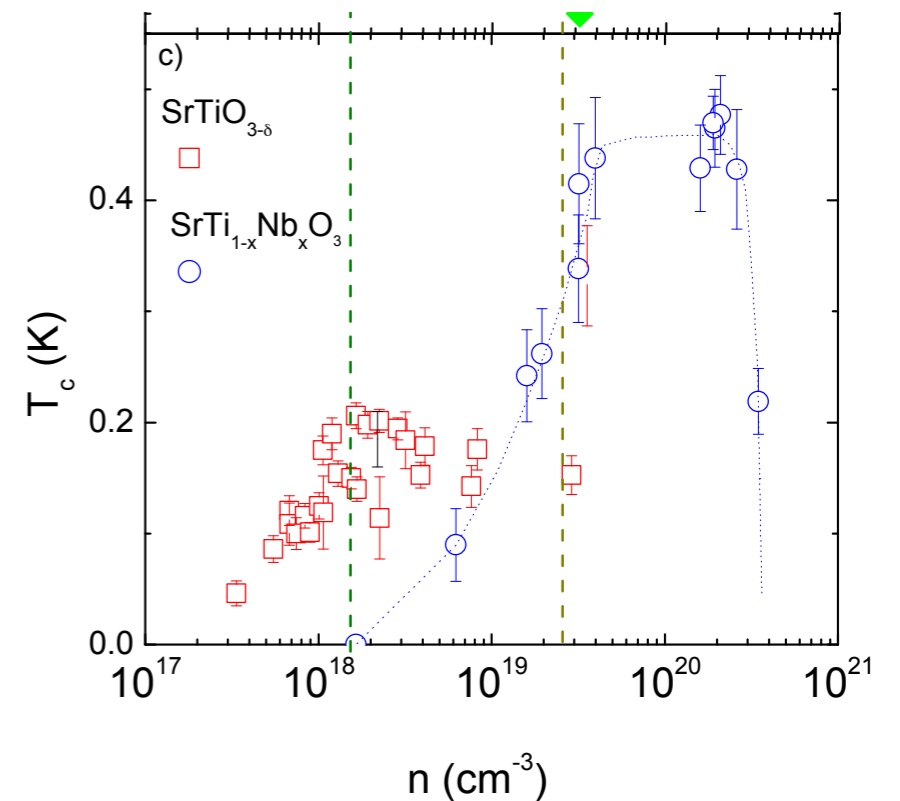
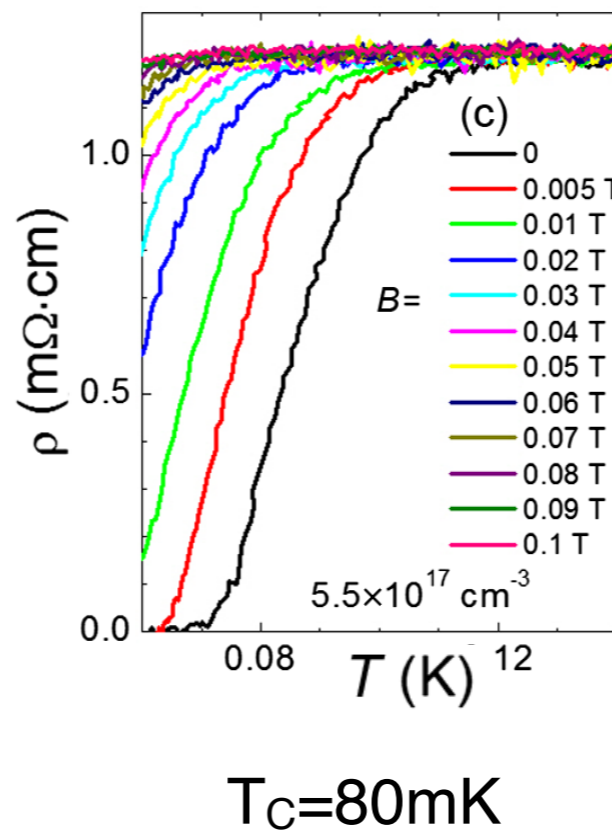
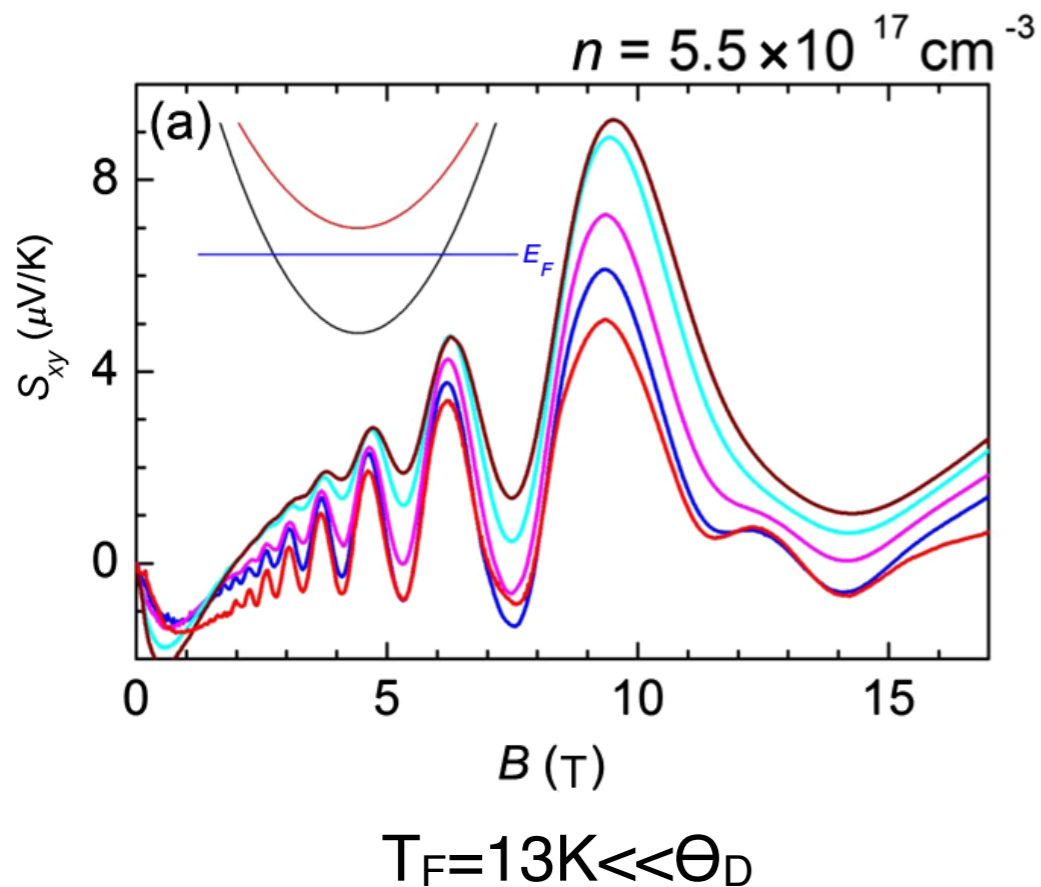
Superconductivity :

anti-adiabatic limit $E_F < k_B \Theta_D$

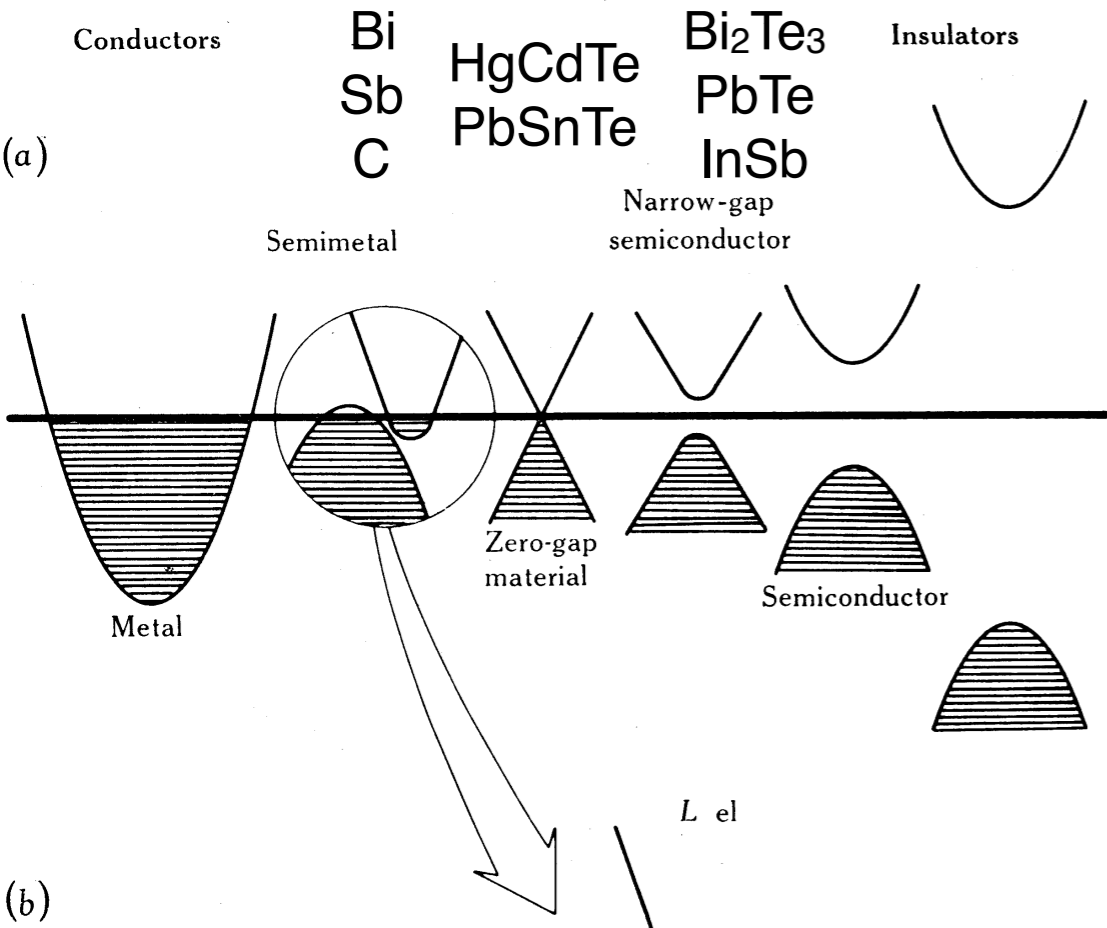
SrTiO_{3-x} : X. Lin and al., PRX, **3**, 021002 (2013)

C. Collignon and al., arXiv1804.07067

Bi : Science (2017)



Motivations



Dilute metals:
 low carrier conductors
 ($n=1e18cm^{-3} \Rightarrow$ small E_F)
 +
 well defined Fermi Surface resolved by QO

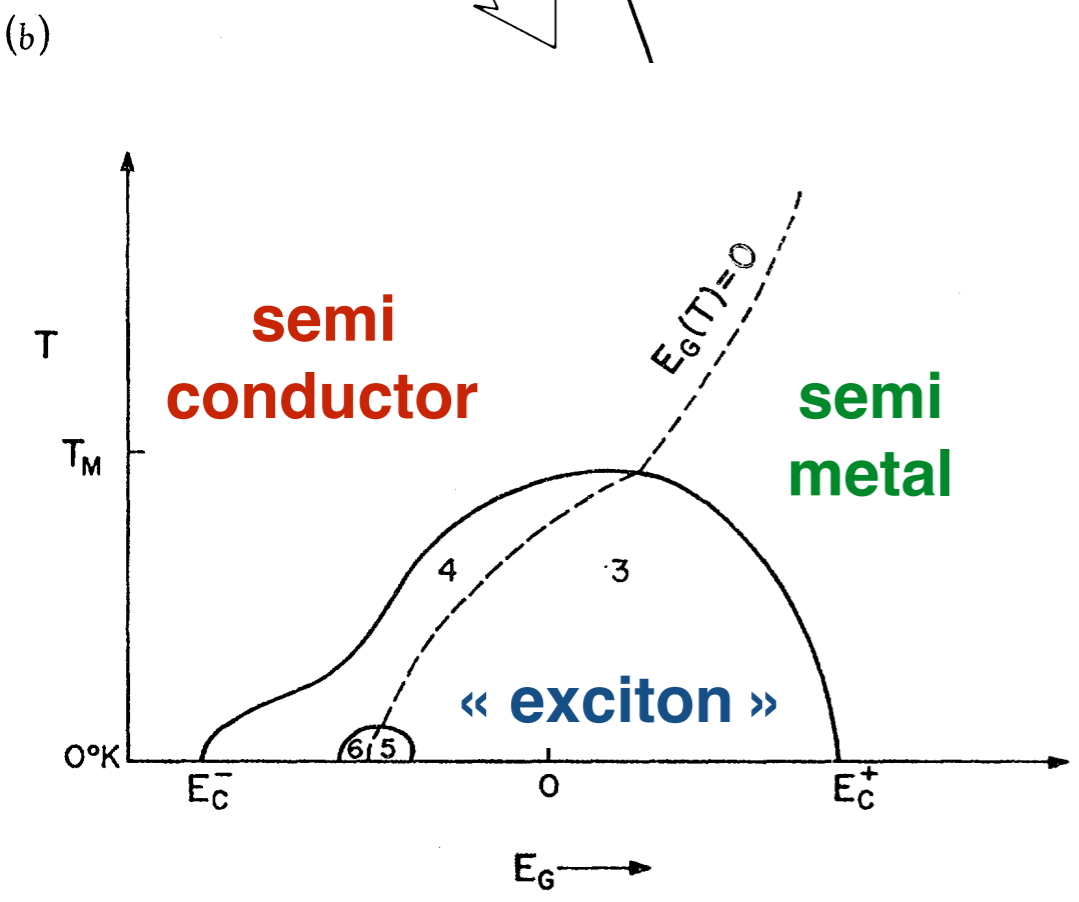
Enhancement of the effect of the interaction: $E_F \sim E_C$
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 $n \rightarrow 0$: electrons-holes form bound pairs -
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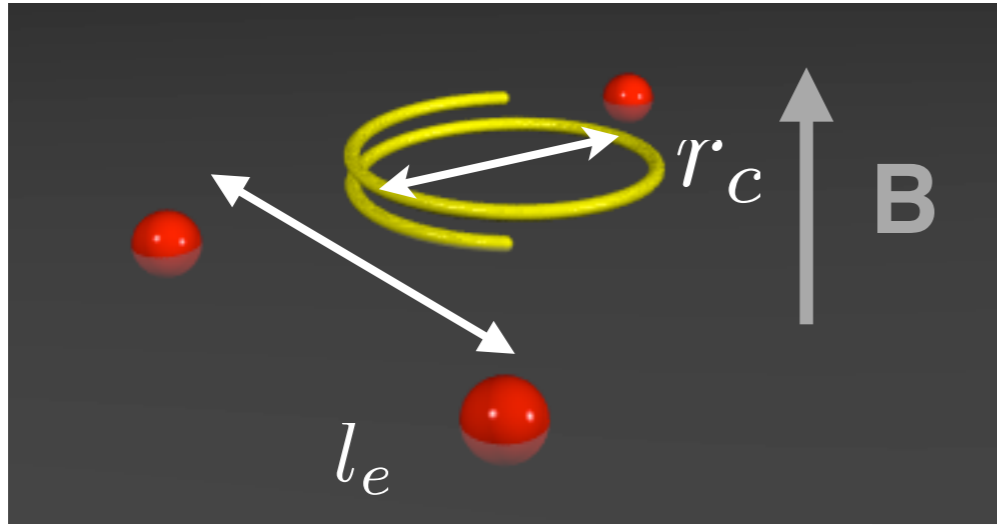
Thermoelectricity : High thermoelectric properties

Superconductivity : anti-adiabatic limit $E_F < k_B \Theta_D$

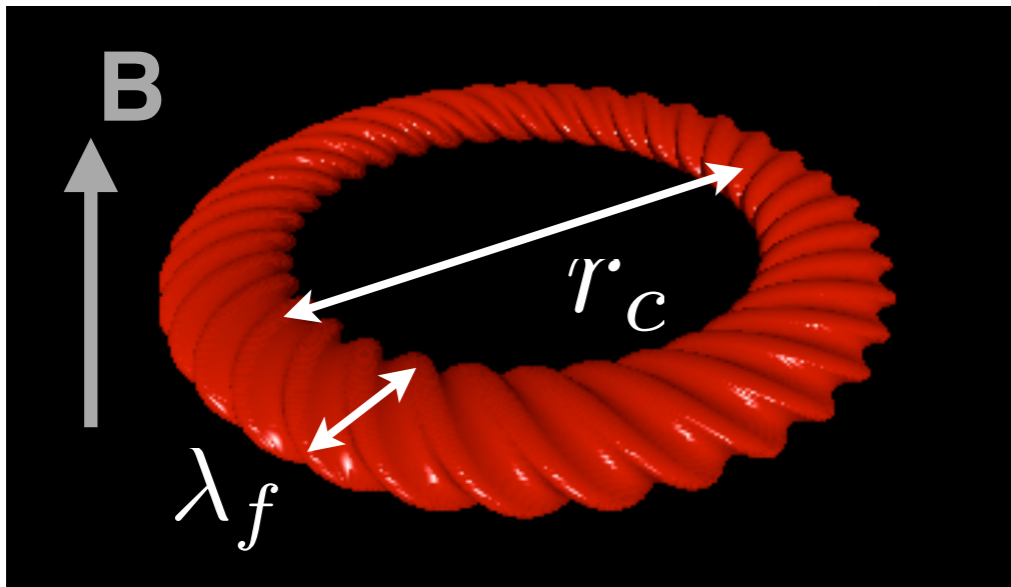
Quantum limit : $E_F \sim \hbar \omega_c$



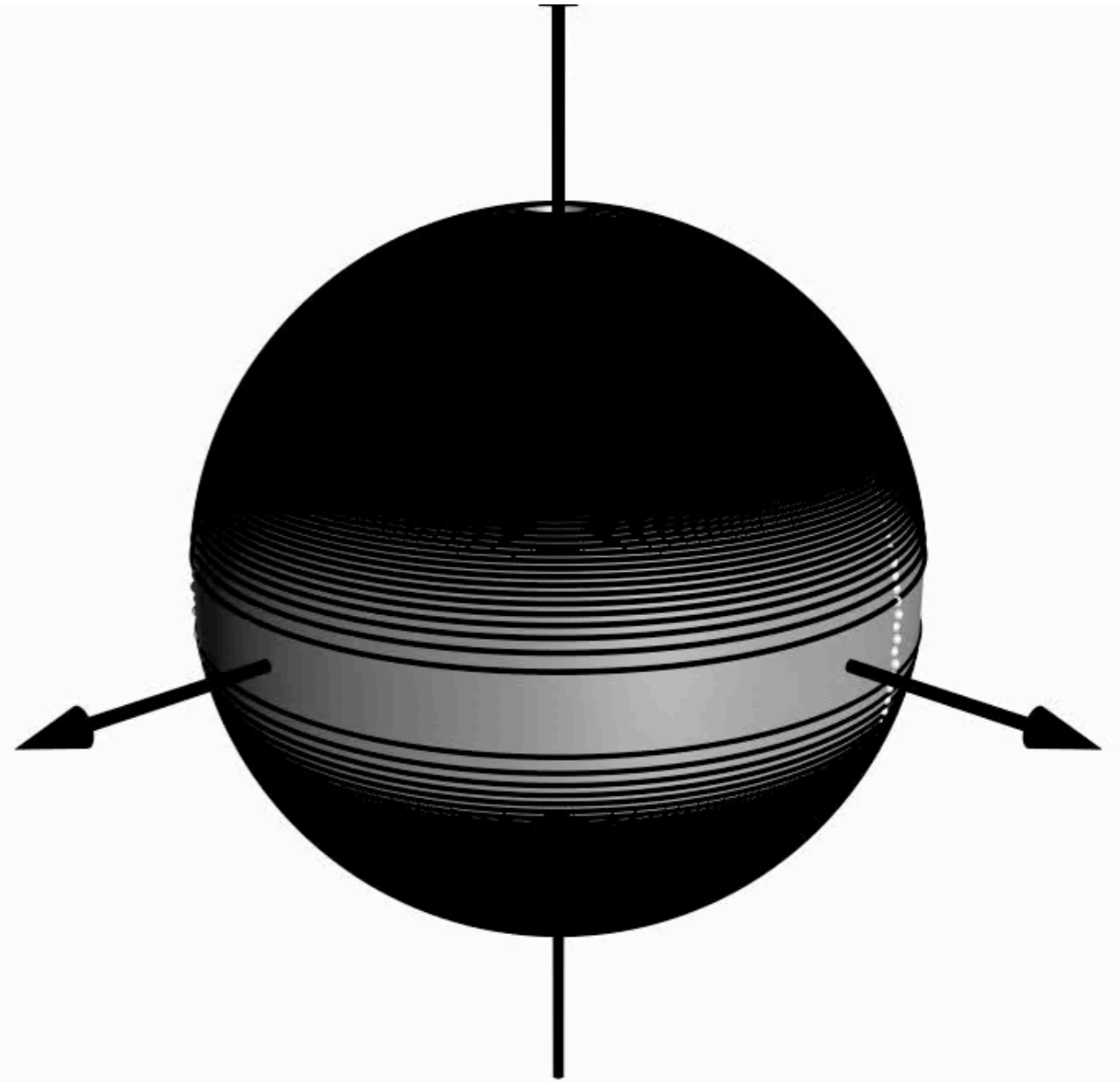
Motivation - Quantum limit



Semi-Classical limit

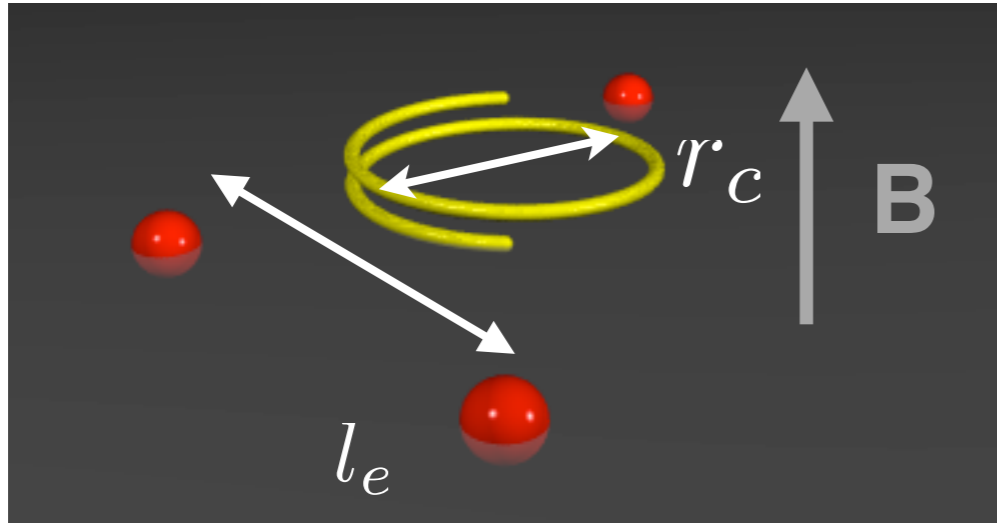


Quantum limit

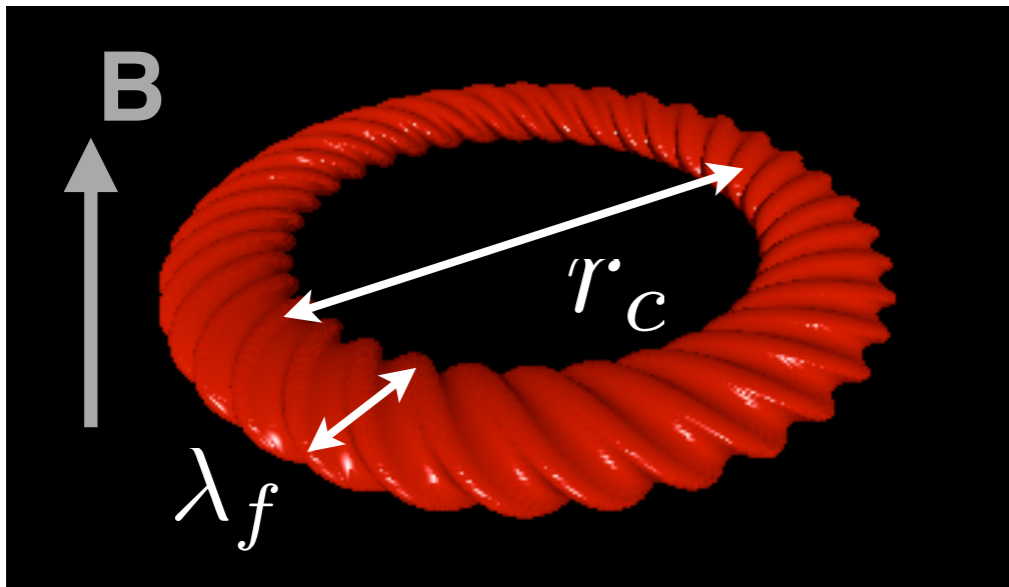


B

Motivation - Quantum Limit (QL)

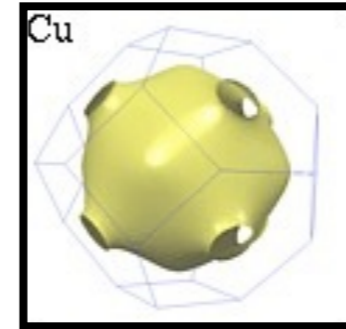


Semi-Classical limit

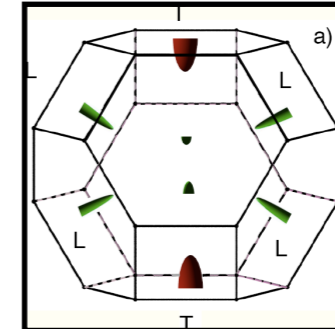


Quantum limit

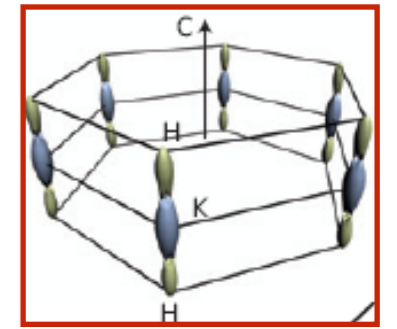
Copper



Bismuth



Graphite



n $8.5 \cdot 10^{22} \text{ cm}^{-3}$ $3 \cdot 10^{17} \text{ cm}^{-3}$ $3 \cdot 10^{18} \text{ cm}^{-3}$

QL $\sim 5000\text{T}$ $\sim 9\text{T}$ $\sim 7.5\text{T}$

but also in doped SC : InSb, PbTe, SrTiO3
and Weyl system i.e TaP

What is the electronic ground state of a 3D metal pushed in the quantum limit ?

B

Acknowledgements



A. Jaoui



W. Rischau



K. Behnia



LNCMI- G/T



D. Le Boeuf (US)



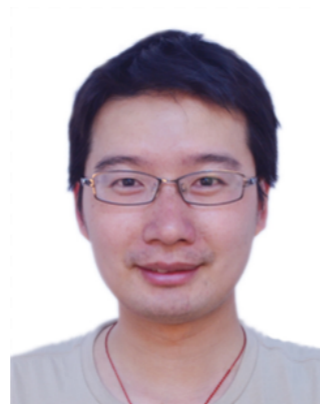
G. Seyfarth



C. Proust



Wuhan



Z. Zhu

Thermoelectrical response beyond the quantum limit of 3D electron gas systems

Introduction

Nernst effect as a probe of quantum oscillations in semi-metals

Transport and thermodynamic measurements in the quantum limit of graphite

Narrow gap semi-conductors in the quantum limit: the case of InAs

Conclusion

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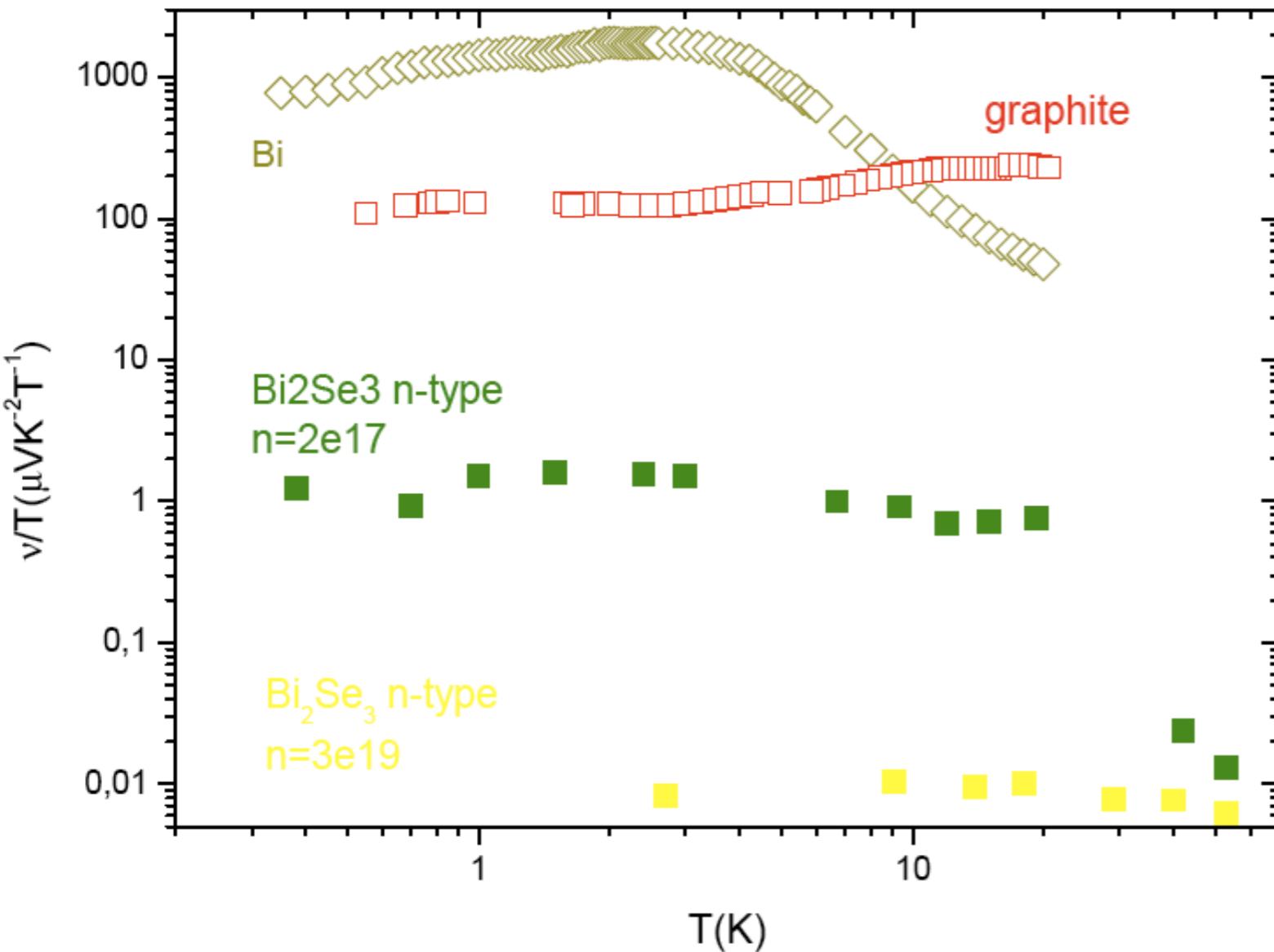
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Nernst effect : low field regime

$$\nu = \frac{-E_y}{\nabla_x T B_z} \quad \nu = N/B = -\frac{\pi^2 k_B^2 T}{3 m^*} \frac{\partial \tau}{\partial \epsilon} \Big|_{\epsilon=\epsilon_F}$$



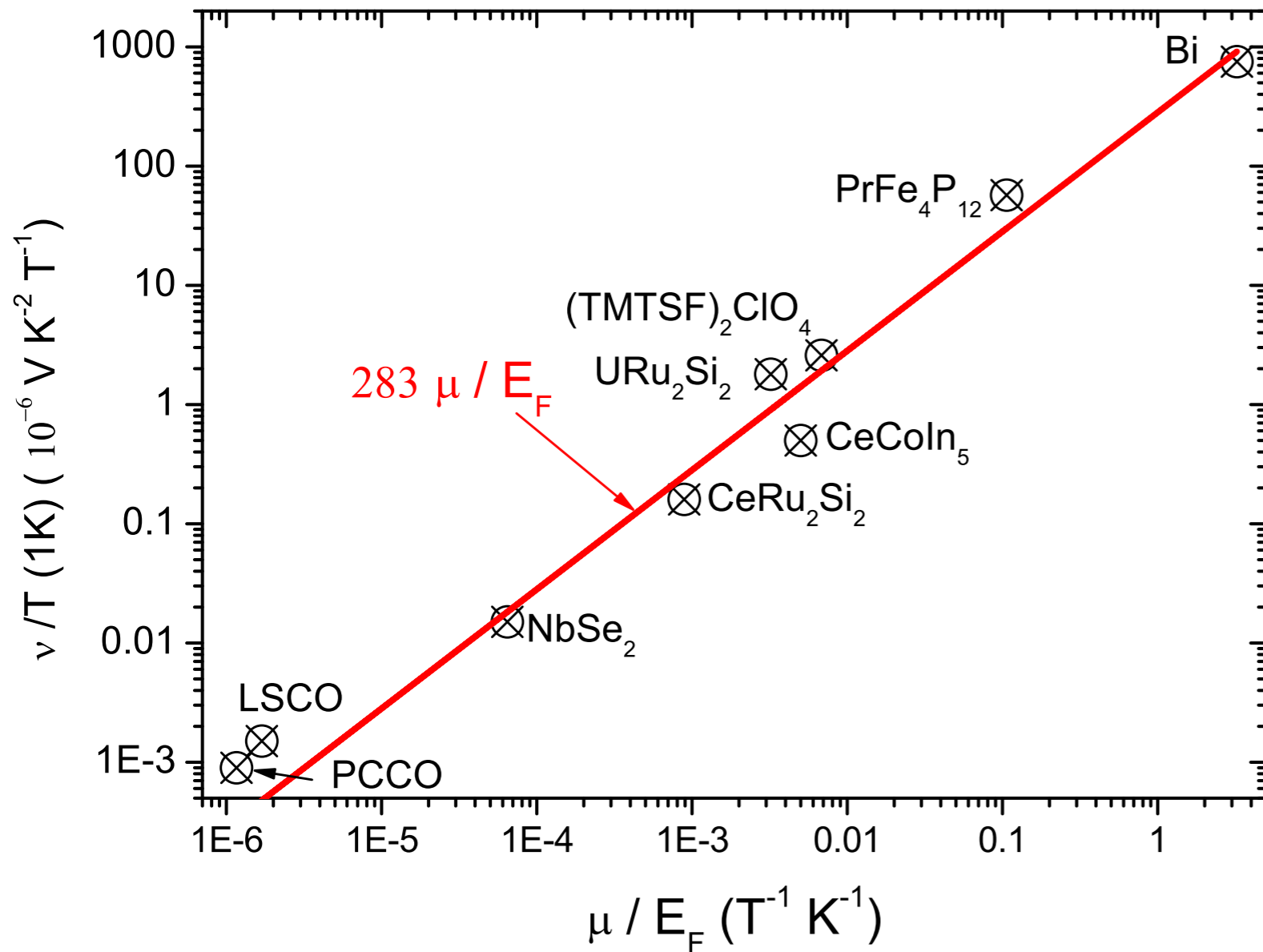
	$\mu(\text{T}^{-1})$	E_f (K ⁻¹)	286 μ/T_f ($\mu\text{V K}^{-2}$)
Bi	500	130	914
C	30	220	40
Bi ₂ Se ₃ 3	0.45	90	1.4
Bi ₂ Se ₃ 3	0.05	1200	0.04

$$\frac{\nu}{T} = -\frac{\pi^2 k_B^2}{3 m^*} \frac{\partial \tau}{\partial \epsilon} \Big|_{\epsilon=\epsilon_f} \approx 283 \frac{\mu}{\epsilon_f} [\mu\text{V K}^{-1}\text{T}^{-1}]$$

K. Behnia, J. Phys.: Condens. Matter
21, 113101 (2009)

Nernst effect : low field regime

$$\nu = \frac{-E_y}{\nabla_x T B_z}$$



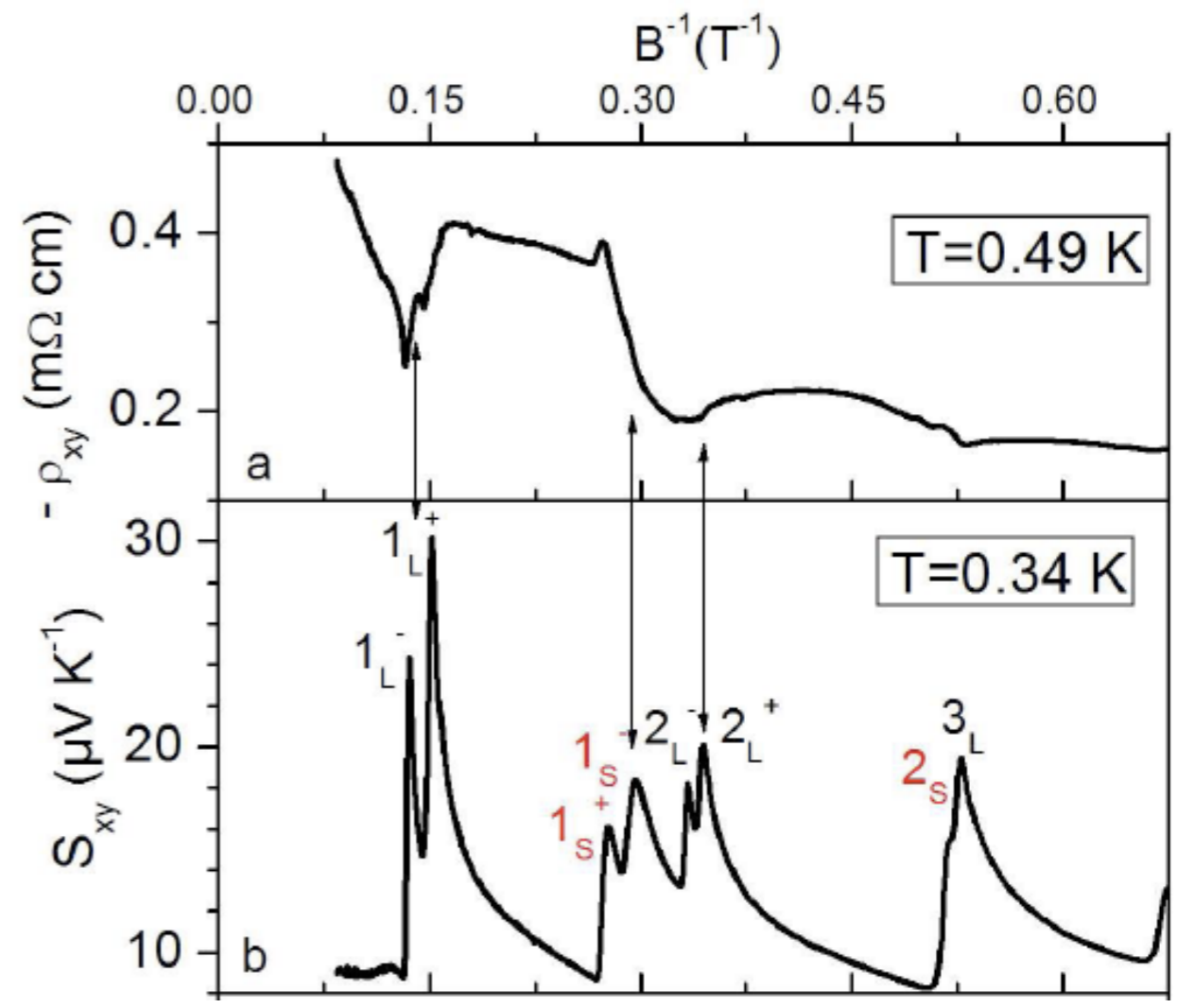
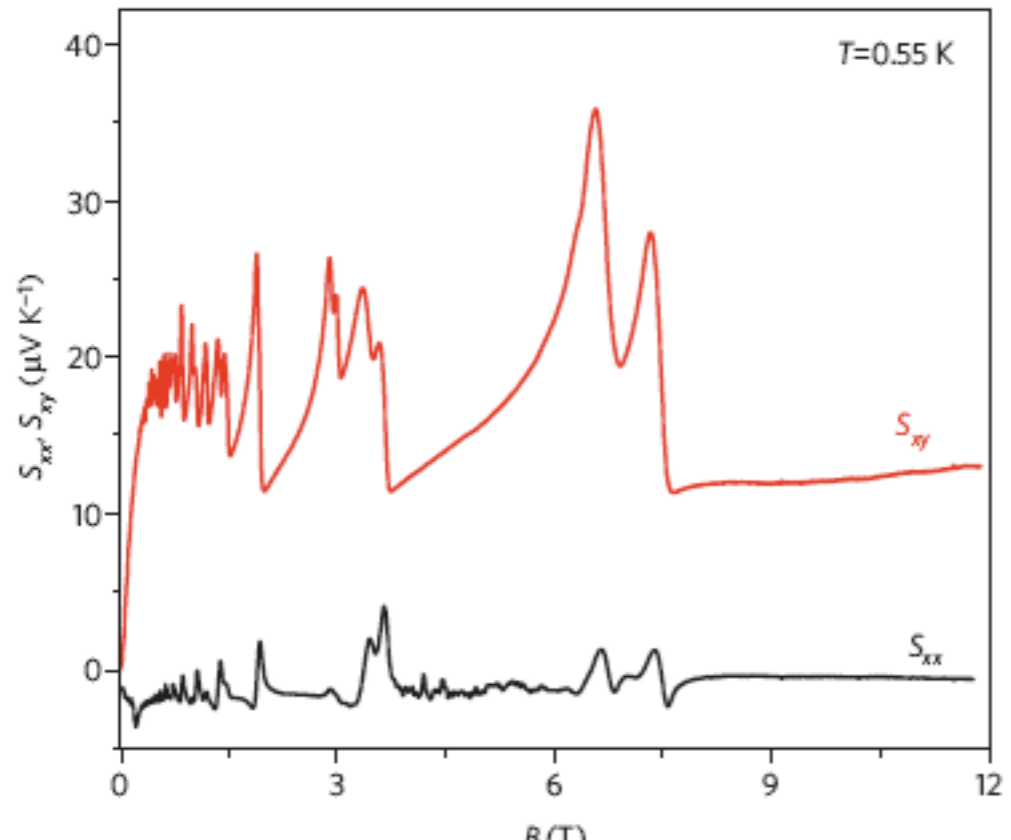
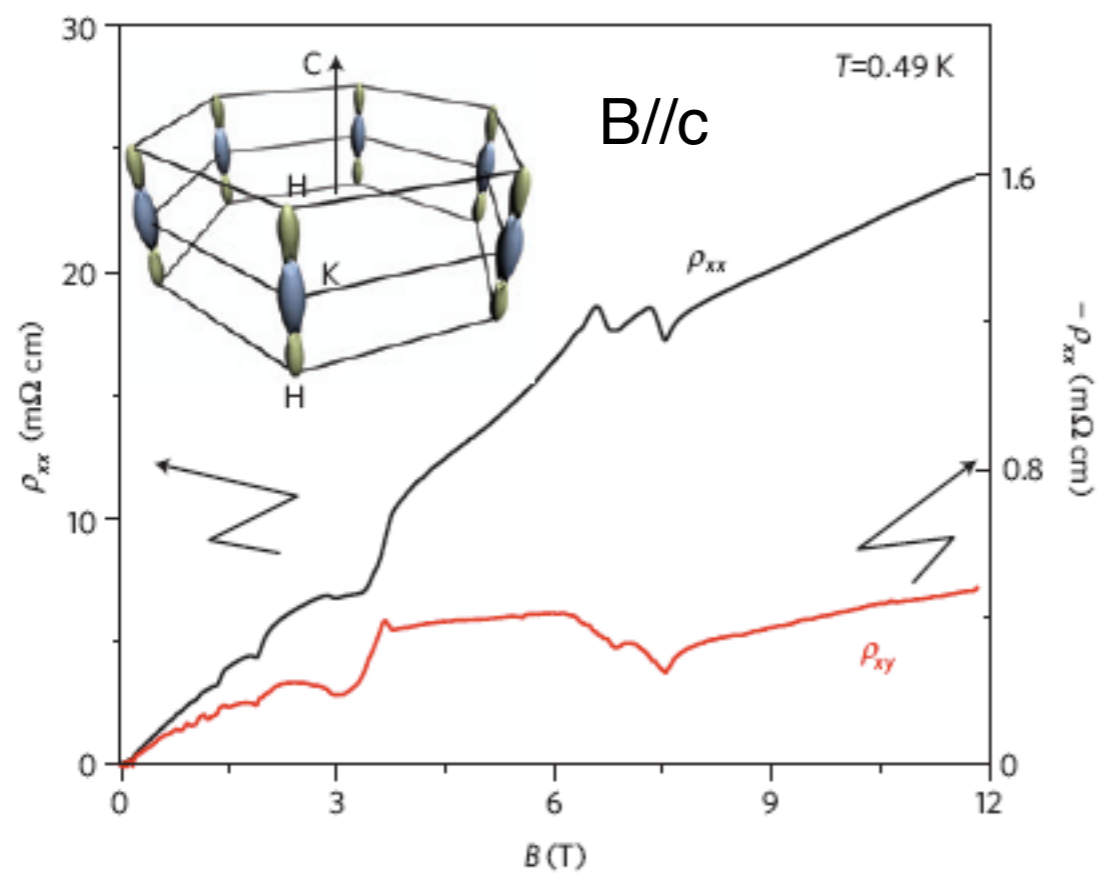
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Bi	500	130	914
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Bi_2Se_3	0.45	90	1.4
Bi_2Se_3	0.05	1200	0.04

In the case of bismuth
the combination of
(i) high mobility
(ii) low Fermi energy
give a large Nernst effect

K. Behnia, J. Phys.: Condens.
Matter
21, 113101 (2009)

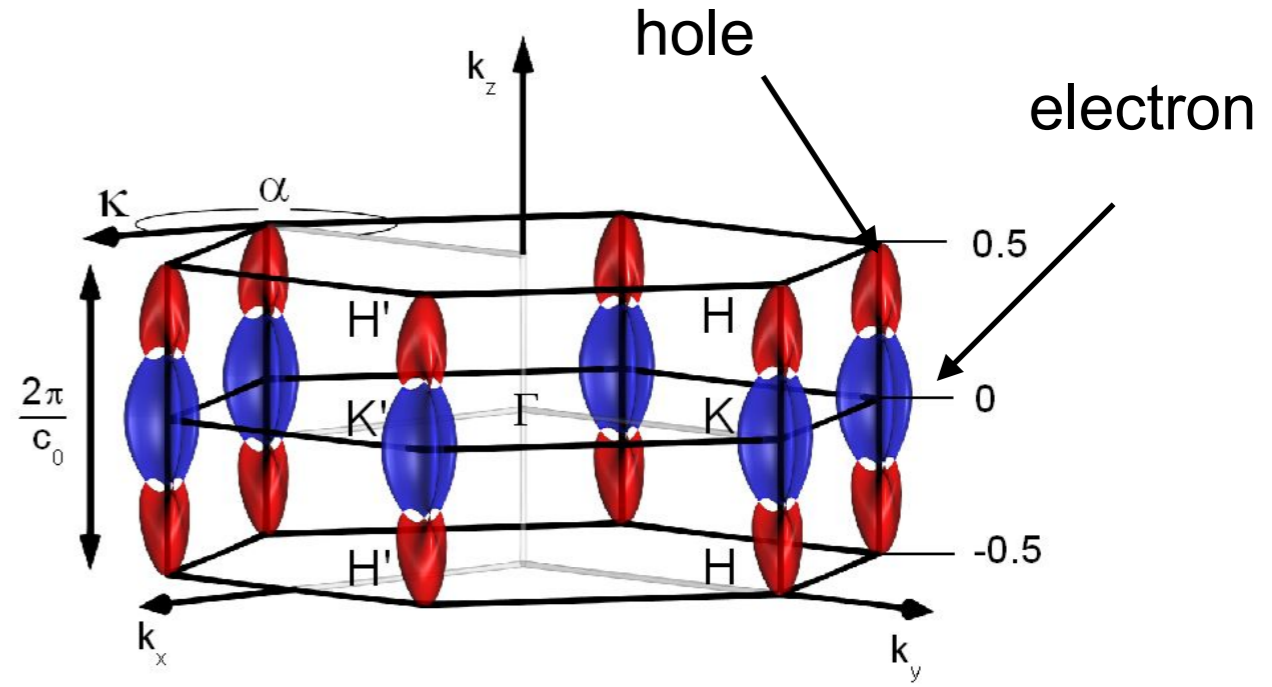
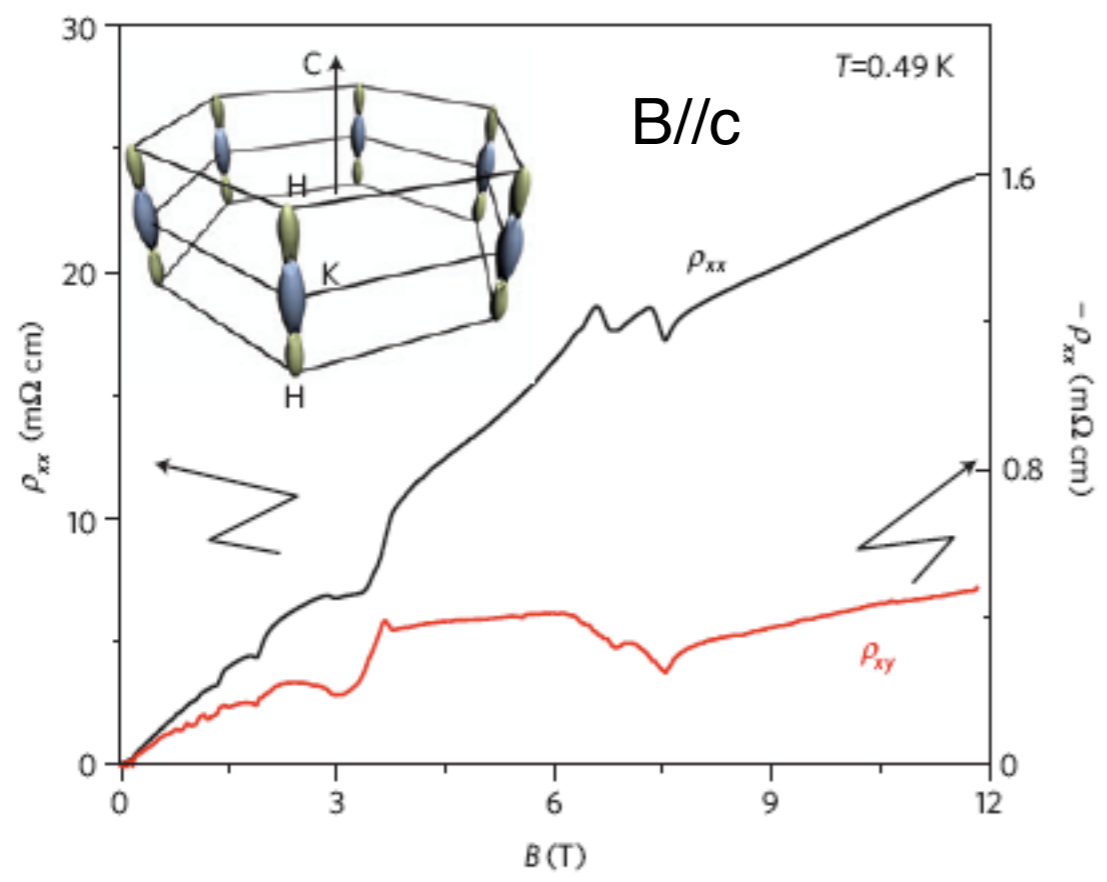
Graphite : Nernst effect up to 12T

(HOPG) Z.Zhu et al., Nature Physics 4,166602 (2009)



Fermi surface of Graphite

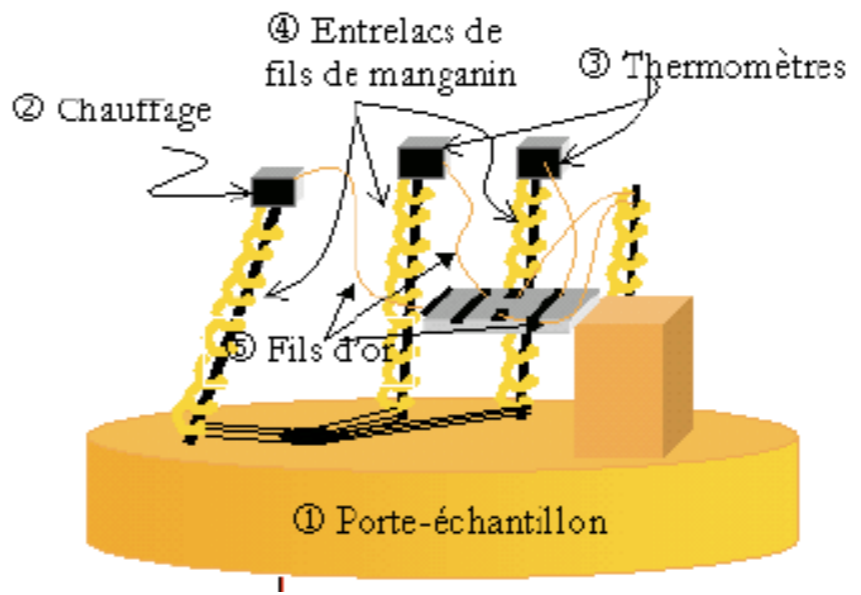
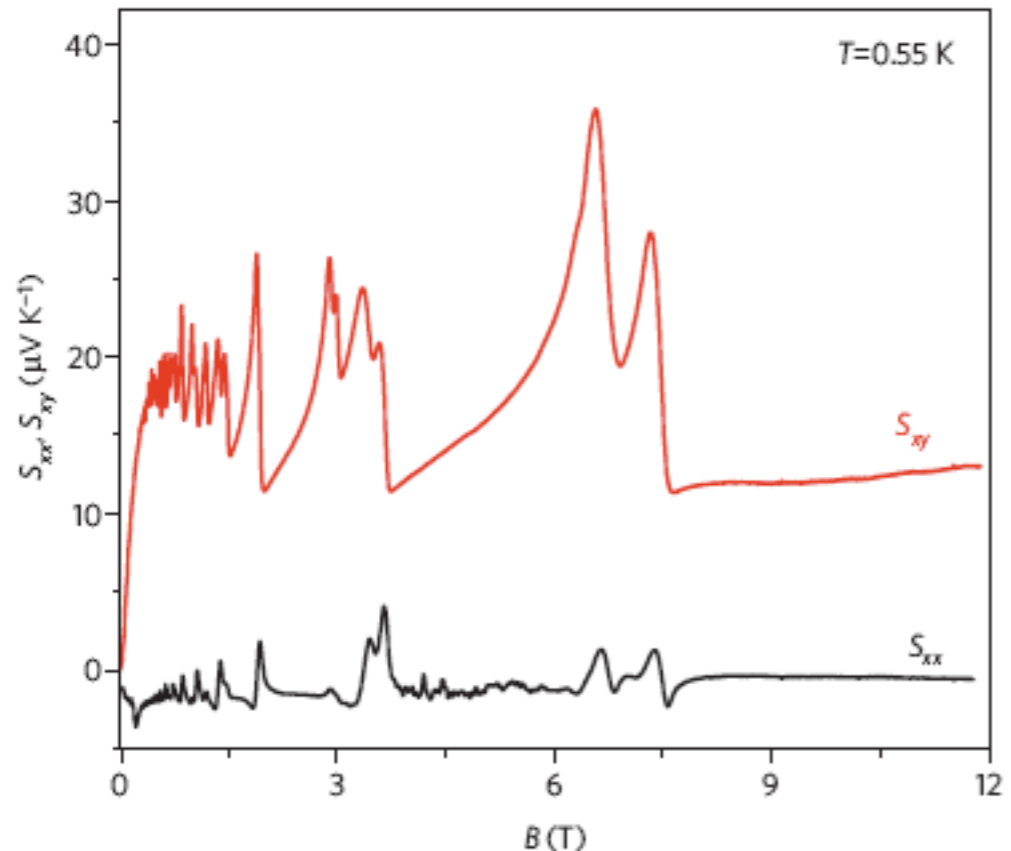
(HOPG) Z.Zhu et al., Nature Physics 4,166602 (2009)



S.B. Hubbard and al., PRB (2011)

hole : Schrödinger type

electron : Schrödinger or Dirac type ?

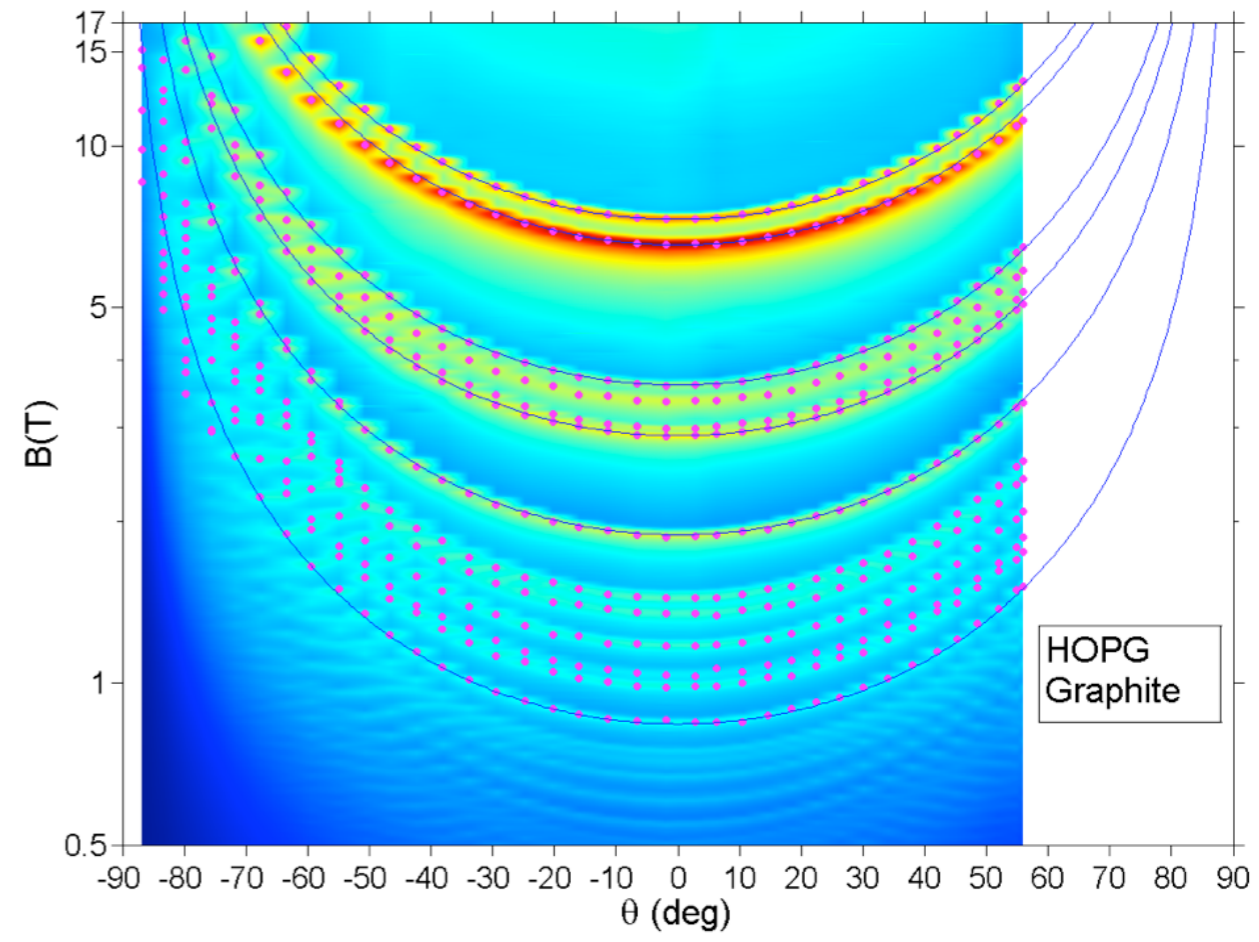
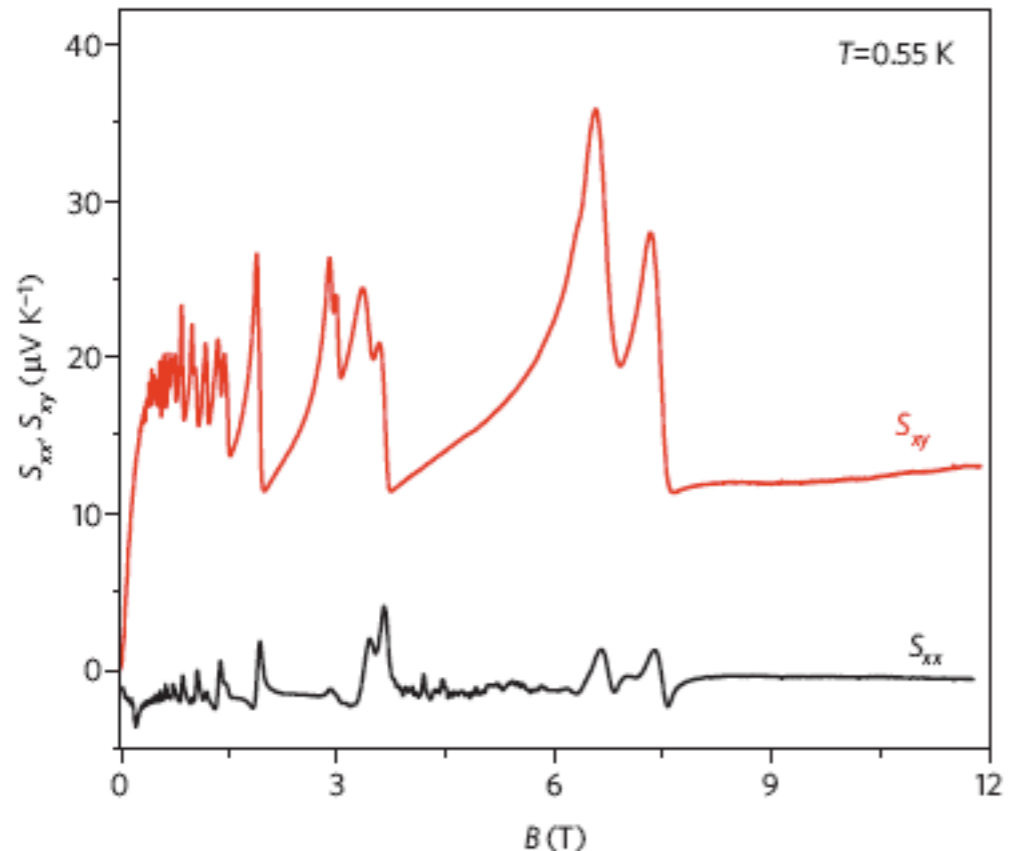
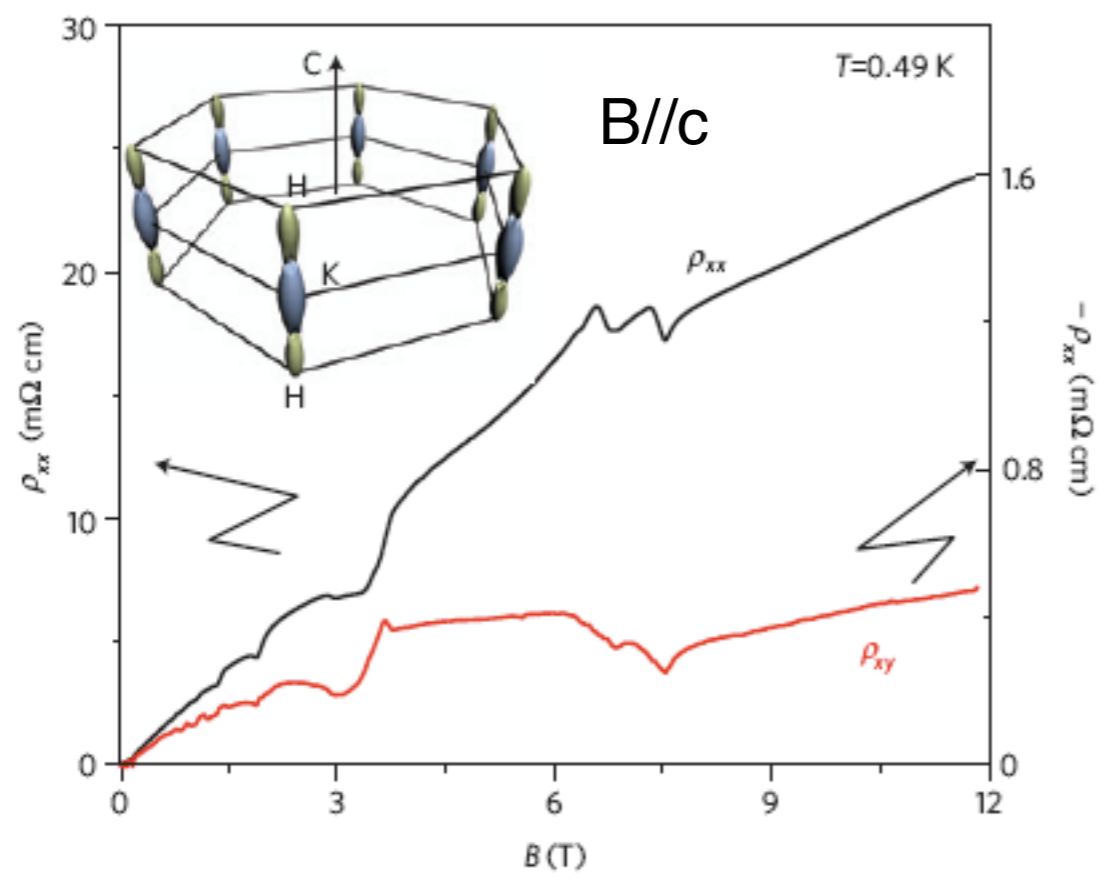


$$S_{xx} = \frac{-E_x}{\nabla_x T}$$

$$S_{xy} = \frac{-E_y}{\nabla_x T}$$

Graphite : Nernst effect up to 12T

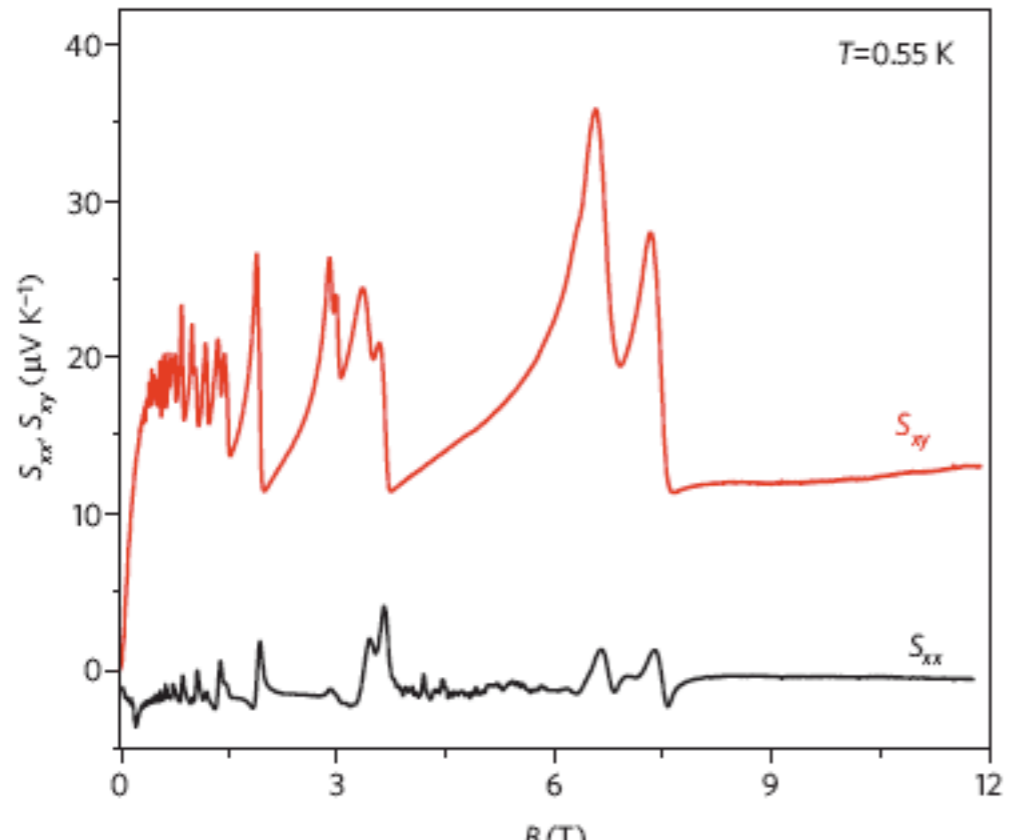
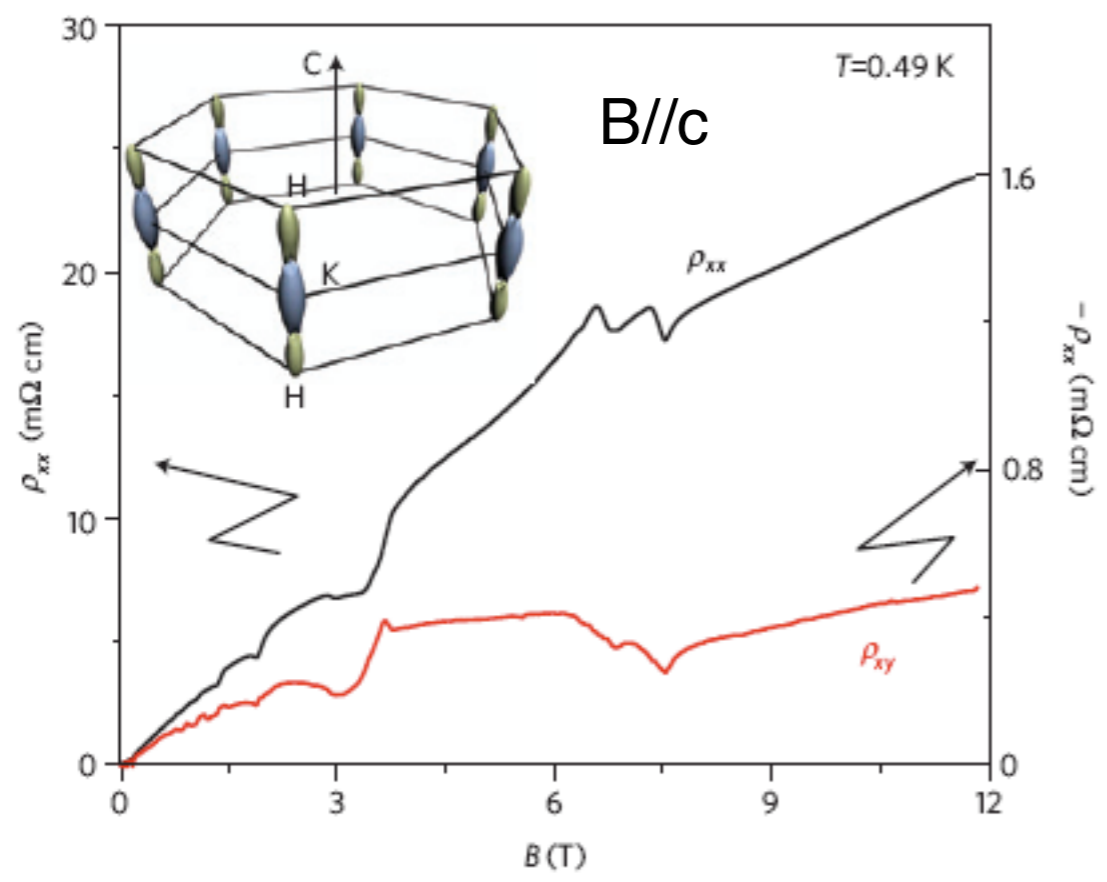
(HOPG) Z.Zhu et al., Nature Physics 4,166602 (2009)



Nernst effect as a probe of the quantum limit in graphite

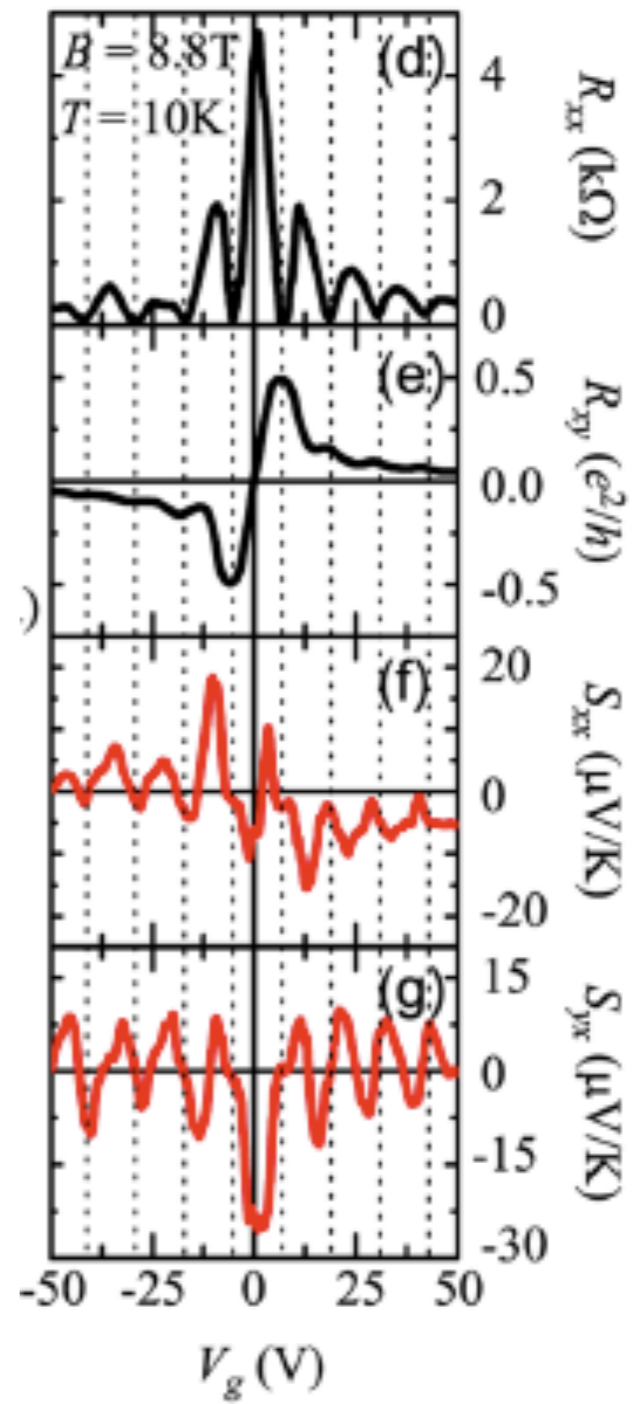
Graphite vs graphene

(HOPG) Z.Zhu et al., Nature Physics 4,166602 (2009)



Y.M Zuev et al, PRL,102, 096807 (2009)

2D : Graphene



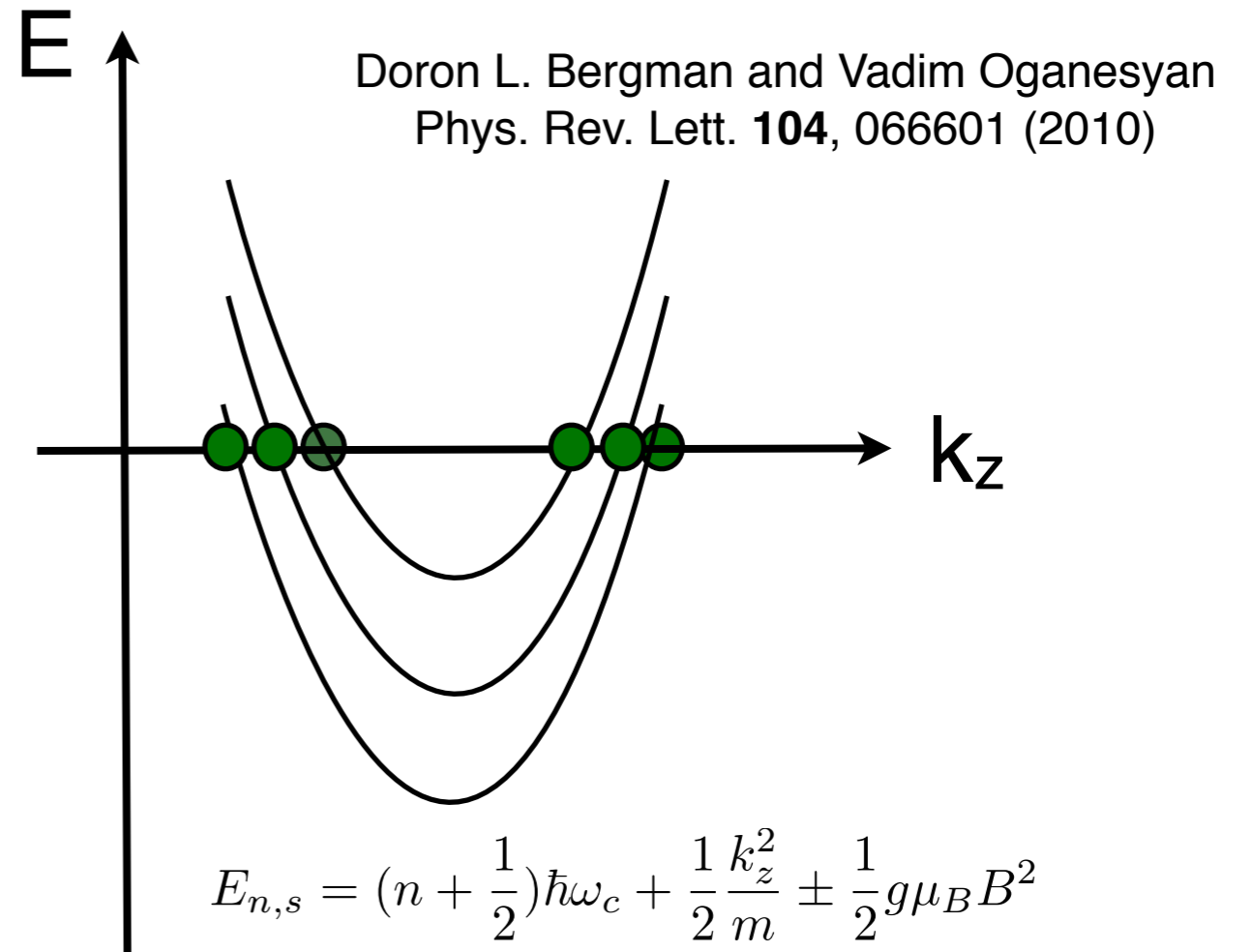
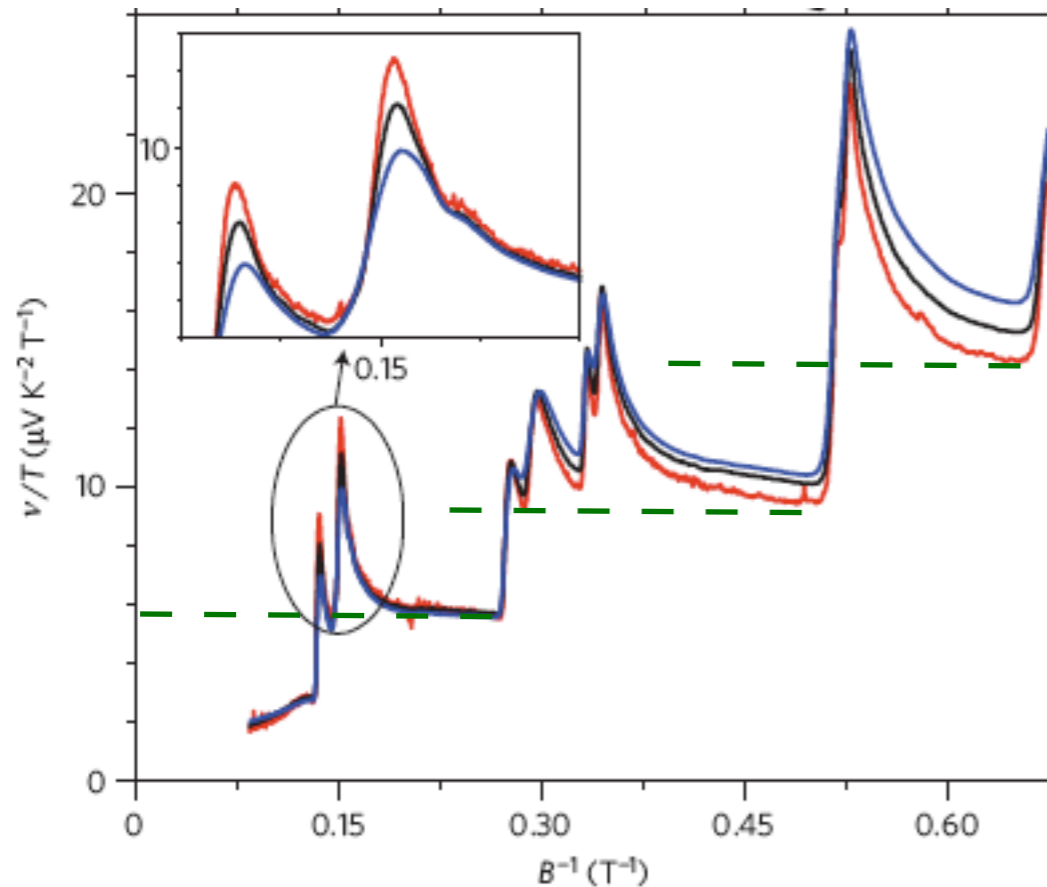
Theory

D. Bergman et al, PRL,104, 066601 (2010)
 I. Luk'yanchuk et al, PRL,107, 016601 (2011)

Origin of the giant Nernst quantum oscillations

Between the peaks, the Nernst effect is “step like”
In this regime, the electronic spectrum is:

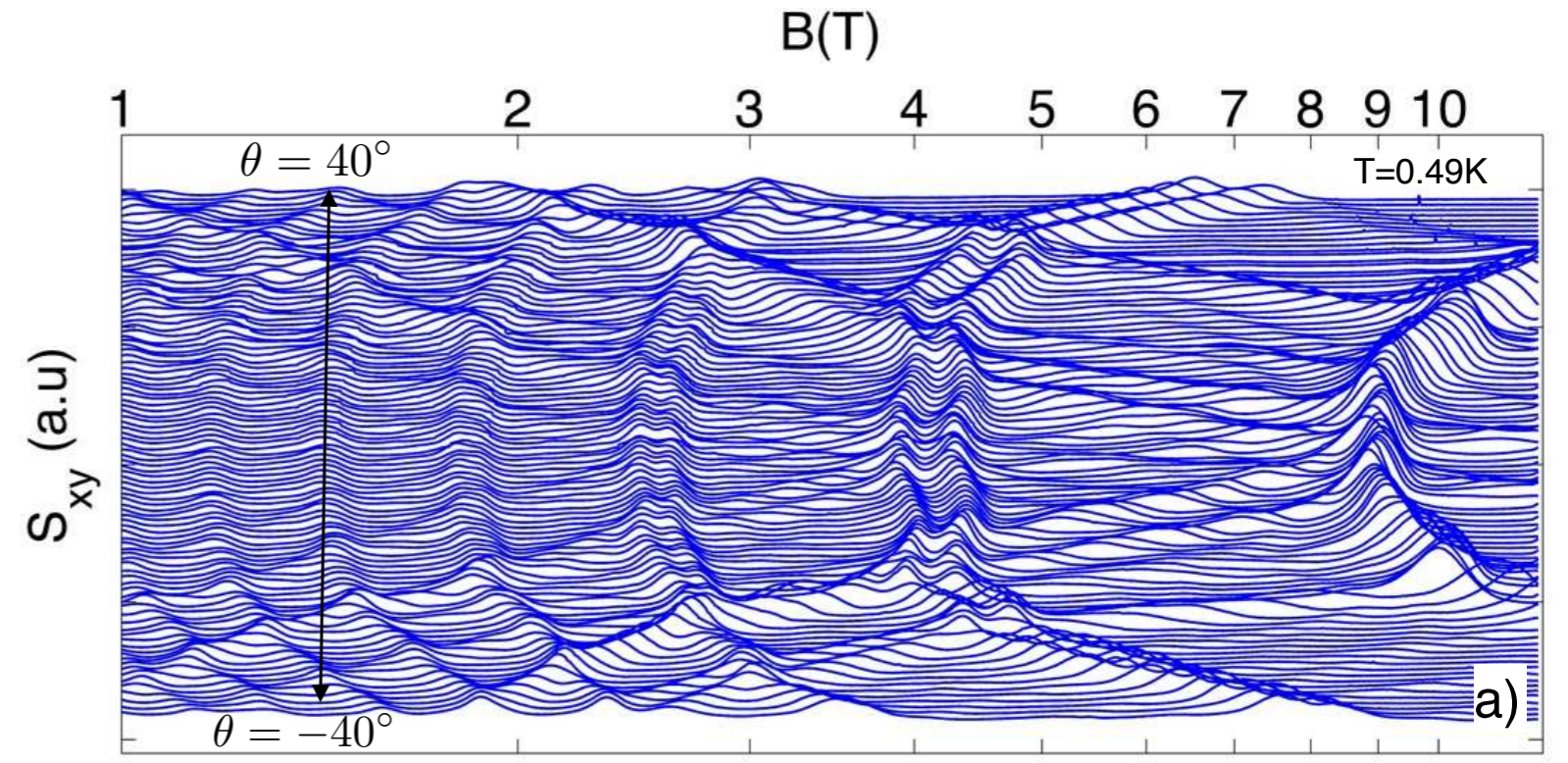
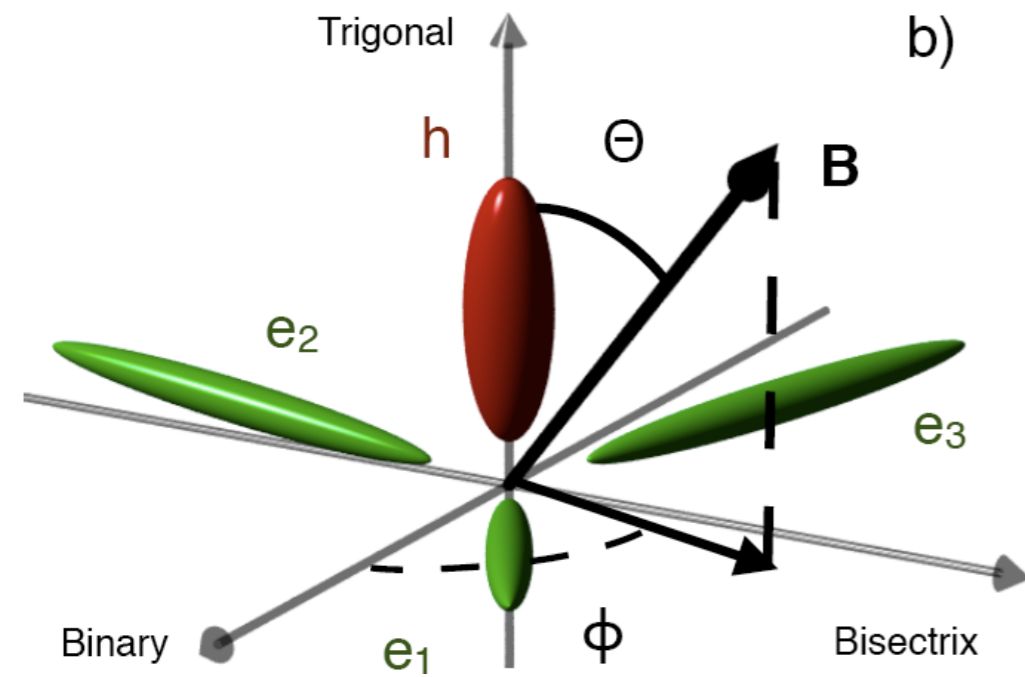
$$\alpha_{xy} = -\frac{ek_B}{h} \frac{\pi^2}{3} \sum_{n=0}^{n_{\max}} \frac{k_B T}{2\pi\hbar v_{Fn}}$$



The Nernst effect is dominated by the green spots of each of the (full) Landau levels.
As long as the chemical potential is far from the bottom of the Landau levels, the Nernst effect is nearly constant.

Landau level spectrum of Bi

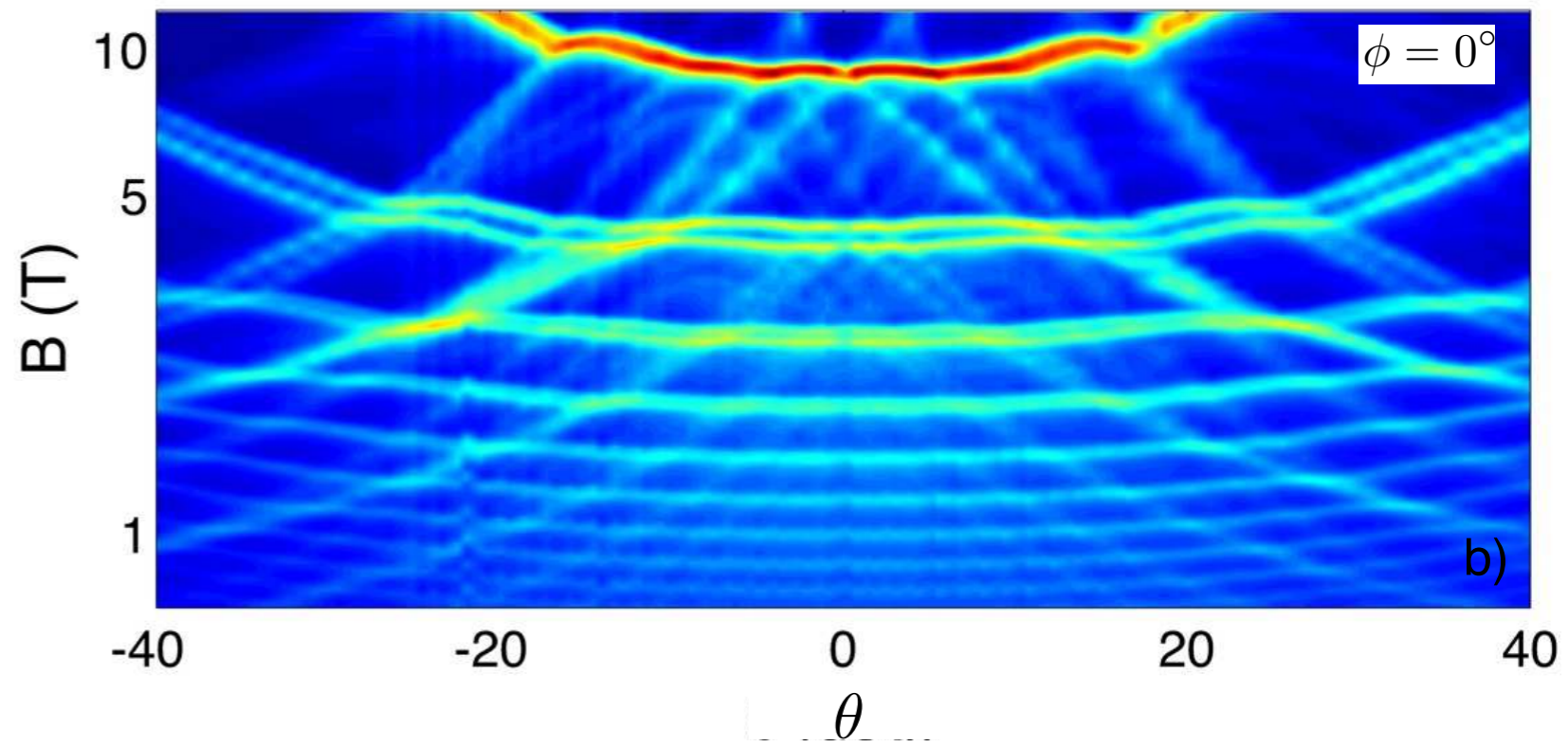
Z.Zhu and al., PRB 84,115137 (2011)

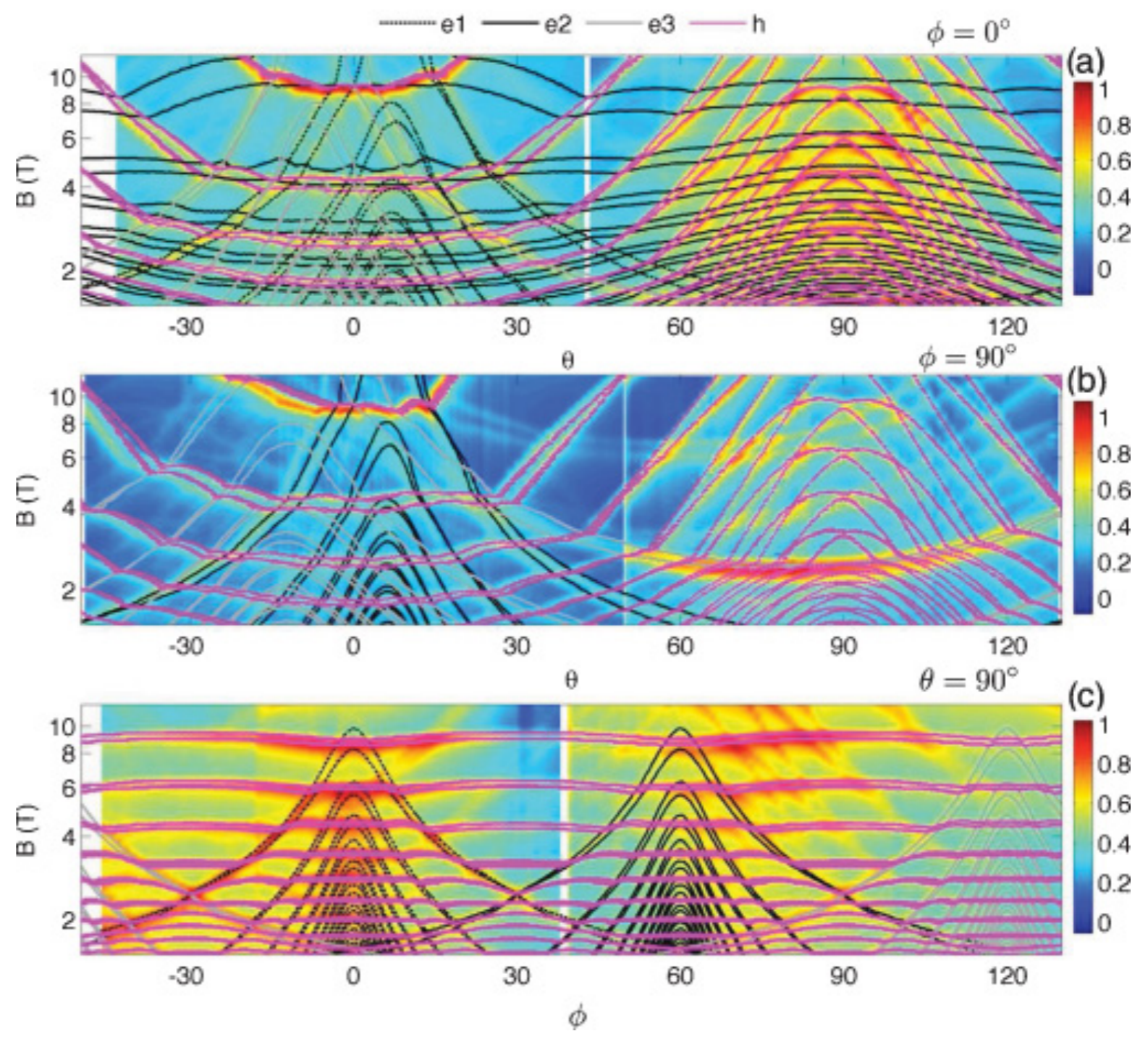
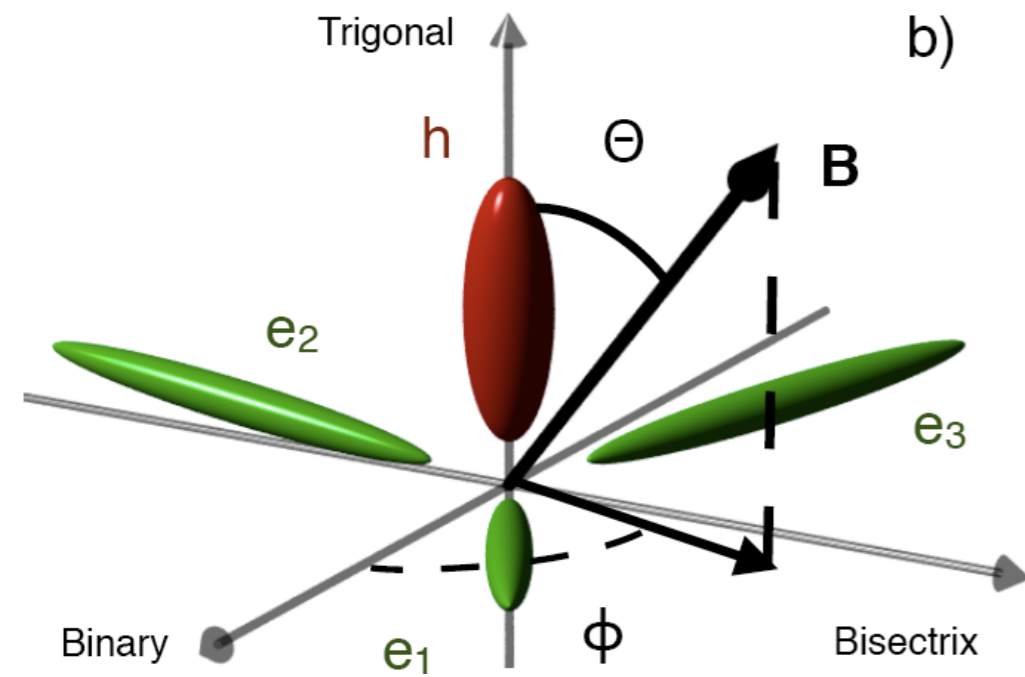


Hole : parabolic dispersion
Spin Mass anisotropic

Electrons (3 valleys):
Dirac dispersion

Charge neutrality :
 $n_{e1} + n_{e2} + n_{e3} = n_h \neq n(0T)$





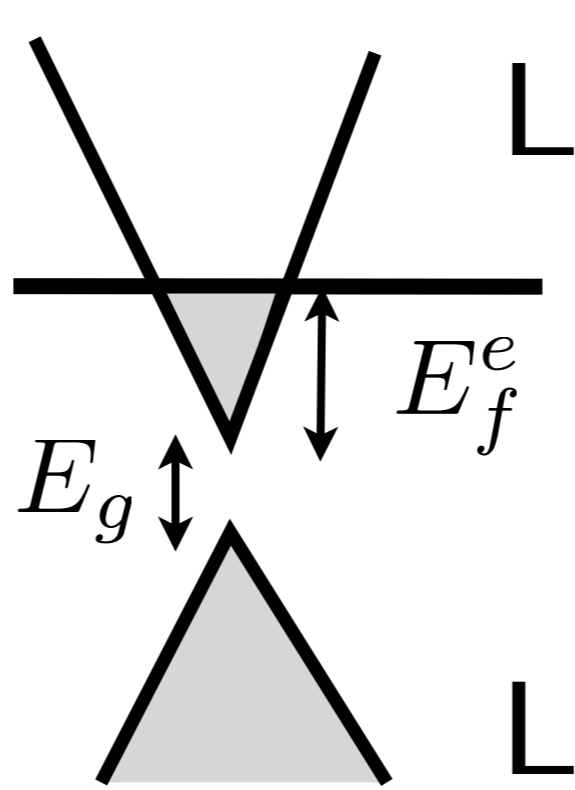
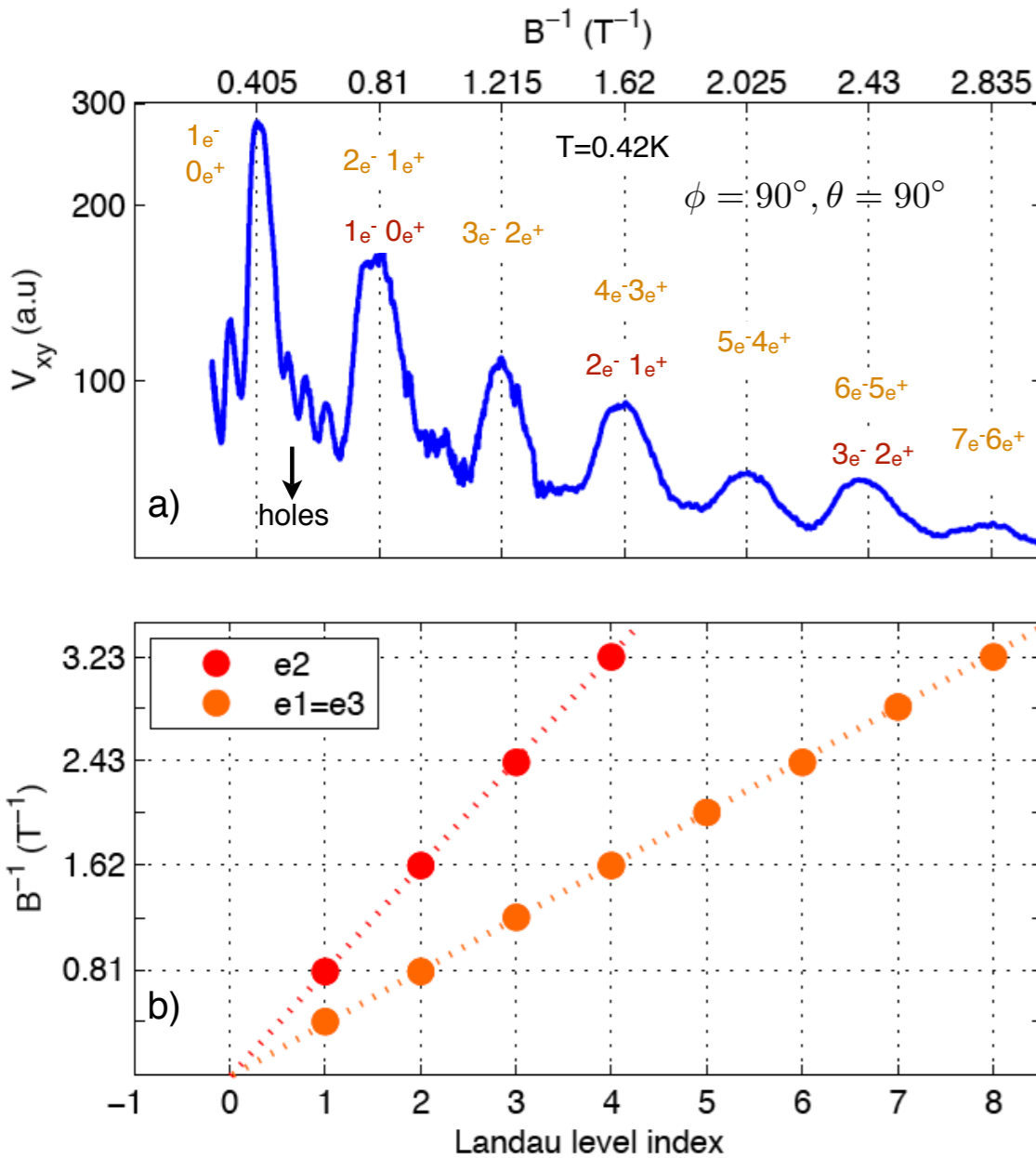
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 $n_{e1} + n_{e2} + n_{e3} = n_h \neq n(0T)$

Dirac electrons in Bismuth : L-point

Z.Zhu and al., PRB 84,115137 (2011)



At the L-point:
two bands with a small gap
+ SO interaction



Dirac like dispersion !

$$\mathcal{S}_F = \frac{2\pi}{\ell_B^2} (n + \gamma),$$

Schrödinger electrons : ($\gamma=1/2$)

$$E_{N,k_z} = (N + \frac{1}{2})\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m},$$

Dirac electrons : ($\gamma=0$)

$$E_{N,k_z} = \sqrt{(m_D v^2)^2 + \left(\frac{\sqrt{2N}\hbar v}{\ell_B}\right)^2 + (\hbar v k_z)^2},$$

Thermoelectrical response beyond the quantum limit of 3D electron gas systems

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Nernst effect as a probe of quantum oscillations in semi-metals : the case of bismuth and graphite

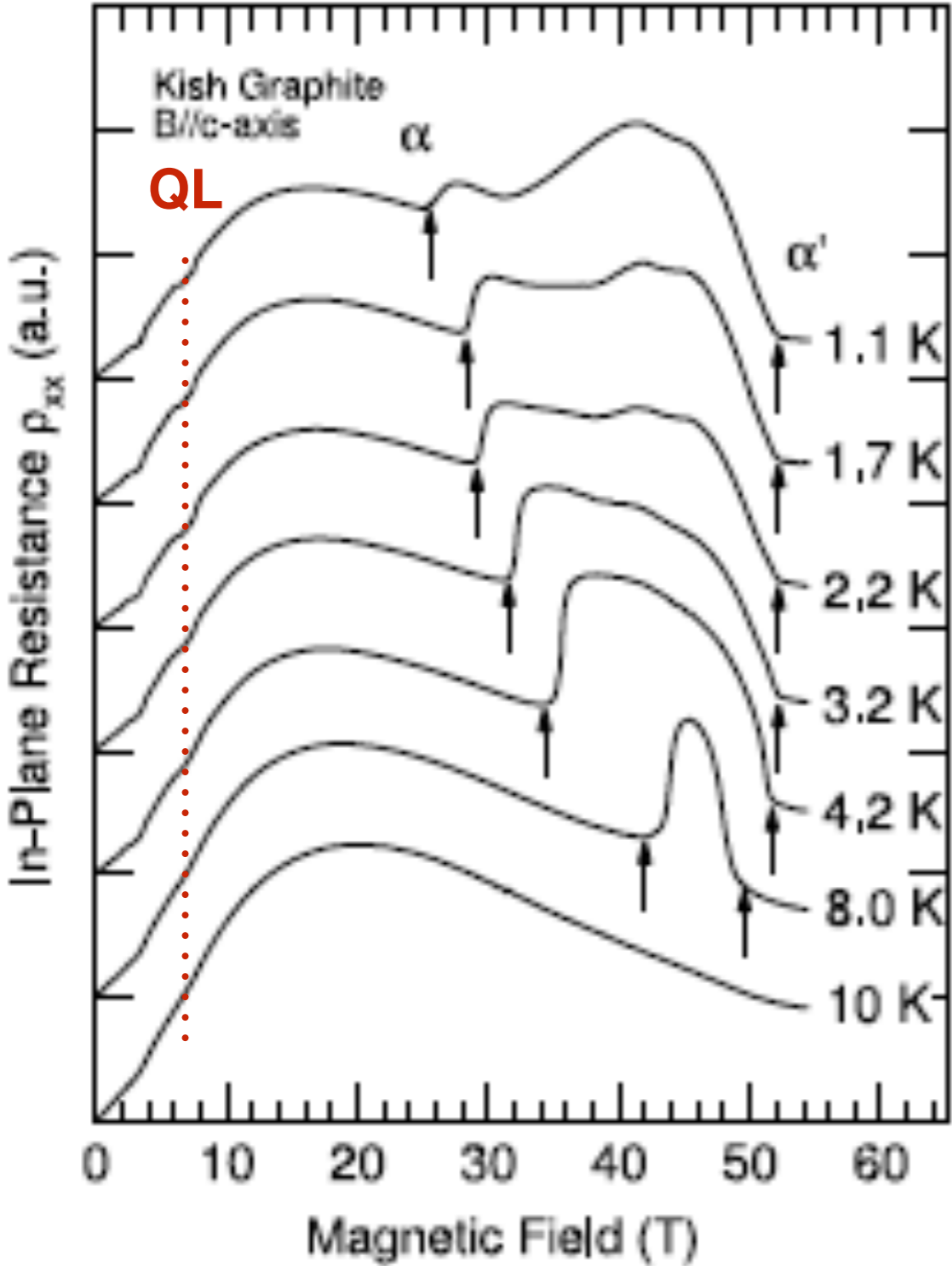
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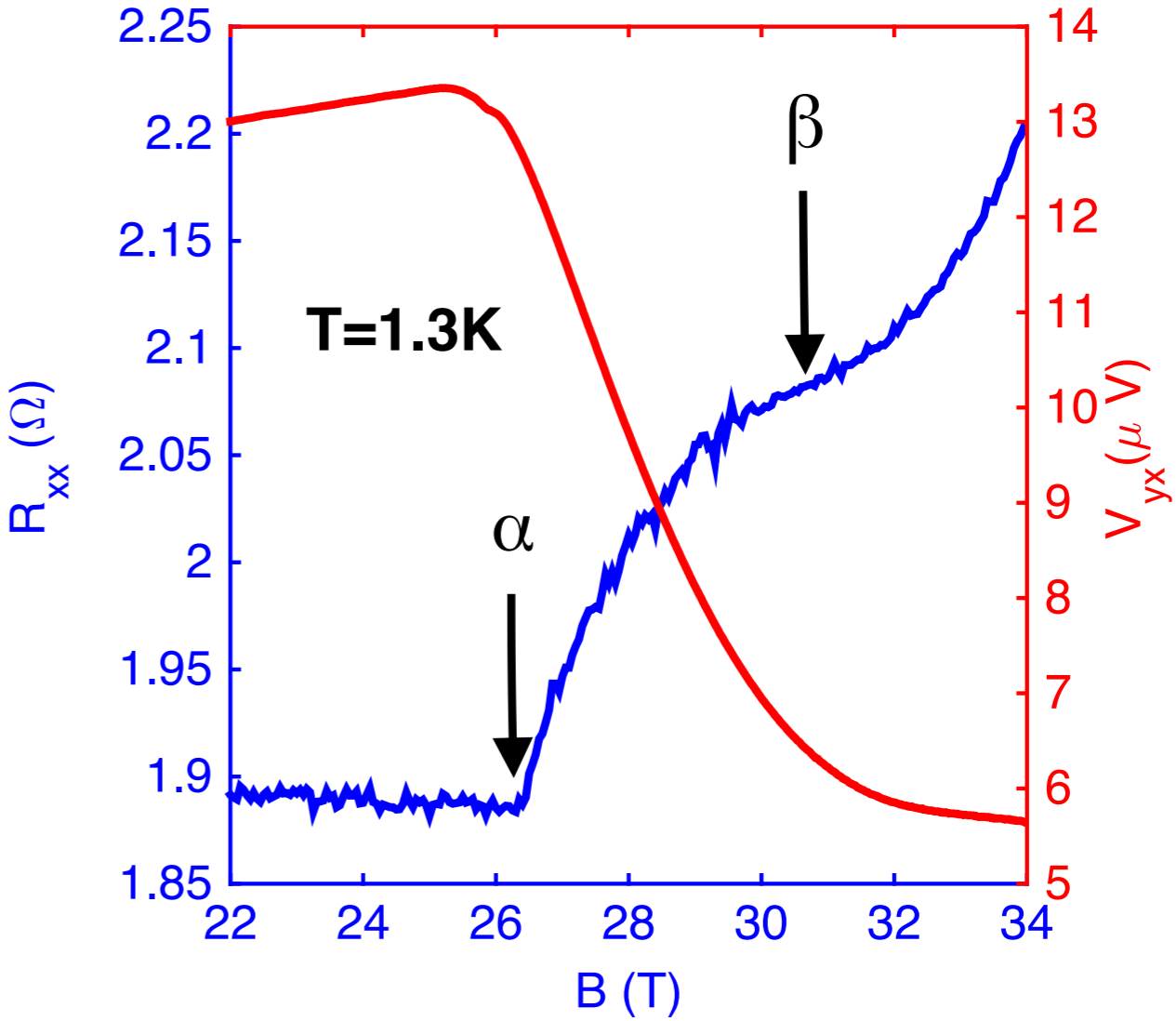
Conclusion

Graphite in the quantum limit regime

H. Yaguchi et al, PRL **81**,5193 (1998)

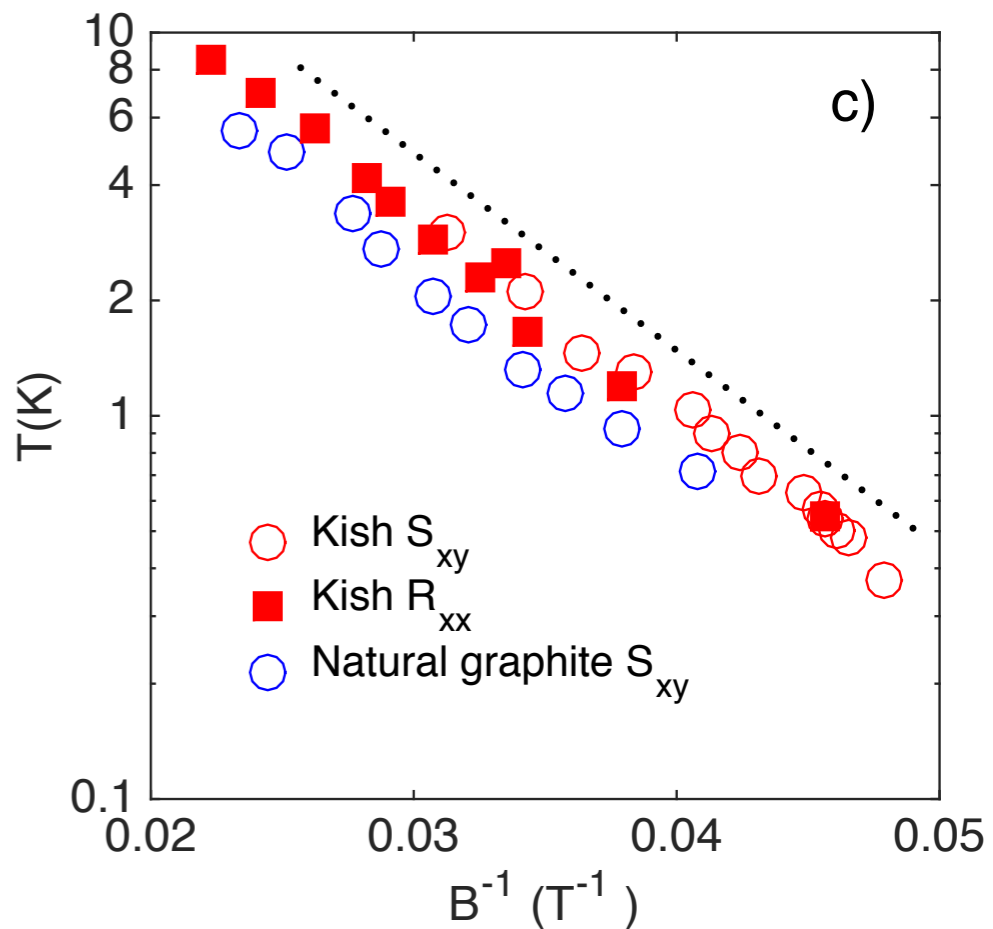
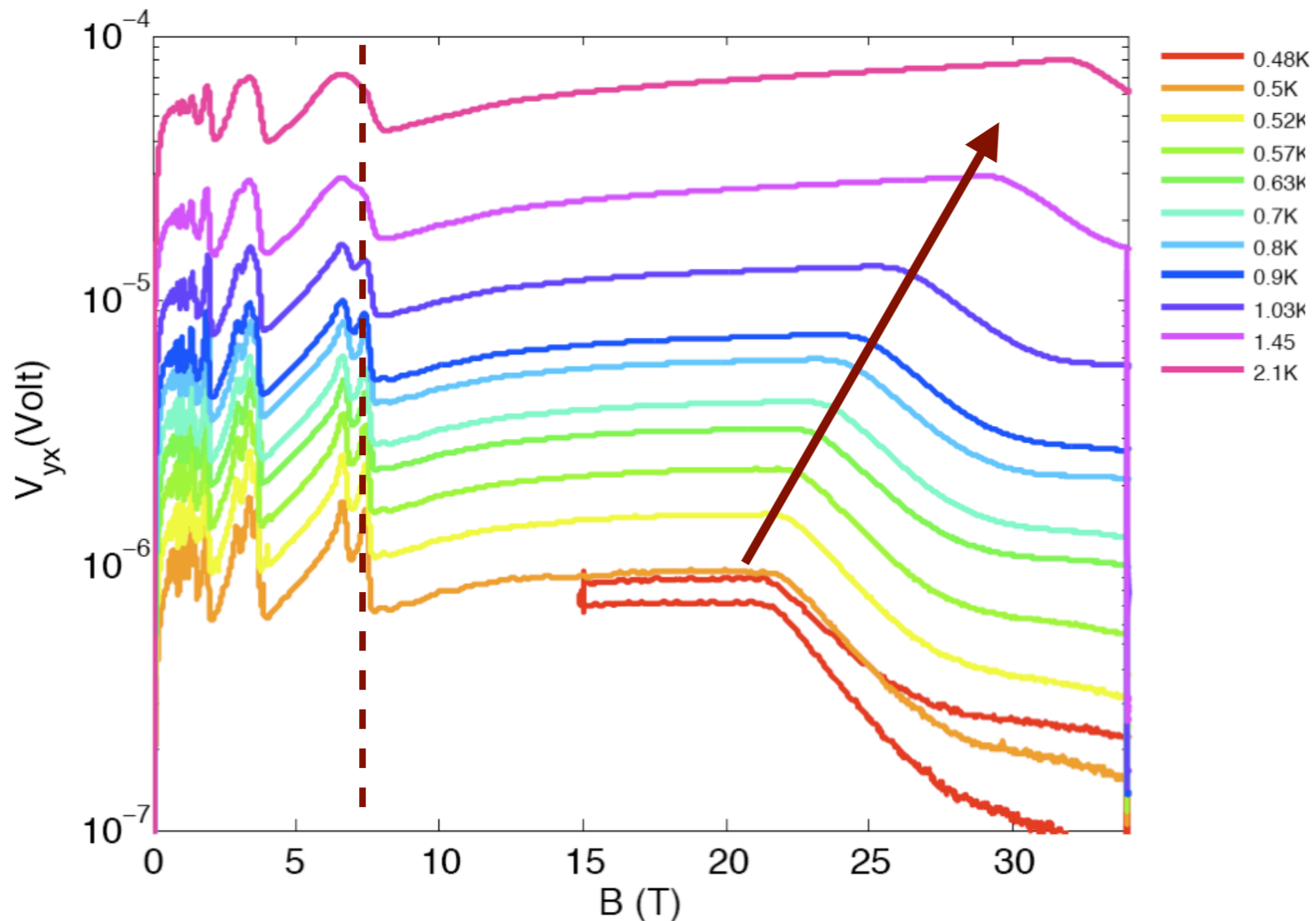


B.Fauqué et K.B,
Basic Physics of Functionalized Graphite,
Springer (2016)



Graphite in the quantum limit regime

B. Fauqué et al, PRL, 106, 246405 (2011)



BCS-type formula for mean-field-type pairing

$$k_B T_c(B) = 1.14 E_F \exp\left(-\frac{1}{N(E_F)V}\right)$$

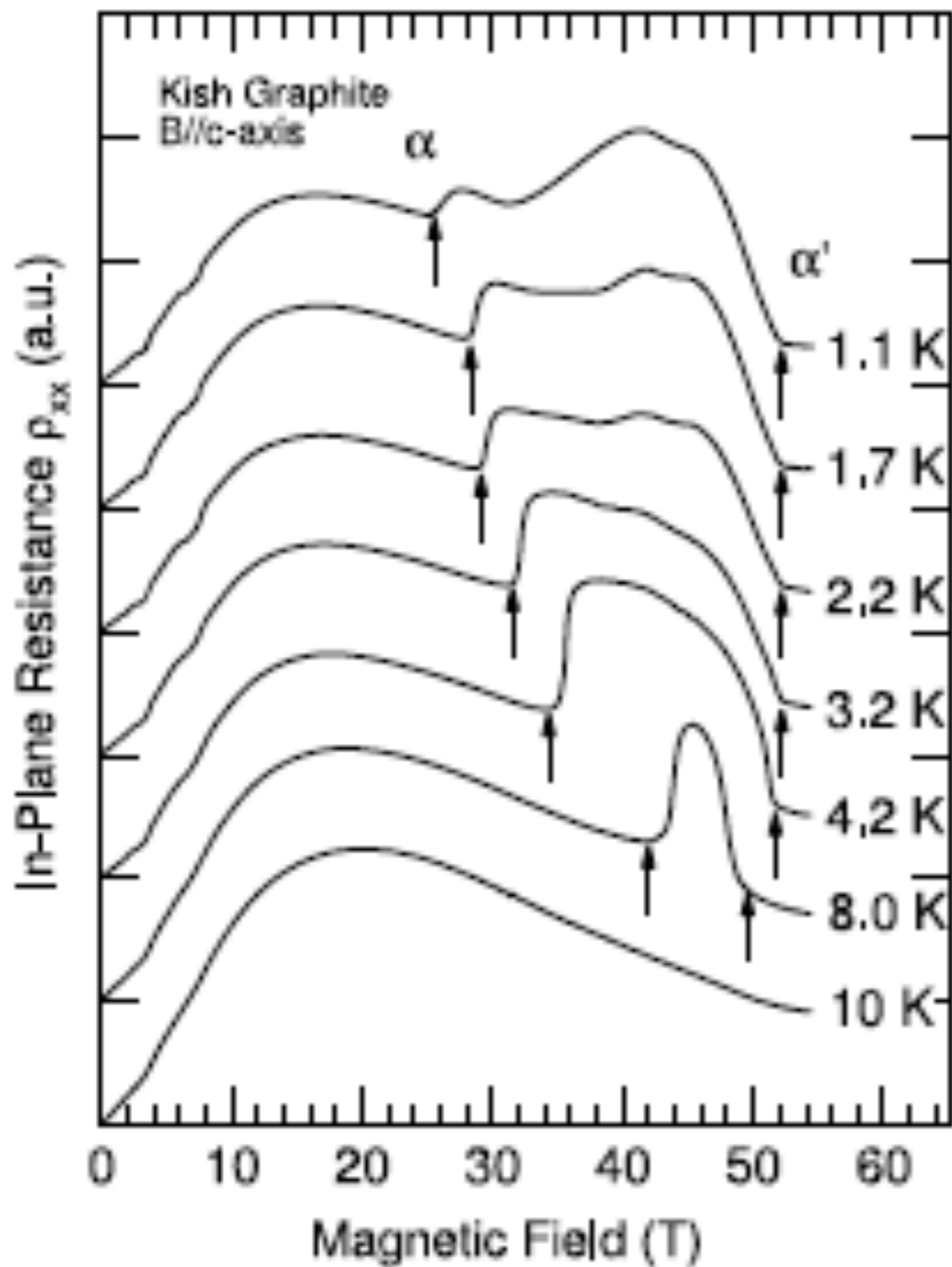
$N(E_F)$ scales with B the Landau level degeneracy

$$T_c = T^* \exp\left(-\frac{B^*}{B}\right)$$

$$T^* = 80\text{K} \sim T_F$$

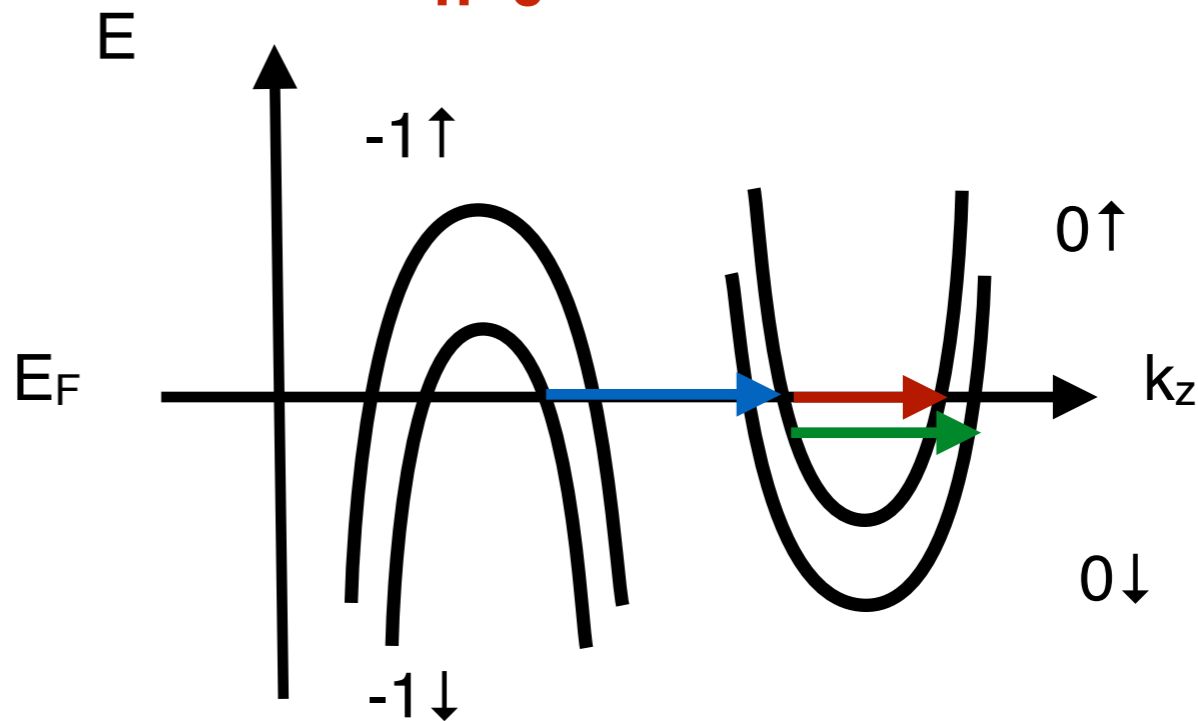
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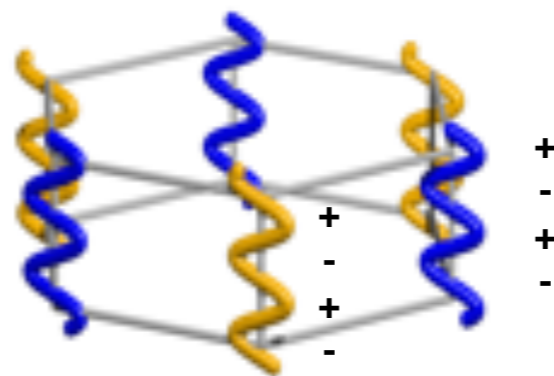


$$E_{n,s} = \left(\chi + \frac{1}{2} \right) \hbar \omega_c + \frac{1}{2} \frac{k_z^2}{m} \pm \frac{1}{2} g \mu_B B$$

n=0



1981 Yoshioka-Fukuyama : **CDW** in the (0,↑)
CDWs are out of phase in each valley to minimize the Coulomb interaction



« Valley Density Wave »
Z. Tezanovic et al.,
Phys. Rev. B **36**,488(1987)

1994 : Takahashi-Takada : transverse **SDW** forms in the (n=0, ↓↑)

2015 : K. Akiba et al., arXiv:1503.04414

excitonic scenario

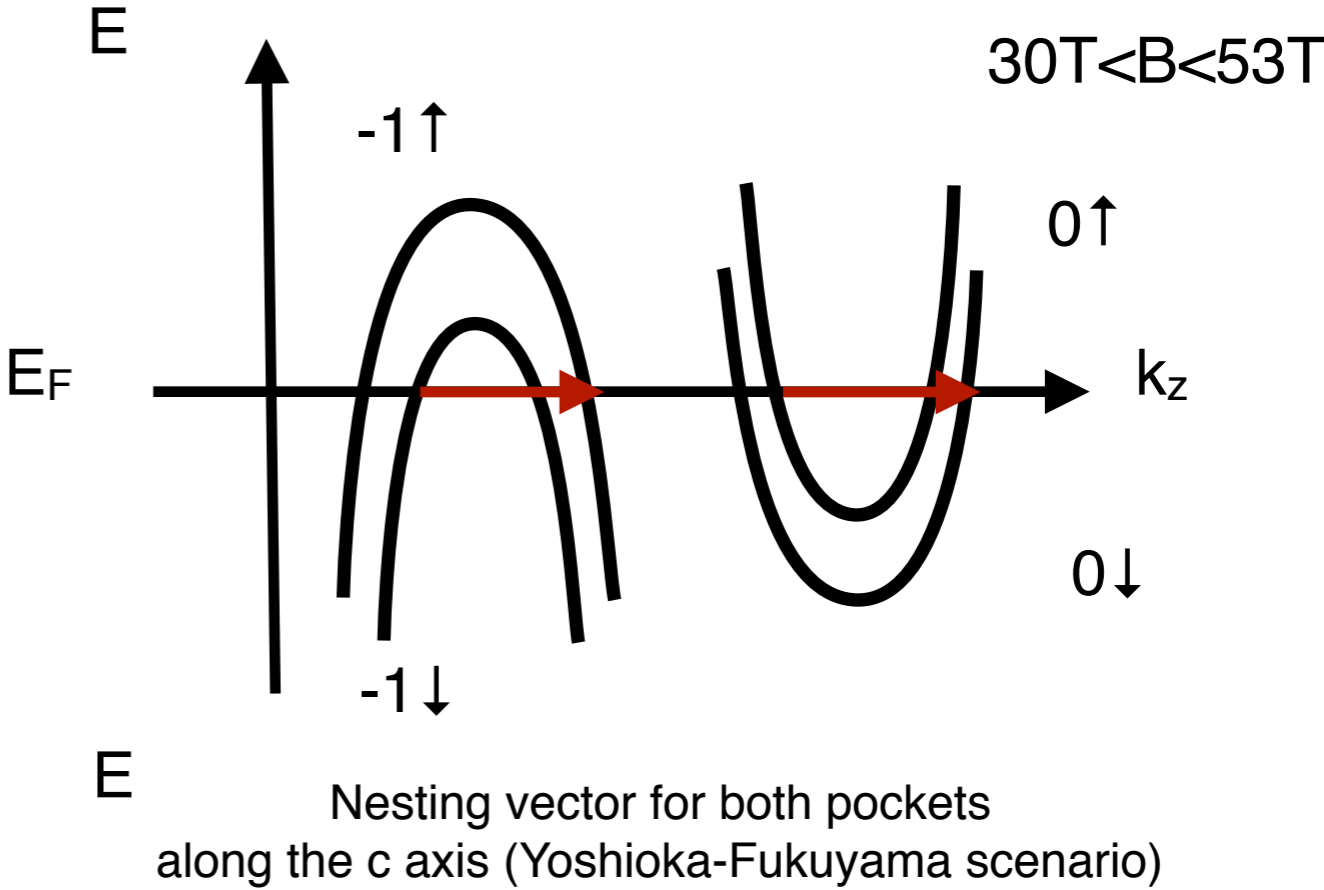
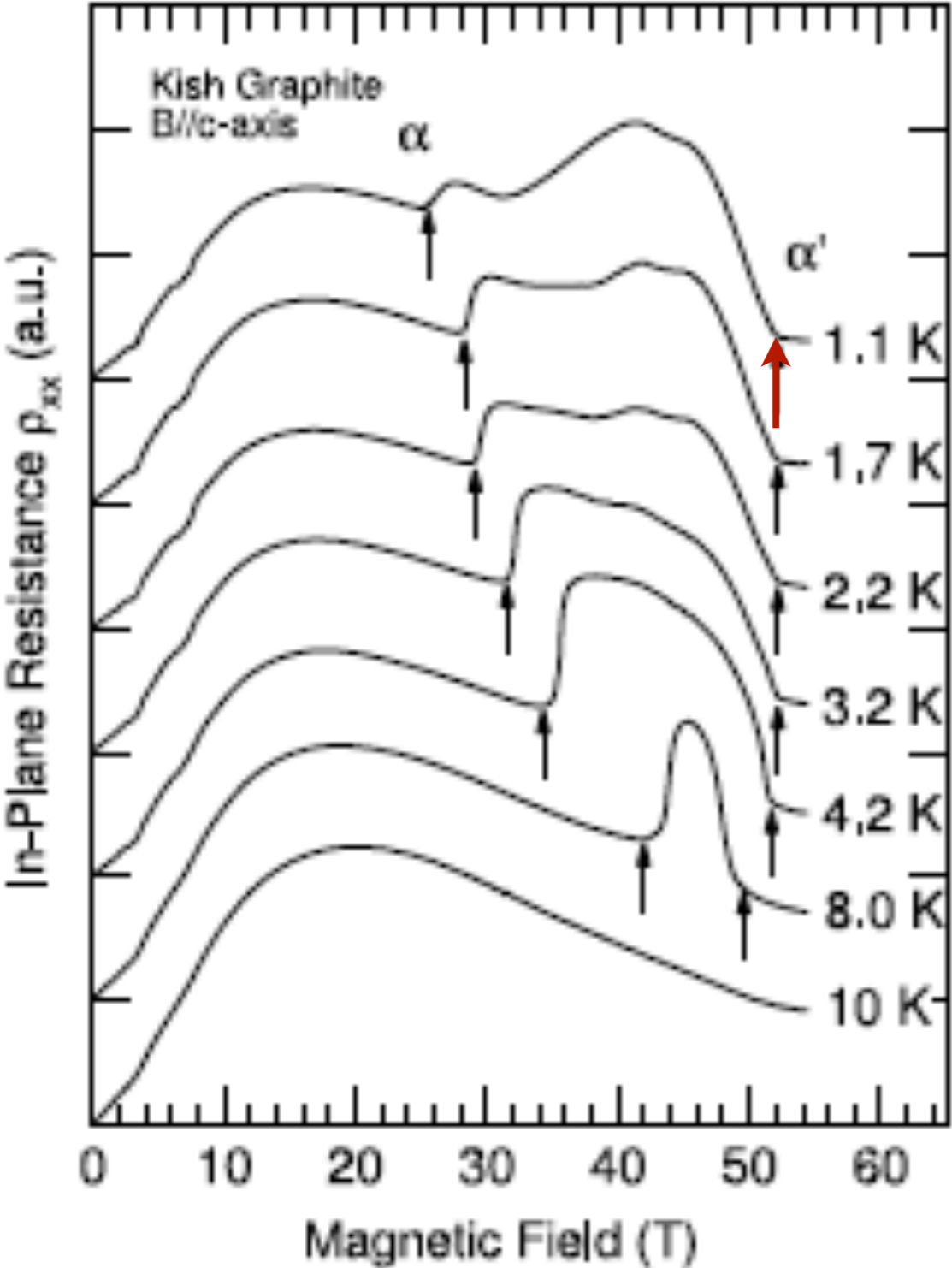
J. Phys. Soc. Jpn. 84, 054709-1-6 (2015)

Graphite in the quantum limit regime

$$E_{n,s} = \left(\mathbf{x} + \frac{1}{2} \right) \hbar \omega_c + \frac{1}{2} \frac{k_z^2}{m} \pm \frac{1}{2} g \mu_B B^2$$

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H. Yaguchi et al, PRL **81**,5193 (1998)

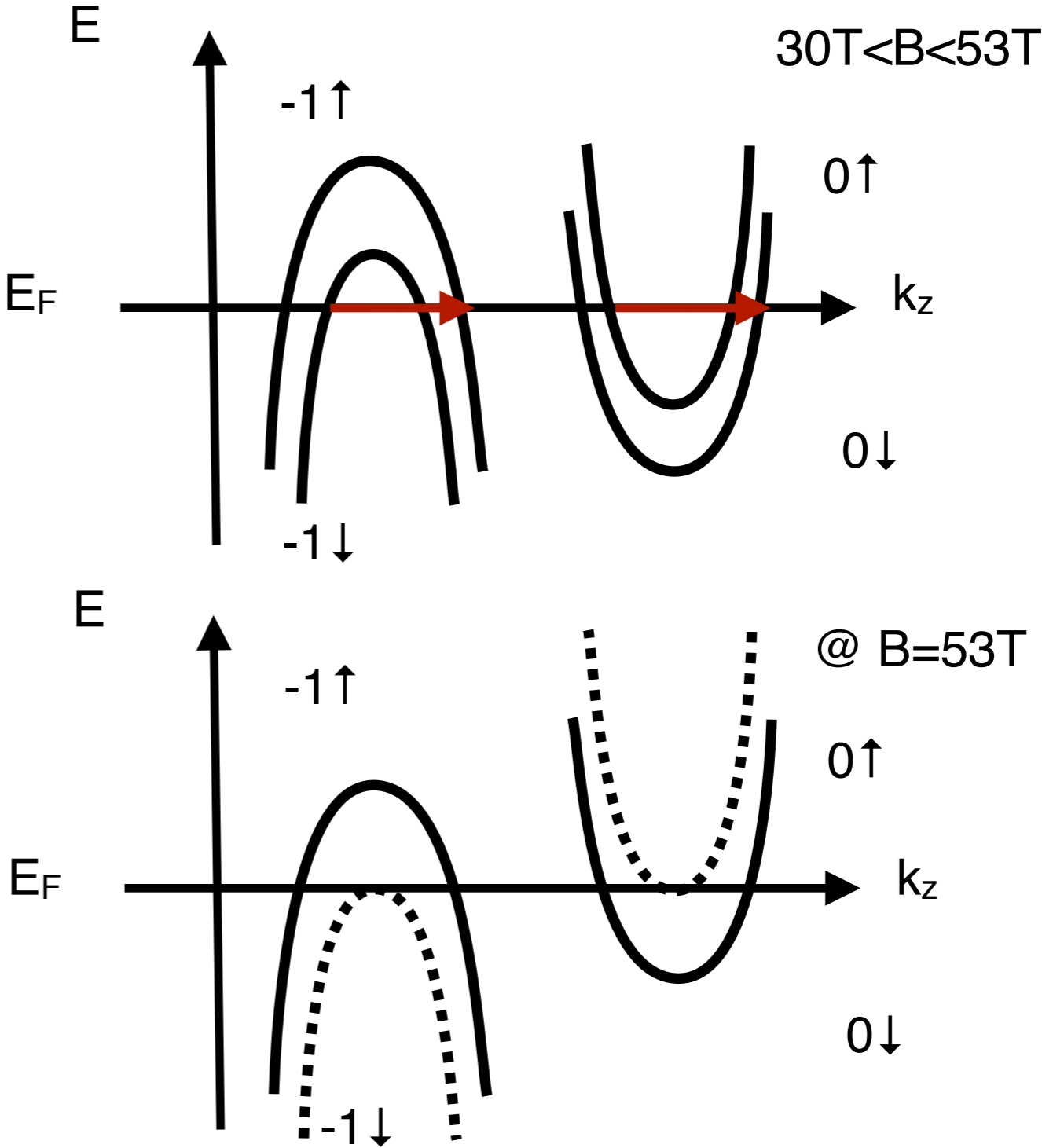
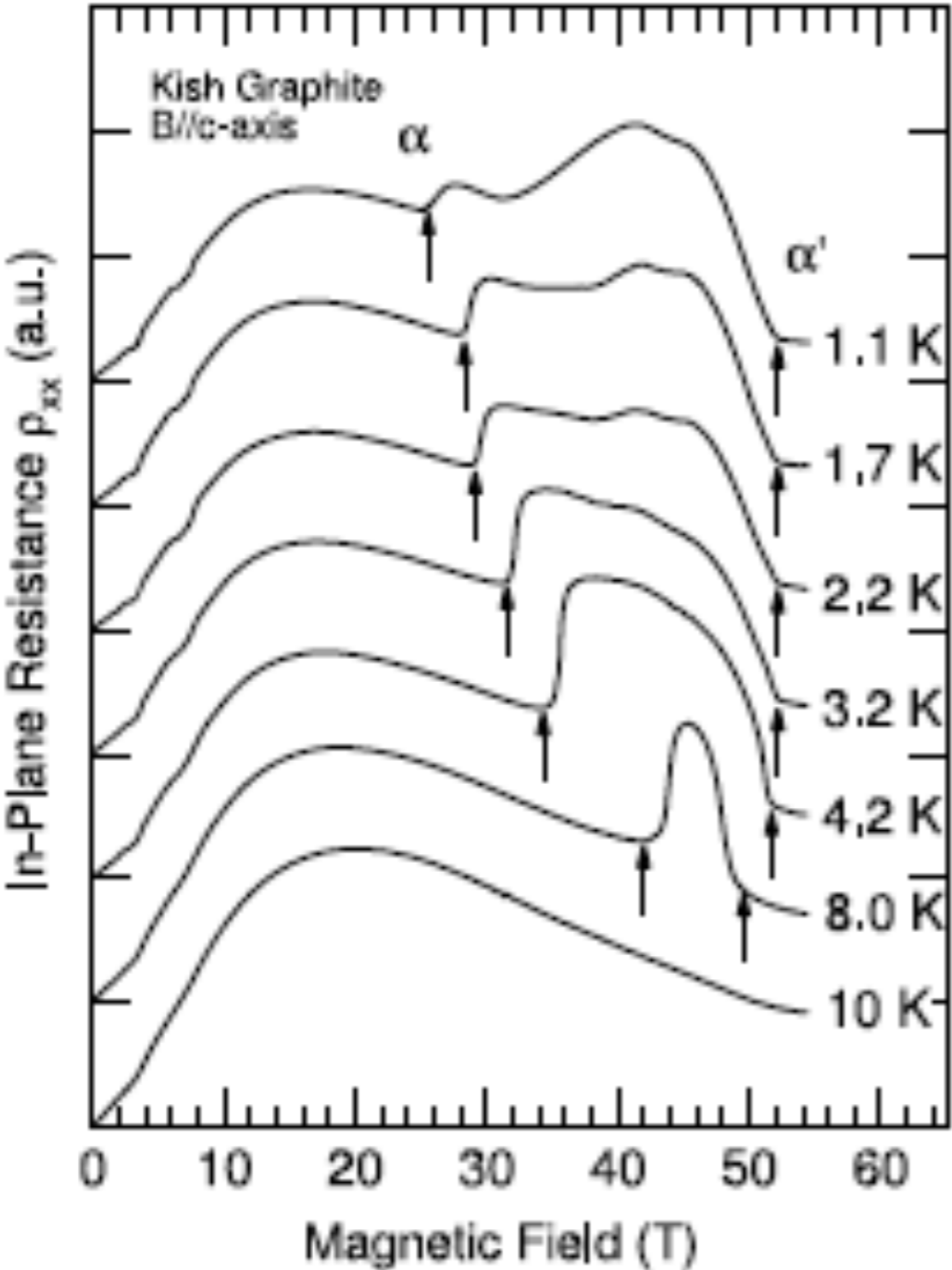


Graphite in the quantum limit regime

$$E_{n,s} = \left(\mathbf{x} + \frac{1}{2} \right) \hbar \omega_c + \frac{1}{2} \frac{k_z^2}{m} \pm \frac{1}{2} g \mu_B B^2$$

n=0

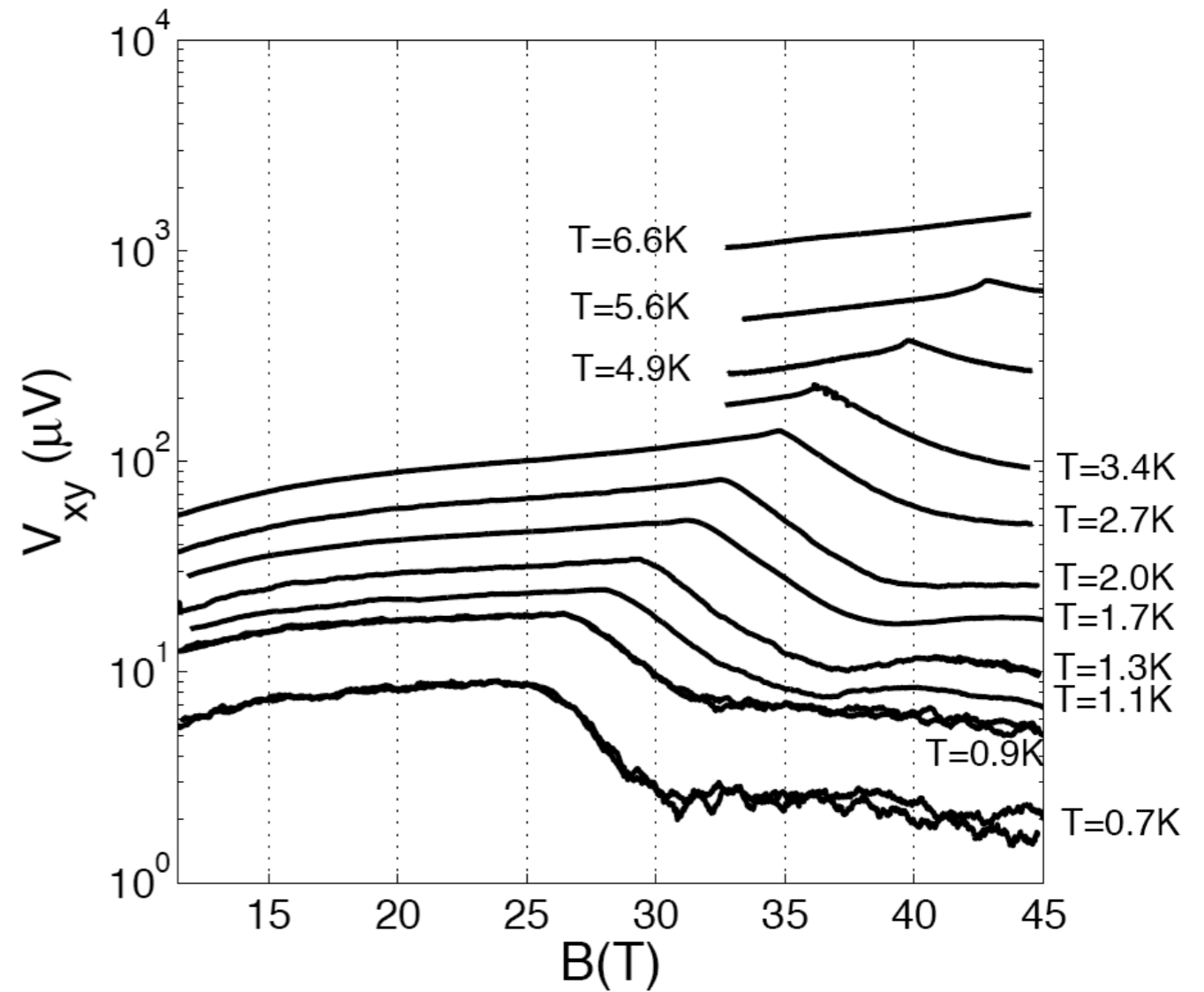
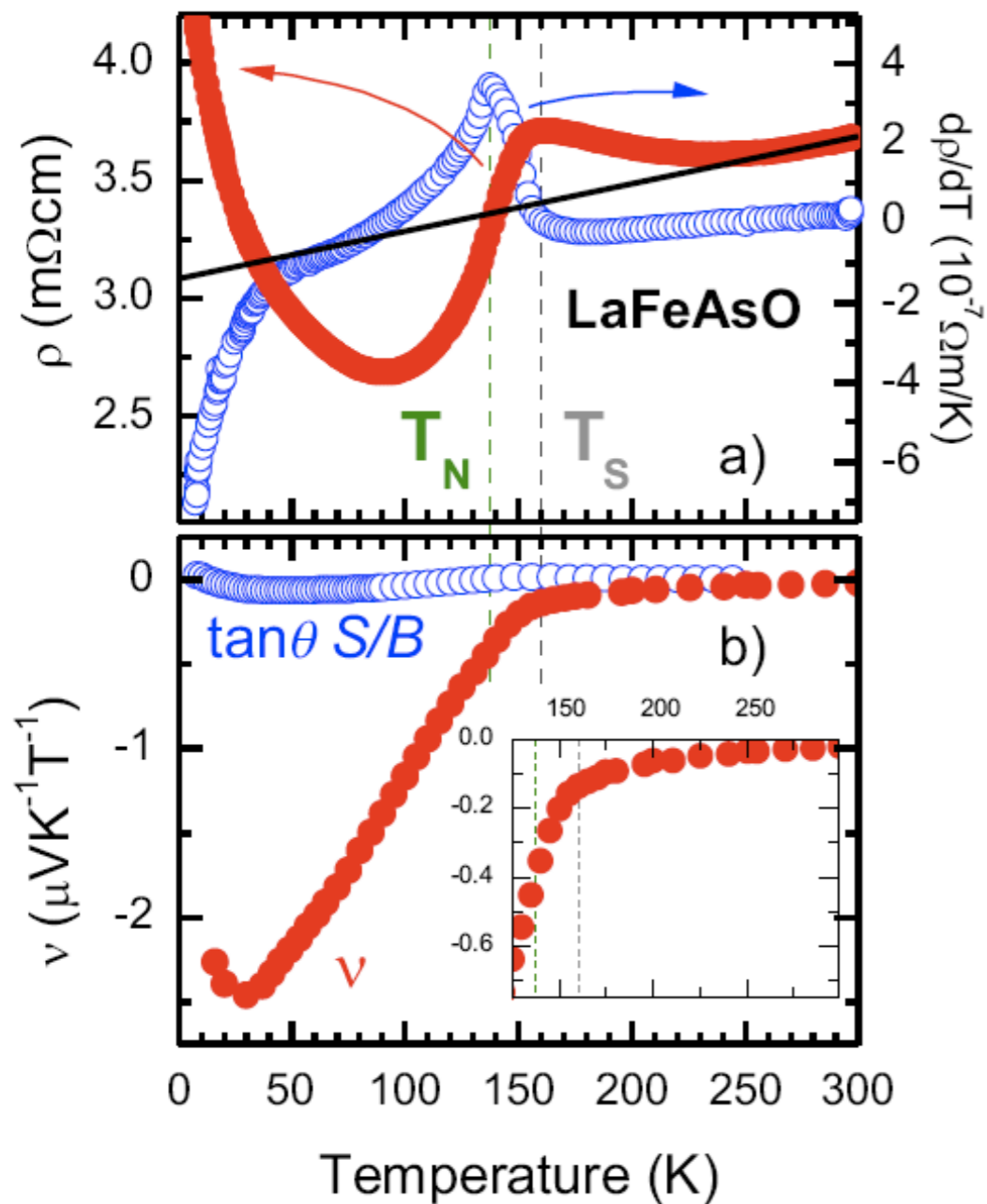
H. Yaguchi et al, PRL **81**,5193 (1998)



The re-entrance transition is associated with the crossing of the Landau levels $(0, \uparrow)$ of electrons and $(-1, \downarrow)$ of holes .

Graphite: origin of the field induced electronic instability ?

C. Hess, arxiv/1202.2959



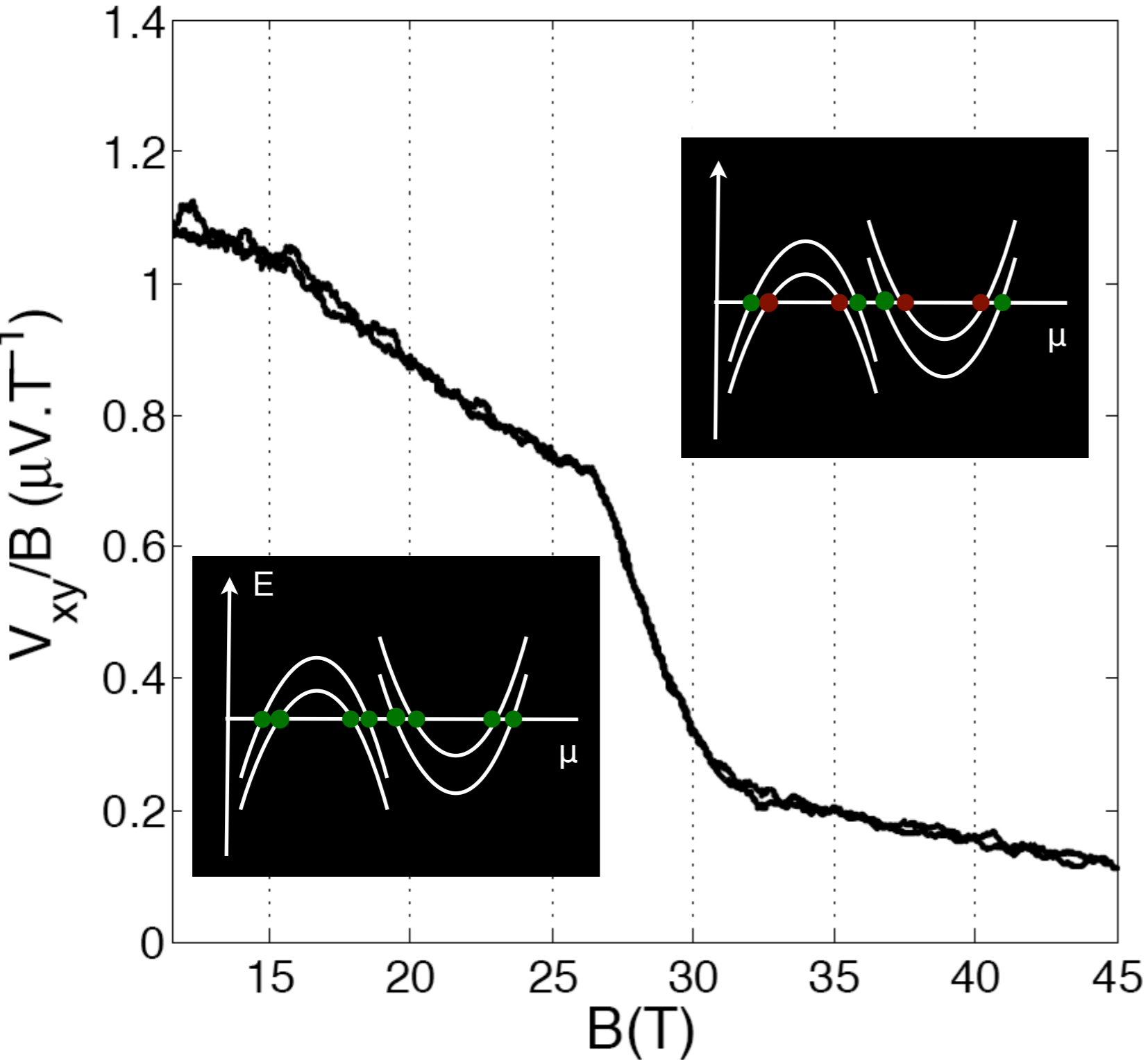
see also the case :

URu_2Si_2 : R. Bel et al., PRB **70**, 220501(R) (2004)

$\text{PrFe}_4\text{P}_{12}$: A.Pourret, PRL **96**, 176402 (2006)

Can a DW decrease the Nernst response in the QL regime?

Graphite: origin of the field induced electronic instability ?



$$E_{n,s} = \left(n + \frac{1}{2}\right)\hbar\omega_c + \frac{1}{2}\frac{k_z^2}{m} \pm \frac{1}{2}g\mu_B B^2$$

When the chemical potential is far from the bottom of one of the Landau level, the Nernst effect is dominated by the green spots of each of the (full) Landau levels.

In a presence of DW, we lost the contribution in the Nernst response of the Landau levels where the gap is opening

The DW scenario in the QL regime can explain the decrease of the Nernst coefficient

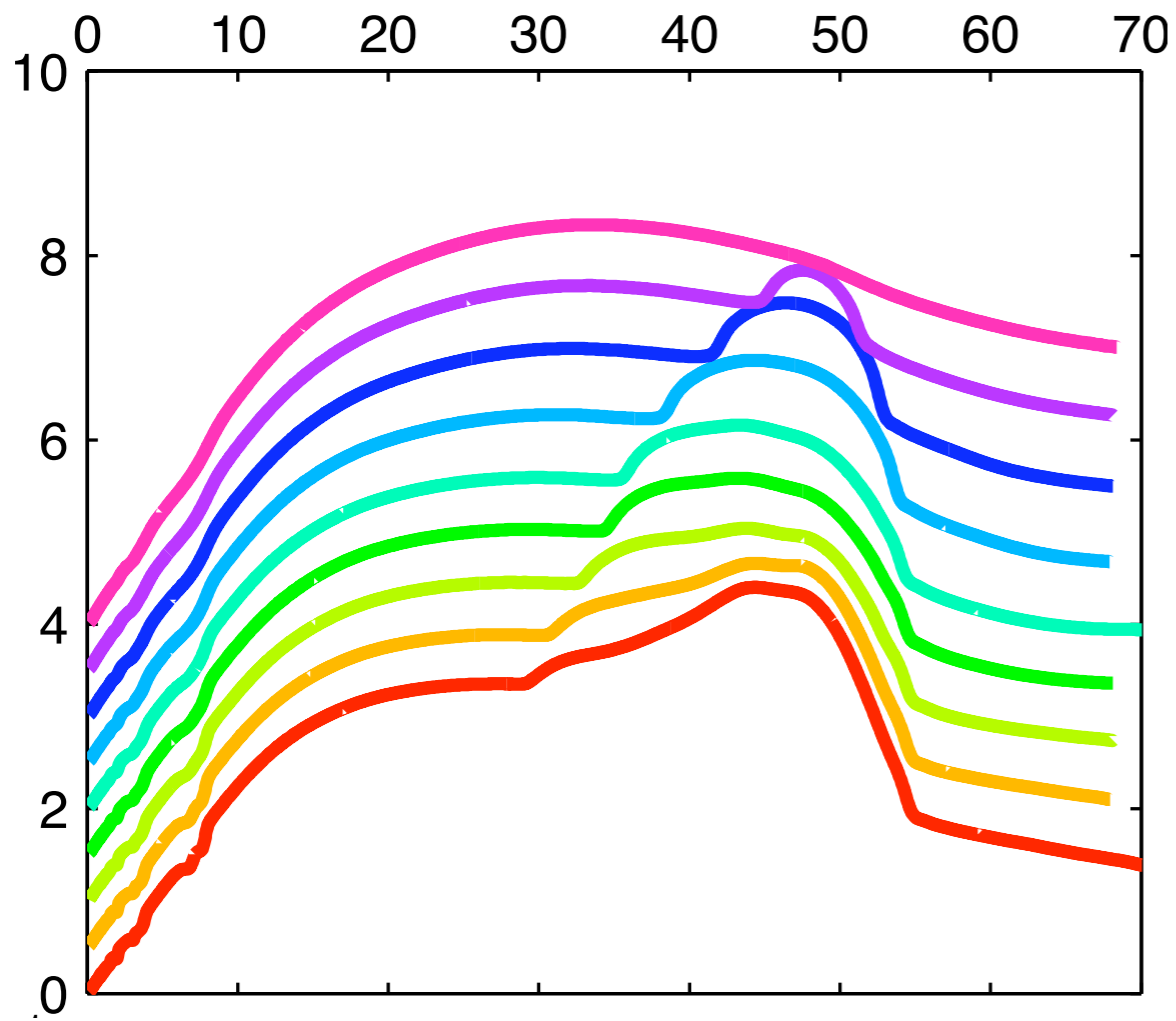
Electrical transport : R_{xx} vs R_{zz}

$B//c$



$j//(a,b)$

B (T)

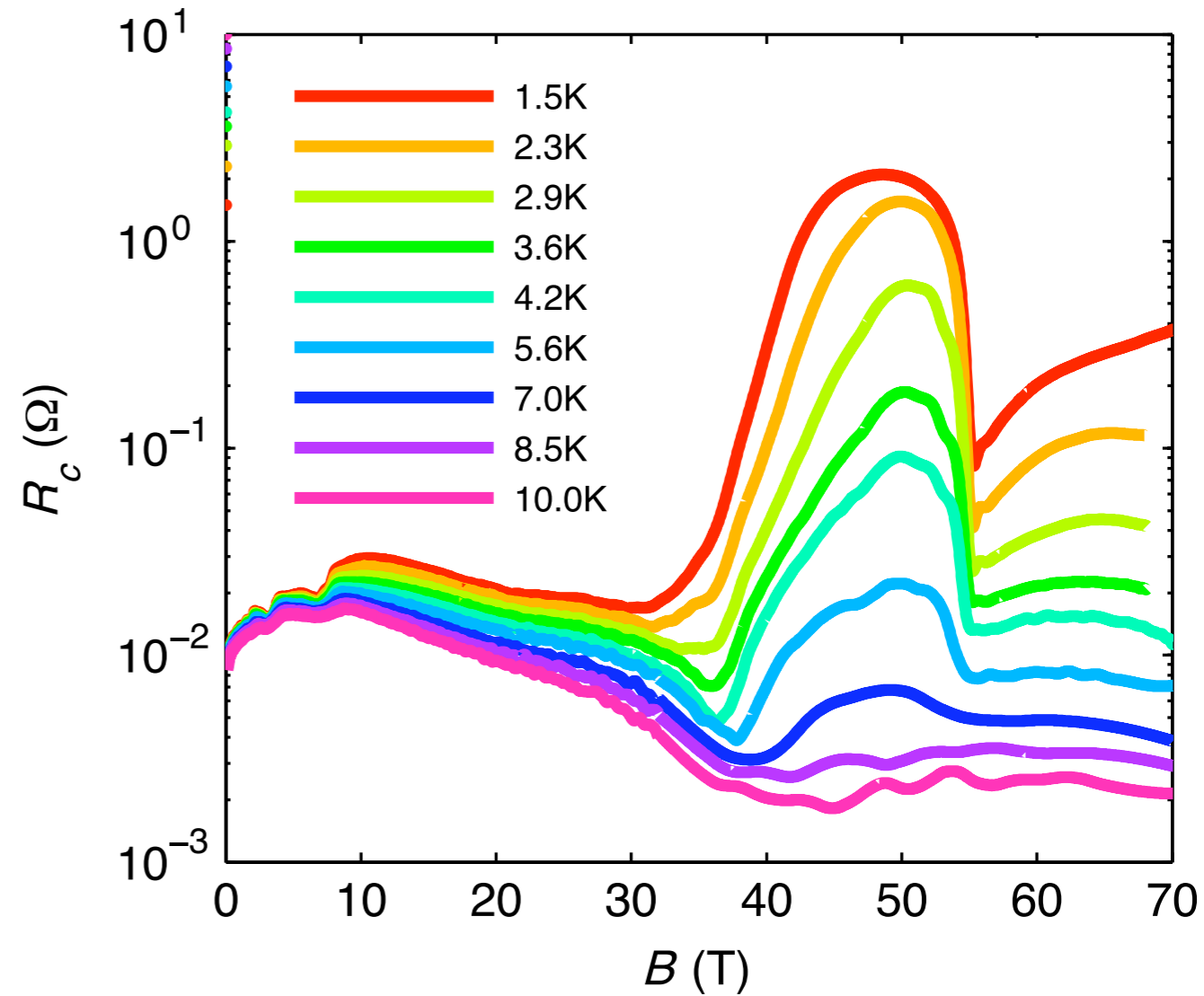


Kish Graphite

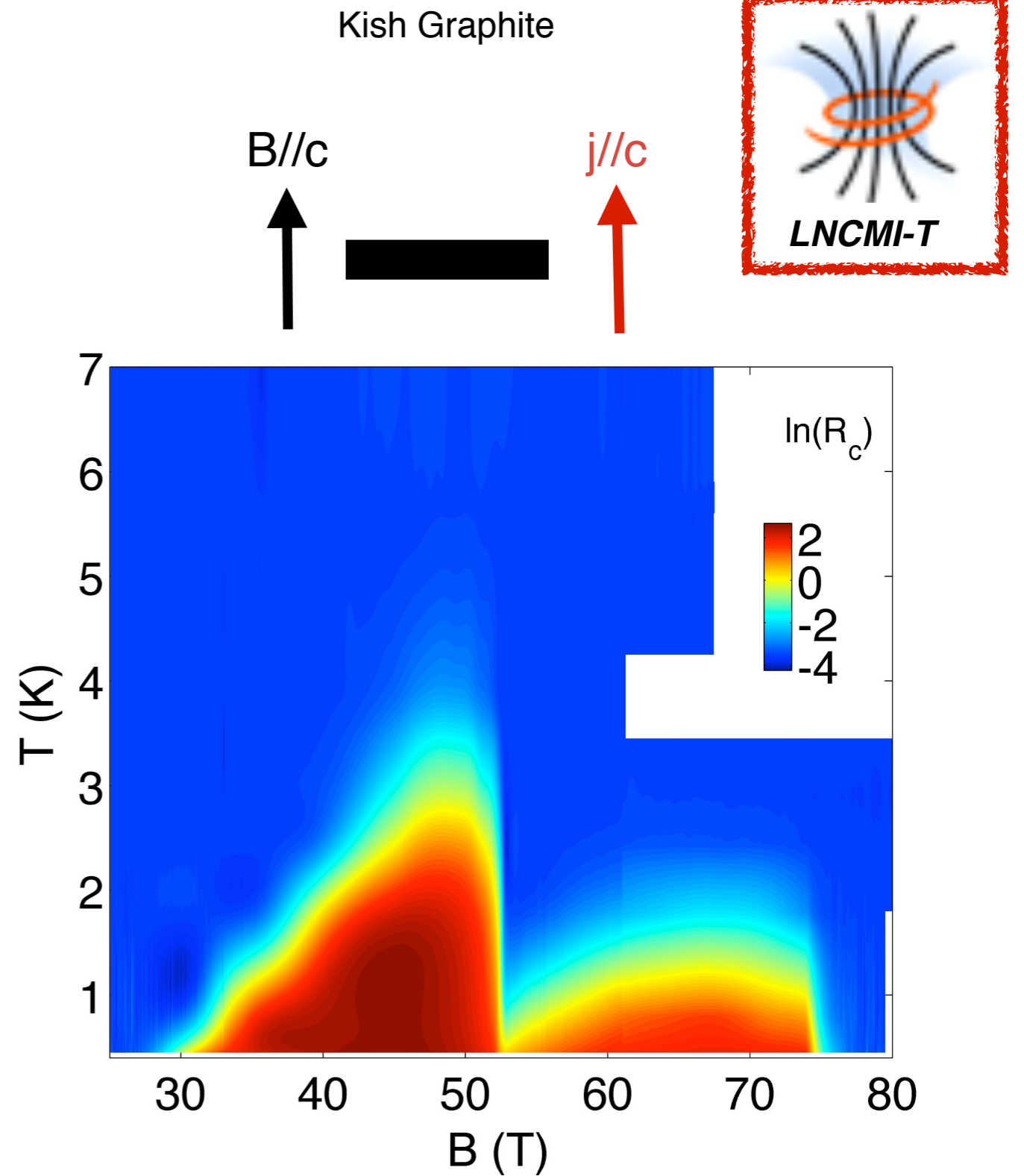
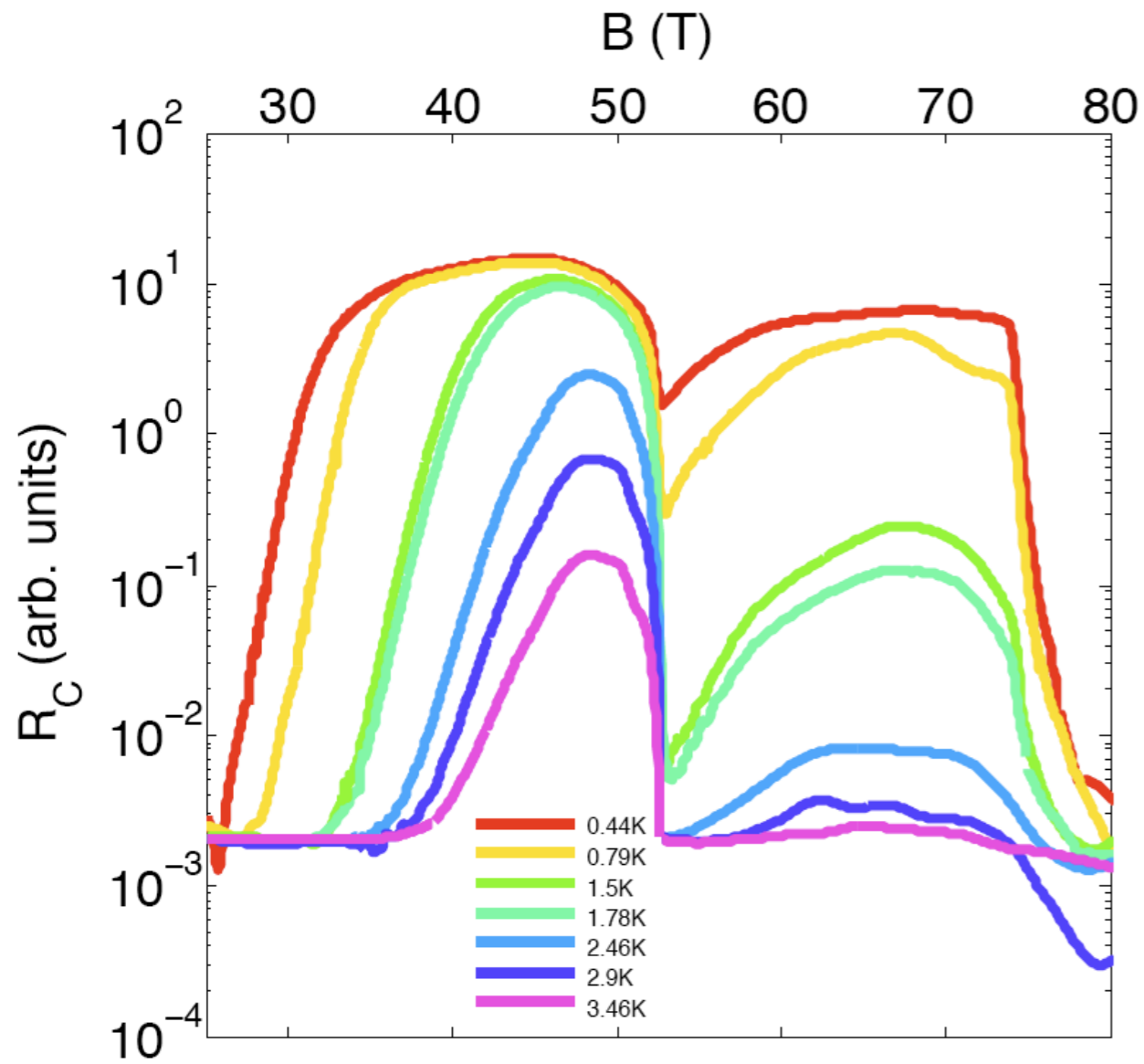
$B//c$



$j//c$



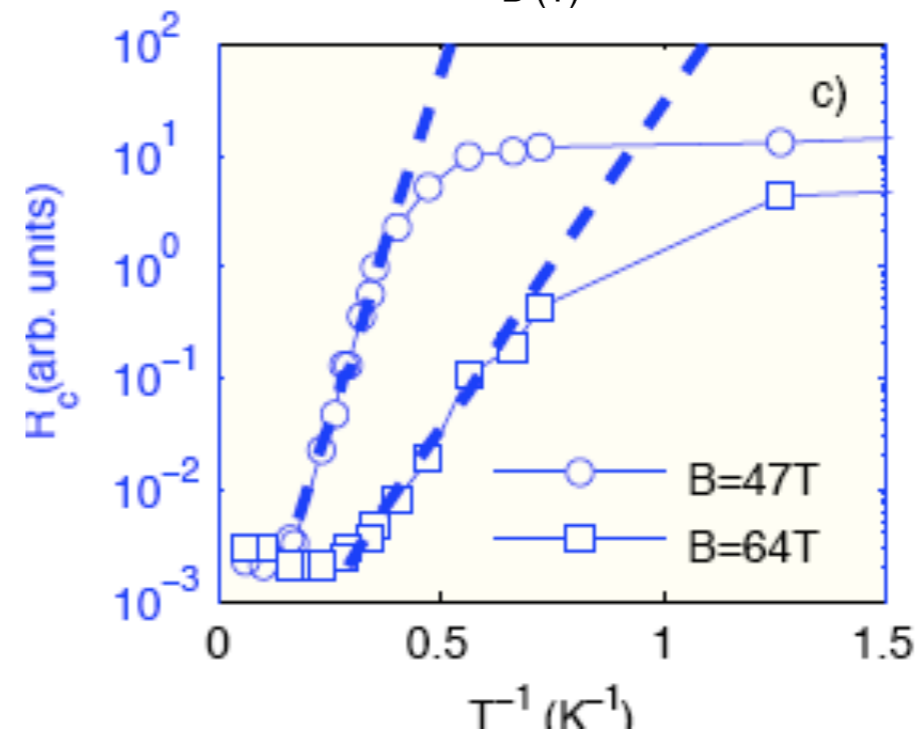
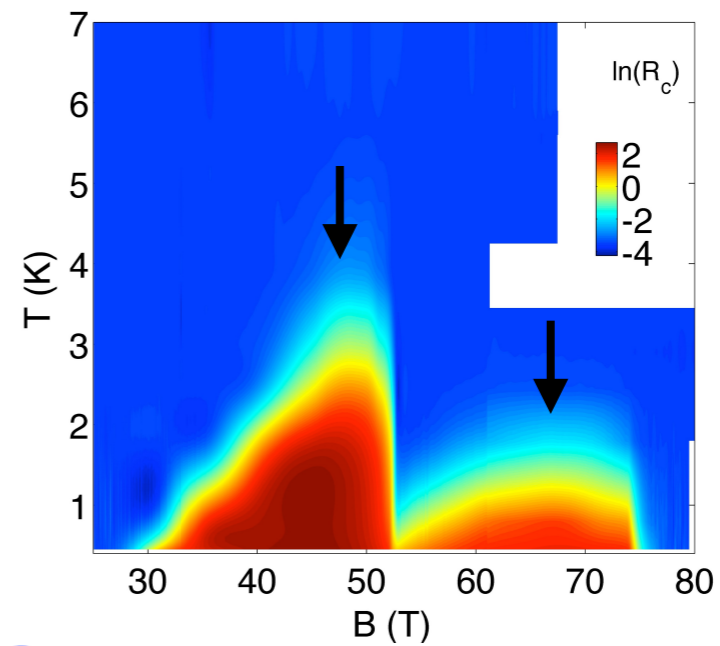
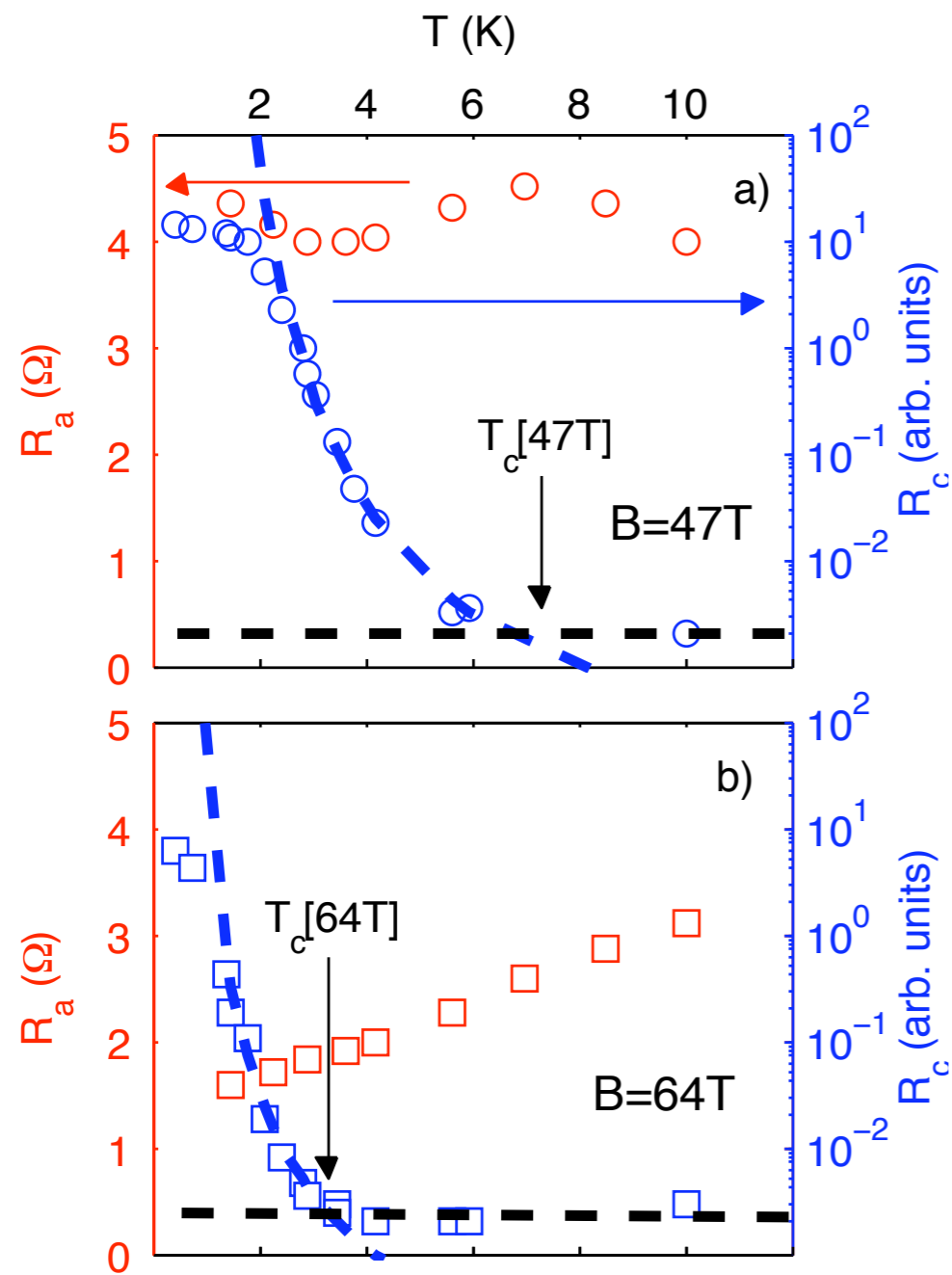
Electrical transport : R_{zz}



There is still a life above 53T !
A second transition induced by a magnetic field in Graphite

Electrical transport : R_{xx} vs R_{zz}

$$R_c \propto \exp\left(\frac{2\Delta}{k_B T}\right)$$

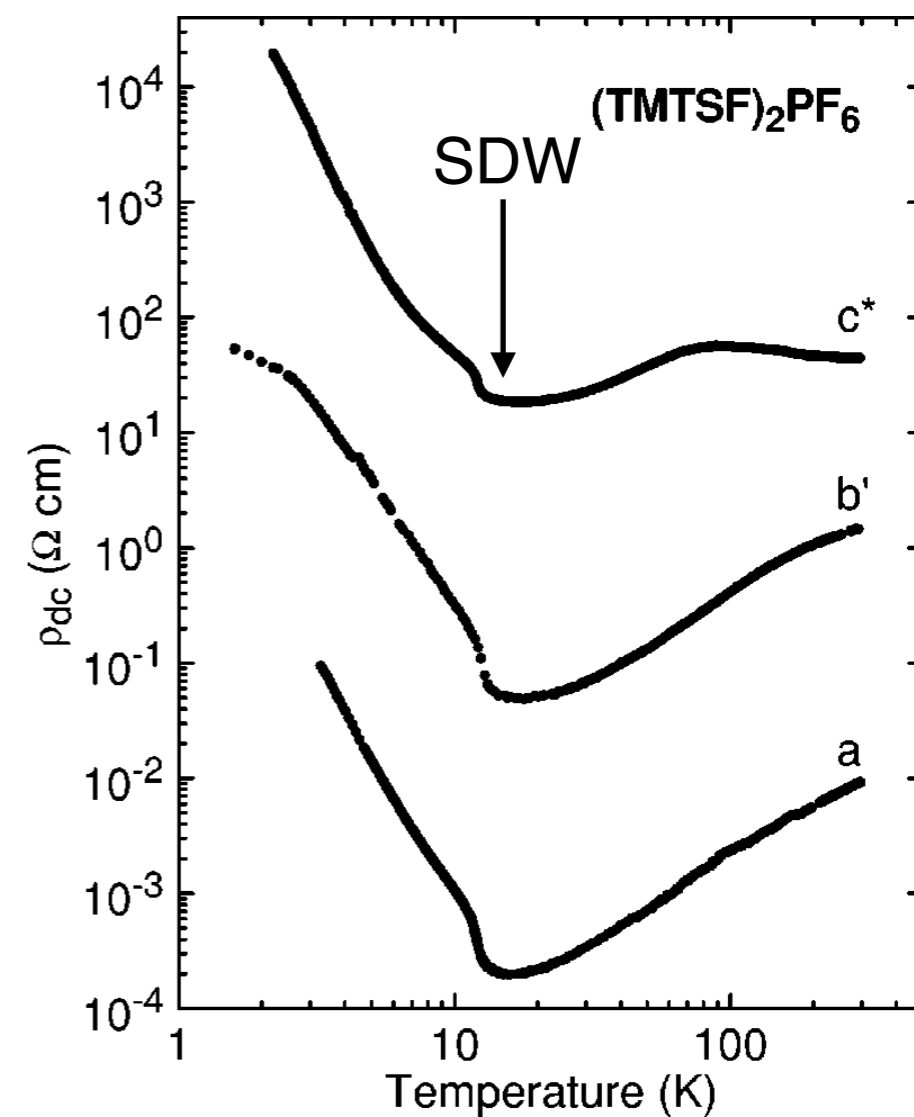
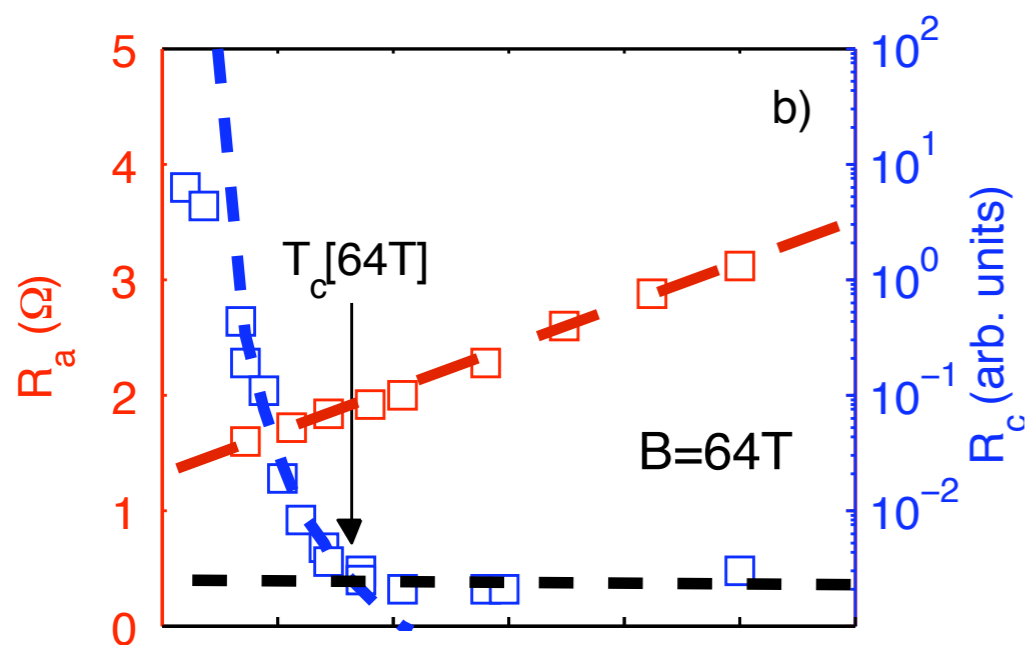
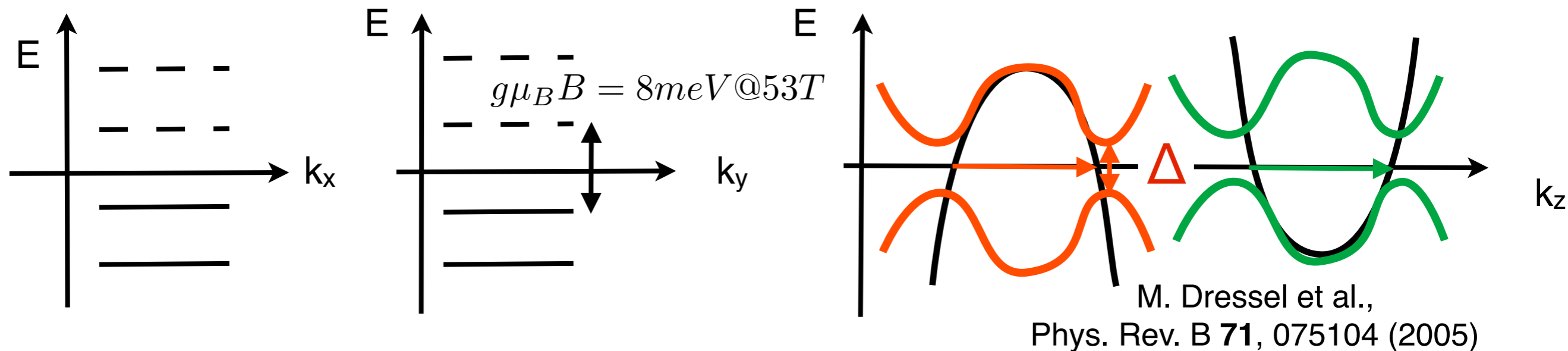


c-axis : all the Landau levels are gapped

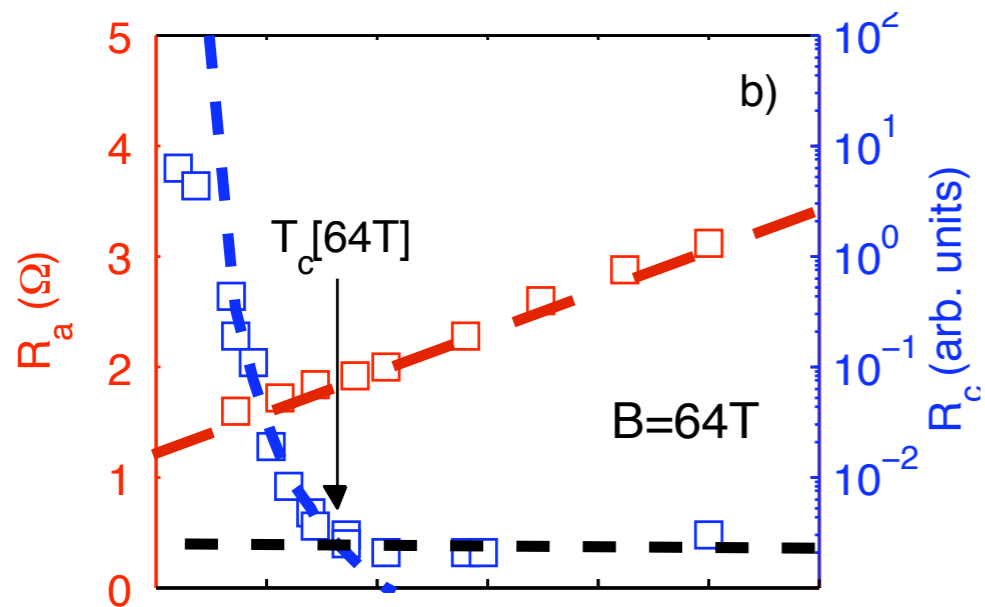
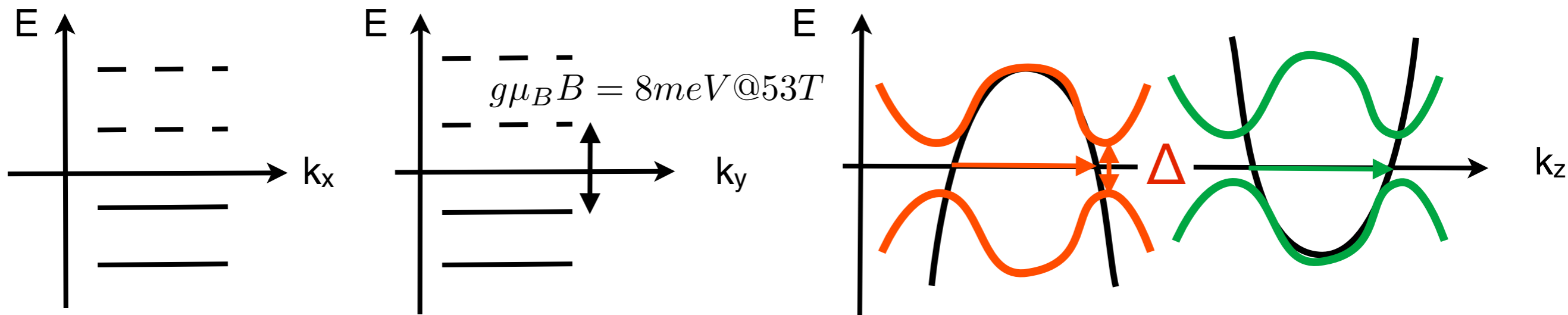
$$2\Delta[47T] = 2.4meV \quad 2\Delta[64T] = 1.1meV$$

Electrical transport : R_{xx} vs R_{zz}

Kish Graphite



Although the electronic spectrum in the two field induced states is gapped, the in-plane resistance is metallic !!



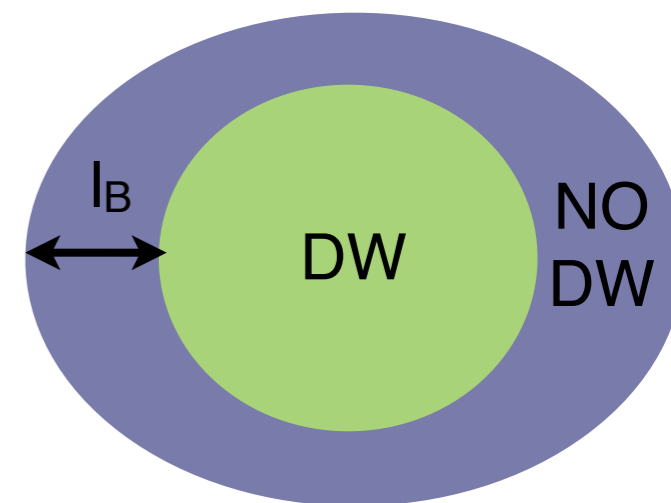
$$\sigma_{xx} = n * \sigma_{xx}^0$$

$$\sigma_{xx}^0 \approx \frac{e^2}{h}$$

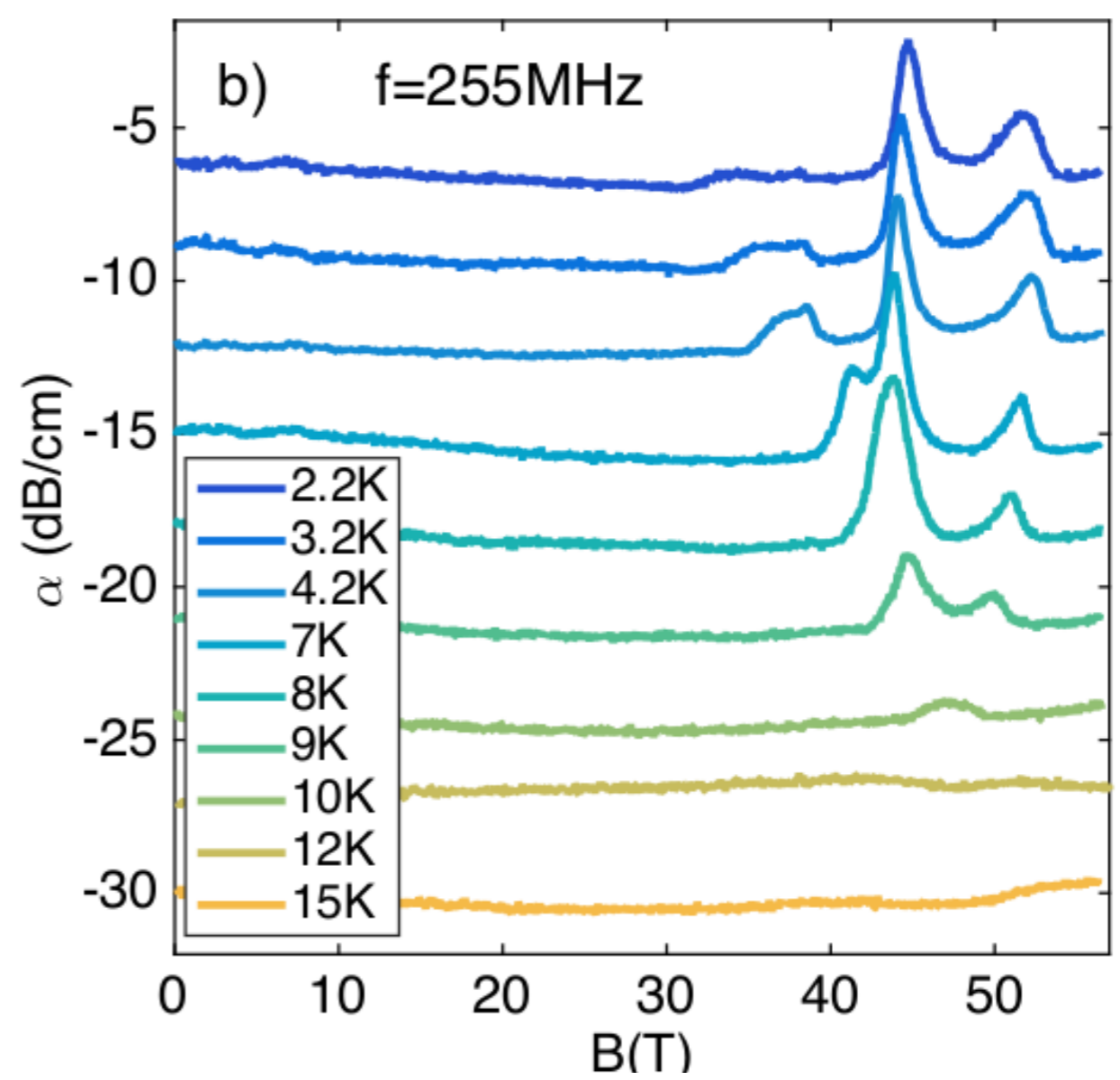
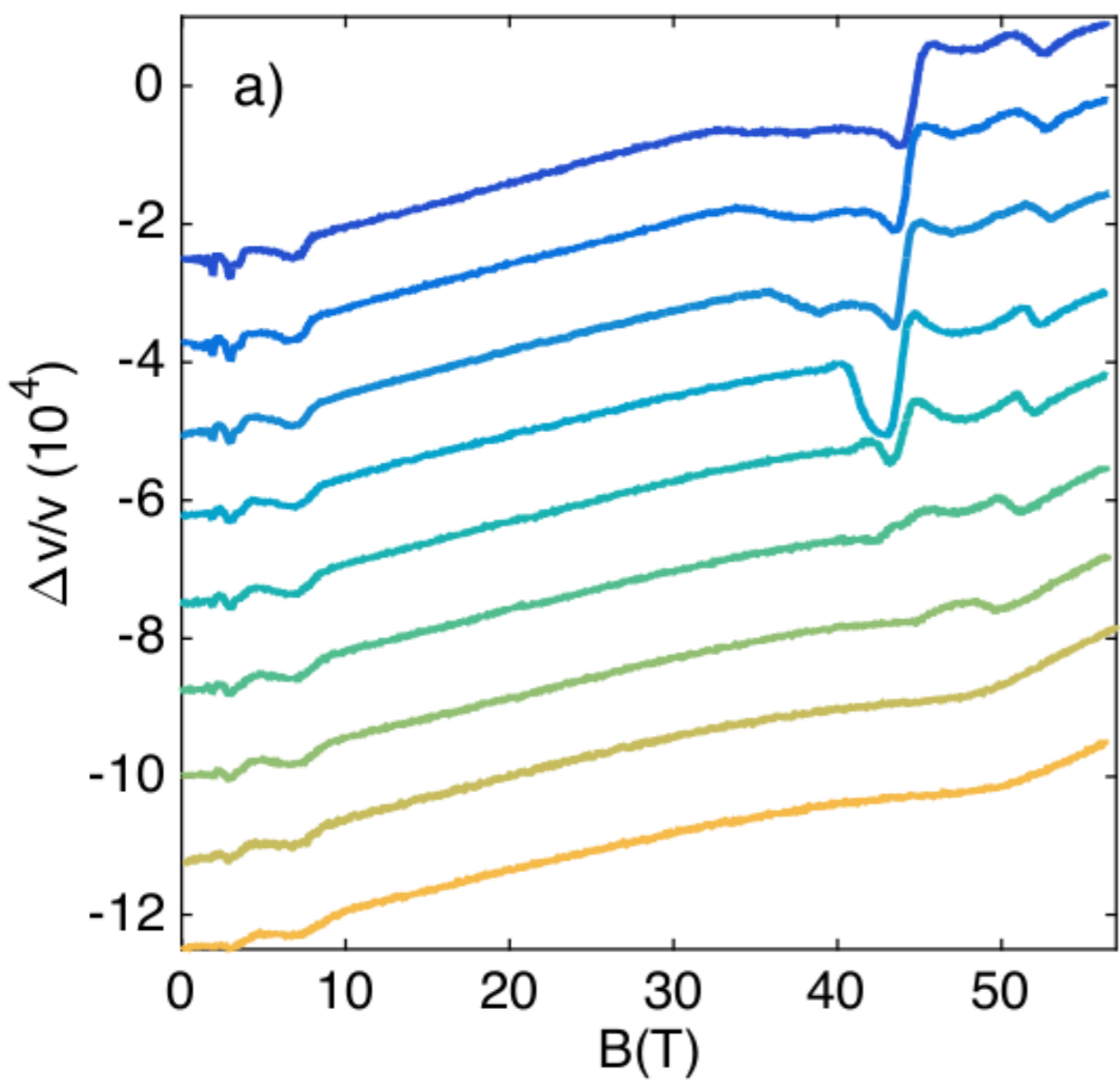
$n=150000$ layers

$$R_{xx} \approx 0.2 \Omega$$

$$l_B = \sqrt{\frac{\hbar}{eB}} \approx 35 \text{ \AA}$$

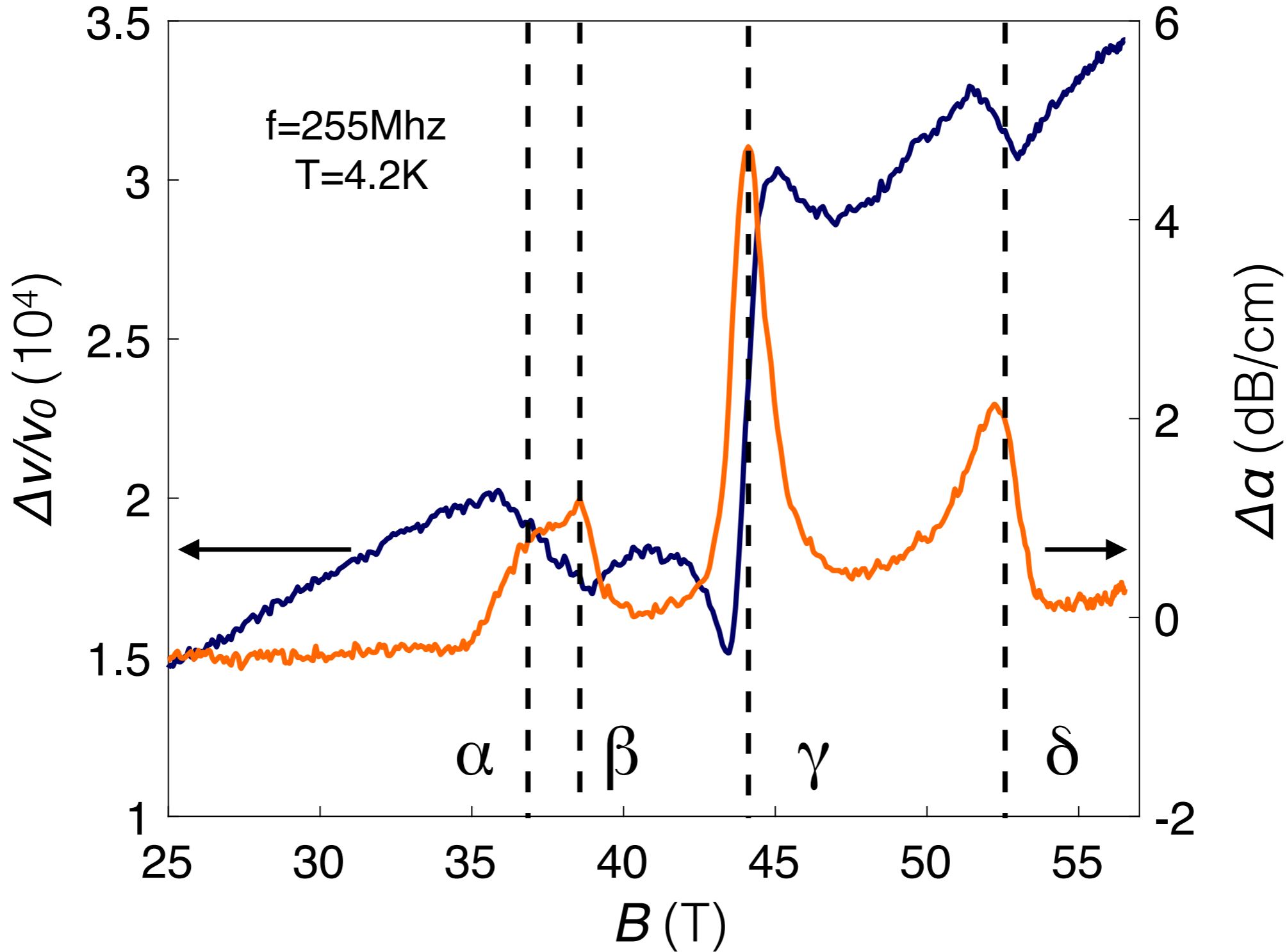


Longitudinal mode : **B//q//c** f=255Mhz



$$v=(C_{33}/\rho)^{1/2}$$

$$\Delta v/v=(v(B)-v(0))/v(0)=1/2\Delta C_{33}/C_{33}(0)$$

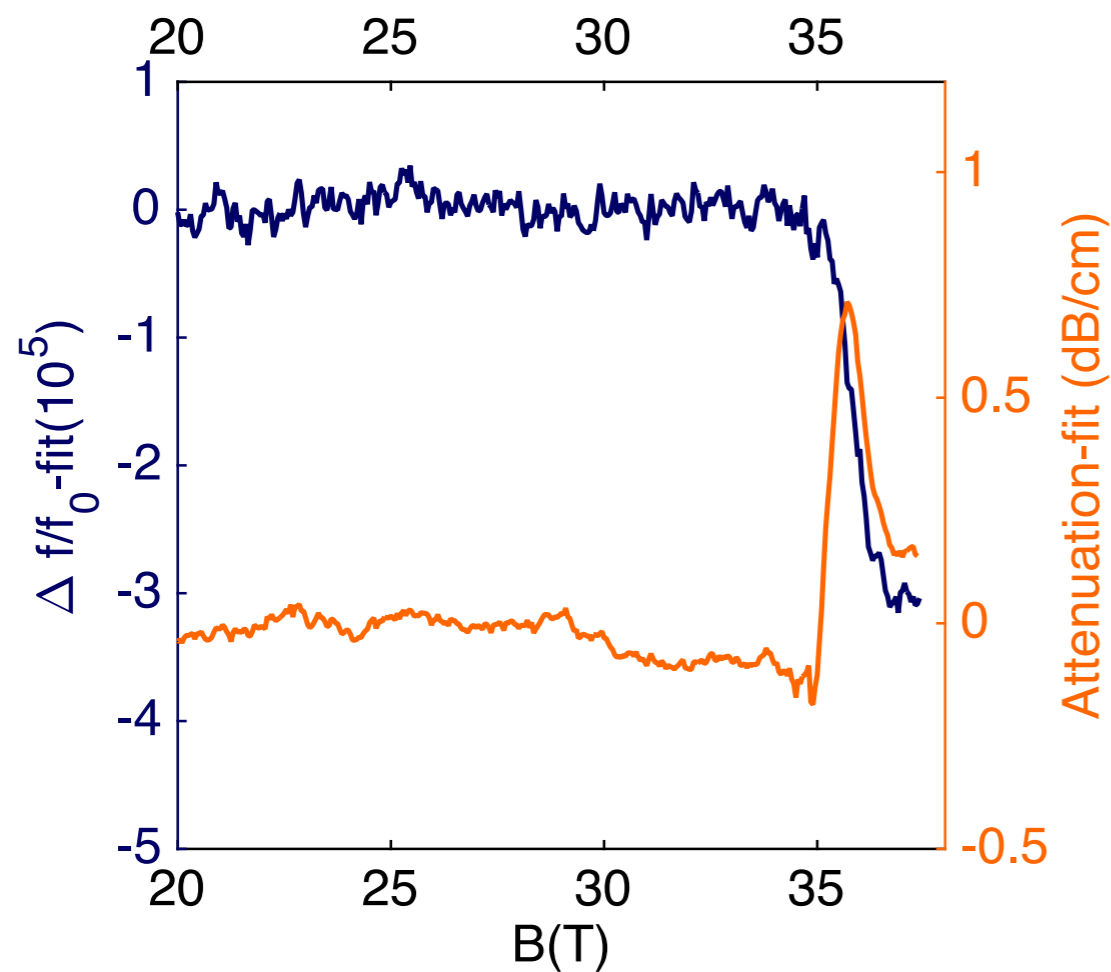


A succession of thermodynamic electronic phase transitions with a large lattice response !

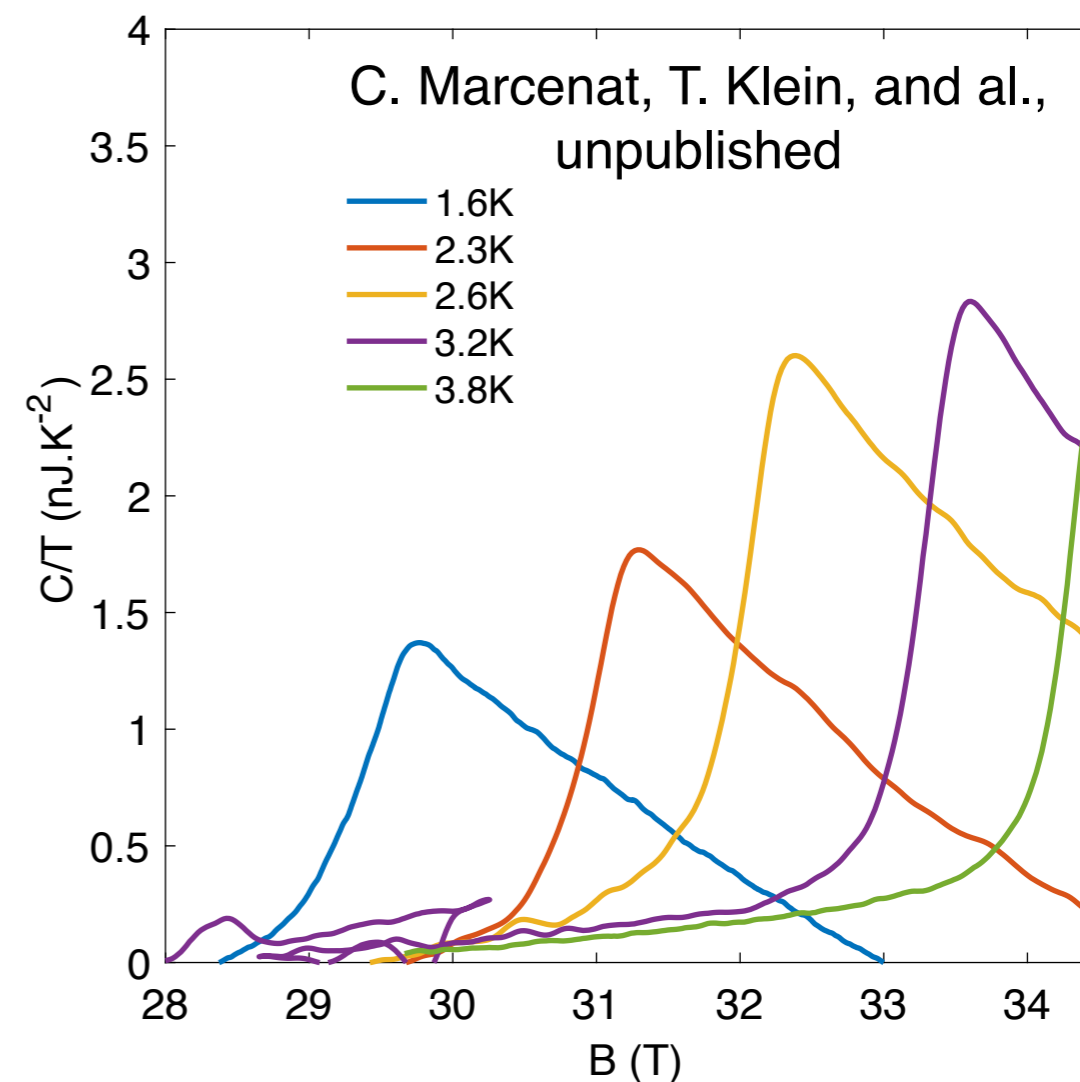
Erhenfest relationships

$$\Delta c_{ii}(T_c) = -\frac{\Delta C_p(T_c)}{V_{\text{mol}} T_c} \left(\frac{dT_c}{d\varepsilon_i} \right)^2$$

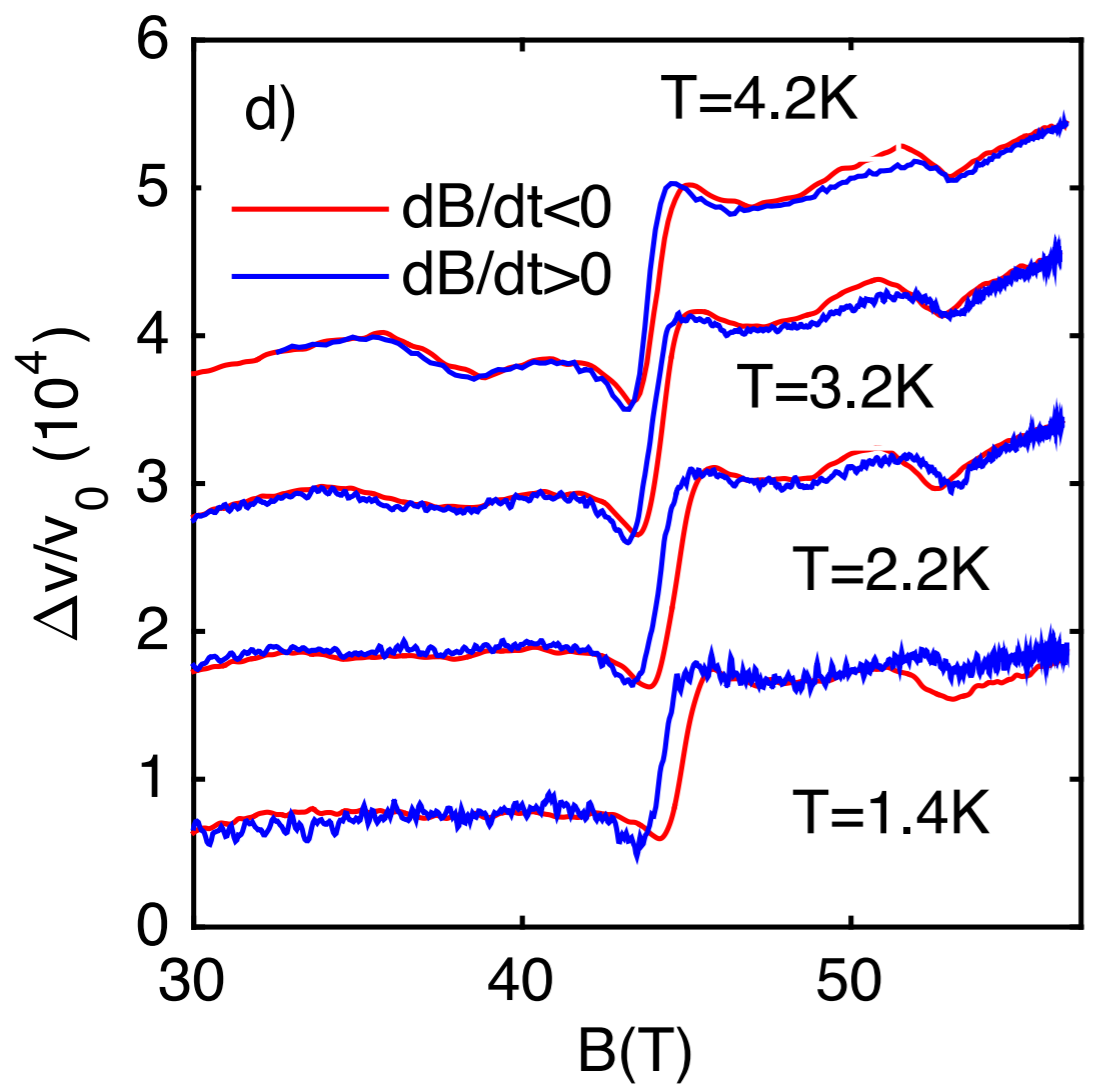
We do find a jump
in the specific heat !



Negative jump of v
 $\Rightarrow >0$ jump in the specific heat

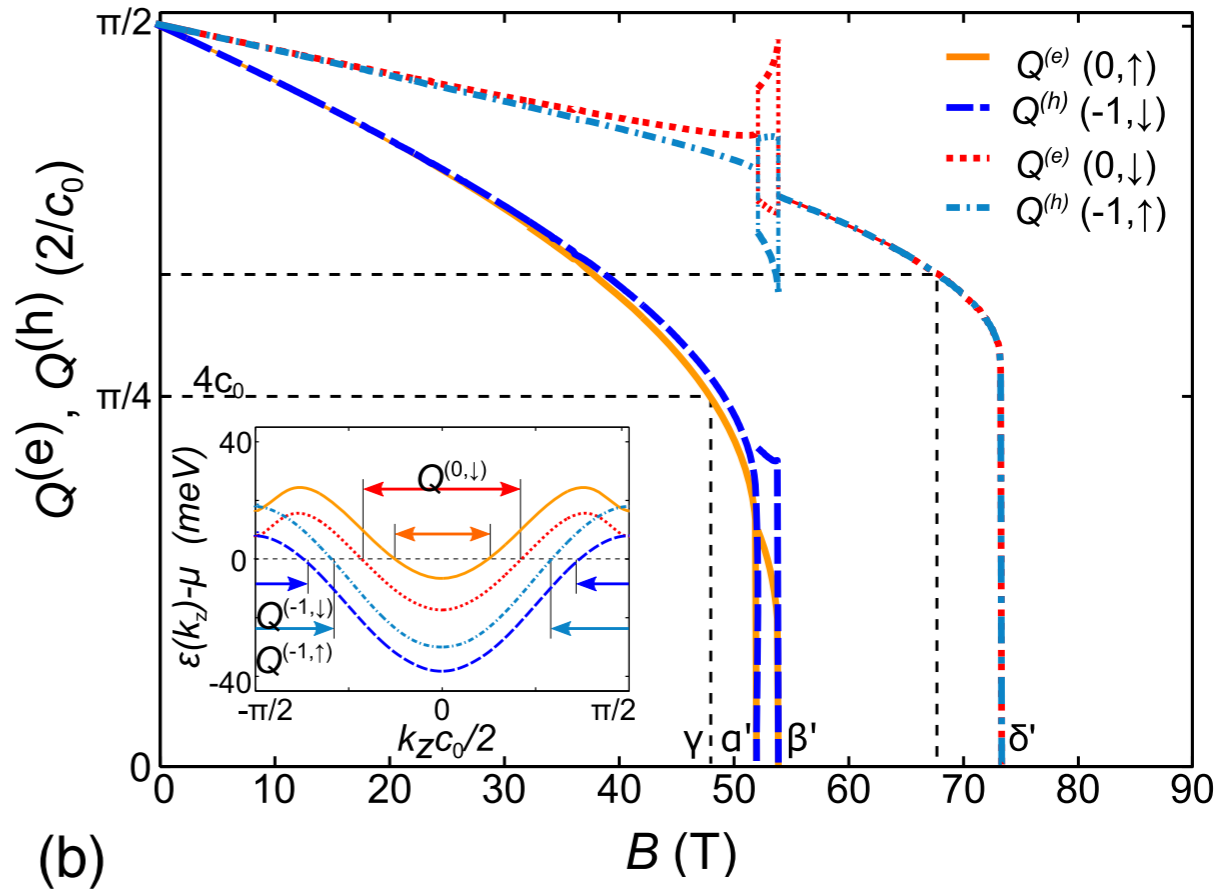


Hysteretic behaviour observed at γ -transition

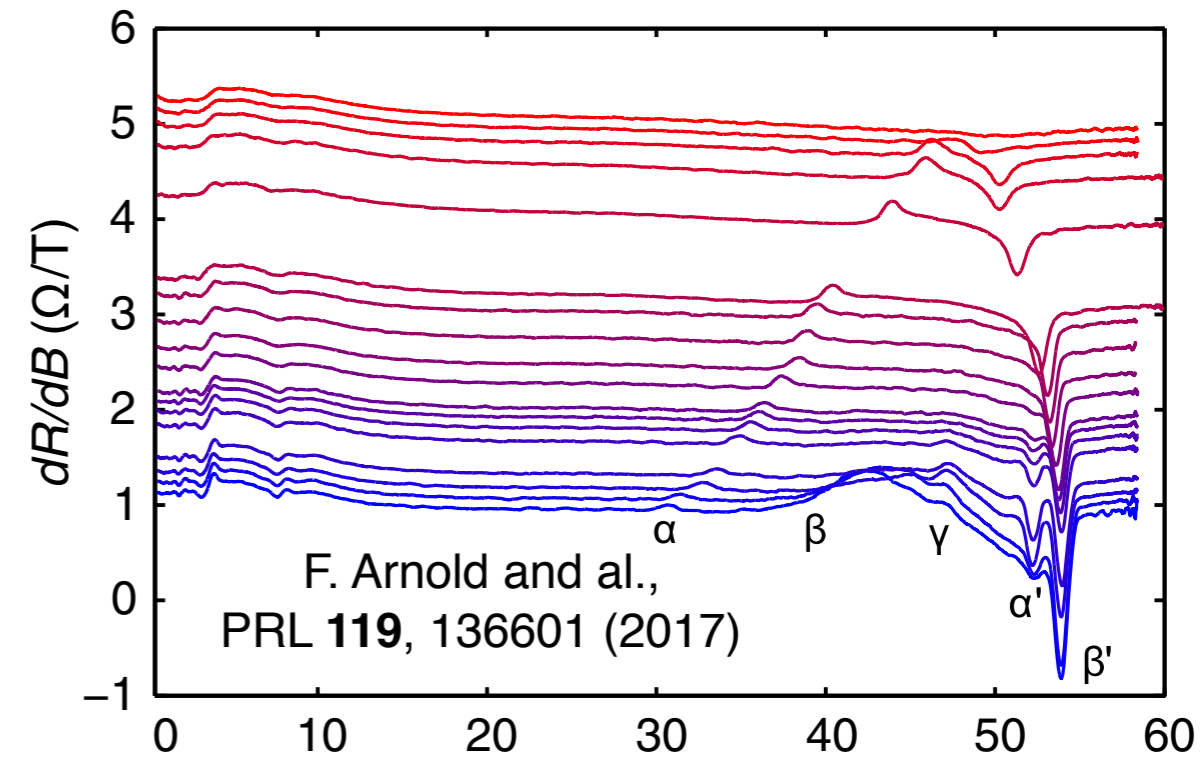


γ -transition :
Lockin transition of the α -transition ?

Nesting vector for two CDW phases



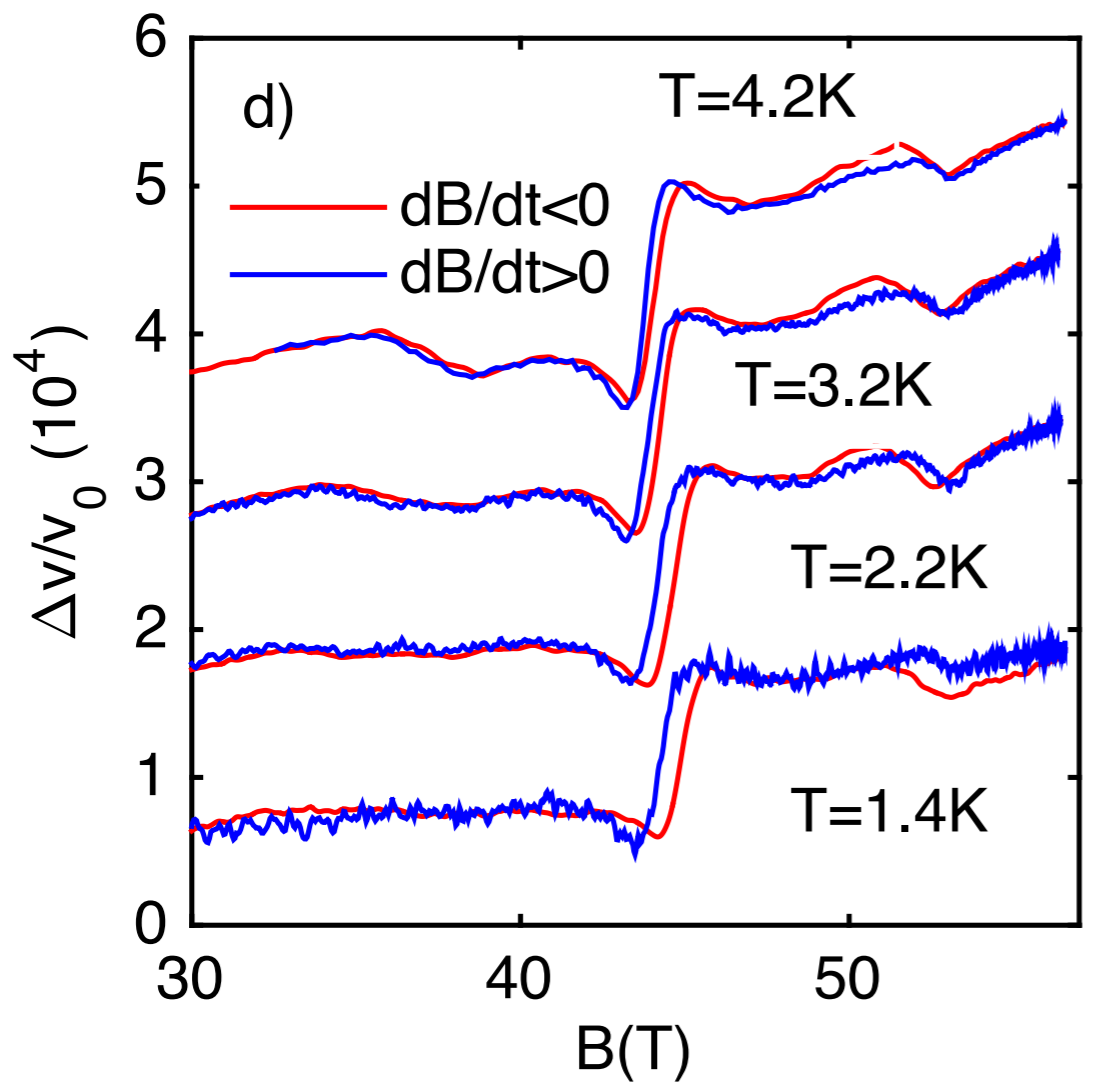
(b)



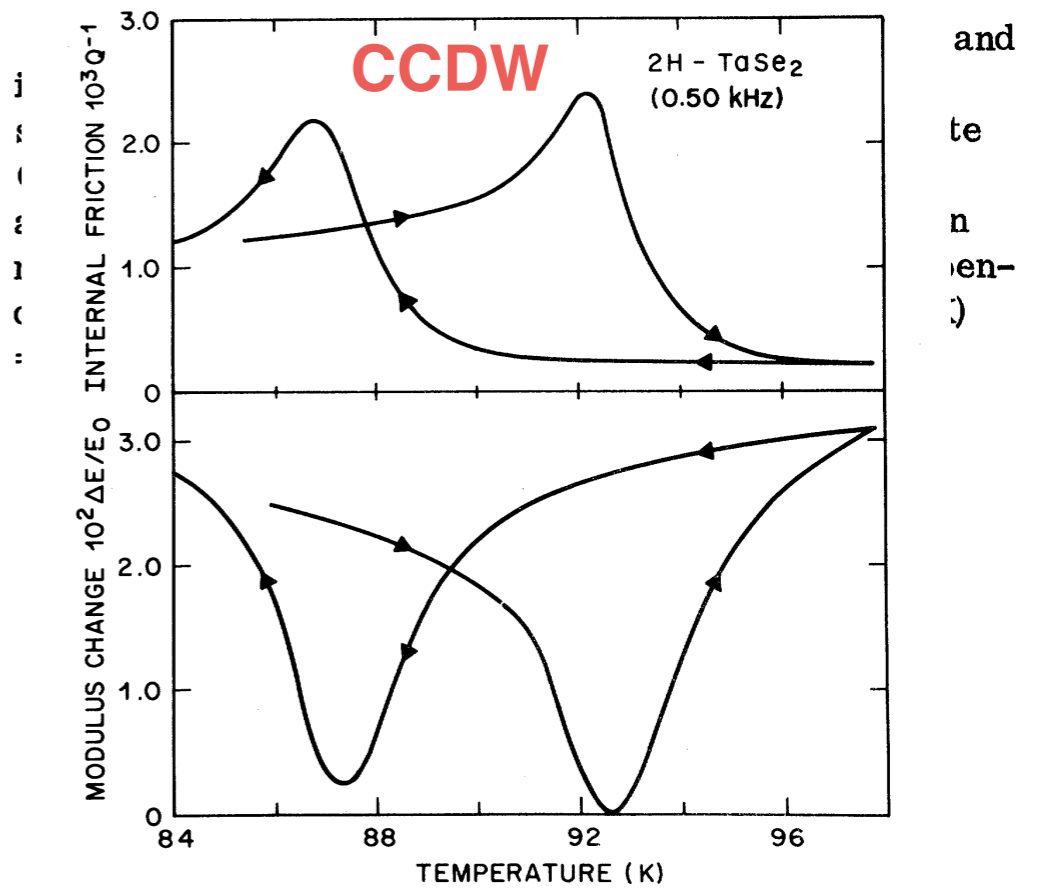
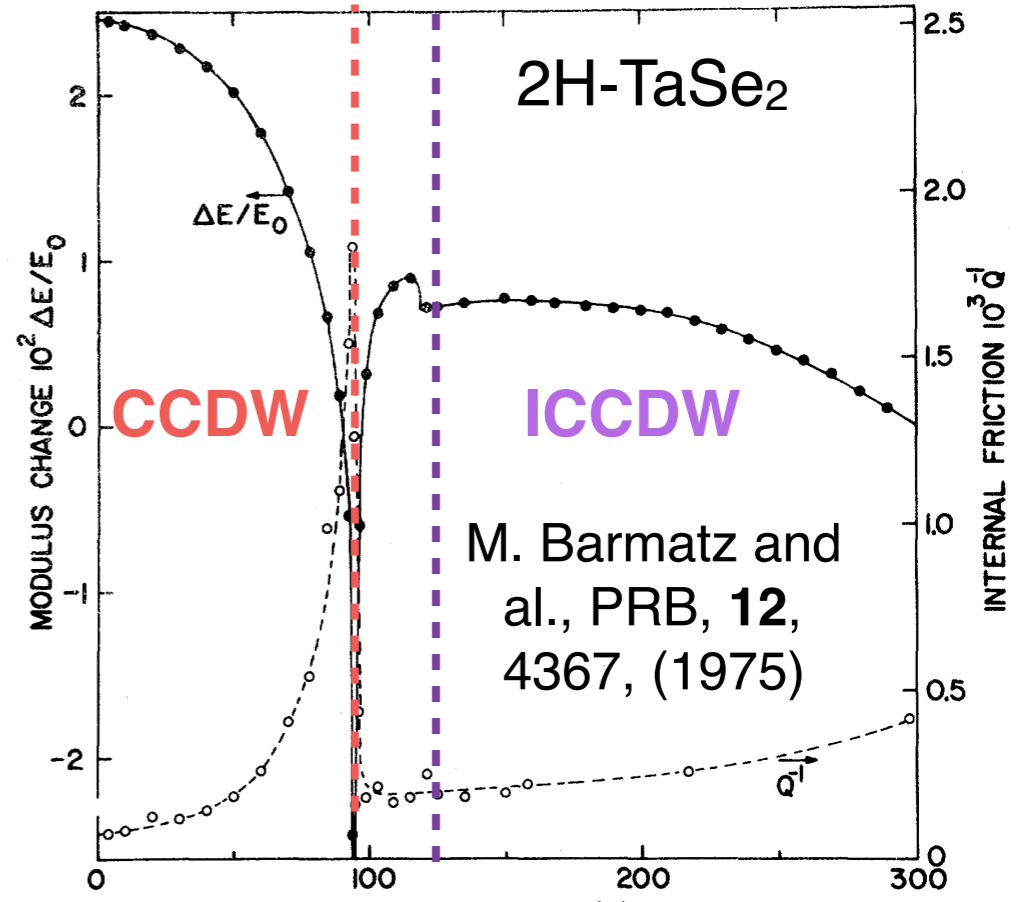
(b)

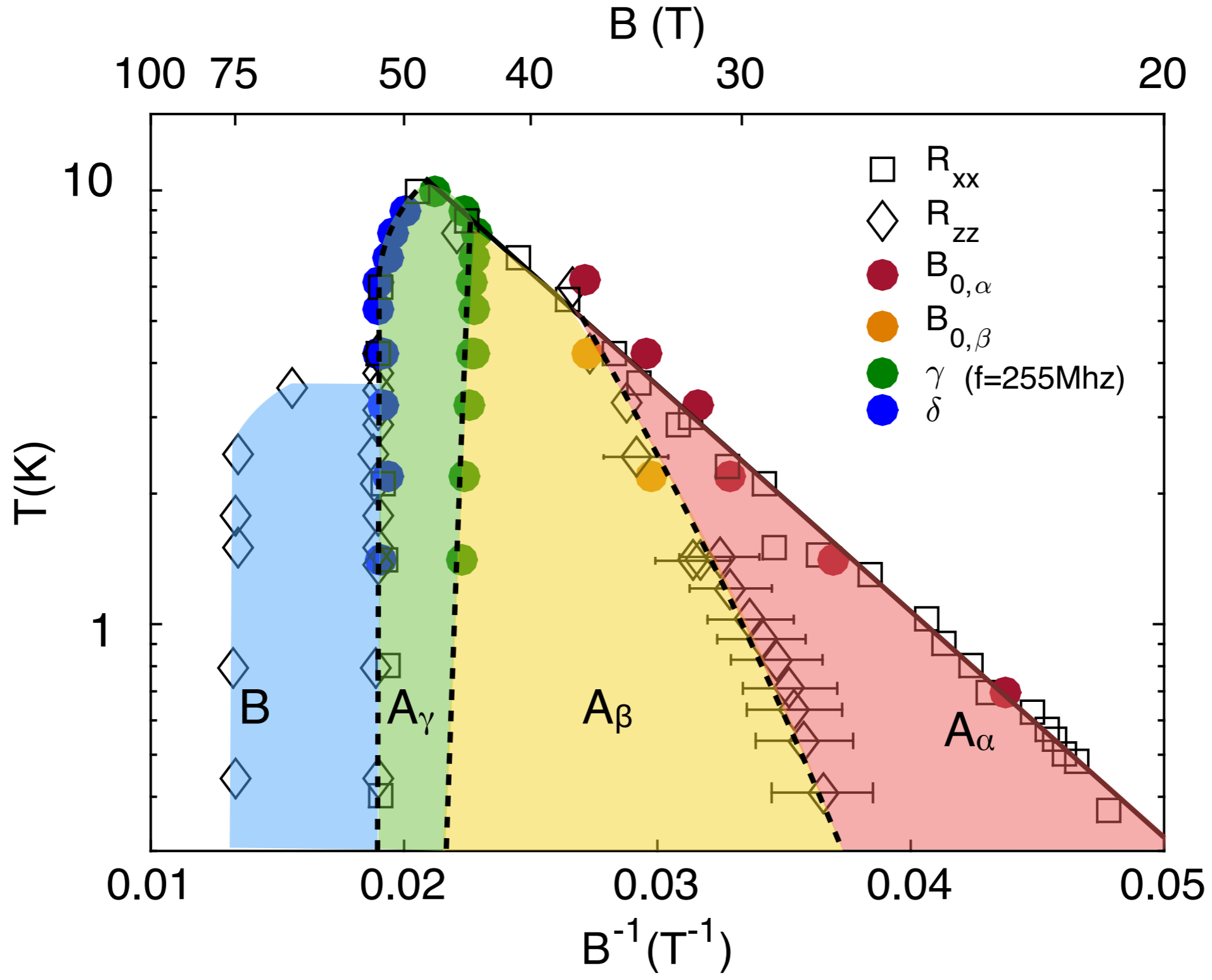
γ -transition : first order transition from ICDW to CCDW

Hysteretic behaviour observed at γ -transition



γ -transition :
from IC to C CDW phase ?

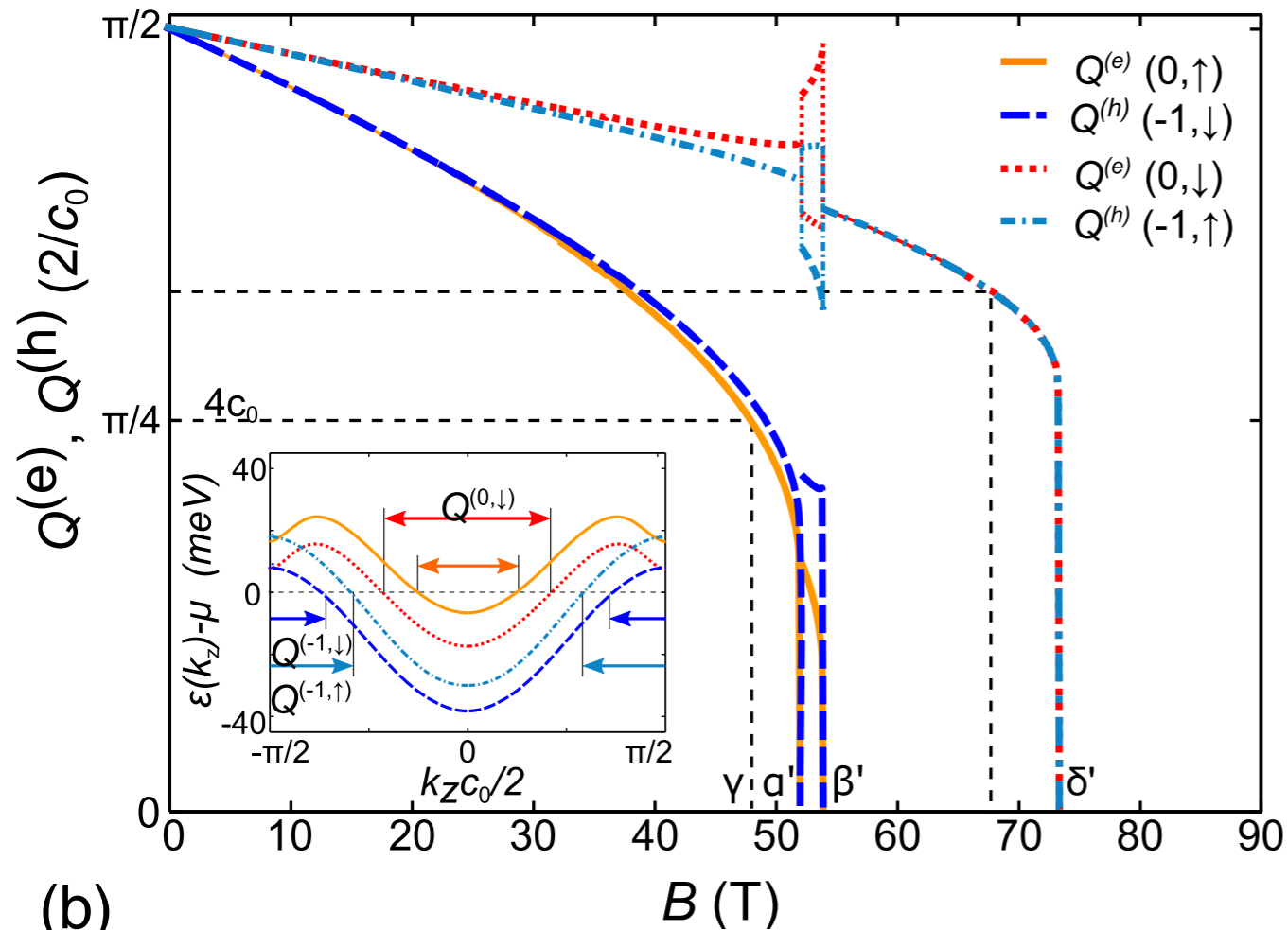
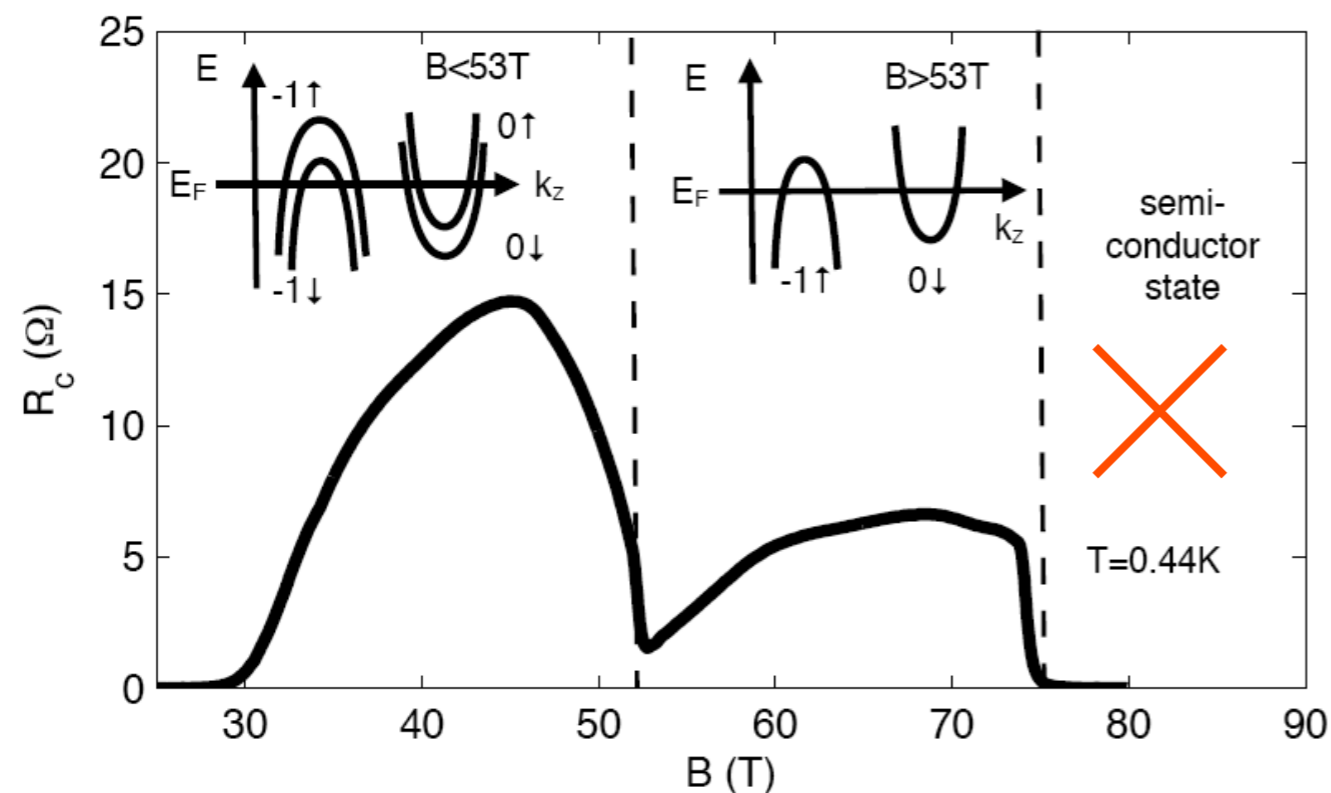




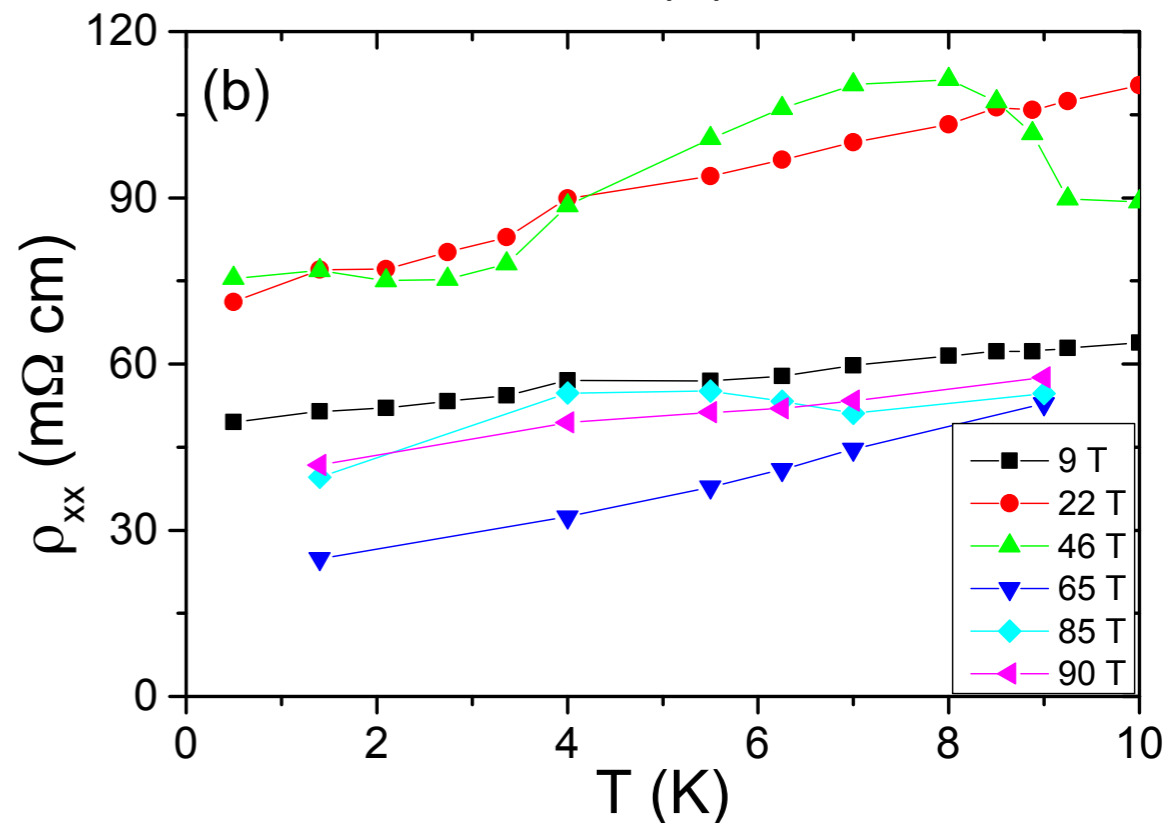
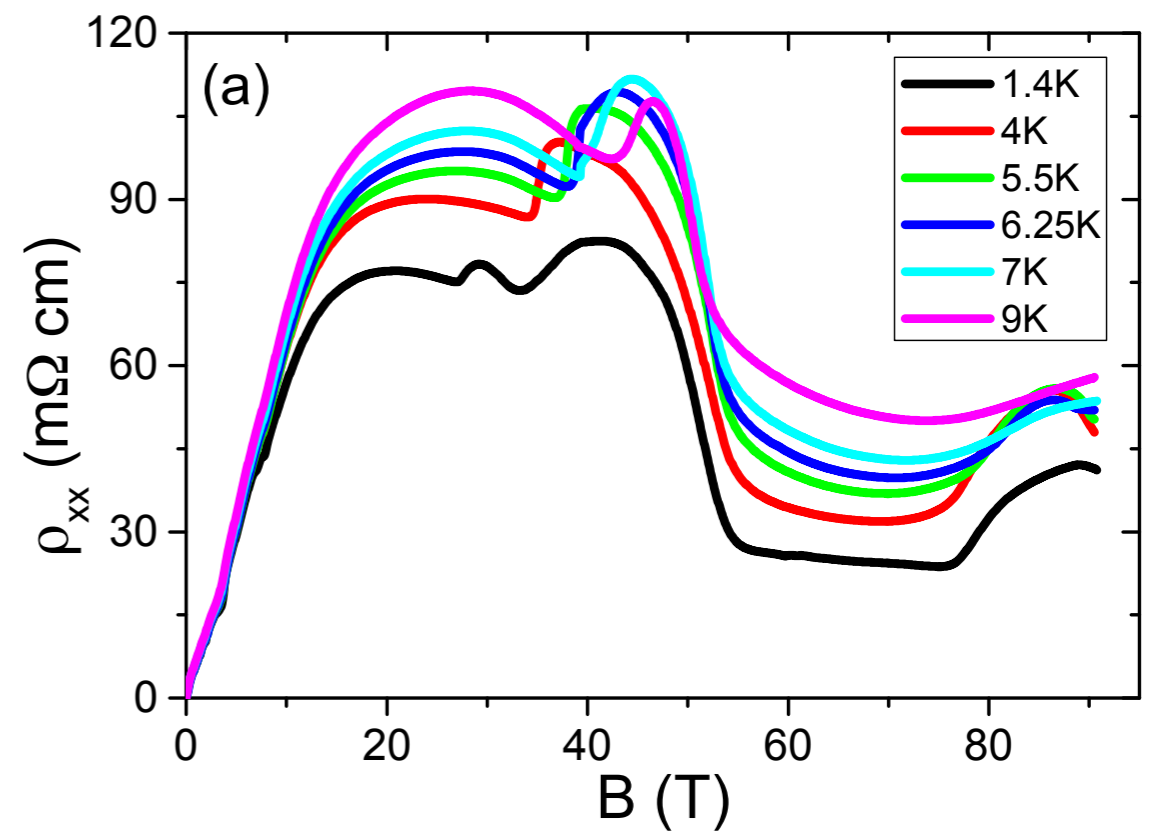
The onset of the field induced state (α -transition) is a **second order thermodynamic phase transition**

Nature of the 75T re-entrance ?

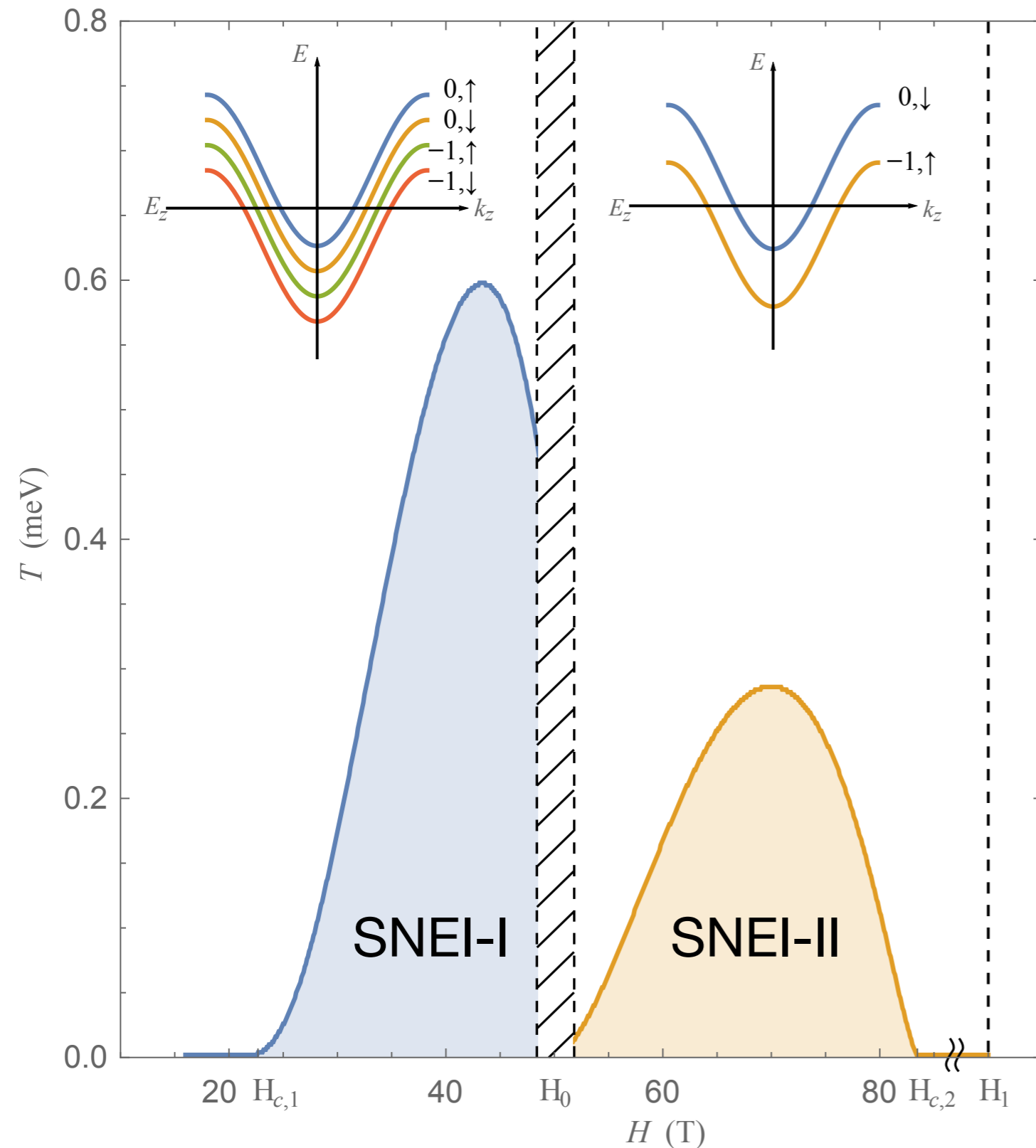
Theory : SM-SC transition @ 75T



Experiment : $B > 75T$ a new (?) metallic phase



Nature of the 75T re-entrance in the EI insulator



arXiv:1802.10253v5

- Umklapp process :
- (i) lock the *total* displacement field
 - (ii) lock the « spin superconducting » phase

In this picture the 75T re-entrance corresponds to a divergence of the quantum fluctuation of the phase induced by the electronic correlation

No LL depopulation at 75T !

$$\omega_g \propto (H_{c,2} - H)^{z/\nu_2} = (H_{c,2} - H)^{1/\nu_2},$$

$$\nu_2 = \frac{1}{2} \sum_{a=2,3} (K_a + K_a^{-1}) - 2.$$

Rem : a CDW can be superposed to the EI!

Thermoelectrical response beyond the quantum limit of 3D electron gas systems

Introduction

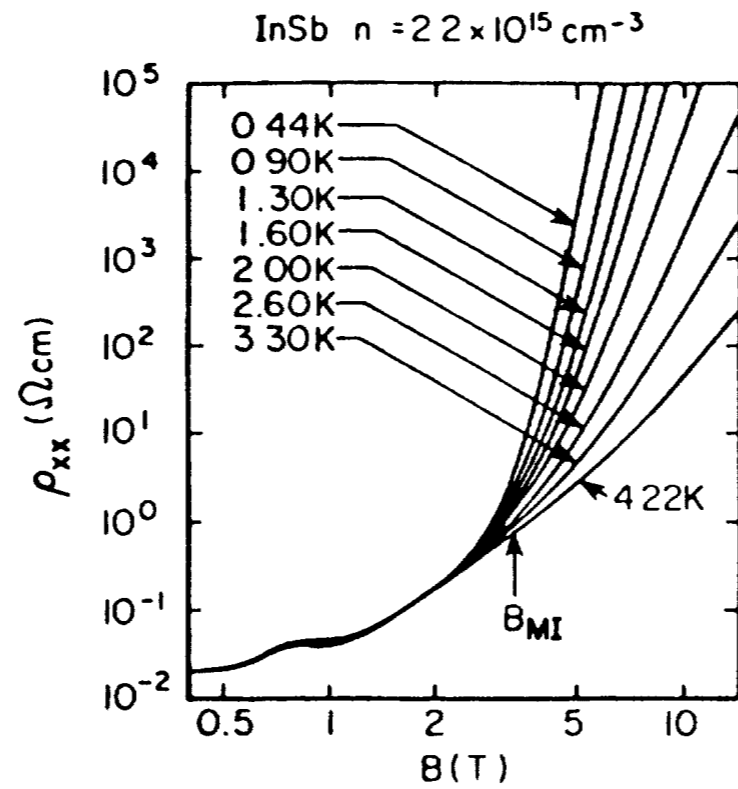
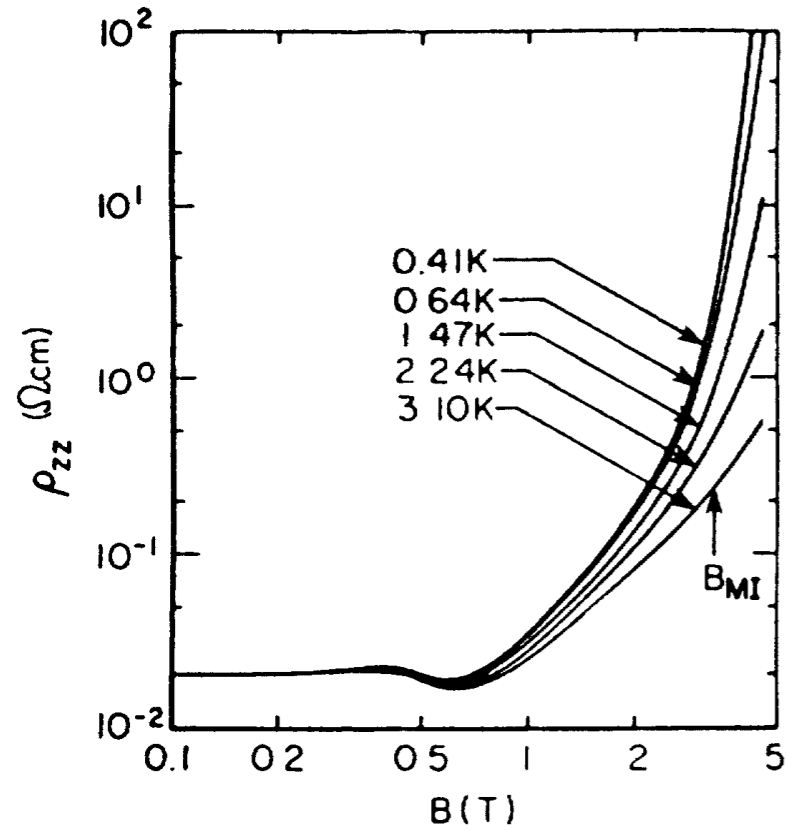
Nernst effect as a probe of quantum oscillations in semi-metals : the case of bismuth and graphite

Transport and thermodynamic measurements in the quantum limit of graphite

Narrow gap semi-conductors in the quantum limit: the case of InAs

Conclusion

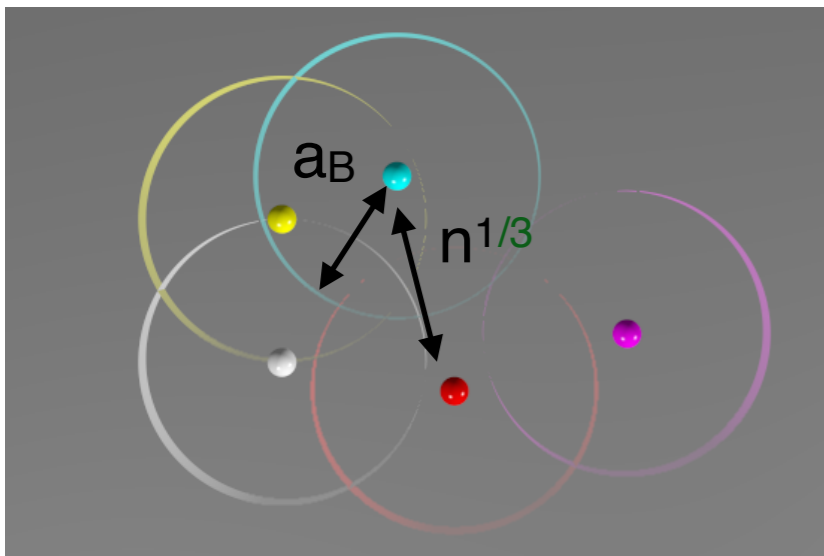
Narrow gap semi-conductors in the quantum limit : *magnetic freeze-out*



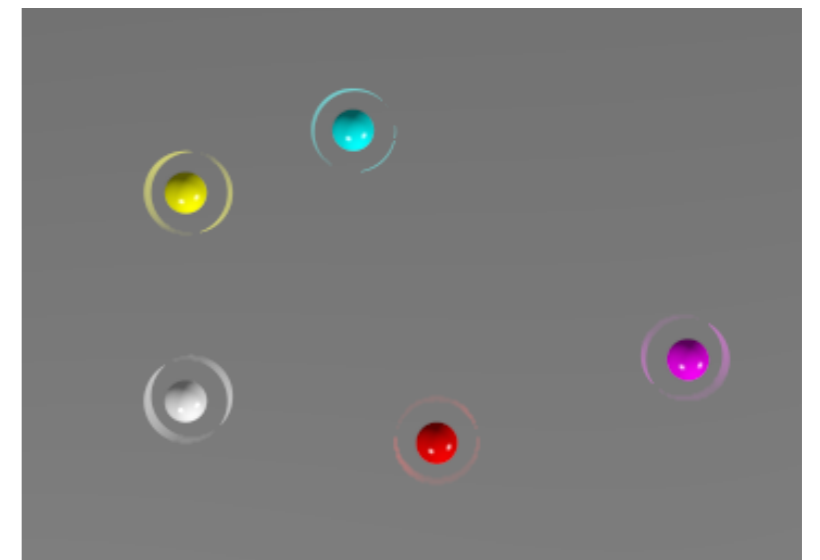
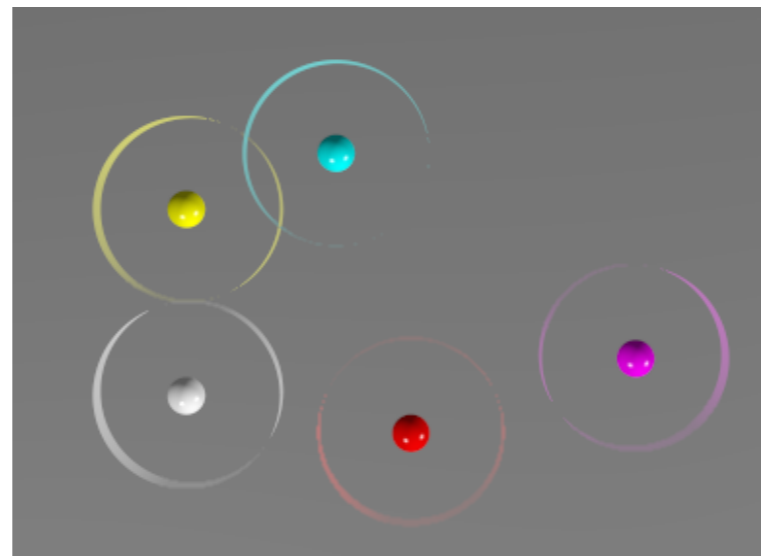
Mott-Anderson transition
assisted by the magnetic field

$$a_B^* n_c^{\frac{1}{3}} \approx 0.25$$

$$a_B^* = (a_{\perp,B}^2 * a_{//,B})$$



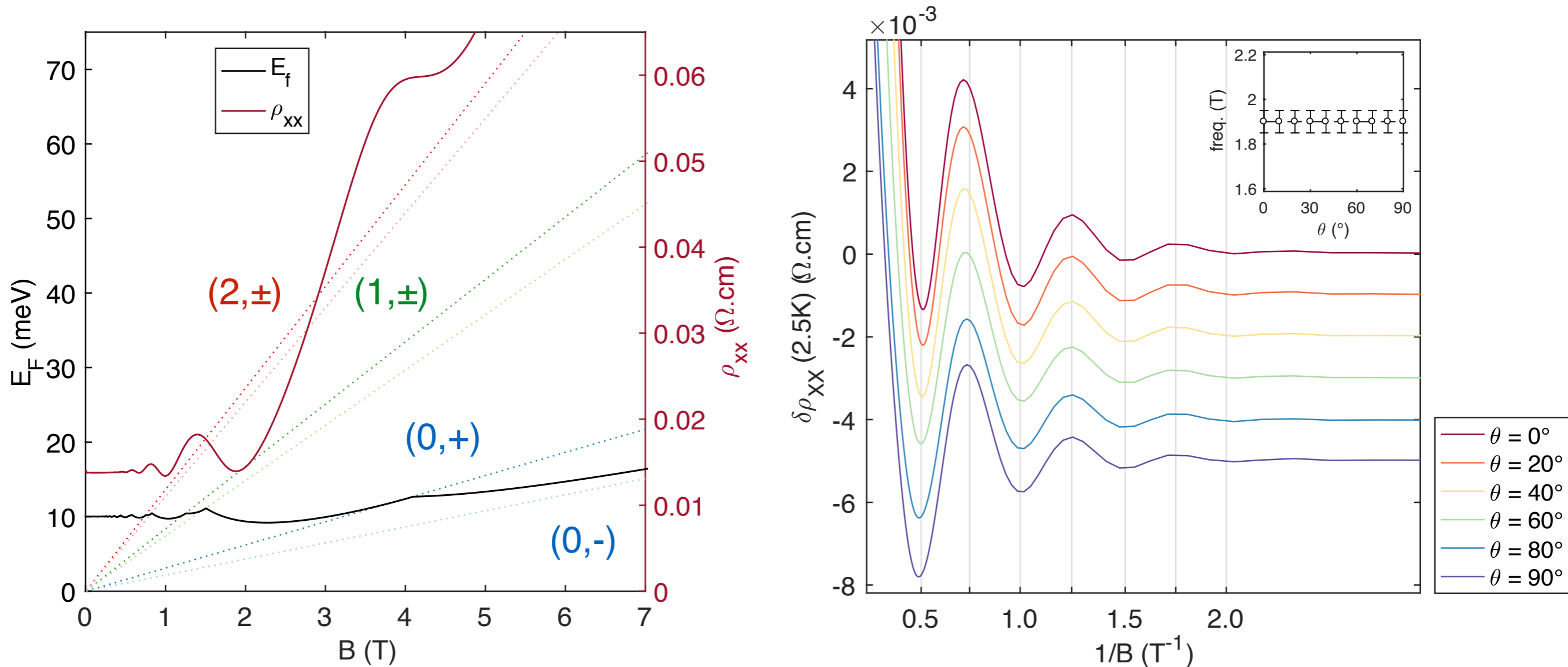
$$a_B^* n^{\frac{1}{3}} \gg 1$$



$$a_B^* n^{\frac{1}{3}} \ll 1$$



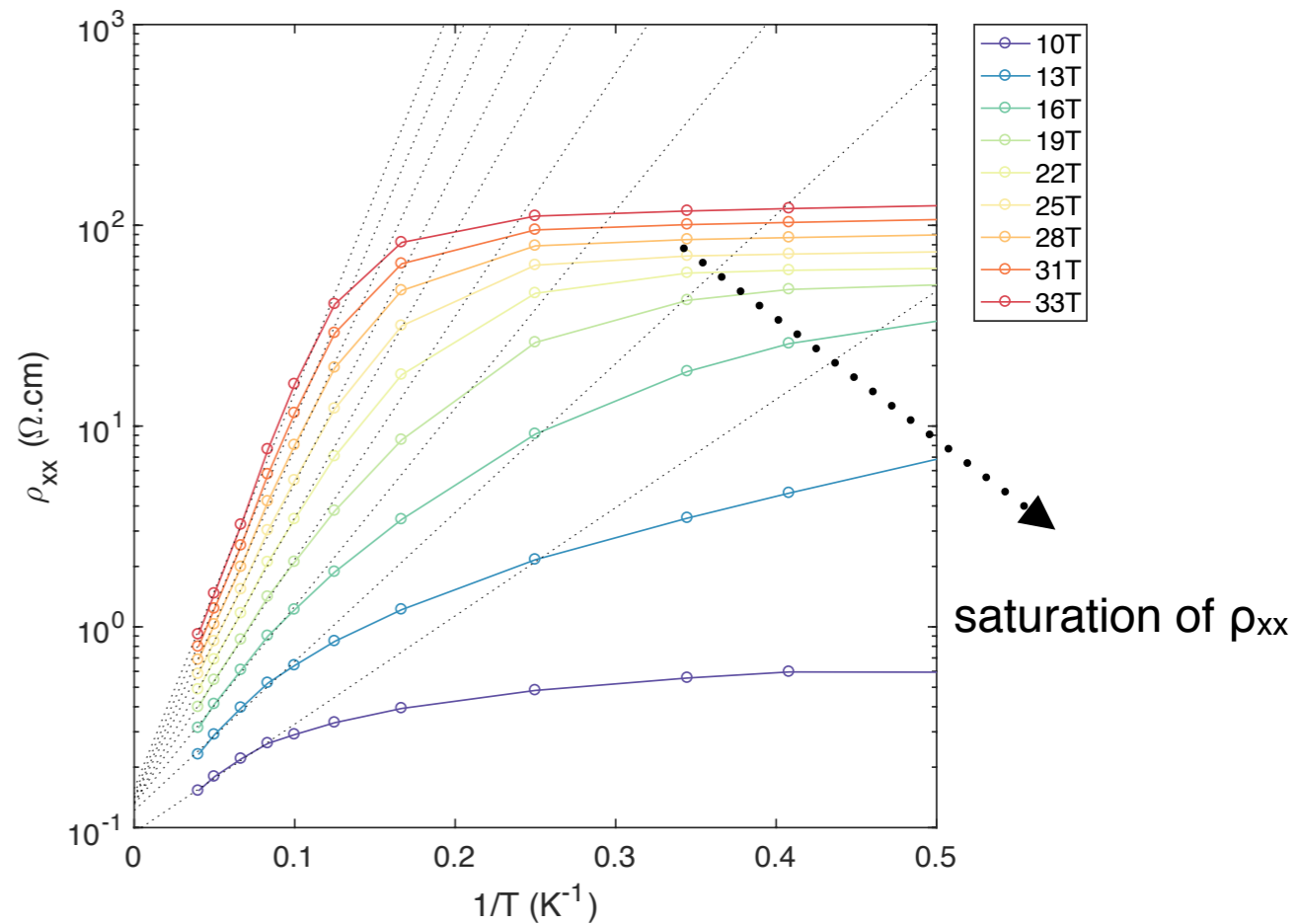
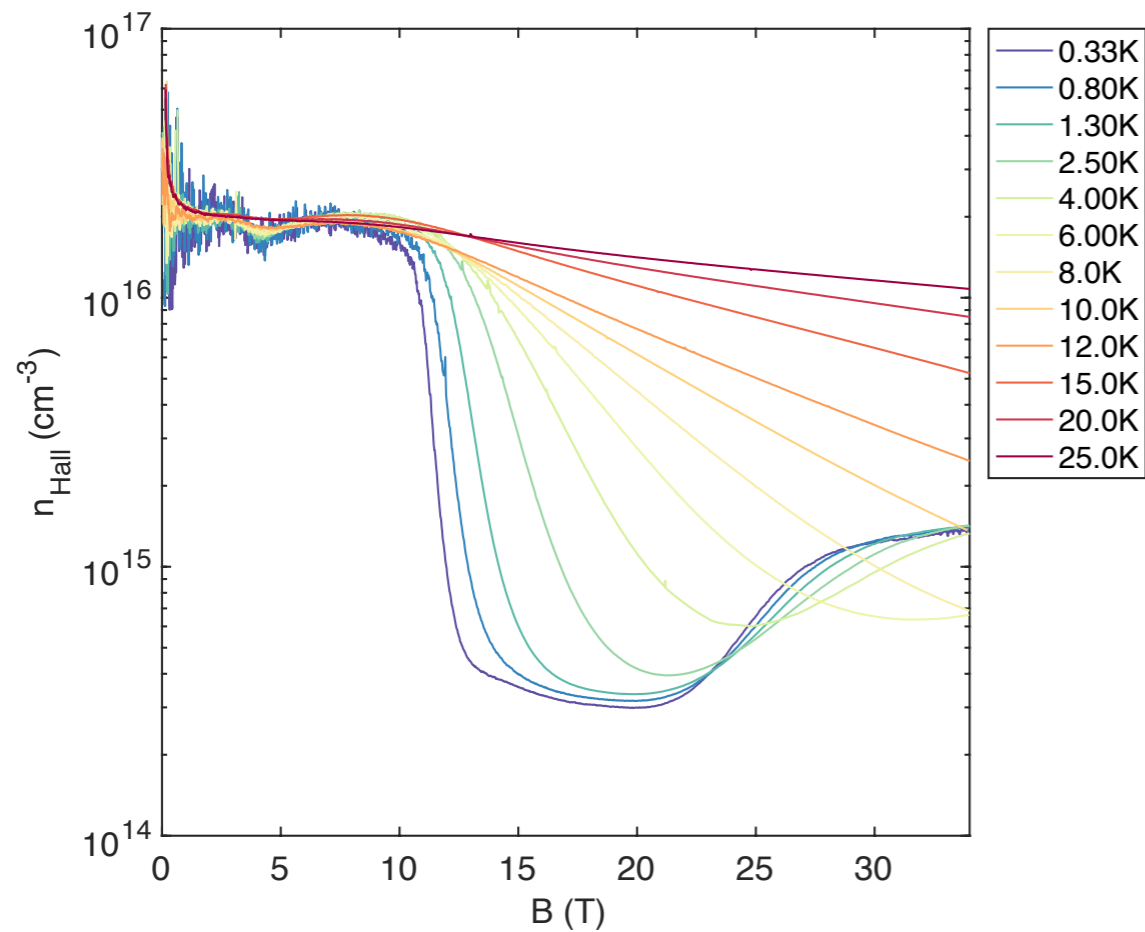
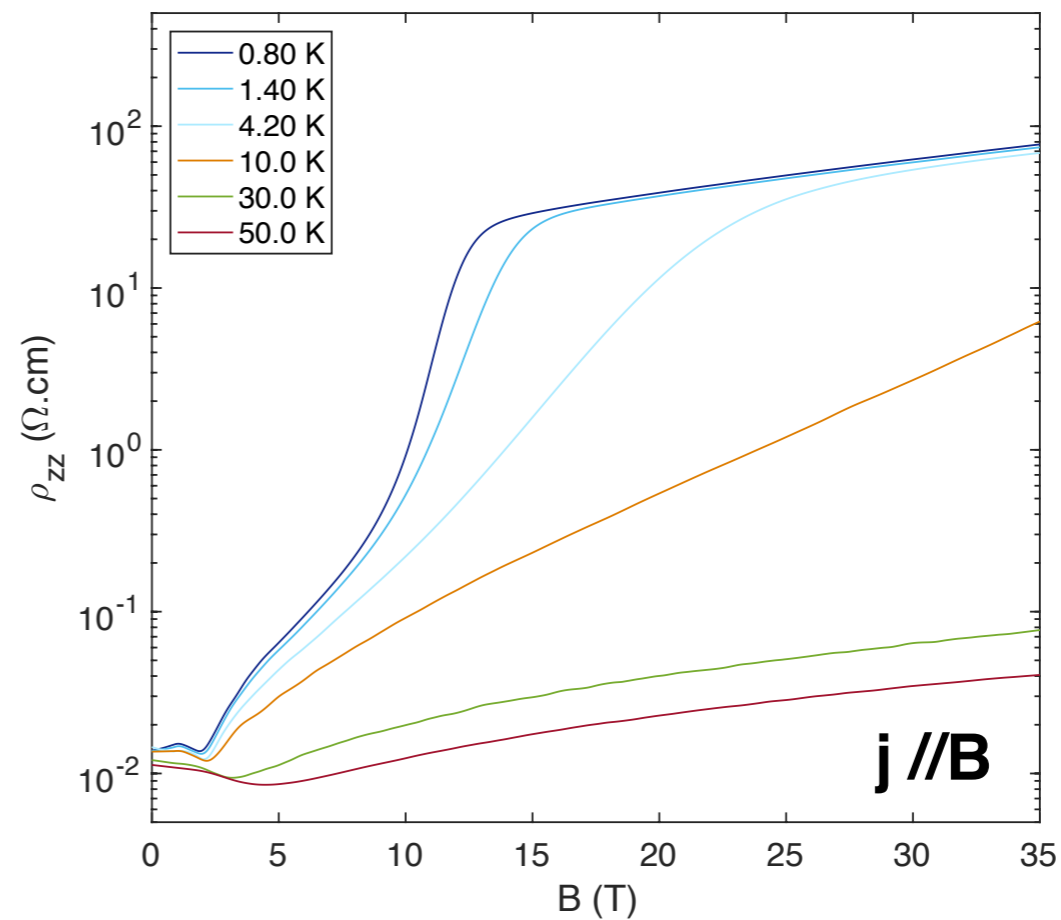
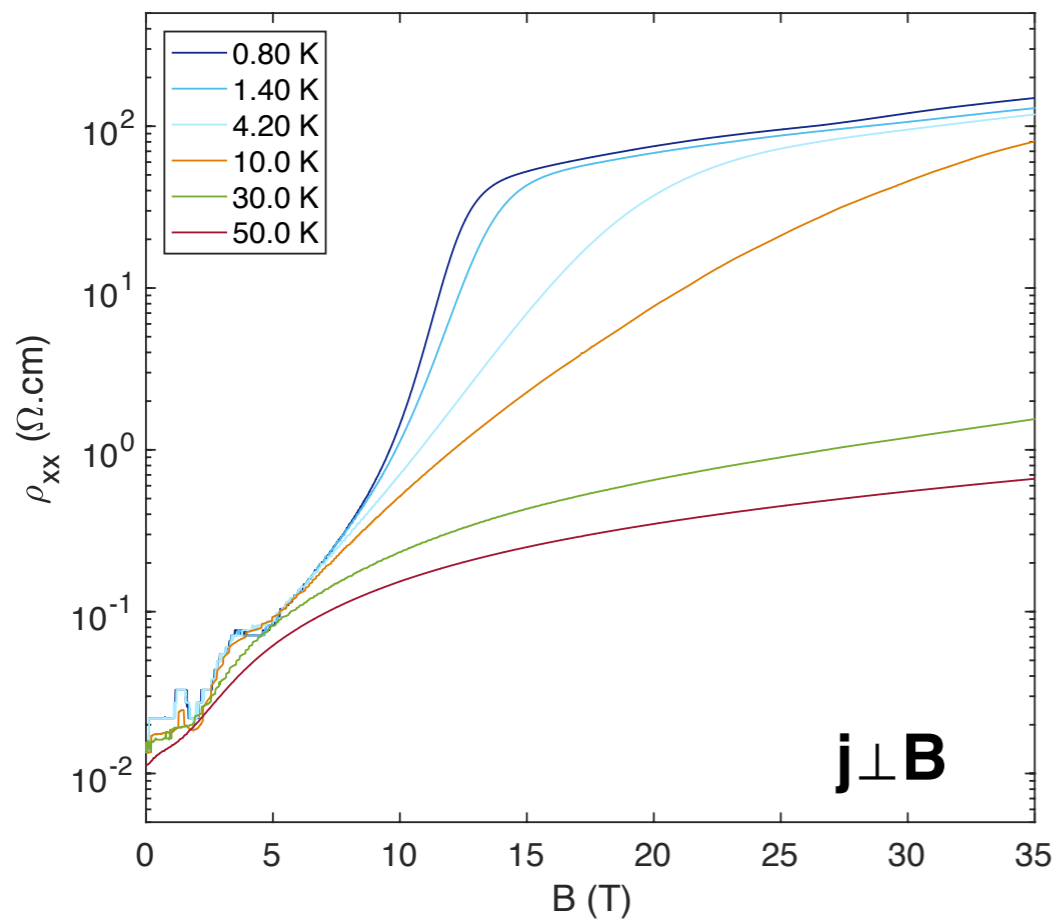
$n=2e16\text{cm}^{-3}$



A spherical Fermi surface where all the electrons are confined in the lowest $(0,-)$ Landau level at $B=4.5\text{T}$

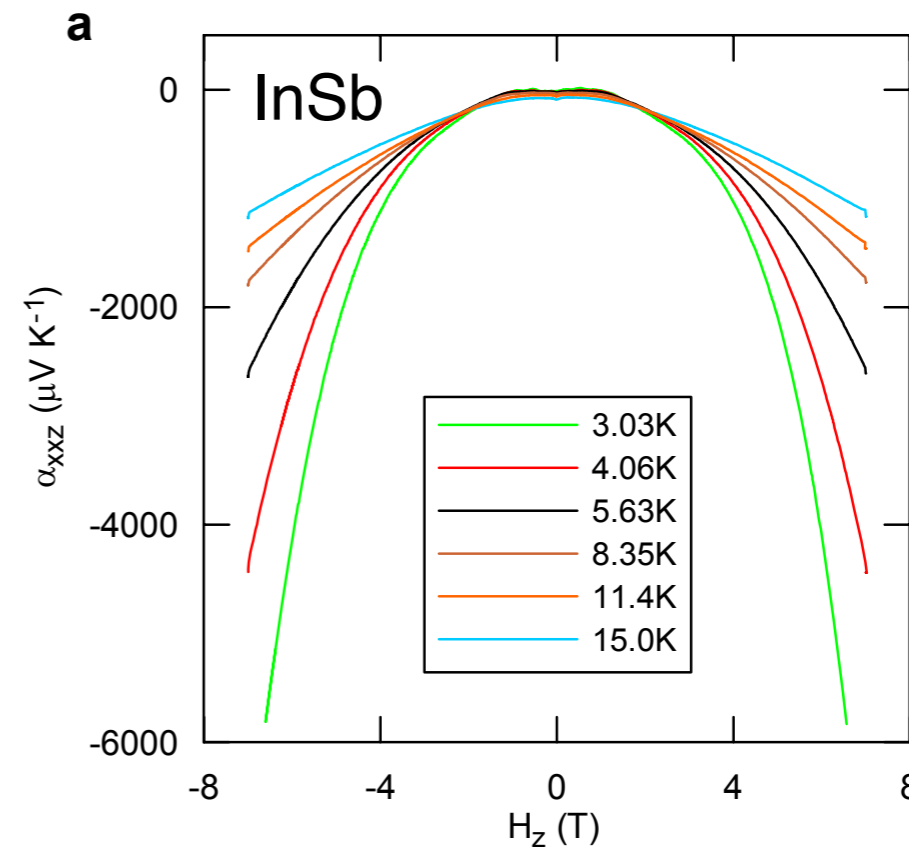
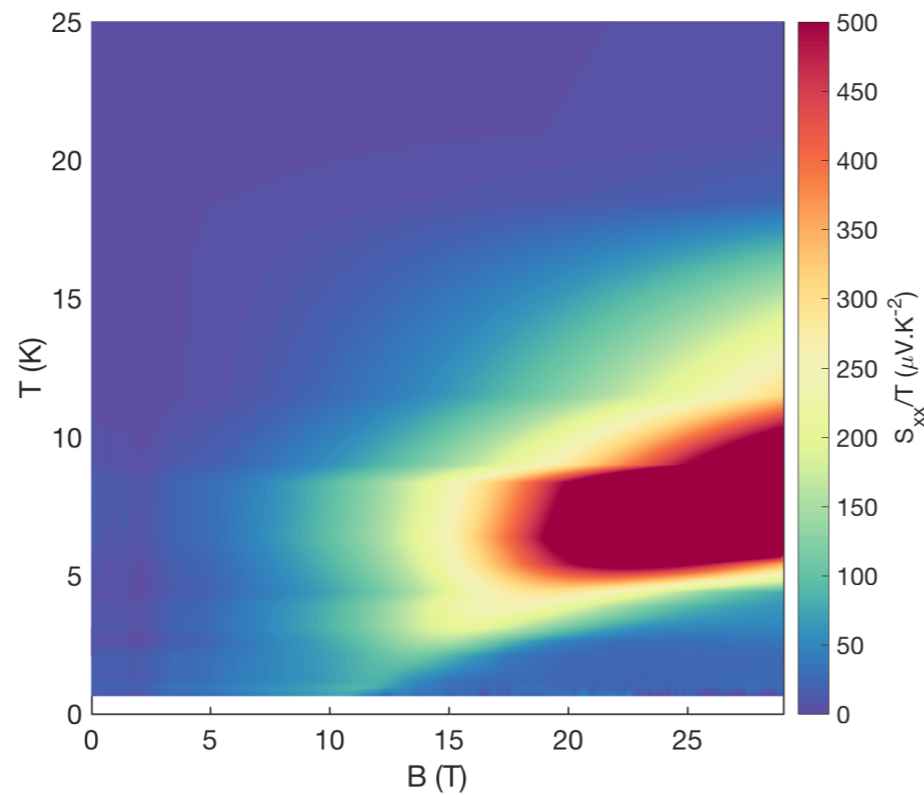
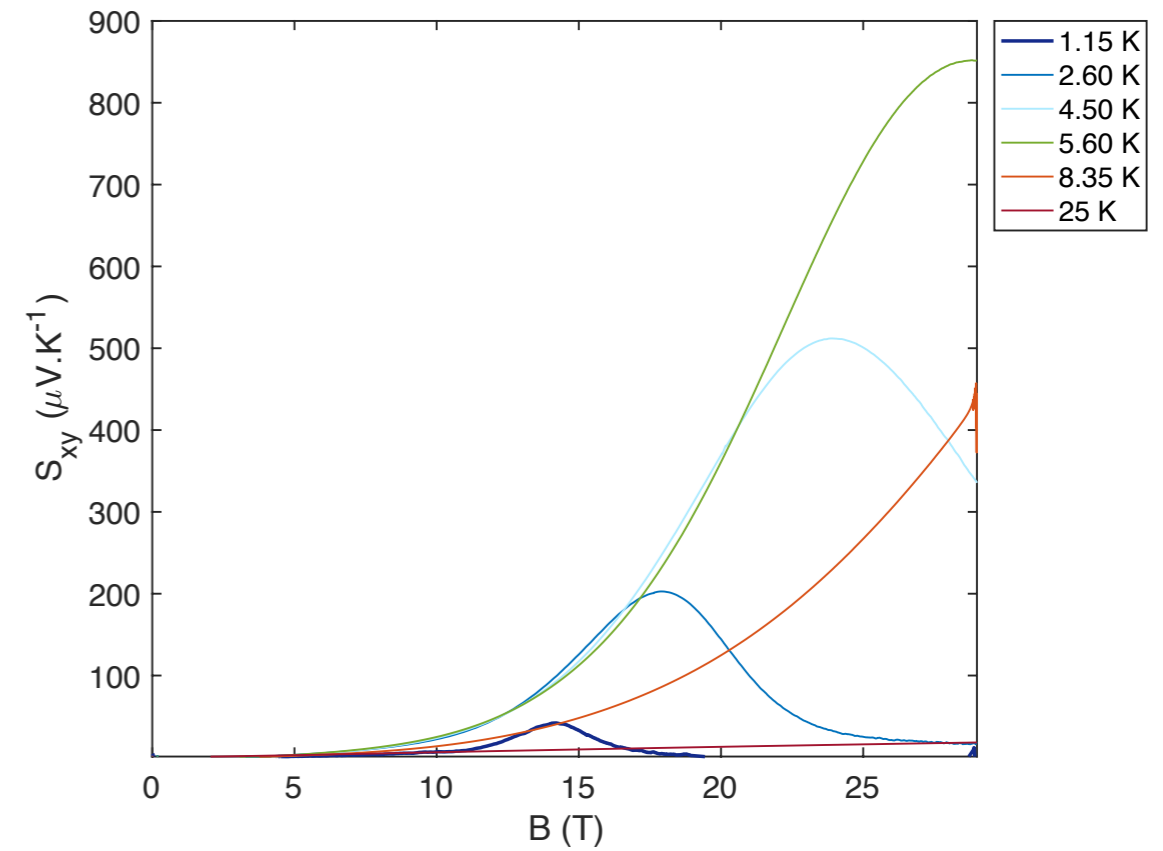
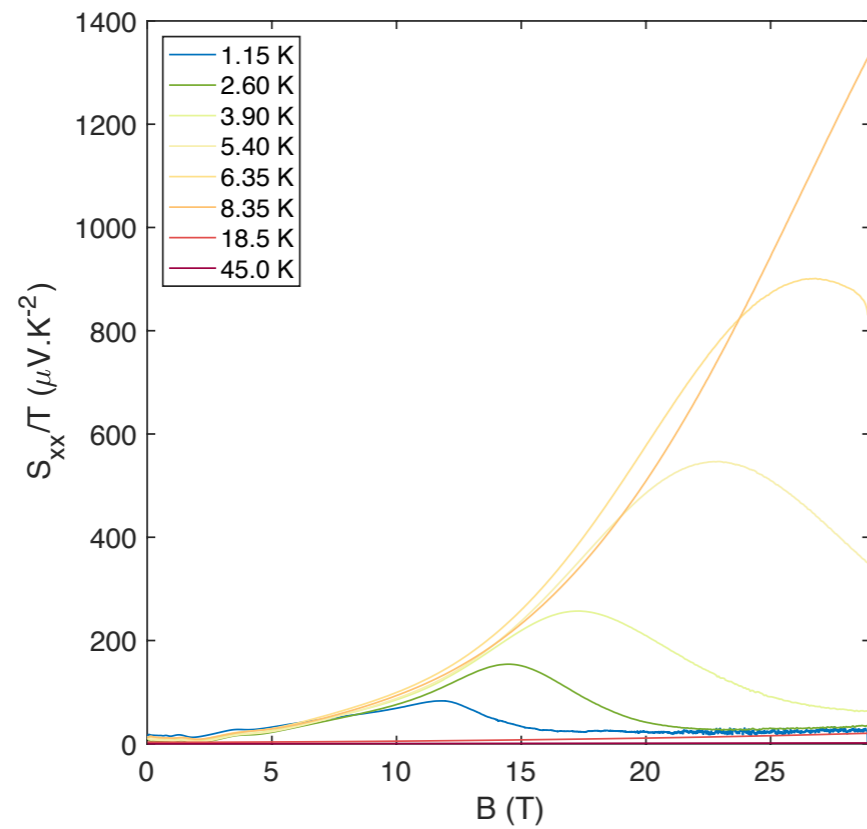
The case of InAs : electrical transport

A. Jaoui and al., unpublished



The case of InAs : Thermopower and Nernst effect

A vanishing thermopower and Nernst effect in the field induced state of InAs !



The case of InAs : a second channel of conduction ?

In presence of two channels of conductance
we have :

$$\sigma_T = \sigma_1 + \sigma_2$$

$$S_T = (\sigma_1 S_1 + \sigma_2 S_2) / \sigma_T$$

If (1), the bulk, goes to insulator :

$$\sigma_1 \propto \exp\left(-\frac{\Delta}{k_B T}\right) \rightarrow 0 \quad \sigma_T \approx \sigma_2$$

$$S_1 \propto \frac{\Delta}{k_B T} \rightarrow \infty \quad S_T \approx S_2$$

Charge Accumulation at InAs Surfaces ?

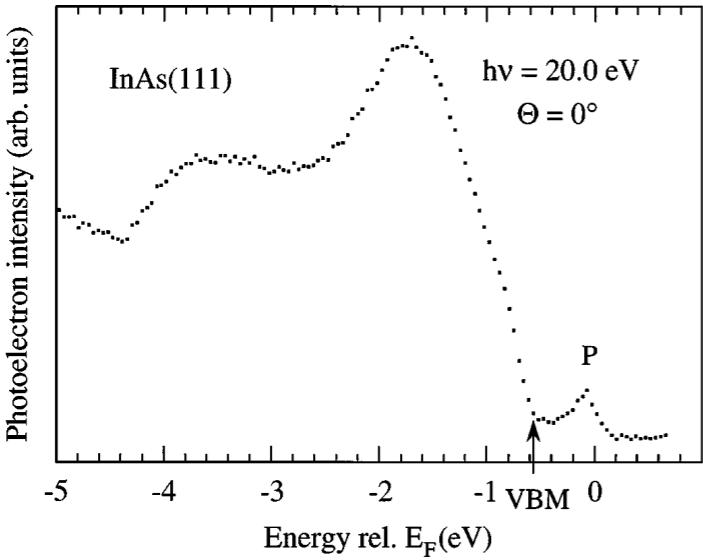
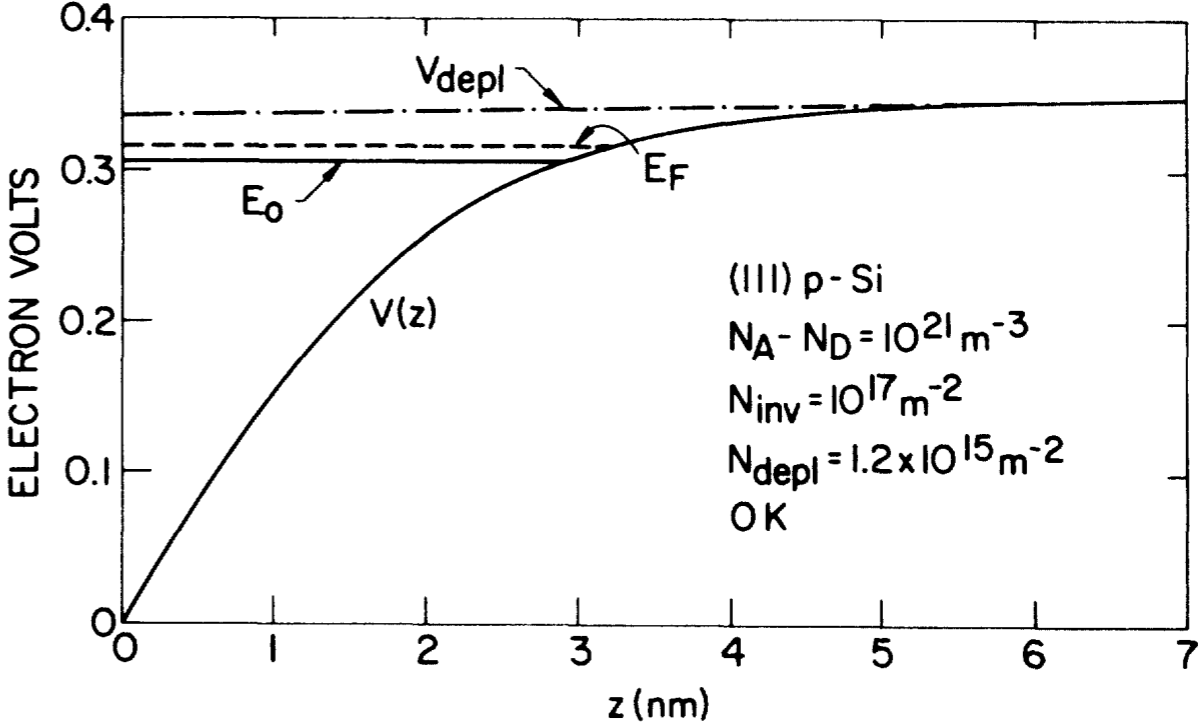


FIG. 1. Photoelectron spectrum from an IBA-prepared InAs(111)-(2 × 2) surface showing the presence of a narrow peak "P" just below E_F . The E_F energy was determined by photoemission from a Mo foil.



F. Stern, PRB 5,4891 (1972)

L. Olsson and al., PRL 76,3626 (1996)

Conclusions

The effect of the magnetic field deep in the QL is generally to turn your metal into an insulating state. However the nature of the insulating state strongly depend of the system under consideration !

For graphite we have a succession thermodynamic transitions with activation gap in charge conductivity along the c-axis which coexists with in-plane metallicity.

In the case of narrow semi-conductor we have an activation gap for all direction of the current injection. Like graphite we have an extra channel of conduction may be due to a charge accumulation layers on the surface.

Theory of the Three Dimensional Quantum Hall Effect in Graphite

B. Andrei Bernevig¹, Taylor L. Hughes², Srinivas Raghu² and Daniel P. Arovas^{2,3}

¹*Princeton Center for Theoretical Physics, Princeton University, Princeton, NJ 08544*

²*Department of Physics, Stanford University, Stanford, CA 94305 and*

³*Department of Physics, University of California at San Diego, La Jolla, CA 92093*

(Dated: February 22, 2012)

We predict the existence of a three dimensional quantum Hall effect plateau in a graphite crystal subject to a magnetic field. The plateau has a Hall conductivity quantized at $\frac{4e^2}{h} \frac{1}{c_0}$ with c_0 the c-axis lattice constant. We analyze the three-dimensional Hofstadter problem of a realistic tight-binding Hamiltonian for graphite, find the gaps in the spectrum, and estimate the critical value of the magnetic field above which the Hall plateau appears. When the Fermi level is in the bulk Landau gap, Hall transport occurs through the appearance of chiral surface states. We estimate the magnetic field necessary for the appearance of the three dimensional quantum Hall Effect to be 15.4 T for electron carriers and 7.0 T for hole carriers.

PACS numbers: 72.25.-b, 72.10.-d, 72.15. Gd



The field actually induces a many body gap !! Phys. Rev. Lett. **99**, 146804 (2007)