Thermodynamic studies of strongly correlated 2D electron system

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V.M. Pudalov, A.Yu. Kuntsevich, M.E. Gershenson, I.S. Burmistrov, M. Reznikov, *Phys. Rev. B* **98**, 155109 (2018).

L.A.Morgun, A.Yu. Kuntsevich, and V.M.P, *Phys. Rev. B* **93,** 235145 (2016).

N.Teneh, A.Yu. Kuntsevich, V.M.P, M.Reznikov, *Phys. Rev. Lett*. **109**, 226403 (2012).

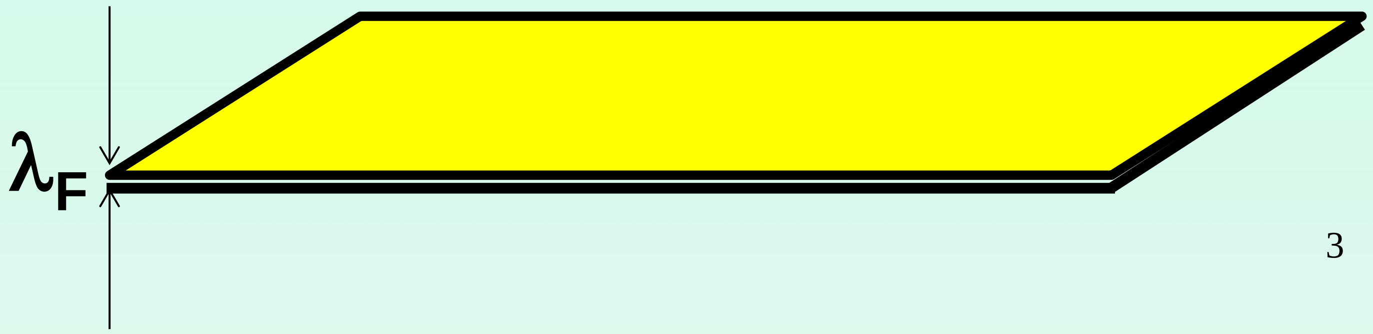
A.Yu.Kuntsevich, Y.V.Tupikov, V.M.P., I.S.Burmistrov, *Nature Comm*. **6,** 7298 (2015).

Y.Tupikov, A.Yu.Kuntsevich, V.M.Pudalov, I.S.Burmistrov, JETP Lett. **101**, 125 (2015)

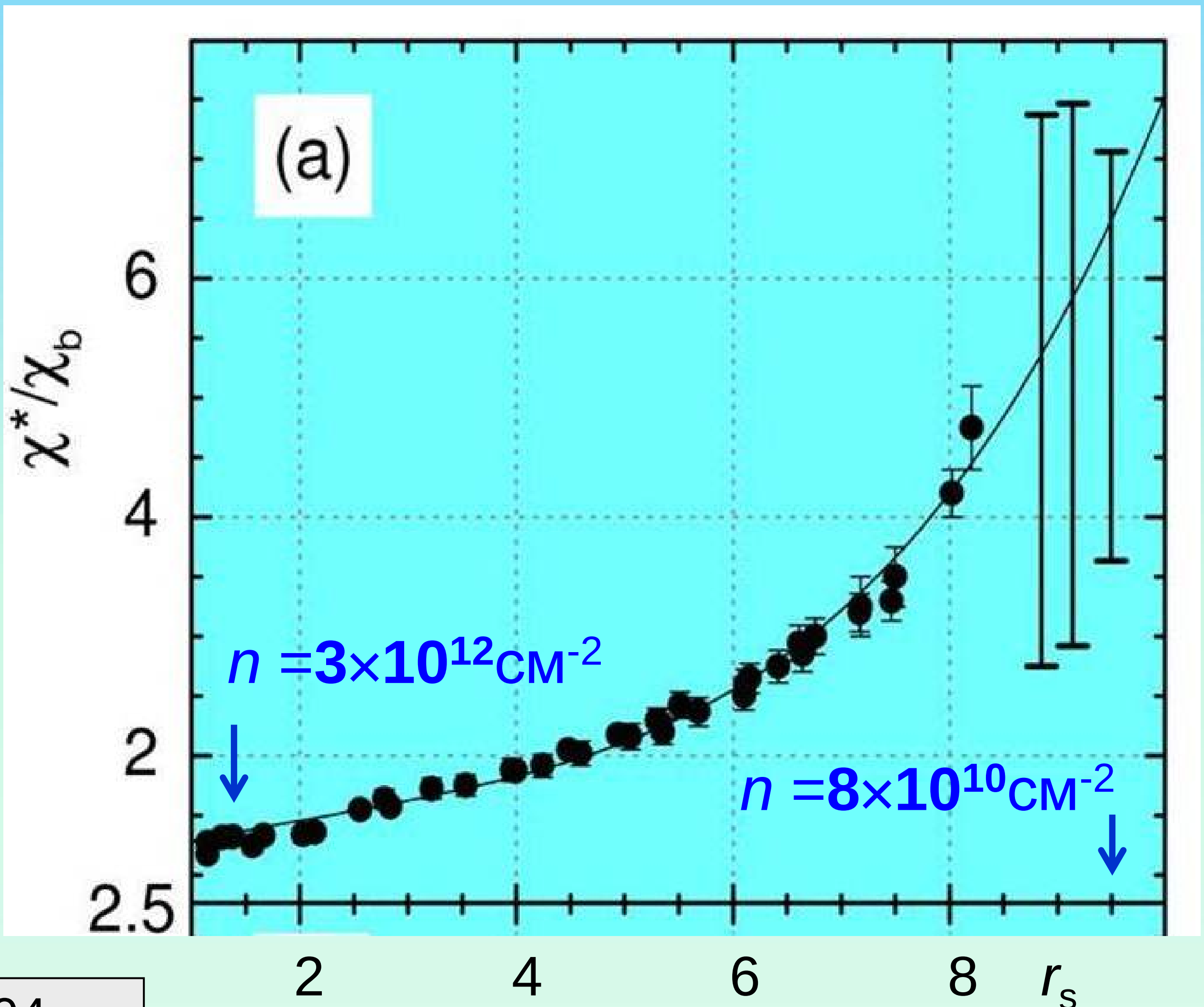
Motivation

- Exper data shows strong growth of χ_s with $\boldsymbol{r}_s$ (i.e. $F_0^\sigma \to -1$). Stoner instability in the 2D FL-state ?
- 2D systems are probed mainly by transport. Can the thermodynamics be measured when the number of particles ~ 10^8 ?
- Transport studies reveal inconsistency with homogeneous FL concepts. Can the thermodynamics shed a light ?

$$r_s = \frac{E_{ee}}{E_F} \propto \frac{1}{\sqrt{n}}$$

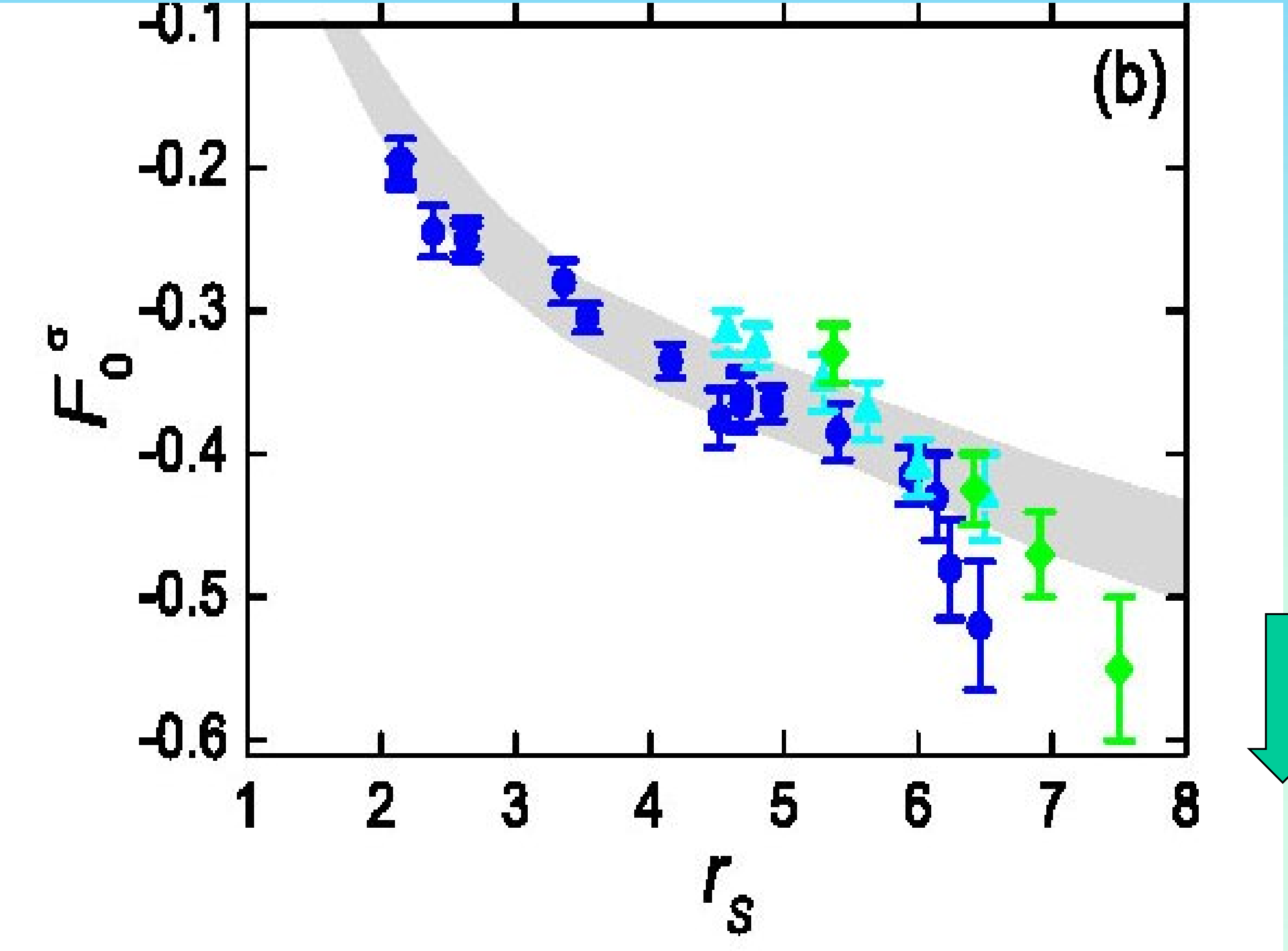


Strong growth of $\chi^* \propto m^*g^*$ with density lowering (r_s growing)



V.M.Pudalov, et al., *PRL* **88**, 196404 (2002); PRB 2008

Strong growth of $|F_0^a|$ with lowering *n* (increase of r_s)



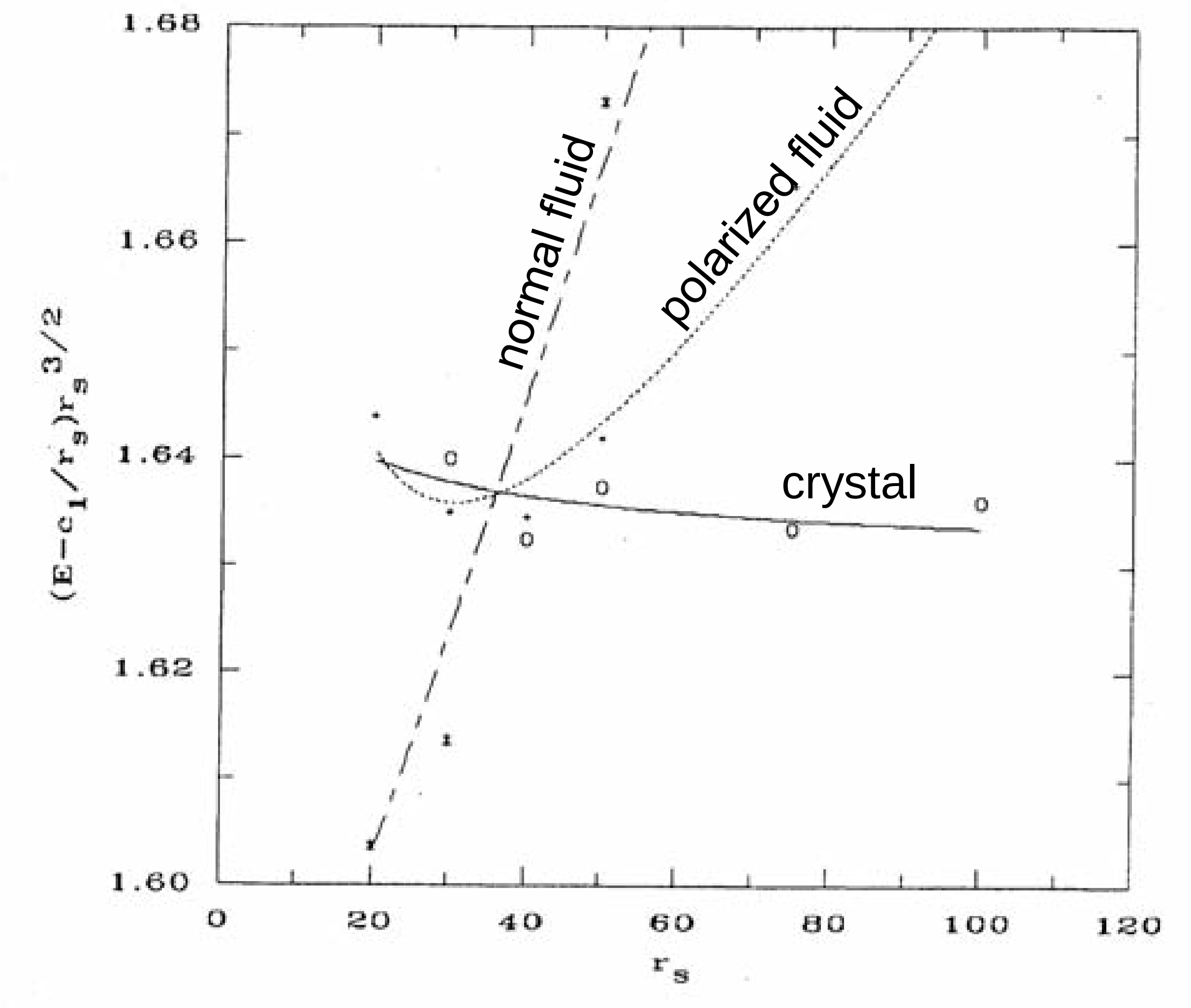
$$g^* = \frac{g_b}{1 + F_0^\sigma}$$

Towards Stoner (or Bloch) instability

N. Klimov, D. Knyazev, O. Omelyanovskii, V. Pudalov, H.Kojima, M. Gershenson, *PRB* 78, 195308 (2008)

Ground state energy of the 2D system

- ✓ Variational and fixed-node MC calculations have insufficient accuracy
- ✓ No way to measure E_g
- ✓ Constructive approach: to measure $\partial E/\partial x$



Tanatar, Ceperley, *PRB* 1989

$r_s = U/E_F \propto n^{-1/2}$

✓First Derivatives $\partial E/\partial x$:

$\partial E / \partial n = \mu$ → chemical potential

$\partial \mu / \partial n$ → compressibility

$\partial \mu / \partial B$ → magnetization

$\partial \mu / \partial T$ → entropy

1. Compressibility $\partial\mu / \partial n$

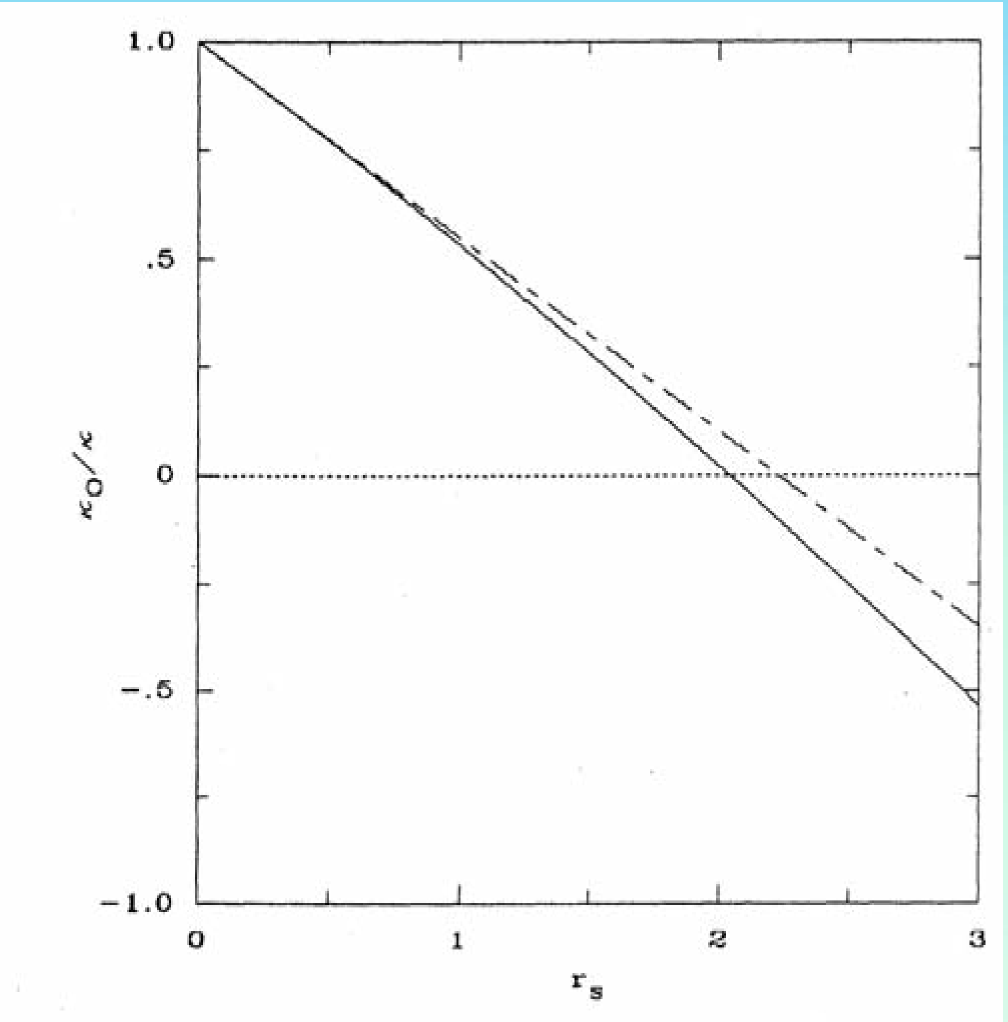
$$\kappa^{-1} = n^2 \frac{\partial^2 E_{tot}}{\partial n^2} = N^2 \left(\frac{\partial \mu}{\partial n}\right).$$

2D F-gas

$$\kappa^{-1} = n^2 \frac{\pi \hbar^2}{g_v m}$$

2D FL

$$\frac{\partial \mu}{\partial n} = \frac{\pi \hbar^2}{m} - \left(\frac{2}{\pi}\right)^{1/2} \frac{e^2}{4\pi\epsilon} \frac{1}{n^{1/2}}$$



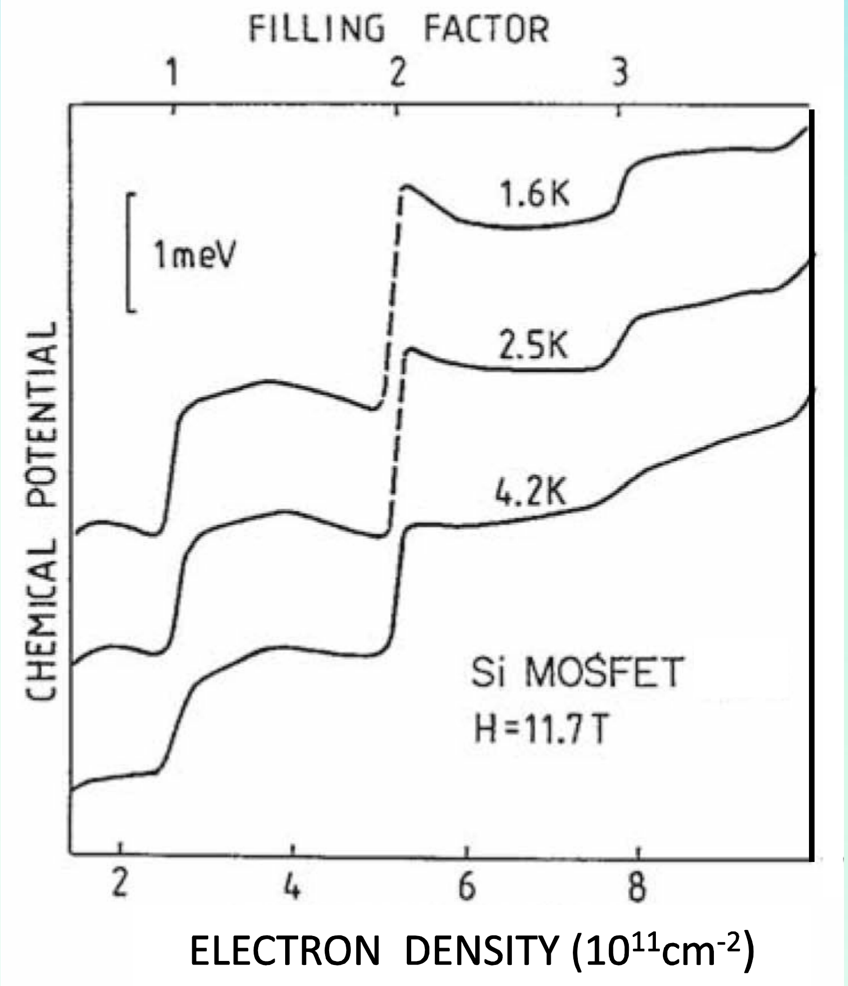
1. Compressibility ∂μ / ∂n

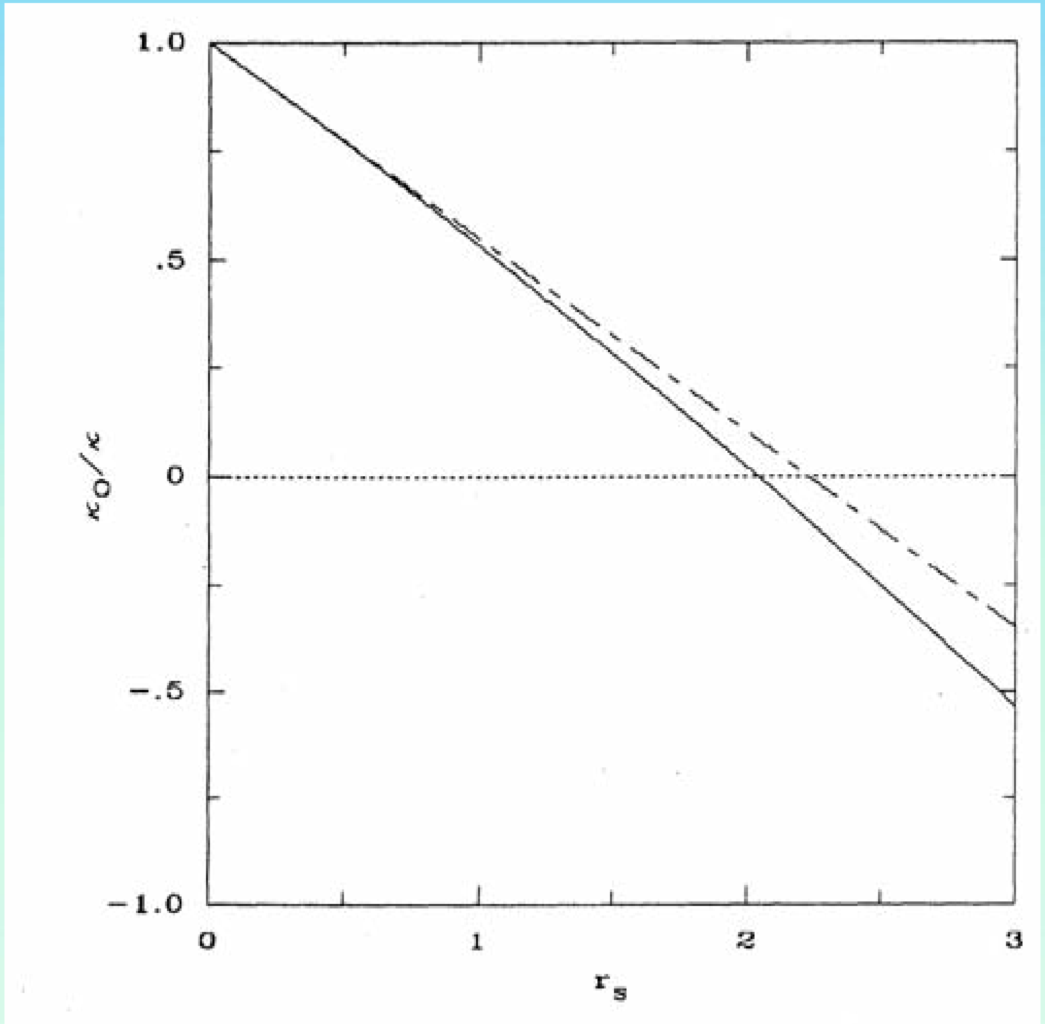
$$\kappa^{-1} = n^2 \frac{\partial^2 E_{tot}}{\partial n^2} = N^2 \left(\frac{\partial \mu}{\partial n} \right).$$

2D F-gas

$$\kappa^{-1} = n^2 \frac{\pi \hbar^2}{g_v m}$$

2D FL

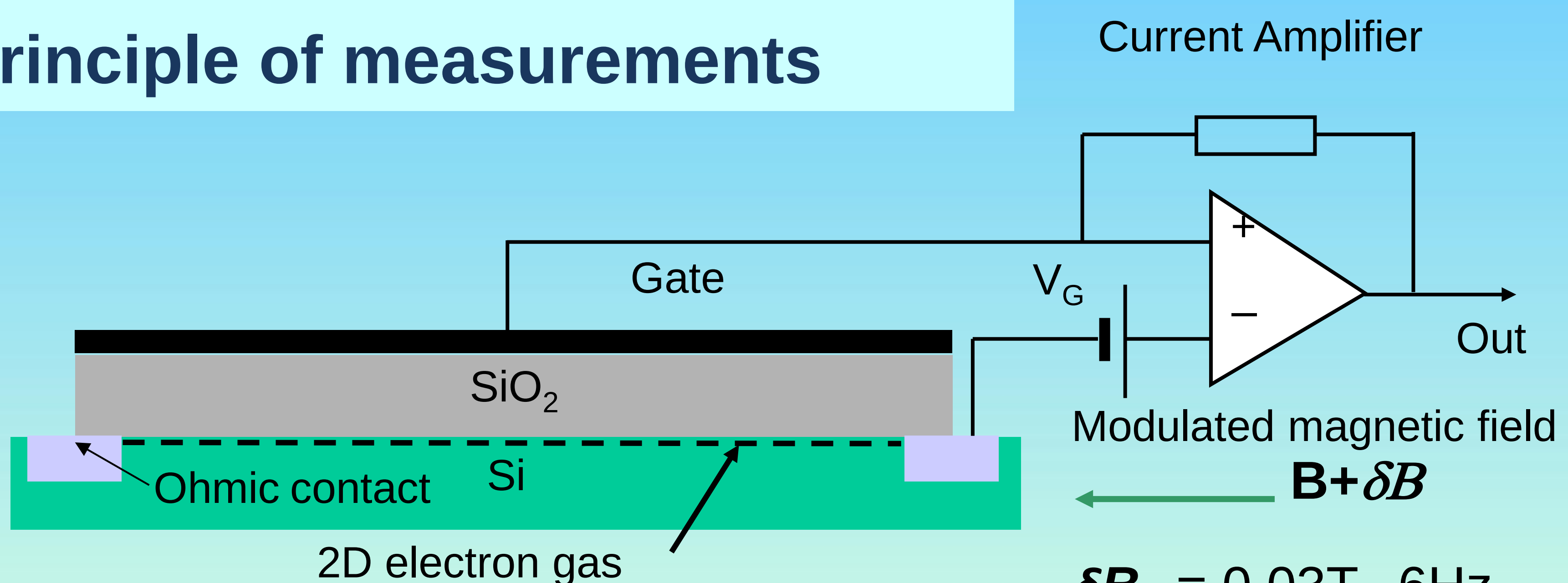




VP et al, *JETPLett.*(1985)

2. Spin magnetization $\partial\mu / \partial B$.

Principle of measurements



$\delta B_{\sim}$ = 0.03T, 6Hz

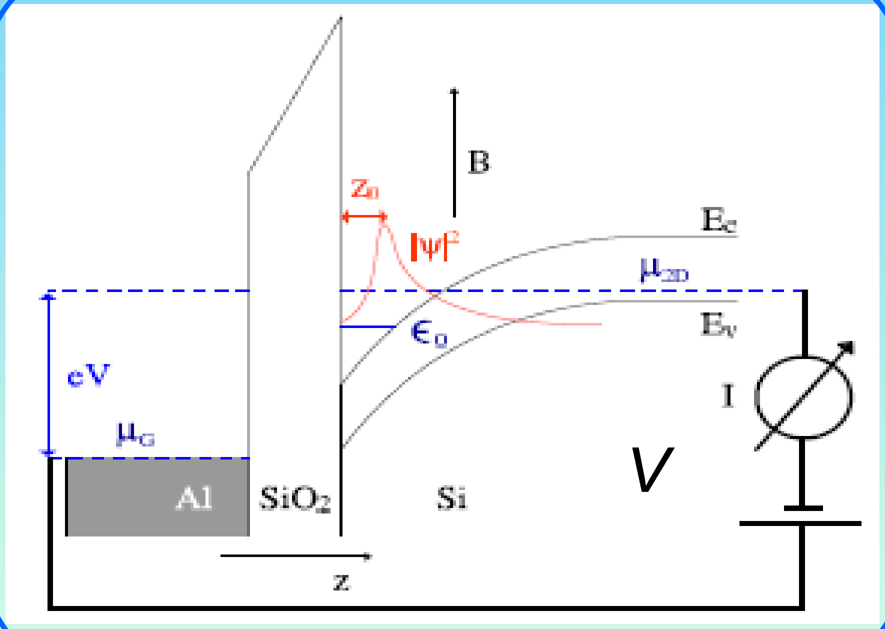
Advantages

- High sensitivity (10^8 spins)
- Measures thermodynamic magnetization
- Accessibility of the Insulating phase
- Low-field measurements

M.Reznikov, A.Yu.Kuntsevich, N.Teneh, V.M.P, *JETP Lett.* (2010).

N.Teneh, A.Yu. Kuntsevich, V.M.P, M.Reznikov, *Phys. Rev. Lett.* 109, 226403 (2012).

Electric circuitry

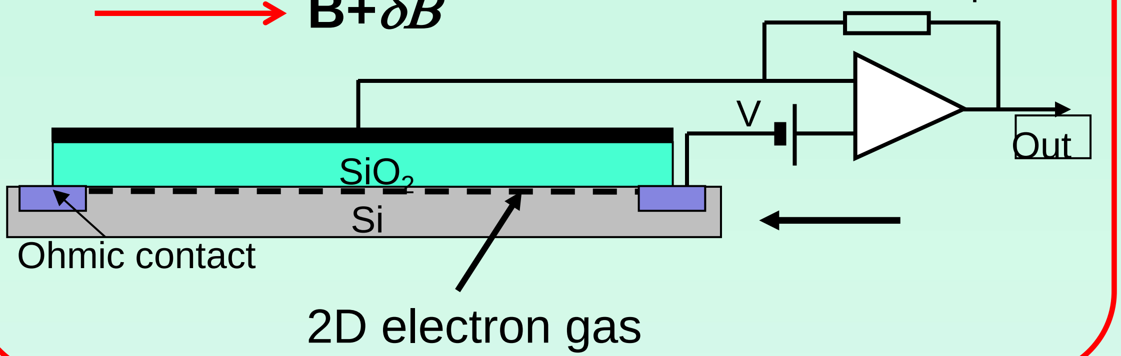


Maxwell relation

$$\partial \mu / \partial B = -\partial M / \partial n,$$

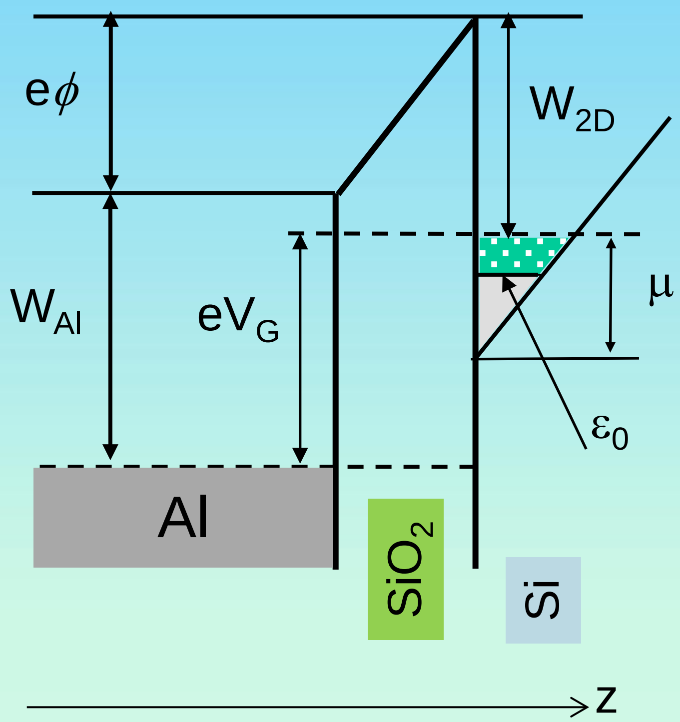
$$V = Q/C_0 + \Delta\mu/e$$

$$\frac{e^2}{\tilde{C}} \frac{dn}{dB} = -\frac{\partial \mu}{\partial B}$$



N.Teneh, AK, VP, M.R., *PRL* **109**, 226403 (2012)

Principle of measurements



$$U = \Delta\varphi + \Delta\mu / e$$

$$\Delta\tilde{\varphi} = -\Delta\tilde{\mu} / e = -\frac{\partial\mu}{\partial B}\frac{\Delta B\cos(\omega t)}{e}$$

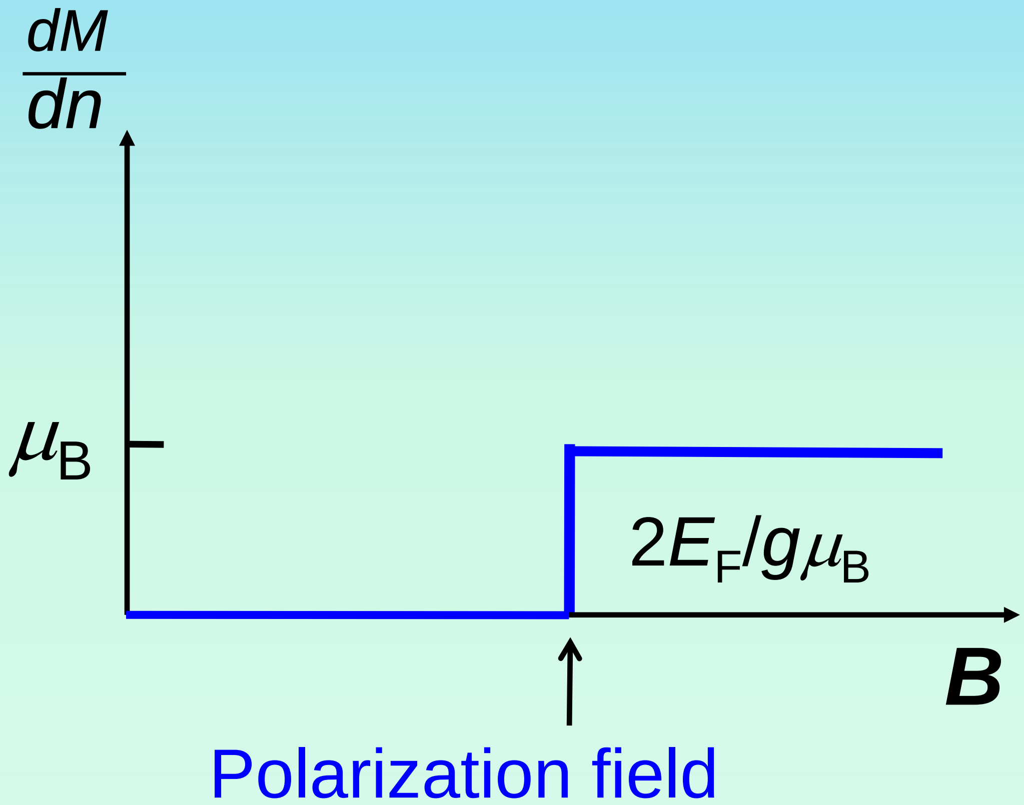
$$\tilde{I} = \frac{\partial\mu}{\partial B}\frac{\Delta B\omega C\sin(\omega t)}{e}$$

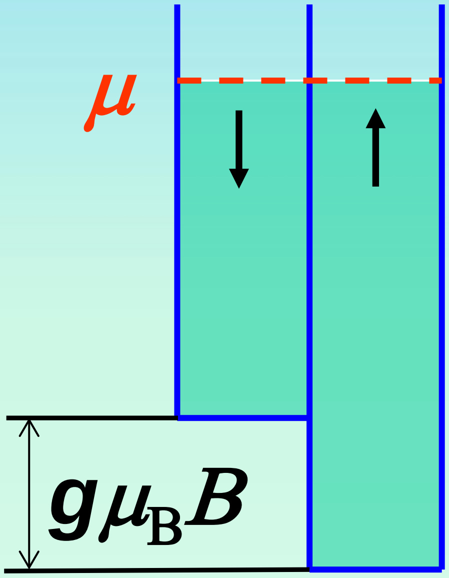
Maxwell relation:

$$\left.\frac{\partial\mu}{\partial B}\right|_n = -\left.\frac{\partial M}{\partial n}\right|_B$$

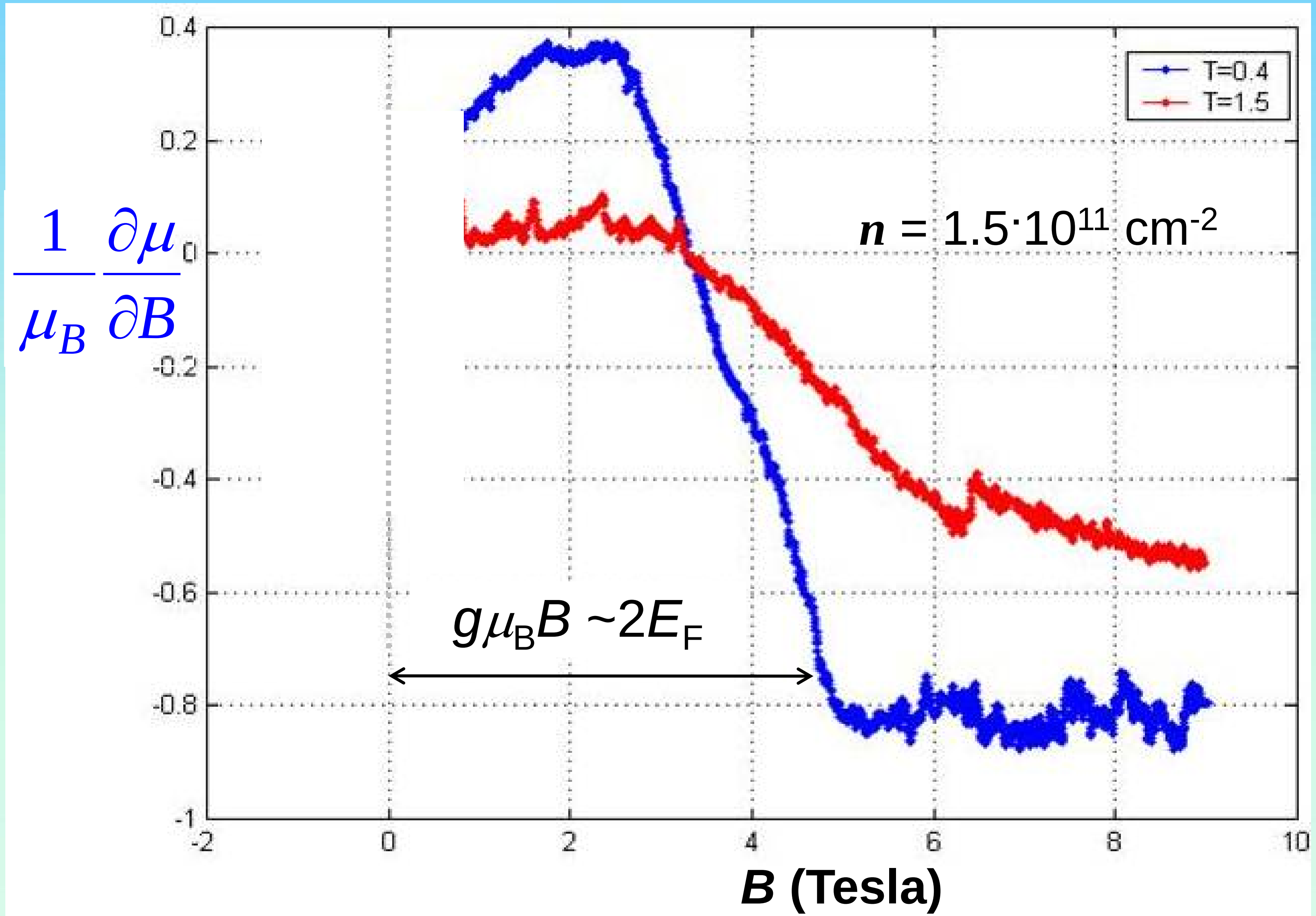
F(n,B) – free energy

dM/dn, expectations for the degenerate Fermi-gas (no interactions)



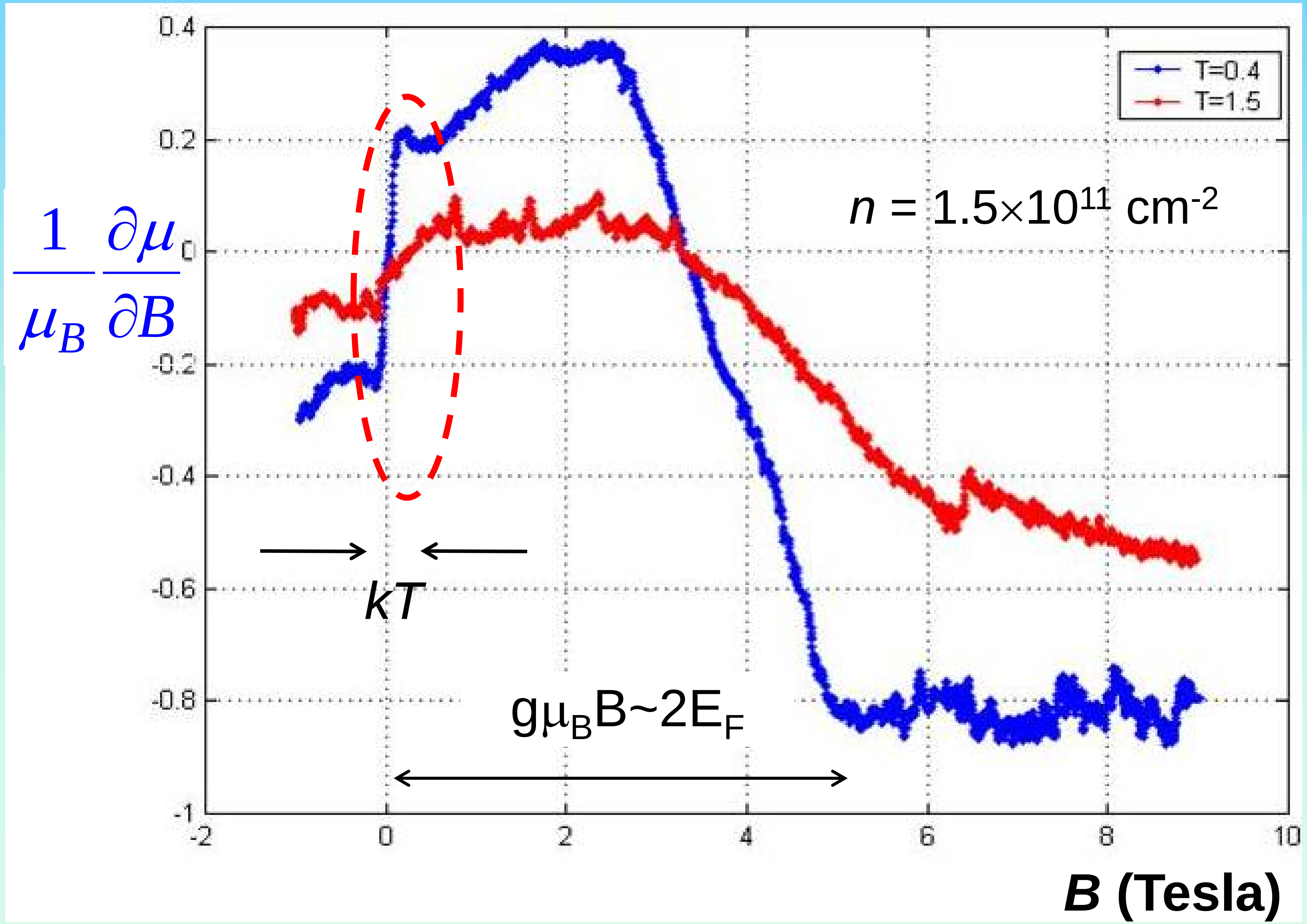


Earlier measurements (high fields).



O.Prus, Y.Yaish, M.Reznikov, U.Sivan, V.Pudalov, *PRB*, 67, 205407 (2003)

Low field measurements: *B* < *T*



N.Teneh, A.Yu. Kuntsevich, V. M. Pudalov, and M. Reznikov, *Phys.Rev.Lett.* 109, 226403 (2012).

$dM/dn > \mu_B$

FM - interaction !

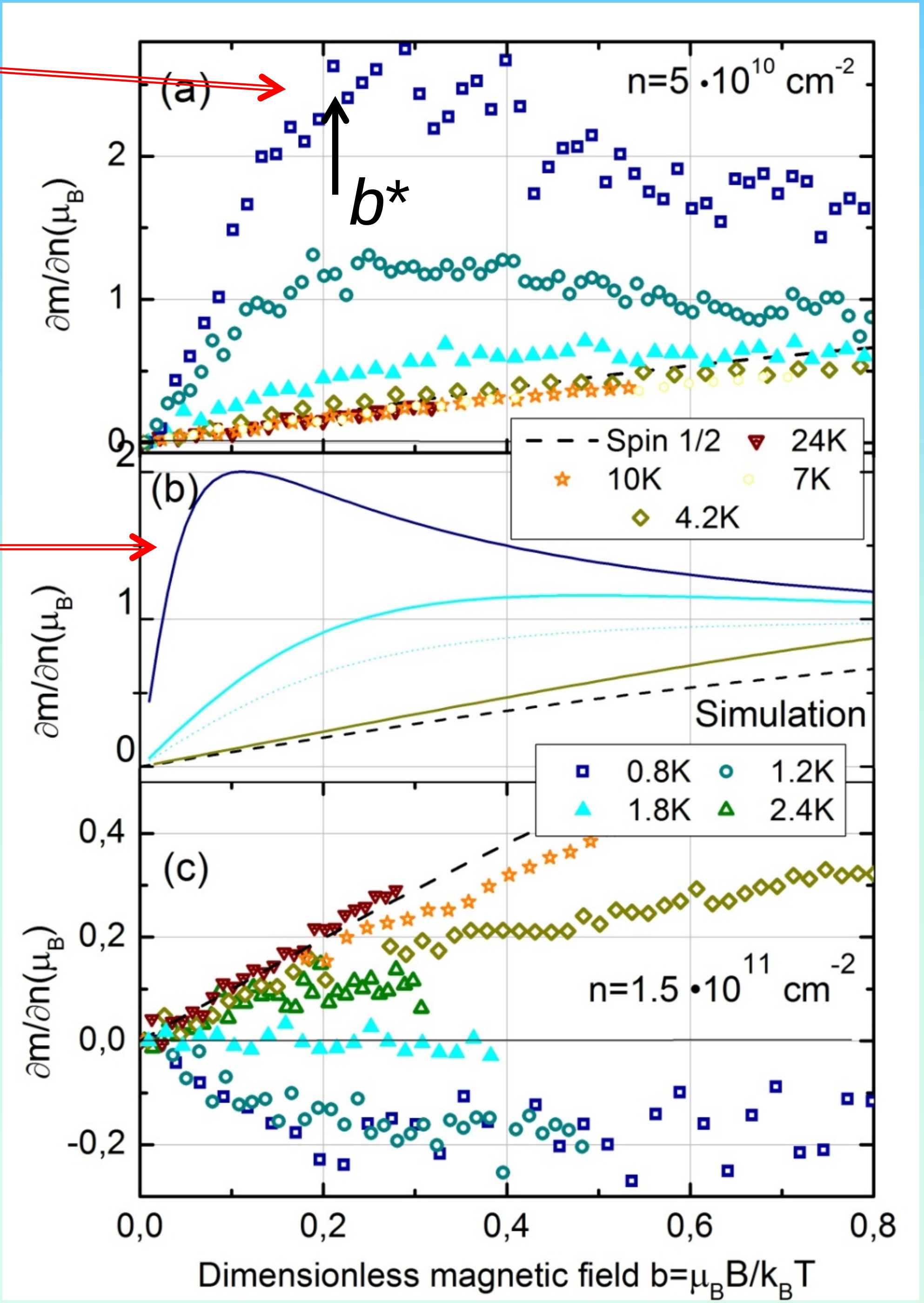
Mean field simulation

$$\tilde{m} = \tanh(b + \tilde{m}/\tilde{t})$$

$t = T/T_c$, $T_c \propto n^k$

$J \sim 1/2b^* \sim 2$

***dM/dn changes sign* with *T* !**



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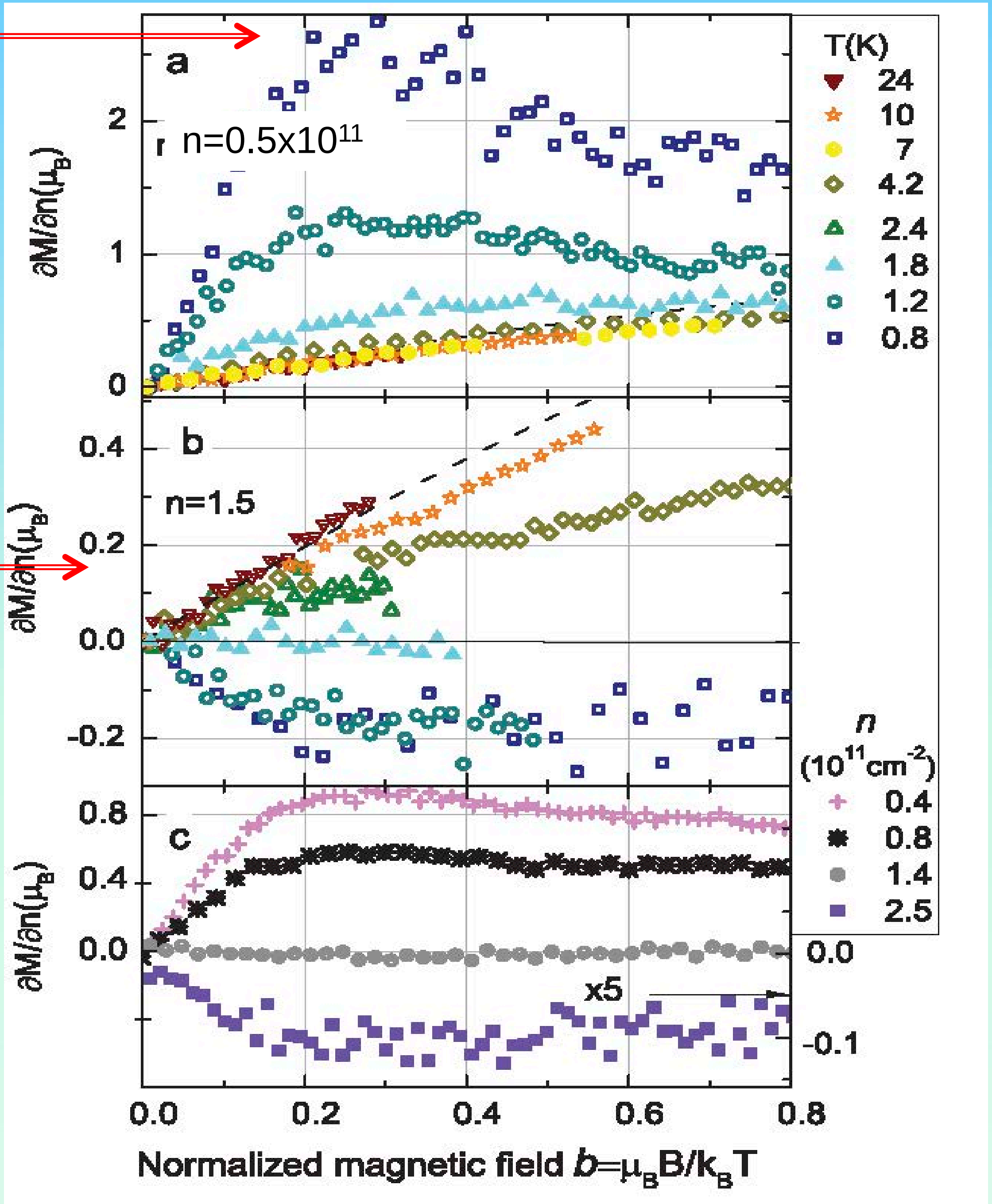
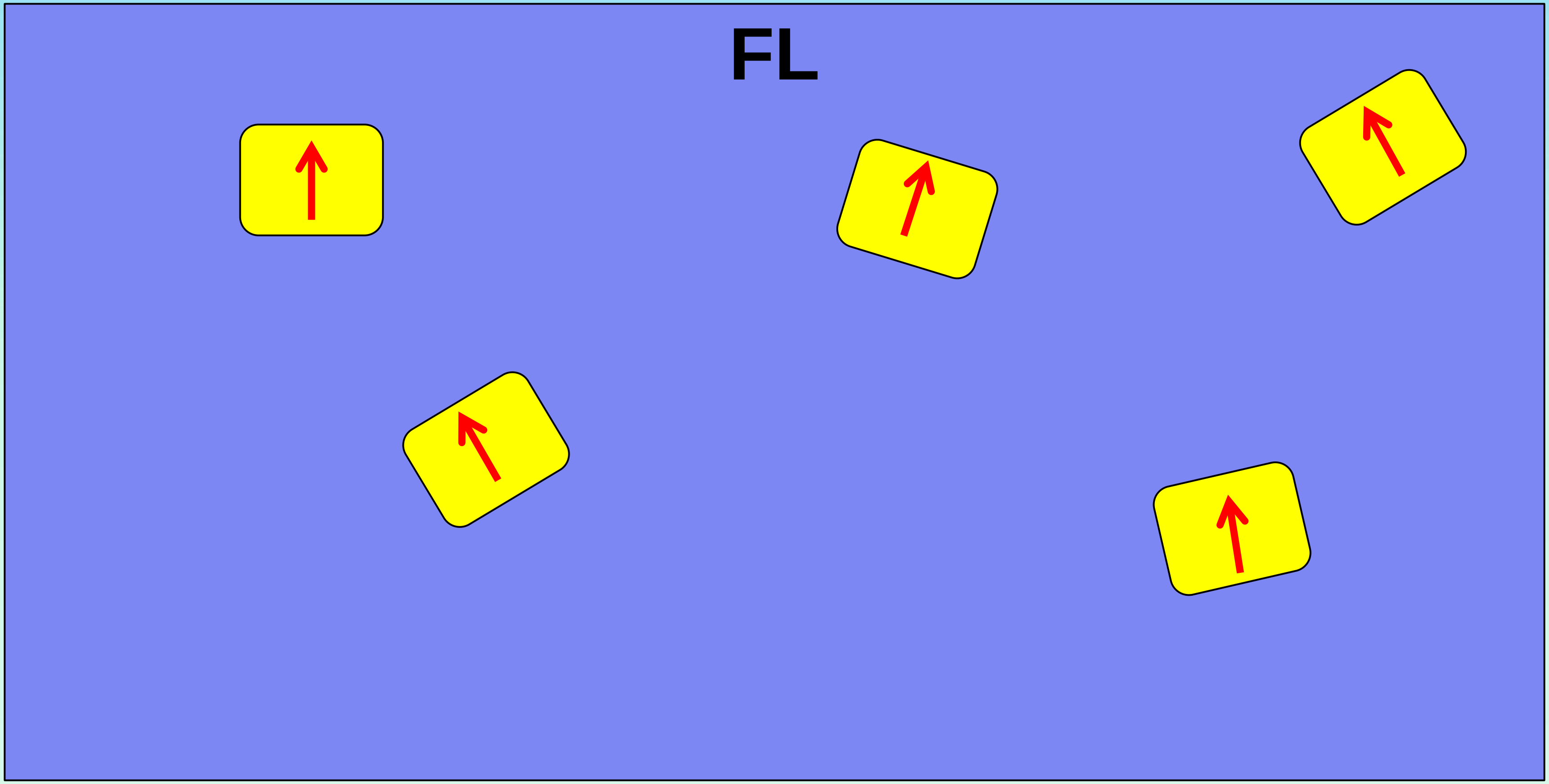
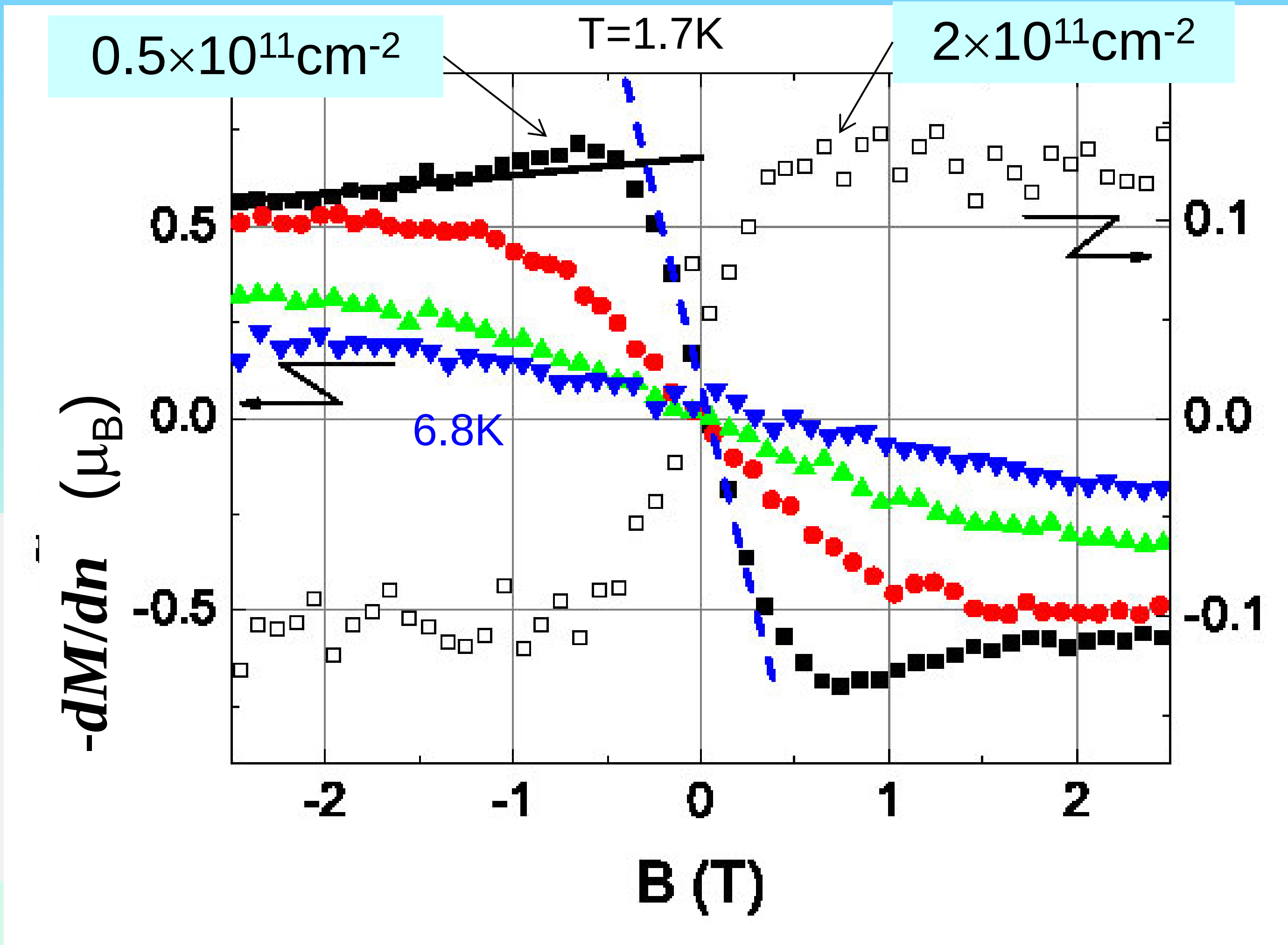
***dM/dn changes sign* with *n* !**



Two phase state



Sign reversal of *dM/dn*



➢ $\partial M/\partial n$ <0 at $n>n_c$ ⇒ each electron added to the 2D system causes decrease in the number of SDs

➢ $\partial M/\partial n$ **> 0** for $n \to 0$

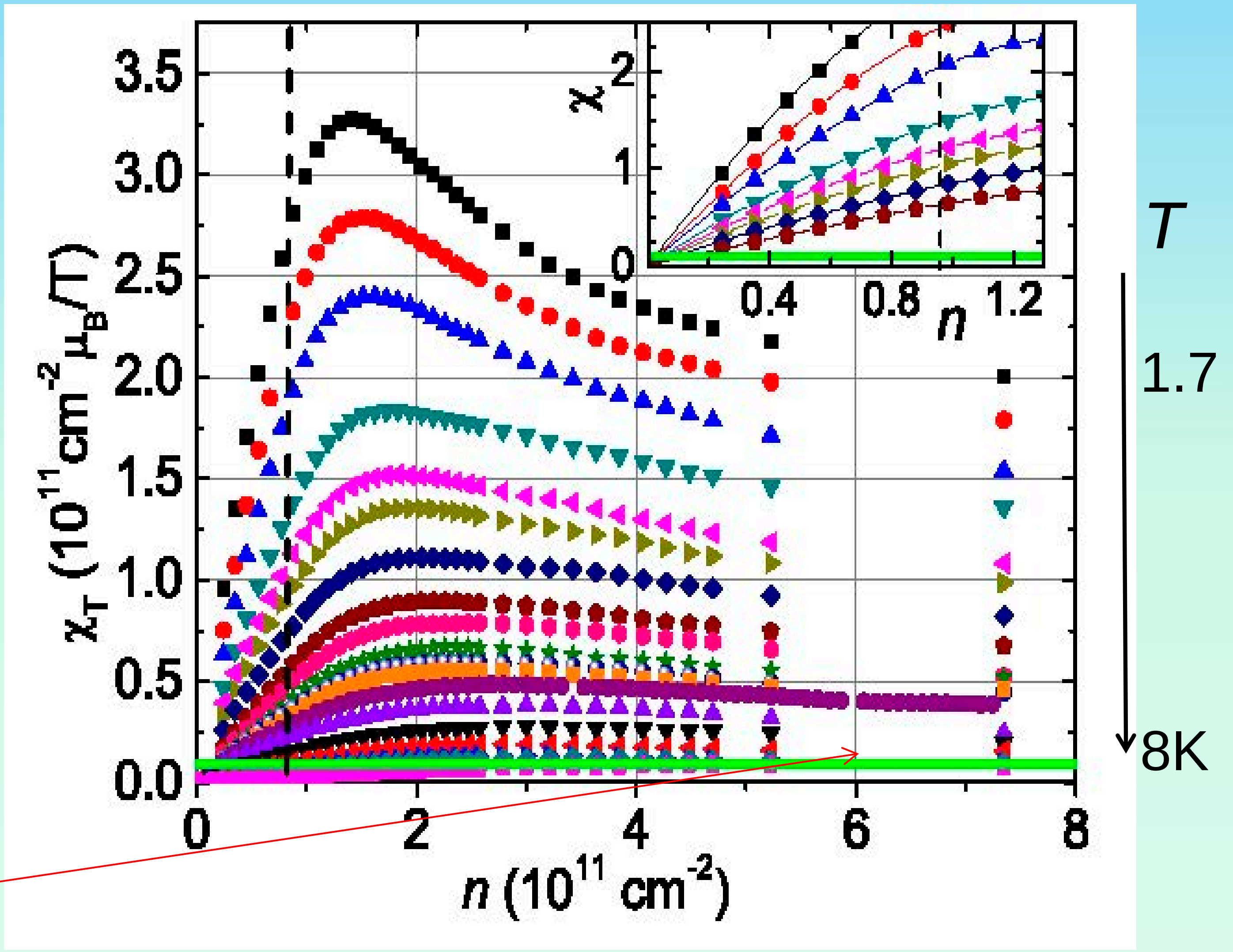
$\partial M/\partial n \to$ **0** at $n = n_c$

$\partial M/\partial n$ **< 0** for $n > n_c$

⇒ A critical behavior of $\partial M/\partial n$

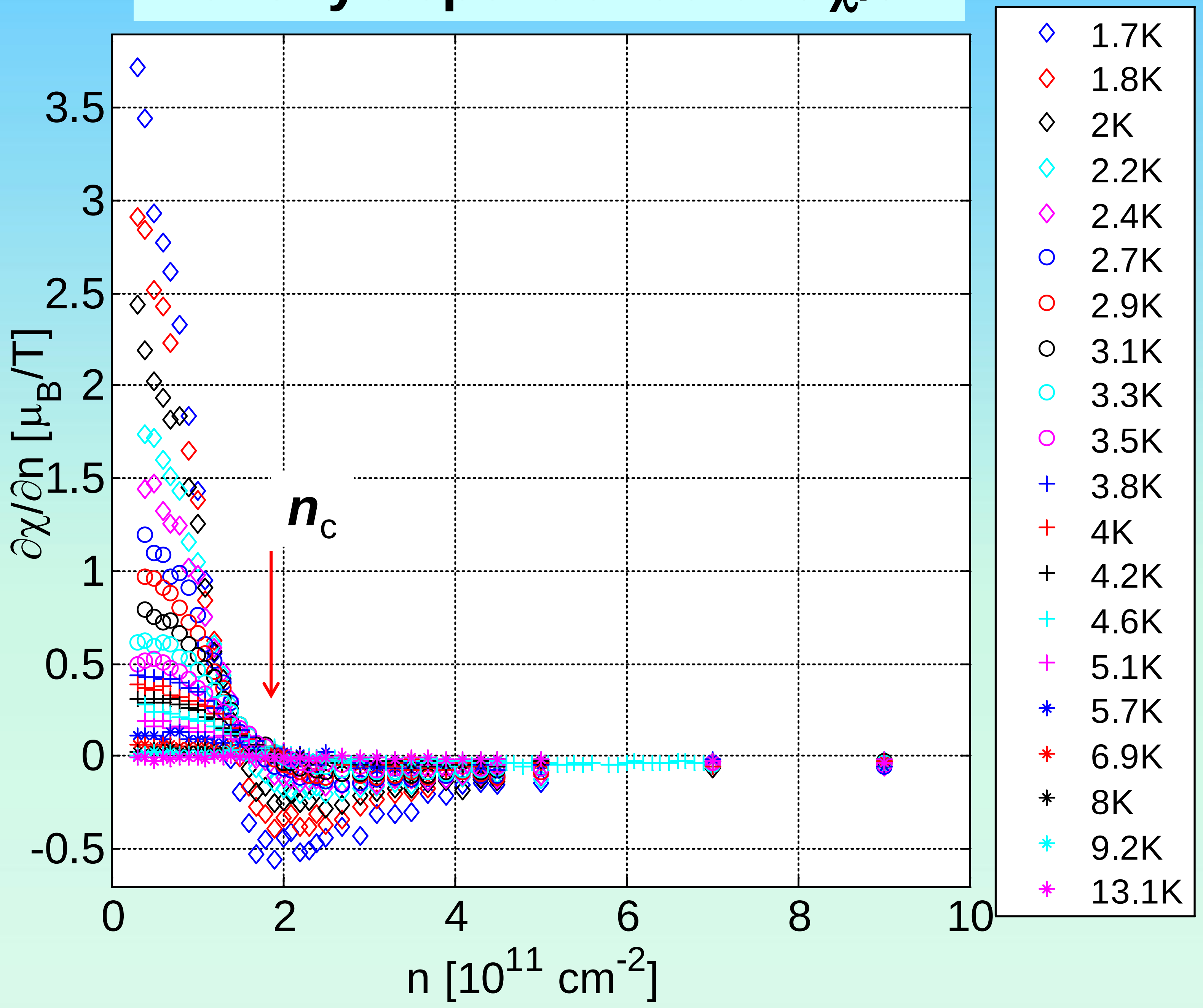
Thermodynamic spin susceptibility

This is the response of the overall electrons

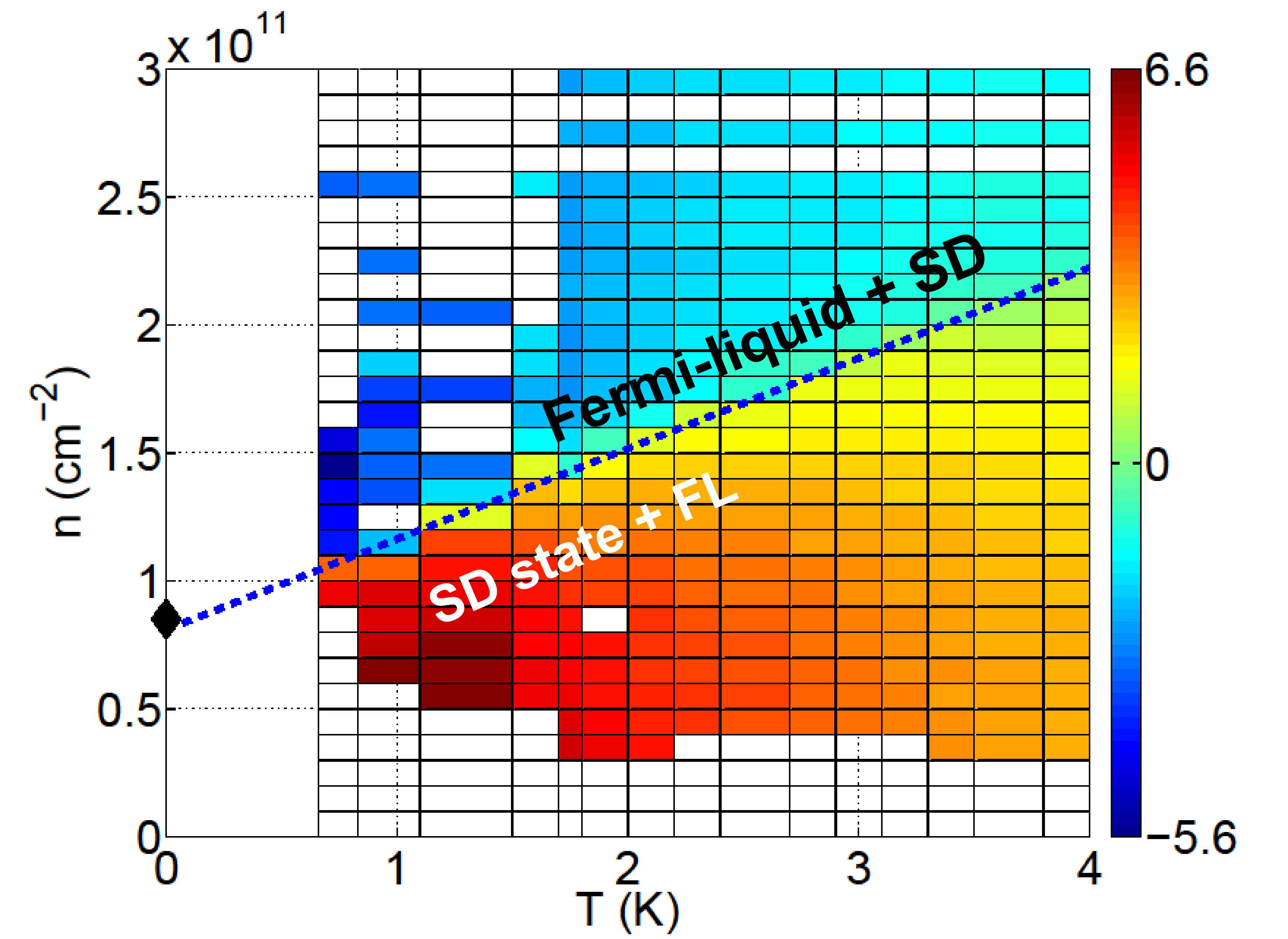


Susceptibility of the localized spins greatly exceeds and masks that of the itinerant electrons

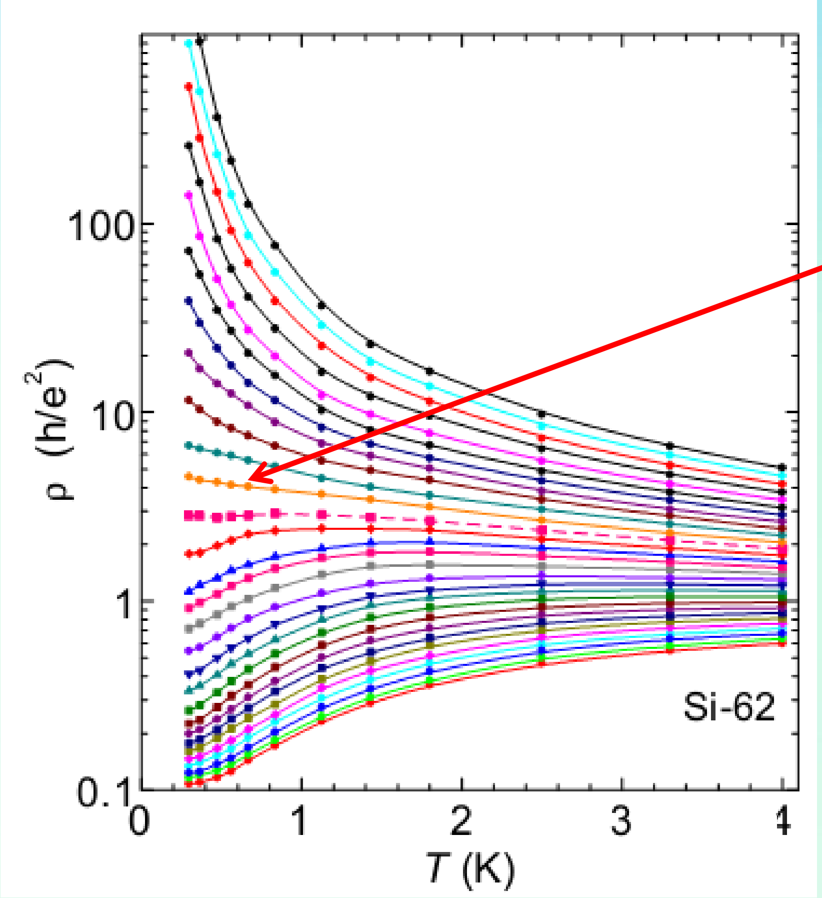
Density dependence of $d\chi/dn$

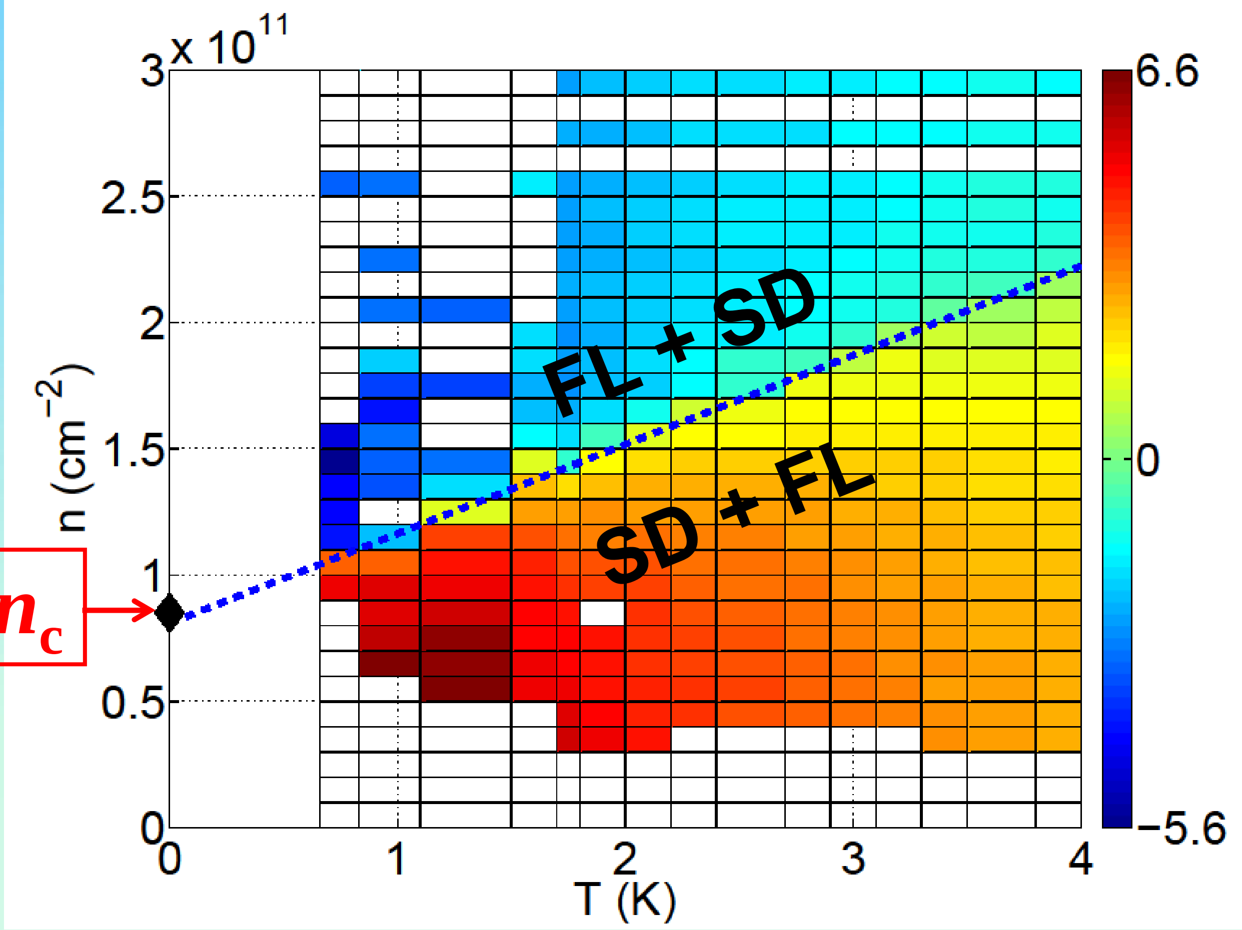


Sign change of $d\chi/dn$ (and dM/dn): a critical behavior



Sign change of *dM/dn*: critical behavior



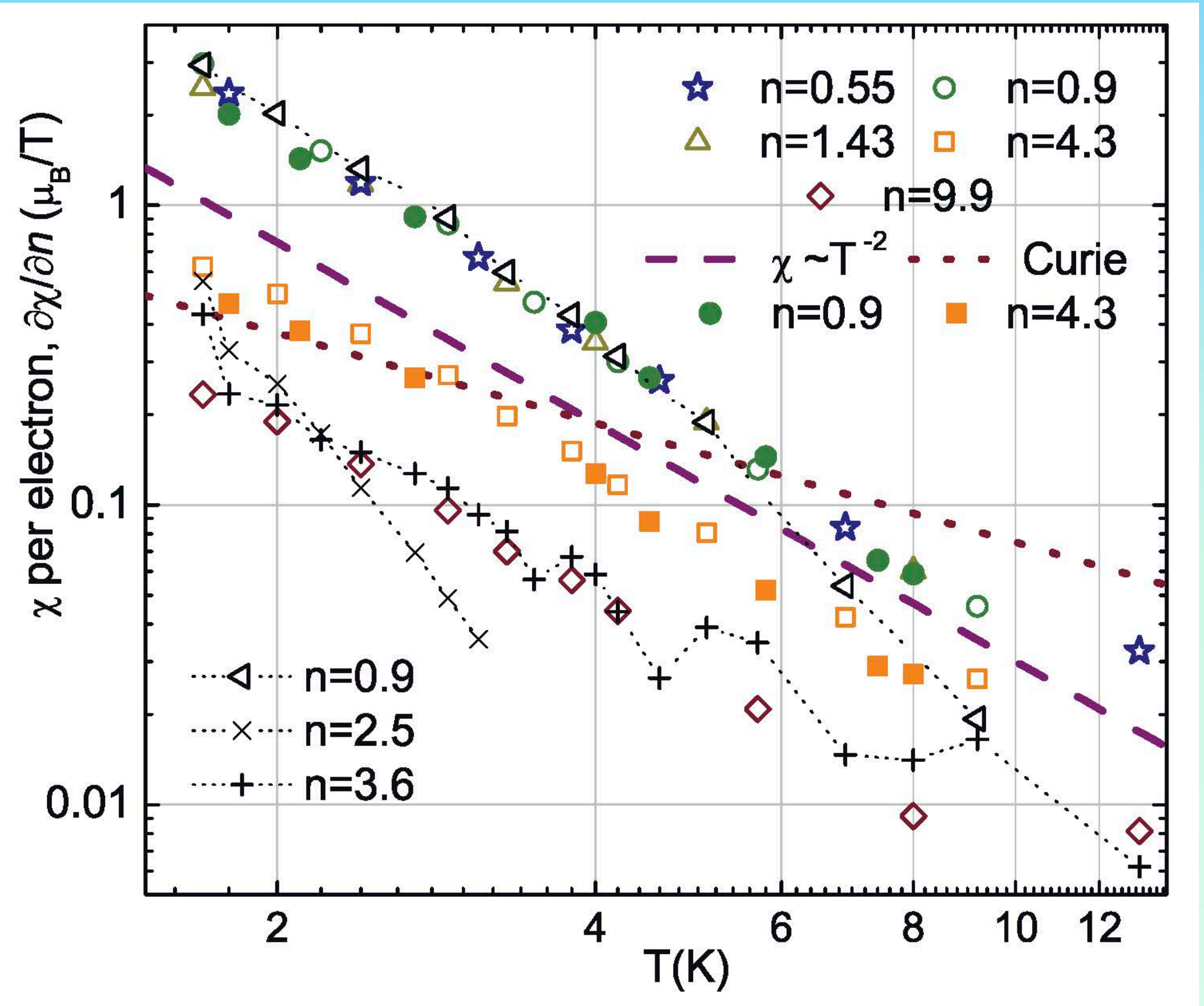


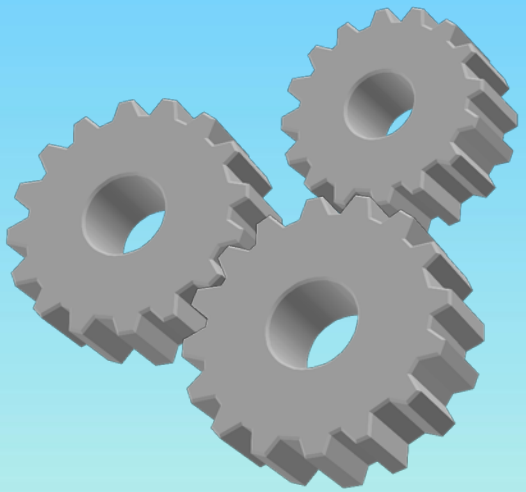
n_c

Thermodynamic spin susceptibility: T-dependence

This is the response of the overall electrons

Susceptibility of the localized spins **diverges as ~$(1/T)^2$**





Part 2: Entropy

Entropy per electron

$$(\partial S/\partial n)_T = -(\partial \mu/\partial T)_n$$

$$S(n) = \int_{n_0}^{n} \frac{\partial S}{\partial n} dn + S(n_0)$$

Problem:
***n* =0** is inaccessible

Differential entropy per electron

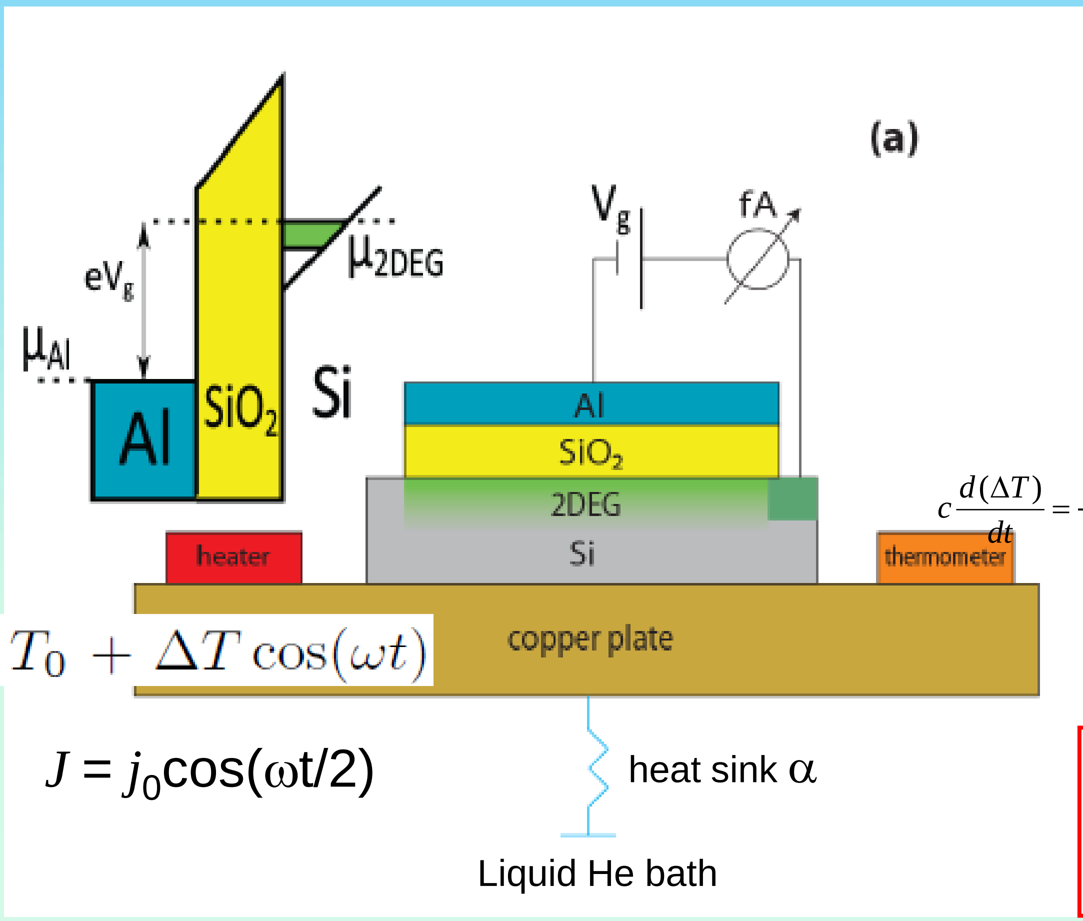
$$(\partial S/\partial n)_T = -(\partial \mu/\partial T)_n$$

$$S(n) = \int_{n_0}^{n} \frac{\partial S}{\partial n} dn + S(n_0)$$

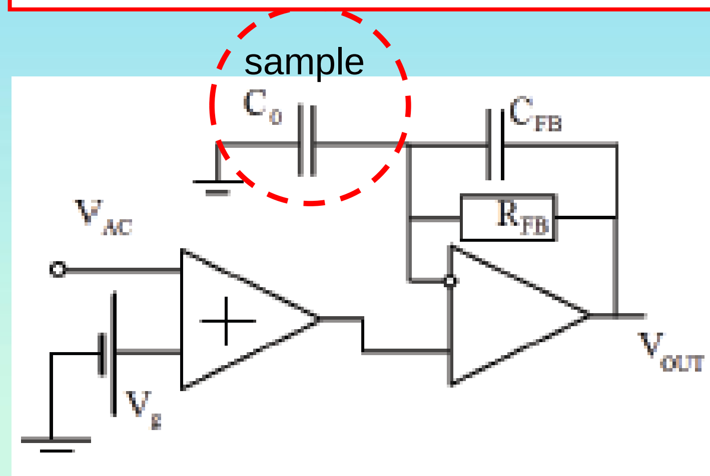
Problem:

***n* =0** is inaccessible

Experimental set-up & principle of measurements



$$i(t) = \frac{\partial\mu}{\partial T}\Delta T C_0 \sin(\omega t)$$



$$C\frac{d(\Delta T)}{dt} - \alpha\Delta T = \frac{1}{2} i_0^2 r\cos(2\pi f t)$$

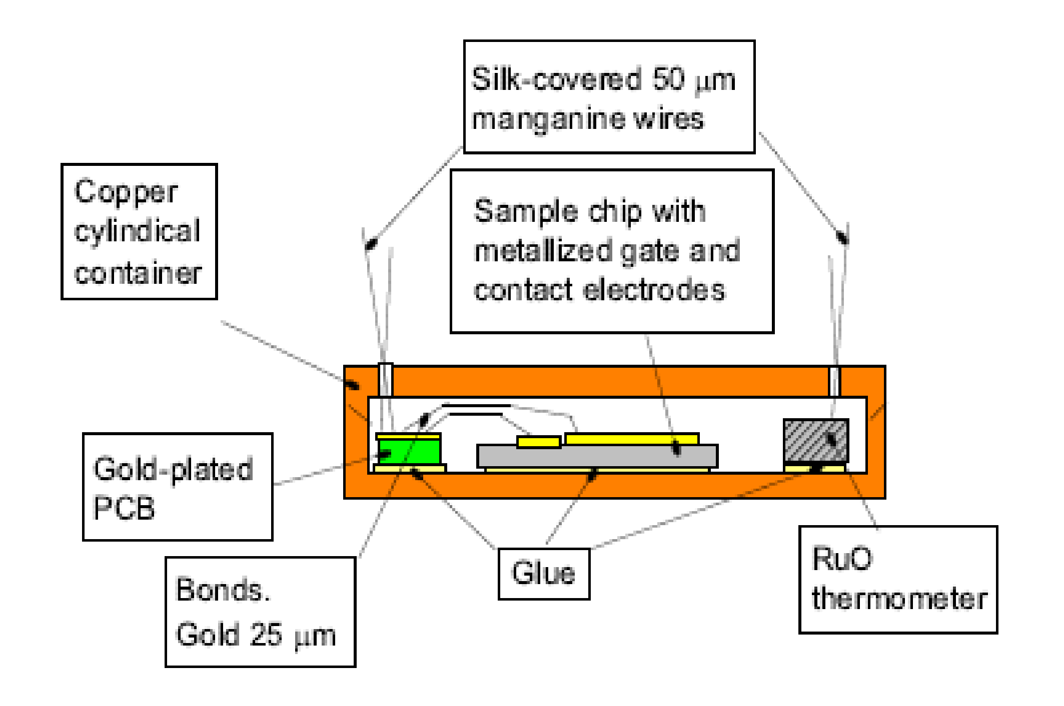
$$\Delta T = \Delta T_0\cos(2\pi f t + \phi)$$

$$\Delta T_0 = \frac{i_0^2 r}{2\sqrt{(2\pi f C)^2 + \alpha^2}}, \qquad \tan\phi = \frac{2\pi f C}{\alpha}$$

For $f > \alpha/C \sim 0.1$ Hz, $\Delta T_0 \sim 1/Cf$ and $i \neq i(f)$

Samples and their parameters

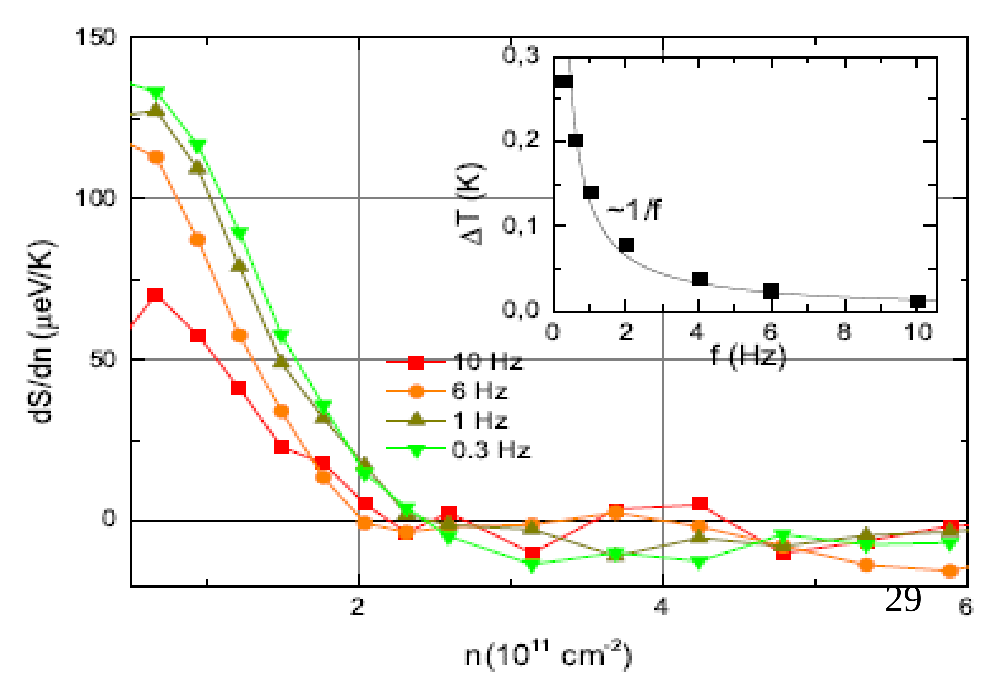
Sample	Density range, 10^{11} cm^{-2}	E_F range, meV	Capacitance, pF	Area, mm^2	Peak mobility, m^2/Vs
SiUW1	0.3-12	0.2-7.5	700	4	3
SiUW2	0.3-12	0.2-7.5	680	4	3
Si8-9	1.5-12	0.9-7.5	630	4	0.5
GaAs1	0.4-5	1.7-20	1100	5	20



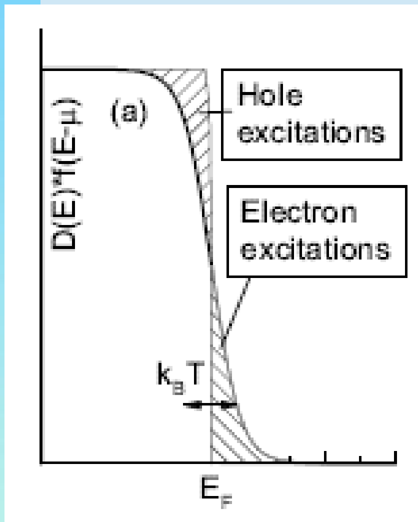
T = 2.5-25K B =0-9Tesla

ΔT~0.05-0.25K

f ~ 0.15 - 5 Hz



Expectations: Entropy for the 2D case



$S = -\Sigma\{ f \cdot ln(f) + (1-f) \cdot ln(1-f)\}$

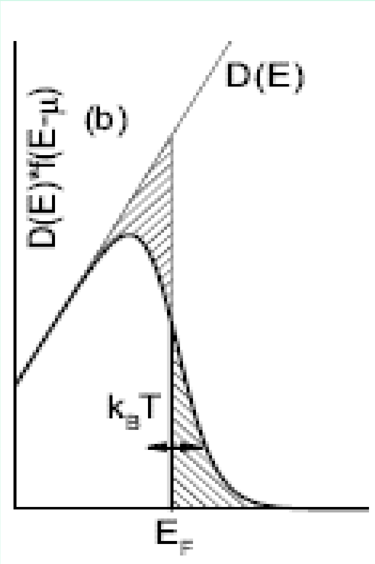
$$S = \pi^2 T D/3$$

For a degenerate 2D Fermi-gas with D = Const

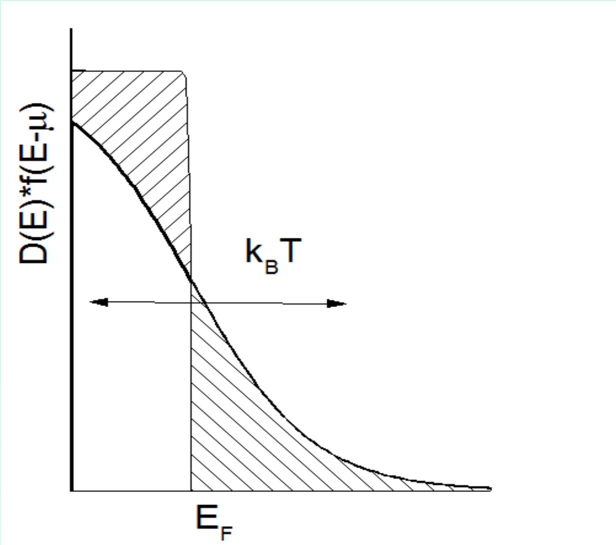
$dS/dn = - d\mu/dT = 0.$

When dS/dn ≠ 0 ?

D depends on the carrier density



Non-degenerate system



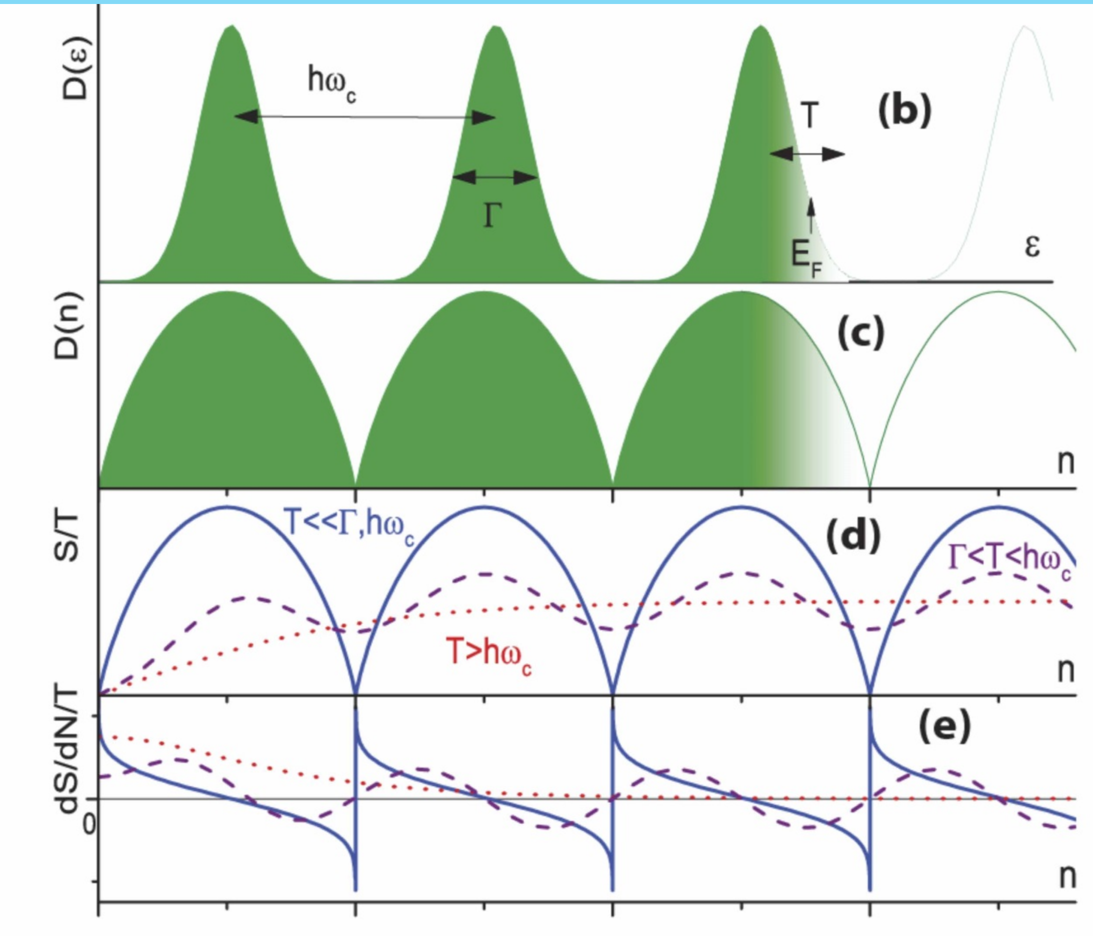
Interacting system

Entropy magneto-oscillations

Ideal degenerate 2D gas

$$S = \pi^2 T D/3$$

$$n = \int_0^\infty D(E)/(1+e^{(E-\mu)/T})dE$$

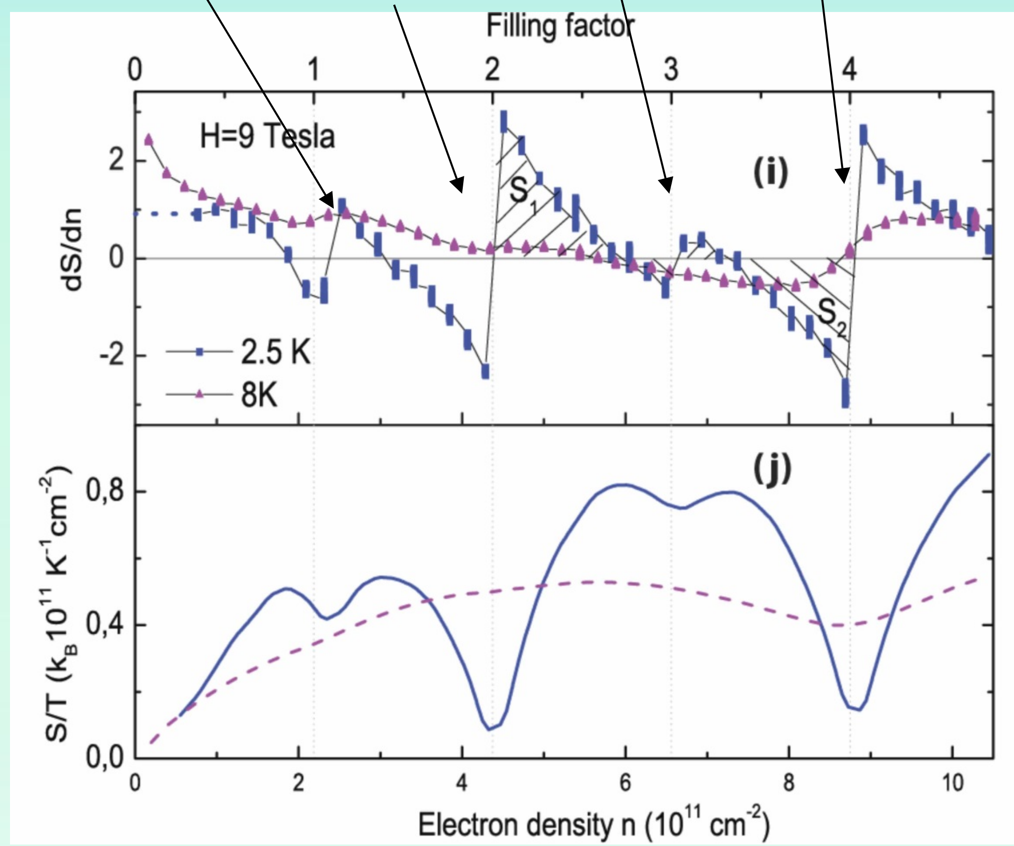


VALLEY GAP

SPIN GAP

VALLEY GAP

CYCLOTRON GAP

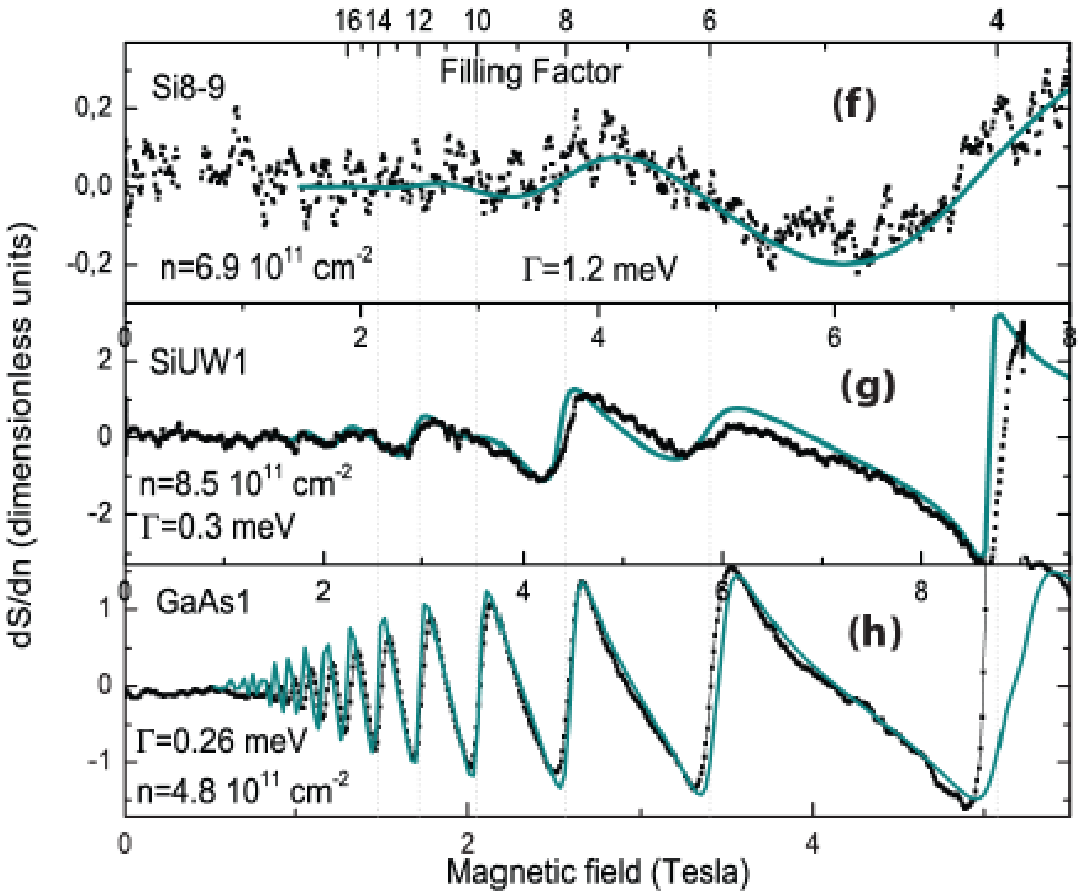


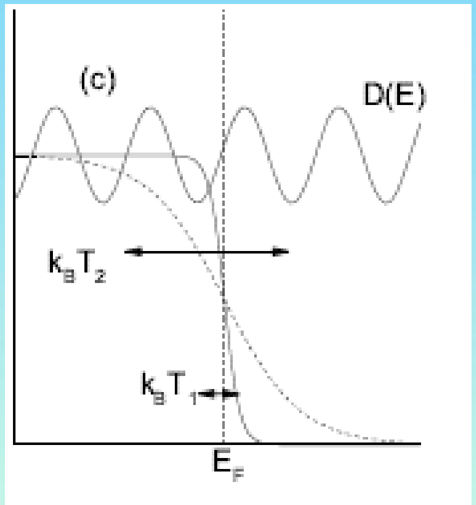
$$S(n) = \int_{n_0}^{n} \frac{\partial S}{\partial n} dn + S(n_0)$$

dS/dn vs *B*

Ideal 2D gas in GaAs/AlGaAs

Y. Tupikov et al, JETPL 2015





Lorentzian	$\mathcal{W}(\varepsilon) = \frac{1}{\pi}\frac{\Gamma}{\varepsilon^2+\Gamma^2}$	$\mathcal{A}_k = e^{-2\pi\Gamma\|k\|/\omega_c}$
Gaussian	$\mathcal{W}(\varepsilon) = \frac{1}{\Gamma\sqrt{\pi}} e^{-\varepsilon^2/\Gamma^2}$	$\mathcal{A}_k = e^{-\pi^2\Gamma^2 k^2/\omega_c^2}$
Semicircle	$\mathcal{W}(\varepsilon) = \frac{2}{\pi\Gamma}\sqrt{1-\varepsilon^2/\Gamma^2}$	$\mathcal{A}_k = \frac{\omega_c}{\pi\Gamma k} J_1\left(\frac{2\pi\Gamma k}{\omega_c}\right)$

$$\left(\frac{\partial S}{\partial n}\right)_T = -\frac{\omega_c}{T}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{\pi k}\mathcal{A}_k\frac{(2\pi^2Tk/\omega_c)}{\sinh(2\pi^2Tk/\omega_c)}\left[1-\frac{2\pi^2Tk}{\omega_c}\coth\frac{2\pi^2Tk}{\omega_c}\right]\sin\frac{2\pi\mu k}{\omega_c}$$
$$\times\left[1-2\sum_{k=1}^{\infty}(-1)^{k-1}\mathcal{A}_k\frac{(2\pi^2Tk/\omega_c)}{\sinh(2\pi^2Tk/\omega_c)}\cos\frac{2\pi\mu k}{\omega_c}\right]^{-1}.$$

What is expected in zero field?

$n = 10^{11}\ cm^{-2}$

$U \sim e^2/\langle r \rangle \sim n^{1/2}$ 70K

$E_F \sim n$ 7K

$r_s \sim U/E_F \sim n^{-1/2}$ 10

$T \sim T_F$. Non-degenerate Fermi-gas

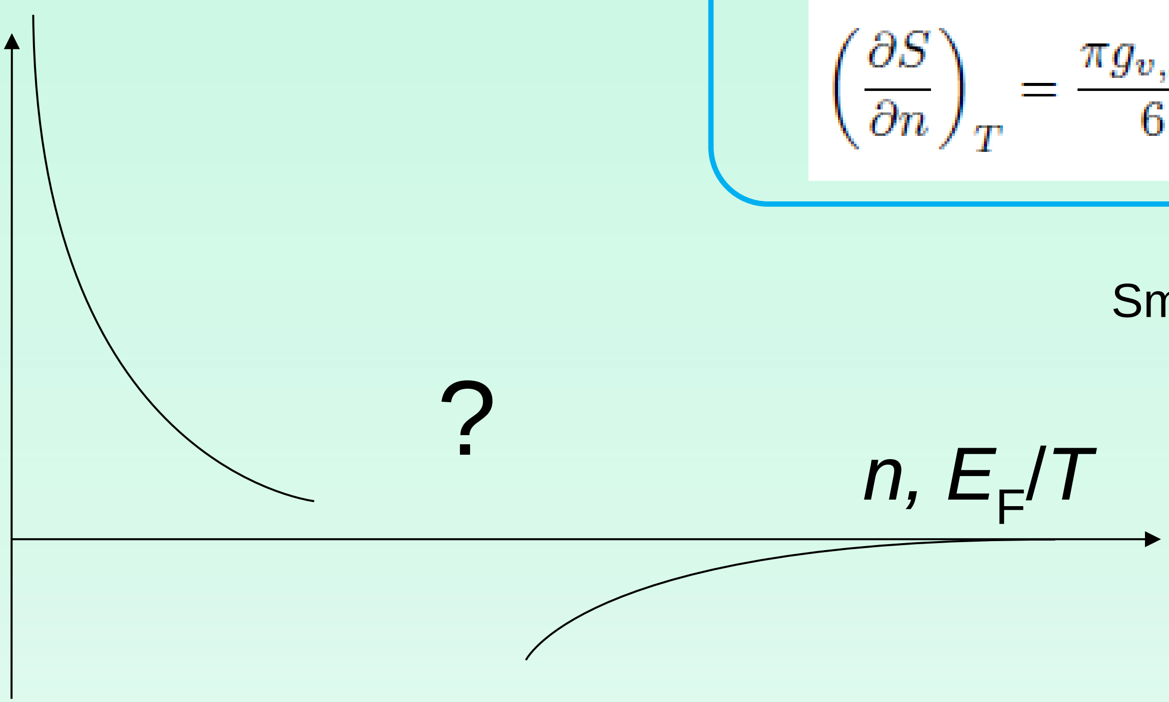
$$\left(\frac{\partial S}{\partial n}\right)_T = \frac{E_F/T}{e^{E_F/T}-1} - \ln(1-e^{-E_F/T}) > 0$$

$T \ll T_F$. Degenerate Fermi liquid

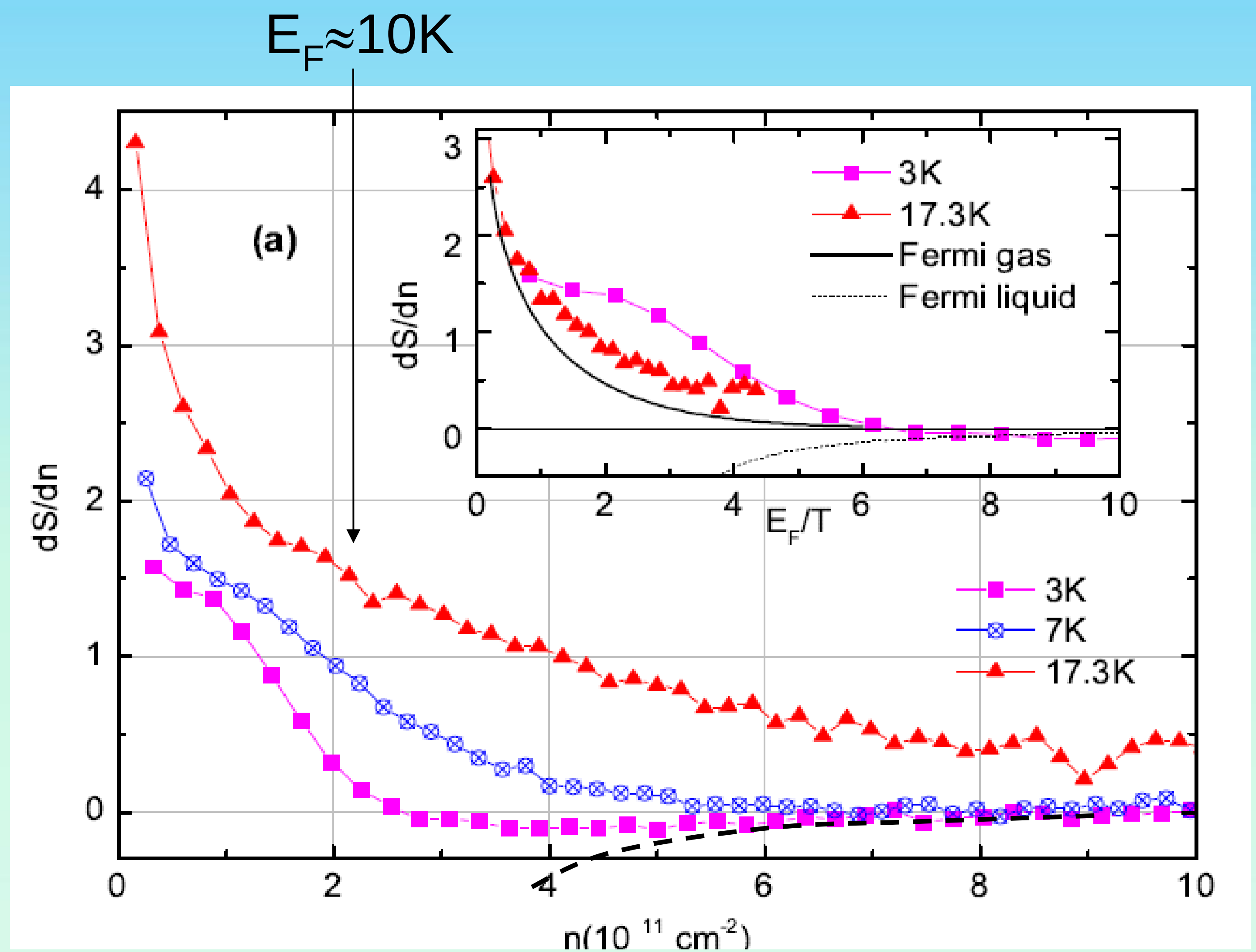
$$\left(\frac{\partial S}{\partial n}\right)_T = \frac{\pi g_{v,s} T}{6}\frac{dm^*}{dn} + \beta(n)T^2 < 0$$

Small corrections

$$\left(\frac{\partial S}{\partial n}\right)$$



Positive & negative (∂S/ ∂n)



In accord with FL:

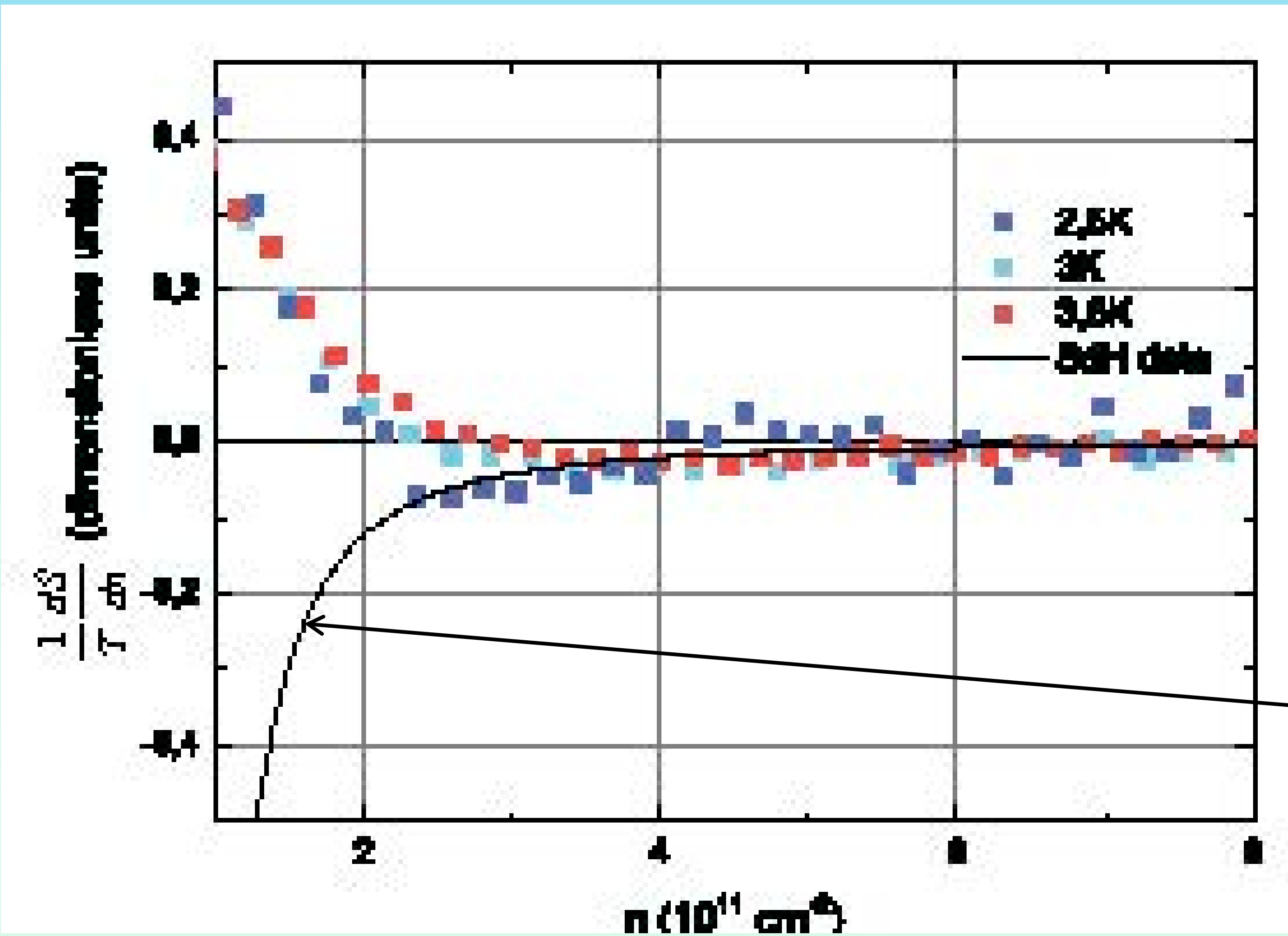
(i) The higher the temperature, the larger is the entropy

(ii) As ***n*** increases, **(d*S/dn)*** decreases to 0

(iii) For the lowest ***T***'s and high densities, ***(dS/dn)*** gets negative

(iv) The effective mass agrees with that extracted from SdH

Negative (∂S/ ∂n)



In accord with FL:

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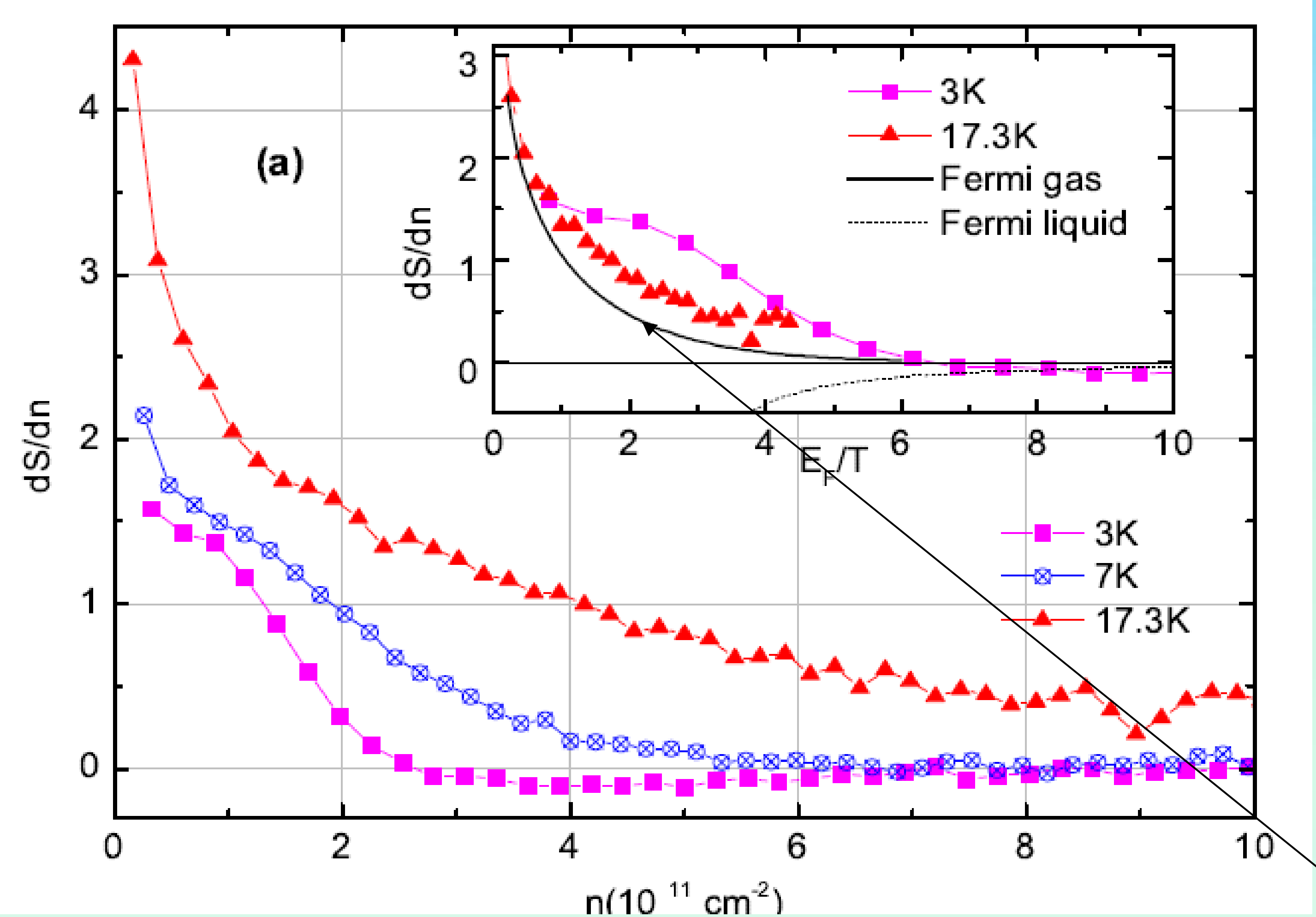
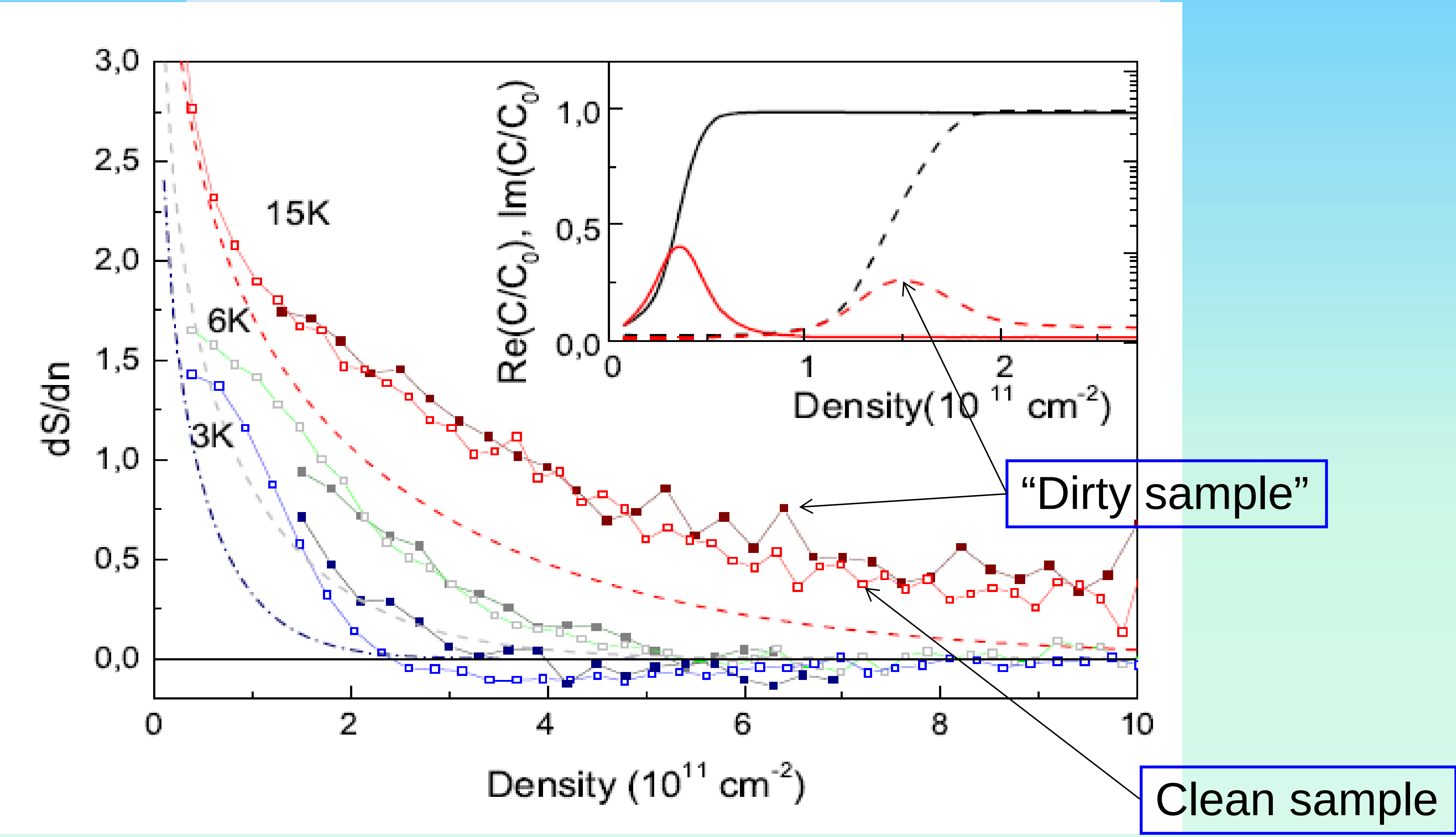
Positive **(∂S/ ∂n)**



$$\left(\frac{\partial S}{\partial n}\right)_T = \frac{E_F/T}{e^{E_F/T}-1} - \ln(1-e^{-E_F/T})$$

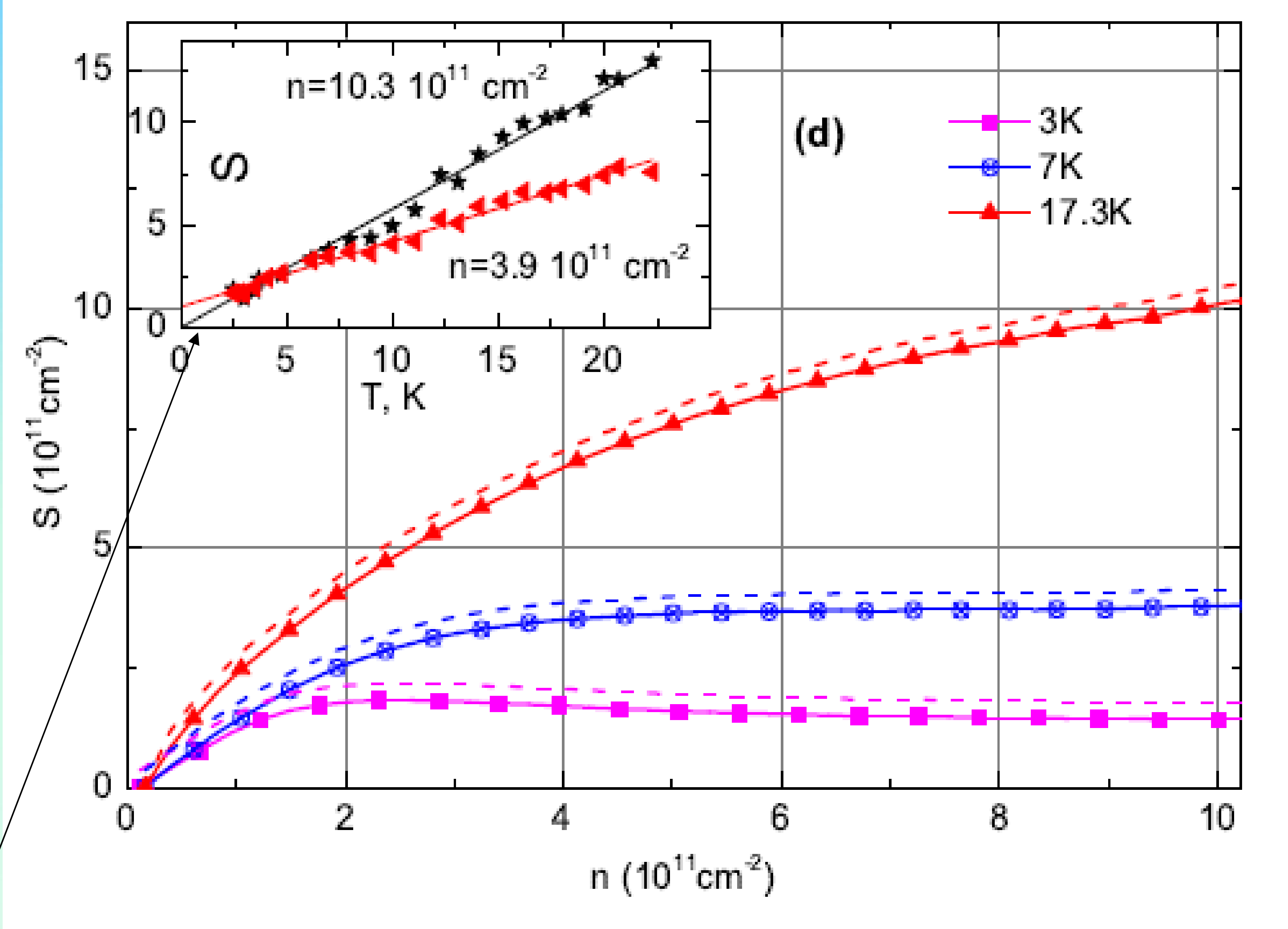
However, (dS/dn) exceeds the value calculated for the ideal Fermi-gas

Role of the disorder



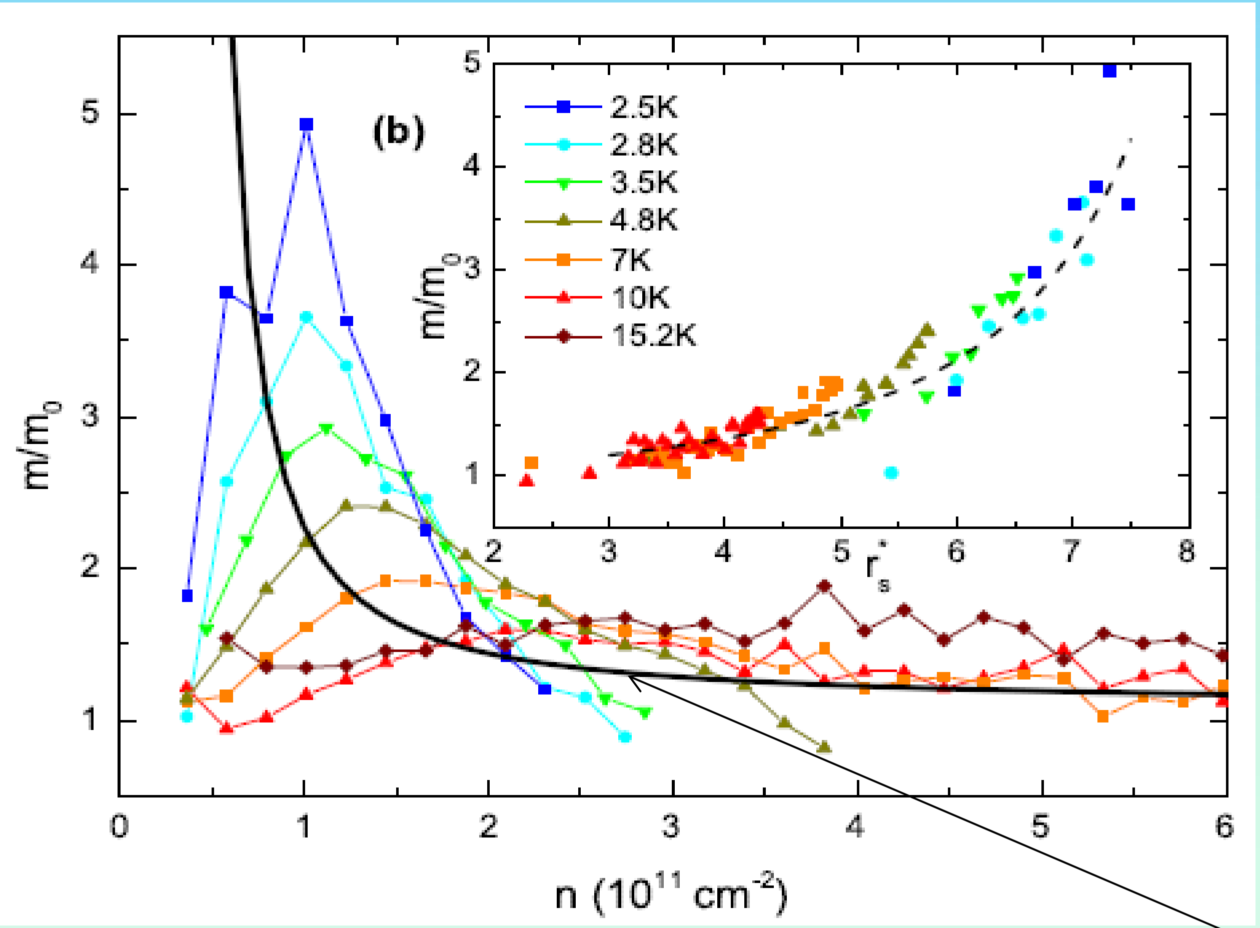
Disorder does not affect the entropy behavior!!

Checking the 3rd low: Entropy integration



The 3rd low of thermodynamics in the Fermi-liquid

Thermodynamic effective mass



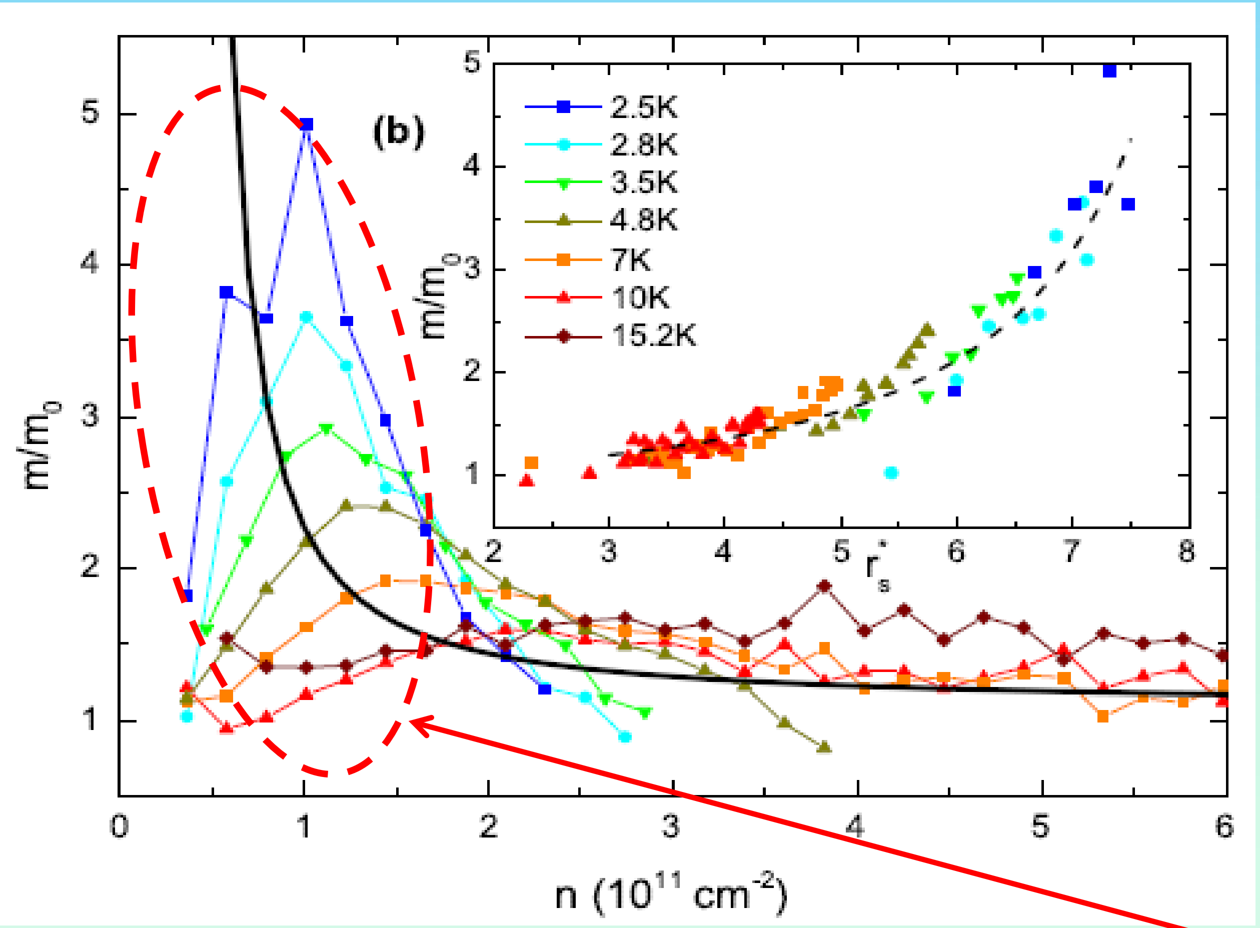
The effective mass ***m*(n)*** shows a reentrant behavior.
It tends to m_b as $n \to 0$.

m(n)* from SdH in the FL regime

$$r_s^* = [\pi a_B^2 n + \alpha T^{\gamma+\beta}/E_F^\gamma U^\beta]^{-1/2}$$

the plasmon frequency at the Fermi wave vector k_F, $\omega_p(k_F) \sim \sqrt{E_F U}$;

Thermodynamic effective mass



The effective mass ***m*(n)*** shows a reentrant behavior. It tends to $\boldsymbol{m}_b$ as $\boldsymbol{n}\to 0$.

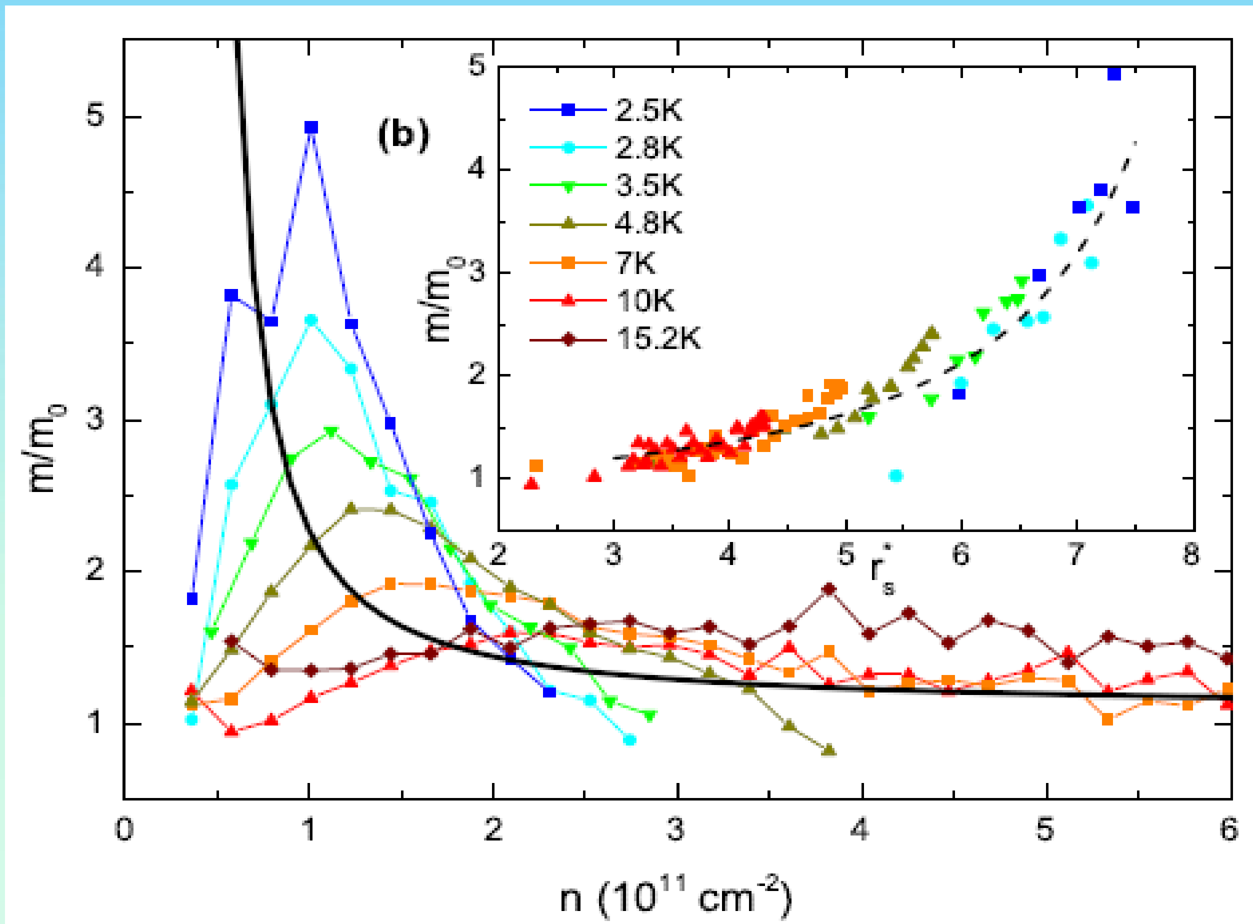
Strongly correlated plasma regime: $E_F<T<<U$

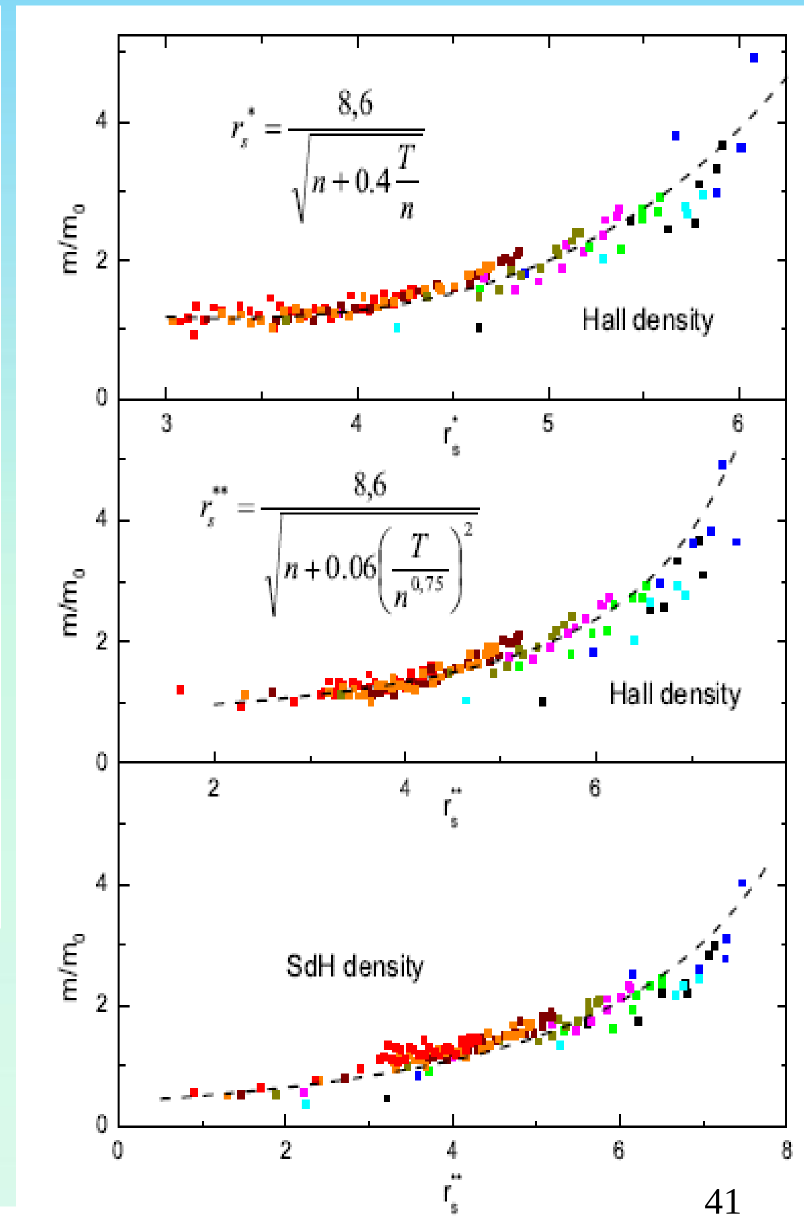
*m**(*n,T*) can be scaled using effective parameter

$$r_s^* = [\pi a_B^2 n + \alpha T^{\gamma+\beta}/E_F^\gamma U^\beta]^{-1/2}$$

the plasmon frequency at the Fermi wave vector k_F, $\omega_p(k_F) \sim \sqrt{E_F U}$;

Thermodynamic effective mass & Plasma regime parametrization





$$r_s^* = [\pi a_B^2 n + \alpha T^{\gamma+\beta} / E_F^{\gamma} U^{\beta}]^{-1/2}$$

the plasmon frequency at the Fermi wave vector k_F, $\omega_p(k_F) \sim \sqrt{E_F U}$;

Summary:

- One can measure ∂S/∂n for a system with $n>10^8$ electrons.
- High densities, low temperature — Fermi-liquid
- Low densities — **strongly correlated plasma**: Novel state of the electronic matter, where interaction parameter is ***T- and n- dependent***.

Thank you for attention!