Twisting the Quantum: from measurement-induced chaos to measurement-powered engines

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The Abdus Salam International Centre for Theoretical Physics (ICTP)

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NSF

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Talk Outline

- Introduction to continuous quantum measurement, quantum trajectories
- Stochastic path integral approach: formalism, most likely paths
- New predictions: Quantum Caustics and Quantum Chaos
- Energy and Measurement: Harnessing wavefunction collapse to build an engine
- Twisted Quantum mechanics: Spooky work at a distance

Fundamental

Application

Twist

Thanks to my great students, and collaborators!

Phil Lewalle

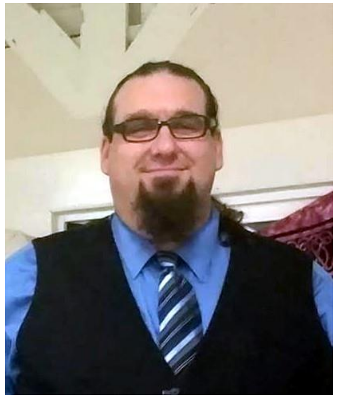
John Steinmetz

Cyril Elouard

Areeya Chantasri
(Griffiths University)

Justin Dressel
(Chapman University)

Cai Waegell
(Chapman University)



Kater Murch
(Washington University,
St. Louis)
+ group

Irfan Siddiqi
(UC Berkeley)
+ group

Benjamin Huard
ENS Lyon

Continuous Quantum Measurements

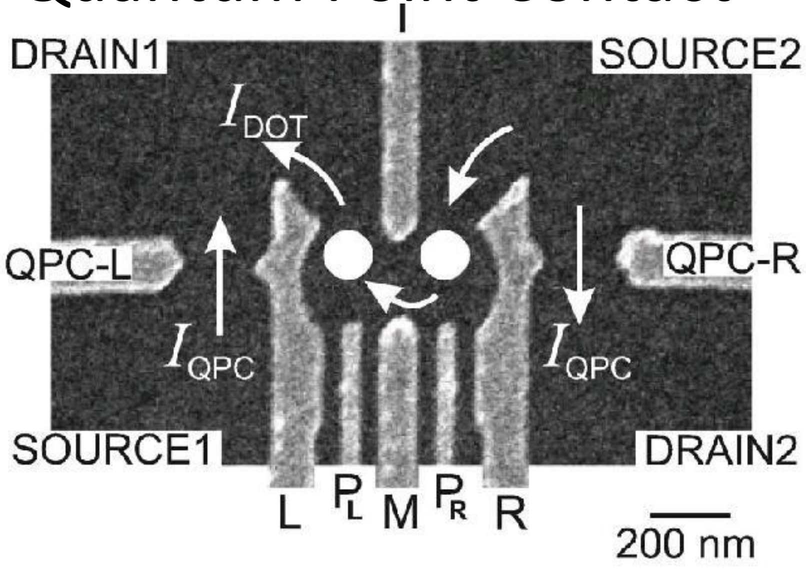
Example 1: Ballistic Electron Detector –Quantum Point Contact

$$I = \begin{cases} I_1 & \text{if } |1\rangle \\ I_2 & \text{if } |2\rangle \end{cases}$$

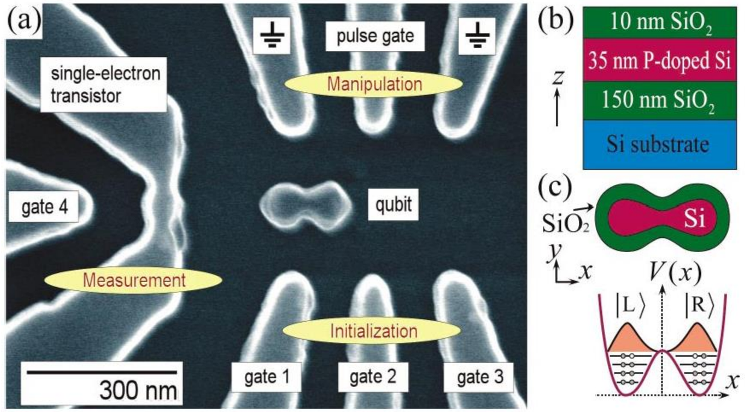
It takes time to measure – there is background detector noise (electron shot noise) of spectral density S_I that must be averaged .

$$T_M = 4S_I/(\Delta I)^2$$

Measurement time – new time scale.



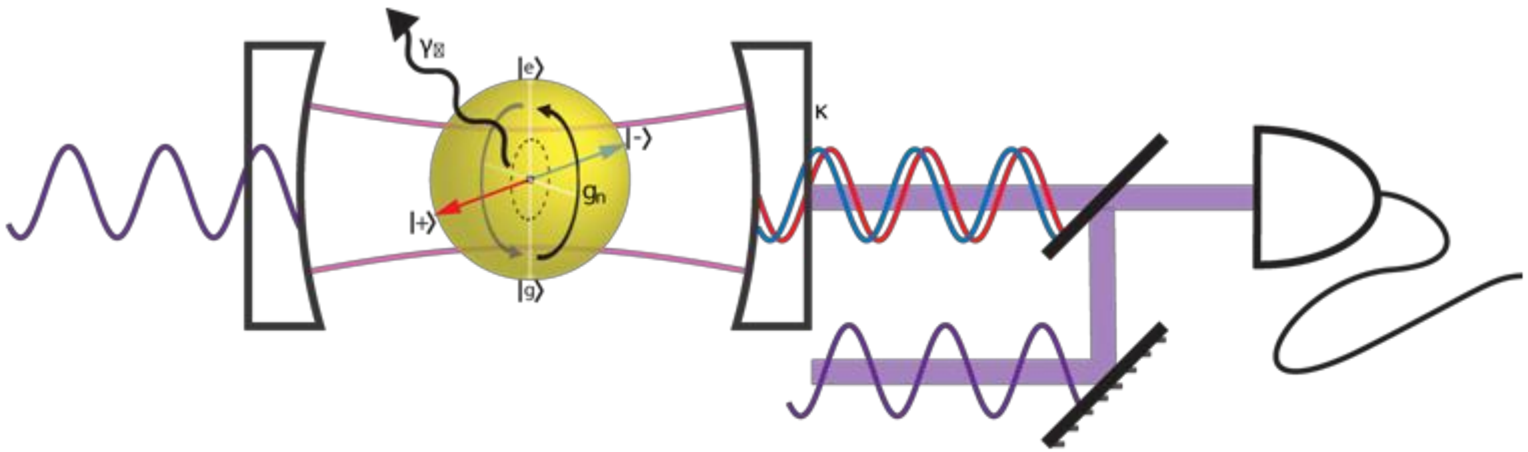
Elzerman *et al.*, PRB(R) (2004)



Gorman, *et al.*, PRL (2005)

Quantum Photonics: Photons interacting with atoms.

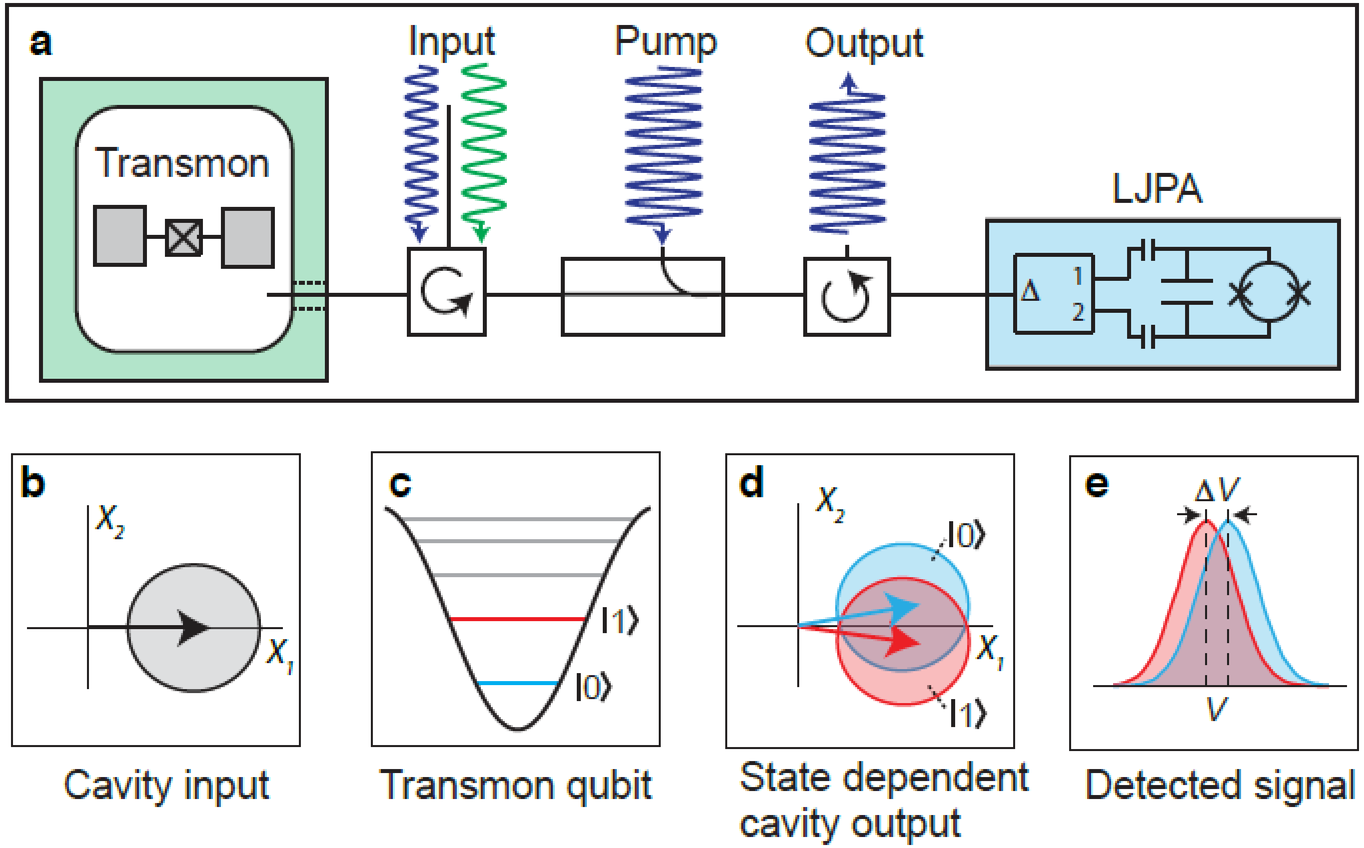
Cavity QED system – photons trapped in a cavity can interact with a single atom



Mabuchi lab image

Superconducting quantum circuit measured by off-resonant microwave tone.

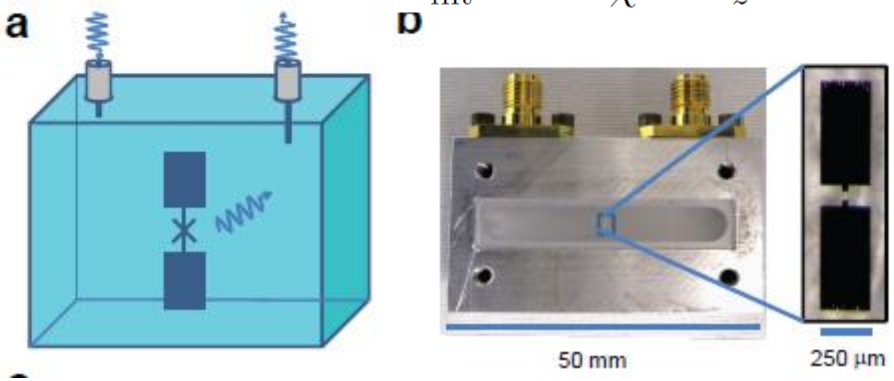
"Transmon" architecture and measurement procedure



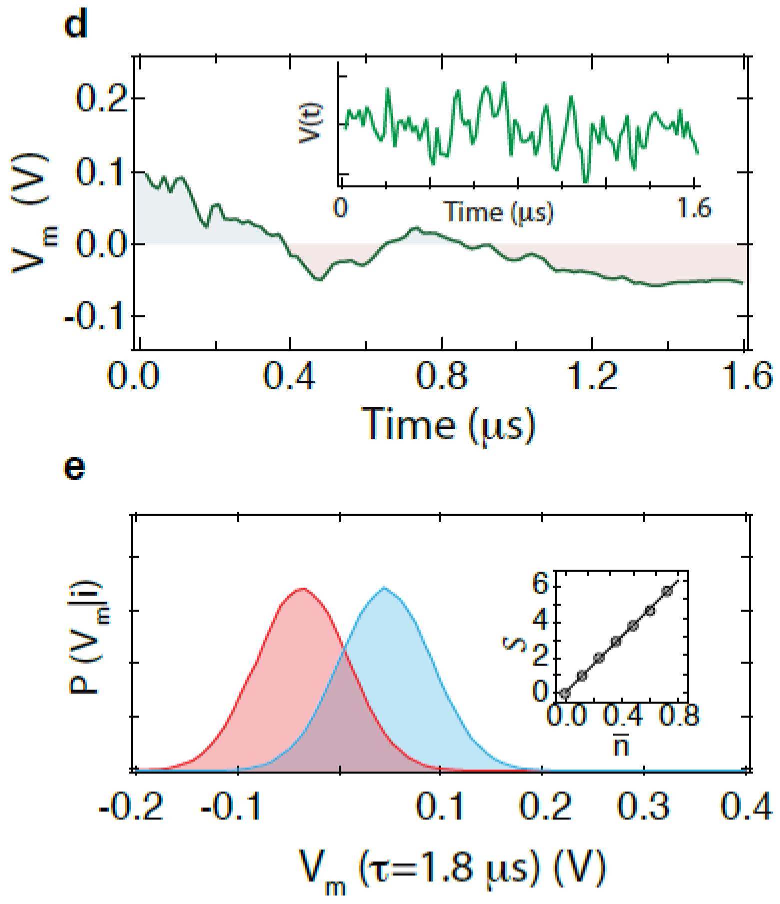
Continuous Quantum Measurements

Example 2: Superconducting cavity QED– 3D Transmons

$$H_{\rm int} = -\hbar\chi a^\dagger a\sigma_z$$



Paik *et al.*, *Phys. Rev. Lett.* **107**, 240501 (2011)

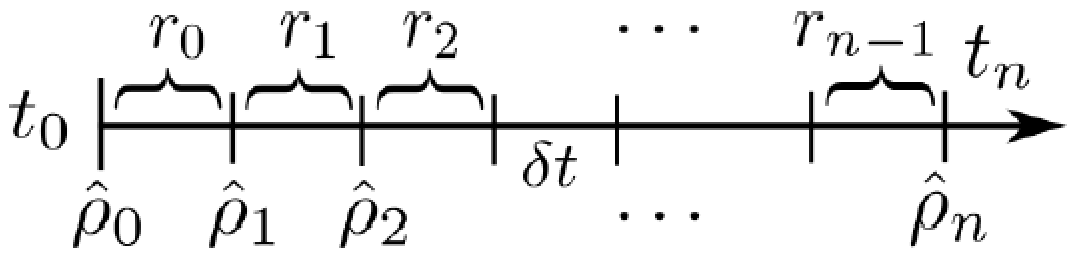


A microwave tone near the resonance frequency of the transmon produces a qubit state dependent phase shift that is amplified and read out in the reflected signal as a voltage.

Murch, Siddiqi, *et al*. Nature 2013

Continuous Quantum Measurement

- A **continuous** measurement is a time series of weak measurements where the measurement results are now effectively continuous.
- The measurement strength grows over time to become a projective measurement.
- Here $r_0, r_1, \ldots, r_n$ are a sequence of continuous measurement results in time bins of size δt.
- Measurement disturbs the state, and given a stochastic measurement readout, the state can be **tracked** in time.

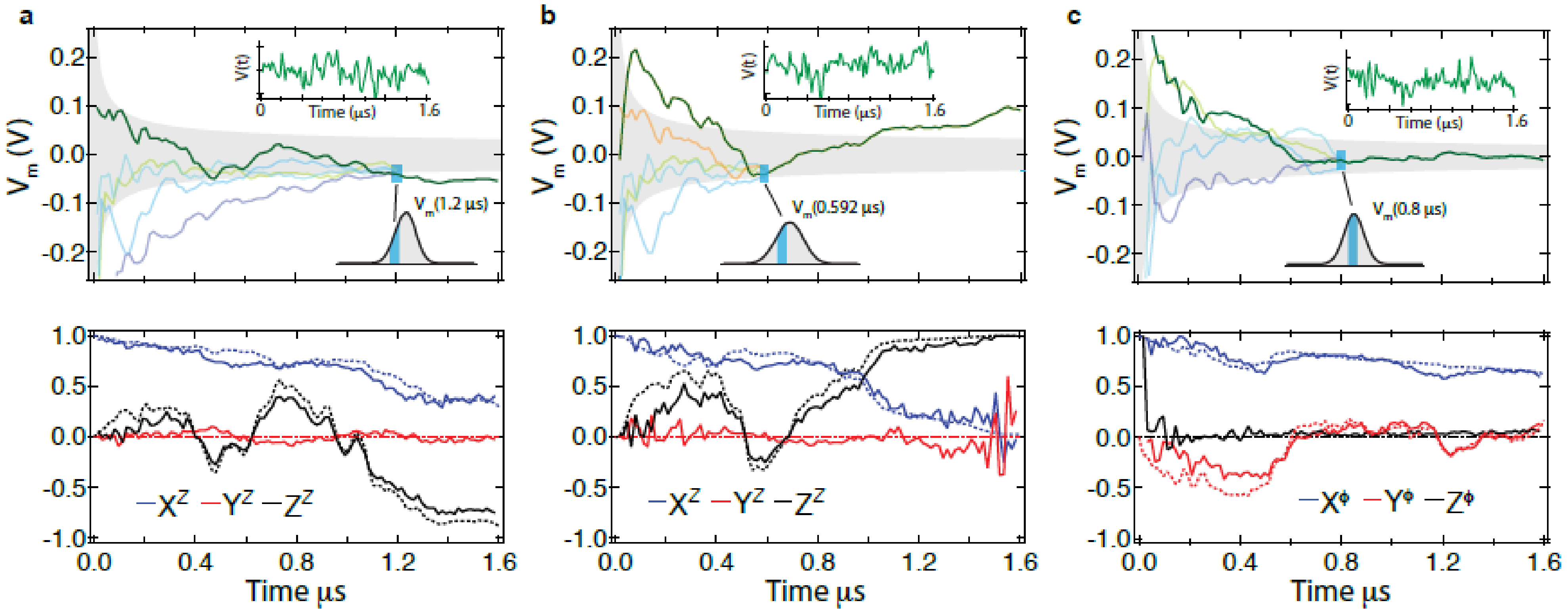


Allow also for Hamiltonian evolution as well as measurement dynamics.

$$\hat{\rho}_{k+1} = \frac{\hat{\mathcal{U}}_{\delta t}\hat{\rho}_k\hat{\mathcal{U}}_{\delta t}^{\dagger}}{\mathrm{Tr}\{\hat{\mathcal{U}}_{\delta t}\hat{\rho}_k\hat{\mathcal{U}}_{\delta t}^{\dagger}\}}$$

The set of ρ's is a **quantum trajectory**.

Quantum Trajectories in 3D Transmons



Murch, Siddiqi, *et al*. Nature 2013

Analogy

ANJ, *Nature* **502**, 177 (2013)

Stochastic Path Integral Formalism and Most Likely (Optimal) Paths

- OPs extremize the probability of stochastic quantum trajectories from weak continuous quantum measurement over a given time interval
  - Stochastic Path Integral Formalism for path probability: $\mathbf{q}$ = quantum state, $\mathbf{r}$ = measurement readout(s).

$$\mathcal{P}(\mathbf{q}(t), \mathbf{r}(t)|\mathbf{q}_0) \propto \int \mathcal{D}\mathbf{p}\, e^{S[\mathbf{q},\mathbf{p},\mathbf{r}]} = \int \mathcal{D}\mathbf{p} \exp\left\{ \int_0^T [H(\mathbf{q},\mathbf{p},\mathbf{r}) - \mathbf{p}\cdot\dot{\mathbf{q}}]dt \right\}$$

- Defines "stochastic Hamiltonian" H and "stochastic action" S.
- Extremize the Path Probability: $\delta S = 0$ leads to OP equations

$$\dot{\mathbf{q}} = \partial_{\mathbf{p}} H, \quad \dot{\mathbf{p}} = -\partial_{\mathbf{q}} H, \quad \partial_{\mathbf{r}} H|_{\mathbf{r}^\star} = 0, \quad \text{with } \{q_i, p_j\} = \delta_{ij}$$

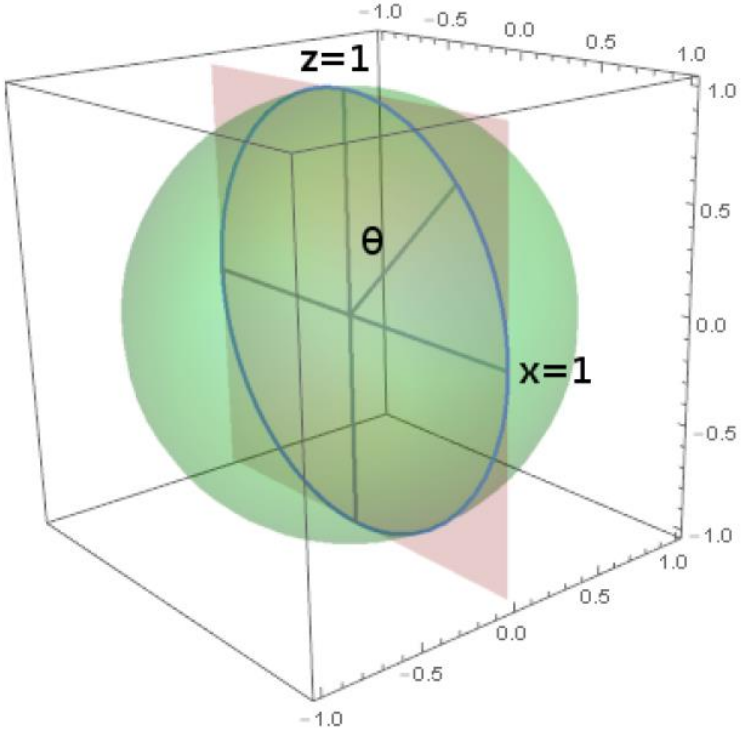
- Role of $\mathbf{p}$:
  - Generalized "momentum" conjugate to $\mathbf{q}$. Implements measurement backaction.
  - Given initial state $\mathbf{q}_0$, different $\mathbf{p}_0$ initialize different optimal readouts $\mathbf{r}^\star(\mathbf{q},\mathbf{p})$, leading to different $\mathbf{q}_T$ on which we could post-select.

A. Chantasri, J. Dressel, A. N. Jordan, Phys. Rev. A **88**, 042110 (2013).

A. Chantasri, A. N. Jordan. Phys. Rev. A **92**, 032125 (2015).

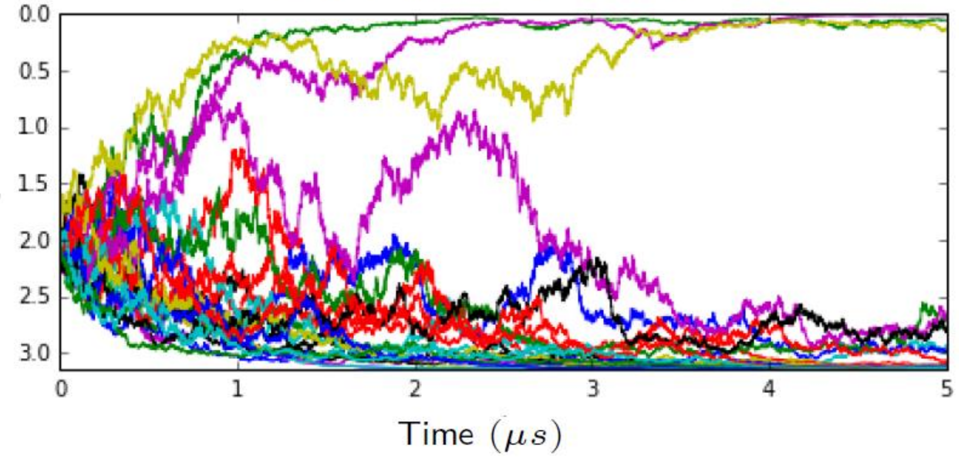
S. J. Weber, A. Chantasri, J. Dressel, A. N. Jordan, K. W. Murch, I. Siddiqi. Nature **511**, 570-573 (2014).

From Stochastic Quantum Trajectories to Optimal Paths



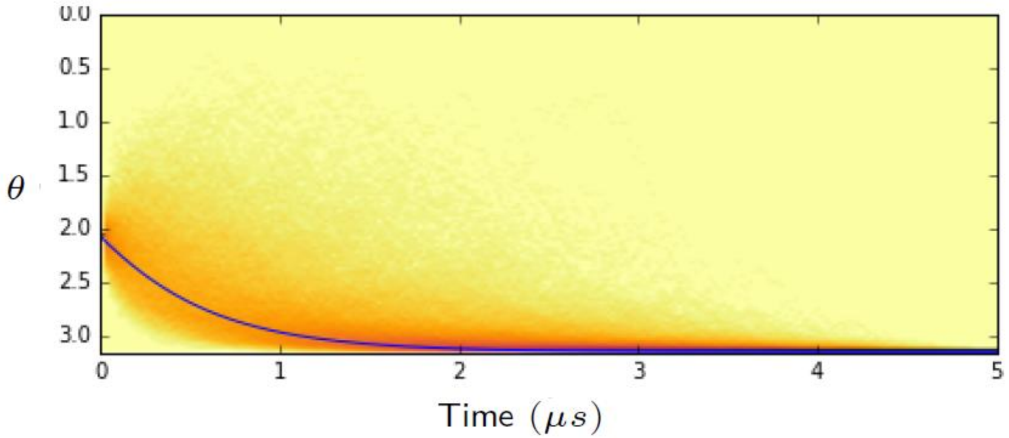
- θ on great circle of xz-plane in Bloch Sphere. ($|1\rangle \sim \theta = 0$, $|0\rangle \sim \theta = \pi$).
- $x = \langle \sigma_x \rangle$, $y = \langle \sigma_y \rangle$, $z = \langle \sigma_z \rangle$.

SQT (Measure σ_z)

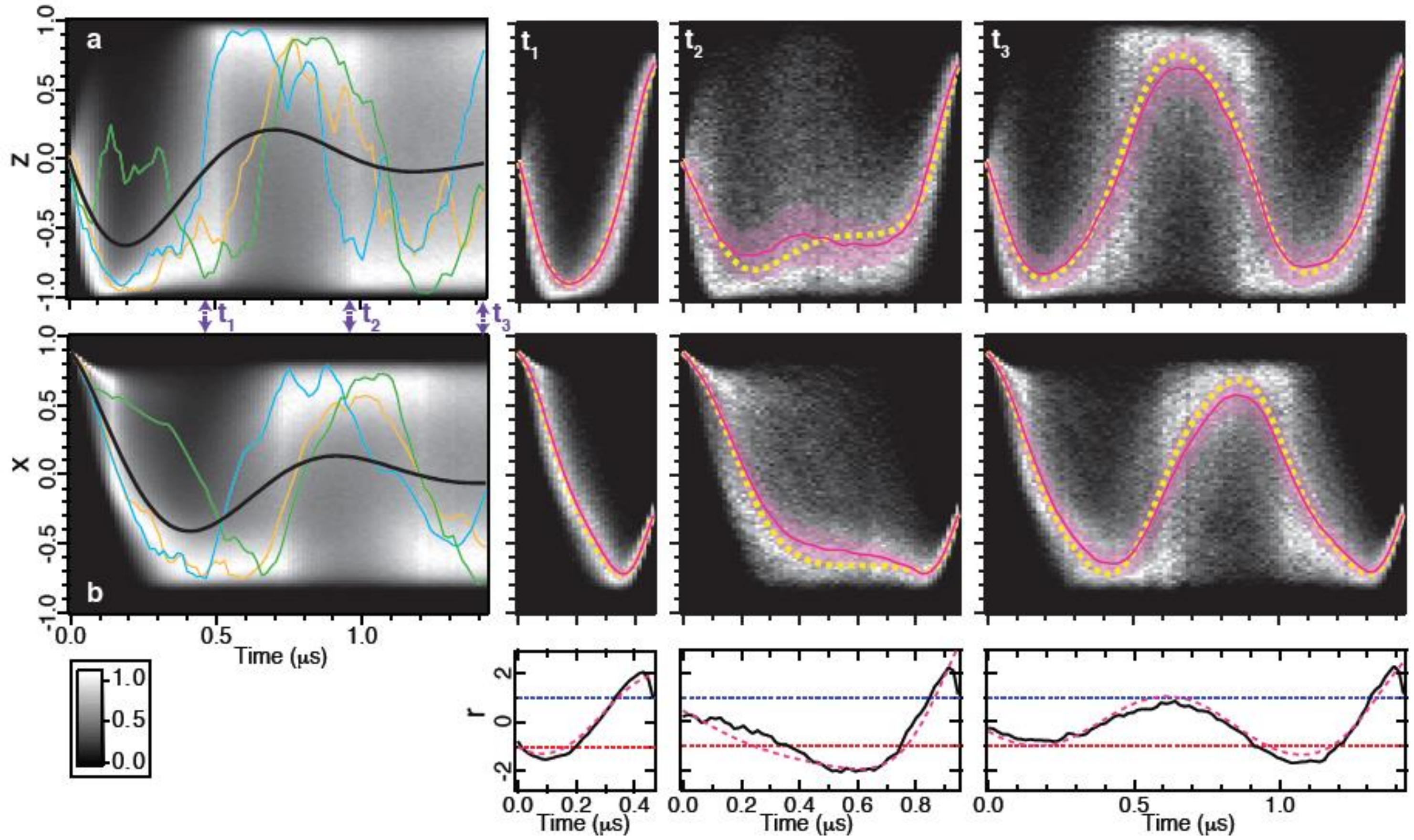


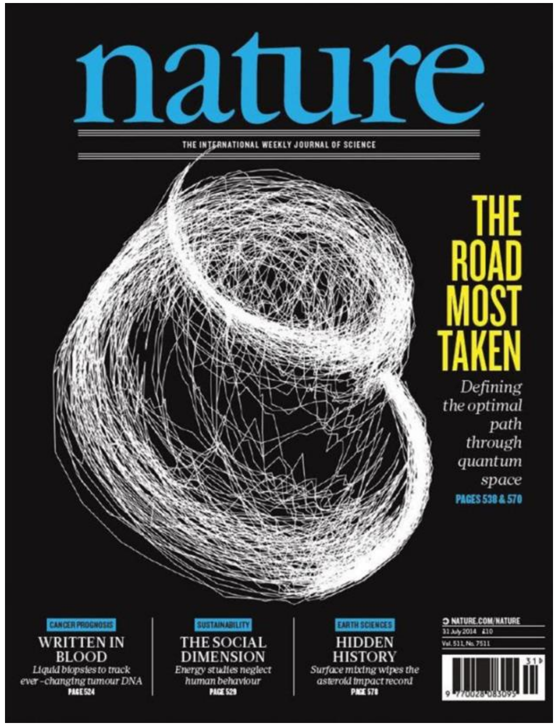
SQTs from indvidual readout realizations

Density with Post-Selection → MLP



Most likely path between two states



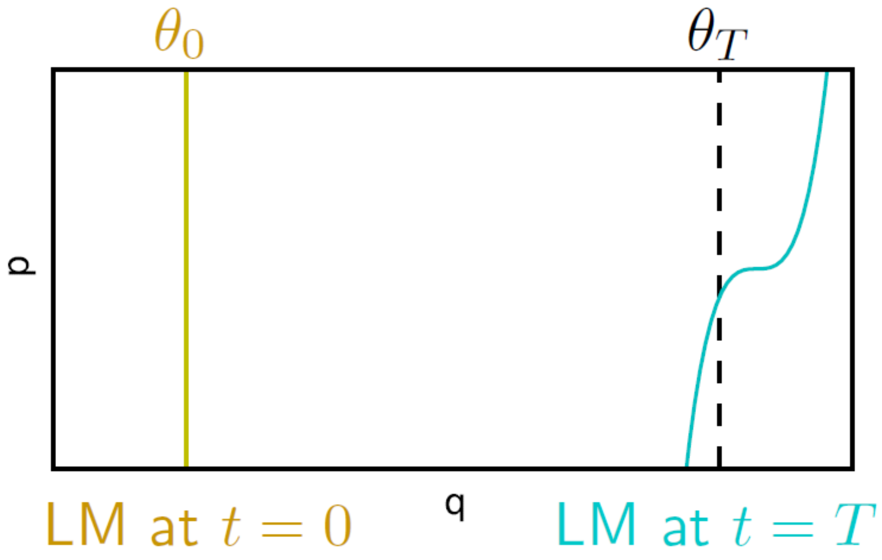


Weber et al, Nature 511, 570–573 (2014)

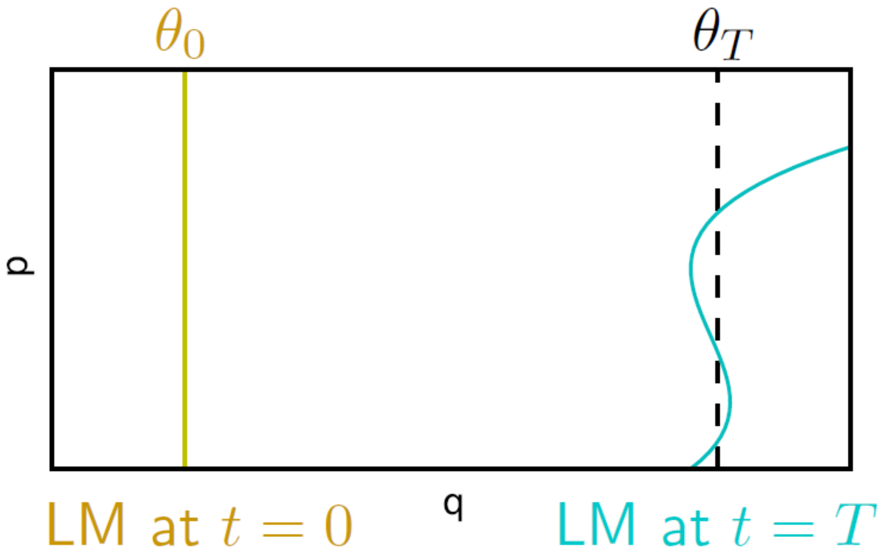
Can there be multiple Optimal Paths?

Multipath Solutions

- Multipaths: Many OPs can meet boundary conditions $\theta_0 \to \theta_T$ over time interval T.
- Use a Lagrangian Manifold (LM) in OP phase space to understand this phenomenon.



Left: One solution from θ_0 to θ_T.



Right: Three solutions from θ_0 to θ_T.

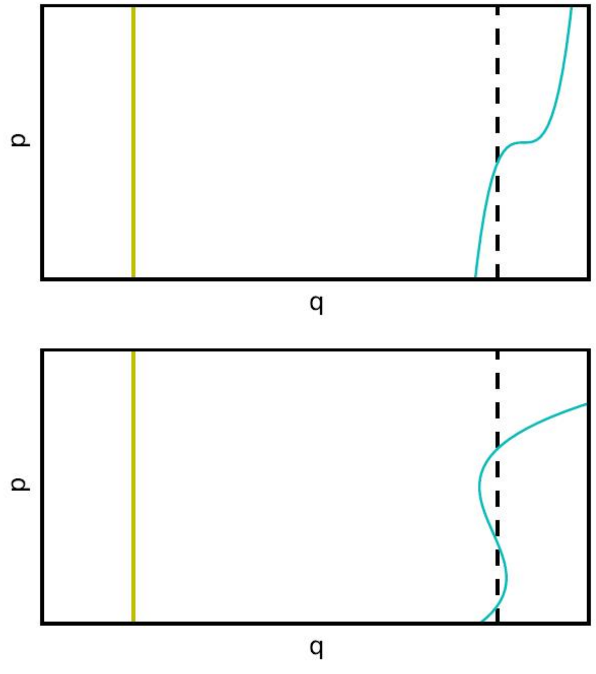
- Winding number multipaths: Physical states are θ mod-2π, not raw θ.

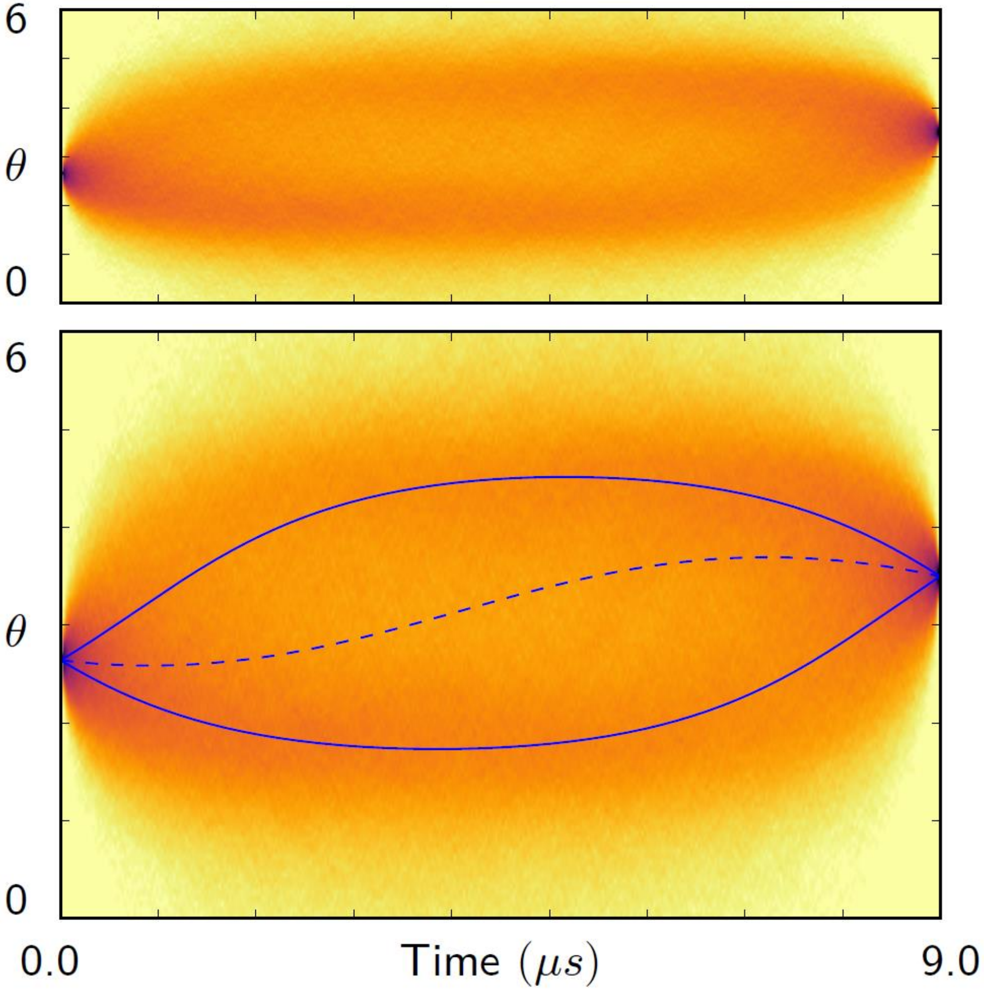
P. Lewalle, A. Chantasri, A. N. Jordan. Phys. Rev. A **95**, 042126 (2017).

M. Naghiloo, D. Tan, P. M. Harrington, P. Lewalle, A. N. Jordan, K. W. Murch, Phys. Rev. A **96** 053807 (2017).

What do we mean by Multipaths?

Lagrange Manifold[13]: all possible p_0 at a fixed q_0. $t = 0$, $t = T$.



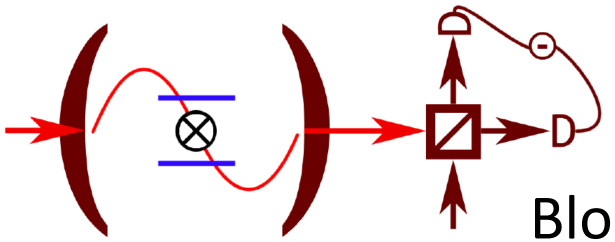


Jacobian ↔ Van-Vleck Det.

$$J = \frac{\partial q_f}{\partial p_0} \quad \leftrightarrow \quad V = \frac{\partial p_0}{\partial q_f} = \frac{\partial^2 S}{\partial q_0 \partial q_f}$$

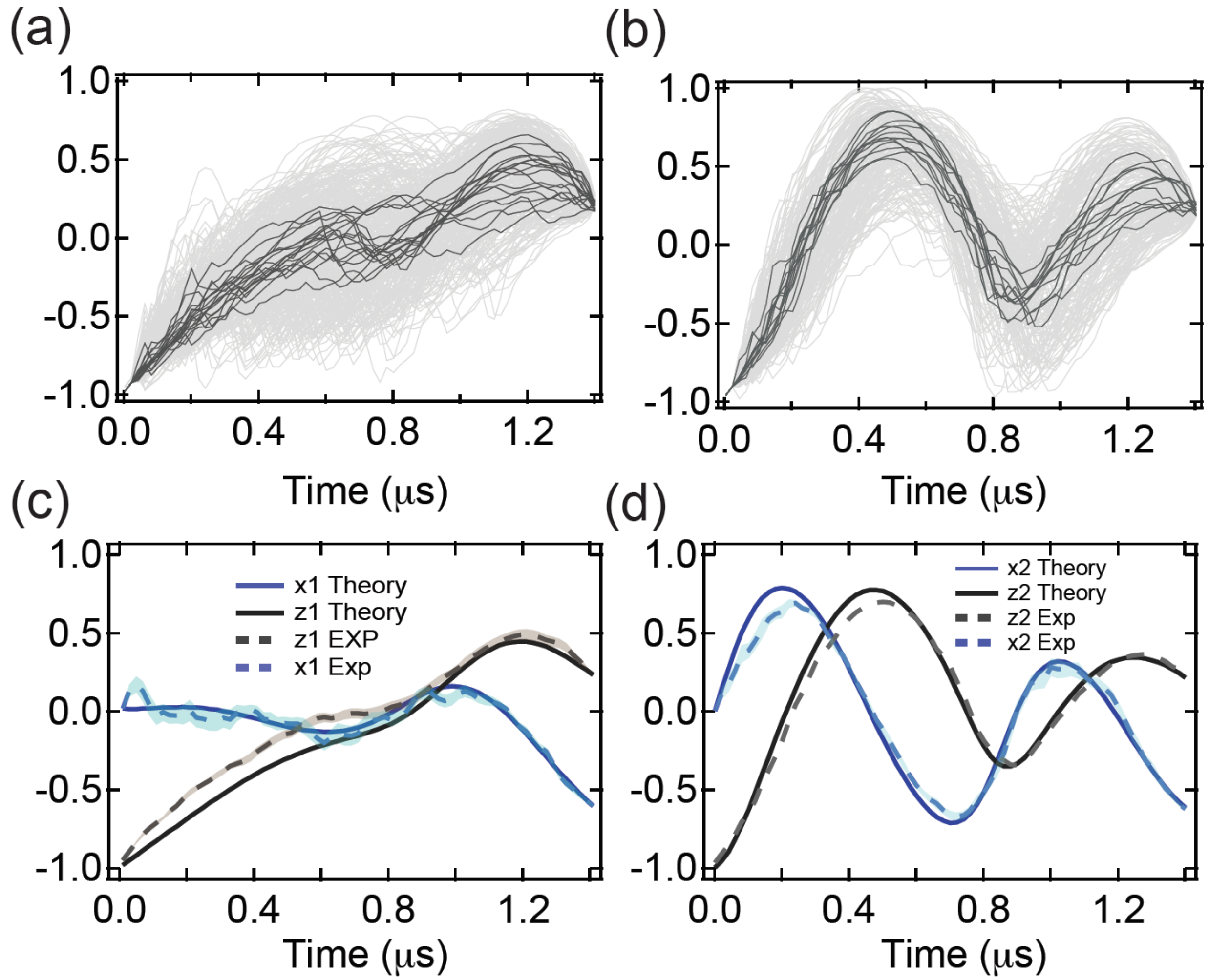
The Van-Vleck determinate will diverge when a caustic forms

Confirmation in Resonance Fluorescence Experiments



- Experiments on Resonance Fluorescence – monitoring the emission of a driven qubit, by the Murch group (Washington University, St. Louis)
- Quantum trajectories of the qubit can be tracked using a modified formalism.
- By performing a sorting algorithm, we can identify clusters of trajectories, and identify the formation of the two most-likely paths.

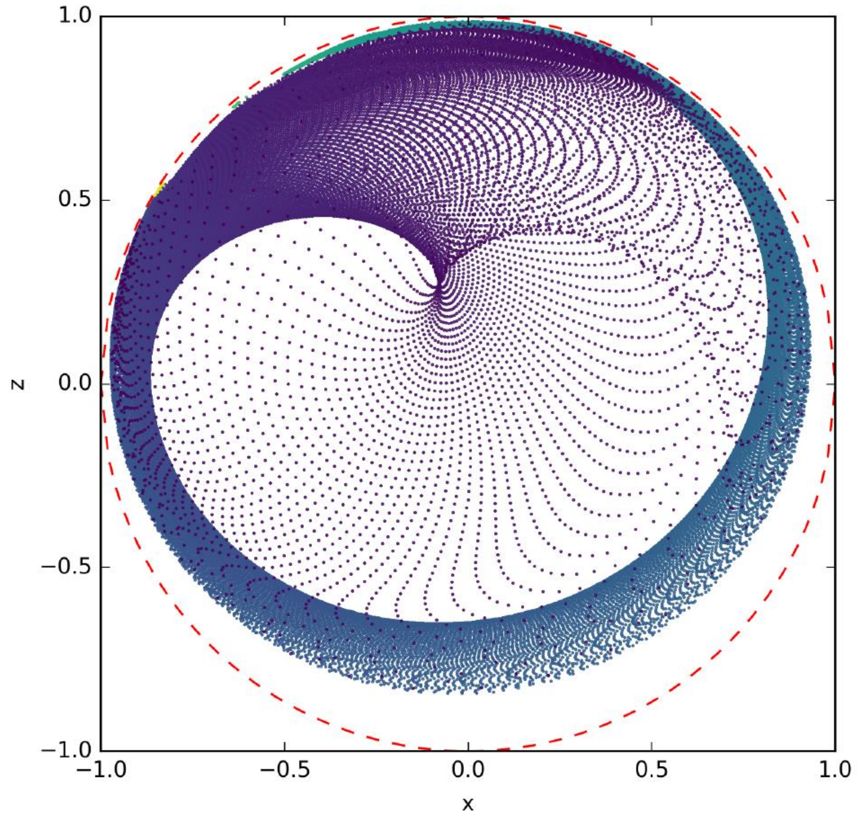
Bloch coordinate z vs. t – many trajectories



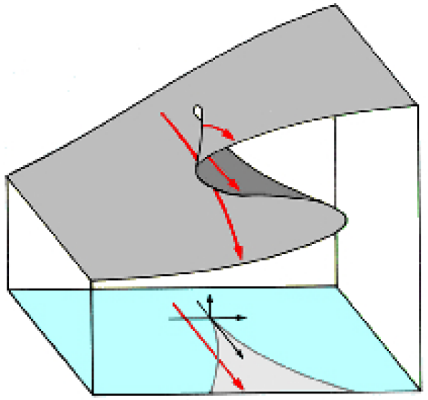
Resonance Fluorescence ("Quantum Caustics")

UNIVERSITY of ROCHESTER

Lagrangian Manifold, Catastrophes, and analogy with Optical Caustics.



http://www.nanowerk.com/spotlight/spotid=19915.php



www.aetheling.com/models/cusp/Intro.htm

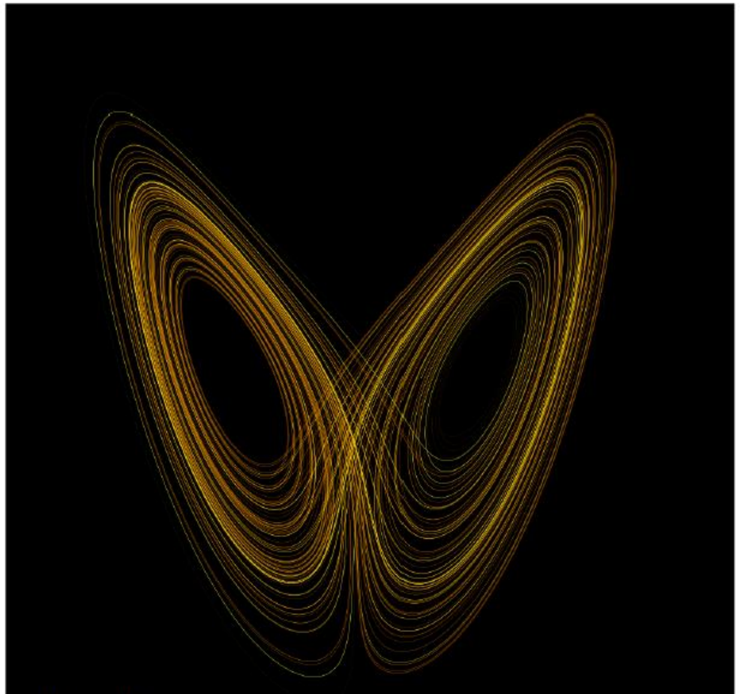
Cusp catastrophe in a Lagrangian manifold.

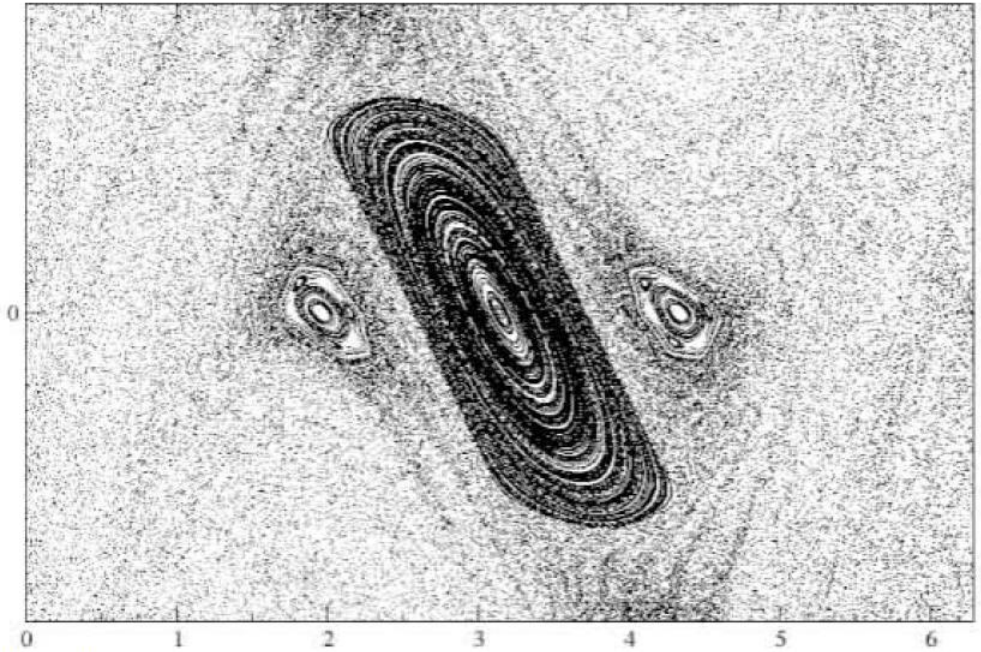
For more details, see the poster by Phil Lewalle

If most likely paths of quantum trajectories can have catastrophes, can they be chaotic?

Introduction & Motivation

- What is classical chaos?
  - Exponential sensitivity to initial conditions; chaotic trajectories diverge, and the deterministic system's long-term behavior is limited by the precision of preparation.


wikipedia



wikipedia

A new type of Quantum Chaos

- Historically, Quantum Chaos has typically considered closed quantum systems with a classical limit that is chaotic.
- The subject was then concerned with statistical properties of energy levels, wavefunctions, connection with random matrix theory, etc.
- However, the quantum mechanics itself is a linear theory where no chaos can occur.
- What about when the system is continuous monitored? Can there be a notion of chaos in the quantum dynamics itself?

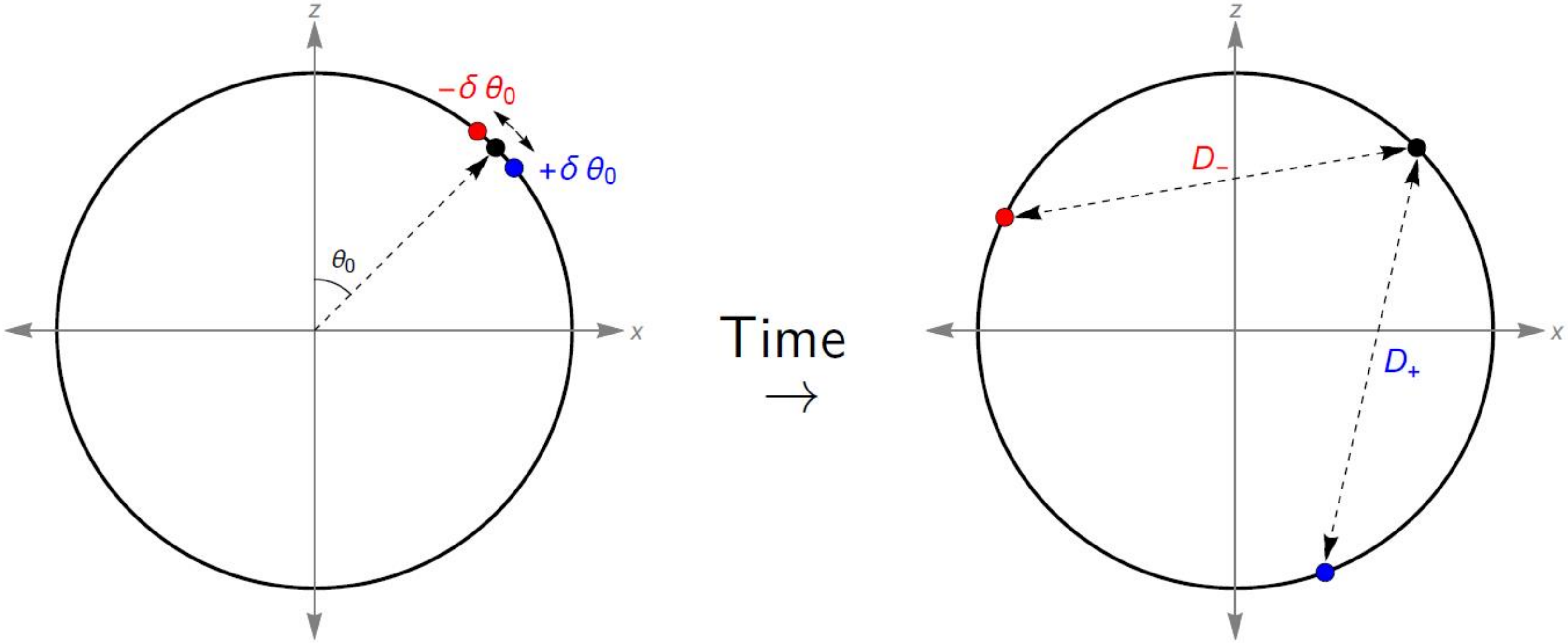
- New Approach: Optimal Paths (OPs) of continuously-monitored quantum systems.

Lyapunov Exponents for Optimal Paths

Lyapunov Exponents λ characeterize exponential divergence of OPs.

$$D(t) = D_0 e^{t\cdot\lambda(t)} \quad \leftrightarrow \quad \lambda(t) = \frac{1}{t}\ln\left(\frac{D(t)}{D_0}\right)$$

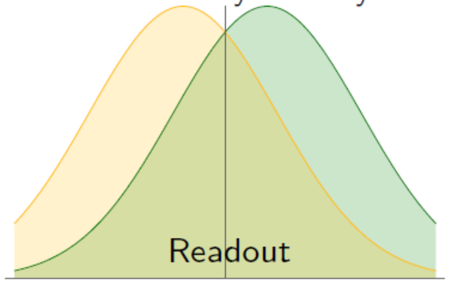
Define $D(t)$ symmetrically on Bloch sphere, with D_0 defined by $\delta\theta_0 = 0.01$ radians.

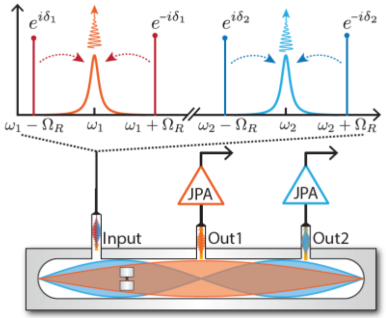


$$D(t) = \tfrac{1}{2}D_-(t) + \tfrac{1}{2}D_+(t)$$

Example: Measuring σ_x and σ_z Simultaneously

- Measurement strength characterized by τ_x and τ_z, timescales for state collapse.





- Hamiltonian for OPs (includes optimization of readouts $r_x^\star$ and $r_z^\star$)

$$H^\star(\theta, p, t) = (p^2 - 1)\left(\frac{\sin^2\theta}{2\tau_z(t)} + \frac{\cos^2\theta}{2\tau_x}\right) + p\,\sin\theta\cos\theta\left(\frac{1}{\tau_x} - \frac{1}{\tau_z(t)}\right)$$

- When $\tau_x = \tau = \tau_z$: OPs follow equations of simple rotor!

$$H^\star \quad \rightarrow \quad H^{(0)} = \frac{p^2 - 1}{2\tau}$$

S. Hacohen-Gourgy, L. S. Martin, E. Flurin, V. V. Ramasesh, K. B. Whaley, I. Siddiqi. Nature **538**, 491–494 (2016).

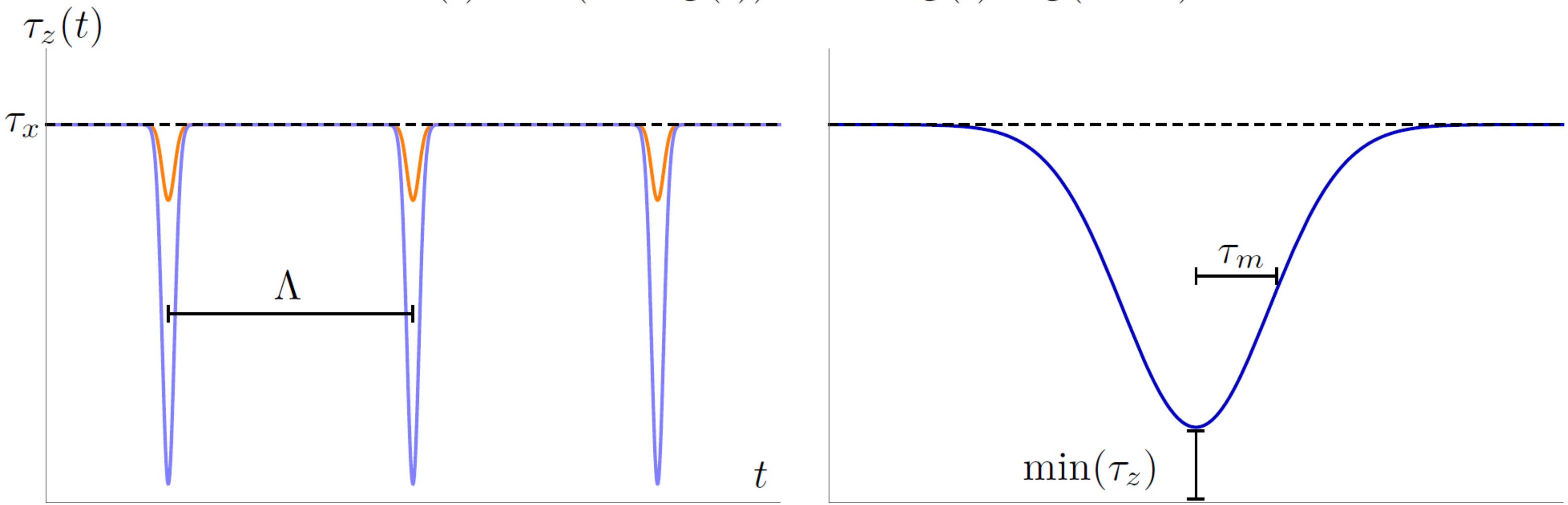
A. N. Jordan, M. Büttiker. Phys. Rev. Lett. **95**, 220401 (2005).

A. Chantasri, J. Atalaya, S. Hacohen-Gourgy, L. S. Martin, I. Siddiqi, and A. N. Jordan, Phys. Rev. A 97, 012118 (2018).

Example: "Kicking" the z-Measurement

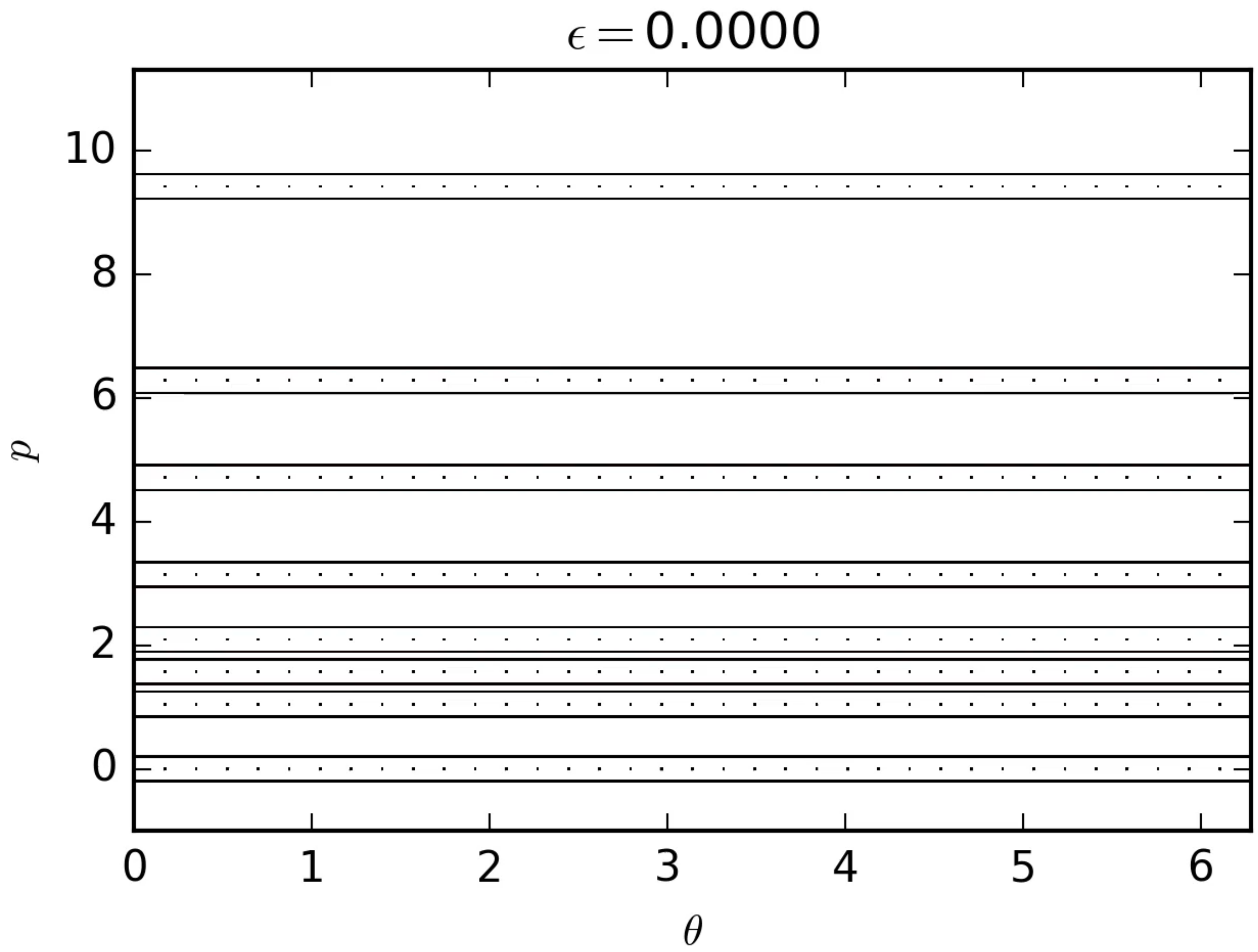
Fix $\tau_x = 1\ \mu\text{s}$, but periodically strengthen the σ_z measurement, with $\tau_z(t)$:

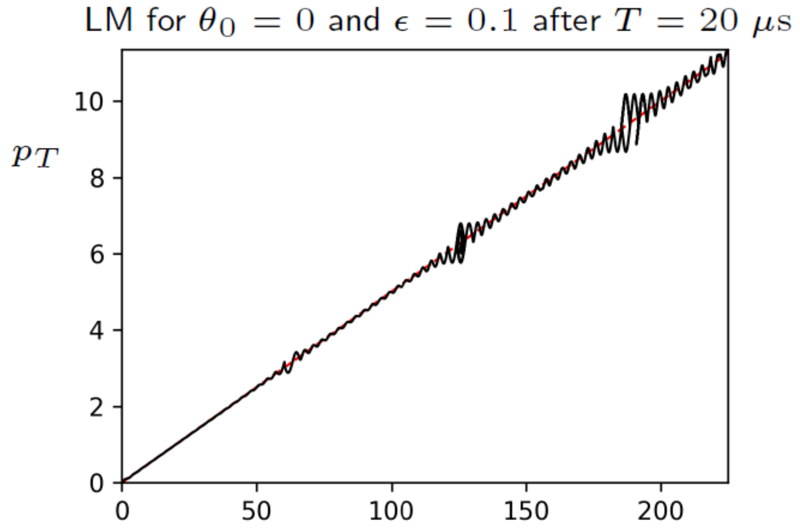
$$\tau_z(t) = \tau_x(1 - \epsilon\, g(t)) \quad \text{with} \quad g(t) = g(t + \Lambda)$$

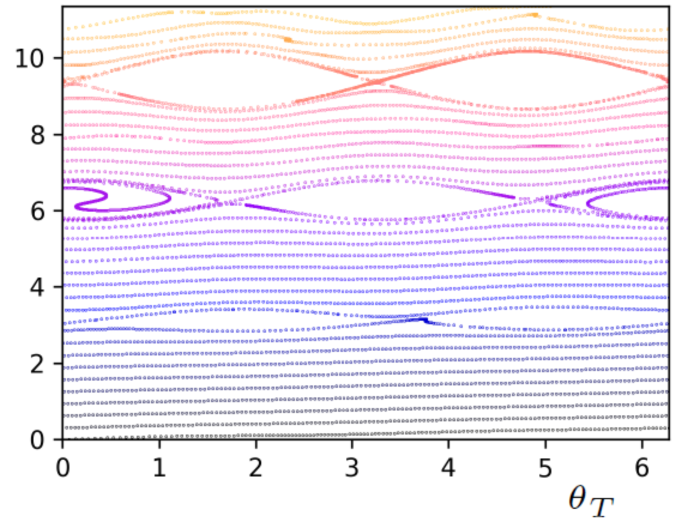


- Weak kicks: $\epsilon \ll 1$ (orange).
- Strong kicks: $\epsilon \to 1$ (lavender). Kick $\approx$ projective when $\tau_m \approx \min(\tau_z)$.

Weak Kicks: Resonances, Chaos, and Multipaths



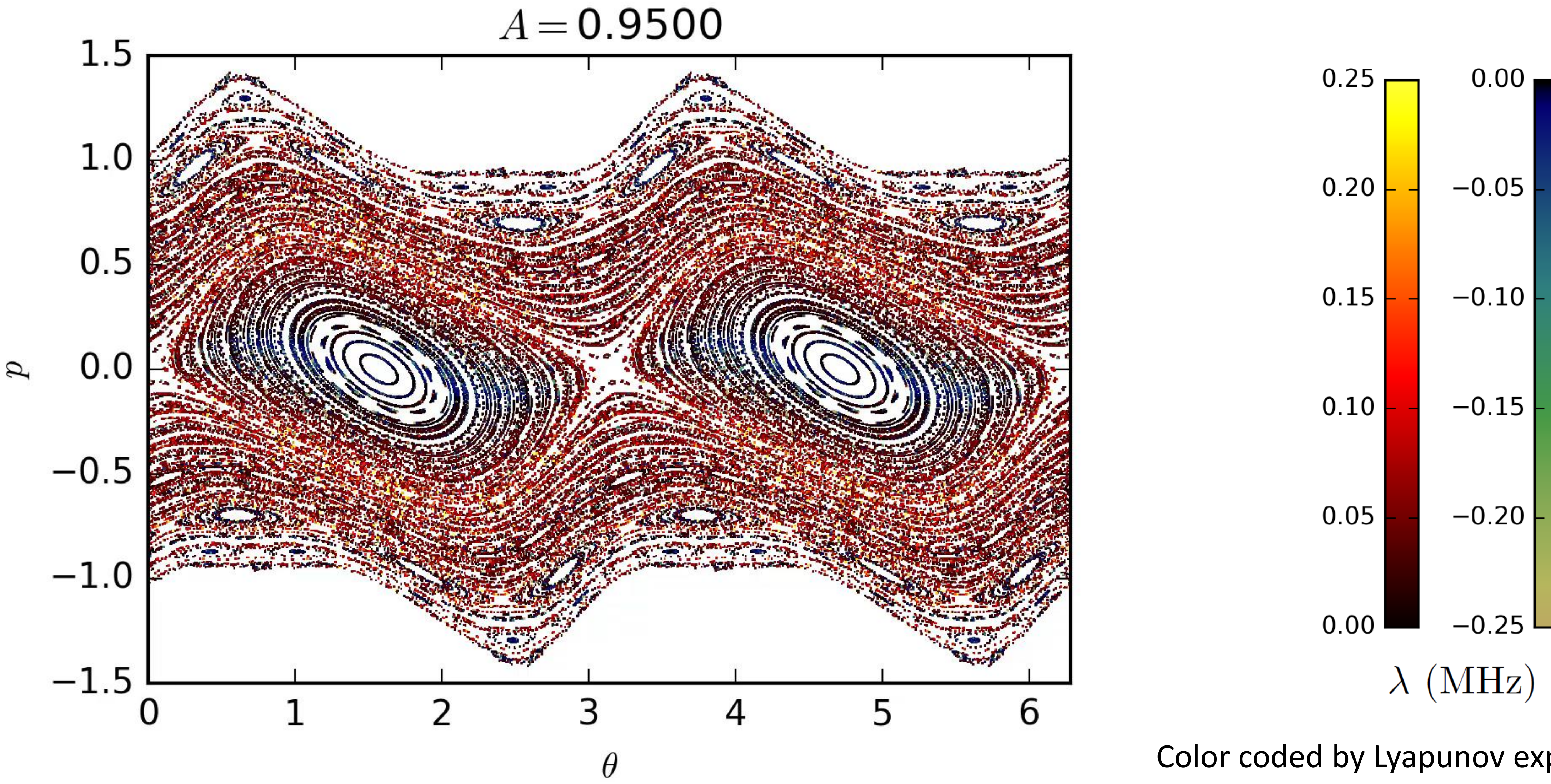




Resonances at $p_0 = k\pi$ for $k \in \mathbb{Z}$

Resonances occur where perturbation theory breaks down – small denominator problem; onset of route to chaos.
Just like kicked rotor!

Strong Kicks I: Chaos Overtakes the OP Phase Space



Color coded by Lyapunov exponent.

For more details, please see the poster by John Steinmetz

Quantum measurement powered engines

Measurement may *randomly perturb* the state of a quantum particle.

Point of view of thermodynamics

Measurement may *randomly change* the energy of a quantum particle.

- We can consider this stochastic energy exchange as analogous to heat "Quantum Heat" – Alexia Auffeves.
- We can further design engines to extract this energy as useful work

PHYSICAL REVIEW LETTERS **120,** 260601 (2018)

Efficient Quantum Measurement Engines

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Science NOW

Quantum measurements could power a tiny, hyperefficient engine | Science

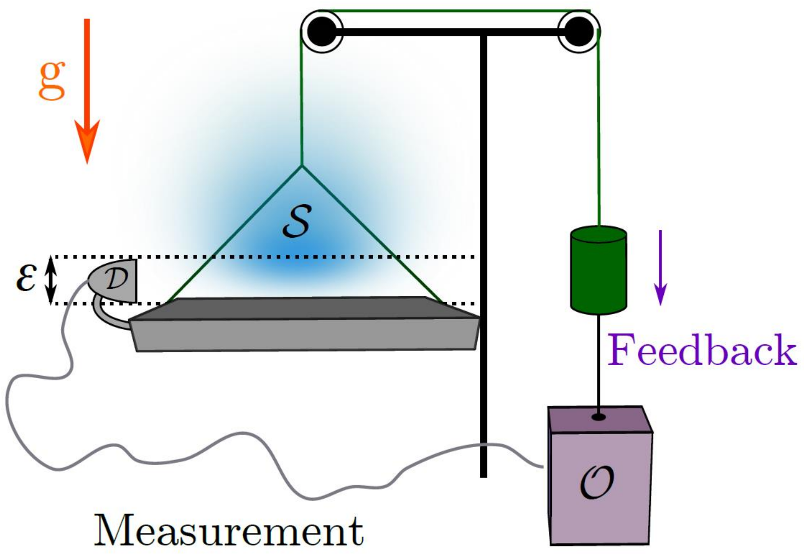
Science/AAAS, 10 Jul 2018

You can't measure an atom without disturbing it, at least according to quantum mechanics.

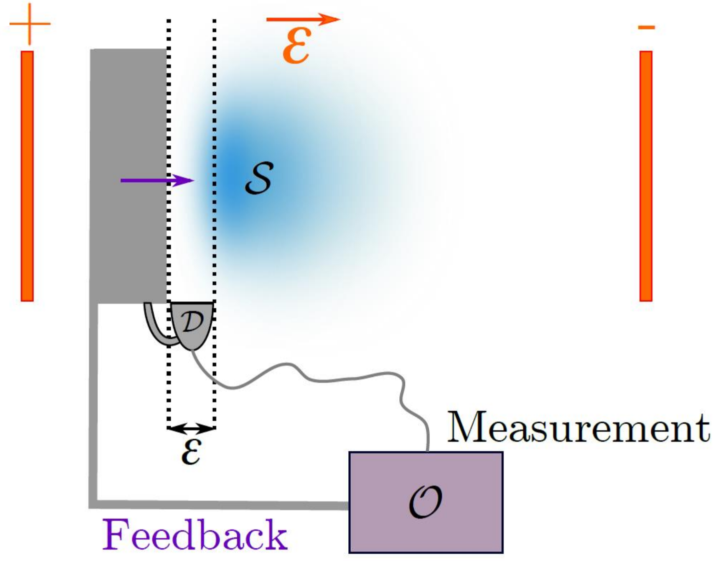
Basic idea: Use quantum measurement as a source of energy to drive an engine.

Elementary Quantum Engines

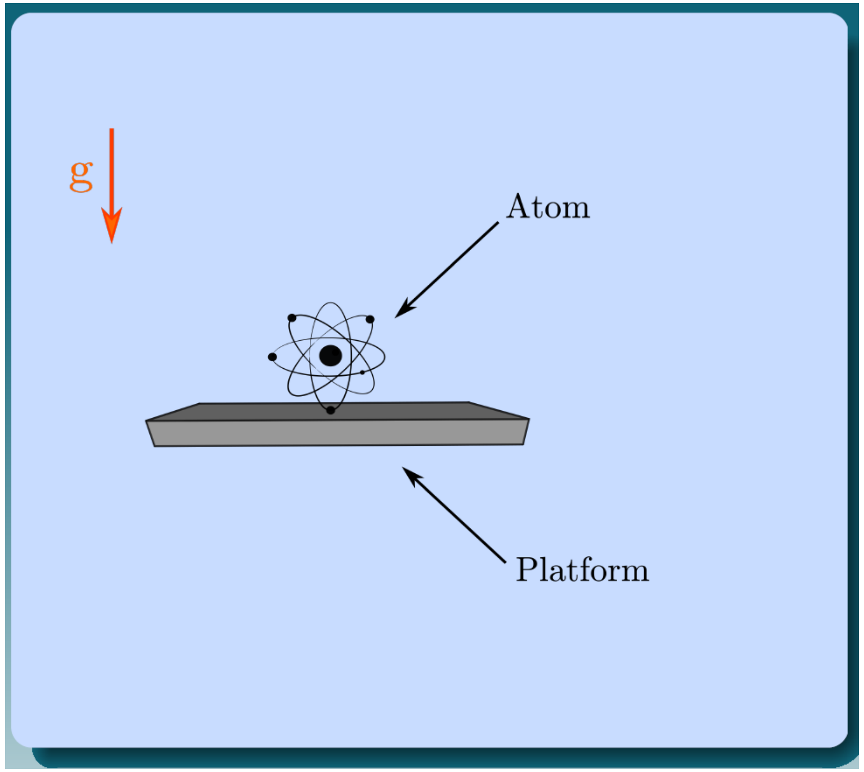
Single atom elevator



Single electron battery

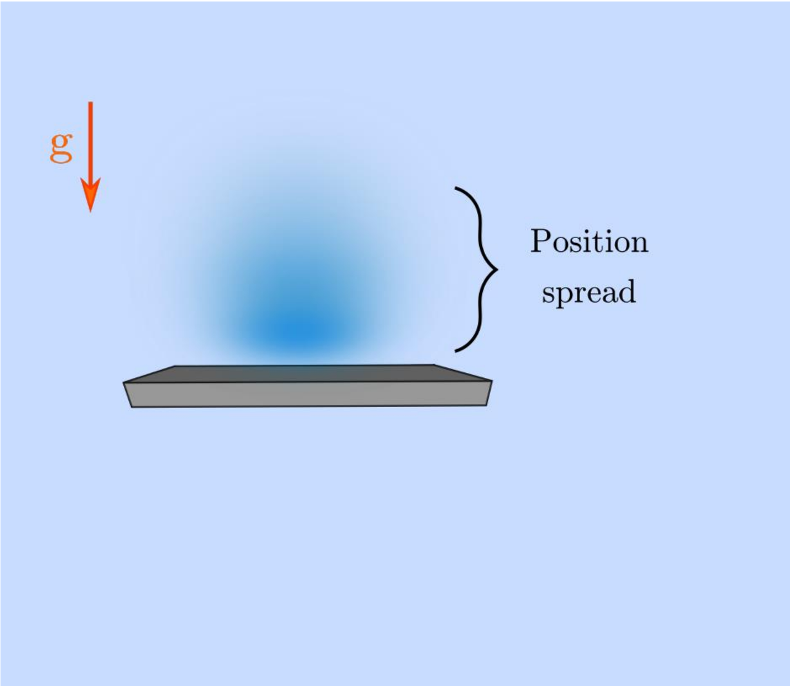


Quantum Measurement Elevator



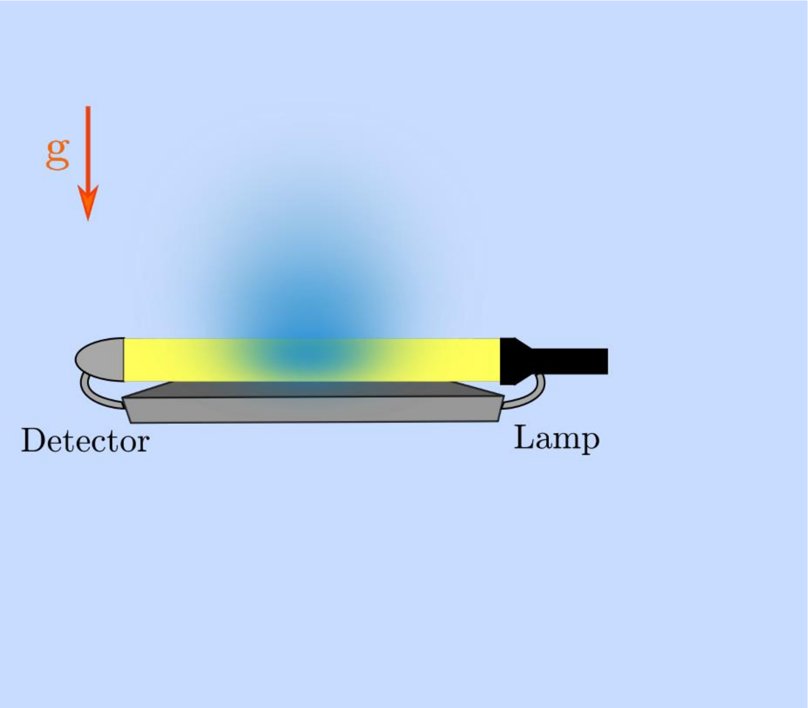
We consider an atom of mass m "placed" on a platform

Quantum Measurement Elevator



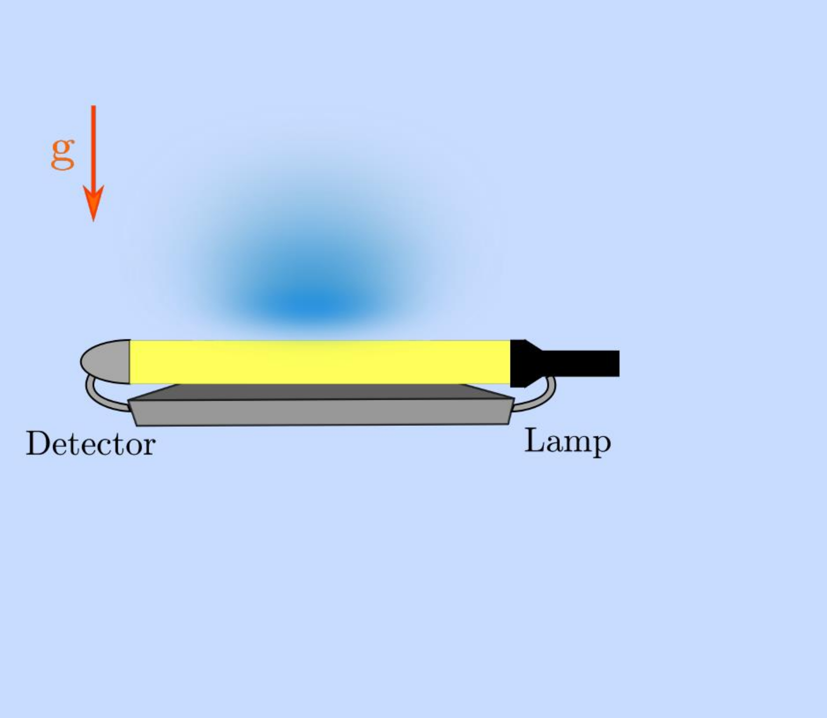
The ground (rest) state of the atom is actually a *superposition of several positions*

Quantum Measurement Elevator



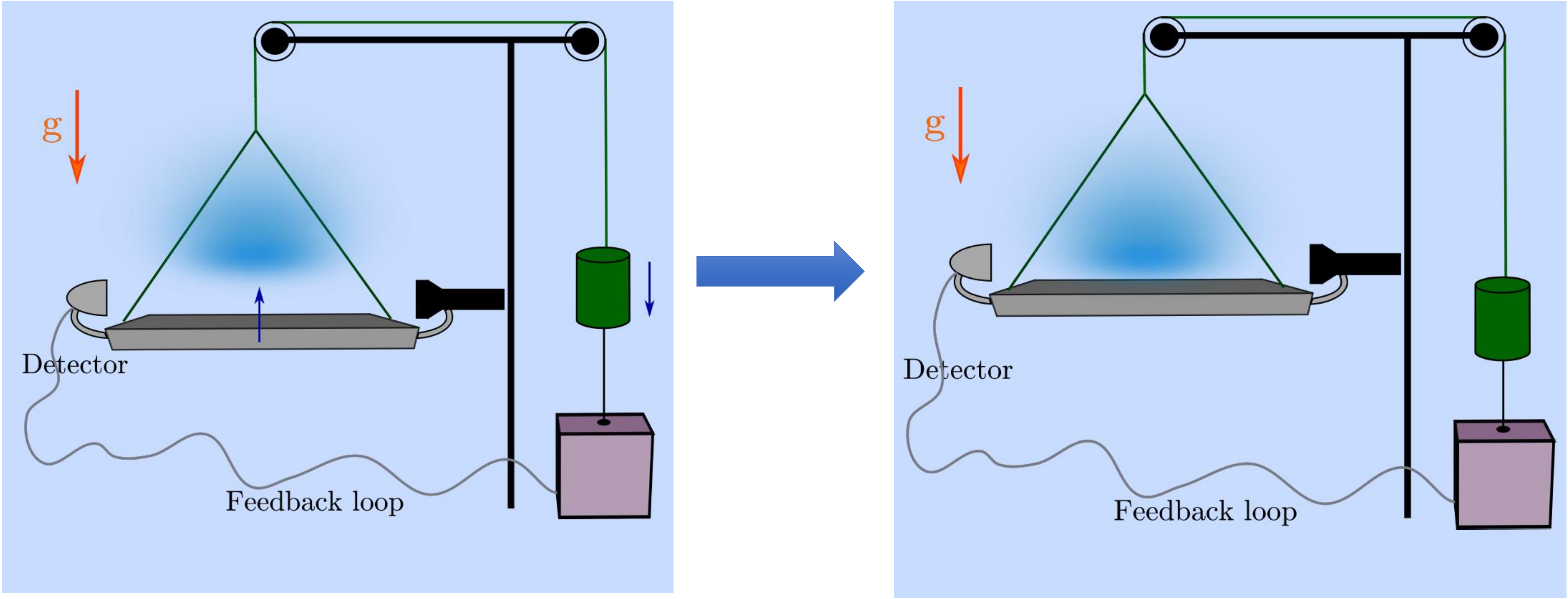
We measure whether the atom is close to the platform (e.g. with light transmission).

Quantum Measurement Elevator



If we find the atom far from the platform, we can raise it without doing work.

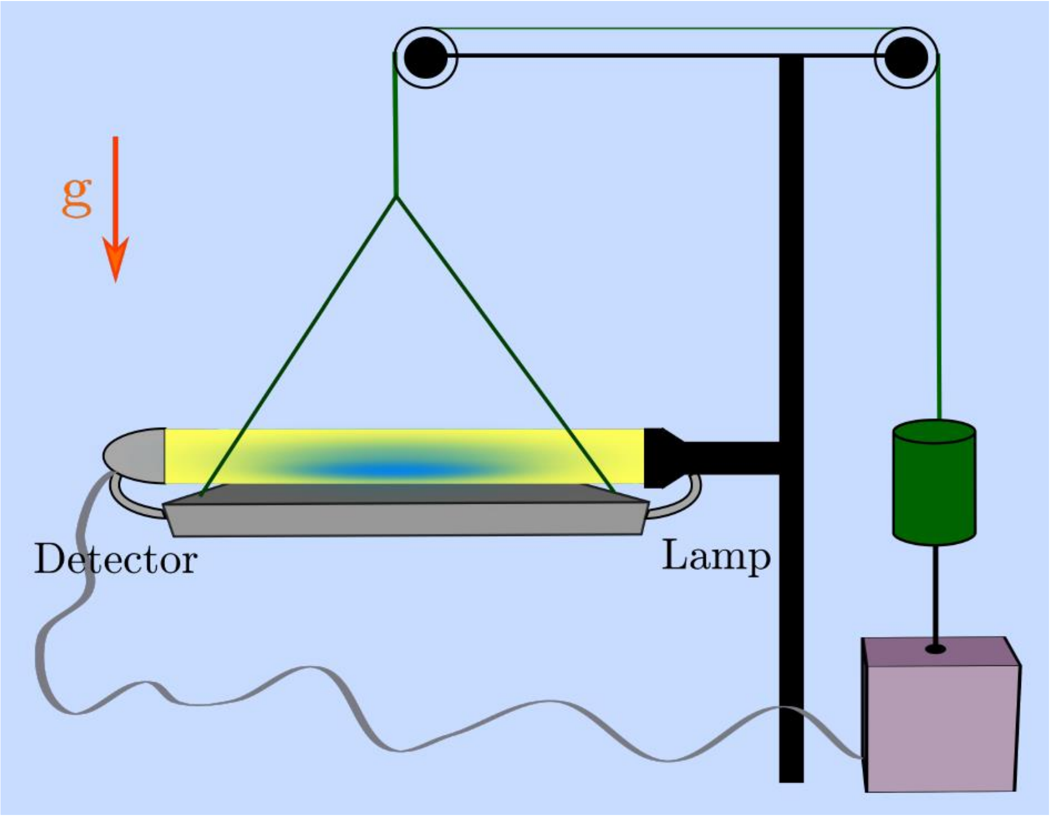
Quantum Measurement Elevator



Same situation as at the beginning but the atom is now higher.

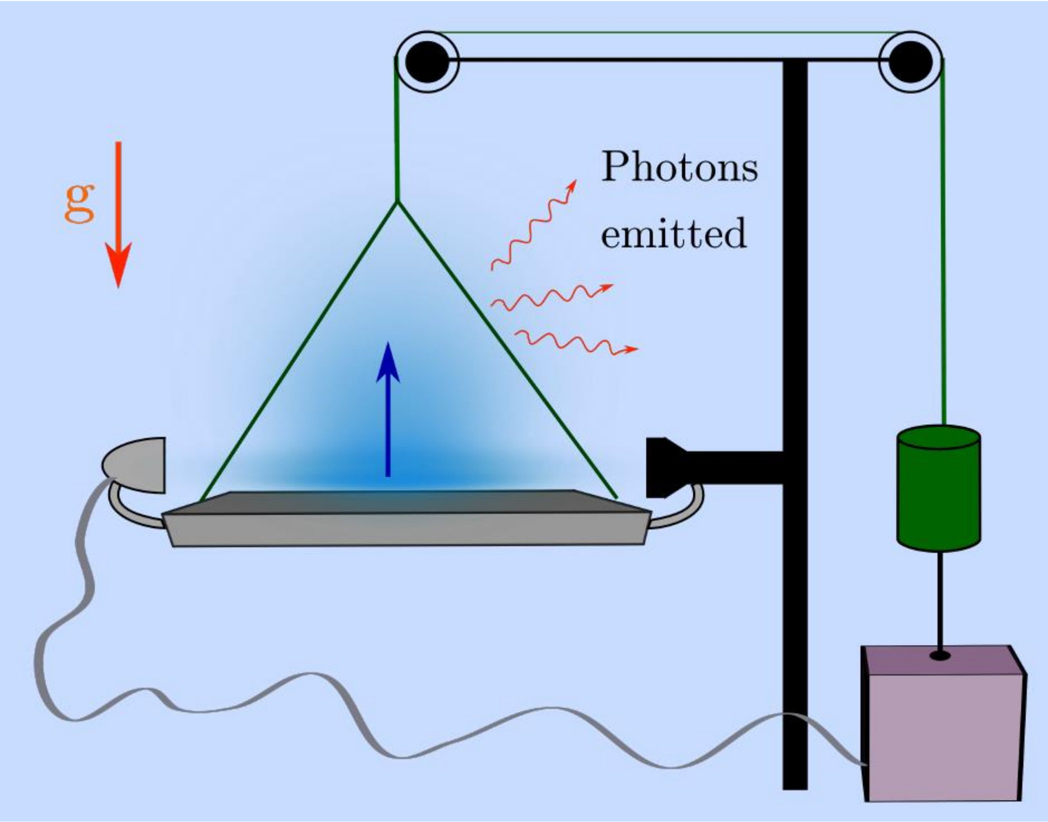
Work has been extracted !

Quantum Measurement Elevator



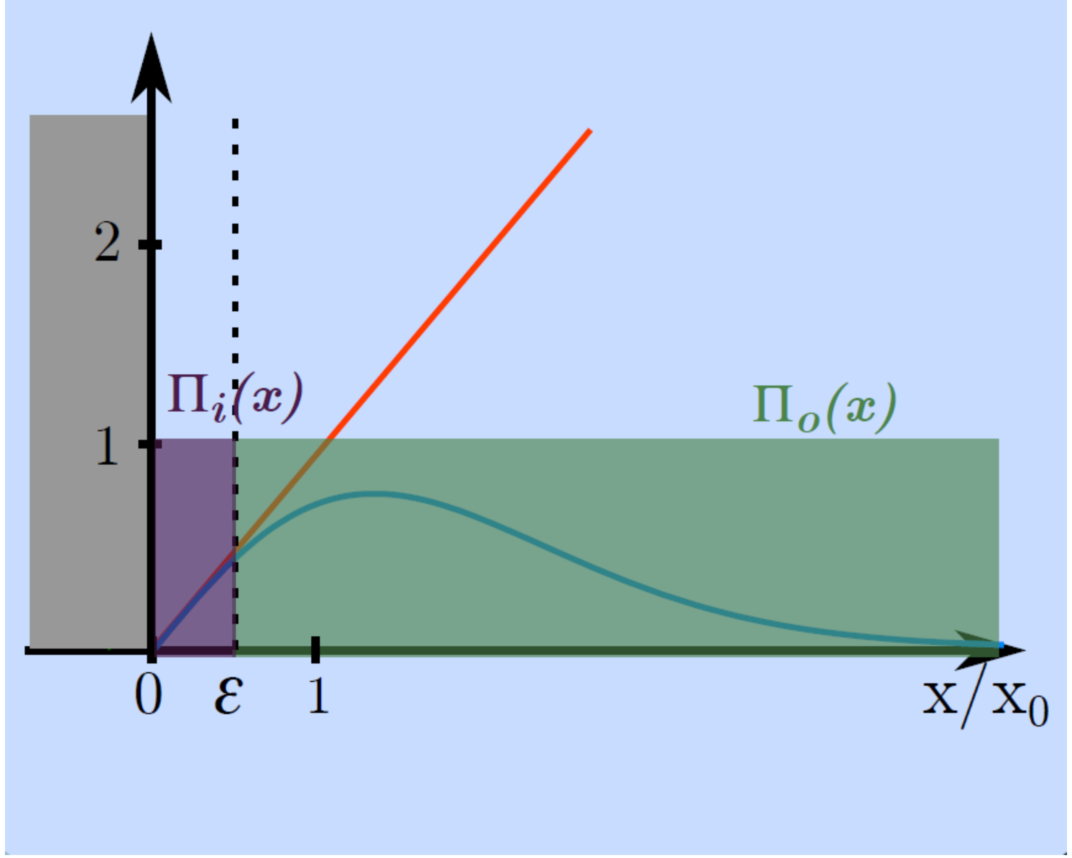
Sometimes the atom is found close to the platform.

Quantum Measurement Elevator



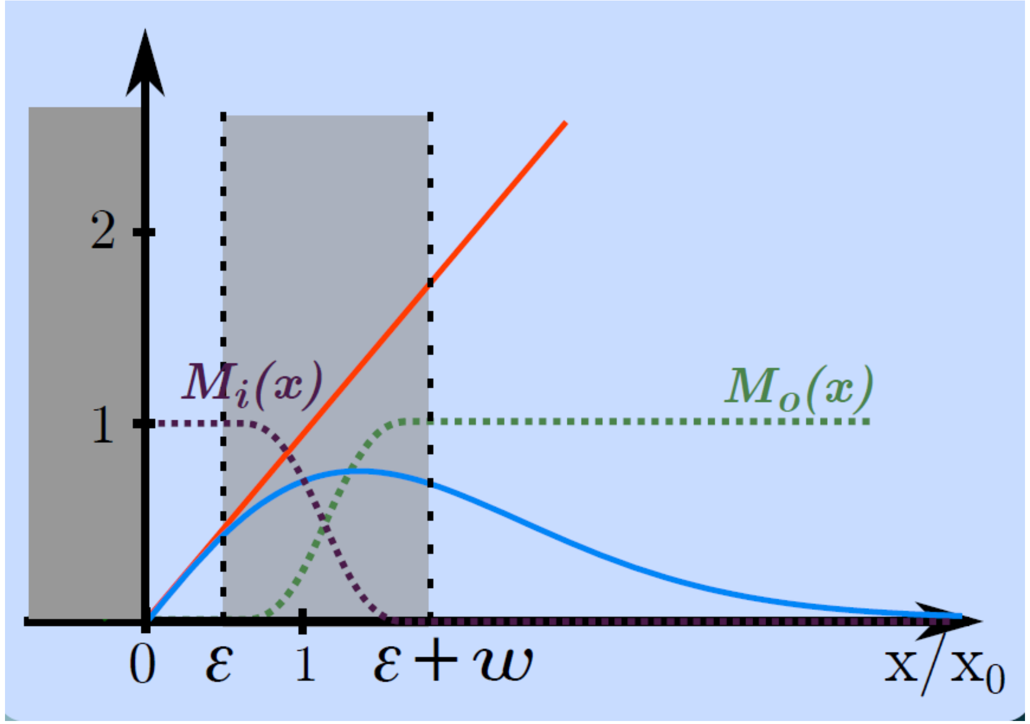
We can then let it relax down to its ground state.

Suppose we make a sharp inside/outside measurement....



The collapsed wavefunction has a sharp edge → Energy cost diverges

Soften the measurement



$\rightarrow$ Requires a smooth transition in region $[\epsilon, \epsilon + w]$

Generalized position measurement

$$M_o(x) = \begin{cases} 0, & x/x_0 < \varepsilon, \\ \sin[\pi(x/x_0 - \varepsilon)/2w], & \varepsilon < x/x_0 < \varepsilon + w, \\ 1, & x/x_0 > \varepsilon + w. \end{cases}$$

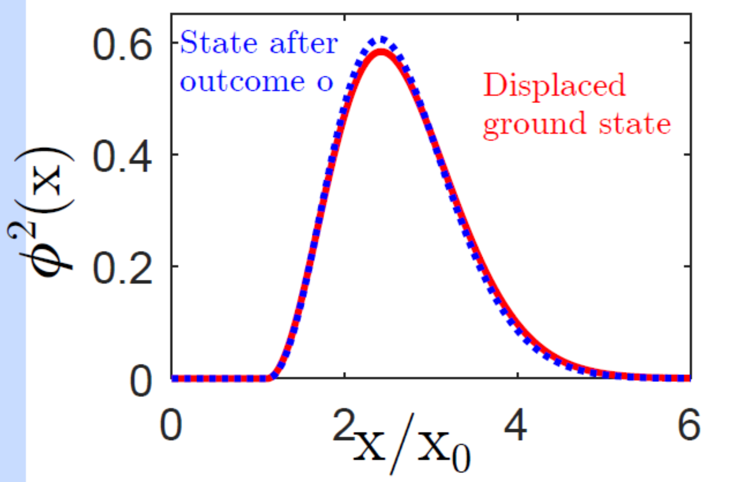
$$M_i(x) = \sqrt{1 - M_o(x)^2}$$

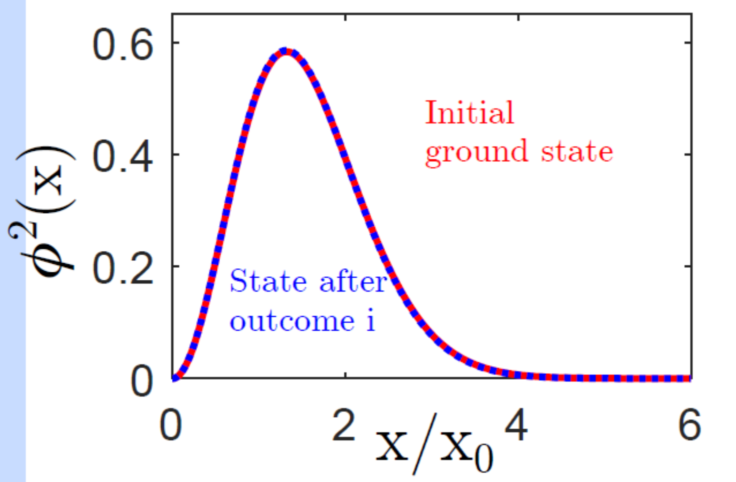
Designer Measurements

Conditions to have no relaxation needed

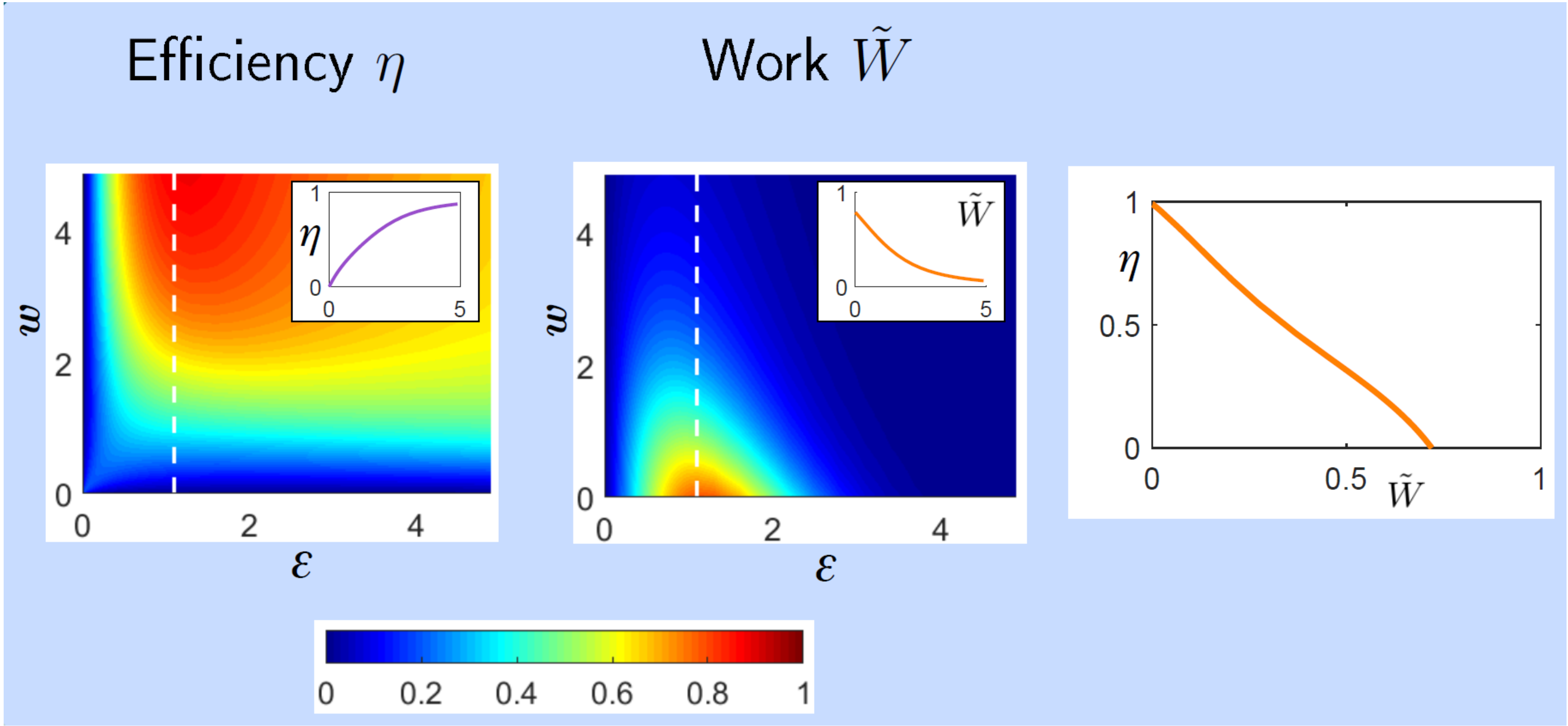
- The state after finding o is the new ground state corresponding to the shifted position of the wall
- The state after finding i is the initial ground state

Can be reached approximately for $\epsilon = \epsilon^* \simeq 1.1$ and $w >> 1$.





Performance tradeoff



Twisted Quantum Mechanics

arXiv: 1904.09289v1

Spooky Work at a Distance: an Interaction-Free Quantum Measurement-Driven Engine

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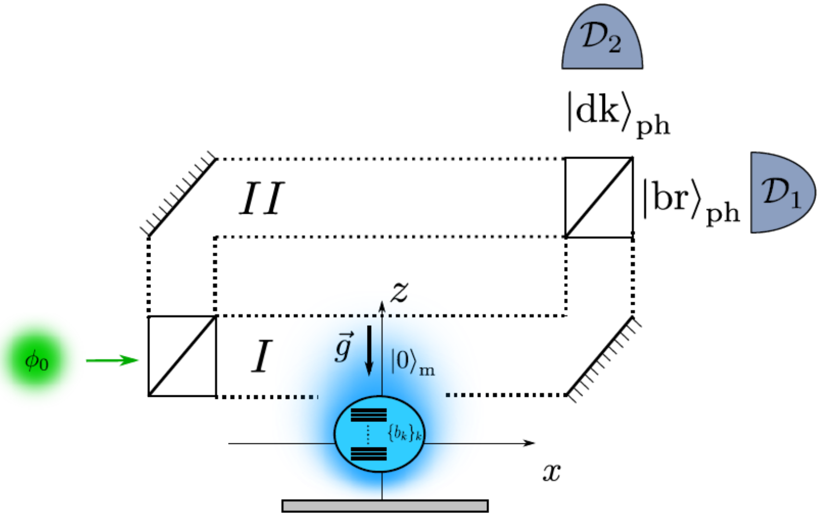
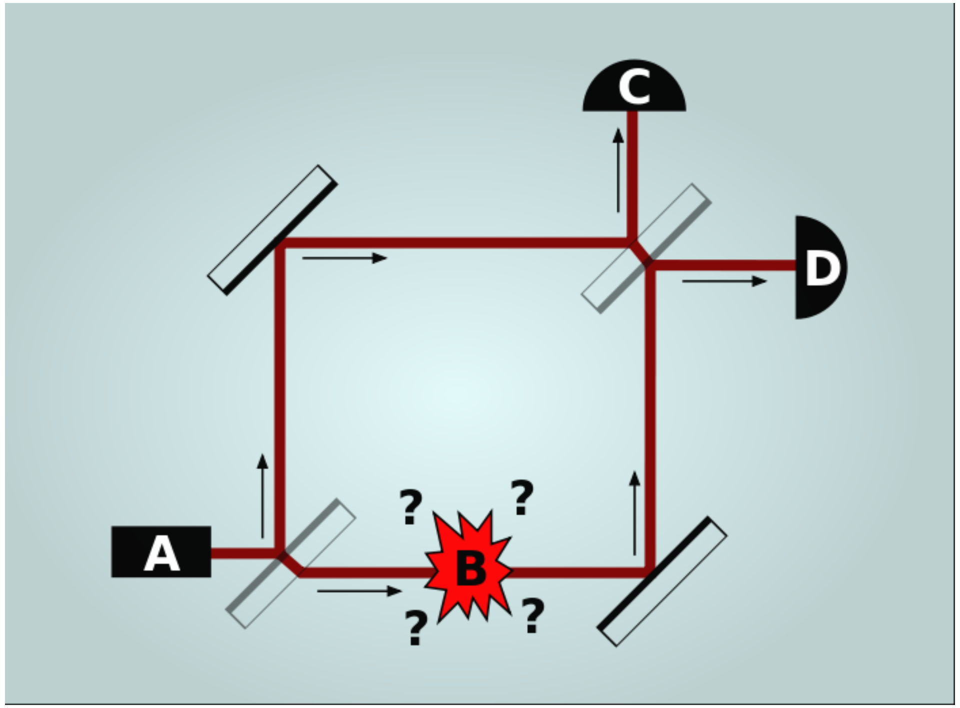
Take the single atom elevator engine, and place it inside a Mach-Zehnder interferometer



“Interaction free measurements”

a.k.a. The Elitzur-Vaidman bomb tester

- Consider a tuned Mach-Zehnder interferometer, so every photon injected in A comes out the D (bright port)
- If something – like the world’s most sensitive bomb sits inside the interferometer - then (a) the bomb can explode, or (b) the bright port can click, or (c) the dark port can click.
- If the dark port clicks, then we can infer the bomb is there, without exploding it.

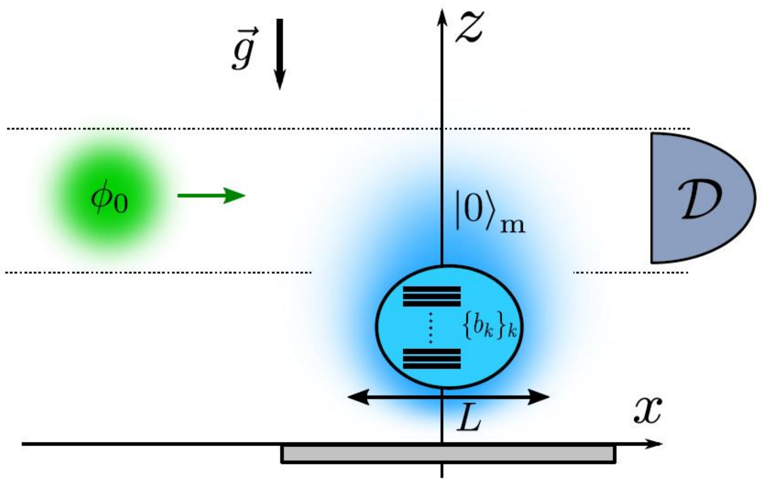


Retrodiction – predicting about the past – would say: The photon must have come through the other arm, otherwise, the bomb would have exploded.

Image credit: Wikipedia

Doing spooky work on the bomb

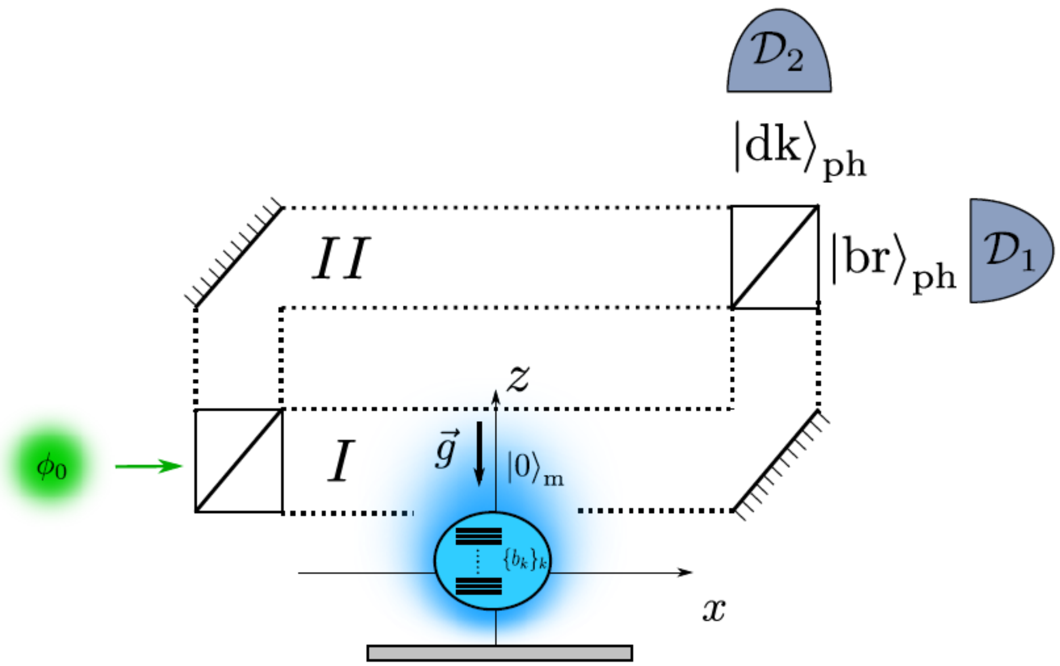
Suppose we let the bomb have its many internal degrees of freedom, but also a motional degree of freedom – it can be lifted against the force of gravity. But…. If a photon is present, it will blow up.



- The bomb can be treated quantum mechanically with its motional degree of freedom and all internal degrees of freedom in its ground state.
- The ground state wavefunction extends for some distance in space.
- We arrange for a photon to pass nearby, so there is a local interaction if the atom extends into the photon's path. Otherwise there is not.
- The other degrees of the bomb are treated as a zero temperature open quantum system, than can absorb the photon, and excite one of its modes.

Doing spooky work on the bomb

- The bomb is now part of the interferometer in its ground state.
- We post select on instances when the dark port clicks.
- If the dark port clicks, then the bomb must have been present inside of the interferometer. It also did not explode.
- Therefore, the bomb's position must be localized inside of the interferometer.
- But, that has a higher energy then it started with, and can be extracted as part of the engine cycle. Where did it come from?
- It must have come from the meter – in this case, the photon. But, the photon passed via the other arm of the interferometer. Or did it?

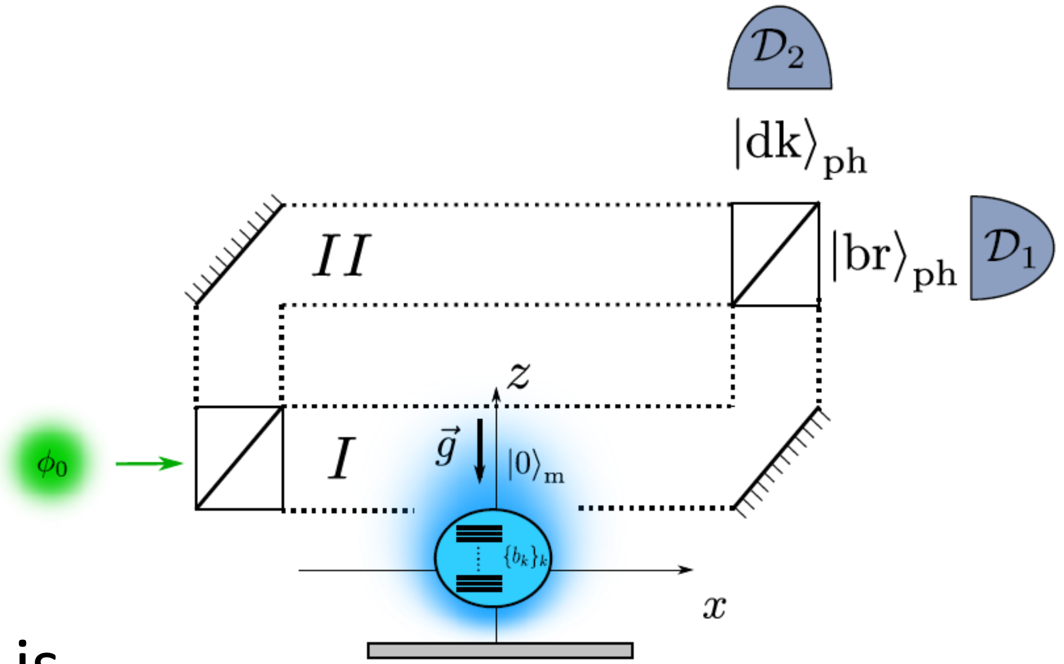


Explicit calculation reveals there is average balance in the system – that is, no energy is created or destroyed in the postselected system.

Where did the energy come from?

Our trilemma:

- It was given to the bomb nonlocally by the photon.
- It was really there, even though the bomb did not explode (i.e. the retrodictive inference is incorrect).
- A virtual photon in arm I locally gave the bomb the energy locally, and interfered with the real photon in arm II at the beam splitter (i.e. the quantum Cheshire cat).



Regardless of interpretation, this system is able to lift the most sensitive bomb, without exploding it.

Conclusions

- Survey of topics in quantum measurement.
- Fundamental aspects of continuous quantum measurement and quantum trajectories.
- Multipaths and a new kind of quantum chaos
- Quantum engines
- Spooky work

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