Multi-Qubit Robustness by Local Encoding

Quantum Information and Quantum Matter Group
www.iip.ufrn.br/qiqm

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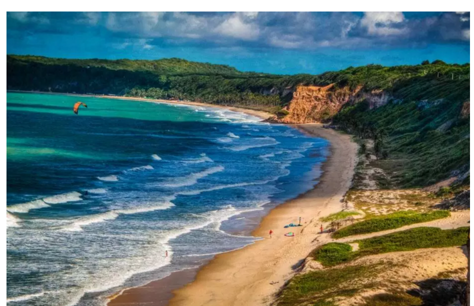
Few words about Natal...



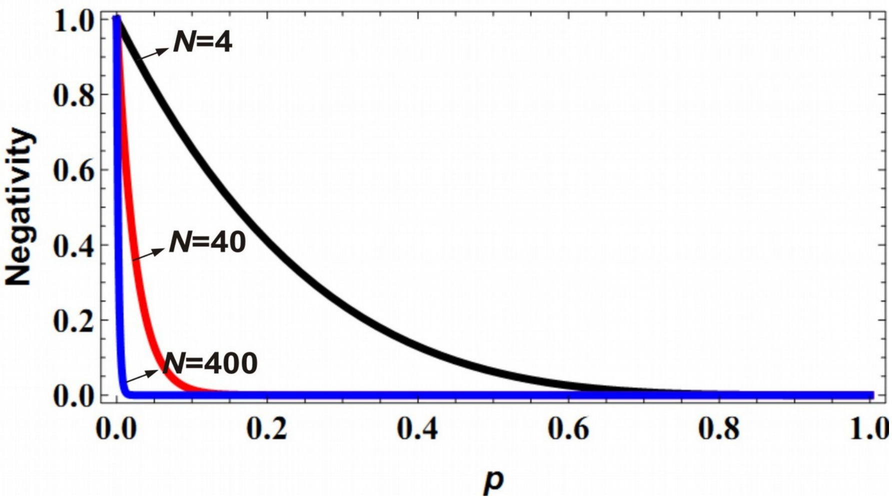
What is this talk about?

Quantum correlations can be fragile…

$$\left|GHZ^N\right\rangle = \sqrt{\tfrac{1}{2}}\left(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}\right)$$

$$\Downarrow$$

$$E\left(\rho^N_{GHZ}\left(p\right)\right) \leq \left(1-p\right)^N \cdot E\left(\rho^N_{GHZ}\left(0\right)\right)$$



*[Aolita, RC, Cavalcanti, Acin, Davidovich PRL 2008]
*[Aolita, Melo, Davidovich, RPP 2015]

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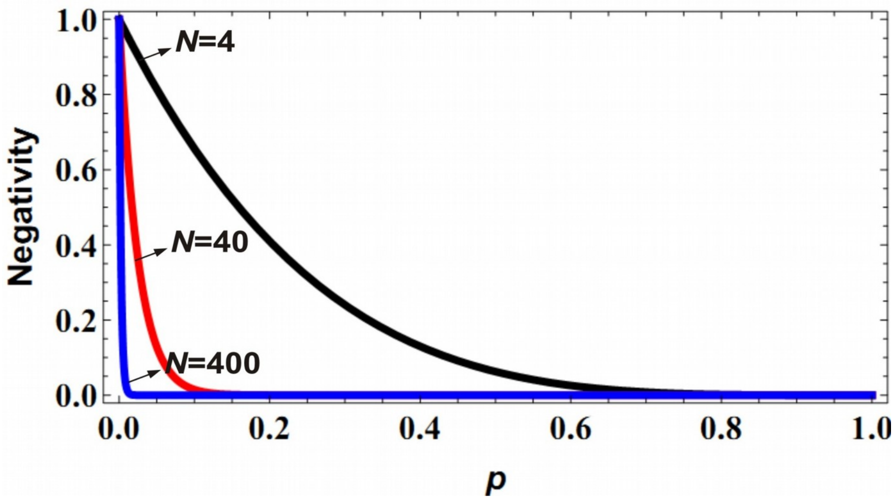

*[Aolita, RC, Cavalcanti, Acin, Davidovich PRL 2008]
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Fragile Entanglement $\stackrel{?}{=}$ Fragile resource

Quantum Metrology ⟹

$$\delta\lambda \geq \frac{1}{2\sqrt{\nu\mathcal{F}}}$$

Apparently, any full-rank noise will spoil scaling quantum advantages.

$$\mathcal{F}\left(\rho_{sep}\right) \leq N$$

$$\mathcal{F}\left(\rho^N_{GHZ}(\rho)\right) = (1-p)^{2N} N^2$$

*[Huelga et al., PRL 1997]
*[Escher et al. NatPhys 2011]
*[Demkowicz-Dobrzanski et al. NatComm 2012]

Quantum correlations can be made robust…

Solution 1 (general state, any noise)

Quantum error-correction codes

- Error Syndrome
- Feedback correction

Quantum correlations can be made robust…

Solution 1 (general state, any noise)

Quantum error-correction codes

- Error Syndrome
- Feedback correction

Solution 2 (GHZ state, white noise)

Encode each logical qubit into a GHZ state

*[Frowis, Duer, PRL 2011]

- Protection is achieved passively
- Experimental overhead

$$|0\rangle \rightarrow |GHZ_{+}^{M}\rangle = |0\rangle^{\otimes M} + |1\rangle^{\otimes M}$$

$$|1\rangle \rightarrow |GHZ_{-}^{M}\rangle = |0\rangle^{\otimes M} - |1\rangle^{\otimes M}$$

Decay rate of entaglement is still

$(1\text{-}p)^N$

but now **p** exponentially decreases with **M**.

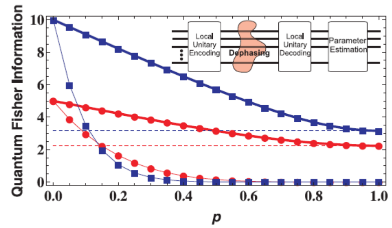
What about noise with a privileged direction?

- Trapped-ion experiments
- Photonic polarization-qubits

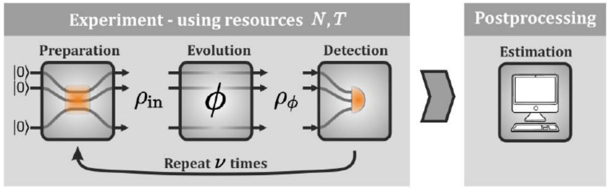
GHZ entanglement can be made exponentially more robust

Outline

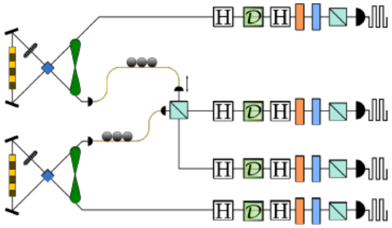
- Experimentally friendly robust GHZ entanglement



- Applications in Metrology



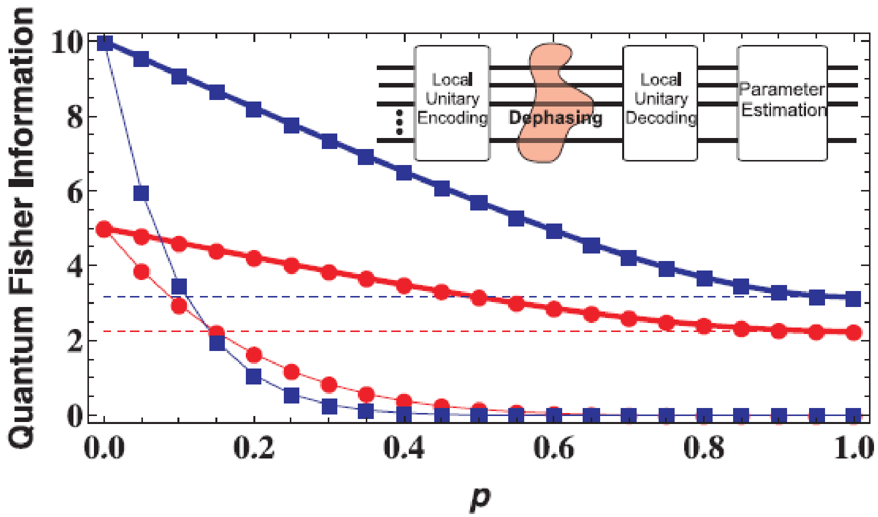
- An experimental investigation



- Some puzzling open questions

Experimentally friendly robust GHZ entanglement

*[RC, Aolita, Acin, PRL 2012]

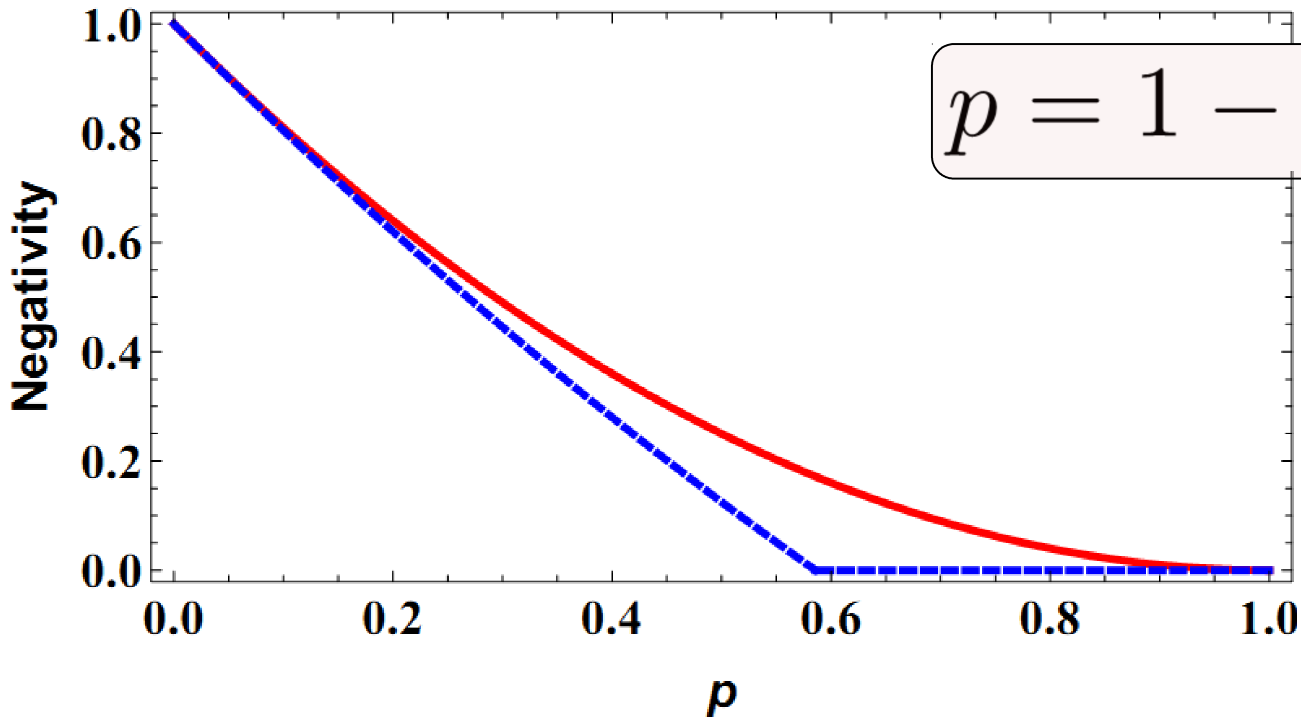


The basic idea

$$\varepsilon(\rho) \rightarrow \left(1 - \frac{p}{2}\right)\rho + \frac{p}{2} Z \rho Z$$

$$|\Phi^+\rangle = |00\rangle + |11\rangle$$

$$(I \otimes H)|\Phi^+\rangle = |0+\rangle + |1-\rangle$$



$$p = 1 - e^{-\gamma \cdot t}$$

What is the most robust local encoding against dephasing?

$$\varepsilon(\rho) \to \left(1-\frac{p}{2}\right)\rho + \frac{p}{2} Z\rho Z$$

$$|\Phi_+^N\rangle \doteq \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N})$$

- Considering the negativity and numerical optimizations up to N=10 show that the optimal states are the transversal states.

$$\left|\Phi_{+T}^N\right\rangle \doteq H^{\otimes N}|\Phi_+^N\rangle = \frac{1}{\sqrt{2}}(|+\rangle^{\otimes N} + |-\rangle^{\otimes N})$$

$$|\pm\rangle \doteq \tfrac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

- Analytically we can prove that for any entanglement measure

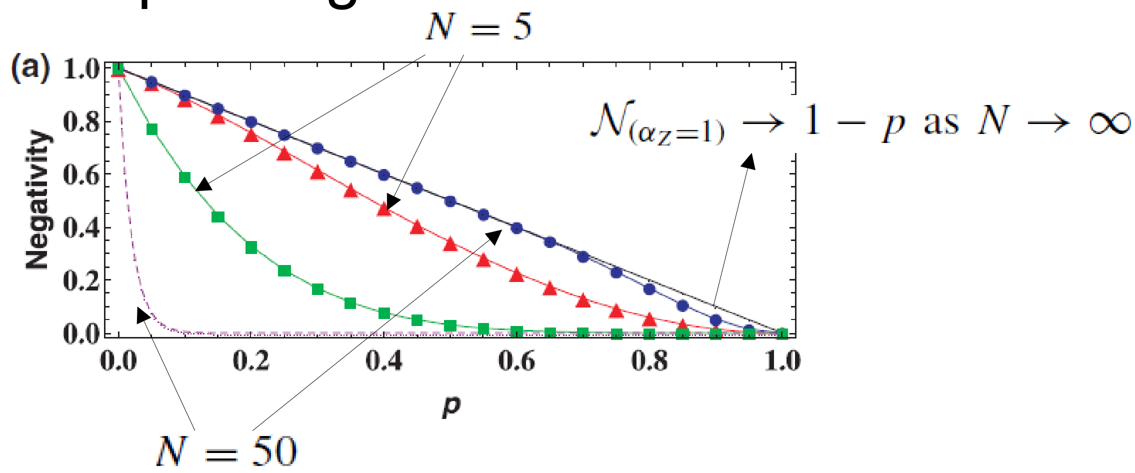
$$E\left(\Lambda^{\mathrm{PD}}\left(\left|\Phi_{+T}^N\right\rangle\right)\right) \geqslant E\left(\Lambda^{\mathrm{PD}}\left(\left|\Phi_{+T}^{N-1}\right\rangle\right)\right) \geqslant \cdots \geqslant E\left(\Lambda^{\mathrm{PD}}\left(\left|\Phi_{+T}^{2}\right\rangle\right)\right)$$

- N entangled qubits are at least as robust as 2 entangled ones!

What is the most robust local encoding against dephasing?

Pure Dephasing

$$\varepsilon\left(\rho\right) \rightarrow \left(1-\frac{p}{2}\right)\rho+\frac{p}{2}Z\rho Z$$



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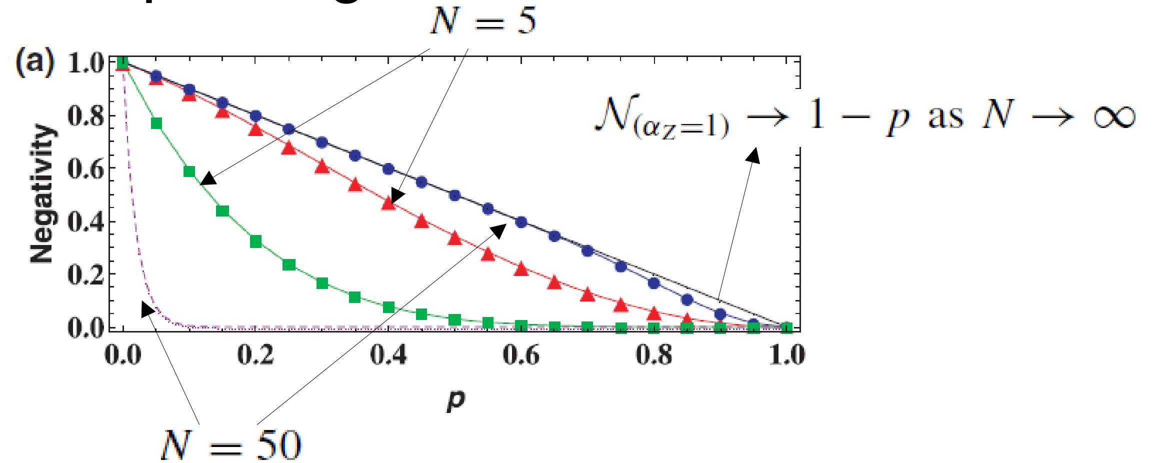
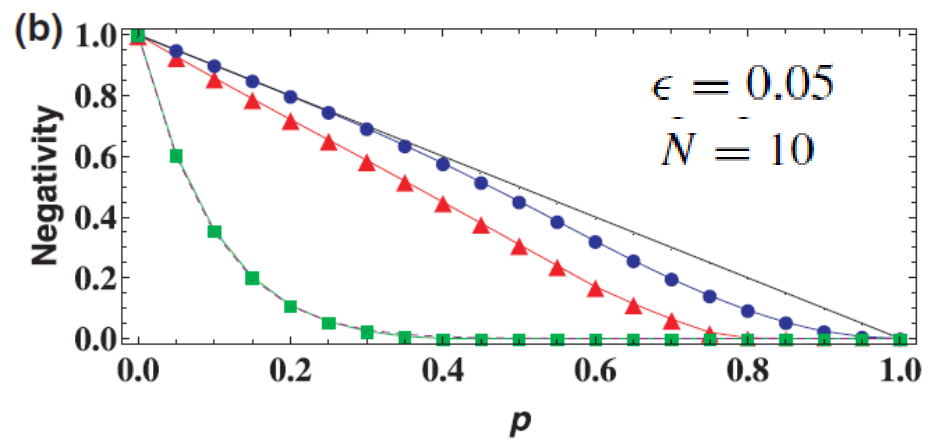


Deviations from pure dephasing

$$\mathcal{E}(\varrho) \doteq \left(1 - \frac{p}{2}\right)\varrho + \frac{p}{2}(\alpha_X X\varrho X + \alpha_Y Y\varrho Y + \alpha_Z Z\varrho Z),$$

$$\epsilon \doteq \alpha_X + \alpha_Y$$

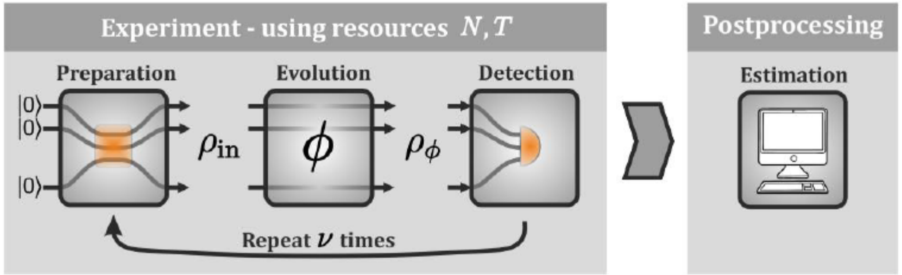
$$\alpha_Z = 1 - \epsilon$$



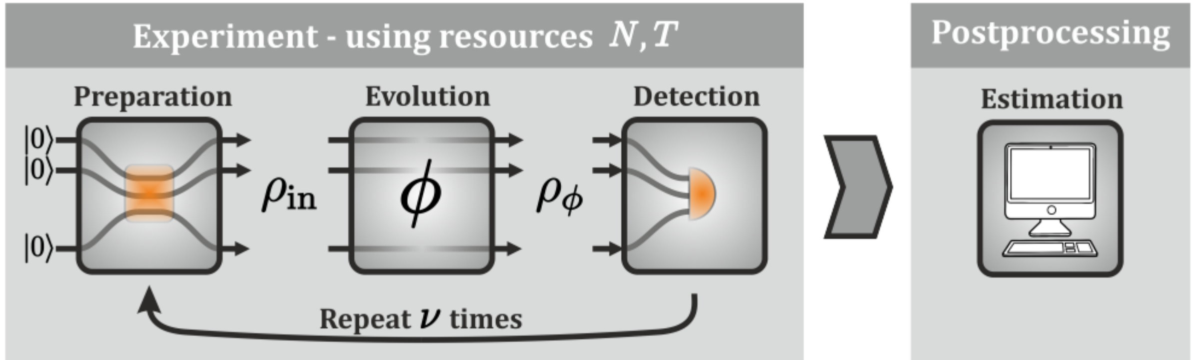
$$\mathcal{N} \approx 1 - p(N\epsilon + 1 - \epsilon) \approx (1-p)^{N\epsilon + 1 - \epsilon}$$

Applications in Metrology

*[RC, Aolita, Acin, PRA 2012]
*[RC, Brask, Marki Acin, PRL 2013]
*[Brask, RC, Markiewicz, Kolodynski, PRX 2014]



Single-Parameter Estimation



Typical experiment: [preparation+interaction+measurement]+postprocessing

As resources we take the ***probe size*** and *the* ***total time N, T***

- Classical strategies based on separable states, the central limit theorem bounds the precision, the ***standard quantum limit*** (**SQL**)

$$\delta\omega \geq c_{cl}\frac{1}{\sqrt{N\nu}}$$

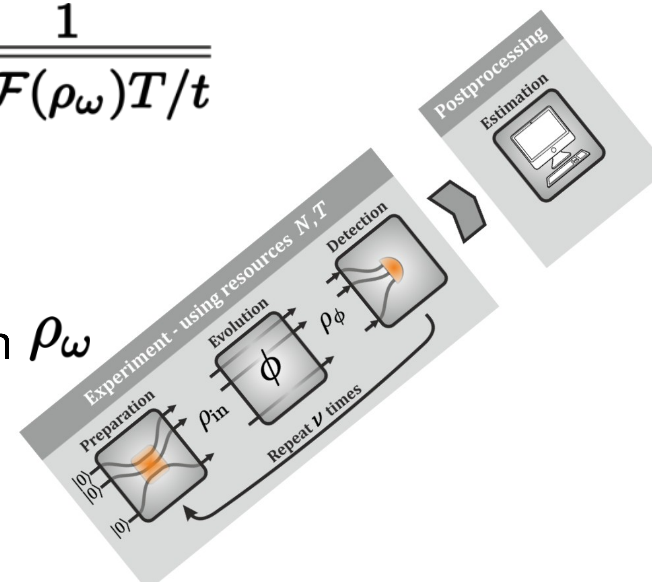
- For entangled states, the precision is limited by the quantum uncertainty relation, the ***Heisenberg bound***

$$\delta\omega \geq c_q\frac{1}{N\sqrt{\nu}} \qquad \nu = T/t$$

Computing the precision

The attainable precision can be bounded by the quantum Fisher information

$$\delta\omega \geq \frac{1}{\sqrt{\mathcal{F}(\rho_\omega)\nu}} = \frac{1}{\sqrt{\mathcal{F}(\rho_\omega)T/t}}$$



$\mathcal{F}(\rho_\omega)$ quantifies the information about ω encoded in ρ_ω

The optimal precision requires optimization over inputs

$$\max_{\rho_{in}} \mathcal{F}(\rho_\omega)$$

hard!

To avoid the optimization we can compute

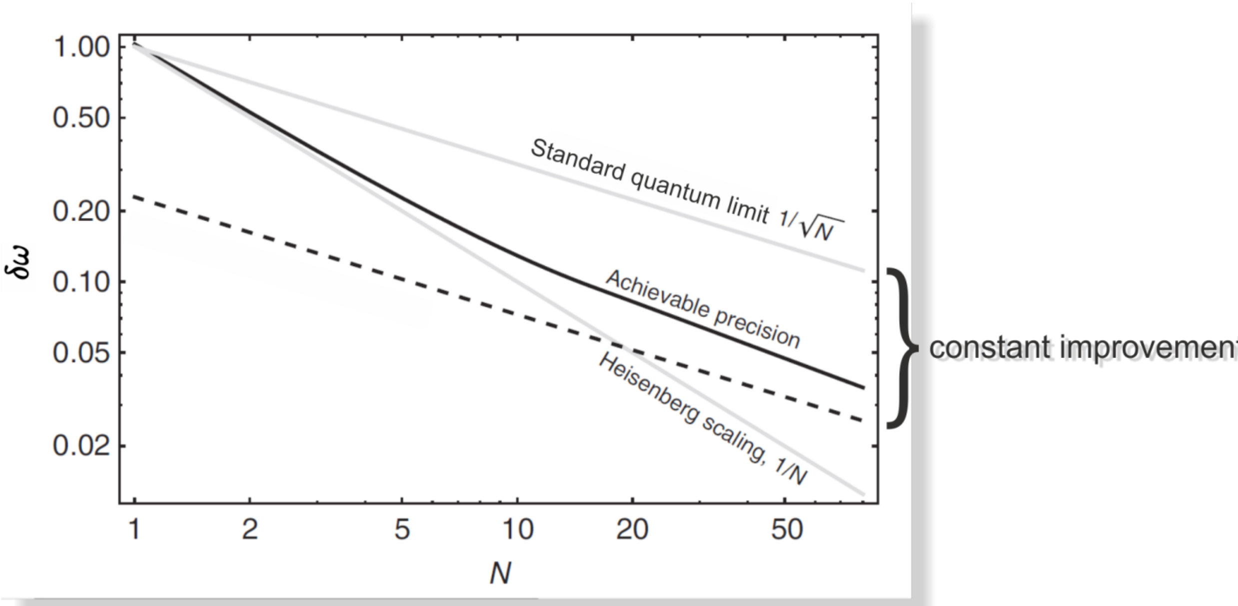
- Fisher information of a ***specific state***
  - gives lower bound on the optimum
- ***Bounds*** that require no or simpler optimization
  - gives upper bounds on the optimum

Metrology versus noise

- The attainable precision depends strongly on the noise model
  - difficult to obtain general results

- Result indicate that even for arbitrarily small amounts of noise, quantum strategies provide only a ***constant improvement over the SQL***

Demkowicz-Dobrzanski ***et al.***, Nat. Comm. 3, 1063 (2012)



- Full rank noise independent of the probe size: SQL like scaling (Fujiwara, Imai, 2008)

- SQL-like scaling for: lossy optical interferometry, atomic spectroscopy under dephasing or spontaneous emission (Knysh ***et al.***, 2010, Escher ***et al.***, 2011, Demkowicz-Dobrzanski ***et al.***, 2012)

Noisy Frequency Estimation Model

We use a master equation description

$$\frac{\partial \rho(t)}{\partial t} = \mathcal{H}(\rho) + \mathcal{L}(\rho)$$

- ***Unitary Evolution***

frequency to be estimated

$$H = \frac{\omega}{2} \sum_{k=1}^{N} \sigma_z^k \qquad \mathcal{H}(\rho) = -i\,[H, \rho]$$

- ***Noise***

$$\mathcal{L}(\rho) = -\frac{\gamma}{2}\left[\rho - \sum_{k=1}^{N} \alpha_x \sigma_x^k \rho \sigma_x^k - \alpha_y \sigma_y^k \rho \sigma_y^k - \alpha_z \sigma_z^k \rho \sigma_z^k\right]$$

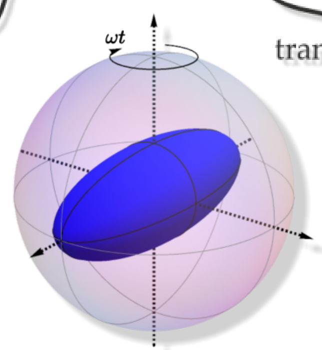
$$\sum_i \alpha_i = 1$$

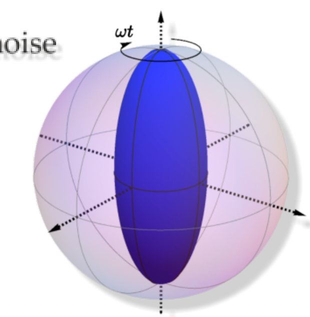
$$\alpha_i \geq 0$$

overall noise strength

transversal noise

parallel noise





$\alpha_z = 1$ → Paralell noise, commutes, studied before

$\alpha_{x,y} > 0$ → Does not commute, noise is full rank

Beating the shot noise limit

We cannot derive analytical bounds – but we can compute the finite-N Channel extension numerically

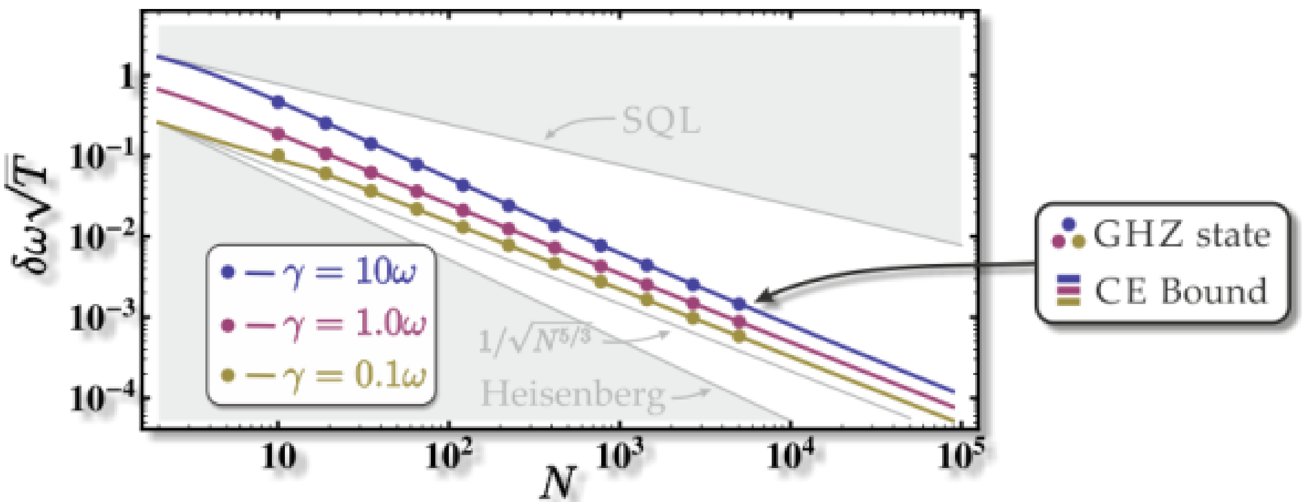
$$\delta\omega\sqrt{T} \geq \sqrt{\frac{c_x(\gamma)}{N^{5/3}}}$$

(based on numerics)

$$c_x(\gamma) = \frac{3^{2/3}}{2}\gamma^{1/3}$$

$$t_x^{\mathrm{opt}} \underset{N\to\infty}{\longrightarrow} 0$$

We can compute the Fisher Information for the GHZ state analytically and optimize the evolution time numerically

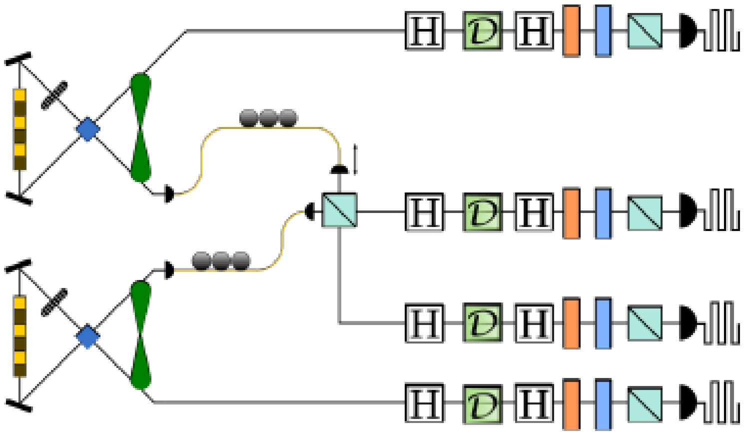


• ***The bound is tight***

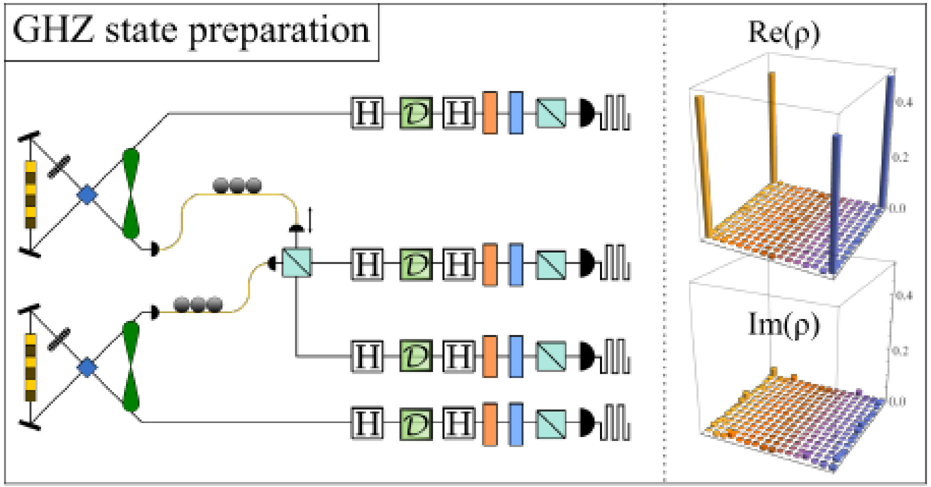
The GHZ state saturates the bound! → • ***Super-classical precision scaling for full-rank noise is indeed possible***

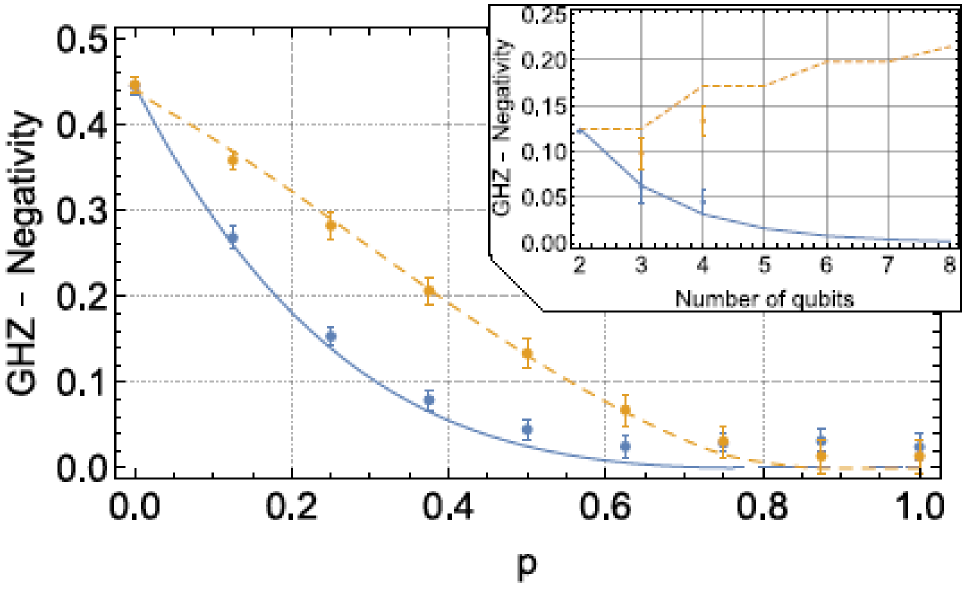
Experimental Investigation

*[M. Proietti, M. Ringbauer, F. Graffitti, P. Barrow, A. Pickston, D. Kundys, D. Cavalcanti, L. Aolita, R. Chaves, and A. Fedrizzi, arXiv: 1903.08667]

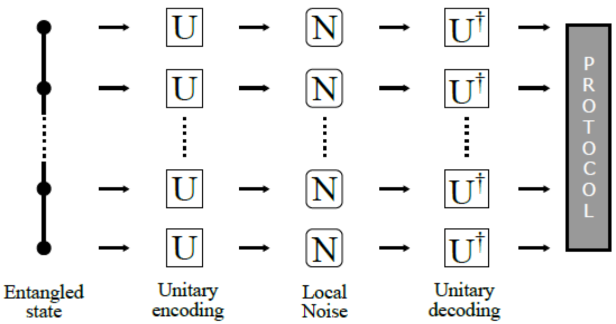


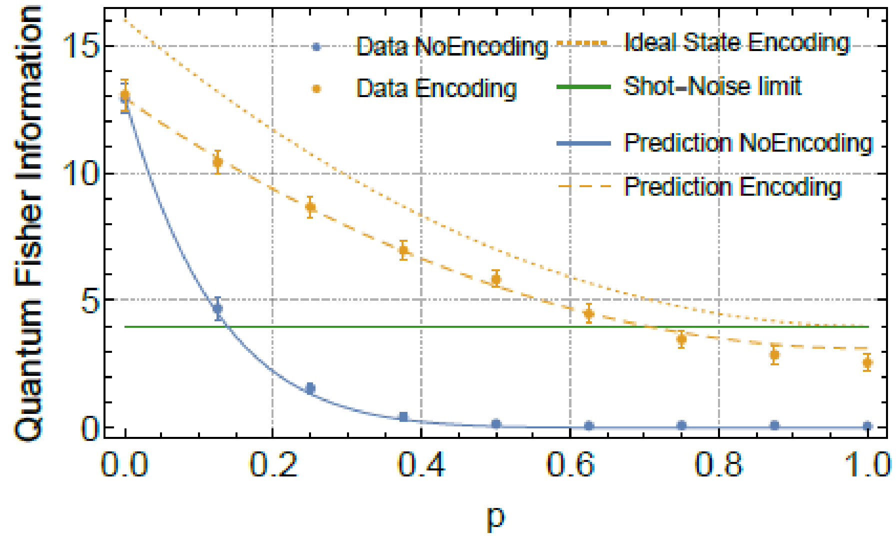
Experimentally Robust GHZ entanglement

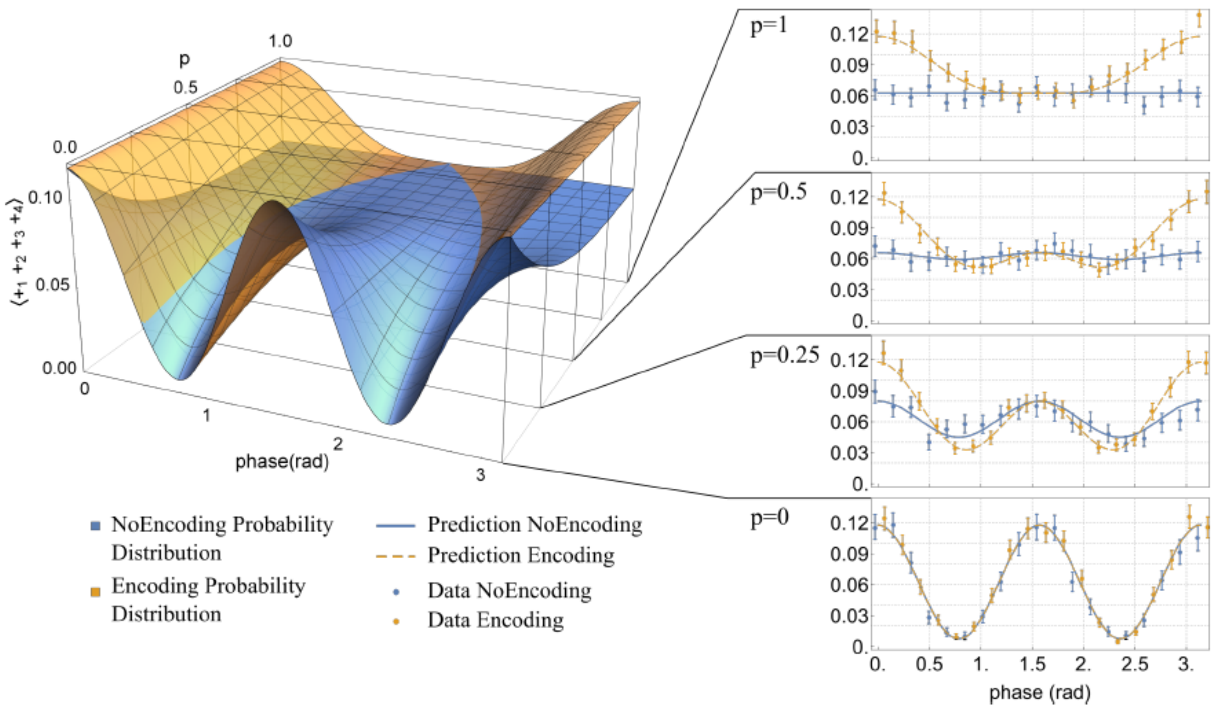




Experimentally Robust metrology

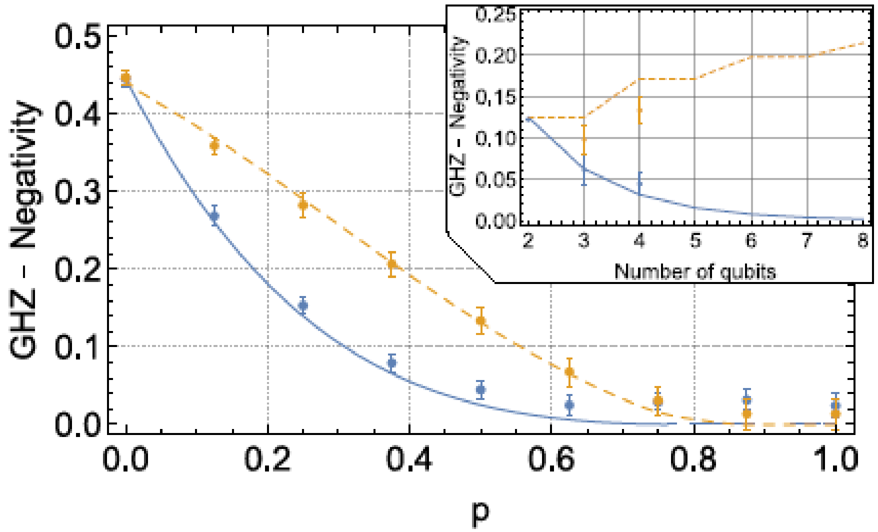


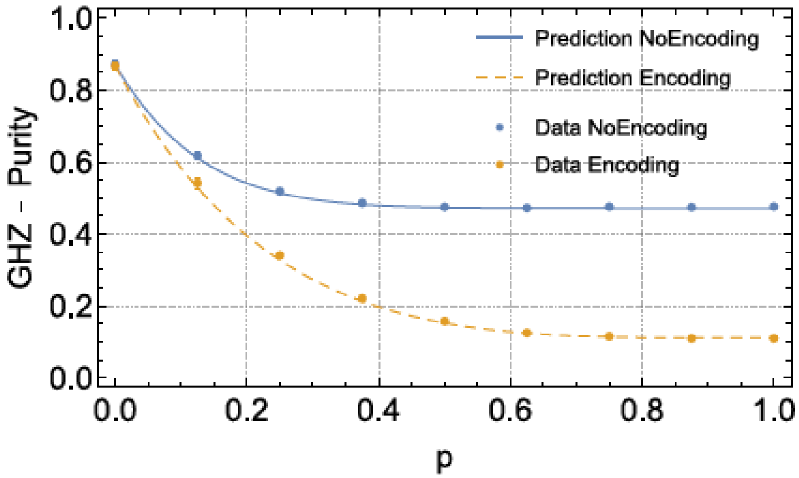




A puzzling question

Puzzle





Most robust state is the one generating far more entanglement with the environment.

What to remember (if anything)

- i) If you have noise with privileged direction local encoding might improve a lot the robustness of correlations

- ii) This robustness is also reflected in the use of these correlations as a resource: metrology, Bell inequalities violation, communication complexity problems, etc

- iii) Is there a principle underlying what is the most robust local basis?