Weak value: a property of a single system

Lev Vaidman

TEL AVIV UNIVERSITY אוניברסיטת תל אביב

Conference on Quantum Measurement: Fundamentals, Twists, and Applications

ICTP

1.5.2019

Outlook

The two-state vector formalism

Weak value as an outcome of a weak measurement:
a little history and controversy

Weak value as a property of a single system

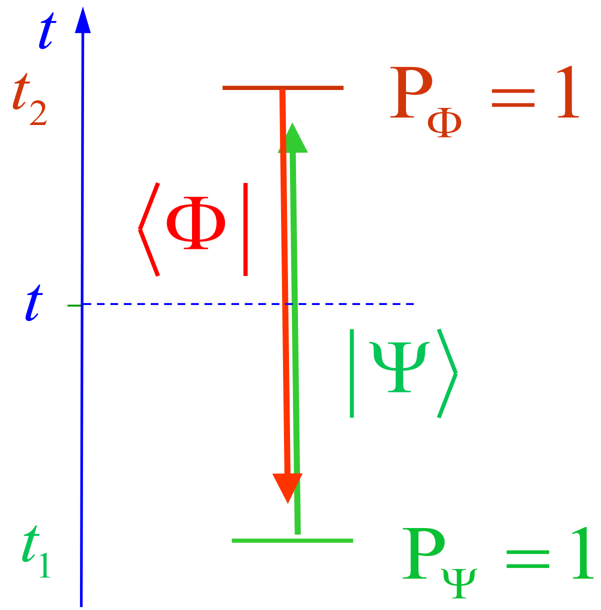
The concept of presence of a quantum particle:
the weak value of a spatial projection operator

Practical application: novel alignment method

The two-state vector formalism of quantum mechanics

The pre- and post-selected particle is described by the two-state vector

$$\langle\Phi| \quad |\Psi\rangle$$

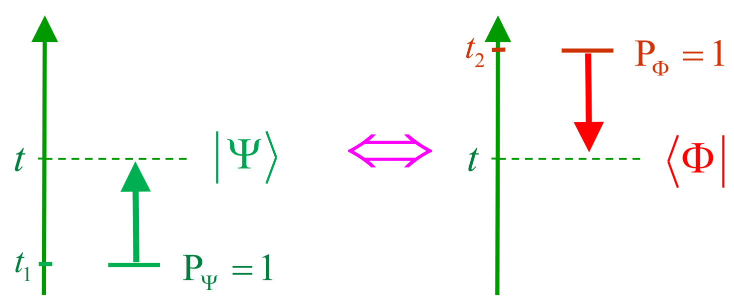


Any coupling of a system described by $\langle\Phi| \; |\Psi\rangle$ **to a variable *O* for a short enough time is a coupling to a weak value**

$$O_w \equiv \frac{\langle\Phi|O|\Psi\rangle}{\langle\Phi|\Psi\rangle}$$

PRL 60, 1351 (1988)

PRA 96, 032114 (2017)
PNAS 116, 2881 (2019)



VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

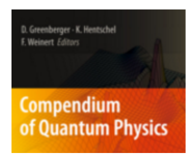
How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

Physics Department, University of South Carolina, Columbia, South Carolina 29208, and
School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel

(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin-$\frac{1}{2}$ particles is presented.



Compendium of Quantum Physics

Concepts, Experiments, History and Philosophy

Editors: **Greenberger**, Daniel, **Hentschel**, Klaus, **Weinert**, Friedel (Eds.)

Weak Value and Weak Measurements

Lev Vaidman

The weak value of a variable O is a description of an effective interaction with that variable in the limit of weak coupling. For a pre- and post-selected system described at time t by the two-state vector $\langle\Phi| \ |\Psi\rangle$ [1], the weak value is [2]:

$$O_w \equiv \frac{\langle\Phi|O|\Psi\rangle}{\langle\Phi|\Psi\rangle}. \quad (1)$$

Colloquium: Understanding quantum weak values: Basics and applications

Justin Dressel, Mehul Malik, Filippo M. Miatto, Andrew N. Jordan, and Robert W. Boyd
Rev. Mod. Phys. **86**, 307 – Published 28 March 2014

ABSTRACT

Since its introduction 25 years ago, the quantum weak value has gradually transitioned from a theoretical curiosity to a practical laboratory tool. While its utility is apparent in the recent explosion of weak value experiments, its interpretation has historically been a subject of confusion. Here a pragmatic introduction to the weak value in terms of measurable quantities is presented, along with an explanation for how it can be determined in the laboratory. Further, its application to three distinct experimental techniques is reviewed. First, as a large interaction parameter it can amplify small signals above technical background noise. Second, as a measurable complex value it enables novel techniques for direct quantum state and geometric phase determination. Third, as a conditioned average of generalized observable eigenvalues it provides a measurable window into nonclassical features of quantum mechanics. In this selective review, a single experimental configuration to discuss and clarify each of these applications is used.

Weak values considered harmful

arXiv.org > quant-ph > arXiv:1307.4016v1

Quantum Physics

Christopher Ferrie, Joshua Combes

*(Submitted on 15 Jul 2013 (this version), **latest version 22 Jan 2014** (v3))*

For the task of parameter estimation, we show using statistically rigorous arguments that the process of postselection (a pre-requisite for so-called weak value amplification) can be no better on average than

PRL 112, 040406 (2014) PHYSICAL REVIEW LETTERS week ending 31 JANUARY 2014

Weak Value Amplification is Suboptimal for Estimation and Detection

Christopher Ferrie and Joshua Combes

Center for Quantum Information and Control, University of New Mexico, Albuquerque, New Mexico 87131-0001, USA

(Received 25 July 2013; revised manuscript received 21 November 2013; published 31 January 2014)

We show by using statistically rigorous arguments that the technique of weak value amplification does not perform better than standard statistical techniques for the tasks of single parameter estimation and

arXiv:1402.0199 [pdf, ps, other]

Comment on "Weak value amplification is suboptimal for estimation and detection"

L.Vaidman

The limiting factor in these and other experiments is not the number of preselected quantum systems (photons) considered by Ferrie and Combes, but the number of detected, post-selected photons. The saturation of the detectors generally hap-

Weak Value Amplification Can Outperform Conventional Measurement in the Presence of Detector Saturation

Jérémie Harris, Robert W. Boyd, and Jeff S. Lundeen

Phys. Rev. Lett. **118**, 070802 – Published 15 February 2017

PRL **113,** 120404 (2014) PHYSICAL REVIEW LETTERS week ending 19 SEPTEMBER 2014

How the Result of a Single Coin Toss Can Turn Out to be 100 Heads

Christopher Ferrie and Joshua Combes
Center for Quantum Information and Control, University of New Mexico, Albuquerque, New Mexico 87131-0001, USA

(Received 16 March 2014; revised manuscript received 18 July 2014; published 18 September 2014)

We show that the phenomenon of anomalous weak values is not limited to quantum theory. In particular, we show that the same features occur in a simple model of a coin subject to a form of classical backaction with pre- and postselection. This provides evidence that weak values are not inherently quantum but rather a purely statistical feature of pre- and postselection with disturbance.

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Home > Science Magazine > 3 October 2014 > Cho, 346 (6205): 22-23

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Science 3 October 2014:
Vol. 346 no. 6205 pp. 22-23
DOI: 10.1126/science.346.6205.22

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IN DEPTH

QUANTUM MECHANICS

Breakthrough lost in coin toss?

Adrian Cho

For 26 years, physicists have argued over an unorthodox quantum measurement technique that seems to circumvent one of the central tenets of quantum mechanics. "Anomalous weak values" bend the rule that you can't

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Are 'weak values' quantum after all?

Oct 9, 2014 8 comments

Flip of the coin: are weak values classical?

known as a "weak measurement", which allows
measure certain properties of a quantum system
rbing it, is being called into question by two physicists
iada and the US. The researchers argue that such
ts, and their counterparts known as "weak values",
inherently quantum mechanical and do not provide any
hts into the quantum world. Indeed, they say that the
such measurements can be replicated classically and
not properties of a quantum system.

years ago, Yakir Aharonov, Lev Vaidman and
t Tel Aviv University in Israel came up with a unique
uring a quantum system without disturbing it to the point
erence occurs and some information is lost. This is
ntional "strong measurements" in quantum mechanics,
system "collapses" into a definite value of the property
red – its position, for example. Instead, the researchers

arXiv.org > quant-ph > arXiv:1703.08870

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Quantum Physics

Weak value controversy

Phil. Trans. R. Soc. A 375: 20160395 (2017)

Lev Vaidman

(Submitted on 26 Mar 2017)

Recent controversy regarding the meaning and usefulness of weak values is reviewed. It is argued that in spite of recent statistical arguments by Ferrie and Combes, experiments with anomalous weak values provide a useful amplification techniques for precision measurements of small effects in many realistic situations. The statistical nature of weak vales was questioned. Although measuring weak value requires an ensemble, it is argued that the weak value, similarly to an eigenvalue, is a property of a single pre- and post-selected quantum system.

Comment on "How the result of a single coin toss can turn out to be 100 heads"

quant-ph > arXiv:1409.5386

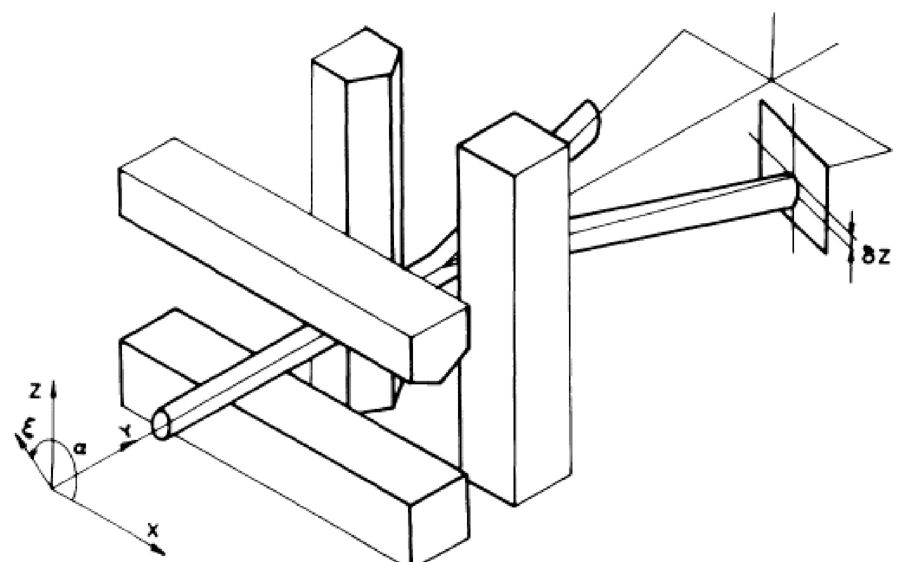
FC: Now we demonstrate that it is possible to find anomalous weak values for pre- and postselected states in the same basis provided there is classical disturbance. In particular, we take $A = Z$, $|\psi\rangle = |+1\rangle$, and later we will postselect on $|\phi\rangle = |-1\rangle$. By using the probabilities in Eq. (11), the

Combes misunderstanding of the concept of weak value.

Weak value of a variable A is a property of a single quantum system pre-selected in a state $|\psi\rangle$ and post-selected in a state $|\phi\rangle$:

$$A_w \equiv \frac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}.$$

$$(\sigma_z)_w = \frac{\langle\uparrow_x|\sigma_z|\uparrow_\xi\rangle}{\langle\uparrow_x|\uparrow_\xi\rangle} = \tan\frac{\alpha}{2} = 100$$



Weak value as a property of a single system

Weak value beyond conditional expectation value of the pointer readings

Lev Vaidman, Alon Ben-Israel, Jan Dziewior, Lukas Knips, Mira Weißl, Jasmin Meinecke, Christian Schwemmer, Ran Ber, and Harald Weinfurter

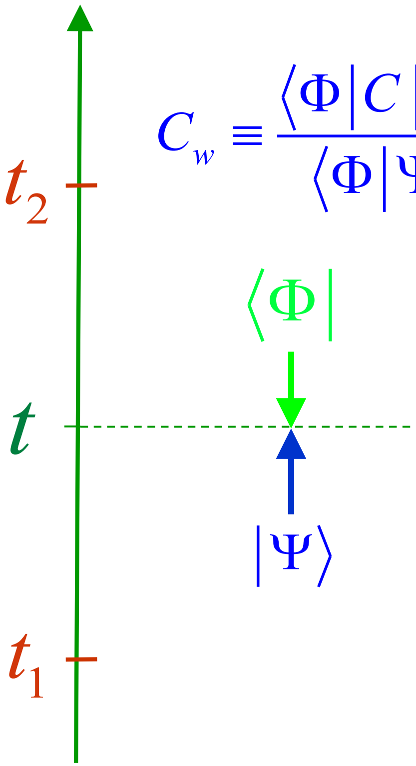
Phys. Rev. A **96**, 032114 – Published 19 September 2017

Weak value is more like an eigenvalue than like an expectation value

The weak value as a property of a single system at a particular time *t*

$\langle\Phi| \; |\Psi\rangle$ **is a complete description at a particular time *t***

$$C_w \equiv \frac{\langle\Phi|C|\Psi\rangle}{\langle\Phi|\Psi\rangle}$$ **is a complete description of coupling to *C* at time *t***



$\chi(t+dt)$

$\chi(t)$

$$H_{\rm int} = \hat{C}\hat{B}$$

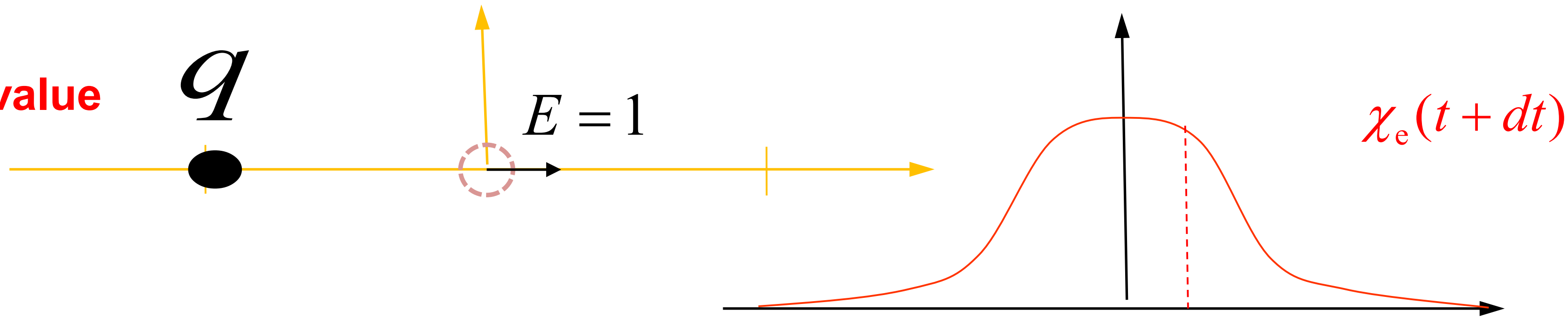
$$H_{\rm int} = C_w\hat{B}$$

$$\chi_{\rm w}(t+dt) \simeq e^{-iC_w\hat{B}dt}\chi(t)$$

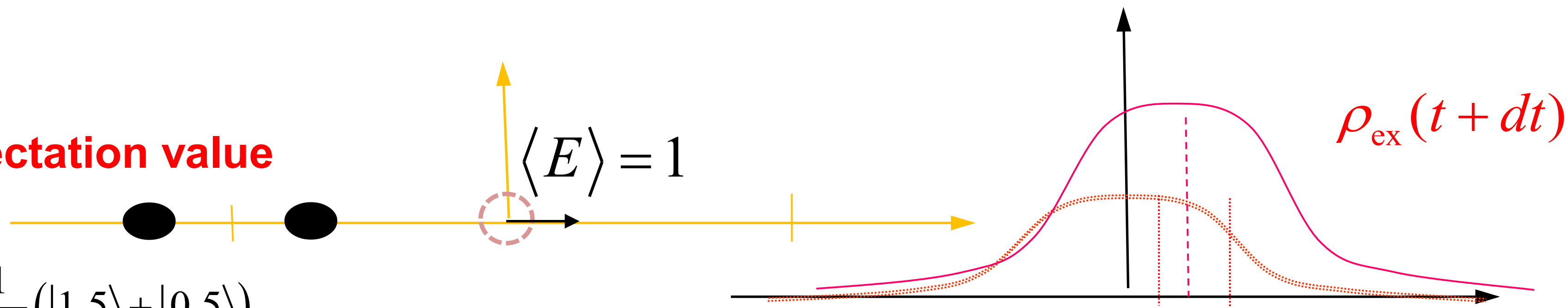
System: charged particle, variable: electric field at the origin

eigenvalue

$$|\Psi\rangle = |1\rangle$$



expectation value

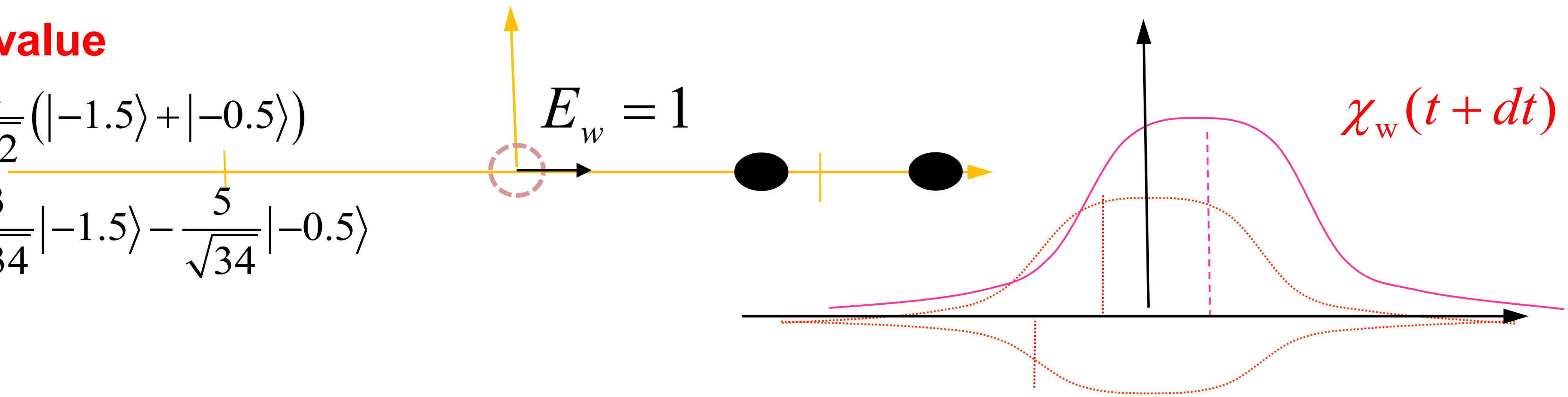


$$|\Psi'\rangle = \frac{1}{\sqrt{2}}\left(|1.5\rangle + |0.5\rangle\right)$$

weak value

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|-1.5\rangle + |-0.5\rangle\right)$$

$$|\Psi\rangle = \frac{3}{\sqrt{34}}|-1.5\rangle - \frac{5}{\sqrt{34}}|-0.5\rangle$$



Comparing states of external system after dt

$$H_{\text{int}} = \hat{C}\hat{B} \qquad C_w = c_k = \langle C \rangle = c$$

weak value

$$C = C_w \equiv \frac{\langle \Phi | C | \Psi \rangle}{\langle \Phi | \Psi \rangle}$$

The system is pre-selected $|\Psi\rangle$ and post-selected $|\Phi\rangle$

$$\chi_{\text{w}}(t+dt) \simeq e^{-iC_w \hat{B} dt} \chi(t)$$

eigenvalue

$$C = c_k$$

The system is pre-selected $|\Psi\rangle = |\Psi_k\rangle$

$$\chi_{\text{e}}(t+dt) = e^{-ic_k \hat{B} dt} \chi(t)$$

expectation value

$$C = \langle C \rangle = \langle \Psi | C | \Psi \rangle$$

The system is pre-selected $|\Psi\rangle = \sum_k \alpha_k |\Psi_k\rangle$

$$\rho_{\text{ex}}(t+dt) \overset{?}{\simeq} e^{-i\langle C \rangle \hat{B} dt} \chi(t)$$

Bures angle distance

$$D(\chi, \xi) \equiv \arccos \left| \langle \chi | \xi \rangle \right|$$

$$D(\chi, \rho) \equiv \arccos \sqrt{\left| \langle \chi | \rho | \chi \rangle \right|}$$

$$D\left(\chi(t), \chi_{\text{e}}(t+dt)\right) \simeq c \Delta B dt$$

$$D\left(\chi_{\text{e}}(t+dt), \chi_{\text{w}}(t+dt)\right) \simeq \frac{1}{2} \left| \left(C^2\right)_w - c^2 \right| \sqrt{\langle B^4 \rangle - \langle B^2 \rangle^2} \, (dt)^2$$

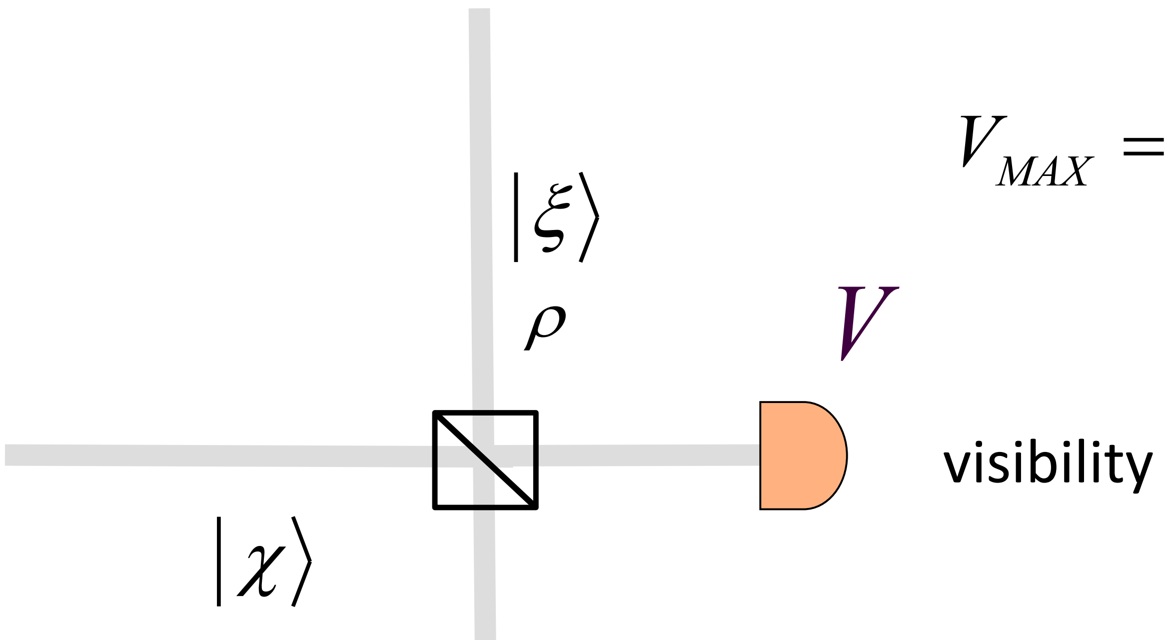
$$D\left(\chi_{\text{e}}(t+dt), \rho_{\text{ex}}(t+dt)\right) \simeq \Delta C \, \Delta B \, dt$$

Experiment

$$D(\chi,\xi) \equiv \arccos V_{MAX}$$

$$D(\chi,\xi) \equiv \arccos\left|\langle \chi \mid \xi \rangle\right|$$

$$V_{MAX} = \left|\langle \chi \mid \xi \rangle\right|$$



$$D(\chi,\rho) \equiv \arccos\sqrt{\left|\langle \chi \mid \rho \mid \chi \rangle\right|}$$

$$D(\chi,\rho) \equiv \arccos V_{MAX}$$

$$V_{MAX} = \sqrt{\left|\langle \chi \mid \rho \mid \chi \rangle\right|}$$

$$V_{MAX} = \sqrt{\left|\langle \chi | \rho | \chi \rangle\right|}$$

$$V_{MAX} = \left|\langle \chi | \xi \rangle\right|$$

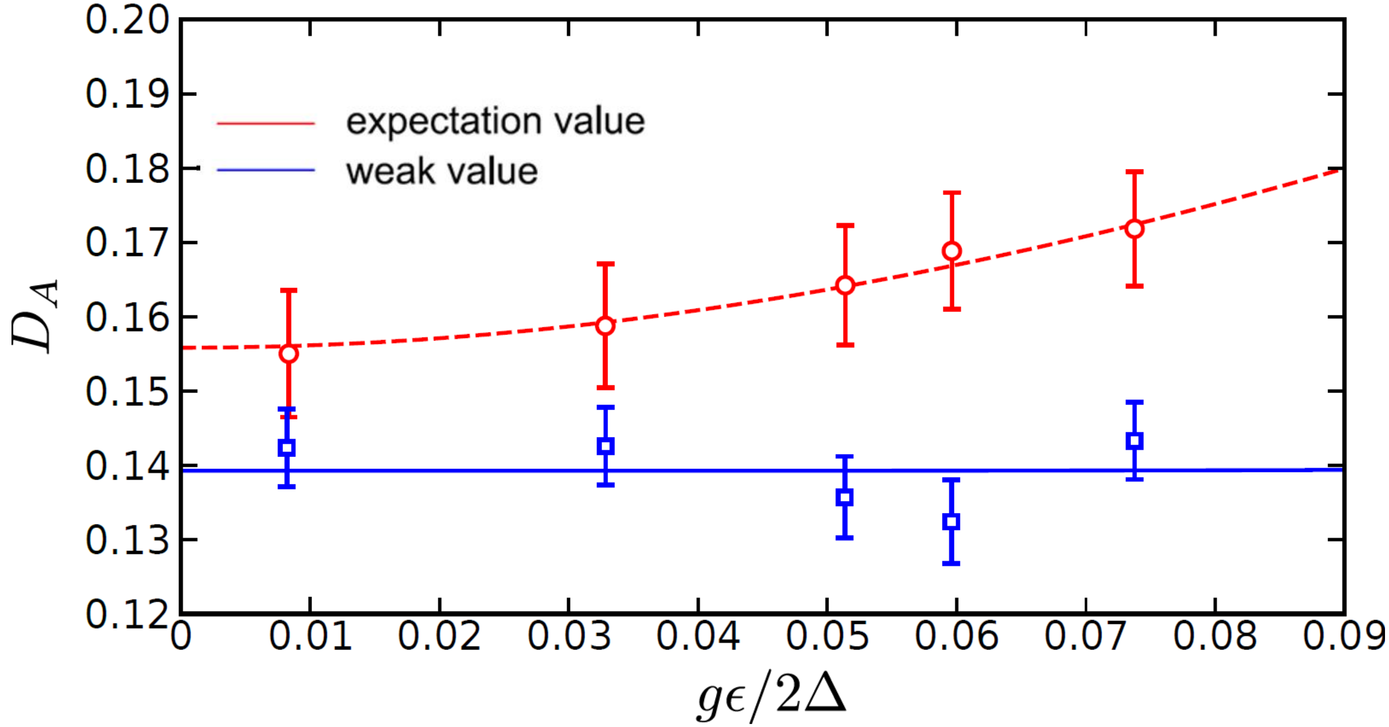
$$D_A\left(\Phi_{\mathrm{e}}, \rho_{\mathrm{ex}}\right) \simeq \frac{g\epsilon}{2\Delta}$$

$$\boxed{D_A = \arccos V_{\max}}$$

$$D_A\left(\Phi_{\mathrm{e}}, \Phi_{\mathrm{w}}\right) \simeq \frac{g^2\epsilon^2}{4\sqrt{2}\Delta^2}$$

$$D_A\left(\Phi_{\mathrm{e}}, \rho_{\mathrm{ex}}\right) = \sqrt{a_1^2 + \left(\frac{g\epsilon}{2\Delta}\right)^2}$$

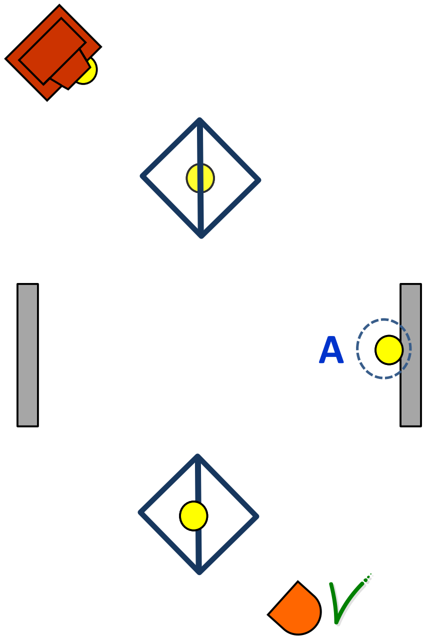
$$D_A\left(\Phi_{\mathrm{e}}, \Phi_{\mathrm{w}}\right) = \sqrt{a_2^2 + \left(\frac{g^2\epsilon^2}{4\sqrt{2}\Delta^2}\right)^2}$$



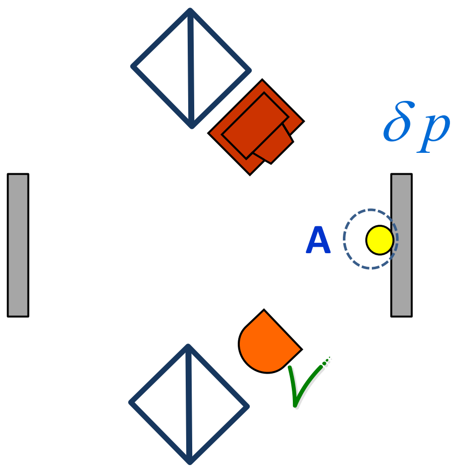
The meaning of weak value of spatial projection operator: the concept of presence of a quantum particle

Was the particle in A or was not?

To be in A = to leave a local trace in A



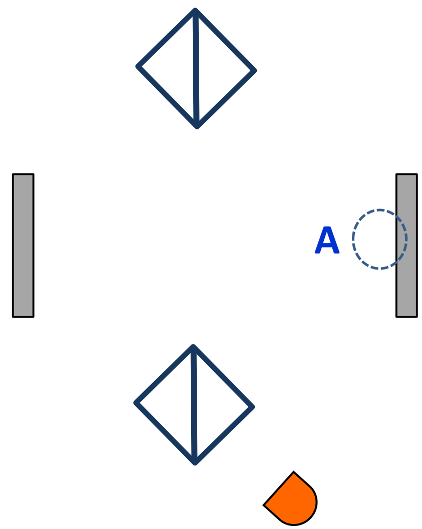
Was the particle in A or was not?



$$|\chi_0\rangle_A \to \mathrm{N}\left(|\chi_0\rangle_A + \varepsilon|\chi_\perp\rangle_A\right)$$

$$\delta p \sim \varepsilon$$

Was in A

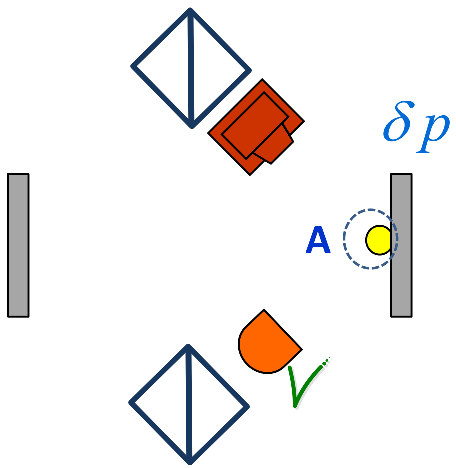


$$|\chi_0\rangle_A \to |\chi_0\rangle_A$$

$$\delta p = 0$$

Was not in A

Was the particle in A or was not?

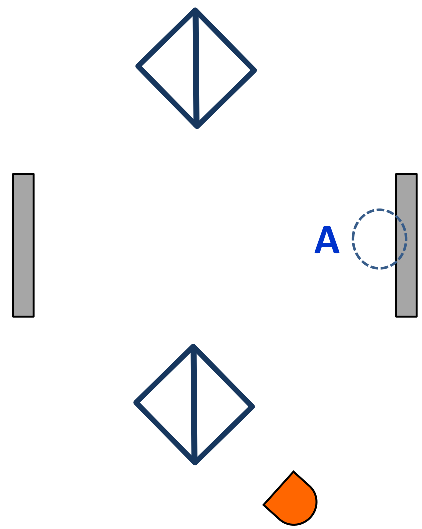


$$|\chi_0\rangle_A \rightarrow \mathrm{N}\left(|\chi_0\rangle_A + \varepsilon|\chi_\perp\rangle_A\right)$$

$$\delta p \sim \varepsilon$$

Was in A

$$(\mathbf{P}_A)_w = 1$$

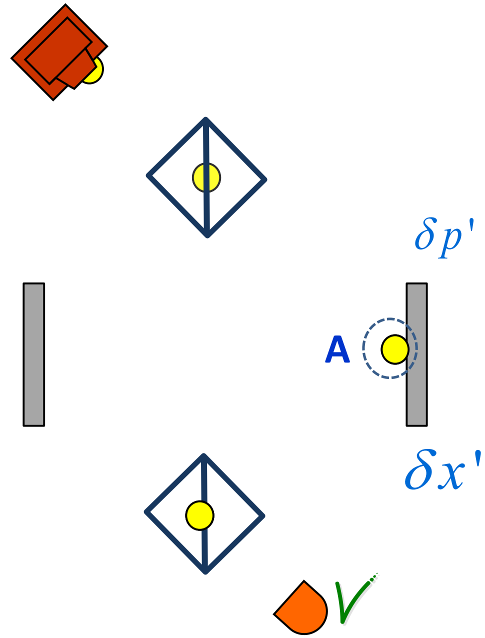


$$|\chi_0\rangle_A \rightarrow |\chi_0\rangle_A$$

$$\delta p = 0$$

Was not in A

$$(\mathbf{P}_A)_w = 0$$



$$|\chi_0\rangle_A \rightarrow \mathrm{N'}\left(|\chi_0\rangle_A + (\mathbf{P}_A)_w\,\varepsilon|\chi_\perp\rangle_A\right)$$

$$\delta p' \sim \mathrm{Re}(\mathbf{P}_A)_w\ \varepsilon$$

$$\delta x' \sim \mathrm{Im}(\mathbf{P}_A)_w\ \varepsilon$$

Was in A with “presence” $(\mathbf{P}_A)_w$

Weak value of the local projection operator $\left(\mathbf{P}_A\right)_w = \frac{\langle \Phi | \mathbf{P}_A | \Psi \rangle}{\langle \Phi | \Psi \rangle}$

is the description of the presence of a quantum particle in a particular place in the past

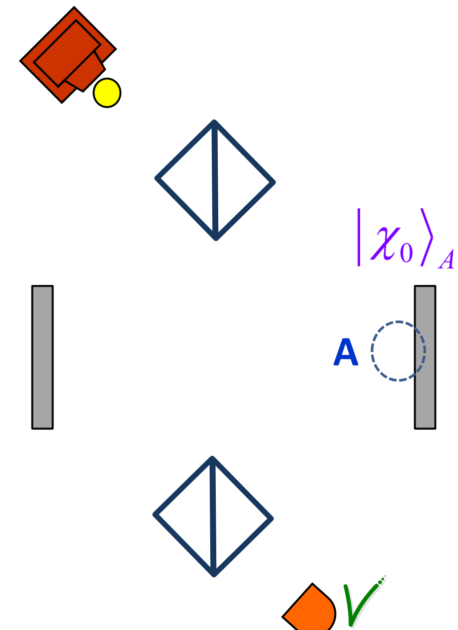
$\left(\mathbf{P}_A\right)_w$ **tells us how the trace in A is modified**

relative to the trace of a particle well localized in A

$$\left|\chi_0\right\rangle_A \rightarrow \mathrm{N}'\left(\left|\chi_0\right\rangle_A + \left(\mathbf{P}_A\right)_w \varepsilon \left|\chi_\perp\right\rangle_A\right)$$

$\left(\mathbf{P}_A\right)_w$ **tells us how effects of all weak (or short) interactions in A are modified**

relative to the effects of a particle well localized in A



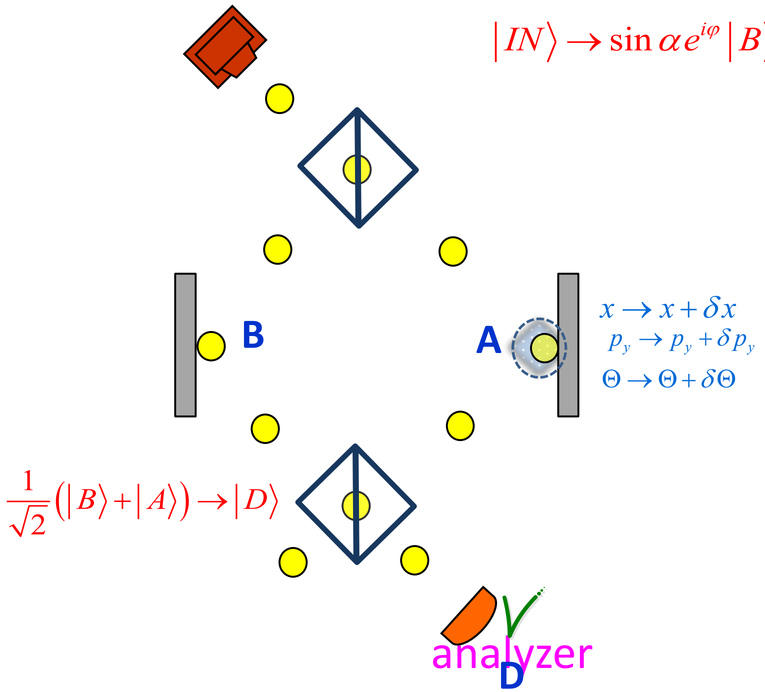
Not to be in A	To be well localized in A	To be with "presence" $\left(\mathbf{P}_A\right)_w$
$\left\|\chi_0\right\rangle_A \rightarrow \left\|\chi_0\right\rangle_A$	$\left\|\chi_0\right\rangle_A \rightarrow \mathrm{N}\left(\left\|\chi_0\right\rangle_A + \varepsilon\left\|\chi_\perp\right\rangle_A\right)$	$\left\|\chi_0\right\rangle_A \rightarrow \mathrm{N}'\left(\left\|\chi_0\right\rangle_A + \left(\mathbf{P}_A\right)_w \varepsilon\left\|\chi_\perp\right\rangle_A\right)$
$Q \rightarrow Q$	$Q \rightarrow Q + \delta Q$	$Q \rightarrow Q + \mathrm{Re}\left(\mathbf{P}_A\right)_w \delta Q$
$P_Q \rightarrow P_Q$	$P_Q \rightarrow P_Q$	$P_Q \rightarrow P_Q + 2\left(\Delta P_Q\right)^2 \mathrm{Im}\left(\mathbf{P}_A\right)_w \delta Q$

Experiment: observing local trace in A

Universality of local weak interactions and its application for interferometric alignment

Jan Dziewior, Lukas Knips, Demitry Farfurnik, Katharina Senkalla, Nimrod Benshalom, Jonathan Efroni, Jasmin Meinecke, Shimshon Bar-Ad, Harald Weinfurter, and Lev Vaidman

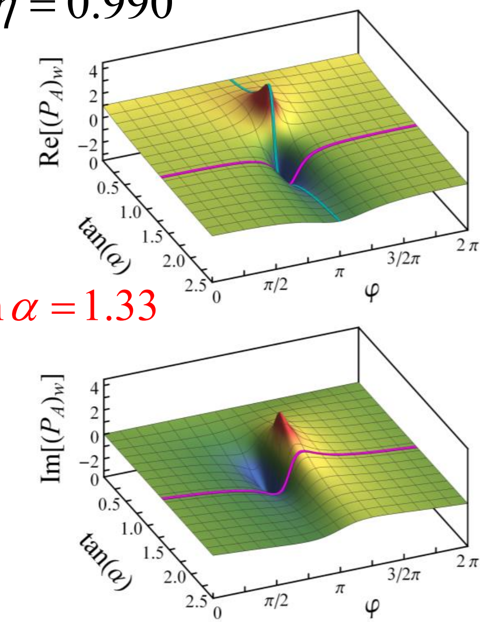
PNAS February 19, 2019 116 (8) 2881-2890; published ahead of print February 19, 2019



$$(\mathbf{P}_A)_w = \frac{1+\eta\tan\alpha e^{i\varphi}}{1+\tan^2\alpha + 2\eta\tan\alpha\cos\varphi}$$

$\eta = 0.990$

$\tan\alpha = 1.33$



To be well localized in A

$$|\chi_0\rangle_A \rightarrow \mathrm{N}\left(|\chi_0\rangle_A + \varepsilon|\chi_\perp\rangle_A\right)$$

$$x \rightarrow x + \delta x$$

$$p_y \rightarrow p_y + \delta p_y$$

$$\Theta \rightarrow \Theta + \delta\Theta$$

To be with "presence" $(\mathbf{P}_A)_w$

$$|\chi_0\rangle_A \rightarrow \mathrm{N}'\left(|\chi_0\rangle_A + (\mathbf{P}_A)_w\,\varepsilon|\chi_\perp\rangle_A\right)$$

$$x \rightarrow x + \mathrm{Re}(\mathbf{P}_A)_w\,\delta x \qquad p_x \rightarrow p_x + 2(\Delta p_x)^2\,\mathrm{Im}(\mathbf{P}_A)_w\,\delta x$$

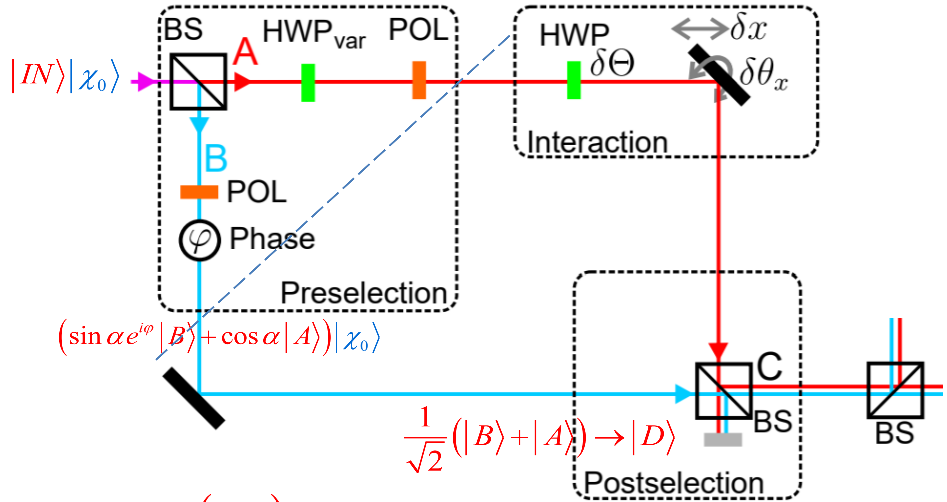
$$p_y \rightarrow p_y + \mathrm{Re}(\mathbf{P}_A)_w\,\delta p_y \qquad y \rightarrow y - 2(\Delta y)^2\,\mathrm{Im}(\mathbf{P}_A)_w\,\delta p_y$$

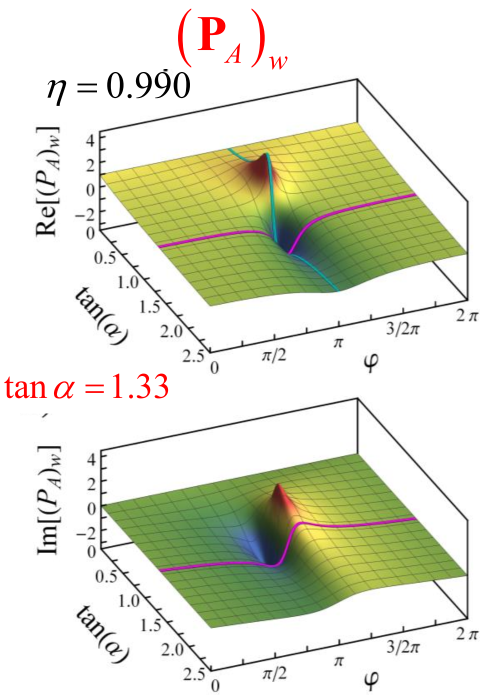
$$\Theta \rightarrow \Theta + \mathrm{Re}(\mathbf{P}_A)_w\,\delta\Theta \qquad \Upsilon \rightarrow \Upsilon + \mathrm{Im}(\mathbf{P}_A)_w\,\delta\Theta$$

Experiment: observing local trace in A

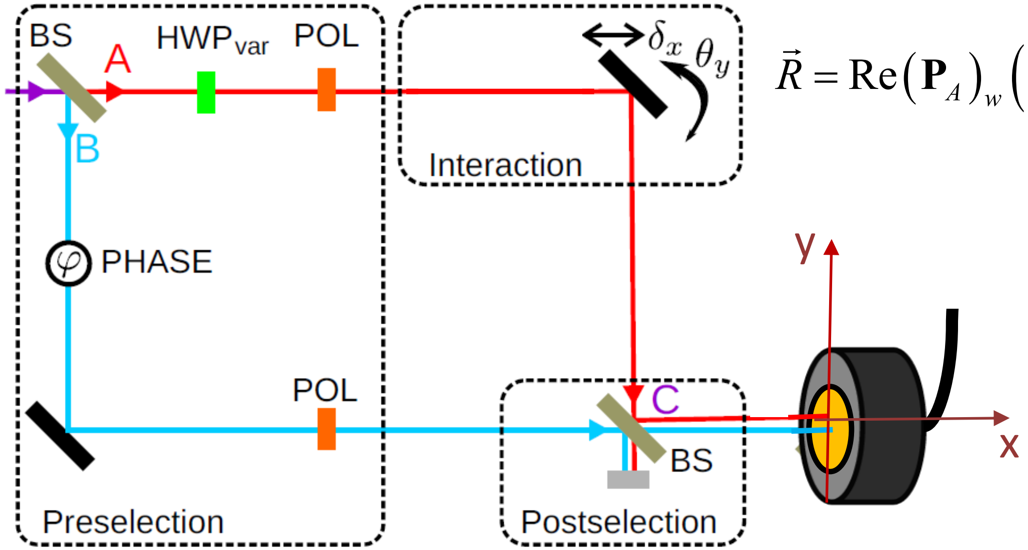
Dziewior, Knips, Farfurnik, Senkalla, Benshalom, Efroni, Meinecke, Bar-Ad, Weinfurter, Vaidman, PNAS, to be published

$$|\chi_0\rangle_A \rightarrow N\left(|\chi_0\rangle_A + (\mathbf{P}_A)_w\,\varepsilon\,|\chi_\perp\rangle_A\right)$$



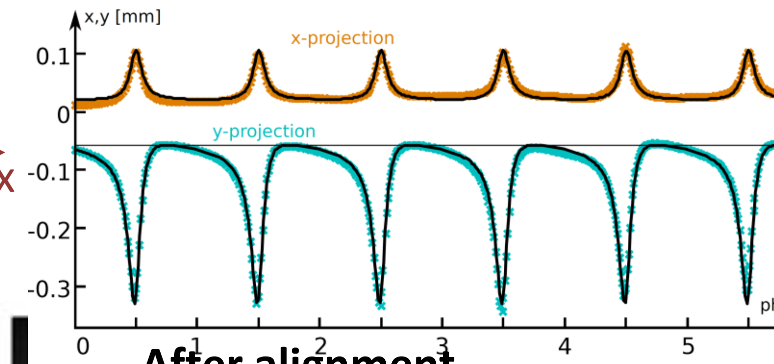


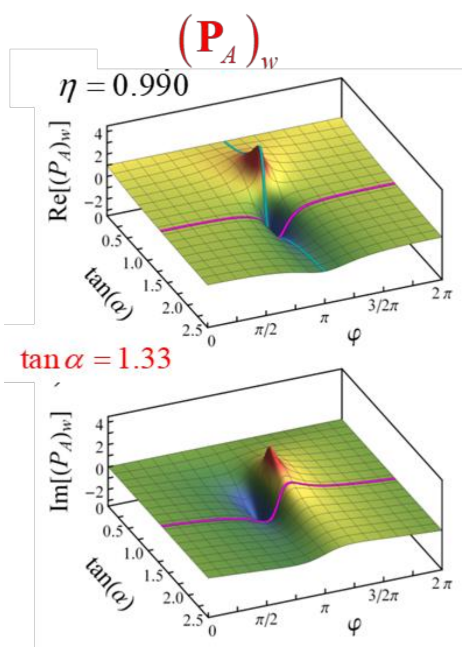
Application for Alignment



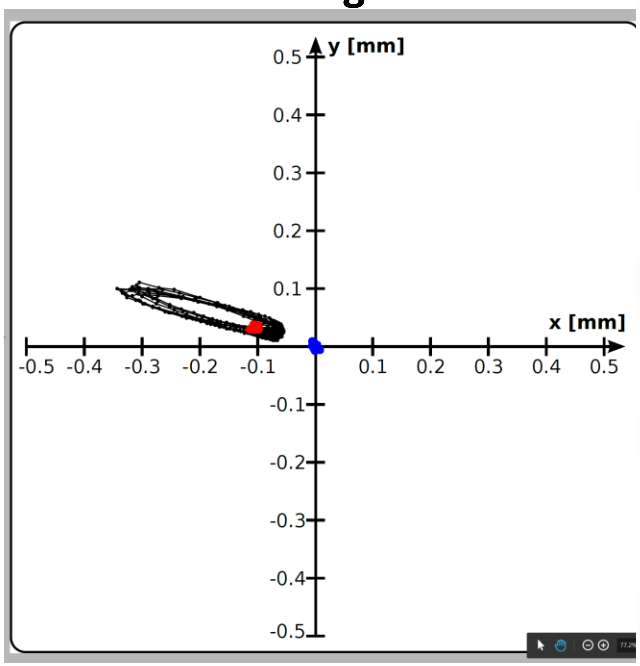
$$\vec{R} = \mathrm{Re}\left(\mathbf{P}_A\right)_w\left(\delta\vec{r} + L\delta\vec{\theta}\right) + \mathrm{Im}\left(\mathbf{P}_A\right)_w\left(\frac{\lambda L}{\pi\sigma^2}\delta\vec{r} - \frac{\pi\sigma^2}{\lambda}\delta\vec{\theta}\right)$$

$$\left(\mathrm{P}_A\right)_w = \frac{1+\eta\tan\alpha\, e^{-i\varphi}}{1+\tan^2\alpha+2\eta\tan\alpha\cos\varphi}$$

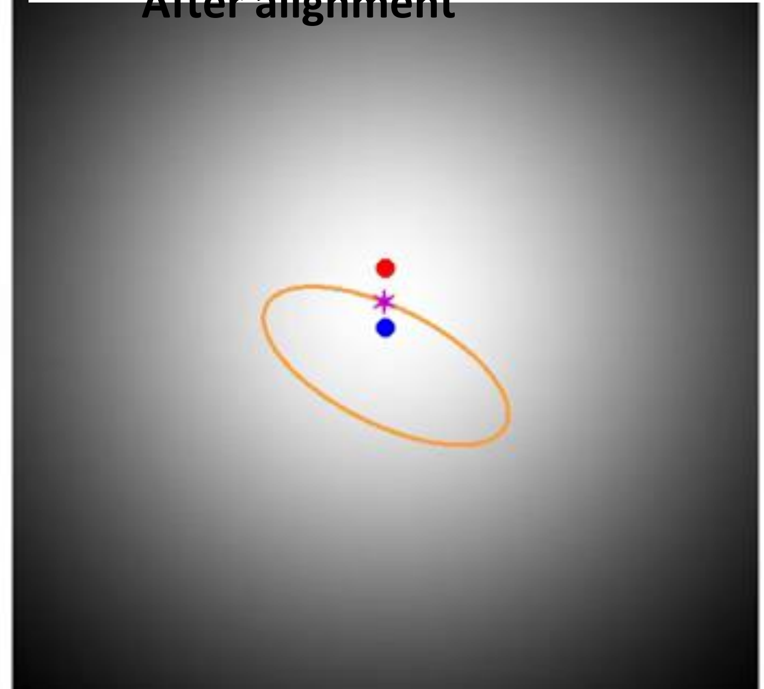




Before alignment



After alignment



Conclusions

Weak value is a property of a single system at a single moment of time

Weak value of *C* is a complex number which can replace, in the limit of short period of time, operator *C* in the Hamiltonian

$$H_{\text{int}} = \ldots\hat{C}\ldots \;\rightarrow\; H_{\text{int}} = \ldots C_w \ldots \qquad C_w = \frac{\langle \Phi | C | \Psi \rangle}{\langle \Phi | \Psi \rangle}$$

Weak value of the projection operator on a particular location $(\mathbf{P}_A)_w$ characterizes the presence of the system there.

It describes a universal modification of all infinitesimally short interactions in this place.