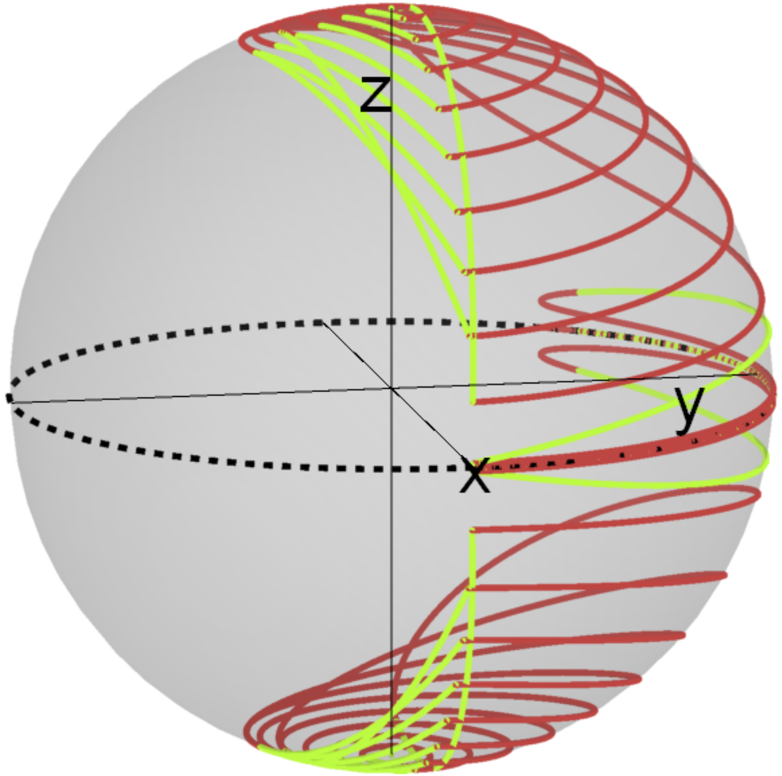
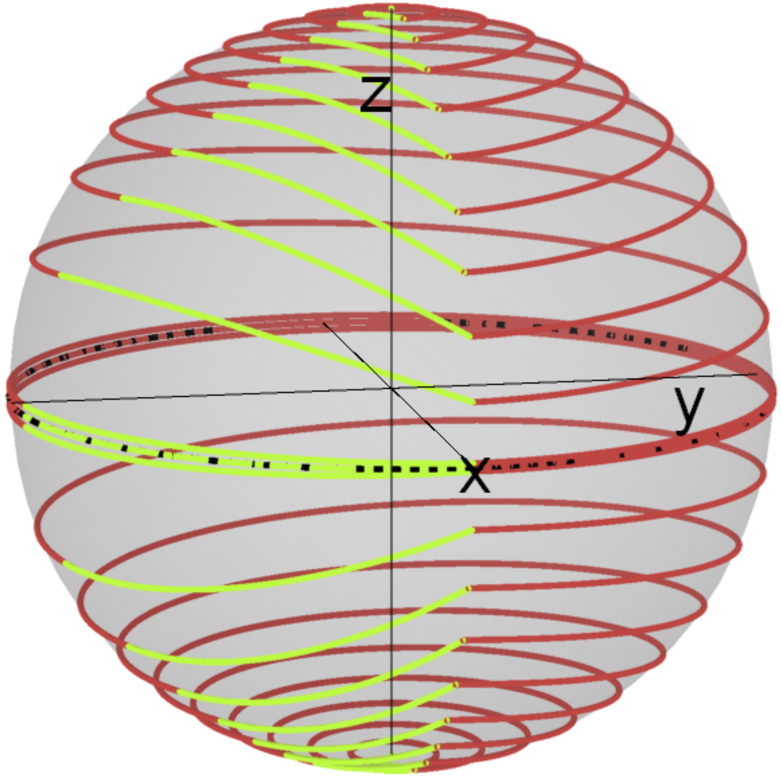
Topological transition in weak-measurement-induced geometric phases



Kyrylo Snizhko

מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE



Conference on Quantum Measurement, ICTP
02.05.2019

Project participants

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Gebhart, KS, Wellens, Buchleitner, Romito, Gefen, arXiv:1905.01147

Outline

1. Berry phase and
   Pancharatnam (measurement-induced) phase
2. Weak measurements and their back action

3. Weak-measurement-induced geometric phase
4. The topological transition

Berry phase

$$H(\boldsymbol{R})|n(\boldsymbol{R})\rangle = E_n(\boldsymbol{R})|n(\boldsymbol{R})\rangle$$

$\boldsymbol{R}(t)$ — slowly-changing

$\boldsymbol{R}(0) = \boldsymbol{R}(T) = \boldsymbol{R}_0$

$$|\psi(0)\rangle = |n(\boldsymbol{R}_0)\rangle \to |\psi(t)\rangle = e^{i\varphi}|n(\boldsymbol{R}_0)\rangle$$

$$\varphi = \varphi_d + \varphi_B,$$

$$\varphi_d = -\int_0^T E_n dt,$$

$$\varphi_B = i\oint d\boldsymbol{R}\langle n(\boldsymbol{R})|\partial_{\boldsymbol{R}}|n(\boldsymbol{R})\rangle$$

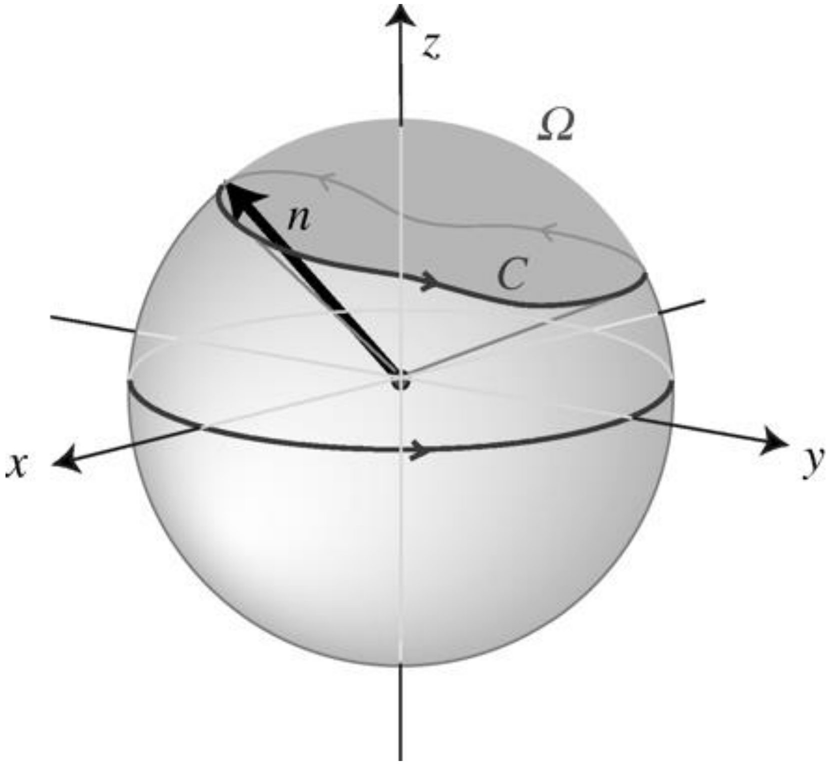
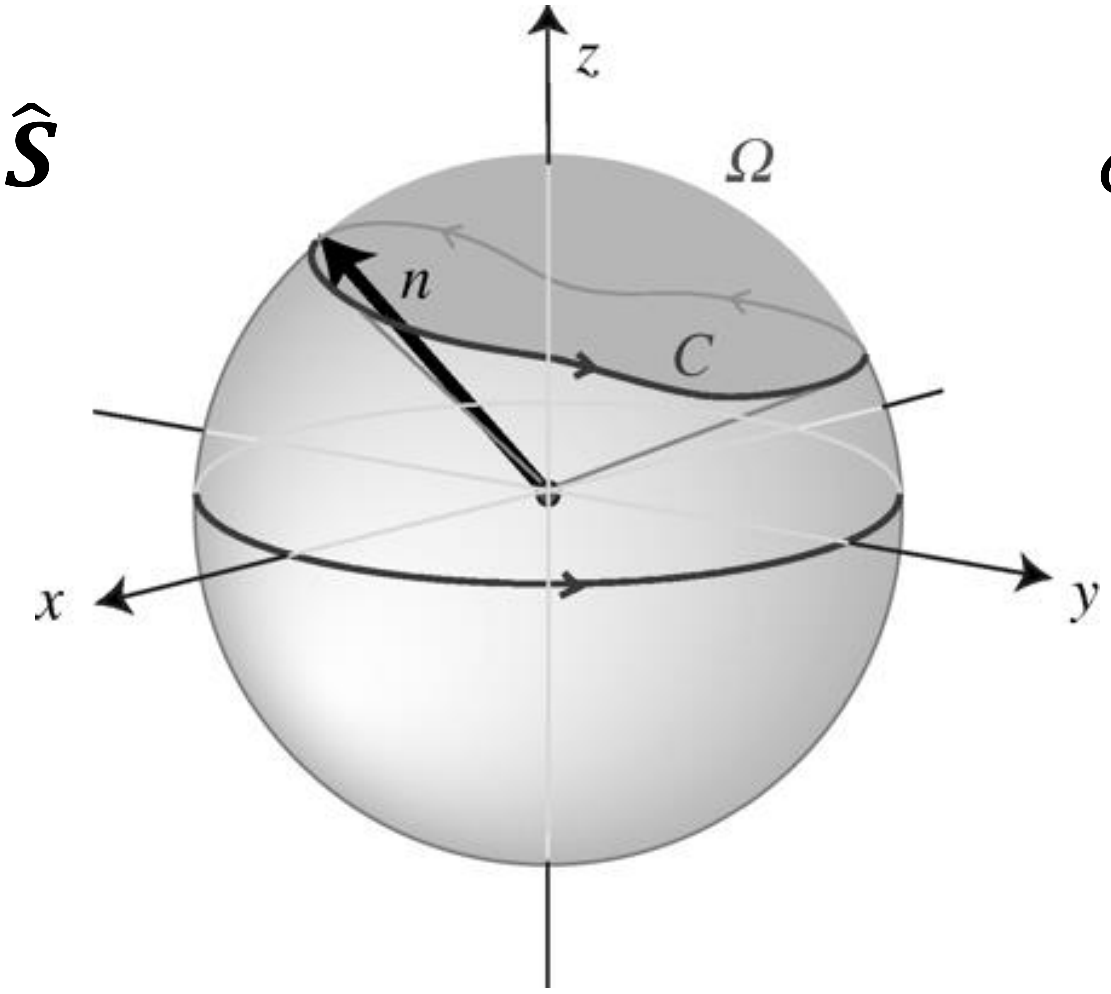


Figure from Pachos, Carollo, Phil. Trans. R. Soc. A **364**, 3463 (2006)

Berry, 1984

Berry phase for spin 1/2

$$H = \boldsymbol{B}(t) \cdot \hat{\boldsymbol{S}}$$



$$\varphi_B = -\Omega/2$$

Figure from Pachos, Carollo, Phil. Trans. R. Soc. A **364**, 3463 (2006)

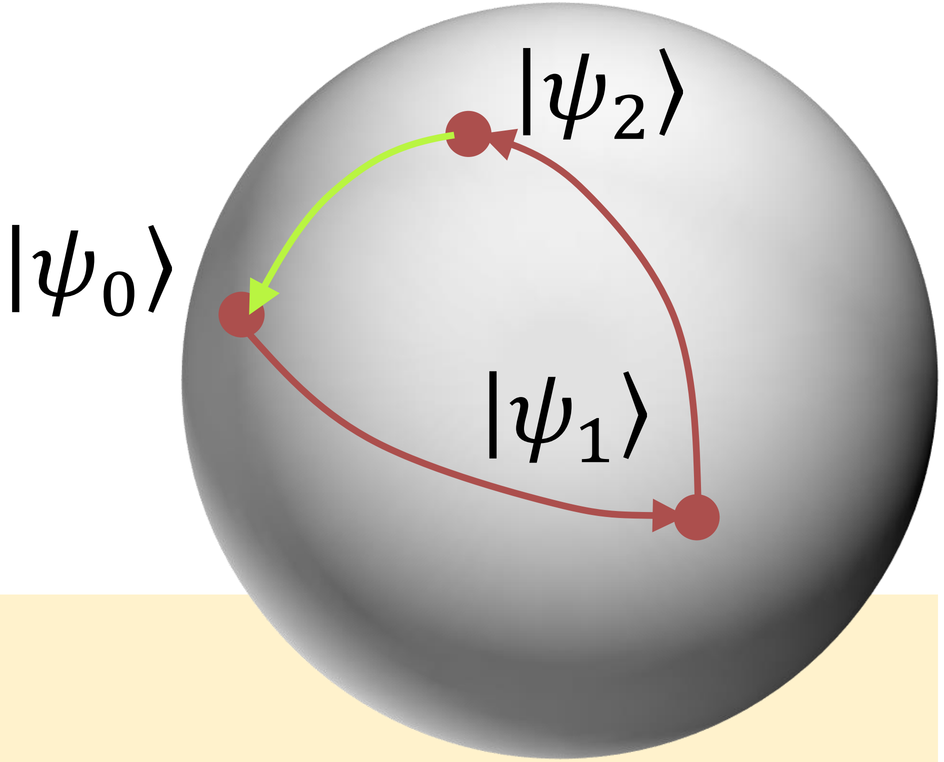
Berry, 1984

Pancharatnam phase and measurement

A sequence of states:

$|\psi_0\rangle, |\psi_1\rangle, \ldots, |\psi_{N-1}\rangle$

$$P_k = |\psi_k\rangle\langle\psi_k|$$



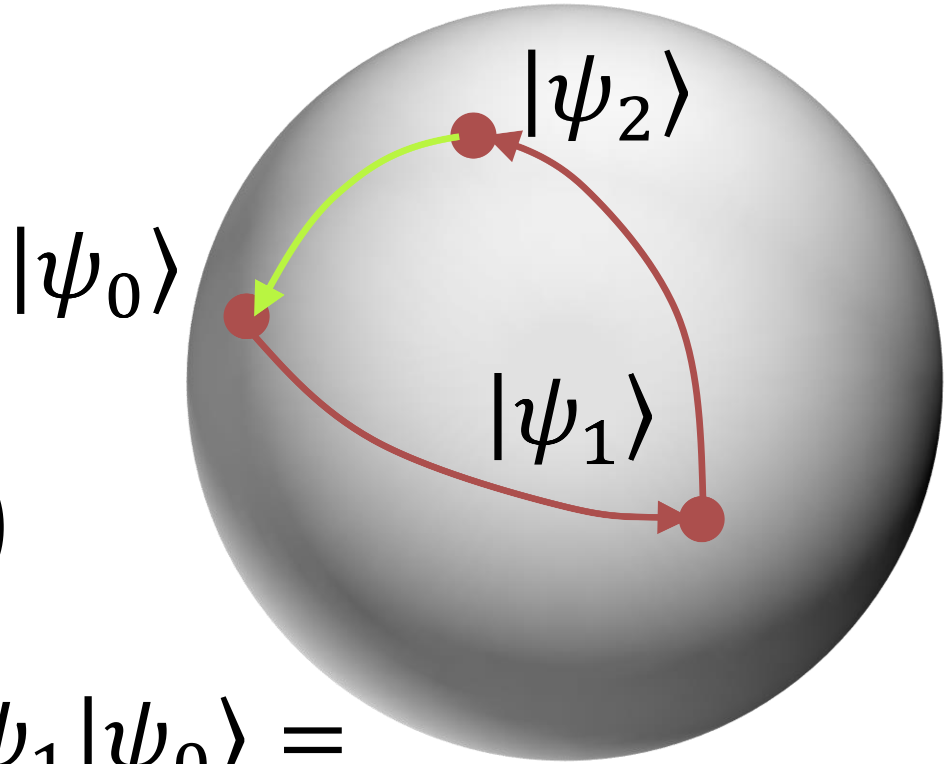
…defines a phase φ_P:

$$e^{i\varphi_P}\sqrt{P} = \langle\psi_0|P_{N-1}\ldots P_2 P_1|\psi_0\rangle$$
$$= \langle\psi_0|\psi_{N-1}\rangle\langle\psi_{N-1}|\psi_{N-2}\rangle\ldots\langle\psi_2|\psi_1\rangle\langle\psi_1|\psi_0\rangle$$

Pancharatnam, 1956

Berry phase = Pancharatnam phase

$$|\psi_k\rangle = |n(\boldsymbol{R}_k)\rangle$$
$$|\psi_{k+1}\rangle = |\psi_k\rangle + |\delta\psi_k\rangle$$
$$|\delta\psi_k\rangle \approx (\boldsymbol{R}_{k+1} - \boldsymbol{R}_k)\partial_{\boldsymbol{R}_k}|n(\boldsymbol{R}_k)\rangle$$



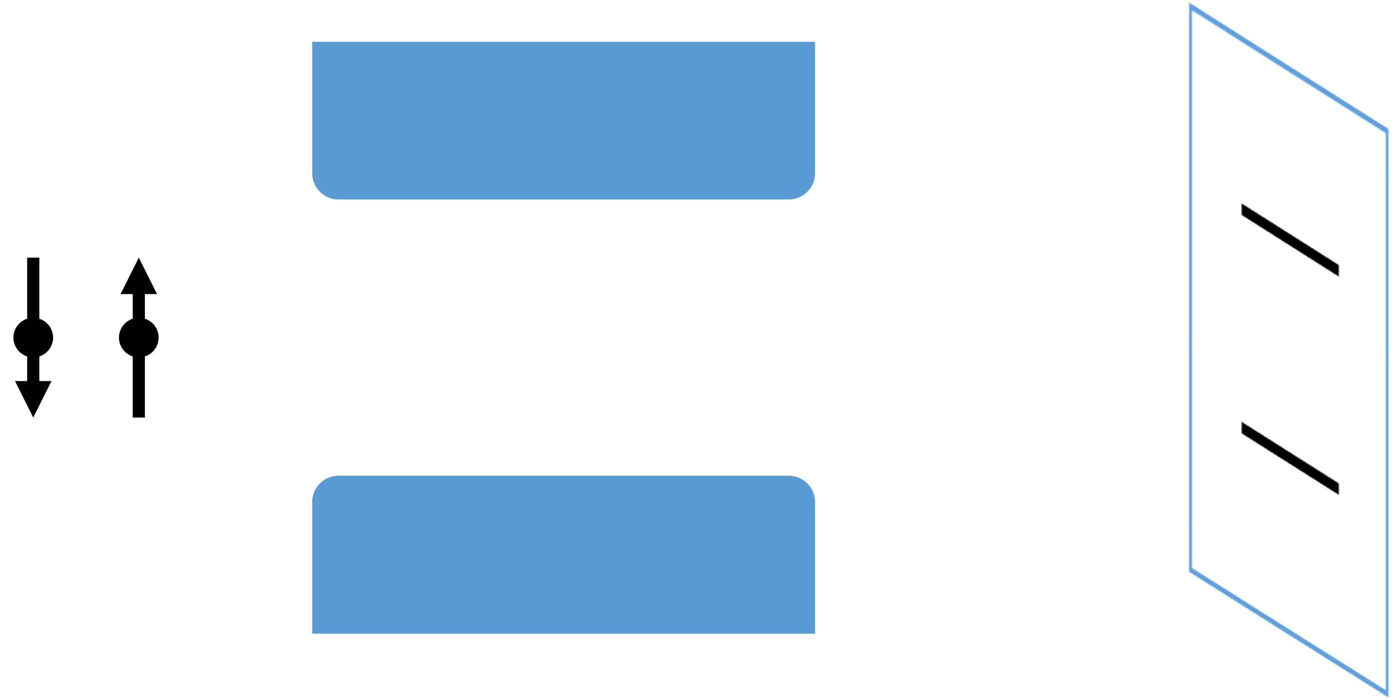
$$\langle\psi_{k+1}|\psi_k\rangle \approx$$
$$\approx \exp\big(-\delta\boldsymbol{R}\langle n(\boldsymbol{R}_k)|\partial_{\boldsymbol{R}_k}|n(\boldsymbol{R}_k)\rangle\big)$$

$$\langle\psi_0|\psi_{N-1}\rangle\langle\psi_{N-1}|\psi_{N-2}\rangle \ldots \langle\psi_2|\psi_1\rangle\langle\psi_1|\psi_0\rangle =$$
$$= \exp\left(-\oint d\boldsymbol{R}\langle n(\boldsymbol{R})|\partial_{\boldsymbol{R}}|n(\boldsymbol{R})\rangle\right) = \exp(i\varphi_B)$$

cf. Chruscinski, Jamiolkowski “Geometric phases in classical and quantum mechanics”

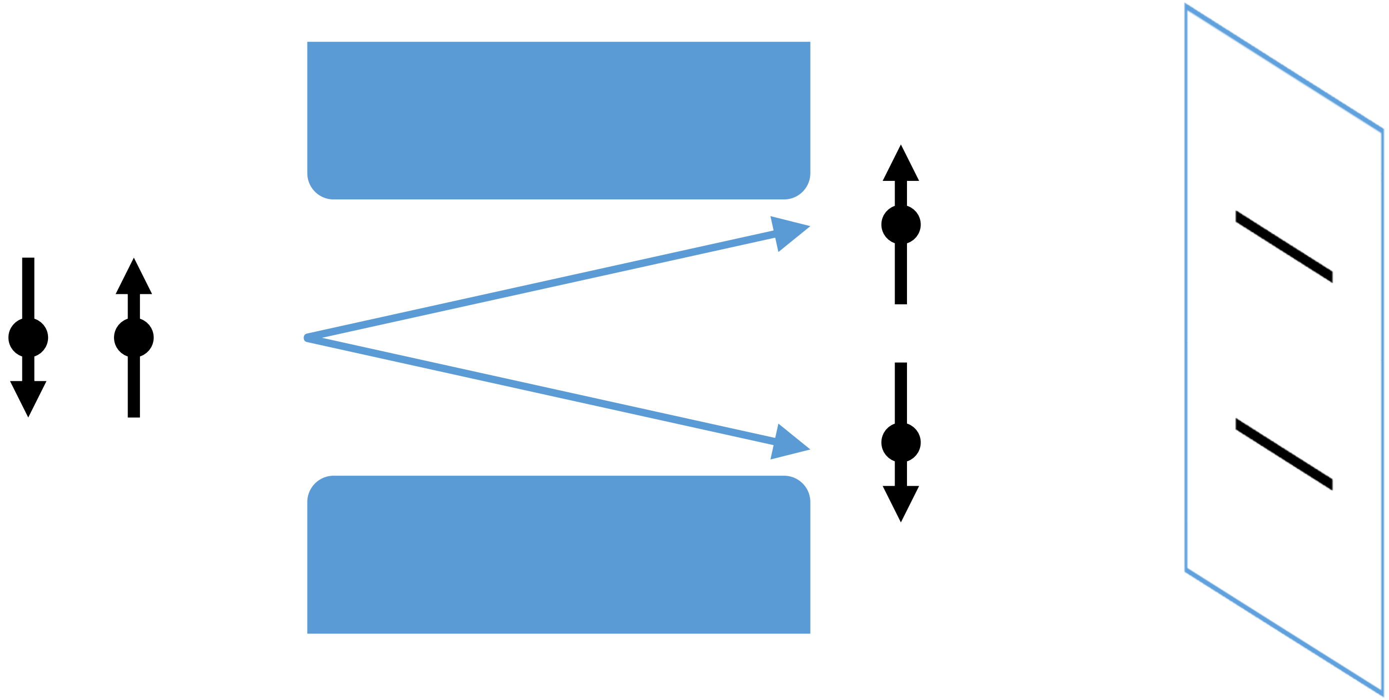
(projective) Quantum measurements

Stern–Gerlach device



(projective) Quantum measurements

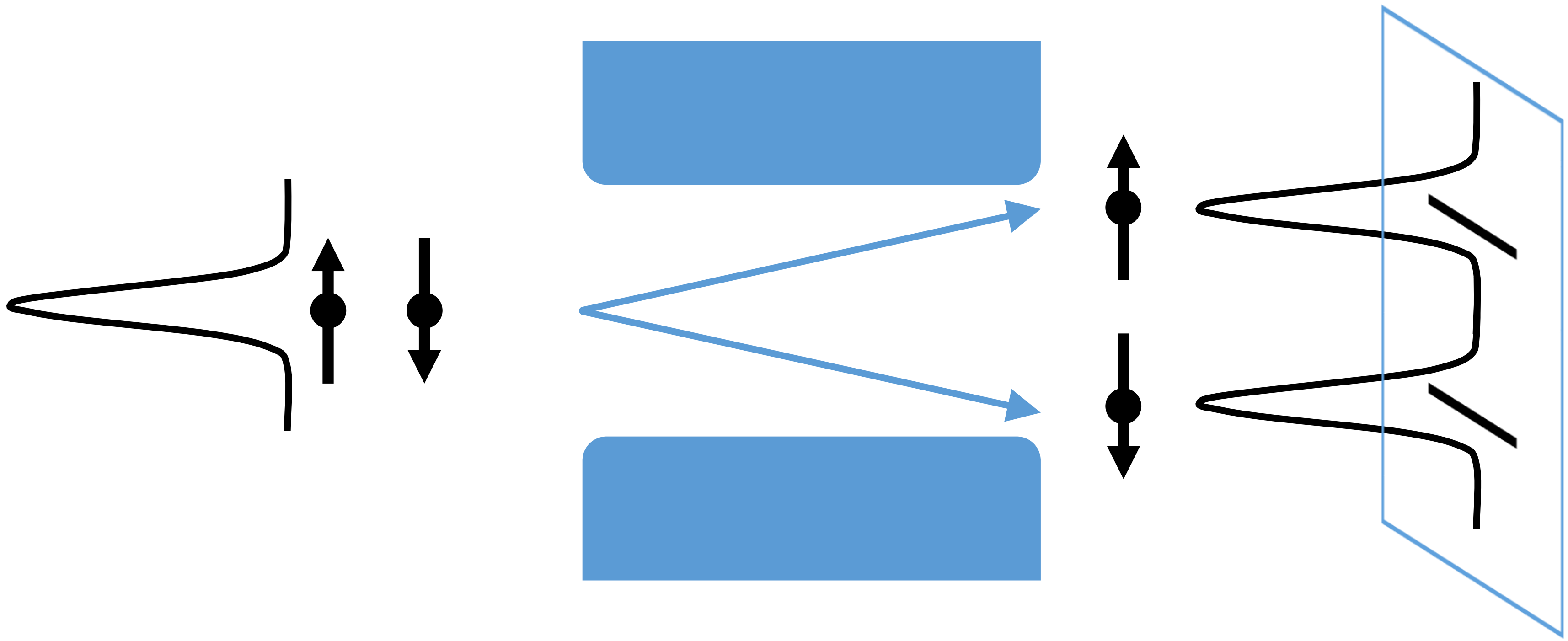
Stern–Gerlach device



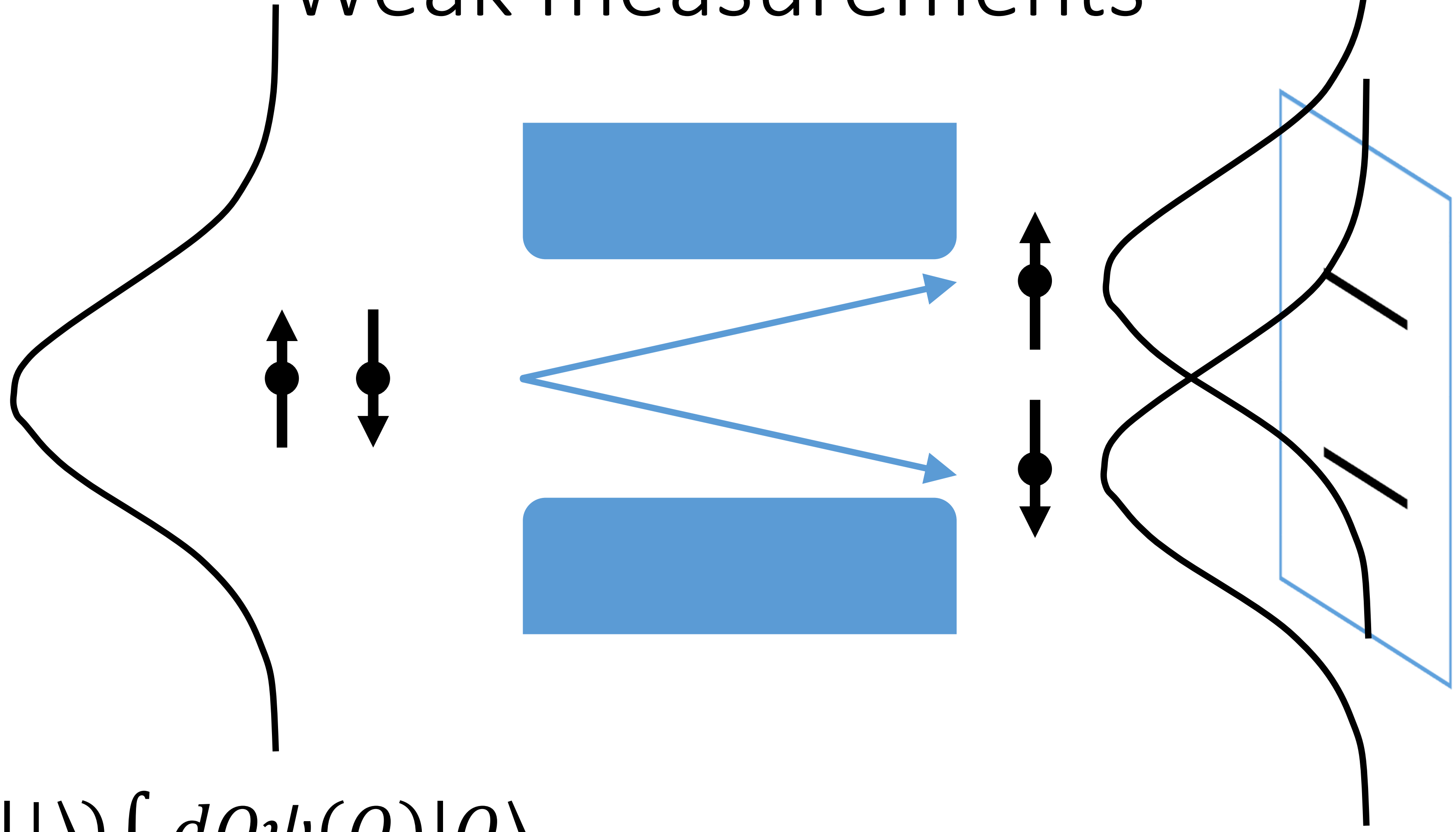
$$(a|\uparrow\rangle + b|\downarrow\rangle)|Q=0\rangle \longrightarrow$$
$$\longrightarrow a|\uparrow\rangle|Q=1\rangle + b|\downarrow\rangle|Q=-1\rangle$$

$$a|\uparrow\rangle = P_\uparrow(a|\uparrow\rangle + b|\downarrow\rangle)$$
$$b|\downarrow\rangle = P_\downarrow(a|\uparrow\rangle + b|\downarrow\rangle)$$

Weak measurements



Weak measurements



$$(a|\uparrow\rangle + b|\downarrow\rangle)\int dQ\psi(Q)|Q\rangle$$

$$\longrightarrow \int dQ\big(a|\uparrow\rangle\psi(Q-1) + b|\downarrow\rangle\psi(Q+1)\big)|Q\rangle$$

$$= \int dQ|Q\rangle M_Q(a|\uparrow\rangle + b|\downarrow\rangle)$$

Formalism of generalized measurements

Initial state $|\Psi\rangle$ (e.g., $a|\uparrow\rangle + b|\downarrow\rangle$)

Measurement readout r (e.g., Q)

System state after measurement $M_r|\Psi\rangle$, e.g.,

$$M_Q(a|\uparrow\rangle + b|\downarrow\rangle) = \left(a|\uparrow\rangle\psi(Q-1) + b|\downarrow\rangle\psi(Q+1)\right)$$

Probability of readout r:

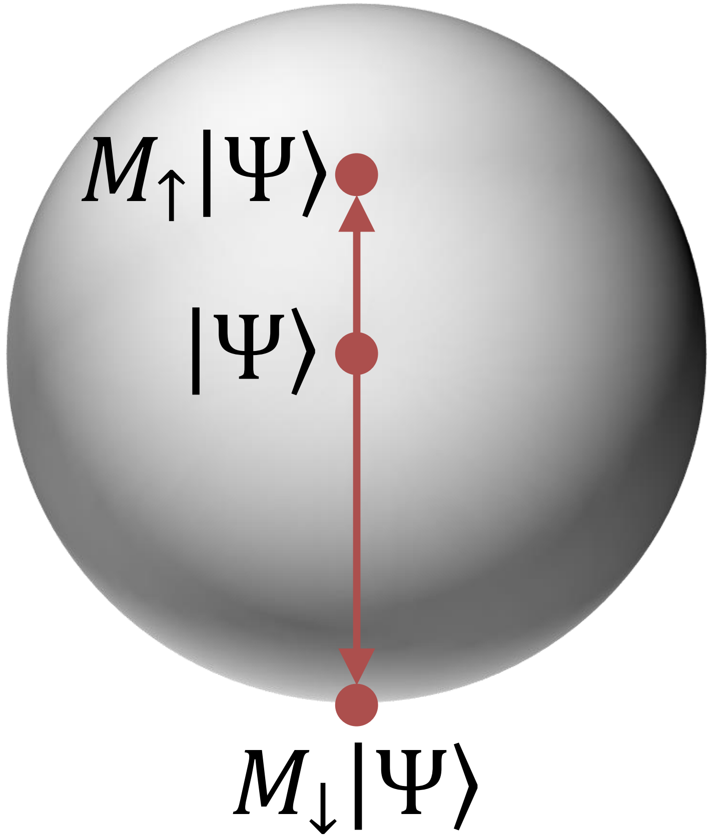
$$P = \langle\Psi|M_r^\dagger M_r|\Psi\rangle = \|M_r|\Psi\rangle\|^2$$

Two-outcome detector: the measurement back-action

Two-outcome detector: $r = \uparrow / \downarrow$

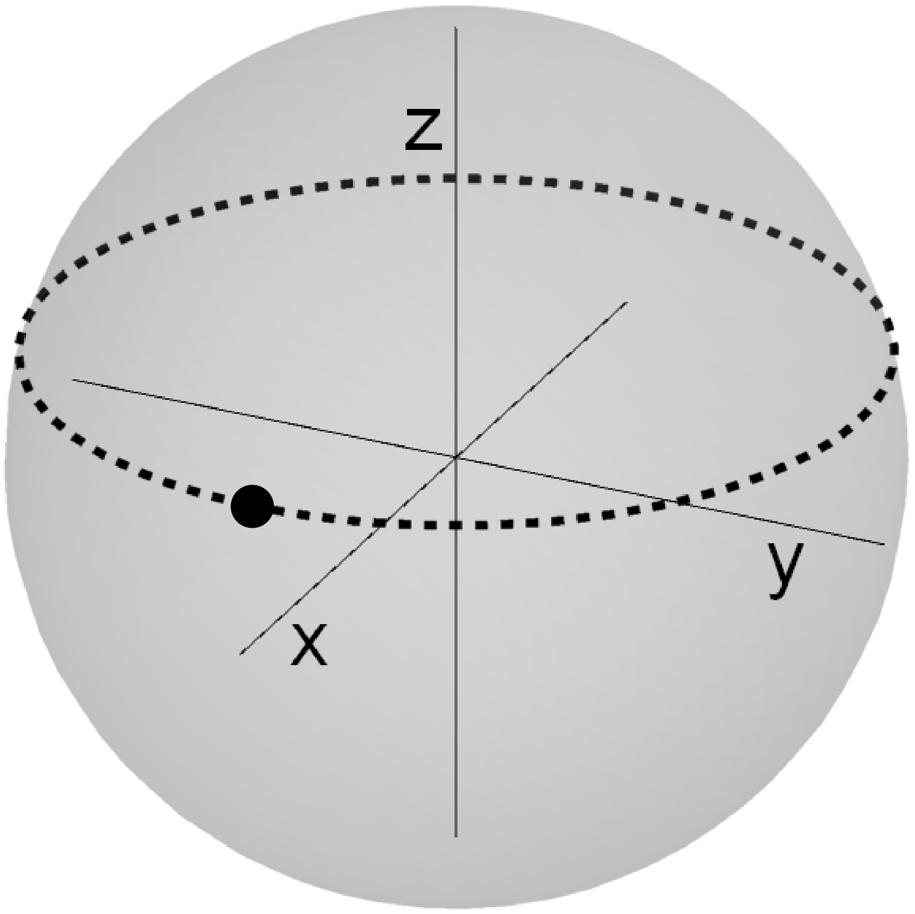
$$M_\uparrow = \begin{pmatrix} 1 & 0 \\ 0 & t \end{pmatrix}$$

$$M_\downarrow = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{1-t^2} \end{pmatrix}$$



Weak-measurement-induced phase

$$\langle \Psi_0 | P_{N-1}^{(\uparrow)} \dots P_1^{(\uparrow)} | \Psi_0 \rangle = e^{i\varphi_P}$$

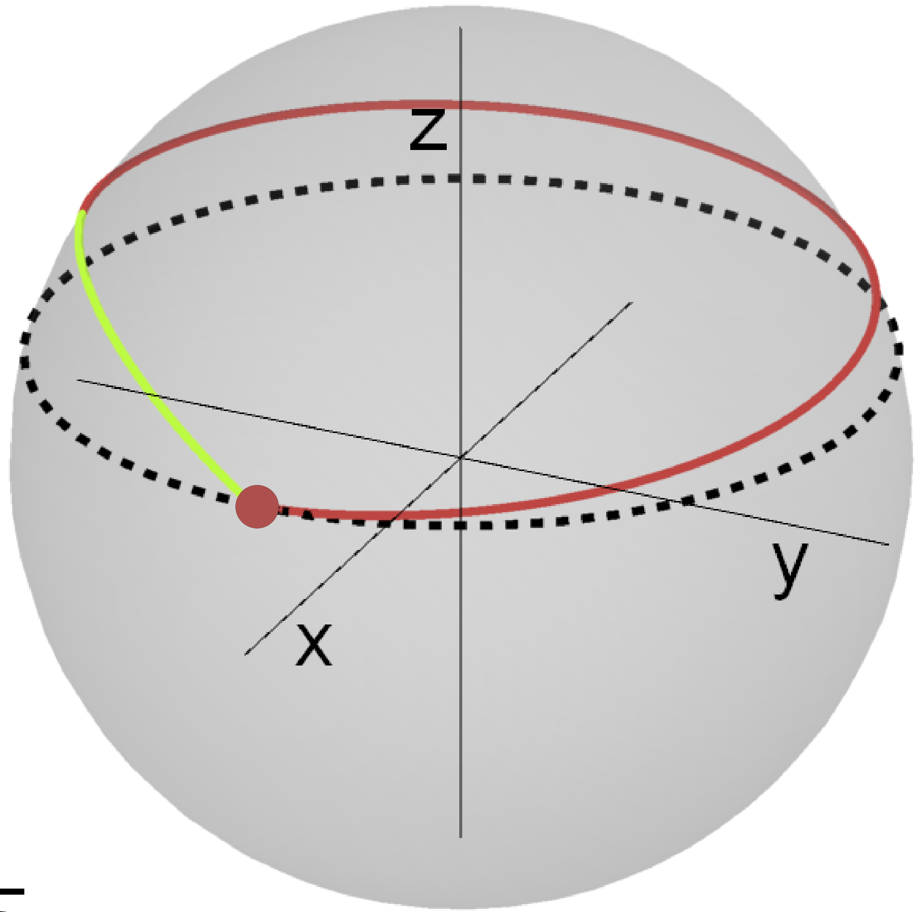


Weak-measurement-induced phase

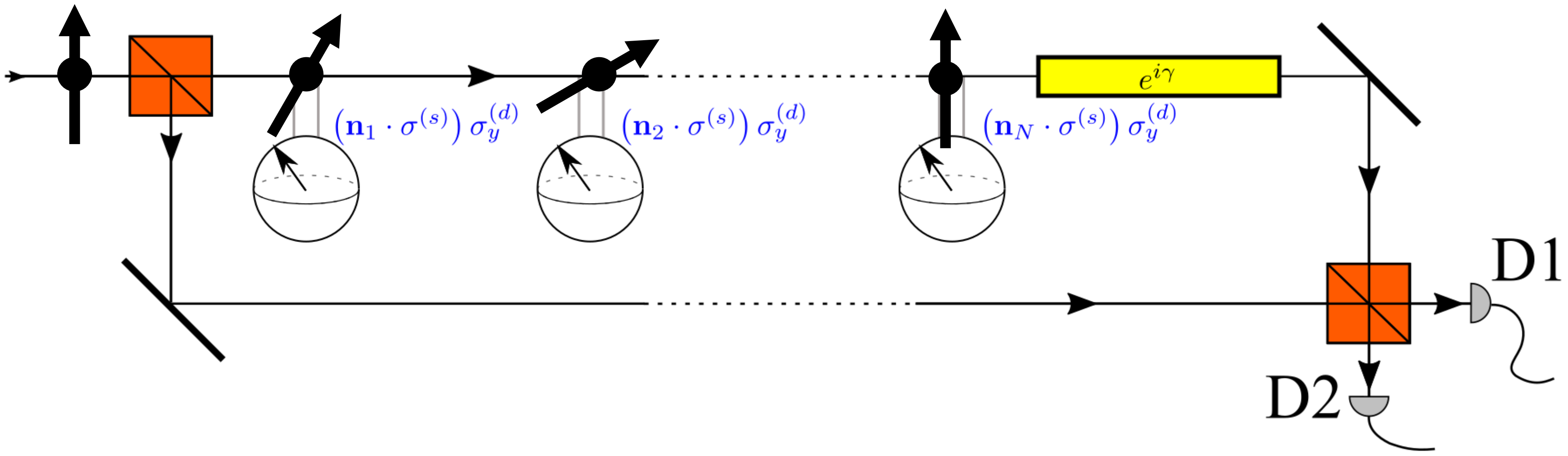
$$\langle\Psi_0|P_{N-1}^{(\uparrow)}\dots P_1^{(\uparrow)}|\Psi_0\rangle = e^{i\varphi_P}$$

$$\langle\Psi_0|M_{N-1}^{(\uparrow)}\dots M_1^{(\uparrow)}|\Psi_0\rangle = e^{i\chi_{\text{geom}}}\sqrt{P}$$

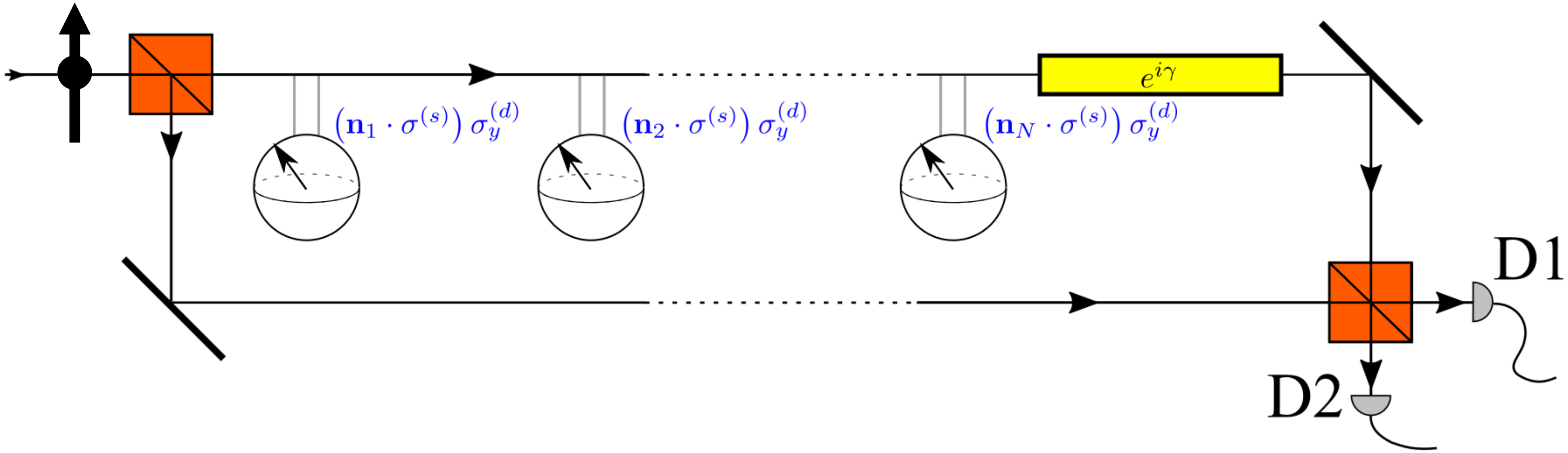
$$\langle\Psi_0|M_{N-1}^{(r_{N-1})}\dots M_1^{(r_1)}|\Psi_0\rangle = e^{i\chi_{\text{geom}}}\sqrt{P}$$



Observing measurement-induced geometric phase



Observing measurement-induced geometric phase



To observe interference, detectors should not click

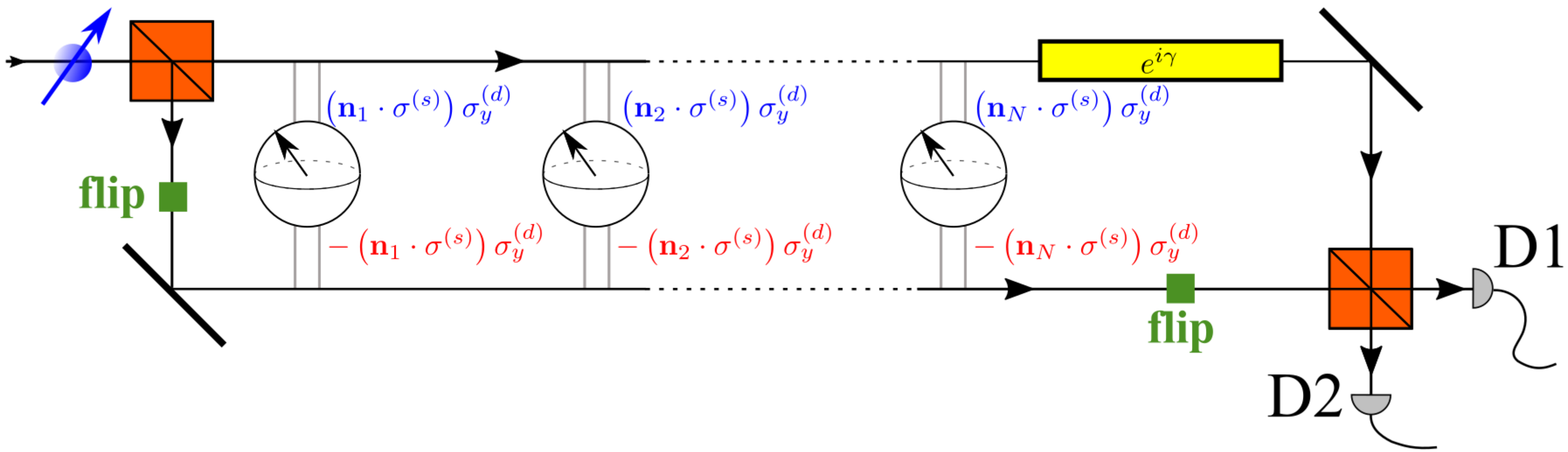
$\Longrightarrow$ Null-(outcome) measurement

↑-postselected phase

$$\langle \Psi_0 | M_{N-1}^{(\uparrow)} \dots M_1^{(\uparrow)} | \Psi_0 \rangle = \sqrt{P} e^{i\chi_{\text{geom}}}$$

$$M_{\uparrow} = \begin{pmatrix} 1 & 0 \\ 0 & 1 - \frac{2c}{N} \end{pmatrix}$$

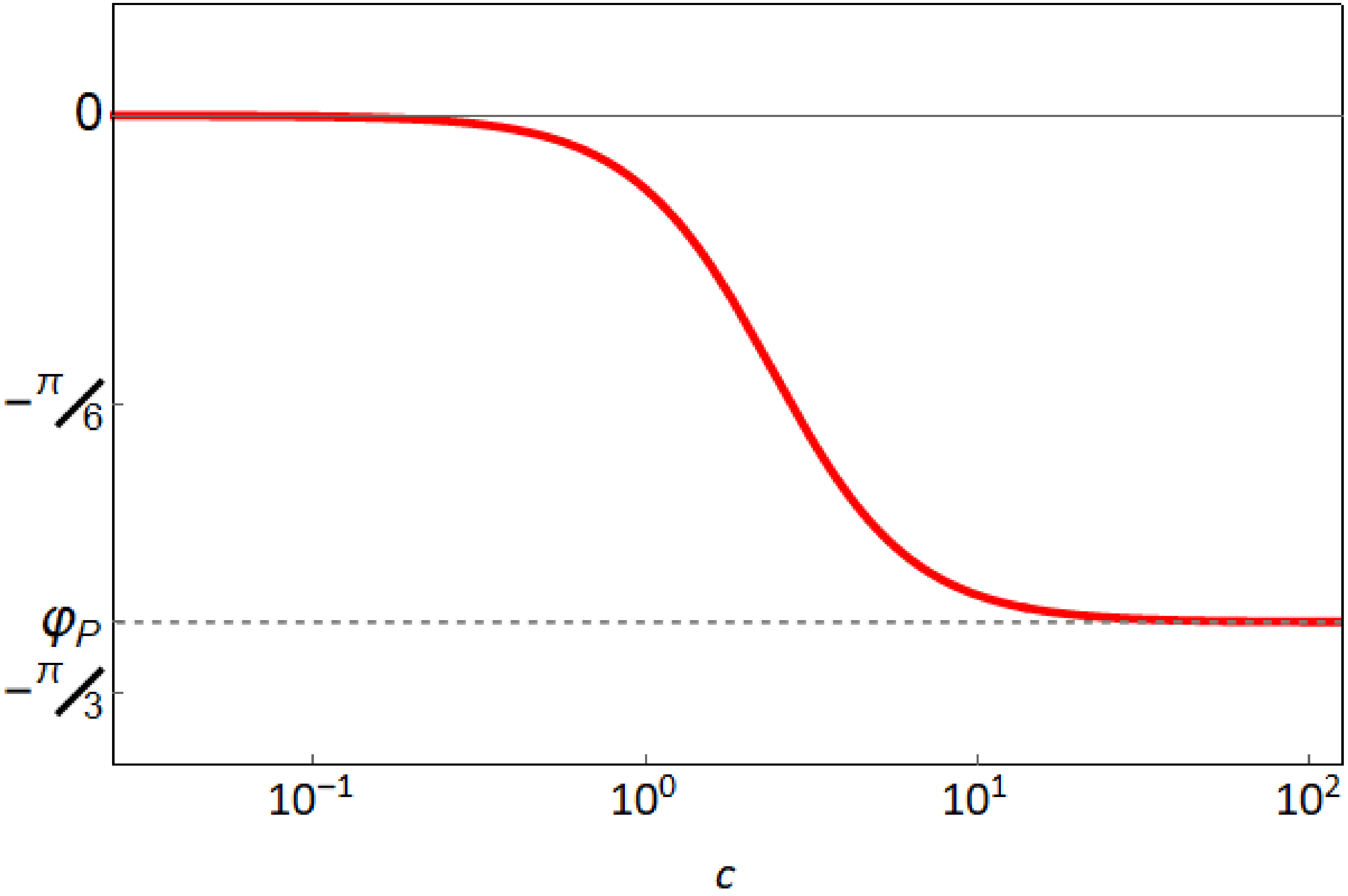
Observing averaged phase



Can measure

$$\sum_{r_1,\ldots,r_{N-1}} \left(\langle \Psi_0 | M_{N-1}^{(r_{N-1})} \ldots M_1^{(r_1)} | \Psi_0 \rangle \right)^2 = e^{2i\bar{\chi}_{\mathrm{geom}} - \alpha}$$

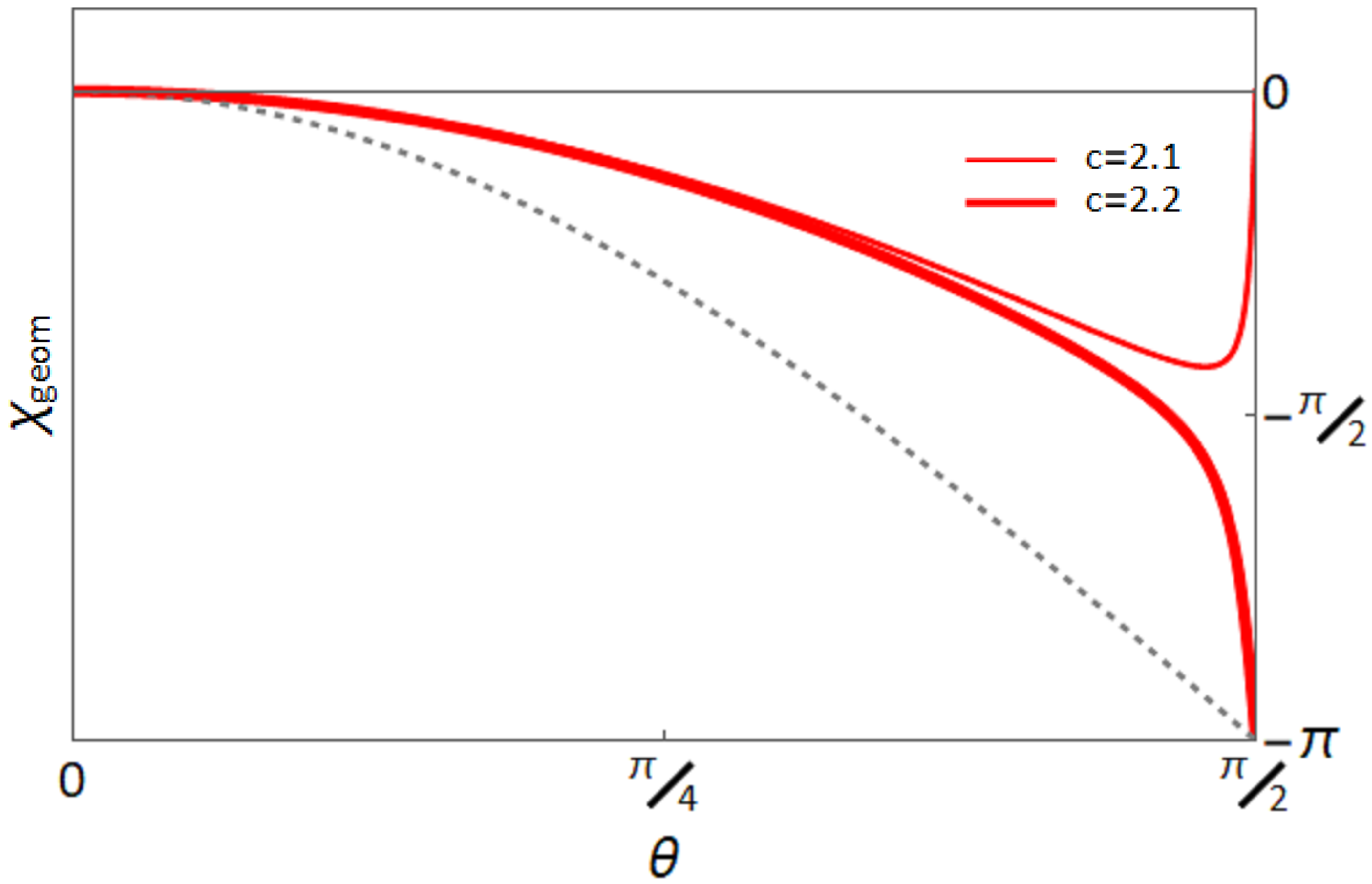
Phase dependence on the measurement strength

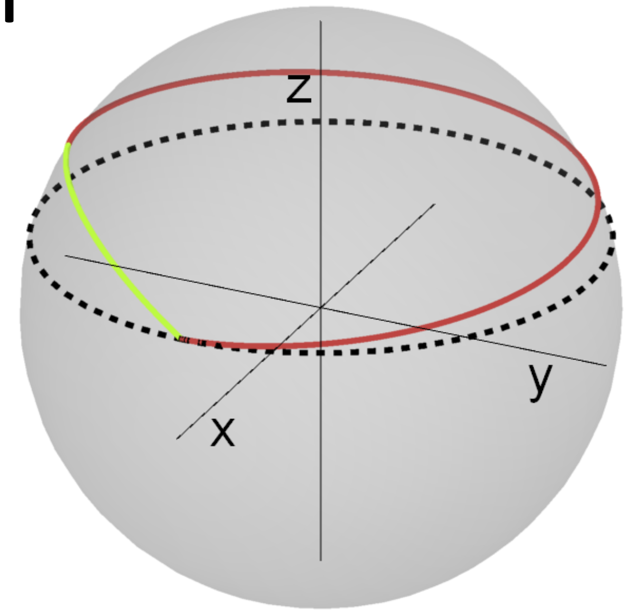


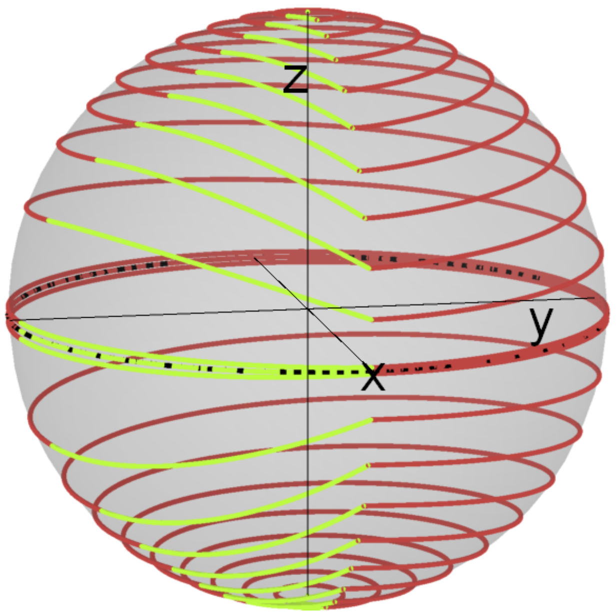
$$M_\uparrow = \begin{pmatrix} 1 & 0 \\ 0 & 1 - \frac{2c}{N} \end{pmatrix}$$

$$\langle \Psi_0 | M_{N-1}^{(\uparrow)} \dots M_1^{(\uparrow)} | \Psi_0 \rangle$$

Surprise: sharp transition

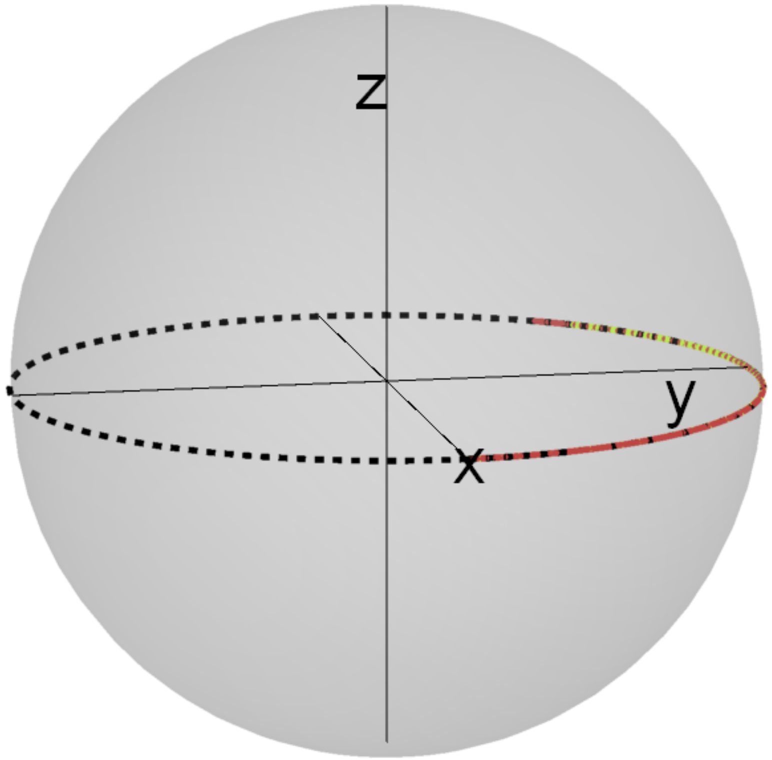






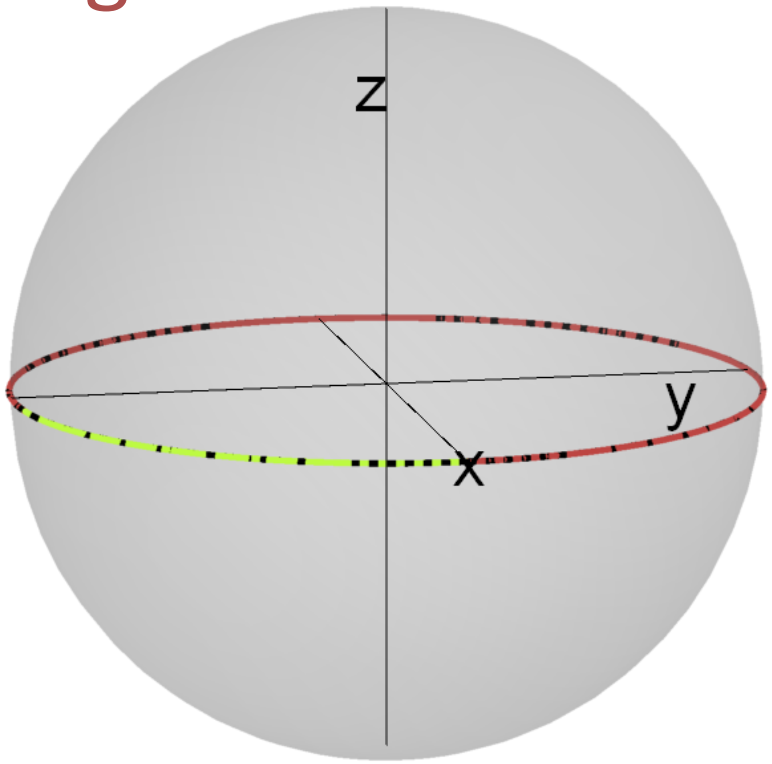
Equator trajectory and the topological transition

weak side



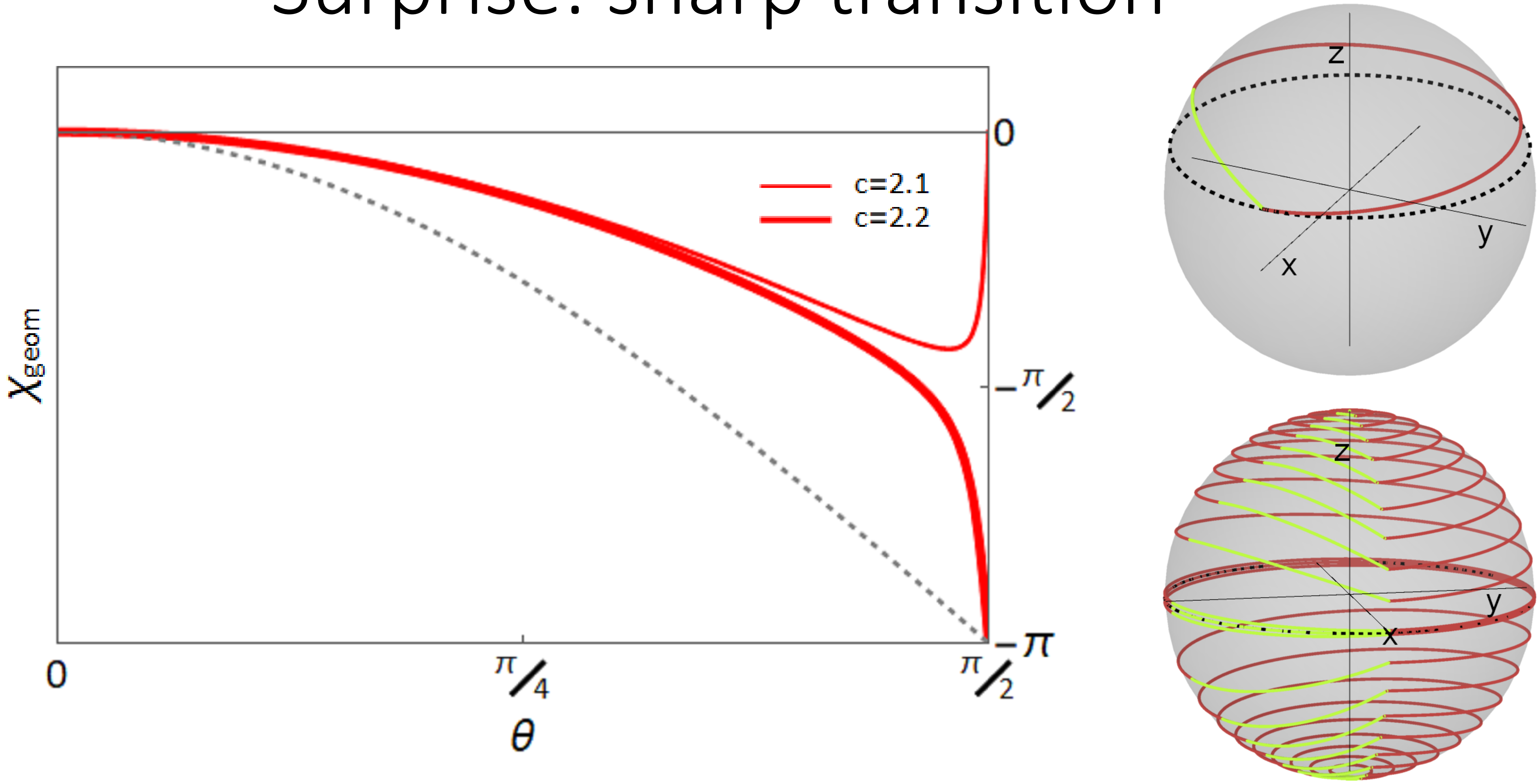
$geometric\ phase = 0$

strong side

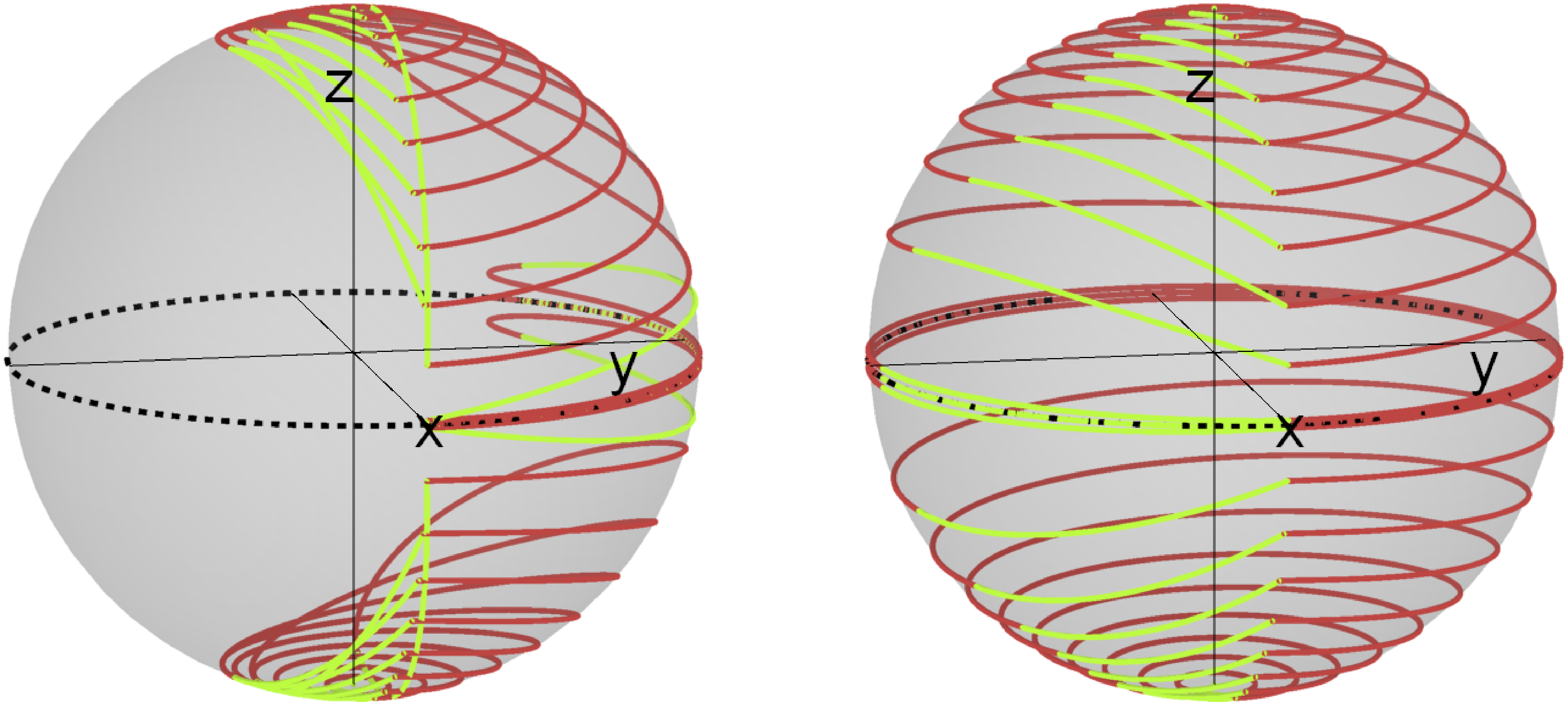


$geometric\ phase = -\pi$

Surprise: sharp transition



Trajectories and the topological transition (postelected)



Conclusions

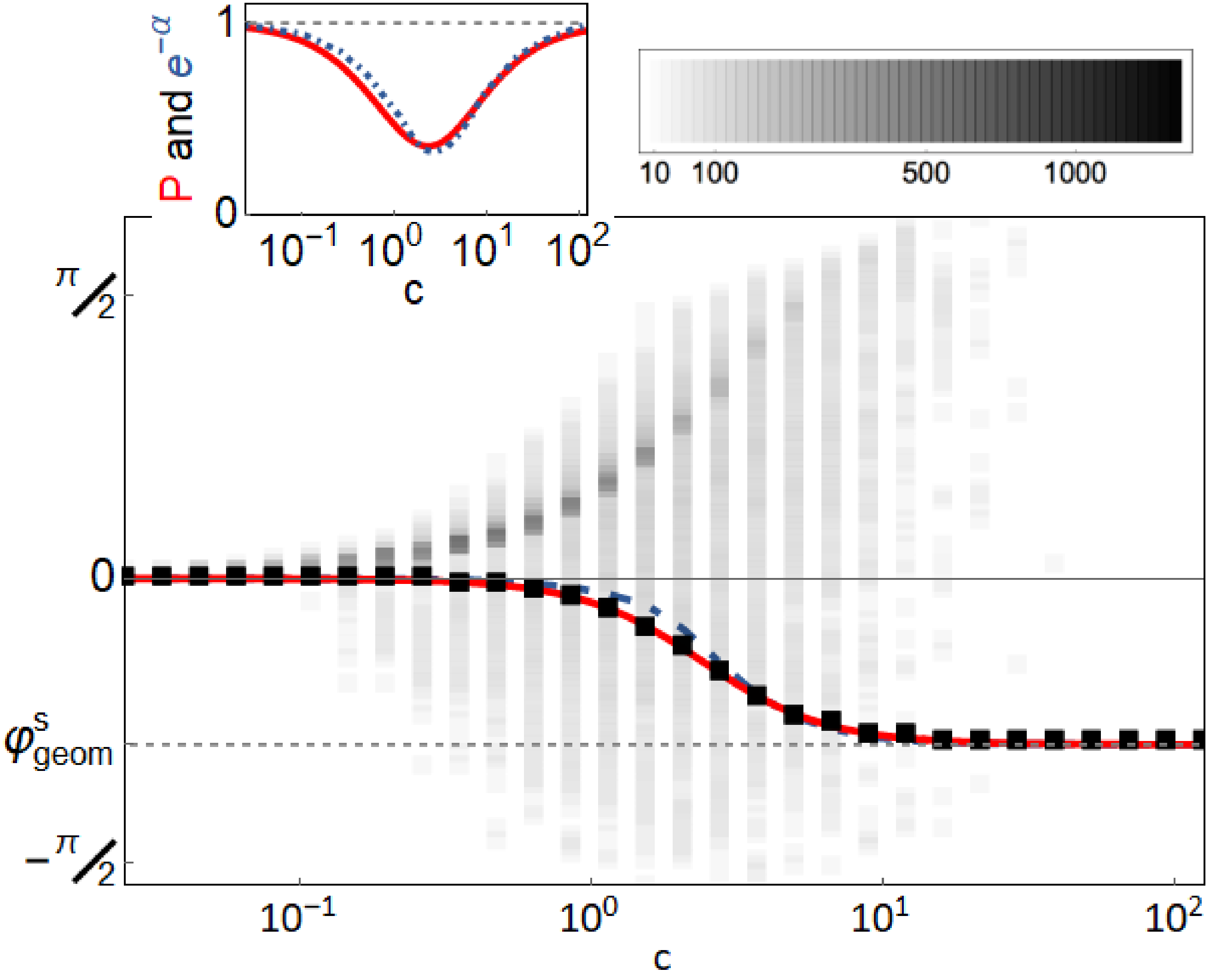
- Use measurement back-action to manipulate states

- Induce geometric phase by weak measurements

- Topological transition in weak-measurement-induced geometric phase
  as a function of measurement strength

Thank you for attention!

Phase dependence on the measurement strength

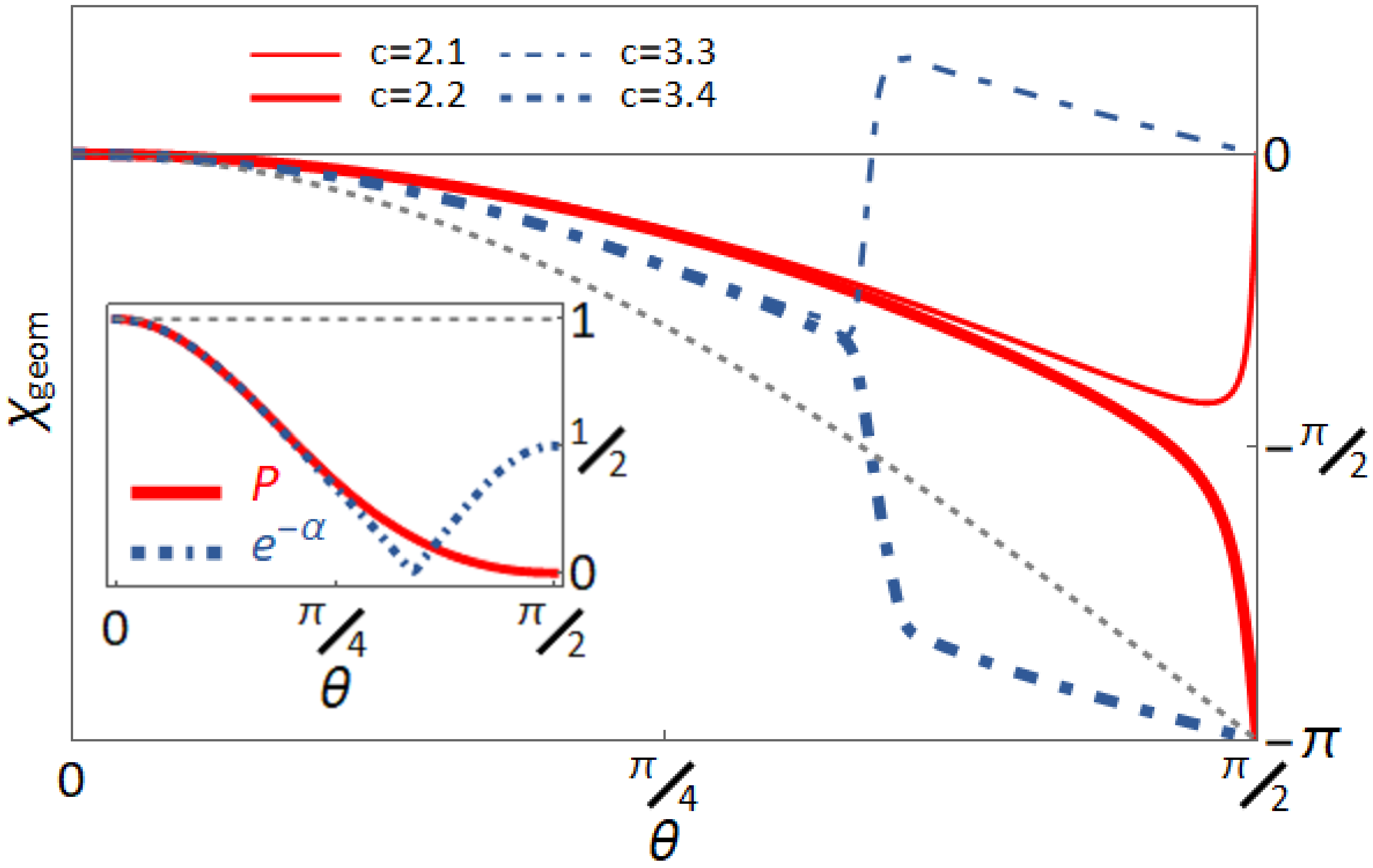


$$M_{\uparrow} = \begin{pmatrix} 1 & 0 \\ 0 & 1 - \frac{2c}{N} \end{pmatrix}$$

$$\langle \Psi_0 | M_{N-1}^{(\uparrow)} \ldots M_1^{(\uparrow)} | \Psi_0 \rangle$$

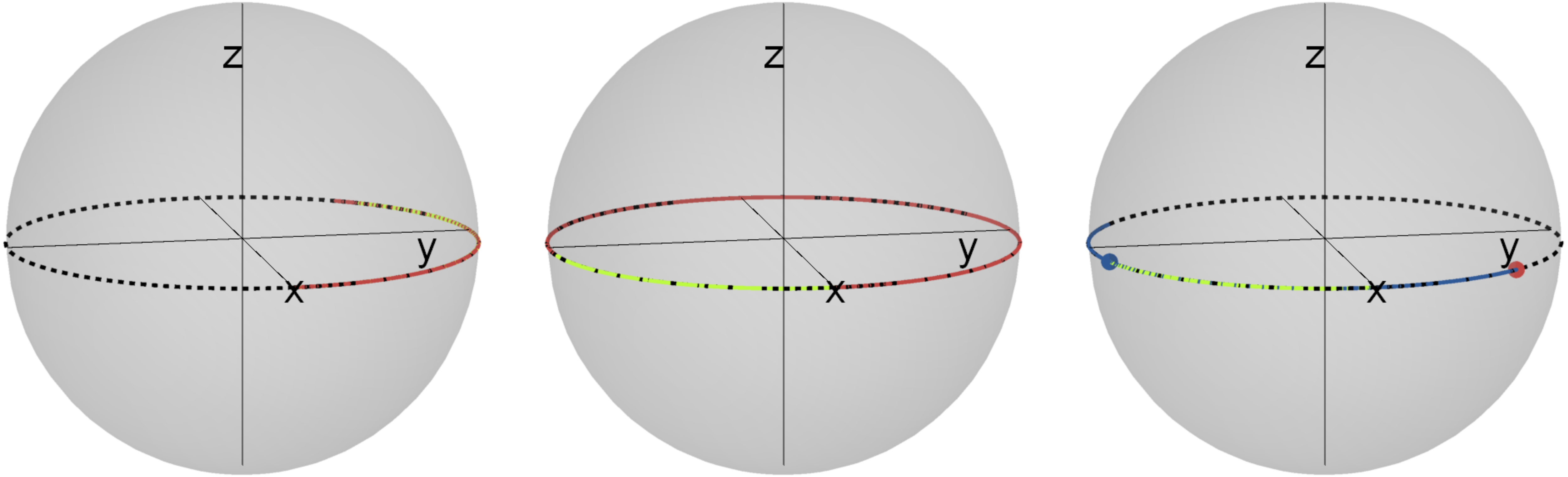
$$\sum_{r_1,\ldots,r_{N-1}} \left(\langle \Psi_0 | M_{N-1}^{(r_{N-1})} \ldots M_1^{(r_1)} | \Psi_0 \rangle \right)^2$$

Surprise: sharp transition(s)



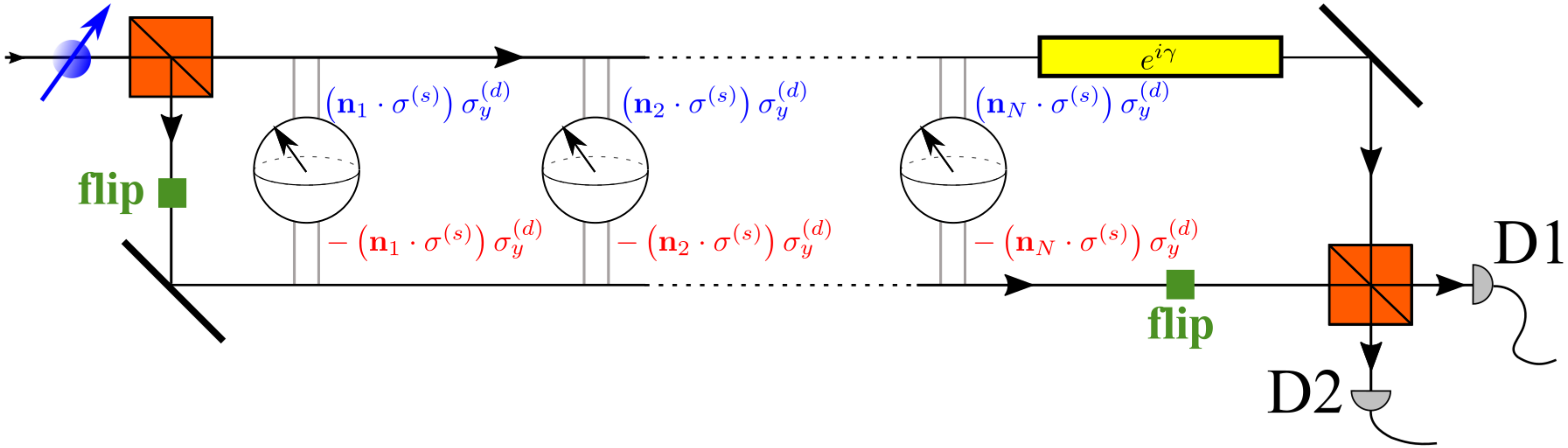
Equator trajectory and the topological transition(s)

$c = 1.9$ $c = 3.0$ $c = 3.0$



$\chi_G = 0$ $\chi_G = -\pi$ $\chi_G = 0$

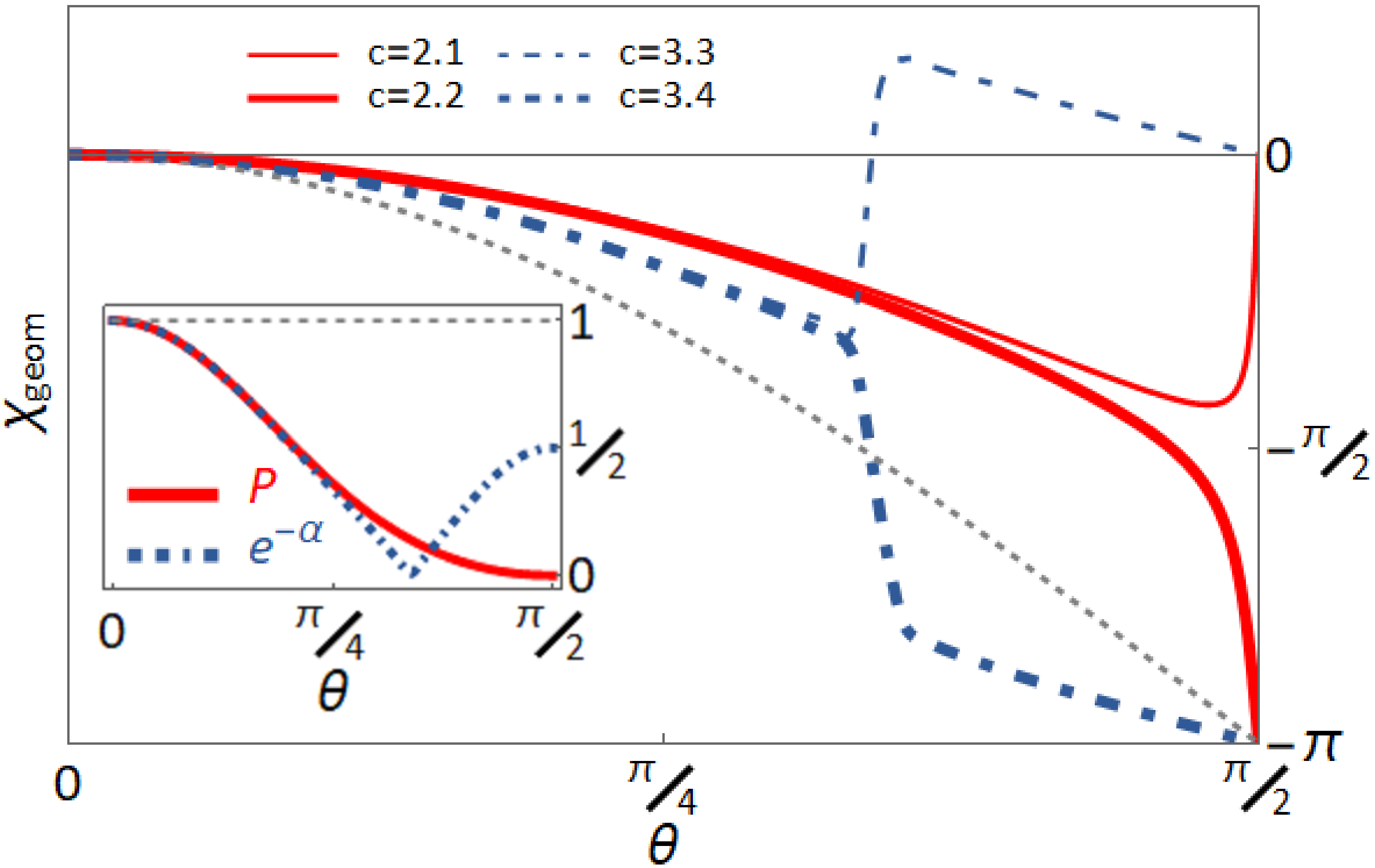
Observing averaged MIGP



Can measure

$$\sum_{r_1,\ldots,r_{N-1}} \left(\langle \Psi_0 | \mathcal{M}_{N-1}^{(r_{N-1})} \ldots \mathcal{M}_1^{(r_1)} | \Psi_0 \rangle \right)^2 = e^{2i\bar{\chi}_G - \alpha}$$

Surprise: sharp transition(s)



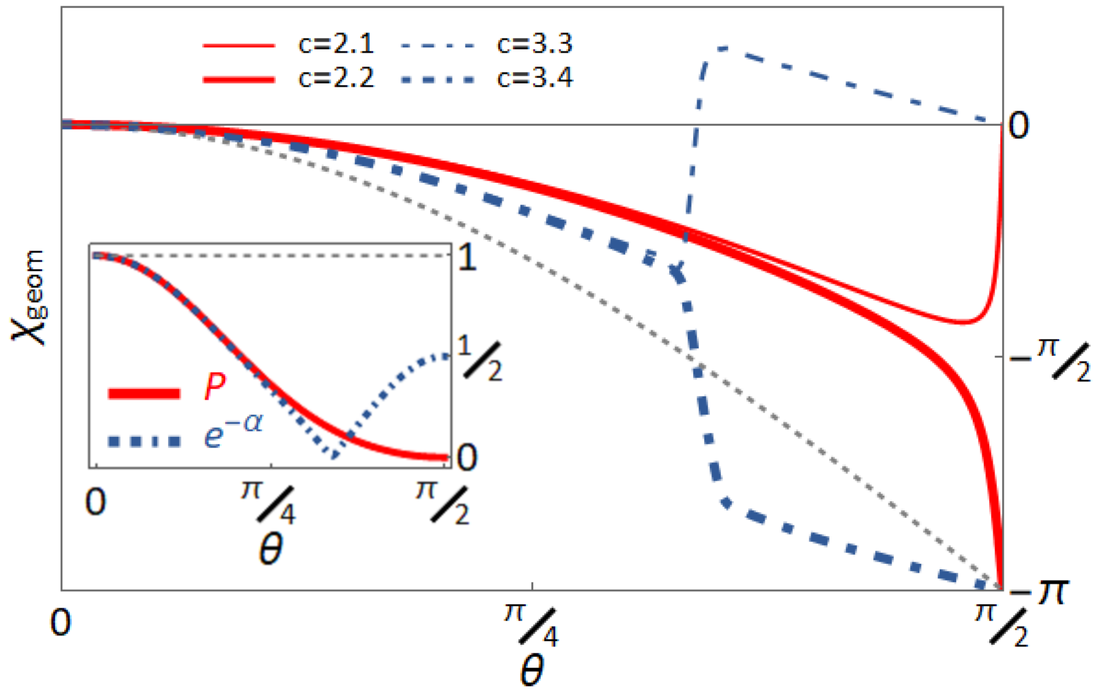
What do the transitions tell about?

- New property of measurements?

  Not really, different critical strengths depending on what one looks

- New way of revealing topological features of the measured system?

  Seems so, topological features of the Bloch sphere in our example.



Recent experiments observe MIGP

nature physics

Article | Published: 15 April 2019

Emergence of the geometric phase from quantum measurement back-action

Young-Wook Cho ✉, Yosep Kim, Yeon-Ho Choi, Yong-Su Kim, Sang-Wook Han, Sang-Yun Lee, Sung Moon & Yoon-Ho Kim ✉

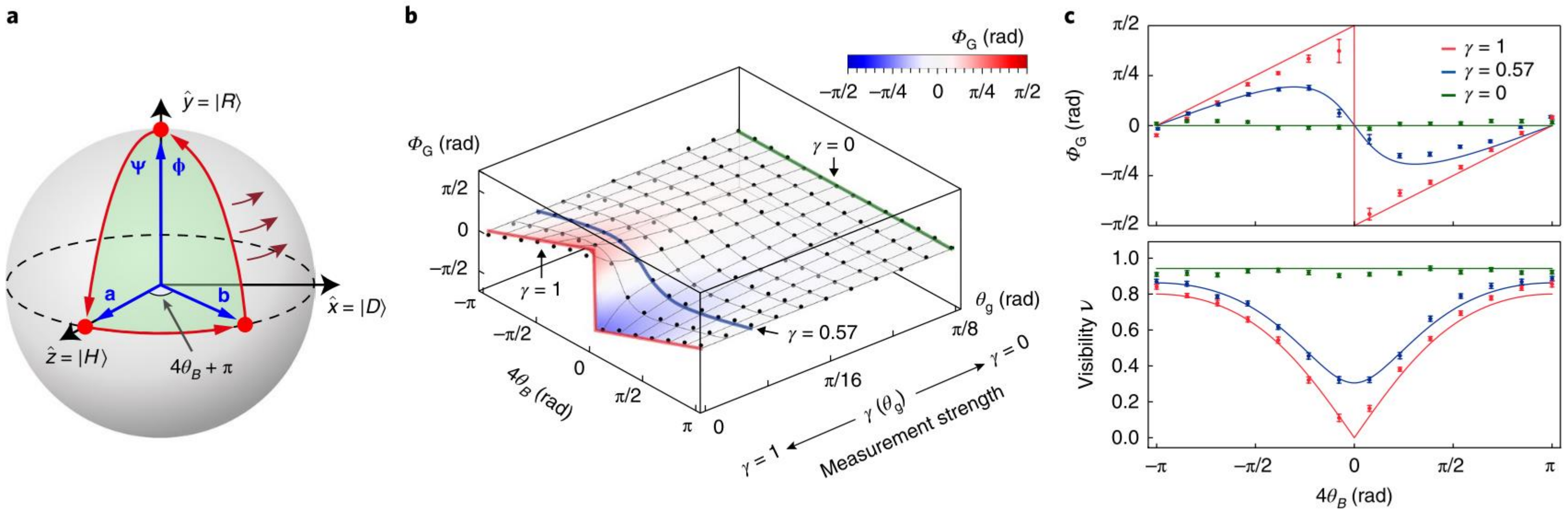
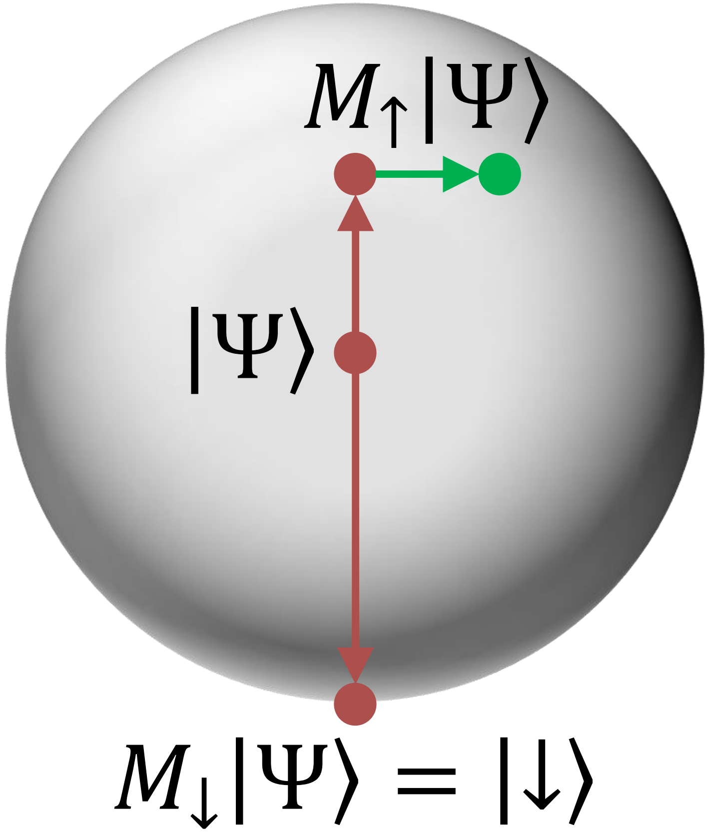


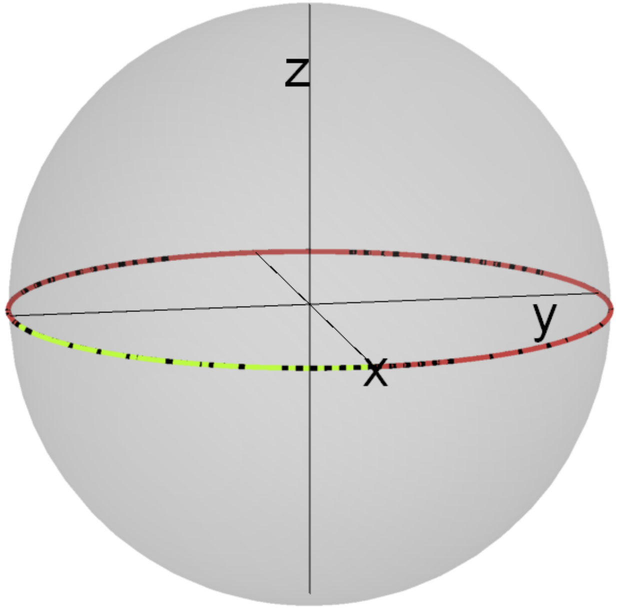
Fig. 4 | Emergence of the geometric phase from quantum measurement back-action. a, The quantum state trajectory based on the geodesic hypothesis

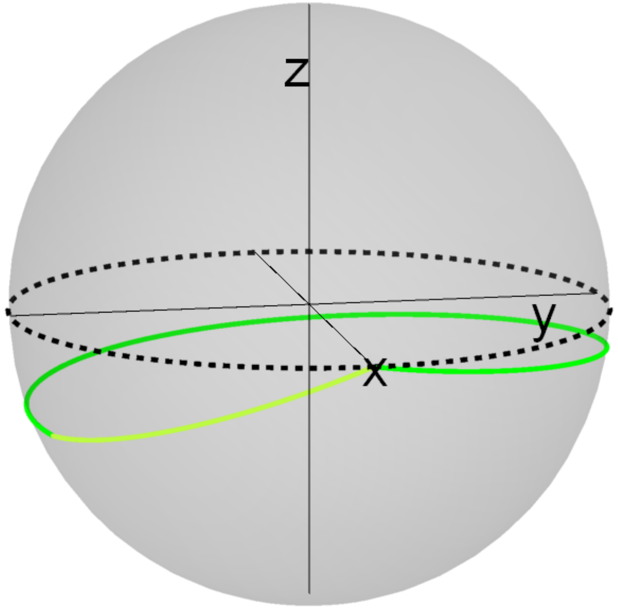
Symmetry protected?

Two-outcome detector: $r = \uparrow / \downarrow$

- $M_\uparrow = \begin{pmatrix} 1 & 0 \\ 0 & te^{i\phi} \end{pmatrix}$
- $M_\downarrow = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{1-t^2} \end{pmatrix}$

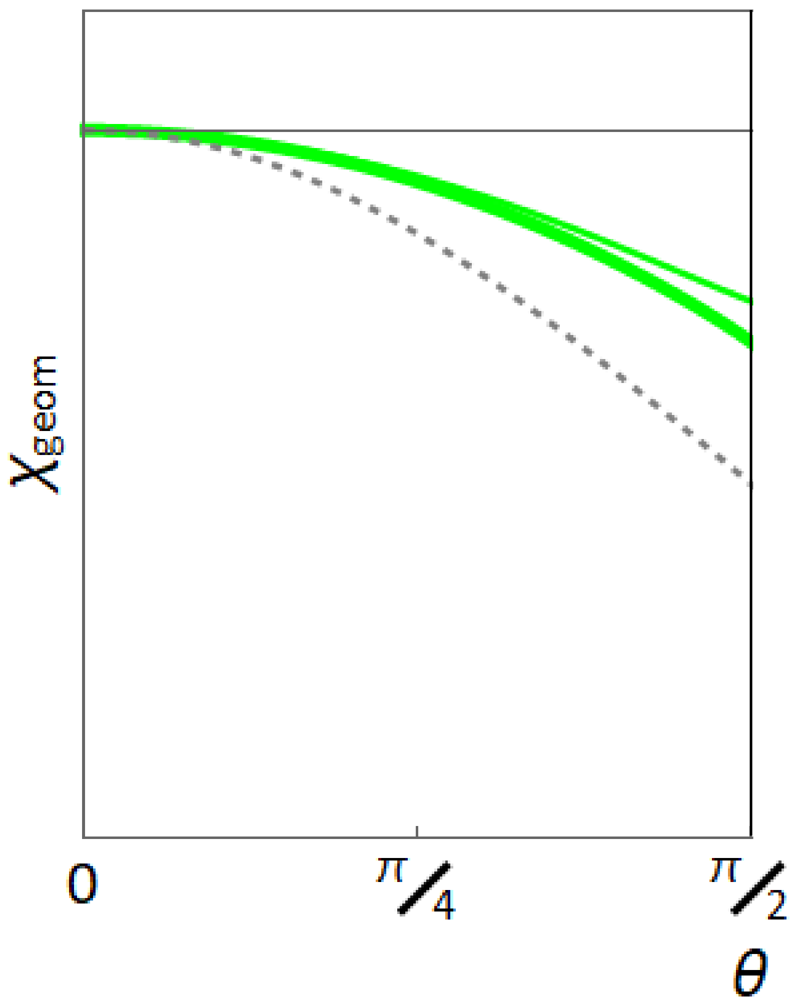






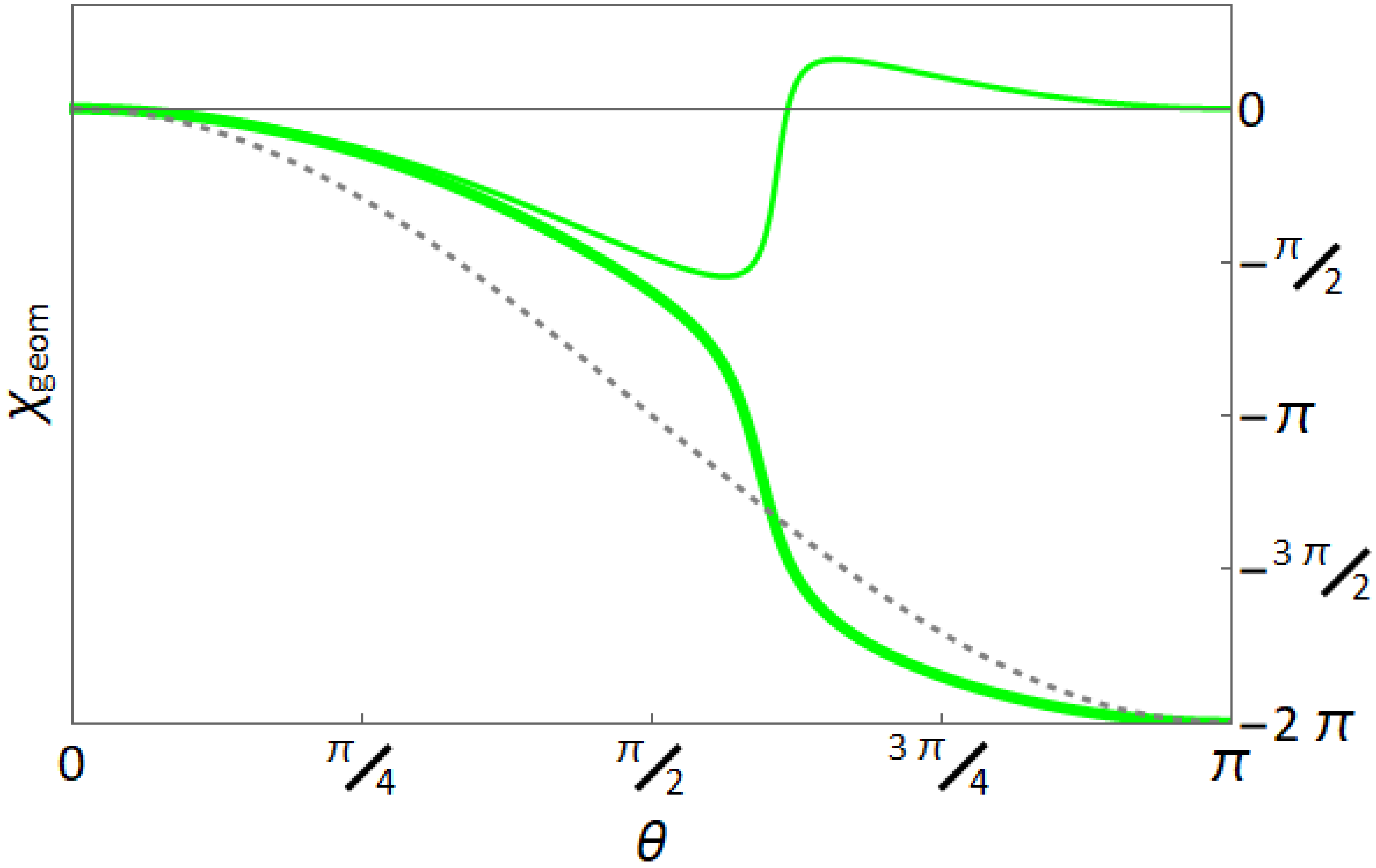
KS, Rao, Kumar, Gefen, in preparation

Symmetry protected?



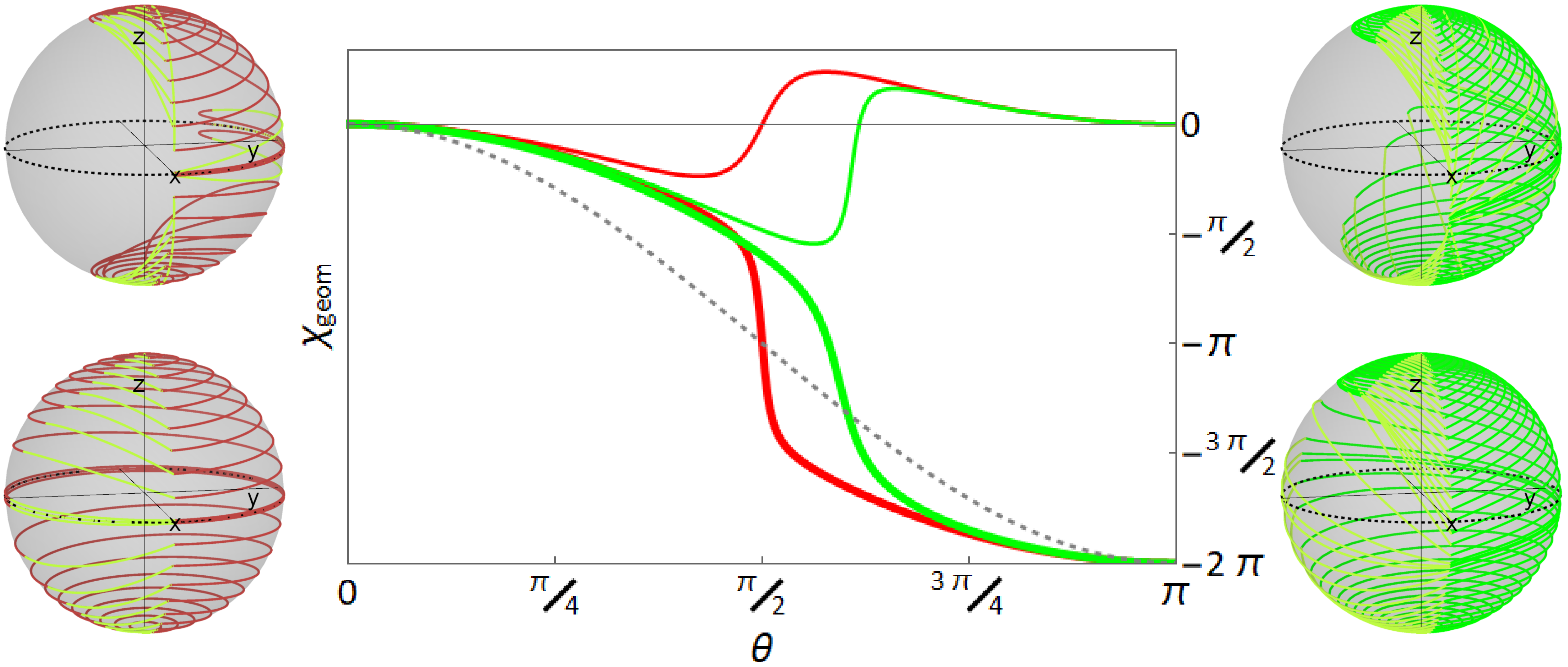
KS, Rao, Kumar, Gefen, in preparation

Symmetry protected? No.



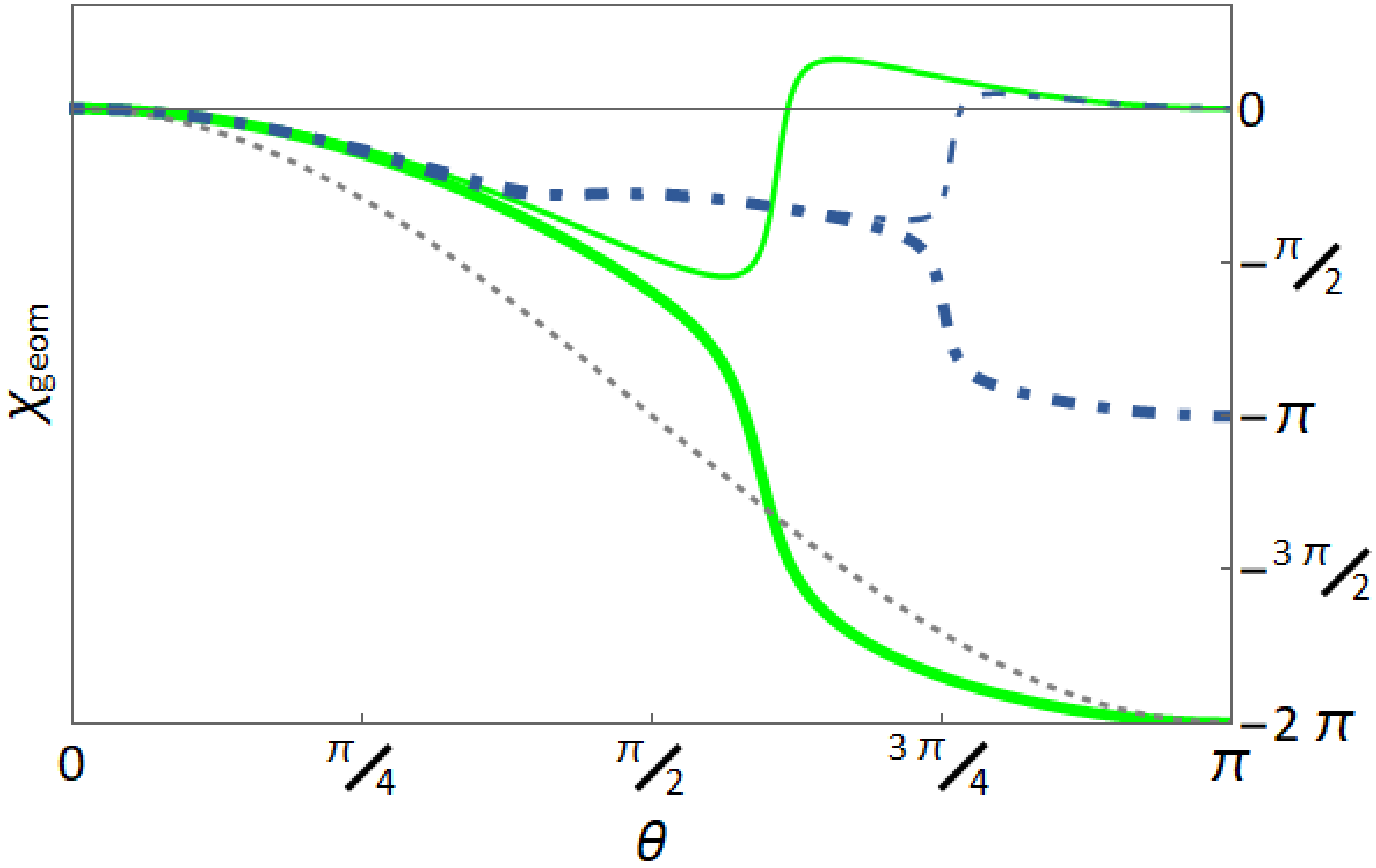
KS, Rao, Kumar, Gefen, in preparation

Symmetry protected? No.



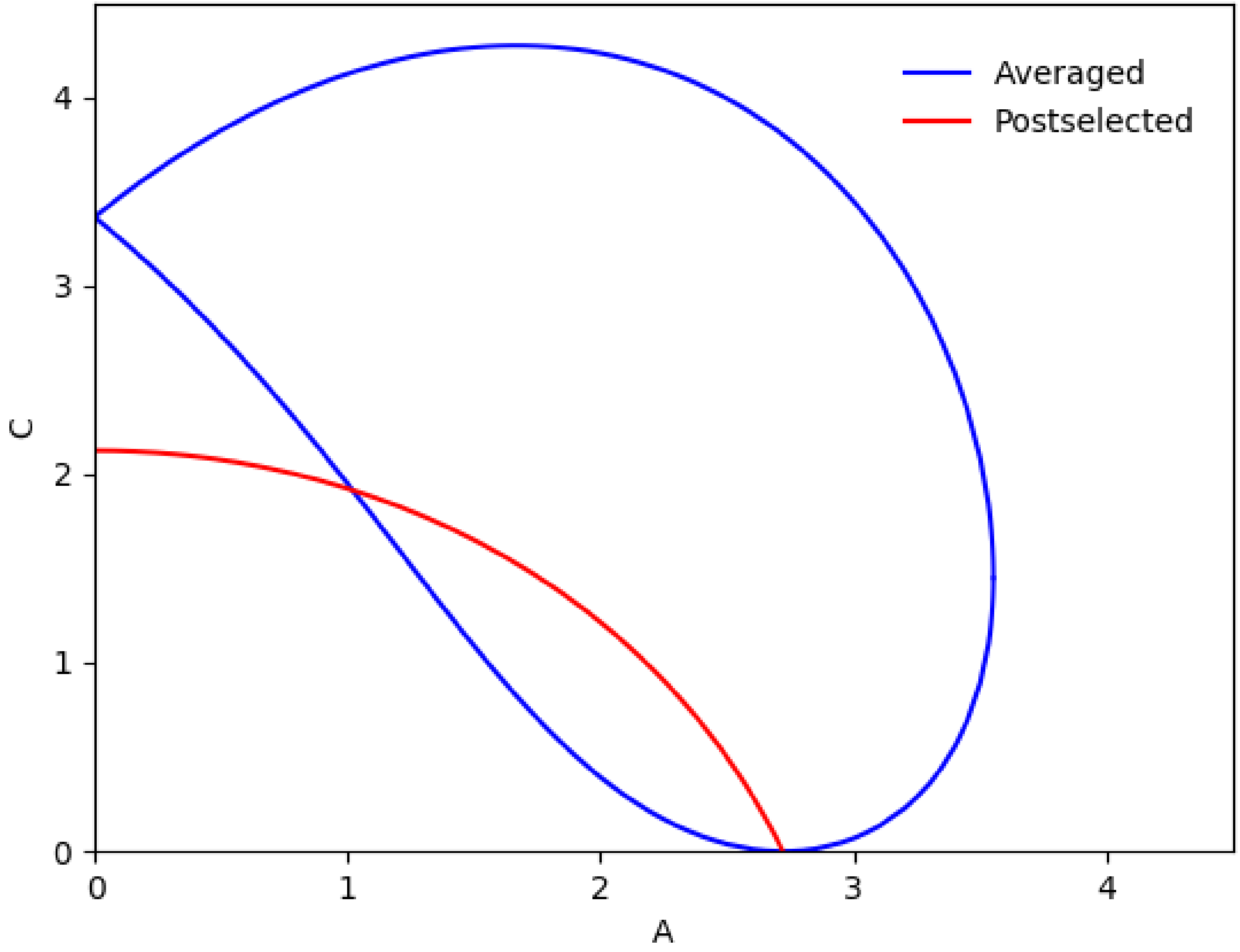
KS, Rao, Kumar, Gefen, in preparation

Also for the averaged phase



KS, Rao, Kumar, Gefen, in preparation

Phase diagram



KS, Rao, Kumar, Gefen, in preparation