Quantum states of mechanical resonators in optomechanics

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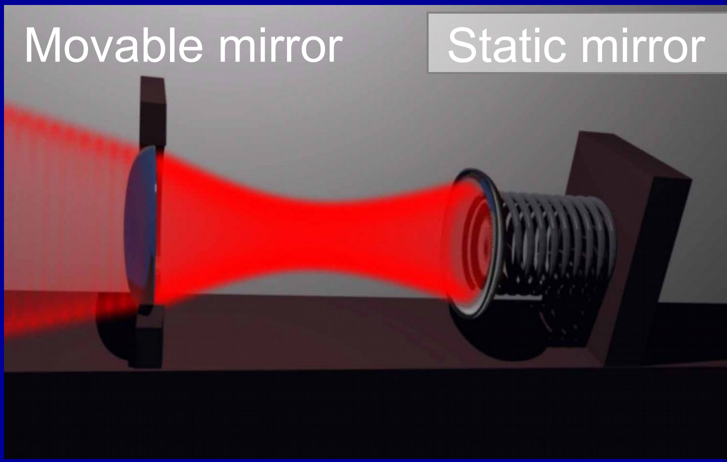
With: João Pereira Machado; Rutger Slooter

- Cavity optomechanics
- Membrane in the middle
- Quantum effects

J. D. P. Machado, R. J. Slooter, and YMB, Phys. Rev. A **99**, 053801 (2019)

Cavity optomechanics



Kippenberg's Group website

Radiation pressure coupling

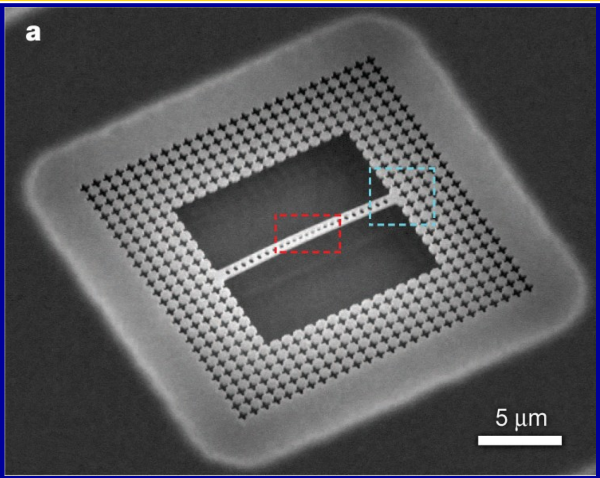
$\omega_{cav}(x)$

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_{m}\hat{b}^{\dagger}\hat{b} - \hbar g_{0}\hat{a}^{\dagger}\hat{a}(b^{\dagger} + b)$$

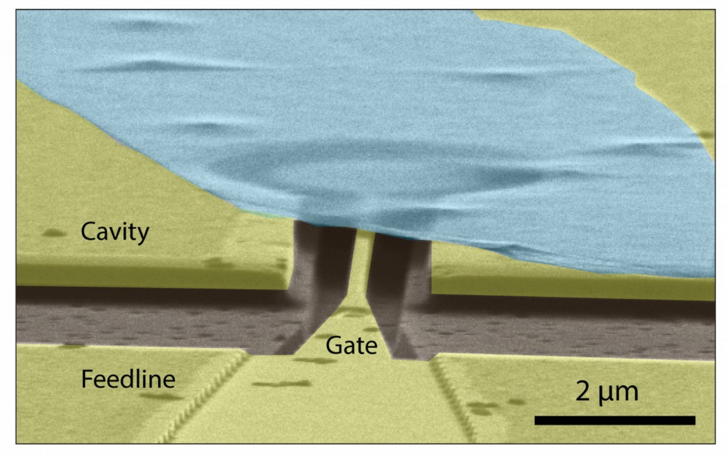
Cavity

Mechanical resonator

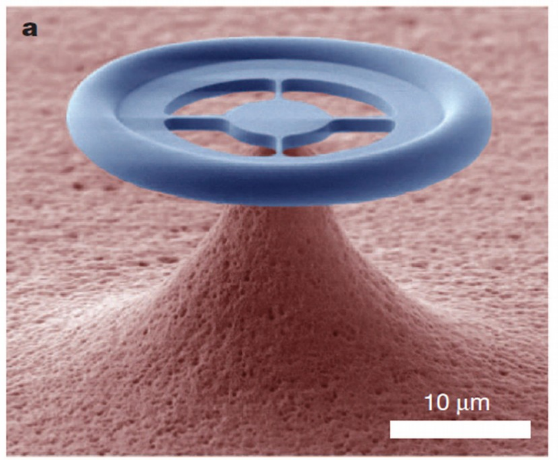
Cavity optomechanics



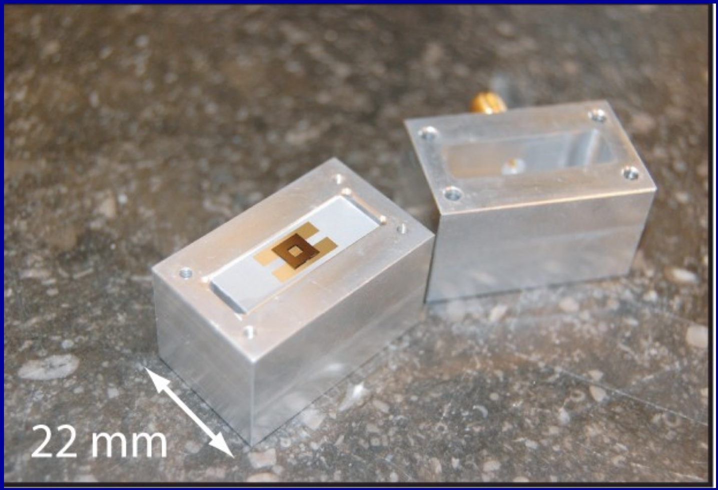
Chan et al, Nature **478**, 89 (2011)



Singh et al, Nature Nanotech. **9**, 820 (2014)



Verhagen et al, Nature **482**, 63 (2012)



Yuan et al, Nature Comms. **6**, 8491 (2015)

Coupling

$$H = \hbar\omega_{cav}\hat{a}^\dagger\hat{a} + \hbar\omega_m\hat{b}^\dagger\hat{b} - \hbar g_0\hat{a}^\dagger\hat{a}(b^\dagger + b)$$

Dissipation rate in the cavity

Sideband-resolved regime

$$\Gamma, \kappa \ll \omega_m \ll \omega_{cav}$$

Where is g_0?

Weak coupling ← → Strong coupling

Driving and linearization: $g = g_0\sqrt{n_{cav}}$

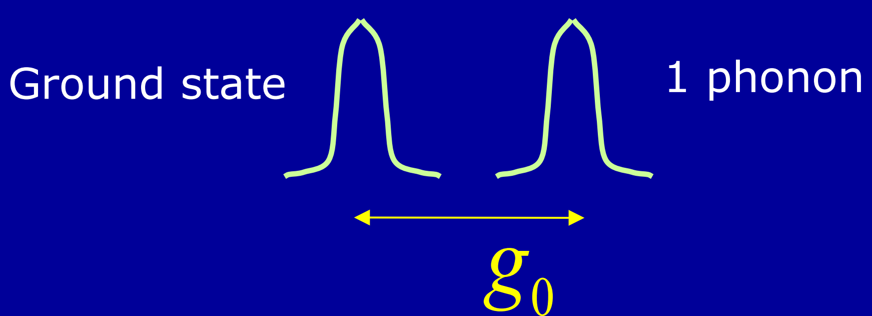
Single-photon strong coupling

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} - \hbar g_0 \hat{a}^{\dagger}\hat{a}(b^{\dagger} + b)$$

Dissipation rate in the cavity

$$\Gamma, \kappa \ll g_0 \ll \omega_m \ll \omega_{cav}$$

Shift of the cavity frequency due to addition of one phonon is bigger than the linewidth



Coupling

$$H_{\text{int}} = -\hbar g_0 \hat{a}^\dagger \hat{a}(b^\dagger + b) \to -\hbar g(\hat{a}^\dagger + \hat{a})(b^\dagger + b)$$

Non-resonant? Depends how we drive.

$$g = g_0 \sqrt{n_{cav}}$$

In the rotating frame: $\sqrt{n_{cav}} \propto e^{i\omega_d t}; a \propto e^{i\omega_{cav} t}; b \propto e^{i\omega_m t}$

Red-detuned drive: $\omega_d = \omega_{cav} - \omega_m$

$$H_{\text{int}} = -\hbar g(\hat{a}^\dagger b + \hat{a} b^\dagger)$$

Blue-detuned drive: $\omega_d = \omega_{cav} + \omega_m$

$$H_{\text{int}} = -\hbar g(\hat{a}^\dagger b^\dagger + \hat{a} b)$$

Quantum detection of mechanical oscillations

Can we see quantum effects in mechanical motion?

Issues:

1. Need low temperatures $k_B T \ll \hbar\omega$

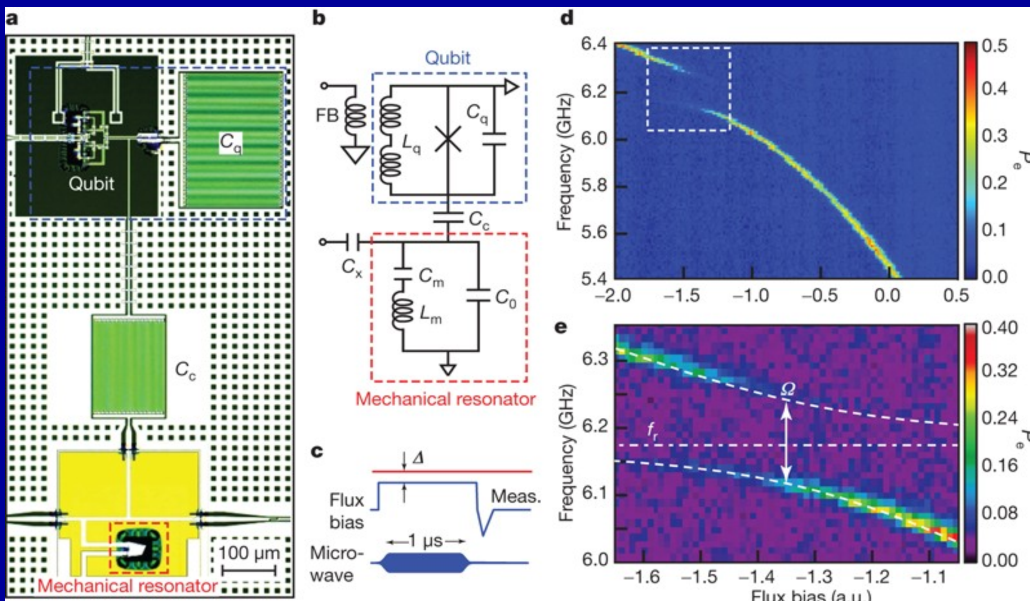
$T = 1K \Rightarrow \omega \gg 100\ \text{GHz}$

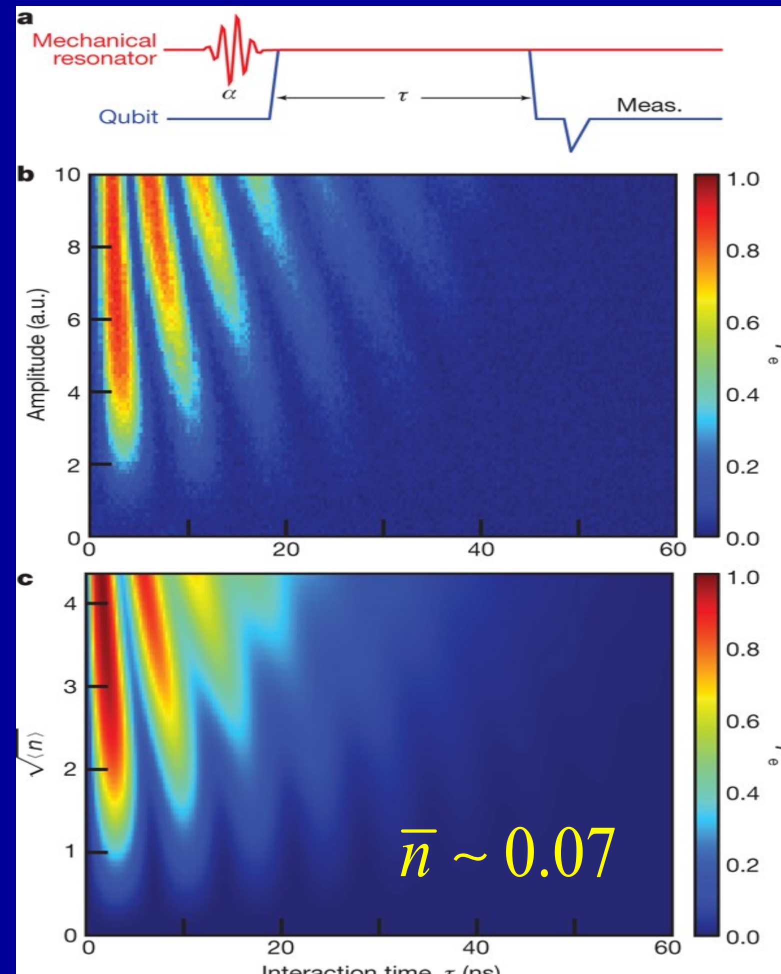
Either need to cool the mechanical resonator down or need to work with very high frequerncies

2. Need to decide what are the signatures of the quantum behavior and need a quantum detector to measure them (technically: can not measure quantum phonons)

Most proposals for quantum effects involve single-photon strong coupling and non-linear systems

Quantum detection of mechanical oscillations





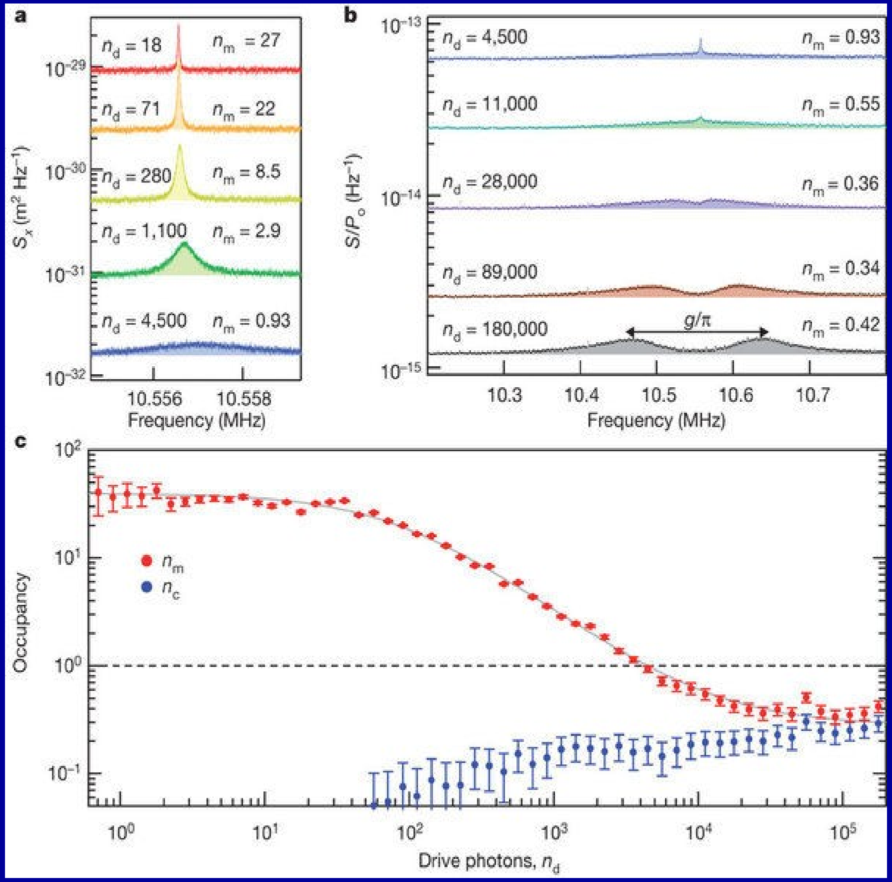
$\bar{n} \sim 0.07$

A. D. O'Connell, M. Hofheinz, M. Ansmann, R. C. Bialczak, M. Lenander, E. Lucero, M. Neeley, D. Sank, H. Wang, M. Weides, J. Wenner, J. M. Martinis, A. N. Cleland
Nature **464**, 697 (2010)

A mechanical resonator capacitively coupled to a superconducting qubit

$f \sim 6\,\text{GHz}$

Quantum detection of mechanical oscillations



J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, R. W. Simmonds
Nature **475**, 359 (2011)

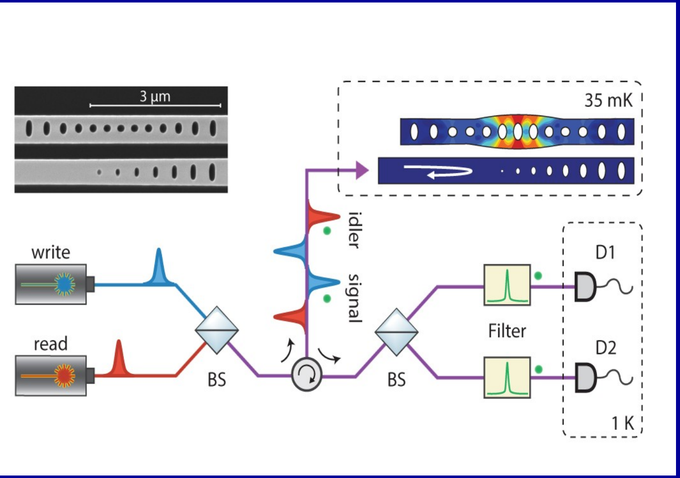
Cavity: $f_c \sim 7.5$ GHz

Mechanical resonator: $f \sim 10$ MHz

Sideband cooling

Quantum behavior of mechanical resonator

S. Hong, R. Riedinger, I. Marinkovic, A. Wallucks, S. G. Hofer, R. A. Norte, M. Aspelmeyer, S. Gröblacher, Science **358**, 203 (2017)



Two-point correlation function:

$$g^{(2)}(\tau) = \frac{\left\langle b^\dagger(t) b^\dagger(t+\tau) b(t) b(t+\tau) \right\rangle}{\left\langle b^\dagger(t) b(t) \right\rangle^2}$$

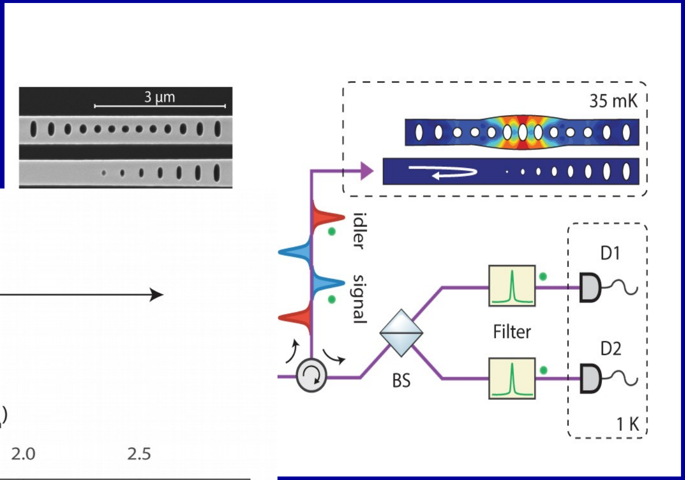
Signature of non-classical states: $g^{(2)}(0) < 1$

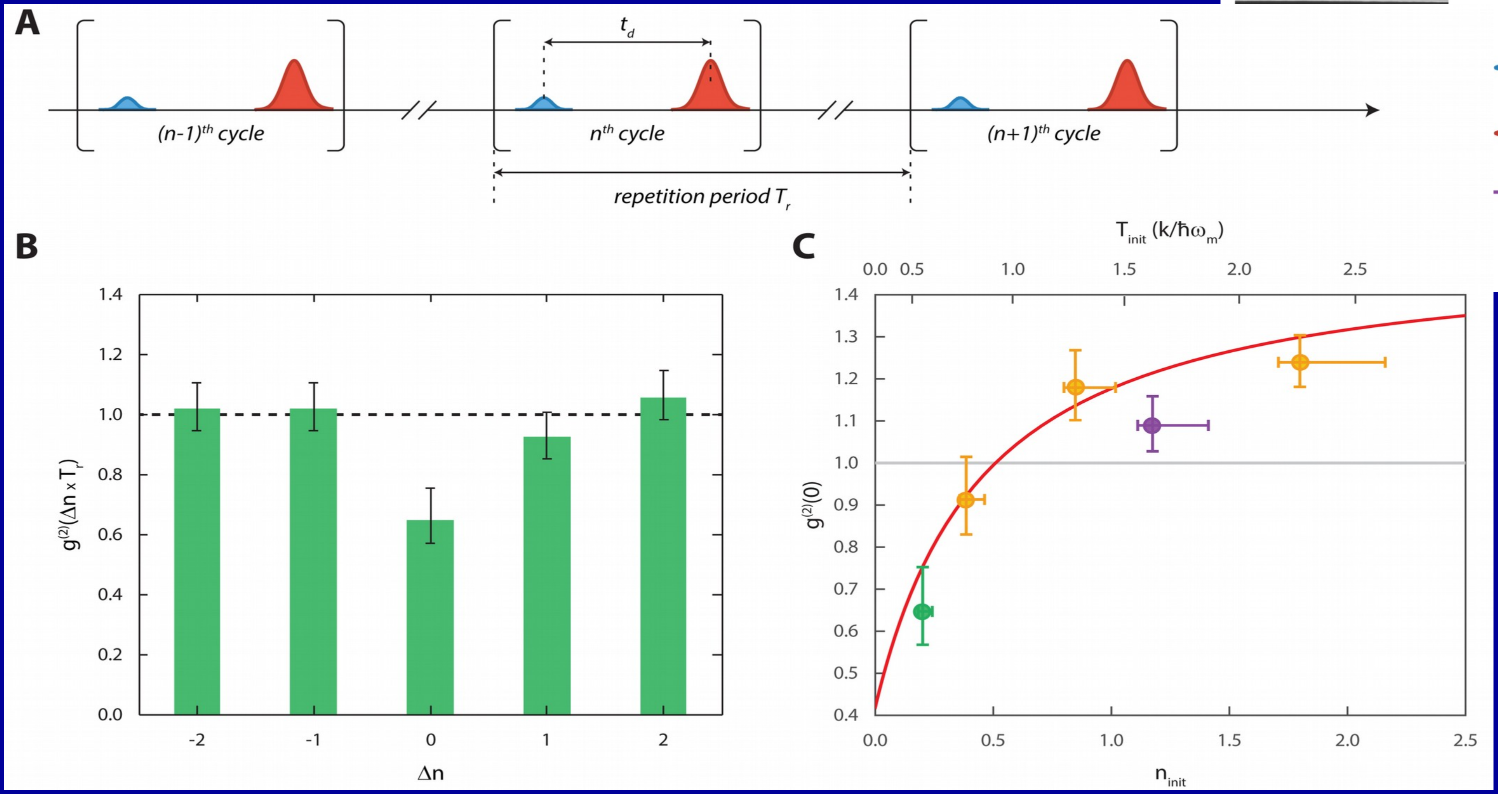
Generally: $0 < g^{(2)}(0) < 2$

Quantum behavior of mechanical resonator

S. Hong, R. Riedinger, I. Marinkovic, A. Wallucks, S. G. Hofer, R. A. Norte, M. Aspelmeyer, S. Gröblacher, arXiv:1706.03777

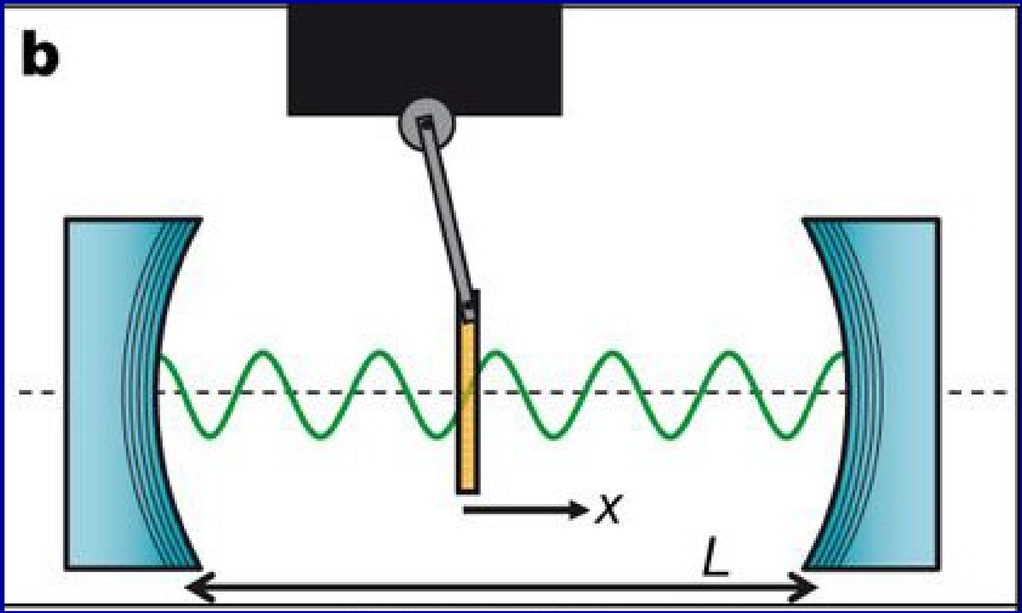
Signature of non-classical states: $g^{(2)}(0) < 1$





Membrane in the middle

J.D. Thompson, B.M. Zwickl, A.M. Jayich, F. Marquardt, S.M. Girvin, and J.G.E. Harris, Nature 452, 72 (2008)



$\omega_{cav}(x)$ - periodic function of the position of the membrane

Quadratic coupling!

Quadratic coupling

- Much weaker than linear coupling

- But one does not need to go to the single-photon coupling regime

Isolated cavity

Can be exactly diagonalized

Zero-point fluctuations

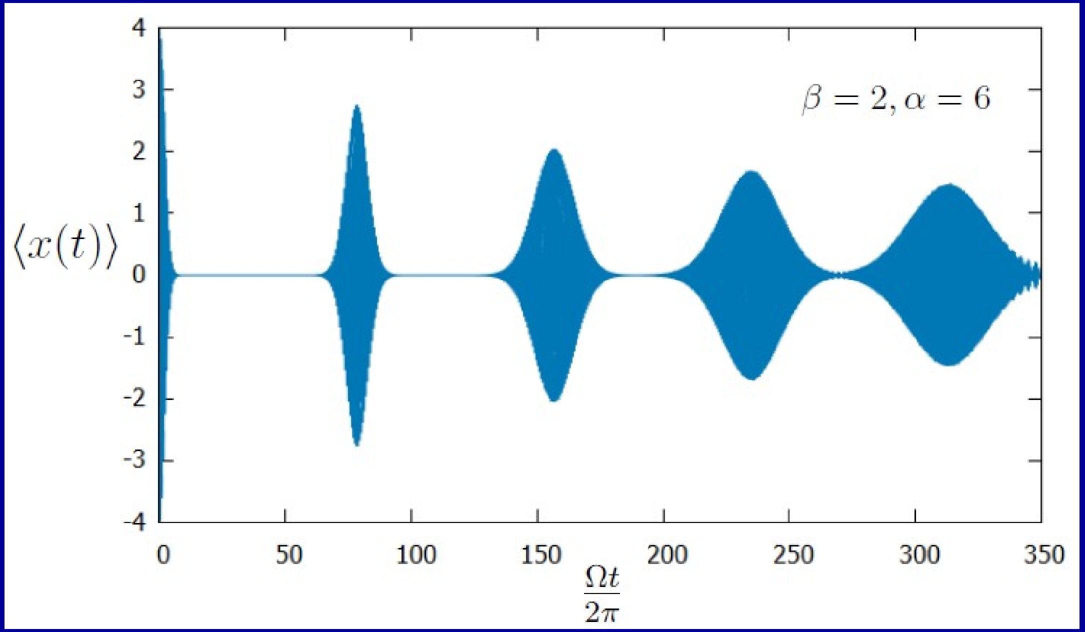
$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_m\hat{b}^{\dagger}\hat{b} + \hbar g_0\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right)(b^{\dagger} + b)^2$$

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\sqrt{\omega_m^2 + 4g_0\omega_m\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right)}\left(b^{\dagger}b + \frac{1}{2}\right)$$

A. Rai and G.S. Agarwal, Phys. Rev. A 78, 013831 (2008)

Isolated cavity: Collapses and revivals

Initial coherent state $|\alpha, \beta\rangle$



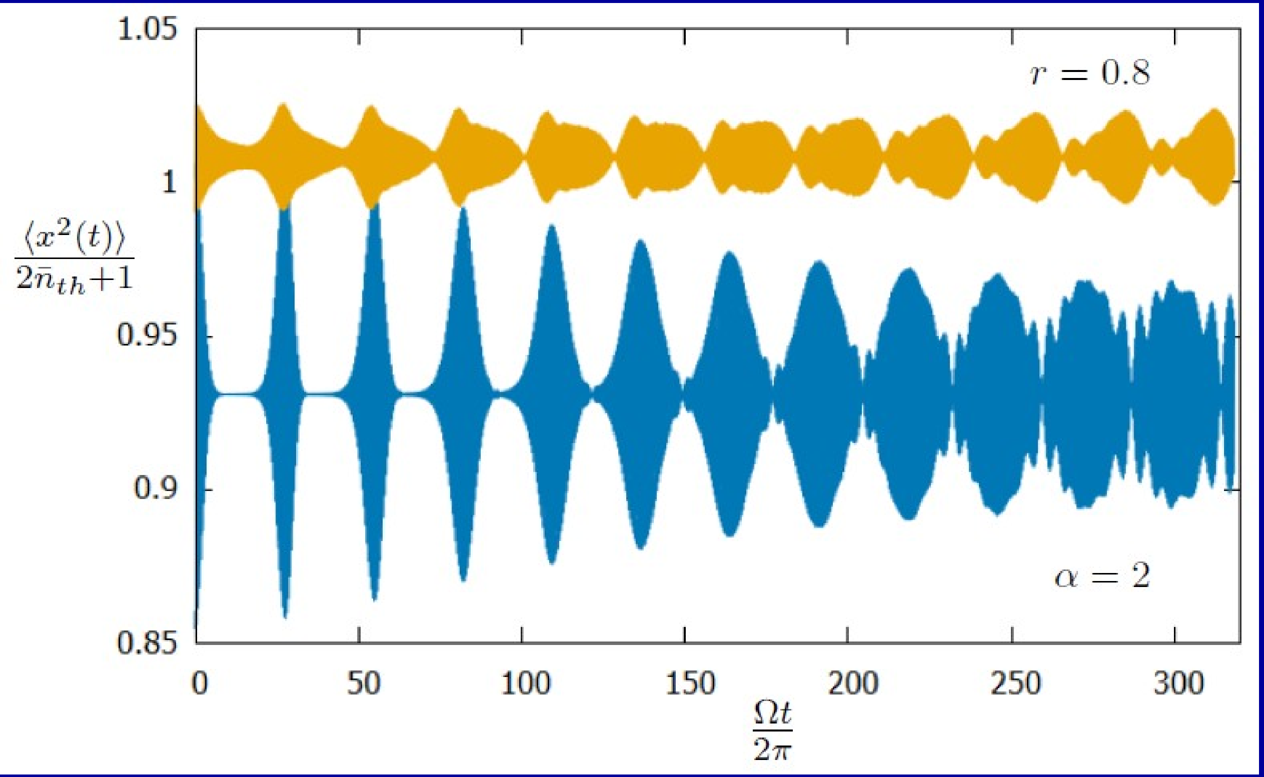
$$T_{rev} = 2\pi|\alpha| T_{coll} = \frac{\pi\sqrt{\omega_m^2 + 4g_0\omega_m|\alpha|^2}}{g_0\omega_m}$$

A. Rai and G.S. Agarwal, Phys. Rev. A 78, 013831 (2008);
J. D. P. Machado, R.J. Slooter, and YMB, Phys. Rev. A **99**, 053801 (2019

Isolated cavity: Collapses and revivals

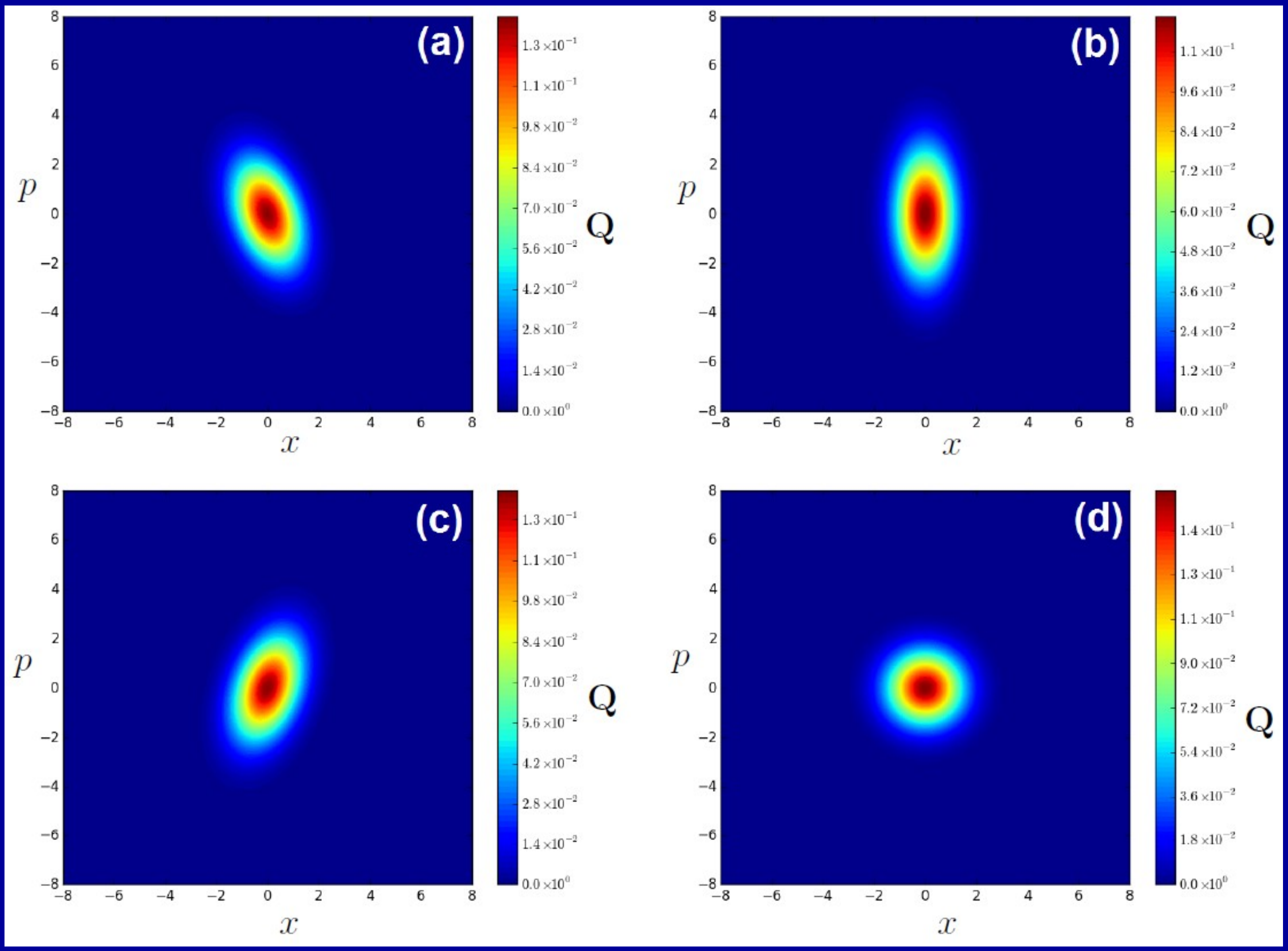
Initial thermal state of phonons
Coherent or vacuum-squeezed state of the cavity

Seen in all properties of the mechanical resonator



J. D. P. Machado, R.J. Slooter, and YMB, Phys. Rev. A **99**, 053801 (2019

Quantum states

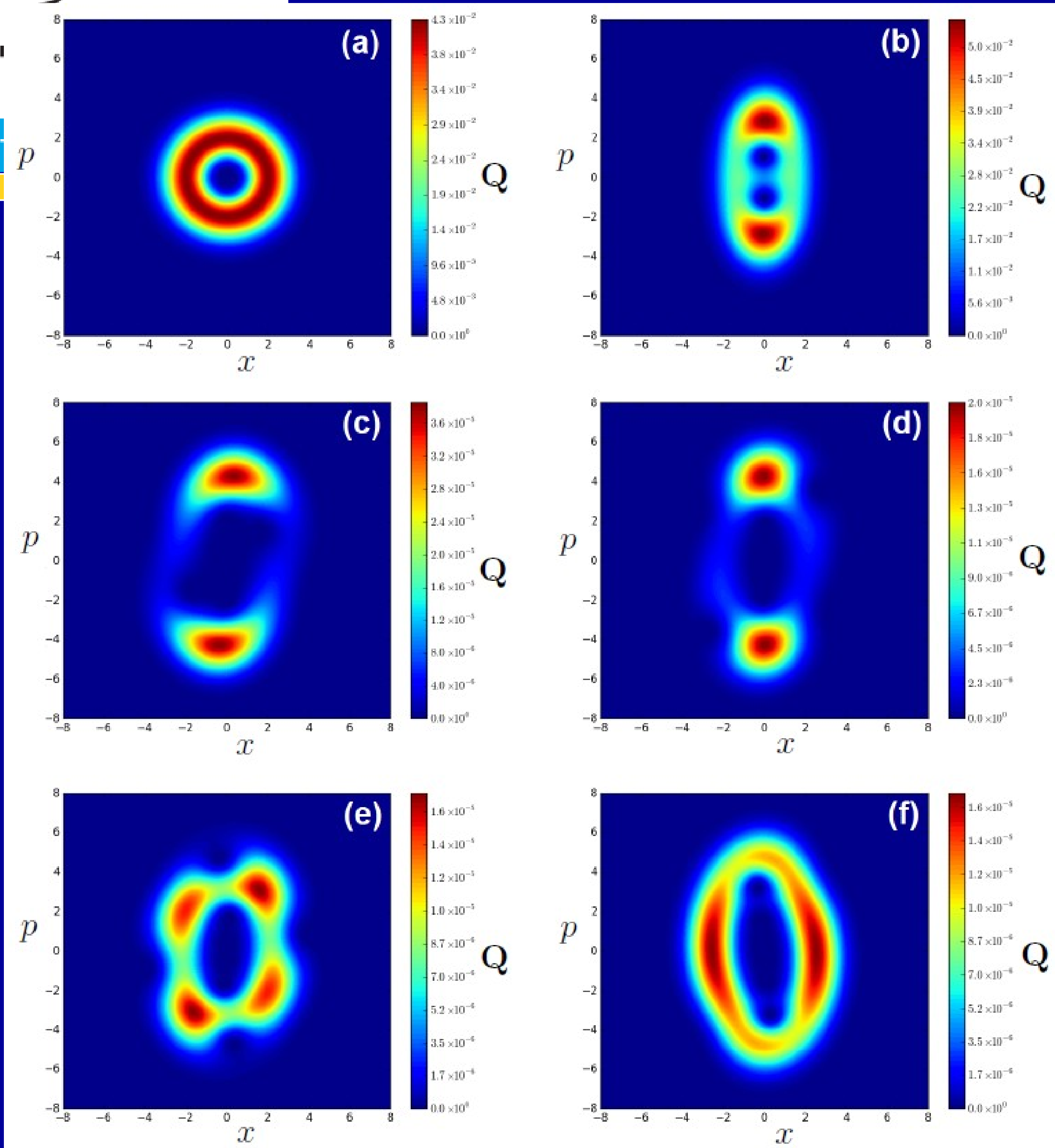


$$g_0 = 0.01\omega_m$$

Initial:
Phonon ground state
Cavity Fock state n=100

After ¼, ½, ¾, 1 period

Quantum states



$$g_0 = 0.01\omega_m$$

Initial:
Phonon Fock state n=2
Cavity coherent state

$$\alpha = \sqrt{40}$$

After 0, 1.5, 130. 260, 260.25, 261 mechanical periods

How to measure zero-point fluctuations?

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\sqrt{\omega_m^2 + 4g_0\omega_m\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right)}\left(b^{\dagger}b + \frac{1}{2}\right)$$

Frequency is shifted even of there are no photons in the cavity:

$$\tilde{\omega}_m = \sqrt{\omega_m^2 + 2g_0\omega_m} \approx \omega_m + g_0$$

Can be measured by putting the membrane first in the middle and then in a generic position

(can be generalized to many cavity modes)

J. D. P. Machado, R.J. Slooter, and YMB, Phys. Rev. A **99**, 053801 (201

Driven cavity

Rotating wave approximation:

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_{m}\hat{b}^{\dagger}\hat{b} + 2\hbar g_{0}\hat{a}^{\dagger}\hat{a}b^{\dagger}b$$

Solving: master equation for the Q-function

$$Q_{n}(\alpha) = \frac{1}{\pi}\langle n\alpha | \rho | n\alpha \rangle$$

Phonons

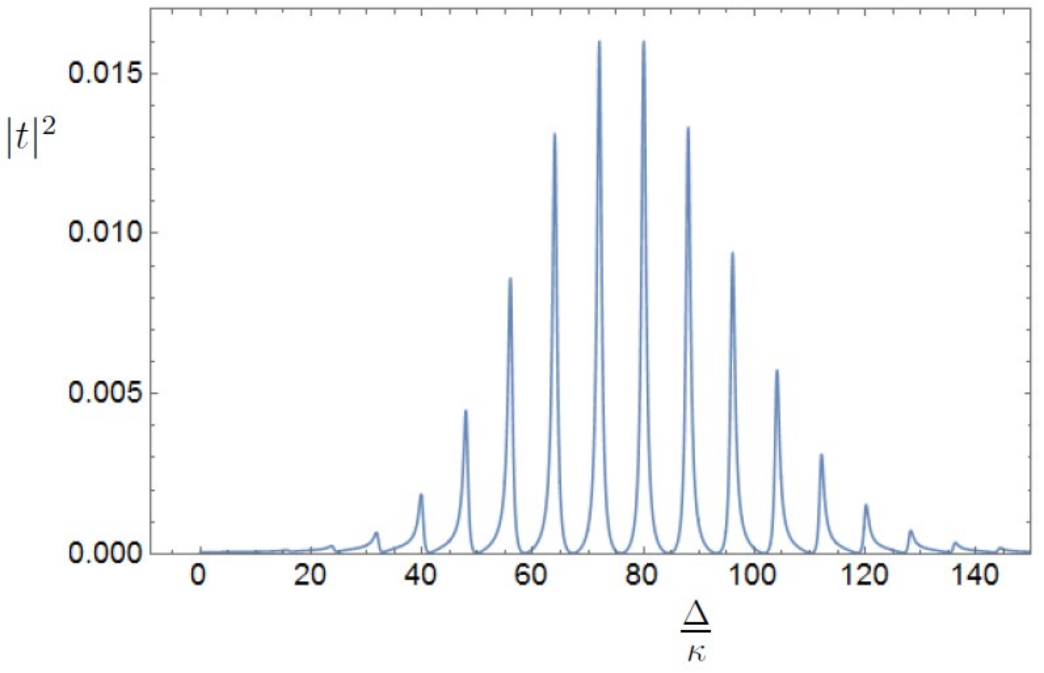
Photons

Phonon statistics

Intracavity field amplitude (stationary state):

$$\langle \hat{a} \rangle = \sum_n \frac{E p_n}{\frac{\kappa}{2} - i\left(\Delta - 2 g_0 n\right)} \qquad \Delta = \omega_{dr} - \omega_{cav} - g_0$$

Transmission:

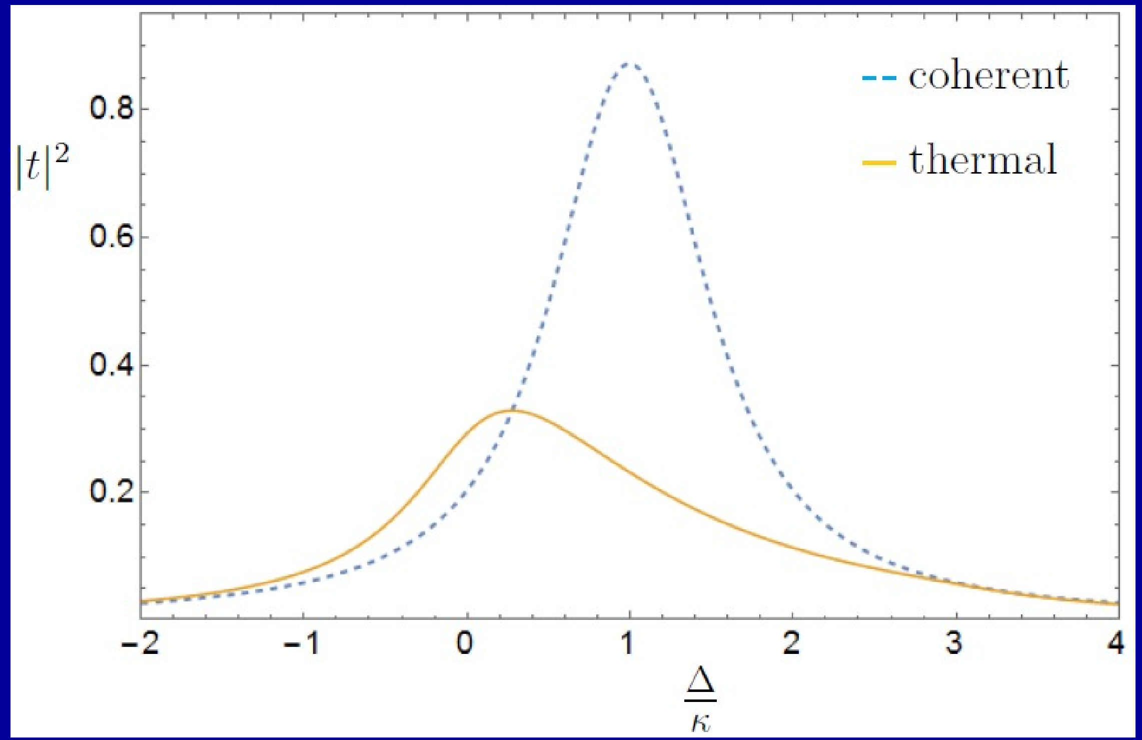


(Need single-photon strong coupling)

Phonon state

Multi-photon strong coupling: Can distinguish the phonon state and estimate the temperature

Transmission:



Conclusions

- Collapse and revivals
- Squeezing and non-trivial quantum states
- Measurements of zero-point fluctuations
- Driven cavity: Phonon statistics and phonon state