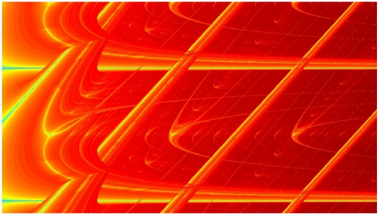
Complexity in nonlinear delay dynamics for chimera states

Laurent Larger

FEMTO-ST institute / Optics Dpt
CNRS / University Bourgogne Franche-Comté
Besançon, France



May 8, 2019 / Trieste, Italy
ICPT School and Workshop on
Patterns of Synchrony: Chimera States and Beyond

Complexity in nonlinear delay dynamics for chimera states

Laurent Larger

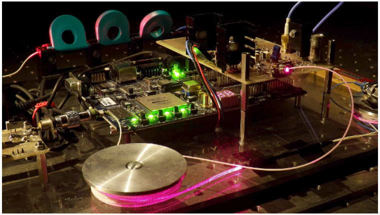
FEMTO-ST institute / Optics Dpt
CNRS / University Bourgogne Franche-Comté
Besançon, France

May 8, 2019 / Trieste, Italy
ICPT School and Workshop on
Patterns of Synchrony: Chimera States and Beyond

Complexity in nonlinear delay dynamics for chimera states

Laurent Larger

FEMTO-ST institute / Optics Dpt
CNRS / University Bourgogne Franche-Comté
Besançon, France



May 8, 2019 / Trieste, Italy
ICPT School and Workshop on
Patterns of Synchrony: Chimera States and Beyond

Many collaborators, many disciplines

FEMTO-ST Y.K. Chembo, M. Jacquot, D. Brunner, J.M. Dudley
PhD students: R. Martinenghi, B. Penkovskyi, B. Marquez

LMB J.-P. Ortega (Sankt Gallen), L. Grigoryeva (Konstanz)

TU Berlin E. Schöll, Y. Maistrenko, R. Levchenko

IFISC I. Fischer, P. Colet, C.R. Mirasso, M.C. Soriano
PhD student: R.M. Nguimdo, N. Oliver

ULB T. Erneux
PhD student: L. Weicker

U. Maryland R. Roy, T.E. Murphy, Y.K. Chembo
PhD student: J.D. Hart

VUB J. Danckaert, G. Van der Sande
PhD student: L. Appeltant

IFCAS L. Pesquera
PhD student: S. Ortin

Take-home message

Delay dynamics can be lovely simple in their equation of motion. . .

Take-home message

Delay dynamics can be lovely simple in their equation of motion. . .

. . . They can also be amazingly complex in their solutions

Outline

Introduction

NLDDE in theory and practice

Space-Time analogy: From DDE to Chimera

DDE Apps: chaos communications, μwave radar, photonic AI

Hidden bonus slides

How familiar are we with delays?

How familiar are we with delays?

Actually every day, everywhere!

- Living systems (population dynamics, blood cell regulation mechanisms, people reaction after perception and neural system processing,. . .)

How familiar are we with delays?

Actually every day, everywhere!

- Living systems (population dynamics, blood cell regulation mechanisms, people reaction after perception and neural system processing,. . .)
- Traffic jam, accordeon car flow



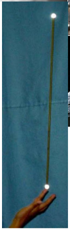


How familiar are we with delays?

Actually every day, everywhere!

- Living systems (population dynamics, blood cell regulation mechanisms, people reaction after perception and neural system processing,. . .)
- Traffic jam, accordeon car flow
- Distant control of satelites or rockets in space

How familiar are we with delays?



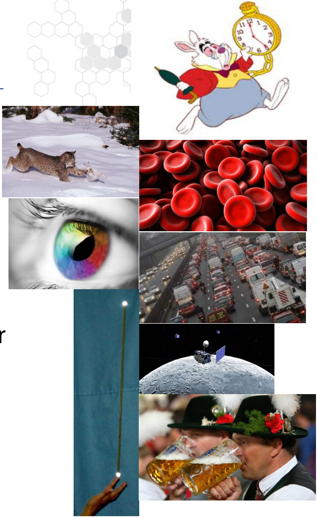
Actually every day, everywhere!

- Living systems (population dynamics, blood cell regulation mechanisms, people reaction after perception and neural system processing,. . .)
- Traffic jam, accordeon car flow
- Distant control of satelites or rockets in space
- Game of vertical stick control at the tip of a finger

How familiar are we with delays?

Actually every day, everywhere!

- Living systems (population dynamics, blood cell regulation mechanisms, people reaction after perception and neural system processing,. . .)
- Traffic jam, accordeon car flow
- Distant control of satelites or rockets in space
- Game of vertical stick control at the tip of a finger
- Human stand-up position control (and effects of increased perception delay after alcoolic drinks)



How familiar are we with delays?

Actually every day, everywhere!

- Living systems (population dynamics, blood cell regulation mechanisms, people reaction after perception and neural system processing,. . .)
- Traffic jam, accordeon car flow
- Distant control of satelites or rockets in space
- Game of vertical stick control at the tip of a finger
- Human stand-up position control (and effects of increased perception delay after alcoolic drinks)

How familiar are we with delays?

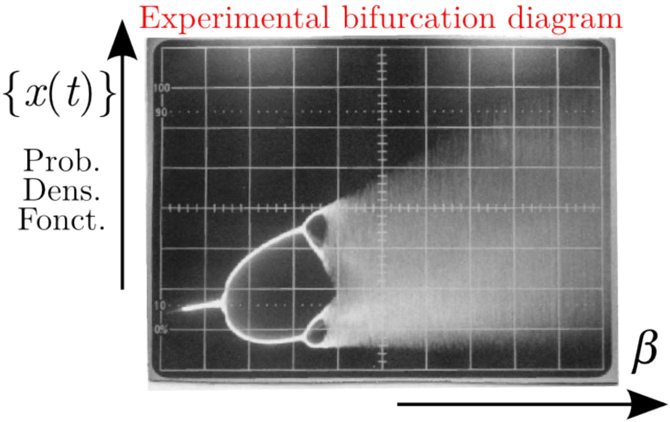
Actually every day, everywhere!

- Living systems (population dynamics, blood cell regulation mechanisms, people reaction after perception and neural system processing,. . .)
- Traffic jam, accordeon car flow
- Distant control of satelites or rockets in space
- Game of vertical stick control at the tip of a finger
- Human stand-up position control (and effects of increased perception delay after alcoolic drinks)
- Hot and cold oscillations at shower start

. . . Any time when information transport occurs (at finite speed), thus resulting in longer propagation time compared to intrinsic dynamical time scales

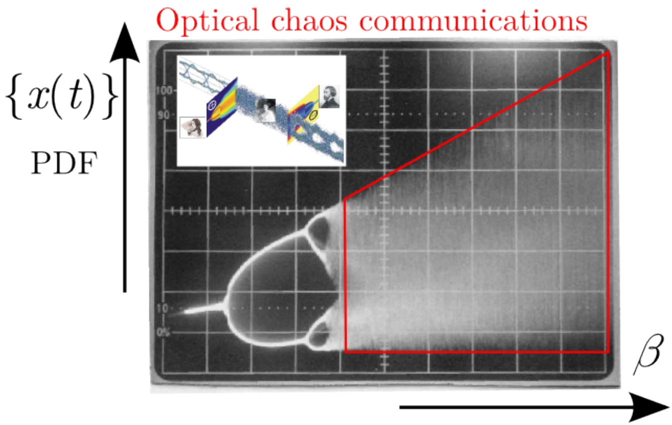
Delay equations, complexity & apps

$$\varepsilon \dot{x}(t) = -x(t) + \beta \sin^2[x(t-1) + \Phi_0]$$



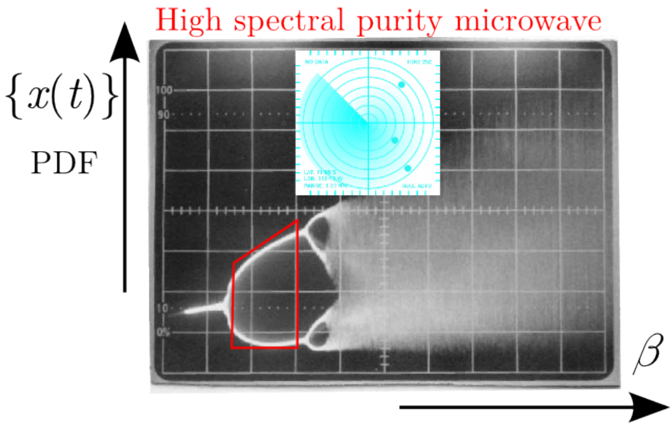
Delay equations, complexity & apps

$$\varepsilon \dot{x}(t) = -x(t) + \beta \sin^2[x(t-1) + \Phi_0]$$



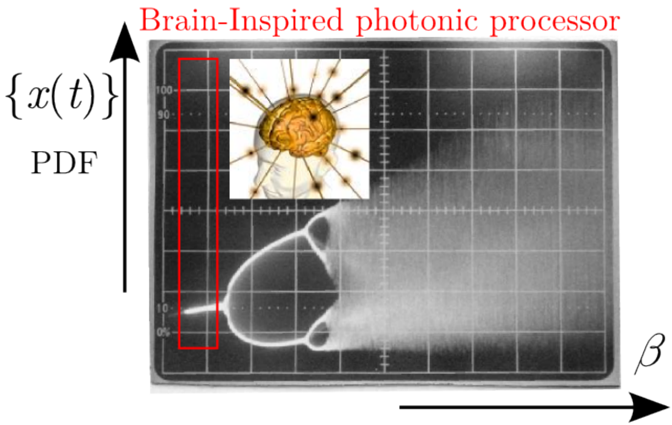
Delay equations, complexity & apps

$$\varepsilon \dot{x}(t) = -x(t) + \beta \sin^2[x(t-1) + \Phi_0]$$



Delay equations, complexity & apps

$$\varepsilon \dot{x}(t) = -x(t) + \beta \sin^2[x(t-1) + \Phi_0]$$



Outline

Introduction

NLDDE in theory and practice

NLDDE modeling through signal theory

Implementation of NLDDE in Photonic

Space-Time analogy: From DDE to Chimera

DDE Apps: chaos communications, μwave radar, photonic AI

Hidden bonus slides

NLDDE modeling through signal theory

Linear first order scalar dynamics

$$\tau \frac{\mathrm{d}x}{\mathrm{d}t}(t) + x(t) = 0, \qquad \tau\text{: response time}$$

$$\dot{x} = -\gamma \cdot x, \qquad \gamma = 1/\tau\text{: rate of change}$$

Simplest modeling of the un-avoidable continuous time (finite speed, or rate) physical transients

NLDDE modeling through signal theory

Linear first order scalar dynamics

$$\tau \frac{\mathrm{d}x}{\mathrm{d}t}(t) + x(t) = 0, \qquad \tau\text{: response time}$$

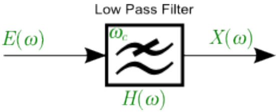
$$\dot{x} = -\gamma \cdot x, \qquad \gamma = 1/\tau\text{: rate of change}$$

Simplest modeling of the un-avoidable continuous time (finite speed, or rate) physical transients

Time and Fourier domains (FT$\equiv$ Fourier Transform)

$$H(\omega) = \frac{H_0}{1+i\omega\tau} = \frac{X(\omega)}{E(\omega)}$$

with $X(\omega)$ =FT$[x(t)]$, and $E(\omega)$ =FT$[e(t)]$, & $\omega_c = 1/\tau$



NLDDE modeling through signal theory

Linear first order scalar dynamics

$$\tau \frac{dx}{dt}(t) + x(t) = 0, \qquad \tau\text{: response time}$$

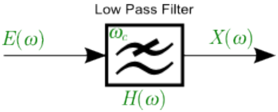
$$\dot{x} = -\gamma \cdot x, \qquad \gamma = 1/\tau\text{: rate of change}$$

Simplest modeling of the un-avoidable continuous time (finite speed, or rate) physical transients

Time and Fourier domains (FT$\equiv$ Fourier Transform)

$$H(\omega) = \frac{H_0}{1+i\omega\tau} = \frac{X(\omega)}{E(\omega)}$$

with $X(\omega)$ =FT$[x(t)]$, and $E(\omega)$ =FT$[e(t)]$, & $\omega_c = 1/\tau$



$$(1 + i\omega\tau) \cdot X(\omega) = H_0 \cdot E(\omega) \qquad \xrightarrow{\text{FT}^{-1}}$$

NLDDE modeling through signal theory

Linear first order scalar dynamics

$$\tau \frac{\mathrm{d}x}{\mathrm{d}t}(t) + x(t) = 0, \qquad \tau\text{: response time}$$

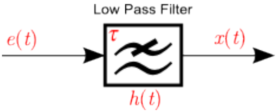
$$\dot{x} = -\gamma \cdot x, \qquad \gamma = 1/\tau\text{: rate of change}$$

Simplest modeling of the un-avoidable continuous time (finite speed, or rate) physical transients

Time and Fourier domains (FT$\equiv$ Fourier Transform)

$$H(\omega) = \frac{H_0}{1+i\omega\tau} = \frac{X(\omega)}{E(\omega)}$$

with $X(\omega)$ =FT$[x(t)]$, and $E(\omega)$ =FT$[e(t)]$, & $\omega_c = 1/\tau$



$$(1 + i\omega\tau) \cdot X(\omega) = H_0 \cdot E(\omega) \quad \xrightarrow{\mathrm{FT}^{-1}} \quad x(t) + \tau \frac{\mathrm{d}x}{\mathrm{d}t}(t) = H_0 \cdot e(t)$$

(remember FT$^{-1}[i\omega \times (\cdot)] = \frac{\mathrm{d}}{\mathrm{d}t}FT^{-1}[(\cdot)]$)

NLDDE modeling through signal theory

Linear first order scalar dynamics

$$\tau\,\frac{\mathrm{d}x}{\mathrm{d}t}(t) + x(t) = 0, \qquad \tau\text{: response time}$$

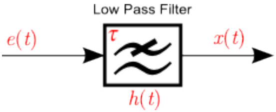
$$\dot{x} = -\gamma \cdot x, \qquad \gamma = 1/\tau\text{: rate of change}$$

Simplest modeling of the un-avoidable continuous time (finite speed, or rate) physical transients

Time and Fourier domains (FT$\equiv$ Fourier Transform)

$$H(\omega) = \frac{H_0}{1+i\omega\tau} = \frac{X(\omega)}{E(\omega)}$$

with $X(\omega)$ =FT$[x(t)]$, and $E(\omega)$ =FT$[e(t)]$, & $\omega_c = 1/\tau$



$$(1+i\omega\tau)\cdot X(\omega) = H_0 \cdot E(\omega) \quad \xrightarrow{\mathrm{FT}^{-1}} \quad x(t) + \tau\,\frac{\mathrm{d}x}{\mathrm{d}t}(t) = H_0 \cdot e(t)$$

(remember FT$^{-1}[i\omega \times (\cdot)] = \frac{\mathrm{d}}{\mathrm{d}t}FT^{-1}[(\cdot)]$)

$h(t)$ =FT$^{-1}[H(\omega)]$ [(causal) impulse reponse] $\rightarrow$ $x(t) = \int_{-\infty}^{t} h(t-\xi)\cdot e(\xi)\,\mathrm{d}\xi$

Solutions, initial conditions, phase space

Autonomous case ($e(t) = e_0$, $\Leftrightarrow e \equiv 0$ with $z = x - e_0$)

$$\tau\,\dot{x} + x = 0,$$
$$\Rightarrow x(t) = x_0\, e^{-t/\tau} = x_0\, e^{-\gamma t},$$

0: (dead) fixed point ($\dot{x} = 0$)

γ: convergence rate $\to 0$, $\forall x_0$

$-\gamma : < 0$ eigenvalue (stable);

Size of the init. cond., dim $x_0 = 1$ $\Rightarrow$ 1D dynamics (or phase space)

Solutions, initial conditions, phase space

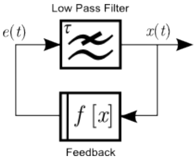
Autonomous case ($e(t) = e_0$, $\Leftrightarrow e \equiv 0$ with $z = x - e_0$)

$$\tau\,\dot{x} + x = 0, \qquad \text{0: (dead) fixed point } (\dot{x} = 0)$$

$$\Rightarrow x(t) = x_0\, e^{-t/\tau} = x_0\, e^{-\gamma t}, \qquad \gamma\text{: convergence rate} \to 0,\ \forall x_0$$

$-\gamma :< 0$ eigenvalue (stable); Size of the init. cond., dim $x_0 = 1$ $\Rightarrow$ 1D dynamics (or phase space)

Feedback ($e(t) = f[x(t)]$): stability, multi-stability



Fixed point(s): $\{x_F \mid x = f[x]\}$

(Graphics: intersect(s) between $y = f[x]$ and $y = x$)

Stability @ x_F: linearization for $x(t) - x_F = \delta x(t) \ll 1$,

$$f[x] = x_F + \delta x \cdot f'[x_F] \quad \Rightarrow \quad \dot{\delta x} = -\gamma(1 - f'_{x_F}) \cdot \delta x = -\gamma_{\text{fb}} \cdot \delta x$$

$f'_{x_F} < 0 \equiv$ negative feedback, speed up the rate; $f'_{x_F} > 0$, slow down the rate, possibly unstable if > 1

Solutions, initial conditions, phase space

Autonomous case ($e(t) = e_0$, $\Leftrightarrow e \equiv 0$ with $z = x - e_0$)

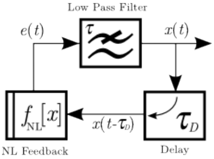
$$\tau\, \dot{x} + x = 0, \qquad \Rightarrow x(t) = x_0\, e^{-t/\tau} = x_0\, e^{-\gamma t},$$

0: (dead) fixed point ($\dot{x} = 0$)

γ: convergence rate $\to 0$, $\forall x_0$

$-\gamma$: < 0 eigenvalue (stable); Size of the init. cond., dim $x_0 = 1$ $\Rightarrow$ 1D dynamics (or phase space)

Feedback ($e(t) = f[x(t)]$): stability, multi-stability



Fixed point(s): $\{x_F \mid x = f[x]\}$

(Graphics: intersect(s) between $y = f[x]$ and $y = x$)

Stability @ x_F: linearization for $x(t) - x_F = \delta x(t) \ll 1$,

$$f[x] = x_F + \delta x \cdot f'[x_F] \quad \Rightarrow \quad \dot{\delta x} = -\gamma(1 - f'_{x_F}) \cdot \delta x = -\gamma_{\text{fb}} \cdot \delta x$$

$f'_{x_F} < 0 \equiv$ negative feedback, speed up the rate; $f'_{x_F} > 0$, slow down the rate, possibly unstable if > 1

Delayed feedback ($e(t) = f[x(t - \tau_D)]$): $\infty-$dimensional

Fixed point(s): $\{x_F \mid x = f[x]\}$

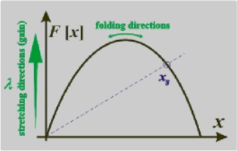
Stability: $\delta x(t) = a \cdot e^{\sigma t}$, eigenvalues: $\{\sigma \in \mathbb{C} \mid 1 + \sigma\tau = e^{-\sigma\tau_D} \cdot f'_{x_F}\}$,

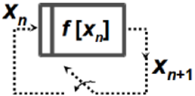
Size of initial conditions: $\{x(t),\ t \in [-\tau_D; 0]\}$ $\Rightarrow \infty$D **phase space**

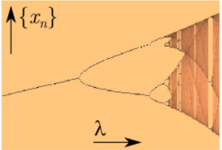
Discrete time dynamics: Mapping

Large delay case ($\tau/\tau_D \to 0$): simplified to a 1D (Map)!!!

- Logistic map (feedback + sample & hold) $x_{n+1} = \lambda x_n(1 - x_n)$



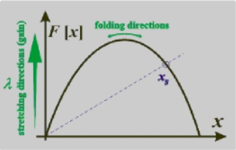


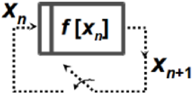


Discrete time dynamics: Mapping

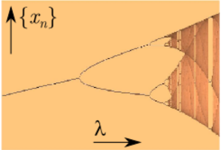
Large delay case ($\tau/\tau_D \to 0$): simplified to a 1D (Map)!!!

- Logistic map (feedback + sample & hold) $x_{n+1} = \lambda x_n(1 - x_n)$

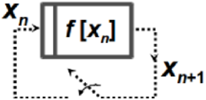


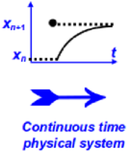


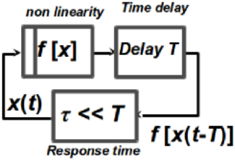
Delay: step n to $n+1$

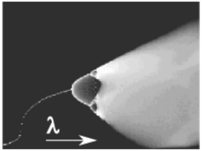


- DDE (large, but finite, delay with a feedback loop)





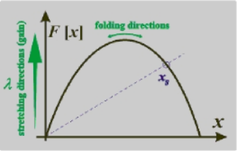


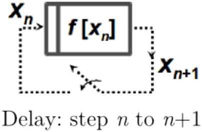


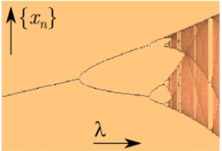
Discrete time dynamics: Mapping

Large delay case ($\tau/\tau_D \to 0$): simplified to a 1D (Map)!!!

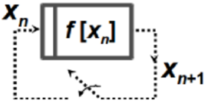
- Logistic map (feedback + sample & hold) $x_{n+1} = \lambda x_n(1 - x_n)$

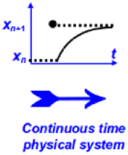


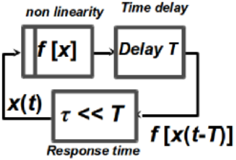


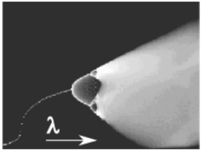


- DDE (large, but finite, delay with a feedback loop)









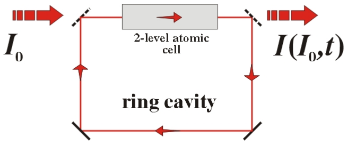
- Similarities, but still strong differences (**singular** limit map)

Design tips for an NLDDE in Optics

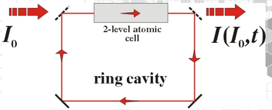
Concepts of the first chaotic optical setup

A closed loop oscillator architecture:

- All-optical Ikeda ring cavity



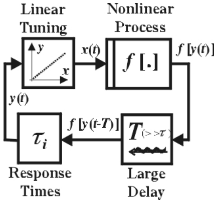
Design tips for an NLDDE in Optics



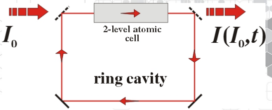
Concepts of the first chaotic optical setup

A closed loop oscillator architecture:

- All-optical Ikeda ring cavity
- Generic bloc diagram setup



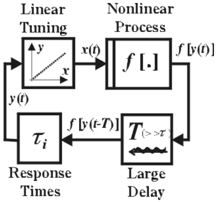
Design tips for an NLDDE in Optics



Concepts of the first chaotic optical setup

A closed loop oscillator architecture:

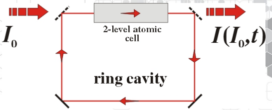
- All-optical Ikeda ring cavity
- Generic bloc diagram setup



Modeling, DDE

$$\tau \frac{dx}{dt}(t) = -x(t) + F_{NL}[x(t - \tau_D)]$$

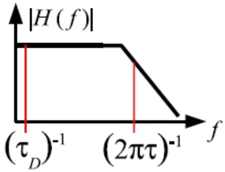
Design tips for an NLDDE in Optics



Concepts of the first chaotic optical setup

A closed loop oscillator architecture:

- All-optical Ikeda ring cavity
- Generic bloc diagram setup



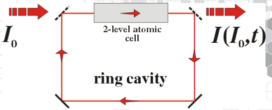
Modeling, DDE

$$\tau \frac{\mathrm{d}x}{\mathrm{d}t}(t) = -x(t) + F_{\mathrm{NL}}[x(t-\tau_D)]$$

- Instantaneous part (<u>linear</u> filter): atomic level life time, Kerr time scale

$$\tau \frac{\mathrm{d}x}{\mathrm{d}t}(t) + x(t) = z(t) \quad \leftrightarrow \quad H(f) = \mathrm{FT}[h(t)] = \frac{X(f)}{Z(f)} = \frac{1}{1 + i2\pi f\tau}$$

Design tips for an NLDDE in Optics



Concepts of the first chaotic optical setup

A closed loop oscillator architecture:

- All-optical Ikeda ring cavity
- Generic bloc diagram setup

Modeling, DDE

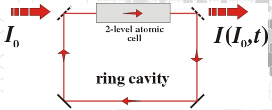
$$\tau \frac{\mathrm{d}x}{\mathrm{d}t}(t) = -x(t) + F_{\mathrm{NL}}[x(t-\tau_D)]$$

- Instantaneous part (linear filter): atomic level life time, Kerr time scale

$$\tau \frac{\mathrm{d}x}{\mathrm{d}t}(t) + x(t) = z(t) \quad \leftrightarrow \quad H(f) = \mathrm{FT}[h(t)] = \frac{X(f)}{Z(f)} = \frac{1}{1 + i2\pi f \tau}$$

- Time delayed feedback: τ_D, time of flight of the light in the cavity

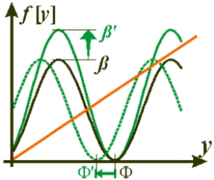
Design tips for an NLDDE in Optics



Concepts of the first chaotic optical setup

A closed loop oscillator architecture:

- All-optical Ikeda ring cavity
- Generic bloc diagram setup



Modeling, DDE

$$\tau \frac{\mathrm{d}x}{\mathrm{d}t}(t) = -x(t) + F_{\mathrm{NL}}[x(t-\tau_D)]$$

- Instantaneous part (linear filter): atomic level life time, Kerr time scale

$$\tau \frac{\mathrm{d}x}{\mathrm{d}t}(t) + x(t) = z(t) \quad \leftrightarrow \quad H(f) = \mathrm{FT}[h(t)] = \frac{X(f)}{Z(f)} = \frac{1}{1 + i2\pi f \tau}$$

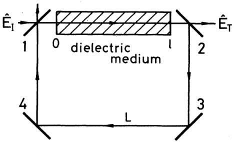
- Time delayed feedback: τ_D, time of flight of the light in the cavity
- Nonlinear delayed driving force: input and feedback interference

$$z(t) = F_{\mathrm{NL}}[x(t-\tau_D)] = \beta \cos^2[x(t-\tau_D) + \Phi]$$

A paradigm for the study of NLDDE complexity

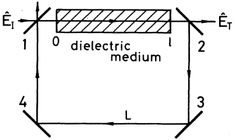
From an Optics Gedanken experiment. . .
. . . to flexible and powerful photonic systems

- The Ikeda ring cavity
  (Ikeda, *Opt.Commun. 1979*).

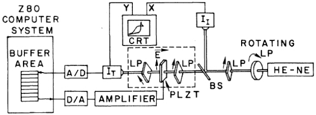


A paradigm for the study of NLDDE complexity

From an Optics Gedanken experiment...
...to flexible and powerful photonic systems

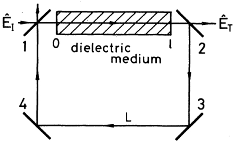


- The Ikeda ring cavity
  (Ikeda, *Opt.Commun. 1979*).
- Bulk electro-optic
  (Gibbs *et al.*, *Phys.Rev.Lett. 1981*).



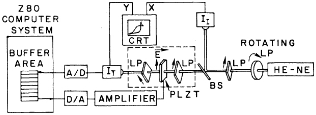
A paradigm for the study of NLDDE complexity

From an Optics Gedanken experiment. . .
. . . to flexible and powerful photonic systems



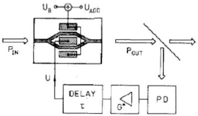
- The Ikeda ring cavity

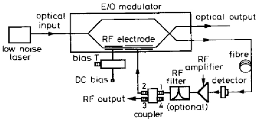
  (Ikeda, *Opt.Commun. 1979*).



- Bulk electro-optic

  (Gibbs *et al.*, *Phys.Rev.Lett. 1981*).



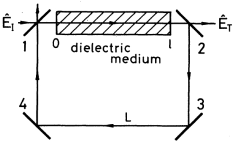


- Integrated optics Mach-Zehnder

  (Neyer and Voges, *IEEE J.Quant.Electron.* 1982;
  Yao and Maleki, *Electr. Lett.* 1994).

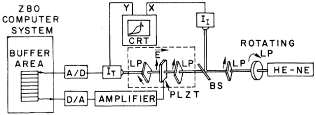
A paradigm for the study of NLDDE complexity

From an Optics Gedanken experiment. . .
. . . to flexible and powerful photonic systems



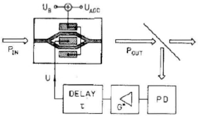
- The Ikeda ring cavity

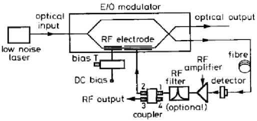
  (Ikeda, *Opt.Commun. 1979*).



- Bulk electro-optic

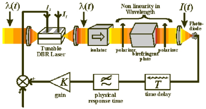
  (Gibbs *et al.*, *Phys.Rev.Lett. 1981*).

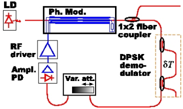




- Integrated optics Mach-Zehnder

  (Neyer and Voges, *IEEE J.Quant.Electron.* 1982;
  Yao and Maleki, *Electr. Lett.* 1994).





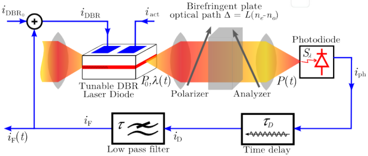
- Wavelength & EO intensity (or phase) delay dynamics

  (Larger *et al.*, *IEEE J.Quant.Electron.* 1998;
  Lavrov *et al.*, *Phys. Rev. E* 2009).

Laser wavelength dynamics

2-wave imbalanced interferometer:

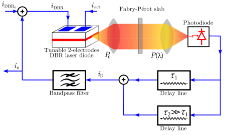
$$f_{\mathrm{NL}}(x) = \beta \sin^2[x + \Phi]$$

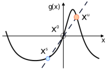


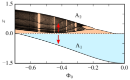
Laser wavelength dynamics

2-wave imbalanced interferometer:

$$f_{\mathsf{NL}}(x) = \beta \sin^2[x + \Phi]$$







Fabry-Pérot interferometer:

$$f_{\mathsf{NL}}(x) = \beta/[1 + m \cdot \sin^2(x + \Phi)]$$

with $x = \pi\Delta/\lambda$

Laser wavelength dynamics

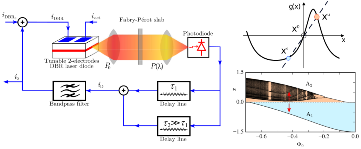
2-wave imbalanced interferometer:

$$f_{\mathrm{NL}}(x) = \beta \sin^2[x + \Phi]$$

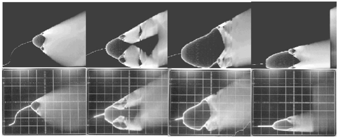
Fabry-Pérot interferometer:

$$f_{\mathrm{NL}}(x) = \beta / [1 + m \cdot \sin^2(x + \Phi)]$$

with $x = \pi\Delta/\lambda$



- **Nicely matched exp. & num. bifurcation diagrams** (increasing Φ_0)
- **Record non linearity strength** up to 14 extrema



Laser wavelength dynamics

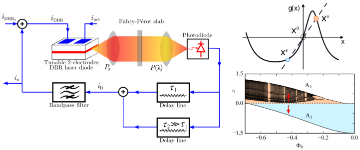
2-wave imbalanced interferometer:

$$f_{\mathrm{NL}}(x) = \beta \sin^2[x + \Phi]$$

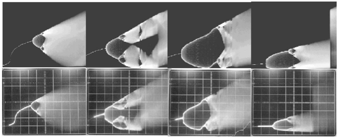
Fabry-Pérot interferometer:

$$f_{\mathrm{NL}}(x) = \beta / [1 + m \cdot \sin^2(x + \Phi)]$$

with $x = \pi \Delta / \lambda$

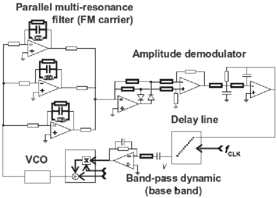


- **Nicely matched exp. & num. bifurcation diagrams** (increasing Φ_0)



- **Record non linearity strength** up to 14 extrema
- **FM chaos:** operating principles transfered to electronics

  $\rightarrow$ 1st bandpass delay dynamics



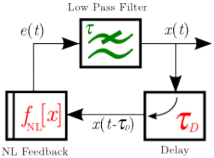
Summary about DDE physics & concepts

Mackey–Glass- or Ikeda-like DDE

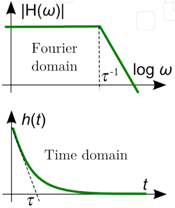
$$\tau \cdot \frac{\mathrm{d}x}{\mathrm{d}t}(t) = -x(t) + f_{\mathrm{NL}}[x(t - \tau_D)]$$

Summary about DDE physics & concepts

Mackey–Glass- or Ikeda-like DDE



$$\tau \cdot \frac{\mathrm{d}x}{\mathrm{d}t}(t) = -x(t) + f_{\mathrm{NL}}[x(t-\tau_D)]$$

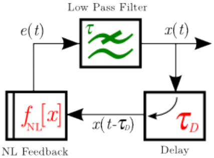


Non-delayed (instantaneous) terms:

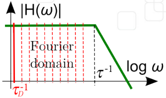
- Linear differential equation, rate of change $\gamma = 1/\tau$
- Stable linear Fourier filter, frequency cut-off $(2\pi\tau)^{-1}$
- A few degrees of freedom $\equiv$ filter or diff.eq. order

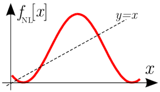
Summary about DDE physics & concepts

Mackey–Glass- or Ikeda-like DDE



$$\tau \cdot \frac{\mathrm{d}x}{\mathrm{d}t}(t) = -x(t) + f_{\mathrm{NL}}[x(t-\tau_D)]$$





Non-delayed (instantaneous) terms:

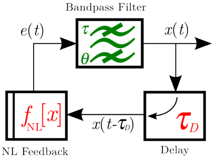
- Linear differential equation, rate of change $\gamma = 1/\tau$
- Stable linear Fourier filter, frequency cut-off $(2\pi\tau)^{-1}$
- A few degrees of freedom $\equiv$ filter or diff.eq. order

Delayed (feedback) term:

- Non-linearity (slope sign, # extrema, multi-stability),
- Delay (infinite degrees of freedom, stability)
- Large delay case, $\tau_D \gg \tau$

Summary about DDE physics & concepts

Mackey–Glass- or Ikeda-like DDE



$$\tau \cdot \frac{\mathrm{d}x}{\mathrm{d}t}(t) + \frac{1}{\theta} \int_{t_0}^{t} x(\xi)\,\mathrm{d}\xi = -x(t) + f_{NL}[x(t-\tau_D)]$$

Non-delayed (instantaneous) terms:

- Linear differential equation, rate of change $\gamma = 1/\tau$
- Stable linear Fourier filter, frequency cut-off $(2\pi\tau)^{-1}$
- A few degrees of freedom $\equiv$ filter or diff.eq. order

Delayed (feedback) term:

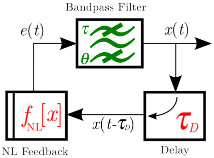
- Non-linearity (slope sign, # extrema, multi-stability),
- Delay (infinite degrees of freedom, stability)
- Large delay case, $\tau_D \gg \tau$

Unusual features for DDE models

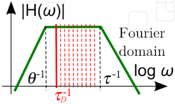
- Bandpass Fourier filter, or integro-differential delay equation

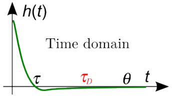
Summary about DDE physics & concepts

Mackey–Glass- or Ikeda-like DDE



$$\tau \cdot \frac{dx}{dt}(t) = -x(t) - y(t) + f_{NL}[x(t-\tau_D)]$$
$$\theta \cdot \frac{dy}{dt}(t) = x(t)$$





Non-delayed (instantaneous) terms:

- Linear differential equation, rate of change $\gamma = 1/\tau$
- Stable linear Fourier filter, frequency cut-off $(2\pi\tau)^{-1}$
- A few degrees of freedom $\equiv$ filter or diff.eq. order

Delayed (feedback) term:

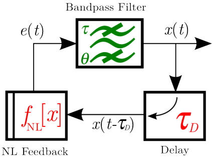
- Non-linearity (slope sign, # extrema, multi-stability),
- Delay (infinite degrees of freedom, stability)
- Large delay case, $\tau_D \gg \tau$

Unusual features for DDE models

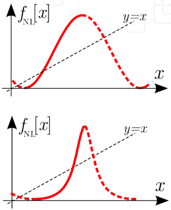
- Bandpass Fourier filter, or integro-differential delay equation

Summary about DDE physics & concepts

Mackey–Glass- or Ikeda-like DDE



$$\tau \cdot \frac{dx}{dt}(t) = -x(t) - y(t) + f_{NL}[x(t-\tau_D)]$$
$$\theta \cdot \frac{dy}{dt}(t) = x(t)$$



Non-delayed (instantaneous) terms:

- Linear differential equation, rate of change $\gamma = 1/\tau$
- Stable linear Fourier filter, frequency cut-off $(2\pi\tau)^{-1}$
- A few degrees of freedom $\equiv$ filter or diff.eq. order

Delayed (feedback) term:

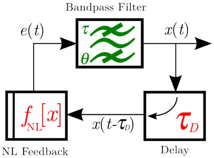
- Non-linearity (slope sign, # extrema, multi-stability),
- Delay (infinite degrees of freedom, stability)
- Large delay case, $\tau_D \gg \tau$

Unusual features for DDE models

- Bandpass Fourier filter, or integro-differential delay equation
- Positive slope operating point

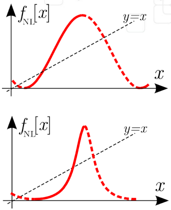
Summary about DDE physics & concepts

Mackey–Glass- or Ikeda-like DDE



$$\tau \cdot \frac{\mathrm{d}x}{\mathrm{d}t}(t) \quad = \quad -x(t) - y(t) + f_{\mathrm{NL}}[x(t-\tau_D)]$$

$$\theta \cdot \frac{\mathrm{d}y}{\mathrm{d}t}(t) \quad = \quad x(t)$$



Non-delayed (instantaneous) terms:

- Linear differential equation, rate of change $\gamma = 1/\tau$
- Stable linear Fourier filter, frequency cut-off $(2\pi\tau)^{-1}$
- A few degrees of freedom $\equiv$ filter or diff.eq. order

Delayed (feedback) term:

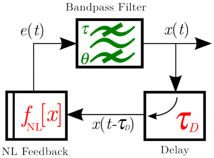
- Non-linearity (slope sign, # extrema, multi-stability),
- Delay (infinite degrees of freedom, stability)
- Large delay case, $\tau_D \gg \tau$

Unusual features for DDE models

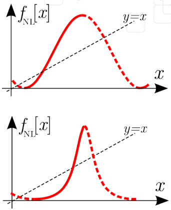
- Bandpass Fourier filter, or integro-differential delay equation
- Positive slope operating point
- Carved nonlinear function profile (e.g. min/max assymmetry)

Summary about DDE physics & concepts

Mackey–Glass- or Ikeda-like DDE



$$\tau \cdot \frac{\mathrm{d}x}{\mathrm{d}t}(t) = -x(t) - y(t) + f_{\mathrm{NL}}[x(t-\tau_D)]$$
$$\theta \cdot \frac{\mathrm{d}y}{\mathrm{d}t}(t) = x(t)$$



Non-delayed (instantaneous) terms:

- Linear differential equation, rate of change $\gamma = 1/\tau$
- Stable linear Fourier filter, frequency cut-off $(2\pi\tau)^{-1}$
- A few degrees of freedom $\equiv$ filter or diff.eq. order

Delayed (feedback) term:

- Non-linearity (slope sign, # extrema, multi-stability),
- Delay (infinite degrees of freedom, stability)
- Large delay case, $\tau_D \gg \tau$

Unusual features for DDE models

- Bandpass Fourier filter, or integro-differential delay equation
- Positive slope operating point
- Carved nonlinear function profile (e.g. min/max assymmetry)
- Multiple delay architectures

Outline

Space-Time representation of DDE

Normalization wrt Delay τ_D: $s = t/\tau_D$, and $\varepsilon = \tau/\tau_D$

$$\varepsilon\,\dot{x}(s) = -x(s) + f_{\mathrm{NL}}[x(s-1)], \quad \text{where} \quad \dot{x} = \frac{\mathrm{d}x}{\mathrm{d}s}.$$

Large delay case: $\varepsilon \ll 1$, potentially high dimensional attractor

$\infty-$dimensional phase space, initial condition: $x(s), s \in [-1, 0]$

Space-Time representation of DDE

Normalization wrt Delay τ_D: $s = t/\tau_D$, and $\varepsilon = \tau/\tau_D$

$$\varepsilon\,\dot{x}(s) = -x(s) + f_{\mathrm{NL}}[x(s-1)], \quad \text{where} \quad \dot{x} = \frac{\mathrm{d}x}{\mathrm{d}s}.$$

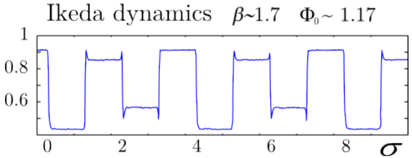
Large delay case: $\varepsilon \ll 1$, potentially high dimensional attractor

$\infty-$dimensional phase space, initial condition: $x(s), s \in [-1, 0]$

Space-time representation

- Virtual space variable σ,

  $\sigma \in [0; 1+\gamma]$ with $\gamma = O(\varepsilon)$.



Space-Time representation of DDE

Normalization wrt Delay τ_D: $s = t/\tau_D$, and $\varepsilon = \tau/\tau_D$

$$\varepsilon\, \dot{x}(s) = -x(s) + f_{\mathrm{NL}}[x(s-1)], \quad \text{where} \quad \dot{x} = \frac{\mathrm{d}x}{\mathrm{d}s}.$$

Large delay case: $\varepsilon \ll 1$, potentially high dimensional attractor

$\infty-$dimensional phase space, initial condition: $x(s), s \in [-1, 0]$

Space-time representation

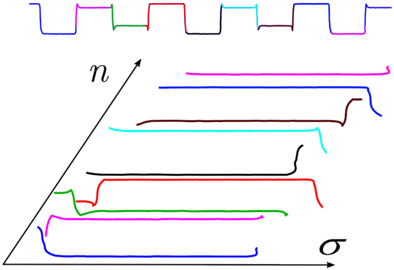
- Virtual space variable σ,

  $\sigma \in [0; 1+\gamma]$ with $\gamma = O(\varepsilon)$.

- Discrete time n

$$n \quad \rightarrow \quad (n+1)$$

$$s = n(1+\gamma) + \sigma \quad \rightarrow \quad s = (n+1)(1+\gamma) + \sigma$$



Space-Time representation of DDE

Normalization wrt Delay τ_D: $s = t/\tau_D$, and $\varepsilon = \tau/\tau_D$

$$\varepsilon\,\dot{x}(s) = -x(s) + f_{\mathrm{NL}}[x(s-1)], \quad \text{where} \quad \dot{x} = \frac{\mathrm{d}x}{\mathrm{d}s}.$$

Large delay case: $\varepsilon \ll 1$, potentially high dimensional attractor

$\infty-$dimensional phase space, initial condition: $x(s), s \in [-1, 0]$

Space-time representation

- Virtual space variable σ,

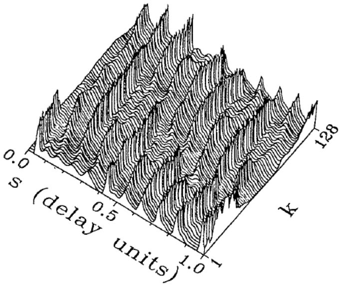
  $\sigma \in [0; 1+\gamma]$ with $\gamma = O(\varepsilon)$.

- Discrete time n

$$n \quad \rightarrow \quad (n+1)$$

$$s = n(1+\gamma) + \sigma \quad \rightarrow \quad s = (n+1)(1+\gamma) + \sigma$$

F.T. Arecchi, *et al.* Phys. Rev. A, 1992



Space-Time representation of DDE

Normalization wrt Delay τ_D: $s = t/\tau_D$, and $\varepsilon = \tau/\tau_D$

$$\varepsilon\, \dot{x}(s) = -x(s) + f_{\mathrm{NL}}[x(s-1)], \quad \text{where} \quad \dot{x} = \frac{\mathrm{d}x}{\mathrm{d}s}.$$

Large delay case: $\varepsilon \ll 1$, potentially high dimensional attractor

$\infty-$dimensional phase space, initial condition: $x(s), s \in [-1, 0]$

Space-time representation

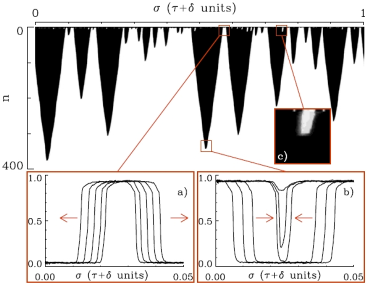
- Virtual space variable σ,

  $\sigma \in [0; 1+\gamma]$ with $\gamma = O(\varepsilon)$.

- Discrete time n

$$n \quad \rightarrow \quad (n+1)$$

$$s = n(1+\gamma) + \sigma \quad \rightarrow \quad s = (n+1)(1+\gamma) + \sigma$$



G. Giacomelli, *et al.* EPL, 2012

Space-time analogy: analytical support

Convolution product involving the linear impulse response,

$$h(t) = \mathbf{FT}^{-1}[H(\omega)]$$

$$x(s) = \int_{-\infty}^{s} h(s-\xi) \cdot f_{\mathrm{NL}}[x(\xi-1)]\, \mathrm{d}\xi \quad \text{with} \quad s = n(1+\gamma) + \sigma$$

LL, Penkovsky, Maistrenko, *Nat. Commun.* 2015, DOI: 10.1038/ncomms8752

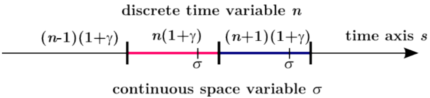
Space-time analogy: analytical support

Convolution product involving the linear impulse response,

$$h(t) = \mathbf{FT}^{-1}[H(\omega)]$$

$$x(s) = \int_{-\infty}^{s} h(s-\xi)\cdot f_{\mathsf{NL}}[x(\xi-1)]\,\mathrm{d}\xi \quad \text{with} \quad s = n(1+\gamma)+\sigma$$

...partitioning the time domain:



$$]-\infty; s] \;=\;]-\infty\,;\, n(1+\gamma)+\sigma\,] \quad \cup \quad]n(1+\gamma)+\sigma\,;\,(n+1)(1+\gamma)+\sigma\,]$$

LL, Penkovsky, Maistrenko, *Nat. Commun.* 2015, DOI: 10.1038/ncomms8752

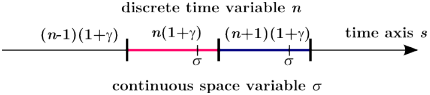
Space-time analogy: analytical support

Convolution product involving the linear impulse response,

$$h(t) = \mathbf{FT}^{-1}[H(\omega)]$$

$x(s) = \int_{-\infty}^{s} h(s-\xi)\cdot f_{\mathsf{NL}}[x(\xi-1)]\,\mathrm{d}\xi$ with $s = n(1+\gamma)+\sigma$

. . . partitioning the time domain:



$$]-\infty; s] \;=\;]-\infty\,;\, n(1+\gamma)+\sigma\,] \quad \cup \quad]n(1+\gamma)+\sigma\,;\,(n+1)(1+\gamma)+\sigma\,]$$

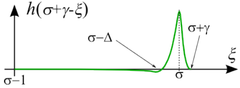
and make a change of integration variable $\xi \leftrightarrow \xi-(n+1)(1+\gamma)+\gamma$

LL, Penkovsky, Maistrenko, *Nat. Commun.* 2015, DOI: 10.1038/ncomms8752

Space-time analogy: analytical support

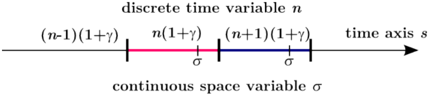
Convolution product involving the linear impulse response,

$$h(t) = \textbf{FT}^{-1}[H(\omega)]$$



$$x(s) = \int_{-\infty}^{s} h(s-\xi)\cdot f_{\mathrm{NL}}[x(\xi-1)]\,\mathrm{d}\xi \quad \text{with} \quad s = n(1+\gamma)+\sigma$$

...partitioning the time domain:



$$]-\infty; s] \; = \;]-\infty\,;\, n(1+\gamma)+\sigma\,] \quad \cup \quad]n(1+\gamma)+\sigma\,;\,(n+1)(1+\gamma)+\sigma\,]$$

and make a change of integration variable $\xi \leftrightarrow \xi-(n+1)(1+\gamma)+\gamma$

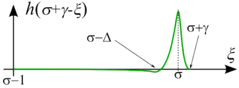
$$\Rightarrow \quad x_{n+1}(\sigma) = I_\epsilon(n,\sigma) + \int_{\sigma-1}^{\sigma+\gamma} h(\sigma+\gamma-\xi)\cdot f_{\mathrm{NL}}[x_n(\xi)]\,\mathrm{d}\xi, \quad \text{with} \quad I_\epsilon \ll x_n(\sigma)$$

LL, Penkovsky, Maistrenko, *Nat. Commun.* 2015, DOI: 10.1038/ncomms8752

Space-time analogy: analytical support

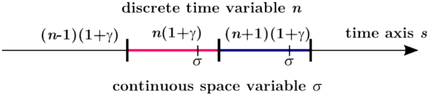
Convolution product involving the linear impulse response,

$$h(t) = \mathbf{FT}^{-1}[H(\omega)]$$



$$x(s) = \int_{-\infty}^{s} h(s-\xi) \cdot f_{\mathrm{NL}}[x(\xi-1)]\,\mathrm{d}\xi \quad \text{with} \quad s = n(1+\gamma)+\sigma$$

...partitioning the time domain:

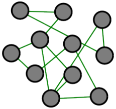


$$]-\infty;s] \ = \ ]-\infty\,;\, n(1+\gamma)+\sigma\,] \quad \cup \quad]n(1+\gamma)+\sigma\,;\,(n+1)(1+\gamma)+\sigma\,]$$

and make a change of integration variable $\xi \leftrightarrow \xi-(n+1)(1+\gamma)+\gamma$

$$\Rightarrow \quad x_{n+1}(\sigma) = I_\epsilon(n,\sigma) + \int_{\sigma-1}^{\sigma+\gamma} h(\sigma+\gamma-\xi)\cdot f_{\mathrm{NL}}[x_n(\xi)]\,\mathrm{d}\xi, \quad \text{with} \ \ I_\epsilon \ll x_n(\sigma)$$

$$\left\{ \frac{\partial\phi}{\partial t} = \omega - \int_{-\pi}^{\pi} G(x-x')\cdot\sin[\phi(x,t)-\phi(x',t)+\alpha]\,\mathrm{d}x \right\}$$

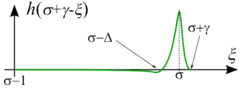


LL, Penkovsky, Maistrenko, *Nat. Commun.* 2015, DOI: 10.1038/ncomms8752

Space-time analogy: analytical support

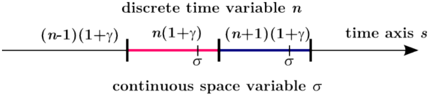
Convolution product involving the linear impulse response,

$$h(t) = \mathbf{FT}^{-1}[H(\omega)]$$



$$x(s) = \int_{-\infty}^{s} h(s-\xi)\cdot f_{\text{NL}}[x(\xi-1)]\,\mathrm{d}\xi \quad \text{with} \quad s = n(1+\gamma)+\sigma$$

…partitioning the time domain:



$$]-\infty;s] =]-\infty\,;\, n(1+\gamma)+\sigma\,] \quad \cup \quad]n(1+\gamma)+\sigma\,;\,(n+1)(1+\gamma)+\sigma\,]$$

and make a change of integration variable $\xi \leftrightarrow \xi-(n+1)(1+\gamma)+\gamma$

$$\Rightarrow \quad x_{n+1}(\sigma) = I_\epsilon(n,\sigma) + \int_{\sigma-1}^{\sigma+\gamma} h(\sigma+\gamma-\xi)\cdot f_{\text{NL}}[x_n(\xi)]\,\mathrm{d}\xi, \quad \text{with} \;\; I_\epsilon \ll x_n(\sigma)$$

$$\left\{ \frac{\partial\phi}{\partial t} = \omega - \int_{-\pi}^{\pi} G(x-x')\cdot\sin[\phi(x,t)-\phi(x',t)+\alpha]\,\mathrm{d}x \right\}$$

Remark: the NL dynamics and coupling features of each virtual oscillator are by construction identical at any position σ!!!

Chimera states. . .

Y. Kuramoto and D. Battogtokh, *Nonlinear Phenom. Complex Syst.* **5**, 380 (2002); D. M. Abrams and S. H. Strogatz, *Phys. Rev. Lett.* **93**, 174102 (2004); I. Omelchenko *et al. Phys. Rev. Lett.* **106** 234102 (2011); A. M. Hagerstrom *et al.* & M. Tinsley *et al.*, *Nat. Phys.* 8, 658 & 662 (2012)

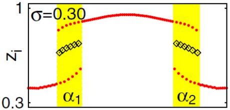
Chimera states...

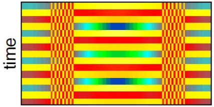
What is a Chimera state?

- Network of coupled oscillators with clusters of incongruent motions
- Predicted numerically in 2002, derived for a particular case in 2004, and 1st observed experimentally in 2012
- Not observed (initially) with local coupling, neither with global one

Y. Kuramoto and D. Battogtokh, *Nonlinear Phenom. Complex Syst.* **5**, 380 (2002); D. M. Abrams and S. H. Strogatz, *Phys. Rev. Lett.* **93**, 174102 (2004); I. Omelchenko *et al. Phys. Rev. Lett.* **106** 234102 (2011); A. M. Hagerstrom *et al.* & M. Tinsley *et al.*, *Nat. Phys.* 8, 658 & 662 (2012)

Chimera states...





What is a Chimera state?

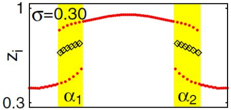
- Network of coupled oscillators with clusters of incongruent motions
- Predicted numerically in 2002, derived for a particular case in 2004, and 1st observed experimentally in 2012
- Not observed (initially) with local coupling, neither with global one

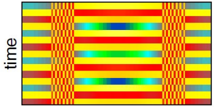
Features allowing for Chimera states?

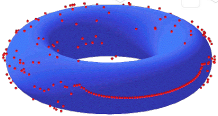
- Network of coupled <u>identical</u> oscillators, spatio-temporal dynamics
- Requires <u>non-local</u> nonlinear coupling between oscillator nodes
- Important parameters: <u>coupling strength</u>, and <u>coupling distance</u>

Y. Kuramoto and D. Battogtokh, *Nonlinear Phenom. Complex Syst.* **5**, 380 (2002); D. M. Abrams and S. H. Strogatz, *Phys. Rev. Lett.* **93**, 174102 (2004); I. Omelchenko *et al. Phys. Rev. Lett.* **106** 234102 (2011); A. M. Hagerstrom *et al.* & M. Tinsley *et al.*, *Nat. Phys.* 8, 658 & 662 (2012)

Chimera states...







What is a Chimera state?

- Network of coupled oscillators with clusters of incongruent motions
- Predicted numerically in 2002, derived for a particular case in 2004, and 1st observed experimentally in 2012
- Not observed (initially) with local coupling, neither with global one

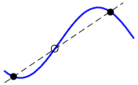
Features allowing for Chimera states?

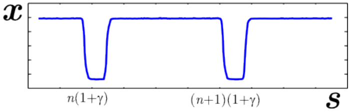
- Network of coupled identical oscillators, spatio-temporal dynamics
- Requires non-local nonlinear coupling between oscillator nodes
- Important parameters: coupling strength, and coupling distance

Y. Kuramoto and D. Battogtokh, *Nonlinear Phenom. Complex Syst.* **5**, 380 (2002); D. M. Abrams and S. H. Strogatz, *Phys. Rev. Lett.* **93**, 174102 (2004); I. Omelchenko *et al. Phys. Rev. Lett.* **106** 234102 (2011); A. M. Hagerstrom *et al.* & M. Tinsley *et al.*, *Nat. Phys.* 8, 658 & 662 (2012)

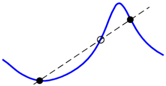
DDE recipe for chimera states

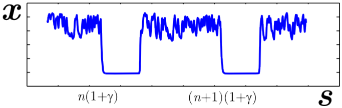
Symmetric $f_{NL}[x]$: Similar $\sigma-$"clusters" for $x < 0$ and $x > 0$





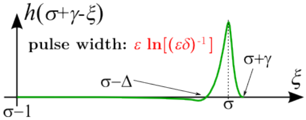
Asymmetric $f_{NL}[x]$: Distinct $\sigma-$clusters for $x < 0$ and $x > 0$





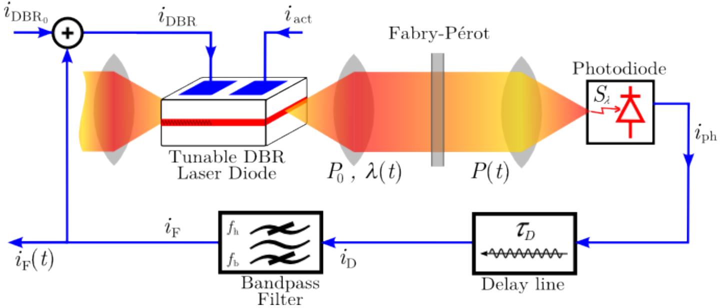
And i DDE

$$\delta \int_{s_0}^{s} x(\xi)\,\mathrm{d}\xi + x(s) + \varepsilon \frac{\mathrm{d}x}{\mathrm{d}s}(s) = f_{NL}[x(s-1)]$$



Laser based delay dynamics experiment

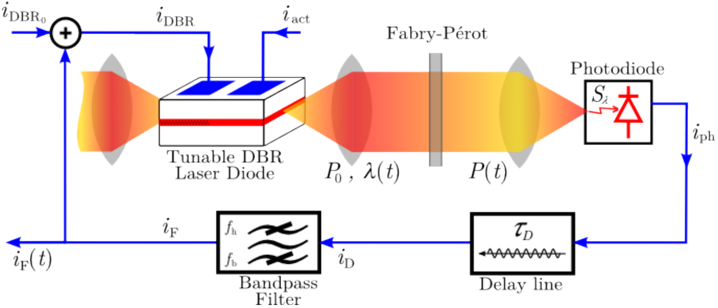
Tunable SC Laser setup, for i DDE Chimera



LL, Penkovsky, Maistrenko, *Nat. Commun.* 2015, DOI: 10.1038/ncomms8752

Laser based delay dynamics experiment

Tunable SC Laser setup, for i DDE Chimera



$f_{NL}[x]$: the Airy function of a Fabry-Pérot interferometer

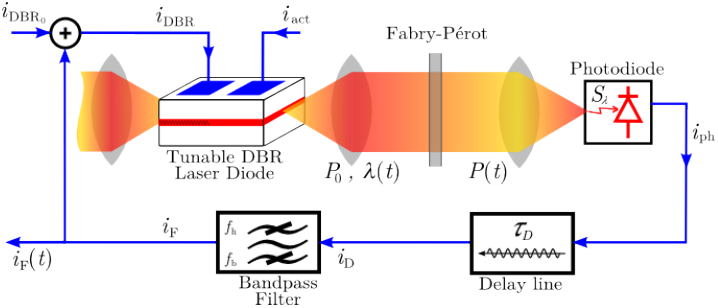
$$\Rightarrow f_{NL}[\lambda] = \frac{\beta}{1+m\,\sin^2(2\pi ne/\lambda)} = \frac{\beta}{1+m\,\sin^2(x+\Phi_0)}$$

with $x = \frac{2\pi ne}{\lambda_0^2}\,\delta\lambda$ and $\Phi_0 = \frac{2\pi ne}{\lambda_0 + S_{tun.}\, i_{DBR_0}}$

LL, Penkovsky, Maistrenko, *Nat. Commun.* 2015, DOI: 10.1038/ncomms8752

Laser based delay dynamics experiment

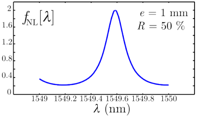
Tunable SC Laser setup, for i DDE Chimera

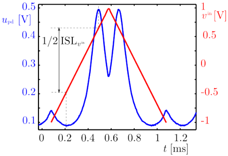


$f_{NL}[x]$: the Airy function of a Fabry-Pérot interferometer

$$\Rightarrow f_{NL}[\lambda] = \frac{\beta}{1+m\ \sin^2(2\pi ne/\lambda)} = \frac{\beta}{1+m\ \sin^2(x+\Phi_0)}$$

with $x = \frac{2\pi ne}{\lambda_0^2}\,\delta\lambda$ and $\Phi_0 = \frac{2\pi ne}{\lambda_0 + S_{tun.}\, i_{DBR_0}}$



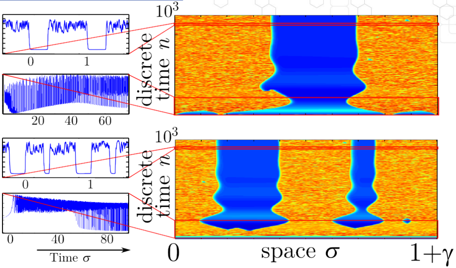


LL, Penkovsky, Maistrenko, *Nat. Commun.* 2015, DOI: 10.1038/ncomms8752

1st Chimera in $(\sigma, n)-$space

Numerics:

- $\beta = 0.6$, $\nu_0 = 1$, $\varepsilon = 5.10^{-3}$, $\delta = 1.6 \times 10^{-2}$
- Initial conditions: small amplitude white noise (repeated several times with different noise realizations)
- Calculated durations: Thousands of n

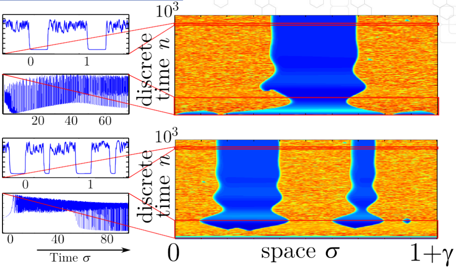


LL *et al. Phys. Rev. Lett.* **111** 054103 (2013)

1^{st} Chimera in $(\sigma, n)-$space

Numerics:

- $\beta = 0.6$, $\nu_0 = 1$, $\varepsilon = 5.10^{-3}$, $\delta = 1.6 \times 10^{-2}$
- Initial conditions: small amplitude white noise (repeated several times with different noise realizations)
- Calculated durations: Thousands of n

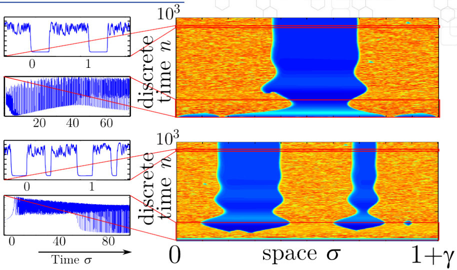


Experiment...

LL *et al. Phys. Rev. Lett.* **111** 054103 (2013)

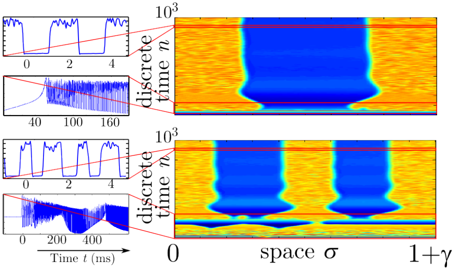
1^{st} Chimera in $(\sigma, n)-$space

Numerics:



- $\beta = 0.6$, $\nu_0 = 1$, $\varepsilon = 5.10^{-3}$, $\delta = 1.6 \times 10^{-2}$
- Initial conditions: small amplitude white noise (repeated several times with different noise realizations)
- Calculated durations: Thousands of n

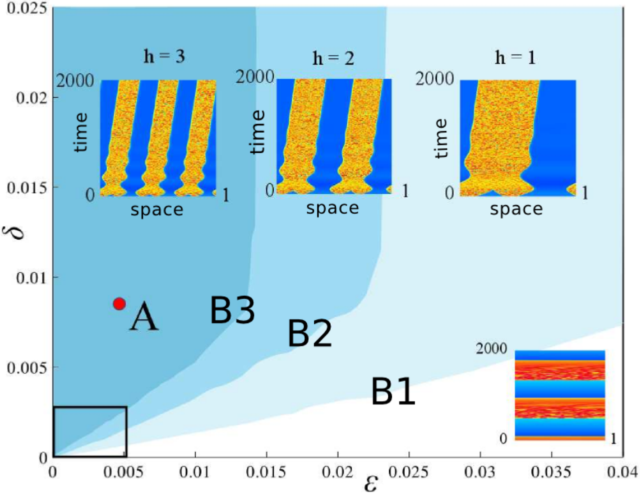
Experiment...



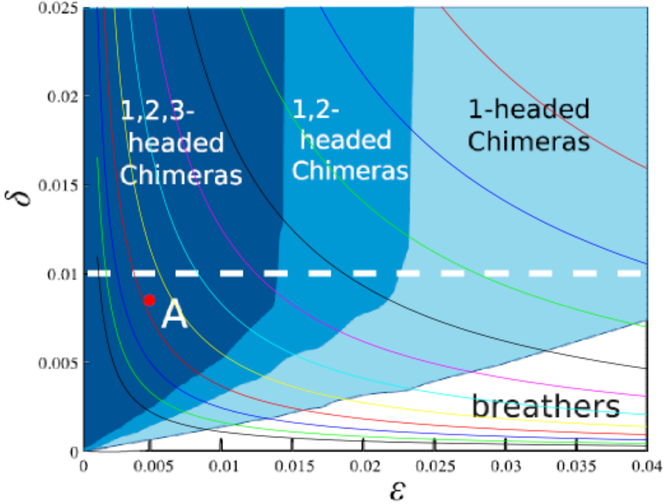
- Very close amplitude and time parameters, $\tau_D = 2.54$ms, $\theta = 160$ms, $\tau = 12.7\mu$s
- Initial conditions forced by background noise
- Recording of up to 16×10^6 points, allowing for a few thousands of n

LL *et al. Phys. Rev. Lett.* **111** 054103 (2013)

Bifurcations in $(\varepsilon, \delta)-$space

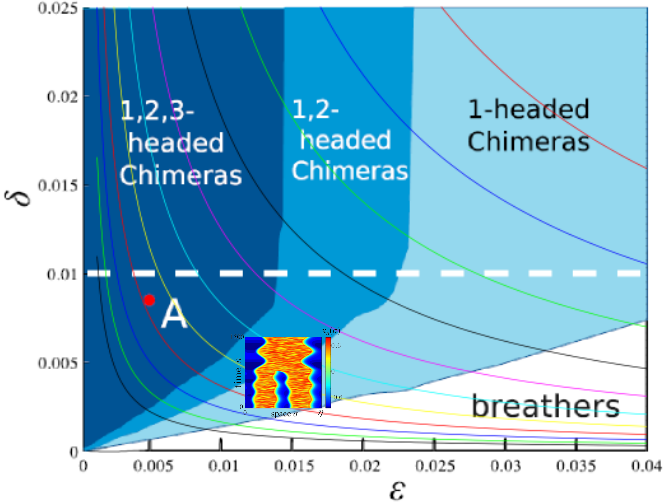


Bifurcations in $(\varepsilon, \delta)-$space



$\varepsilon = \tau / \tau_D$
$\delta = \tau_D / \theta$
$\beta \simeq 1.5$
$\Phi_0 \simeq -0.4$

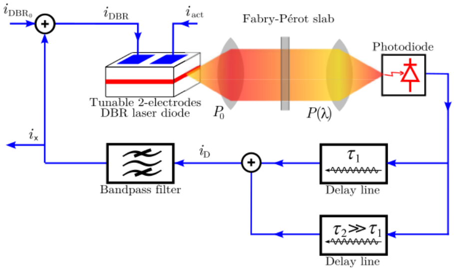
Bifurcations in $(\varepsilon, \delta)-$space

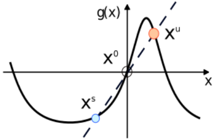


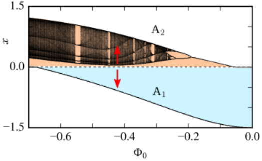
$\varepsilon = \tau/\tau_D$
$\delta = \tau_D/\theta$
$\beta \simeq 1.5$
$\Phi_0 \simeq -0.4$

Double delay dynamics: toward 2D chimera

Setup and delay dynamics features



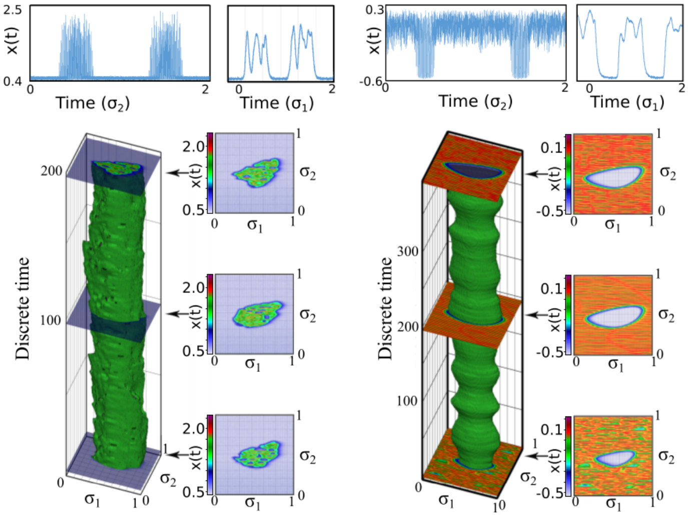




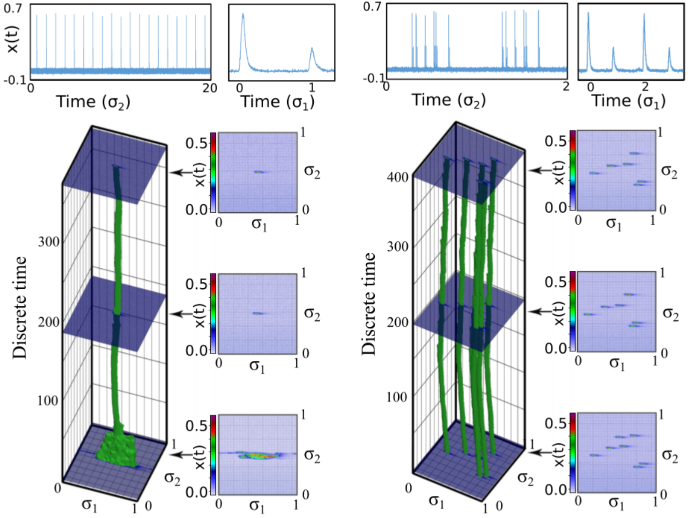
Double delay nonlinear integro-differential equation

$$\varepsilon \frac{\mathrm{d}x}{\mathrm{d}t}(t) + x(t) + \delta \int x(\xi)\mathrm{d}\xi = (1-\gamma) f_{\mathrm{NL}}[x(t-\tau_1)] + \gamma f_{\mathrm{NL}}[x(t-\tau_2)]$$

2D-chimera with chaotic sea, or chaotic island



Isolated pulses



Outline

Introduction

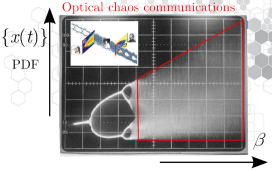
NLDDE in theory and practice

Space-Time analogy: From DDE to Chimera

DDE Apps: chaos communications, μwave radar, photonic AI

Hidden bonus slides

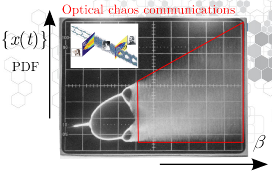
Optical Chaos Communications



Emitter-Receiver architecture

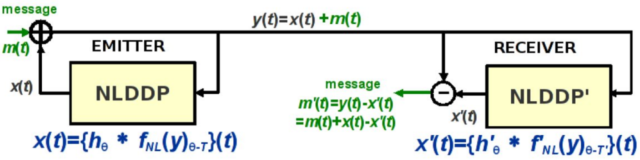
- Fully developed chaos (strong feedback gain, highly NL operation)

Optical Chaos Communications

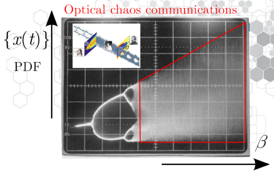


Emitter-Receiver architecture

- Fully developed chaos (strong feedback gain, highly NL operation)
- In-loop message insertion (message-perturbed chaotic attractor, with comparable amplitude)

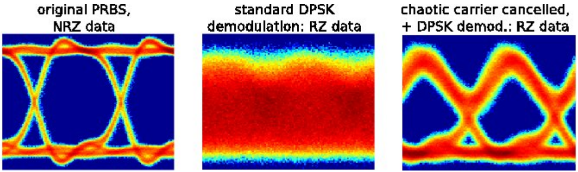


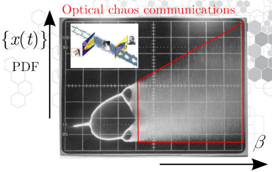
Optical Chaos Communications



Emitter-Receiver architecture

- Fully developed chaos (strong feedback gain, highly NL operation)
- In-loop message insertion (message-perturbed chaotic attractor, with comparable amplitude)
- Real-time encoding and decoding up to 10 Gb/s

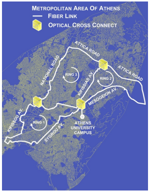


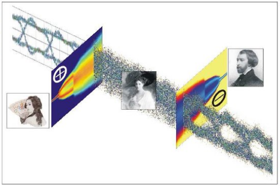


Optical Chaos Communications

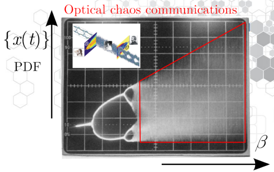
Emitter-Receiver architecture

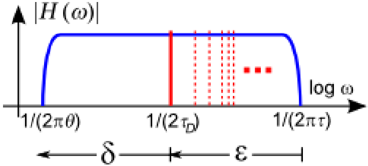
- Fully developed chaos (strong feedback gain, highly NL operation)
- In-loop message insertion (message-perturbed chaotic attractor, with comparable amplitude)
- Real-time encoding and decoding up to 10 Gb/s
- Field experiment over more 100 km, robust vs. fiber channel issues





Optical Chaos Communications



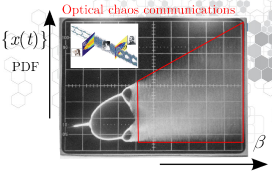


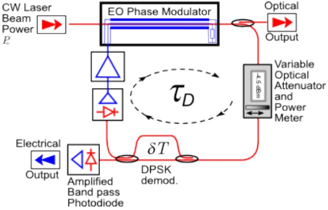
$$\varepsilon\,\dot{x} = -x(s) + \delta\,y(s) + \beta\cos^2[x(s-1) + \Phi]$$
$$\dot{y} = x(s)$$

Application resulted in a modified Ikeda model

- Broadband bandpass feedback (imposed by the high data rate; introduces an integral term with a slow time scale; time scales spanning over 6 orders of magnitude)

Optical Chaos Communications

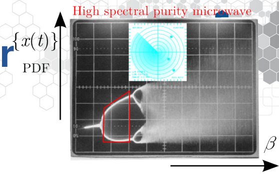




Application resulted in a modified Ikeda model

- Broadband bandpass feedback (imposed by the high data rate; introduces an integral term with a slow time scale; time scales spanning over 6 orders of magnitude)
- Design of multiple delays dynamics (to improve the SNR of the transmission, electro-optic phase setup $\rightarrow$ 4 time scale dynamics)

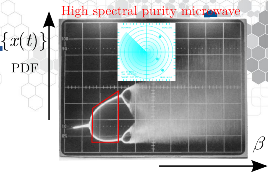
High spectral purity μwave for Radar



Modified physical parameters

- Limit cycle operation (reduced feedback gain)

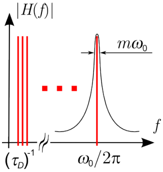
High spectral purity μwave for Radar

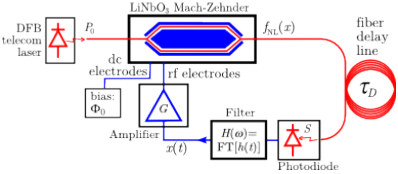


Modified physical parameters

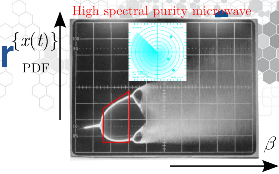
- Limit cycle operation (reduced feedback gain)
- Narrow bandpass feedback, or weakly damped feedback filtering (central freq. 10 GHz, bandwidth 40 MHz)

$$\frac{2m}{\omega_0}\int_{t_0}^{t} x(\xi)\,\mathrm{d}\xi + x(t) + \frac{1}{2m\omega_0}\frac{\mathrm{d}x}{\mathrm{d}t}(t) = \beta\{\cos^2[x(t-\tau_D)+\Phi] - \cos^2\Phi\}$$





High spectral purity μwave for Radar

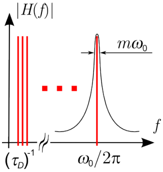


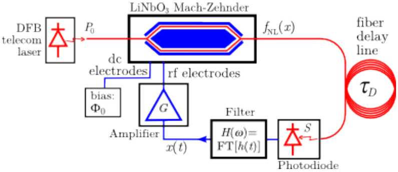
Modified physical parameters

- Limit cycle operation (reduced feedback gain)
- Narrow bandpass feedback, or weakly damped feedback filtering (central freq. 10 GHz, bandwidth 40 MHz)

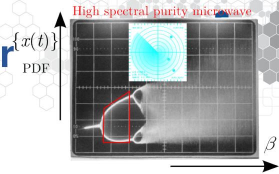
$$\frac{2m}{\omega_0}\int_{t_0}^{t} x(\xi)\,\mathrm{d}\xi + x(t) + \frac{1}{2m\omega_0}\frac{\mathrm{d}x}{\mathrm{d}t}(t) = \beta\{\cos^2[x(t-\tau_D)+\Phi] - \cos^2\Phi\}$$

- Extremely long delay line (4 km vs a few meters)





High spectral purity μwave for Radar

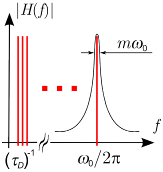


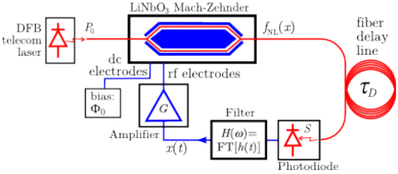
Modified physical parameters

- Limit cycle operation (reduced feedback gain)
- Narrow bandpass feedback, or weakly damped feedback filtering (central freq. 10 GHz, bandwidth 40 MHz)

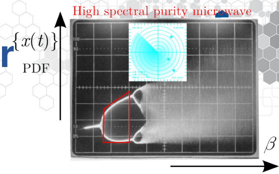
$$\frac{2m}{\omega_0}\int_{t_0}^{t} x(\xi)\,\mathrm{d}\xi + x(t) + \frac{1}{2m\omega_0}\frac{\mathrm{d}x}{\mathrm{d}t}(t) = \beta\{\cos^2[x(t-\tau_D)+\Phi] - \cos^2\Phi\}$$

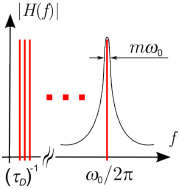
- Extremely long delay line (4 km vs a few meters)
- Dynamics still high dimensional, however forced around a central frequency

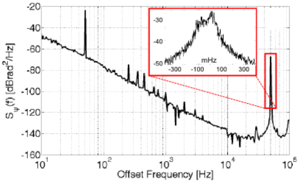




High spectral purity μwave for Radar

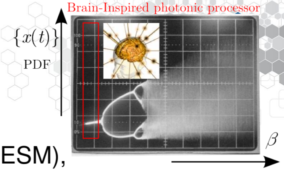






Examples of obtained performances

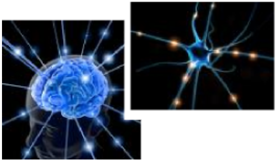
- 10-20dB lower phase noise power spectral density (vs. DRO): -140 dB/Hz @ 10 kHz from the 10 GHz carrier
- Accurate theoretical phase noise modeling (noise $\equiv$ small external perturbation, non-autonomous dynamics)

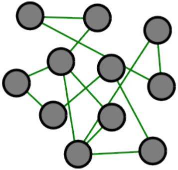


Photonic brain-inspired computing

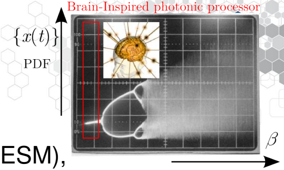
Concepts

- Novel paradigm refered as to Echo State Network (ESM), Liquid State Machine (LSM) and also Reservoir Computing (RC)





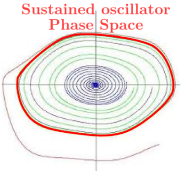
Photonic brain-inspired computing

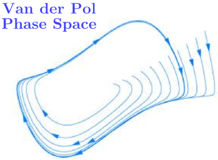


Concepts

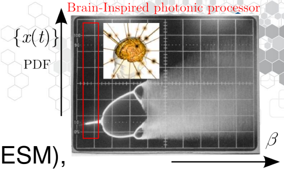
- Novel paradigm refered as to Echo State Network (ESM), Liquid State Machine (LSM) and also Reservoir Computing (RC)
- Processing of time varying information through nonlinear transients observed in a high-dimensional phase space

Asymptotic vs. Transient dynamics (huge space for transients out of the stable solution)



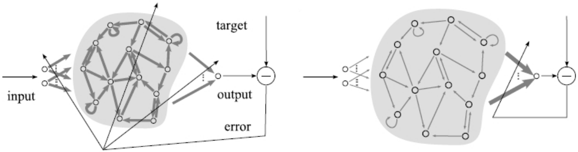


Photonic brain-inspired computing

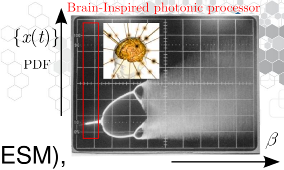


Concepts

- Novel paradigm refered as to Echo State Network (ESM), Liquid State Machine (LSM) and also Reservoir Computing (RC)
- Processing of time varying information through nonlinear transients observed in a high-dimensional phase space
- Derived from RNN, however learning simplified to the output layer only (other weights, input and internal, chosen at random)

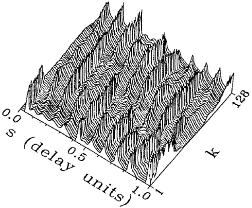


Photonic brain-inspired computing

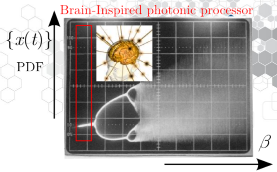


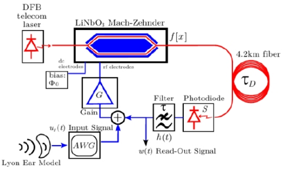
Concepts

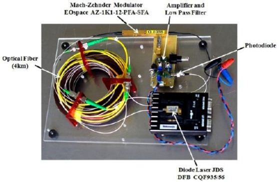
- Novel paradigm refered as to Echo State Network (ESM), Liquid State Machine (LSM) and also Reservoir Computing (RC)
- Processing of time varying information through nonlinear transients observed in a high-dimensional phase space
- Derived from RNN, however learning simplified to the output layer only (other weights, input and internal, chosen at random)
- Instead of the high-dimensional of an RNN, let's try to use a delay dynamics → assumes actual validity of a space-time analogy



Photonic brain-inspired computing







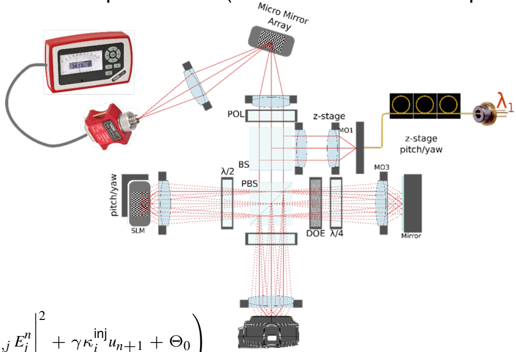
Achievements

- First efficient hardware implementing RC concept (in electronic and optoelectronic delay dynamics)
- Operation around a stable fixed point (fading memory property)
- 400 to 1000 nodes/neurons can be emulated
- Speech recognition successfully demonstrated, with state of the art performances (0% WER, speed up to 1 million words/s)

Real spatio-temporal photonic RC

Experimental setup (D. Brunner, M. Jacquot)

- Nodes are spatially distributed in an image plane
- Coupling between nodes makes use of DOE
- Nonlinear is performed by SLM (polarization filtering)
- Read-Out is full implemented (cascaded DMD and a photodiode)

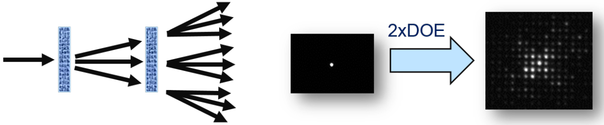


$$I_i^{n+1} = \sin^2\left(\beta\left|\sum_{j=1}^{N}\kappa_{i,j}\,E_j^n\right|^2 + \gamma\kappa_i^{\text{inj}}u_{n+1} + \Theta_0\right)$$

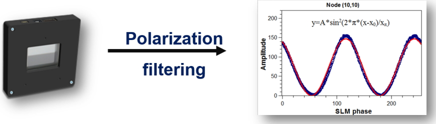
Real spatio-temporal photonic RC

Elements characterization

- Node coupling: two cascaded DOE



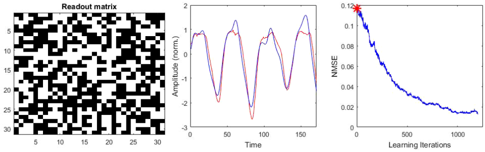
- Nonlinear transformation (SLM)



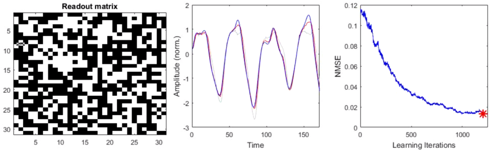
Real spatio-temporal photonic RC

Chaotic time series prediction

- Random initialization and learing

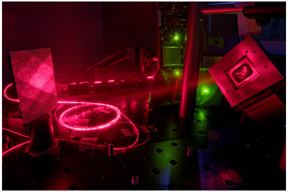


- After re-inforcement learning



Thank you for attention

International
Day of Light
16 May



Outline

A chaotic rainbow. . .

INTERNATIONAL
YEAR OF LIGHT
2015

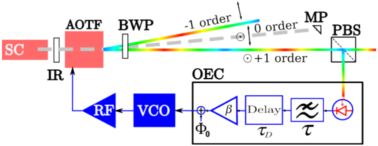
From toy-model to toy-experiment: the (visible) wavelength chaos setup

(Chembo *et al.*, *Phys. Rev. A* **94** 2016)

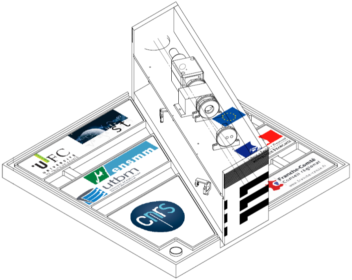
A chaotic rainbow. . .

INTERNATIONAL
YEAR OF LIGHT
2015

From toy-model to toy-experiment: the (visible) wavelength chaos setup



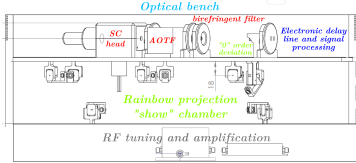
- Delay dynamics on the color sliced by an AOTF from the “rainbow” of a SC white light source
- Friendly “science demo” (many diffracted rainbows with a chaotically moving dark line)
- Easily transportable experiment (no optical table required)
- Setup mimicking the shape of our new FEMTO-ST building in Besançon.



(Chembo *et al.*, *Phys. Rev. A* **94** 2016)

A chaotic rainbow. . .

INTERNATIONAL
YEAR OF LIGHT
2015



From toy-model to toy-experiment: the (visible) wavelength chaos setup

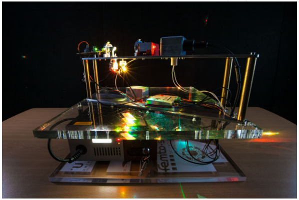
- Delay dynamics on the color sliced by an AOTF from the “rainbow” of a SC white light source
- Friendly “science demo” (many diffracted rainbows with a chaotically moving dark line)
- Easily transportable experiment (no optical table required)
- Setup mimicking the shape of our new FEMTO-ST building in Besançon.

(Chembo *et al.*, *Phys. Rev. A* **94** 2016)

A chaotic rainbow. . .

INTERNATIONAL
YEAR OF LIGHT
2015

From toy-model to toy-experiment: the (visible) wavelength chaos setup



- Delay dynamics on the color sliced by an AOTF from the “rainbow” of a SC white light source
- Friendly “science demo” (many diffracted rainbows with a chaotically moving dark line)
- Easily transportable experiment (no optical table required)
- Setup mimicking the shape of our new FEMTO-ST building in Besançon.

(Chembo *et al.*, *Phys. Rev. A* **94** 2016)