Chimera States in Star Networks

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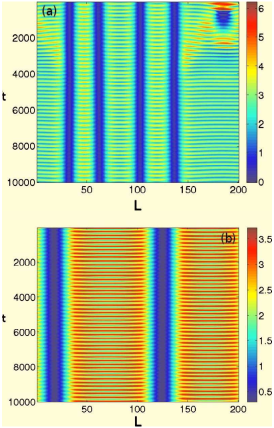
Old stuff... Typical "chimera-like" spatiotemporal patterns arising in a Coupled Map Lattice model of rheological chaos in sheared nematic liquid crystals

S M Kamil, G I Menon & SS *Chaos*, v. 20 (2010) 043123.1-14;
S M Kamil, SS & G I Menon *Physical Review E*, v 78 (2008) 011706



Plan of my talk :

Not to present our old results of such "chimera-like" states!

Rather, I will present some new results that may be of relevance to current practitioners of research on "Chimera States"!

What's in a name... A chimera by any other name is still symmetry-breaking...

Outline of the Talk

- Chimera States in Star Network: Symmetry-Breaking in Dynamically Identical Entities

  Global Stability of Chimeras : an issue that has direct bearing on the observability of such states

- Environment Induced Symmetry-Breaking of the Basin Stability of the Oscillation-Death State

- Emergent Symmetry in Spatiotemporal Patterns Aided by Dissimilarity in the Coupled Dynamical Entities

- Chimera states have been reported primarily in networks that have a regular ring topology, where oscillators are coupled in a non-local or global fashion
- In this work we will show how chimera states also emerge in oscillator networks with a star topology
- This configuration arises extensively in computer networks, where every node connects to a central computer, and the central computer act as a server and the peripheral devices act as clients
- Further, a star-like structure is a primary motif in scale-free networks, which have been reported to arise in wide-ranging phenomena.
- One can also interpret this system as a set of uncoupled oscillators connected to a common drive.

- We will show how the symmetry of the end-nodes, which are indistinguishable in terms of the coupling environment and dynamical equations, is broken and co-existing groups with different dynamical behaviour emerge.
- We will demonstrate the extensive existence of chimeras in the end-nodes of the star network through global stability measures, and show that large parameter regimes of coupling strengths typically yield a chimera state.
- We also confirm the widespread existence of robust chimera states in analog circuit experiments.

C. Meena, K. Murali, SS, *International Journal of Bifurcation and Chaos* (2016)

Star Networks of Chaotic Oscillators

Consider the dynamics of a star network of N identical nonlinear oscillator systems.

- In such networks there is one central hub node (labelled by site index $i = 1$) and $N - 1$ environmentally identical peripheral end-nodes connected to the central node (labelled by node index $i = 2, \ldots N$).
- The focus of this study is the dynamical patterns arising in the $N - 1$ indistinguishable end-nodes of this network.

In order to establish the generality of our results, we consider two prototypical chaotic systems (Rössler system and Lorenz) at the nodes, and three different coupling forms.

First, we consider standard diffusive coupling through similar variables, given by:

$$\begin{aligned} \dot{x}_i &= f_x(x_i, y_i, z_i) + \sum_{j=1}^{N} K_{ij}(x_j - x_i) \\ \dot{y}_i &= f_y(x_i, y_i, z_i) \\ \dot{z}_i &= f_z(x_i, y_i, z_i) \end{aligned} \quad (1)$$

Here coupling matrix element for central node $i = 1$ is $K_{1j} = k/2$ when $j \neq 1$, and for the end-nodes $i = 2, \ldots N$, $K_{i1} = k/2$ and zero otherwise. The coupling strength is given by k.

Then we consider the Conjugate Coupling given as:

$$
\begin{aligned}
\dot{x}_i &= f_x(x_i, y_i, z_i) + \sum_{j=1}^{N} K_{ij}(y_j - x_i) \\
\dot{y}_i &= f_y(x_i, y_i, z_i) \\
\dot{z}_i &= f_z(x_i, y_i, z_i)
\end{aligned}
\tag{2}
$$

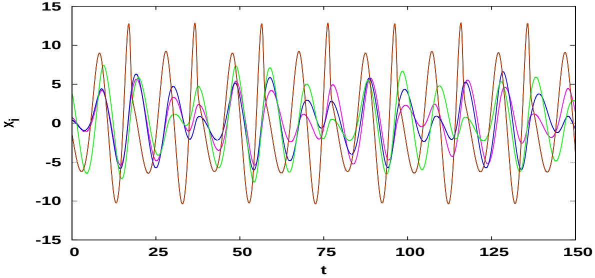
Lastly, we also consider a Mean-Field Coupling, where the dynamics of the central node is given by:

$$\begin{aligned} \dot{x}_1 &= f_x(x_1, y_1, z_1) + \frac{k}{2}(x_m - x_1) \\ \dot{y}_1 &= f_y(x_1, y_1, z_1) \\ \dot{z}_1 &= f_z(x_1, y_1, z_1) \end{aligned} \tag{3}$$

where $x_m = \frac{1}{N-1}\sum_{j=2,\dots N} x_j$ is the mean field of the end-nodes. The dynamics of the end-nodes $i = 2, \dots N$ is given by:

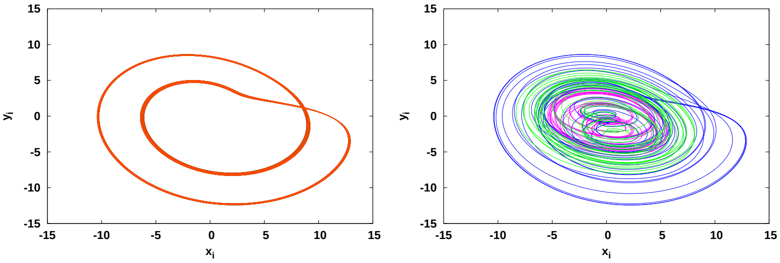
$$\begin{aligned} \dot{x}_i &= f_x(x_i, y_i, z_i) + \frac{k}{2}(x_1 - x_i) \\ \dot{y}_i &= f_y(x_i, y_i, z_i) \\ \dot{z}_i &= f_z(x_i, y_i, z_i) \end{aligned} \tag{4}$$

As coupling strength increases, the end-nodes go from a de-synchronized state to a completely synchronized state, via a large coupling parameter regime yielding chimera states.



Representative chimera state in a star network of diffusively coupled Rössler Oscillators: characterized by the co-existence of synchronized and de-synchronized sets of end-nodes, and are distinct from the fully synchronized state, the fully desynchronized state and the synchronized cluster state.

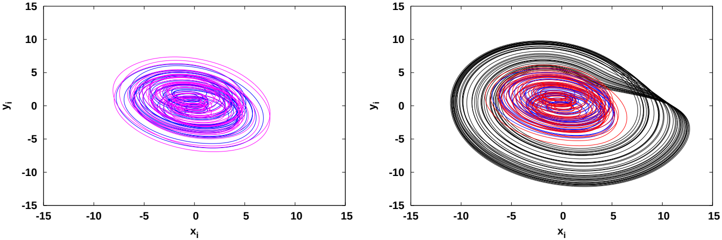
Phase portraits show that the desynchronized nodes are chaotic attractors with different geometries, while the synchronized nodes show periodic behaviour.



So the symmetry of the end-nodes, that have identical dynamical equations and coupling environments, is broken to yield a synchronized periodic group and a desynchronized chaotic group.

Star network of conjugately coupled Rössler systems

Phase portraits

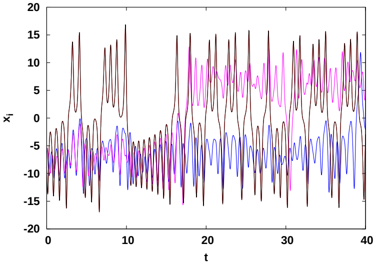


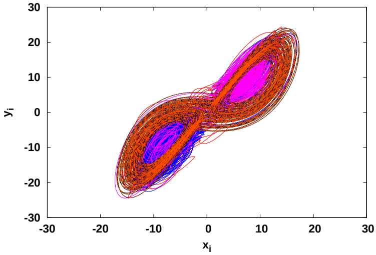
(left) Desynchronized set (blue and magenta) and (right) Three distinct Synchronized Clusters (black, red and blue)

Dynamical Patterns for Coupled Lorenz systems

Here again we find that as coupling strength increases, the end-nodes go from a de-synchronized state to a completely synchronized state, via a large coupling parameter regime yielding chimera states, where the identical end-nodes split into different dynamical groups, thereby breaking symmetry.

It is also evident that in addition to different synchronization properties, the groups also yield different attractor geometries.

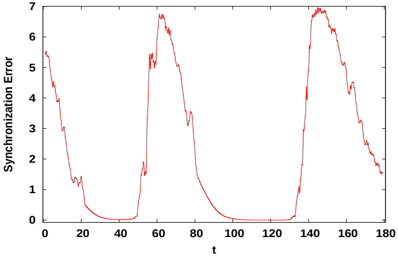




Time evolution and the corresponding phase portrait, for a star network of diffusively coupled Lorenz systems, yielding a synchronized group (shown in brown) and a de-synchronized group (shown in blue and magenta)

Breathing Chimera

Synchronization error of the incoherent group:



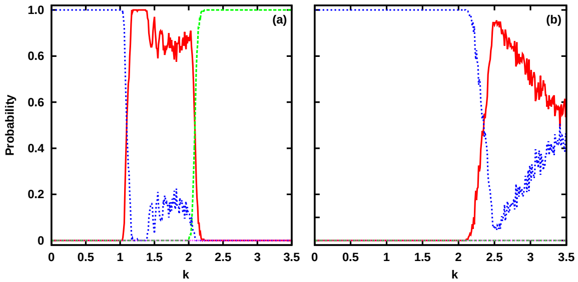
It is clearly evident from the oscillating synchronization error that the nodes move in and out of synchronization

The occurence of breathing chimera states is more common in the coupled Lorenz system than in coupled Rössler systems.

Prevalence of chimera states

- In order to quantify the probability of obtaining chimera states from random initial states we calculate the fraction of initial conditions leading to co-existing synchronized and desynchronized states in the end-nodes, in a large sample of random initial states.
- This provides an estimate of the size of the basin of attraction of the chimera state, and indicates the prevalence of chimeras in this system.
- So this measure is important, as it allows us to gauge the chance of observing chimeras without fixing special initial states.

Global Stability of the Chimera State



Probability of obtaining chimera states (red), synchronized clusters (magenta), fully synchronized states (green), and completely de-synchronized states (blue) in star networks of coupled Lorenz systems, under (a) conjugate coupling and (b) regular diffusive coupling.

- It is clearly evident that there exists extensive regimes of coupling parameter space where the probability of obtaining a chimera state is close to one.
- This quantitively establishes the prevalence of chimeras in the end-nodes of nonlinear oscillators coupled in star configurations.
- Larger networks yield larger basins of attraction for the chimera state.
- Conjugate coupling yields larger parameter bands with high prevalence of chimera states.

Experimental Verification of Chimera States

Now we establish the robustness of these chimera states in experimental situations by demonstrating the occurrence of chimera states in star networks of coupled nonlinear oscillators, evolving from generic initial states.

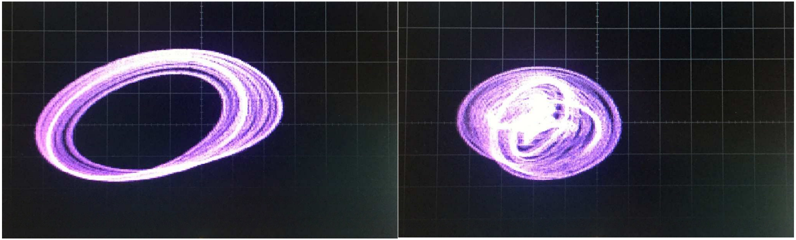
Specifically, we consider a circuit implementation of a chaotic Rössler-type oscillator at the nodes:

$$\frac{d^3x}{dt^3} = -A\frac{d^2x}{dt^2} - \frac{dx}{dt} \pm (|x| - 1)$$

Analog simulation circuit of this equation can be carried out with standard operational amplifiers and diodes.

We then go on to set up 4 diffusively coupled oscillators

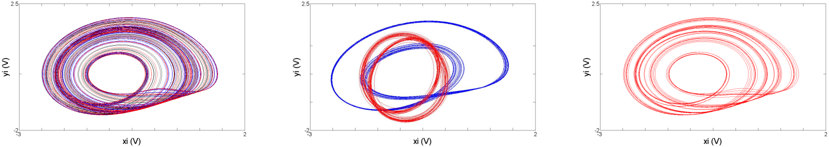
- We observe that for low coupling strength the end-nodes show completely unsynchronized oscillations.
- For large coupling strength, as anticipated, the end-nodes exhibit complete synchronization.
- However, for moderate coupling strengths, the 3 identical end-nodes split into two groups, with two synchronized nodes and one node not in synchrony, thus exhibiting a chimera-like state.
- Note that we have no control over the initial state in the experiment, and these states evolve from generic random initial conditions.



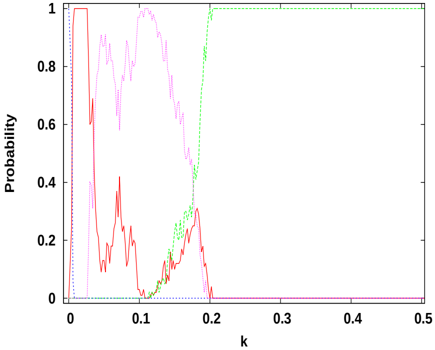
Experimental phase-portraits of the attractor observed from chaotic Rössler-type oscillator circuits diffusively coupled in a star network, yielding a chimera-like state with 2 nodes being synchronized and one node being out of synchrony. Traces display the (a) synchronized set, and (b) desynchronized node. Here X-axis: 50 mV/div and Y-axis: 100 mV/div.

Experimental waveforms of the 3 end-nodes of 4 diffusively coupled chaotic Rossler-type circuits: (a) waveform of the synchronized set; (b) waveform of the distinct node, alongside the waveform of the synchronized set, for comparison. Here X-axis: 2 mS/div and Y-axis: 200 mV/div.

- Further, in order to check the generality of the results, we also investigated the mean-field type of coupling
- Again one finds that for low coupling strengths the end-nodes are completely unsynchronized, while for high coupling strengths they are completely synchronized
- However, in a large window of moderate coupling strengths the identical end-nodes exhibit a chimera-like state.
- Also, one obtains different geometries of the dynamical state in the two synchronized and de-synchronized groups



Circuit simulated phase-portrait of attractors in the $x_i y_i$, plane generated from chaotic chaotic Rössler-type oscillator circuits coupled via mean-field in a star network, for coupling strengths: (a) $k = 0.1$ yielding an unsynchronized state, (b) $k = 1.0$ yielding a chimera-like state and (c) $k = 2.0$ yielding a synchronized state.



Probability of obtaining chimera states (red), synchronized clusters (magenta), fully synchronized states (green), and completely de-synchronized states (blue) in star networks of coupled Rössler systems with mean-field type coupling

- In summary, it is clearly evident from our numerical and experimental investigations that large parameter regimes of moderate coupling strengths yield chimera states from generic random initial conditions in this network topology.

- So star networks provide a promising class of coupled systems, in natural or human-engineered contexts, where chimeras are pervasive.

Environment induced Symmetry Breaking of the Basin Stability of the Oscillation-Death State

S.S. Chaurasia, M. Yadav, and Sudeshna Sinha,
Physical Review E **98**, 032223, (2018)

- Impact of a common external system, which we call a common environment, on the Oscillator Death (OD) states of a group of Stuart-Landau oscillators
- The group of oscillators yield a completely symmetric OD state when uncoupled to the external system, i.e. the two OD states occur with equal probability
- However, remarkably, when coupled to a common external system the Basin Stability symmetry is significantly broken
- This demonstration of an environmental coupling-induced mechanism for the prevalence of certain OD states in a system of oscillators, suggests an underlying process for obtaining certain states preferentially in ensembles of oscillators with environment-mediated coupling

- Oscillation quenching can give rise to oscillation death:

  Oscillators split into two sub-groups, around an unstable fixed point via pitchfork bifurcations, generating a set of stable fixed points

- Oscillation death is very relevant to biological systems, as this oscillation quenching mechanism can lead to the emergence of inhomogeneity in homogeneous medium

- So, for instance, OD has been interpreted as a mechanism for cellular differentiation

- In the context of many real world systems, interactions can occur through a common medium
- For instance, chemical oscillations of catalyst-loaded reactants have been found in a medium of catalyst-free solution, where the coupling is through exchange of chemicals with the surrounding medium
- Similarly, in the context of genetic oscillators coupling occurs by diffusion of chemicals between cells and extracellular medium
- Further, in a collection of circadian oscillators, the concentration of neurotransmitter released by each cell can induce collective behaviour
- In general, such cases occur due to the common medium, referred to as a common environment, interacting with the dynamical systems

A model system mimicking this scenario consists of N identical oscillator systems $\mathbf{x}_i$, $i = 1, \ldots, N$ coupled through a (possibly time-varying) environment, denoted by variables $\mathbf{u}$, whose most general dynamical equations is given as:

$$\dot{\mathbf{x}}_{\mathbf{i}} = f_{\mathbf{x}}(\mathbf{x}_i) + \varepsilon_{\rm ext} \ g(\mathbf{u})$$

and

$$\dot{\mathbf{u}} = f_{\mathbf{u}}(\mathbf{u}) + \varepsilon_{\rm ext} \ h(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$$

Now a variety of models of biochemical oscillators coupled through an environment are described by equations of this form: So this framework unifies many specific models of particular systems, and allows us to obtain some basic general results which potentially apply to all of them

Specifically, consider a group of N globally coupled SL oscillators ($\mathbf{x}_i = (x_i, y_i)$) given by the following evolution equations:

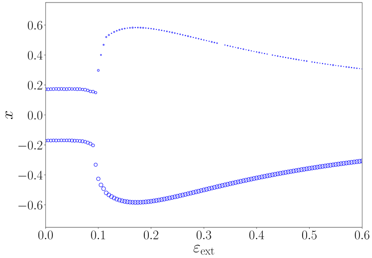
$$\begin{aligned} \dot{x}_i &= (1 - x_i^2 - y_i^2)x_i - \omega y_i + \varepsilon_{\text{intra}}(q\bar{x} - x_i) \\ \dot{y}_i &= (1 - x_i^2 - y_i^2)y_i + \omega x_i + \varepsilon_{\text{ext}} u \\ \dot{u} &= -ku + \varepsilon_{\text{ext}} \bar{y} \end{aligned} \quad (5)$$

Here the oscillators within the group are coupled via the mean field $\bar{x}$ of the x-variable, and $\varepsilon_{\text{intra}}$ reflects the strength of intra-group coupling.

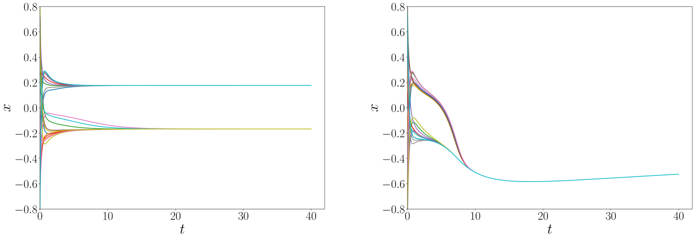
Additionally, this oscillator group also couples to an external common environment, denoted by a single-variable u, and ε_{ext} reflects the strength of the coupling to the external system.

- In this coupling scheme, q is a control parameter for the mean-field interaction, describing the influx and consequent influence of the mean field in the oscillator group
- A similar type of coupling mechanism was suggested in the context of intercell communication of synthetic gene oscillators via a small autoinducer molecule
- As q tends to zero, the effect of the mean-field interaction decreases, suppressing the oscillations of the coupled systems
- The limit $q = 0$ indicates no interaction between the oscillators (i.e. they are simply uncoupled oscillators with self-feedback), while the limit $q = 1$ maximizes the interaction with the mean field.
- At intermediate values of q the oscillators are driven to Amplitude Death/Oscillation Death states
- So in our system the common external environment provides an indirect coupling conjoining the different oscillators in the group, in addition to the direct coupling within the group.

Here one of the oscillator death states has positive x and negative y ("positive state"), and the other oscillator death state has and negative x and positive y ("negative state")

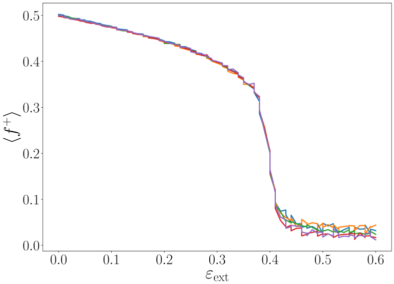


In the bifurcation diagram, the size of the symbols represent the probability of being in that state, with the probability estimated by sampling over a large number of initial states.



Time series of twenty oscillators in the group (shown in distinct colours), (a) in the absence of coupling to an external environment and (b) when the group is connected to the external environment

S S Chaurasia, M Yadav, and S Sinha, *Physical Review E*, **98**, 032223, (2018)



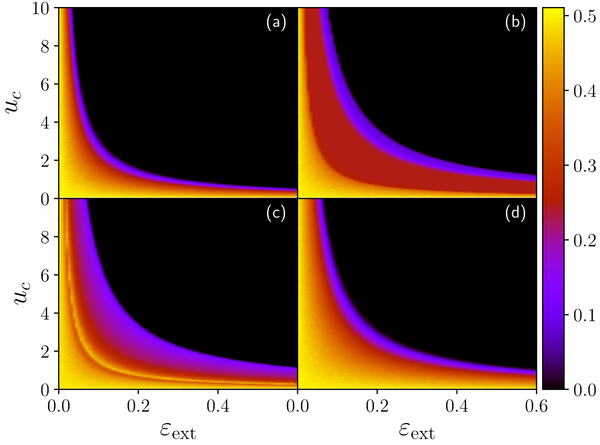
Basin Stability of the positive Oscillator Death state of coupled oscillators with respect to coupling strength ε_{ext}

Groups of oscillators of different sizes N=40, 60, 80, 100, 120 are shown in different colours

- To check the generality of our results we also consider a Constant External Drive

- We also consider time-varying connections to the common external environment, with a fraction of oscillator-environment links switching on and off.

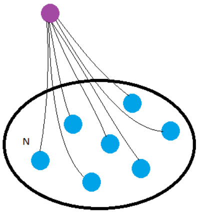
- We found marked breaking of symmetry in the global stability of Oscillator Death states for the case of coupling to a constant external drive as well.
- When the constant environmental drive is large, the Basin Stability asymmetry of the OD states is very large, and the transition between the symmetric and asymmetric state with increasing oscillator-environment coupling is very sharp.
- We also find that the asymmetry induced by environmental coupling decreases as a power law with increase in fraction of on-off connections.
- The suggests that blinking oscillator-environment links can restore the symmetry of the OD state



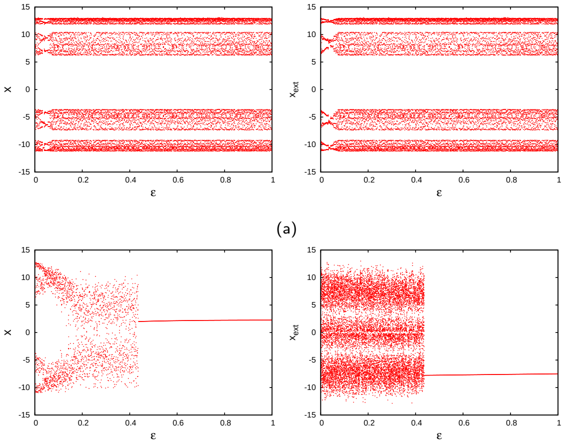
Basin Stability of the positive Oscillator Death state in the parameter space of coupling strength ε_{ext} and constant environment (u_c), with fraction of blinking oscillators (a) $f_{\text{blink}} = 0.0$, (b) $f_{\text{blink}} = 0.25$, (c) $f_{\text{blink}} = 0.50$, (d) $f_{\text{blink}} = 1.0$

- Thus we have demonstrated that this system displays symmetry breaking of the Basin Stability of the Oscillator Death States
- This implies that a specific oscillator death state is preferentially achieved
- This state selection leads to asymmetric distribution of OD states in the ensemble of oscillators, suggesting a natural mechanism that allows the emergence of a favoured set of fixed points

Control of chaotic oscillators to steady states through coupling to a dissimilar external chaotic system

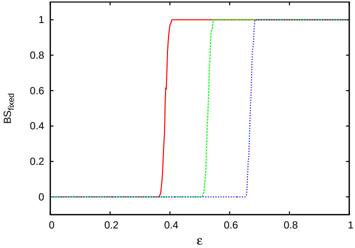


Schematic of a group of N oscillators coupled to an external oscillator



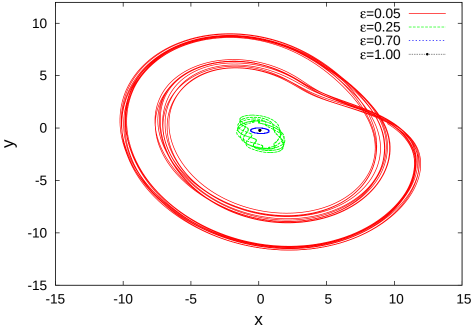
(b)

Bifurcation diagrams, with respect to the coupling strength ε, of one representative oscillator in the group (left) and an external oscillator (right). Here the group consists of chaotic Rössler oscillators and the external oscillator is: (a) a chaotic Rössler oscillator with different parameters and (b) a chaotic Lorenz system



Dependence of the fraction of initial states BS_{fixed} attracted to the fixed point state, on the coupling strength ε, for a group of chaotic Rössler oscillators coupled to a common external chaotic Lorenz system with different parameter values

Note that there is no dependence of BS_{fixed} on the number of oscillators in the group



Phase portrait of a representative Rössler oscillator from the group, coupled to a common external hyper-chaotic oscillator, at different coupling strengths ε.

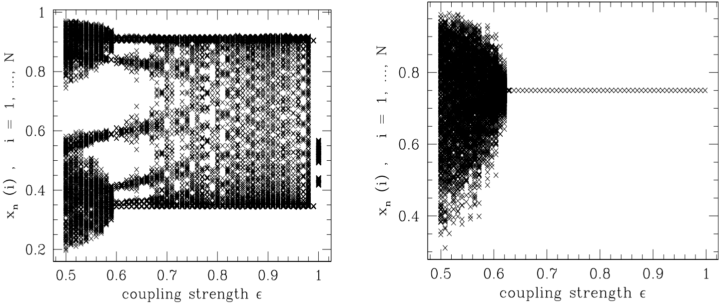
We also investigated the behaviour of a hierarchical network of chaotic oscillators, where at the zeroth level of the hierarchy we have one chaotic external system that is dissimilar to the rest of the oscillators in the network

Remarkably, this external system managed to successfully steer the chaotic oscillators at all levels of the hierarchy onto steady states, at sufficiently high coupling strengths

So this suggests a potent method to efficiently control chaotic dynamics in a hierarchical network to stable steady states, by simply coupling to an external chaotic system.

S S Chaurasia and Sudeshna Sinha, *Europhysics Letters* (2019)

Some old results... Loss of Symmetry in Links leads to more Spatiotemporal Regularity



Regular Ring vs. Random Network

Sudeshna Sinha, Physical Review E, v. 66 (2002) 016209

Outlook... and Outreach...

- Chimera States in Star Network
  - Global Stability of Chimeras: an issue that has direct bearing on the observability of such states
  - Experimental demonstration of robust chinera states
- Environment Induced Symmetry-Breaking of the Basin Stability of the Oscillation-Death State
  - Suggests an underlying process for obtaining certain states preferentially in ensembles of oscillators with environment-mediated coupling
- Emergent Symmetry in Spatiotemporal Patterns Aided by Dissimilarity in the Coupled Dynamical Entities

- C. Meena, K. Murali, Sudeshna Sinha
  Chimera states in Star Networks
  International Journal of Bifurcation and Chaos, **26** (2016) 1630023
- S.S. Chaurasia, M. Yadav, and Sudeshna Sinha
  Environment induced Symmetry Breaking of the Oscillation-Death State
  Physical Review E, **98**, 032223, (2018)
- S.S. Chaurasia and Sudeshna Sinha
  Suppression of chaos through coupling to an external chaotic system
  Nonlinear Dynamics, **87** (2017) 159-167
- S S Chaurasia and Sudeshna Sinha
  Control of Hierarchical Networks by Coupling to an External Chaotic System
  Europhysics Letters, **125**, 50006 (2019)