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Union bound for quantum information processing

The union bound, alternatively known as Boole’s inequality, represents one of the simplest yet non-trivial methods for bounding the probability that either one event or another occurs, in terms of the probabilities of the individual events. By induction, the bound applies to the union of multiple events, and it often provides a good enough bound in a variety of applications whenever the probabilities of the individual events are small relative to the number of events. Recently, the union bound has been listed as the second step to try when attempting to ‘upper-bound the probability of something bad’, with the first step being to determine if the trivial bound of one is reasonable in a given application. Generalizing the union bound to a quantum-mechanical set-up is non-trivial. A natural setting in which we would consider this generalization is when the goal is to bound the probability that two or more successive measurement outcomes do not occur. Clearly, if the projectors representing measurements do not commute, then classical reasoning does not apply and alternative methods are required. Following [1] we report about a quantum union bound that is relevant when performing a sequence of binary outcome quantum measurements on a quantum state. It also involves a tunable parameter that can be optimized. The proof is rather elementary, relying only on basic properties of projectors, the Pythagorean theorem, and the Cauchy–Schwarz inequality. Applications to a variety of situations, including quantum communication theory, quantum algorithms, quantum complexity theory and Hamiltonian complexity theory will be discussed. References [1] S. Khabbazi-Oskouei, S. Mancini, and M. M. Wilde, Proc. Royal Soc. London A 475, 20180612 (2019), arXiv:1804.08144