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Are almost-symmetries almost linear?

It $d$-pends. Wigner's symmetry theorem implies that transformations that preserve transition probabilities of pure quantum states are linear maps on the level of density operators. We investigate the stability of this implication. On the one hand, we show that any transformation that preserves transition probabilities up to an additive $\varepsilon$ in a separable Hilbert space admits a weak linear approximation, i.e. one relative to any fixed observable. This implies the existence of a linear approximation that is $4\sqrt{\varepsilon} d$--close in Hilbert-Schmidt norm, with $d$ the Hilbert space dimension. On the other hand, we prove that a linear approximation that is close in norm and independent of $d$ does not exist in general. To this end, we provide a lower bound that depends logarithmically on $d$.