Outline

- Digital CMOS Design
- Arithmetic Operators
- Floating Point Arithmetic Operators
  - Square root

Square root

- Direct method ⇨ digit-by-digit
- Indirect method ⇨ resolve a non-linear equation

Square root

An real number **y** using floating point representation

Find a real number **x** such as

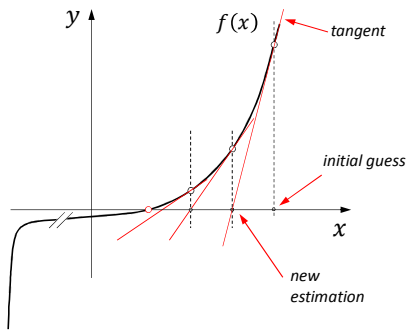
$$(x + \varepsilon)^2 = y$$

- Calculation cannot be implemented in hardware
- Need iterative operation

Square root - indirect

Resolving a non linear equation

$f(x) = 0$



Square root - indirect

Resolving a non linear equation $f(x) = 0$

Taylor series in the neighborhood of x_0

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 + \cdots$$

1st order : $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$

Square root - indirect

Resolving $x = \sqrt{y}$

Find a function f such as $f(x) = 0$ *for* $x = \sqrt{y}$

$f(x) = x^2 - y$

$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$

$f(x) \approx ({x_0}^2 - y) + 2x_0(x - x_0)$

$f(x) = 0 \qquad x = x_0 + \frac{(y - {x_0}^2)}{2x_0}$

$$x = \frac{1}{2}\left(\frac{y}{x_0} + x_0\right)$$

Square root - indirect

Resolving a non linear equation $f(x) = 0$

Iterative resolution starting from an initial guess x_0

$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$

$f(x) = 0$

$f(x_0) + f'(x_0)(x - x_0) = 0$

$$x = \frac{-f(x_0)}{f'(x_0)} + x_0$$

Newton-Raphson method

Square root - indirect

Resolving $x = \sqrt{y}$

Each iteration $x_{i+1} = \frac{1}{2}\left(\frac{y}{x_i} + x_i\right)$

division !!

Hard to implement

~~$f(x) = x^2 - y$~~

Square root - indirect

Resolving $x = \sqrt{y}$

Find a function f such as $f(u) = 0$ *for* $u = \frac{1}{\sqrt{y}}$

$f(u) = u^{-2} - y$

$f(u) \approx f(u_0) + f'(u_0)(u - u_0)$

$f(u) \approx ({u_0}^{-2} - y) - 2{u_0}^{-3}(u - u_0)$

$f(u) = 0 \qquad u = u_0 + \frac{({u_0}^{-2} - y)}{2{u_0}^{-3}}$

$u = \frac{1}{2}u_0(3 - y{u_0}^2)$

Square root - indirect

Resolving $x = \sqrt{y}$

Each iteration $u_{i+1} = \frac{1}{2}u_i(3 - y{u_i}^2)$

multiply !!

$u = \frac{1}{\sqrt{y}}$

$x = \sqrt{y} = \frac{y}{\sqrt{y}} = u \cdot y$