June 11, 2019

ICTP2019, PS # 1,

Question 1: Some small questions

1. What is the symmetry that you can impose such that the period is independent on the amplitude? That is, what is a symettry that forbid higher order terms?
2. Show that adding $m^2\phi^2/2$ to the massless free Lagrangian indeed correspond to particles with mass m, see page 30 of Lecture 1.
3. Estimate $XX \to XY$ in the model we discussed on page 18 of Lecture 2.
4. Find more 3rd and 4th order invariants of the model we discuss on page 28 of Lecture 2.

Question 2: Harmonic oscillator perturbation theory

Consider a system with 3 DOFs with $H = H_0 + H_1$ where

$$H_0 = \frac{p_x^2}{2m} + \frac{m\omega_x^2 x^2}{2} + \frac{p_y^2}{2m} + \frac{m\omega_y^2 y^2}{2} + \frac{p_z^2}{2m} + \frac{m\omega_z^2 z^2}{2}, \qquad H_1 = \lambda_1 xyz + \lambda_2 x^2 z. \tag{1}$$

We further assume that $\omega_y = 3\omega_x$, $\omega_z \gg \omega_y$ and that λ_1 and λ_2 are small and thus can be treated as perturbation. We denote a state of the system as $|n_x, n_y, x_z\rangle$. In this question, you are asked to use two ways to calculate the transition matrix element

$$\mathcal{A}(|0,1,0\rangle \to |3,0,0\rangle). \tag{2}$$

1. Use second order perturbation theory to show that

$$\mathcal{A} = c \times \frac{\lambda_1 \lambda_2}{\omega_z^2 - (2\omega_x)^2} \tag{3}$$

   and find what is c. Use $\hbar = m = 1$ to make the bookkeeping simpler.

2. Use the Feynman diagram for Harmonic oscillator method to get the same result. For that, draw the diagram and calculate it. The amplitude is the product of the following factors
   (a) For each vertex multiply the amplitude by the corresponding coupling constant.
   (b) For each propagator use $-1/(E_z^2 - q^2)$ where E_z is the energy of the internal state and q is the energy that goes out of it.

(c) Use the correct normalization: (i) For each external state use $1/\sqrt{2\omega_i}$ and (ii) for each final state that appear n times multiply the amplitude by $\sqrt{n!}$ (this factor is called symmetry factor).

Compare this result to the one you got in the first item.

Question 3: More Harmonic oscillators

Consider a case with four oscillators, called w, x, y, z and assume that $y \to 2x2w$ is allowed by energy conservation. We use the following interaction

$$\lambda_1 y z^2 + \lambda_2 z x^2 + \lambda_3 z w^2. \tag{4}$$

1. Draw the diagram and calculate the transition matrix element using the Feynman rules and the correct normalization.
2. Calculate the amplitude using perturbation theory. For that find the 6 intermediate states that contribute and calculate the 6 matrix elements and add them up. Verify that the result of the two methods agree.