

# Schwartz: Collider Physics

## Problems

1. How much energy is in the LHC beam, in Joules ( $1 \text{ J} = 6.2 \times 10^{18} \text{ eV}$ )?
2. Show that pseudorapidity differences are *not* boost invariant for massive particles. Calculate the dependence of  $\eta_1 - \eta_2$  on the masses of two massive particles to leading non-vanishing order in the masses.
3. The boost parameter  $\beta$ , with  $\cosh \beta = \frac{1}{1-v^2}$  is also sometimes called the rapidity. Derive a relationship between the rapidity  $y$  and  $\beta$  for a particle of mass  $m$  moving with velocity  $v$  in the  $z$  direction.
4. ATLAS currently has high-resolution tracking information up to  $|\eta| < 2.5$ , but is considering extending it up to  $|\eta| < 4$  in the next long upgrade. Suppose 1000  $Z$  bosons are produced at rest at the LHC and then decay to  $\mu^+\mu^-$ . How many of the events would have both muons leave tracks in the inner tracker with the current ATLAS setup and after the upgrade?
5. One of the goals of a future  $e^+e^-$  collider would be to study Higgs properties.
  - (a) Estimate the cross section for  $e^+e^- \rightarrow \mu^+\mu^-b\bar{b}$  through a  $Zh$  intermediate state at center-of-mass energies 100 GeV, 200 GeV and 300 GeV.
  - (b) Current designs for an  $e^+e^-$  Higgs factory aim to run at 240 GeV. What are the tradeoffs in running at higher and lower energy that might make this the optimal energy?
6. Consider a  $W$  boson decaying to an electron  $e^-$  and a neutrino  $\nu$ . For the electron, we measure the longitudinal momentum  $p_e^z$  and transverse momentum  $\vec{p}_e^T$  and for the neutrino, just the transverse momentum  $\vec{p}_\nu^T$  which we get from missing transverse momentum  $\vec{p}_T^{\cancel{e}}$  in the event:  $\vec{p}_\nu^T = -\vec{p}_T^{\cancel{e}}$ . The transverse energy of the electron and neutrinos are defined as  $E_T^e = \sqrt{m_e^2 + (\vec{p}_e^T)^2}$  and  $E_T^\nu = |\vec{p}_T^{\cancel{e}}|$ . Show that the transverse mass, defined as

$$M_T = \sqrt{(E_T^e + E_T^\nu)^2 - (\vec{p}_e^T + \vec{p}_\nu^T)^2} \quad (1)$$

is less than or equal to the  $W$  boson mass.

7. Suppose there were four flavors of light neutrinos instead of three. What would the  $Z$  width be in this case? Put a bound on the number of massless neutrinos using that  $\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$ .

8. Consider the process  $pp \rightarrow t\bar{t}h$ .

- (a) What fraction of events has 1, 2, 3 or 4 b-jets in it.
- (b) What is some of the largest backgrounds to in the fully leptonic decay mode (i.e. tops decay leptonically, Higgs decays to  $b\bar{b}$ )?
- (c) Consider the fully hadronic channel, with both tops decaying to jets. Estimate the fraction of events in which *all* of the jets are in the central region ( $|\eta| < 3$ ).
- (d) How much  $p_T$  does the Higgs need for the typical distance between the  $b$  jets from its decay to be within  $\Delta R < 1$ ?