Symmetries + ... (3)

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Symmetries: Yesterday and today

Yesterday:

- How to built invariants
- $\mathcal{L}$ has to be invariant
- For $U(1)$ we explain what is a charge
- For $SU(N)$ we explain what is a representation

Today:

- Different kind of symmetries
  - Imposed vs accidental
  - Lorentz vs internal
  - Global vs local

More on symmetries

Accidental symmetries

A symmetry that is an output, and only due to the truncation

- Example: The symmetry that makes the period independent of the amplitude
- Example: $U(1)$ with $X(q=1)$ and $Y(q=-4)$

$$V(XX^*, YY^*) \Rightarrow U(1)_X \times U(1)_Y$$

  - X^4Y breaks this symmetry

Lorentz invariants

Unlike symmetries in the internal space of the field, Lorentz is the symmetry of space-time

- We always impose it
- The representations we care about are
  - Singlet: Spin zero (scalars, denote by ϕ)
  - LH and RH fields: Spin half (fermions, ψ_L, ψ_R)
  - Vector: Spin one (gauge boson, denote by A_μ)
- Fermions are more complicated than scalars

Fermions

- Under Lorentz, the basic fields are left-handed and right-handed
- We define the complex conjugation as $\bar{\psi}$
- The kinetic term is different, only one derivative

$$\mathcal{L}_{\rm kin} \sim \bar{\psi}_L \partial_\mu \gamma^\mu \psi_L$$

- The mass term is also different, it involves LH and RH field

$$\mathcal{L}_m \sim \bar{\psi}_L \psi_R$$

  - Why it is?
  - We can also have Majorana mass
  - What are the conditions to have a mass term?

Dimension

- In mechanics we count with $dim[x] = 1$
- Here it is a bit more complicated. We use $\hbar = c = 1$ and $dim[E] = 1$ that implies $dim[x] = -1$
- What are the dim of
  - S
  - L
  - ϕ
  - ψ

C, P, CP, CPT

There are also discrete transformation on space-time

$$C \qquad P \qquad T$$

- All Lorentz invariant QFT have CPT
- C and P take you from LH to RH fermion
  - They are “easy” to break
- CP is the difference between matter and anti-matter. Breaking arises from phases in $\mathcal{L}$

Local symmetires

Local symmetry

Basic idea: rotations depend on x and t

$$\phi(x_\mu) \to e^{iq\theta}\phi(x_\mu) \quad \xrightarrow{local} \quad \phi(x_\mu) \to e^{iq\theta(x_\mu)}\phi(x_\mu)$$

- It is kind of logical and we think that all imposed symmetries in Nature are local
- The kinetic term $|\partial_\mu \phi|^2$ is not invariant
- We want a kinetic term (why?)
- We can save the kinetic term if we add a field that is
  - Massless
  - Spin 1
  - Adjoint representation: $q = 0$ for $U(1)$, triplet for $SU(2)$, and octet for $SU(3)$

Gauge boson kinetic term

- Generalization of that of a scalar
- We know how it is in classical $E\&M$

$$\mathcal{L} \propto F_{\mu\nu}F^{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- For non-Abelian there is a modification. For SU(2)

$$F^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - ig\epsilon_{abc}G^b G^c$$

- We get gauge boson self interaction

$$F^a_{\mu\nu}F^{\mu\nu}_a \sim gG^3 + g^2G^4$$

Gauge symmetry

New field A_μ. How we couple it?

- Recall classical electromagnetism

$$H = \frac{p^2}{2m} \Rightarrow H = \frac{(p - qA_i)^2}{2m}$$

- In QFT, for a local $U(1)$ symmetry and a field with charge q

$$\partial_\mu \to D_\mu \qquad D_\mu = \partial_\mu + iqA_\mu$$

- We get interaction from the kinetic term

$$\bar{\psi} \not{D} \psi \sim \bar{\psi}(\partial_\mu + iqA_\mu)\gamma^\mu \psi \to q\bar{\psi}\gamma^\mu \psi A_\mu$$

- The interaction $\propto q$ (for $SU(N)$ to matrices)

The two aspects of symmetries

Thinking about E&M

- Charge conservation
- The force proportional to the charge

Q: Which of these come from the “global” aspect and which from the “local” aspect of the symmetry?

More on masses

To summarize masses

- There is no symmetry that forbids generic scalar masses
- Chiral symmetry forbid fermion Dirac masses
- Gauge symmetry forbid gauge boson masses

SSB

Breaking a symmetry

SSB

- A situation that we have when the Ground state is degenerate
- By choosing a ground state we break the symmetry
- We choose to expend around a point that does not respect the symmetry
- PT only works when we expand around a minimum

What is the difference between a broken symmetry and no symmetry?

SSB implies relations between parameters

SSB

Symmetry is $x \to -x$ and we keep up to x^4

$$f(x) = a^2x^4 - 2b^2x^2 \qquad x_{\rm min} = \pm b/a$$

We choose to expand around $+b/a$ and use $u \to x - b/a$

$$f(x) = 4b^2u^2 + 4bau^3 + a^2u^4$$

- No $u \to -u$ symmetry
- The $x \to -x$ symmetry is hidden
- A general function has 3 parameters $c_2u^2 + c_3u^3 + c_4u^4$
- SSB implies a relation between them

$$c_3^2 = 4c_2c_4$$

Partial SSB

Think about a vector in 3d. What symmetry is broken by it?

SSB in QFT

- When we expand the field around a minimum that is not invariant under a symmetry

$$\phi(x_\mu) \to v + h(x_\mu)$$

- It breaks the symmetries that ϕ is not a singlet under
- Then we can get masses that were protected by the broken symmetry
  - Fermions

$$y\phi\bar{\psi}_L\psi_R \to y(v+h)\bar{\psi}_L\psi_R, \qquad m_\psi = yv$$

  - Gauge fields of the broken symmetries

$$|D_\mu\phi|^2 = |\partial_\mu\phi + iqA_\mu\phi|^2 \ni A^2\phi^2 \to v^2A^2$$

Model buildings

Building Lagrangians

- Choosing the generalized coordinates (fields)
- Imposing symmetries and how fields transform (input)
- The Lagrangian is the most general one that obeys the symmetries
- We truncate it at some order, usually fourth

The general $\mathcal{L}$

We write

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\psi} + \mathcal{L}_{\rm Yuk} + \mathcal{L}_{\phi}$$

- Always have a kinetic term
- The task is to find the other terms and see what they lead to