The SM (4)

Yuval Grossman

Cornell

Building Lagrangians

- Choosing the generalized coordinates (fields)
- Imposing symmetries and how fields transform (input)
- The Lagrangian is the most general one that obeys the symmetries
- We truncate it at some order, usually fourth

QED

QED

(i) The symmetry is a local

$$U(1)_{\rm EM}$$

(ii) There are two fermion fields

$$e_L(-1), \qquad e_R(-1)$$

(iii) There are no scalars

What can you say about

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_\psi + \mathcal{L}_{\rm Yuk} + \mathcal{L}_\phi$$

$\mathcal{L}$ and more

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_\psi + \mathcal{L}_{\rm Yuk} + \mathcal{L}_\phi$$

where

$$\mathcal{L}_{\rm Yuk} = \mathcal{L}_\phi = 0$$

$$\mathcal{L}_\psi = m_e \overline{e_L} e_R + \text{h.c.}$$

$$\mathcal{L}_{\rm kin} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - i\overline{e_L} \not{D} e_L - i\overline{e_R} \not{D} e_R$$

$$D^\mu = \partial^\mu + ieqA^\mu$$

- How many parameters?
- What are the Feynman rules?
- What about P and CP?

2 Generations

- What is $\mathcal{L}$?
- How many parameters?
- What are the accidental symmetries?

Experimental test of QED

- Massless photon implies Coulomb potential
- $g - 2$ of the electron and muon
- Many more
- In many cases, we find deviations that indicate we need to go to the UV theory

QCD

Defining QCD

(i) The symmetry is a local

$$SU(3)_C$$

(ii) There are six left-handed and six right-handed fermion fields, “quarks"

$$Q_{Li}(3), \qquad Q_{Ri}(3), \qquad i = 1, ..., 6$$

(iii) There are no scalars

$\mathcal{L}$ for QCD

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\psi} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi}$$

$$\mathcal{L}_{\text{Yuk}} = \mathcal{L}_{\phi} = 0$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu} - i\overline{Q_{Li}} \not{D} Q_{Li} - i\overline{Q_{Ri}} \not{D} Q_{Ri}$$

$$\mathcal{L}_{\psi} = m_{ij} \overline{Q_{Li}} Q_{Rj} + \text{h.c.}$$

- How many parameters?
- What are the Feynman rules?
- What about P and CP?
- Accidental symmetries?

Running and confinements

- Couplings run. What is it?
- For QCD we get confinement
- We can only use perturbative QCD at the UV
- At the IR we have hadrons: baryons and mesons

Experimental tests of QCD

- Need to be done at the UV
- We “see” that the proton is made of quarks
- We “see” the running of α_s
- A lot of other tests

At the IR we need to use some models. The quark model is a model, that tell us what hadrons are made of

Leptonic SM

Defining the LSM

(i) The symmetry is a local

$$SU(2)_L \times U(1)_Y$$

(ii) There are three fermion generations

$$L^i_L(2)_{-1/2}, \qquad E^i_R(1)_{-1}, \qquad i = 1,2,3$$

(iii) There is a single scalar multiplet:

$$\phi(2)_{+1/2}$$

We could have SSB

$$SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$$

$\mathcal{L}$ for the LSM

What can you say about

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\psi} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi}$$

- Kinetic term (next)
- The Yukawa

$$\mathcal{L}_{\text{Yuk}} = Y^{e}_{ij} \overline{L^i_L} E^j_R \, \phi + \text{h.c.}$$

- The Scalar potential

$$-\mathcal{L}_{\phi} = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

$\mathcal{L}_{kin}$ and $SU(2) \times U(1)$

- Four gauge bosons DOFs

$$W_a^\mu \qquad B^\mu$$

- The covariant derivative is

$$D^\mu = \partial^\mu + igT_A W_a^\mu + ig' Y B^\mu$$

  - Two parameters g and g'
  - Y is the $U(1)$ charge of the field D_μ work on
  - T_a is the $SU(2)$ representation of that field
  - $T_a = 0$ for singlets. $T_a = \sigma_a/2$ for doublets
- Write D_μ for $L(1,2)_{-1/2}$ and $E(1,1)_{-1}$

Explicit examples

$$D^\mu = \partial^\mu + igW_a^\mu T_a + ig'YB^\mu$$

- Write D_μ for $L(1,2)_{-1/2}$ and $E(1,1)_{-1}$

$$D^\mu L = \left(\partial^\mu + \frac{i}{2}gW_a^\mu \sigma_a - \frac{i}{2}g'B^\mu\right) L$$

$$D^\mu E = \left(\partial^\mu - ig'B^\mu\right) E$$

- HW: Using $\phi(1,2)_{1/2}$ write $D^\mu \phi$

SSB in the SM

SSB in the SM

$$-\mathcal{L}_{Higgs} = \lambda\phi^4 - \mu^2\phi^2 = \lambda(\phi^2 - v^2)^2$$

- We measure the fact that $\mu^2 > 0$ by having SSB
- The minimum is at $|\phi| = v$
- ϕ has 4 DOF. We can choose the vev in the real part of the down component
- It leads to: $SU(2)_L \times U(1)_Y \;\to\; U(1)$
- We call the remaining symmetry EM
- We left with one real scalar field: the Higgs boson

QED

- Where is QED in all of this?

$$Q = T_3 + Y$$

- We can write explicitly for $L(1,2)_{-1/2}$ and $\phi(1,2)_{1/2}$

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- We can “tell” the differet component because we have SSB

$\mathcal{L}_{\text{Yuk}}$ and fermion masses

- There is no way to write a (bare) mass term for the leptons
- The Yukawa part of the leptons

$$\mathcal{L}_{\text{Yuk}} = y_{ij}\overline{L_{Li}}E_{Rj}\,\phi \Rightarrow m_{ij}\overline{E_{Li}}E_{Rj} \qquad m_{ij} = vy_{ij}$$

- $i, j = 1, 2, 3$ are flavor indices
- y is a general complex 3×3 matrix and we can choose a basis where m is diagonal and real

$$m_{ij} = y\,v = \text{diag}(m_e, m_\mu, m_\tau)$$

- Neurinos are massless

Accidental symmety

The model has accidental symmetries called lepton numbers

- It is a $U(1)_e \times U(1)_\mu \times U(1)_\tau$
- Total lepton number is the sum of all these three
- Dim 5 operators break it to nothing
- This symmetry is related to the fact that the neutrino is massless. Breaking of it gives masses to the neutrinos

Gauge boson masses

- $W_1,\ W_2,\ W_3,\ B$
- Gauge bosons masses from $|D_\mu \phi|^2$ (HW: do it)
- Diagonalizing the mass matrix the masses are

$$M_{W^+}^2 = M_{W^-}^2 = \frac{1}{4} g^2 v^2 \qquad M_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2 \qquad M_A^2 = 0$$

- The mass eigenstates

$$W^\pm = \frac{1}{\sqrt{2}}(W_1 \pm i W_2) \qquad \tan\theta_W \equiv \frac{g'}{g}$$

$$Z = \cos\theta_W W_3 - \sin\theta_W B \qquad A = \sin\theta_W W_3 + \cos\theta_W B$$

- We have a θ_W rotation from (W_3, B) to (Z, A)

The $\rho = 1$ relation

- We can measure θ from interaction
- We can also get if from the masses of the W and A
- We get the following testable relation

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \qquad \tan\theta_W \equiv \frac{g'}{g}$$

The above is a signal of SSB

Experimental tests

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \qquad \tan\theta_W \equiv \frac{g'}{g}$$

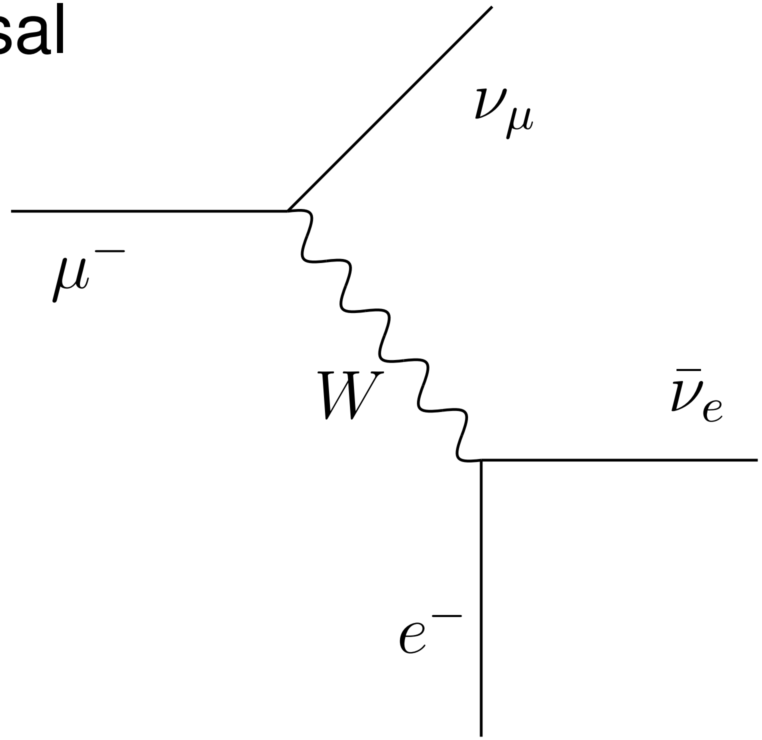
- High energy: Open your pdg and check W and Z decays to leptons. What do you expect to see?
- Z decays to lepton actually measures $\sin^2\theta_W \approx 0.23$
- HW: Calculate $\Gamma(Z \to \nu\bar{\nu})/\Gamma(Z \to e^+e^-)$, get $\sin^2\theta_W$ from the data and check the $\rho = 1$ prediction
- Also many low energy data tests

Interactions

Charged current interactions

$$-\frac{g}{\sqrt{2}}\,\overline{\nu_{eL}}\,W^\mu\gamma_\mu e_L^- + h.c.$$

- Only left-handed fields take part in charged-current interactions. Therefore the W interaction violate parity
- The $W\ell\nu$ interaction is universal
- Can be used to measure g



$$A \sim g^2/m_W^2 \sim G_F$$

Neutral currents

$$\mathcal{L}_{\rm int} = \frac{e}{\sin\theta\cos\theta}(T_3 - \sin^2\theta_W Q)\ \bar\psi\gamma^\mu\psi\, Z_\mu + e\, Q\bar\psi\gamma^\mu\psi\, A_\mu,$$

- We define

$$Q = T_3 + Y \qquad\qquad e = g\sin\theta$$

- Photon coupling is parity invariant
- Z couples to both LH and RH fermions but in a parity violating way
- The coupling to the Z is larger. So why we call it weak interaction?
- Once we know e and g we know θ

The Higgs interaction

- The model predicts one scalar where the couplings are proportional to the mass of the fermion
- We start to see it, but we did not test it yet

One more question

The photon gives rise to the Coulomb force

- Can the Z give rise to a a similar force?
- Can this force be seen in atomic physics?
- Can it give rise to $\nu - \bar{\nu}$ bound state in a way similar to that the photon does for $e^+ - e^-$?

Some summary

- The electro-weak model is rather involved
- Many interesting predictions
- The $\rho = 1$ relation is confirmed
- Leton universality
- Many many more