June 13, 2019

ICTP2019, PS # 2,

Question 1: Gauge boson masses

In this question, you are asked to do some algebra.
We define the vev of the Higss as

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}. \tag{1}$$

Note that in the Lectures I did not care about the factor of $\sqrt{2}$, but here, in order to be consistent with standard notations, I keep it. The covariant derivative is

$$D^\mu\phi = \left(\partial^\mu + \frac{i}{2} g W_a^\mu \sigma_a + \frac{i}{2} g' B^\mu\right)\phi. \tag{2}$$

The masses of the gauge bosons are given by

$$\mathcal{L}_{M_V} = (D_\mu\langle\phi\rangle)^\dagger (D^\mu\langle\phi\rangle). \tag{3}$$

Using Eq. (2) for $D^\mu\phi$, show that

$$m_W^2 = \frac{1}{4} g^2 v^2, \qquad m_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2, \qquad m_A^2 = 0. \tag{4}$$

(Recall that for a complex bosonic field ϕ with mass m, the mass term is $m^2|\phi|^2$ while for a real field it is $m^2\phi^2/2$.) We define an angle θ_W via

$$\tan\theta_W \equiv \frac{g'}{g}. \tag{5}$$

We define four gauge boson states:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_1 \pm iW_2)_\mu, \quad Z_\mu^0 = \cos\theta_W W_{3\mu} - \sin\theta_W B_\mu, \quad A_\mu^0 = \sin\theta_W W_{3\mu} + \cos\theta_W B_\mu. \tag{6}$$

Question 2: The Z force

The photon gives rise to the Coulomb force.

1. Show that the Z gives rise to a similar force and write it down.
2. Can this force be seen in atomic physics? Discuss it with some of the other students.

3. Can it give rise to $\nu - \bar{\nu}$ bound state in a way similar to the way the photon does it for $e^+ - e^-$?

Question 3: Exotic light quarks

Here we study a model that is of historical importance. Before the charm was discovered, only three quarks were known, the up, down and strange. One reason that the charm was predicted was that without it there should have been large FCNCs. Here we discuss some ideas of how to have a world with just three quarks and see why it does not work.

We consider the following model:

(i) The symmetry is a local

$$SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{7}$$

(ii) The quarks fields are

$$Q_L(3,2)_{+1/6}, \quad S_L(3,1)_{-1/3}, \quad U_R(3,1)_{+2/3}, \quad D_R(3,1)_{-1/3}, \quad S_R(3,1)_{-1/3}, \tag{8}$$

and the leptons are like in the two generation SM:

$$L_L^i(1,2)_{-1/2}, \quad E_R^i(1,1)_{-1}, \qquad i = 1,2. \tag{9}$$

(iii) We have one scalar multiplet, the Higgs field,

$$\phi(1,2)_{+1/2} \tag{10}$$

where, we assume that the Higgs acquires a vev like in the SM.

We further denote the upper (lower) component of Q_L as U_L (D_L) and the of L_L as ν_L (e_L).

1. Write down
   (a) The covariant derivative.
   (b) The gauge interactions of the quarks with the charged W bosons (before SSB).
   (c) The Yukawa interactions (before SSB).
   (d) The bare mass terms, that is $\mathcal{L}_\psi$.
   (e) The mass terms after SSB.

   Do it only for the quarks.

2. Show that there are 4 physical flavor parameters in this model. They are 3 masses and one mixing angle that we denote by θ_L.

3. Write down the gauge interactions of the LH-quarks with the Z and W bosons in both the interaction basis and the mass basis. Show that generally the Z coupling to LH-fermion is not diagonal, that is that things like $Zd\bar{s}$ couplings is there.

4. We define
$$R_q \equiv \frac{\Gamma(s \to d\mu^+\mu^-)}{\Gamma(s \to u\mu^+\nu)}. \tag{11}$$
Calculate R_q as a function of the model parameters, keeping only the leading effects.

5. Historically, the model was tested in kaon decays. While we did not discuss mesons yet, all we need to do for this question is that $R_q \approx R_H$ where
$$R_H \equiv \frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K^+ \to \mu^+\nu)}. \tag{12}$$
Find in the PDG the experimental value of R_H, and using the fact that experimentally $\sin\theta_L \sim 0.2$, explain why this result practically ruled out the model. (While the data before the charm was discovered were not as good, it was good enough to exclude the model even then.)