Fundamental Physics from the Precision Frontier

Problem 1: long-range fifth forces

(a) Show that, if a scalar field ϕ of mass μ is sourced by a spherically symmetric object of radius R, centred at the origin, then

$$\phi = C\frac{e^{-\mu r}}{r} \quad \text{for } r > R \tag{1}$$

(b) Suppose that, in a laboratory experiment, the gravitational force between two small objects separated by 1 metre is found to be $0.25(1 \pm 10^{-2})$ of the force when separated by 50 cm. Sketch (approximately) the constraints from this experiment on the coupling of a scalar ϕ to nuclei, $H_{\text{int}} = g\phi\bar{n}n$, as a function of the mass of ϕ (assume that ϕ couples to protons and neutrons roughly equally).

As (b) illustrates, laboratory tests can only place weak bounds on very-long-range fifth forces from their non-inverse-square behaviour. Tests of solar system dynamics can provide stronger constraints, but even these become weak when the range of ϕ exceeds solar system scales.

However, as discussed in the lectures, if ϕ couples to different objects in a way that is not proportional to their inertial mass, then there are stringent bounds on how strongly ϕ can couple, from tests of the weak equivalence principle. These bounds apply even if ϕ has vanishing mass. This raises the question of whether such bounds can be avoided.

The simplest solution would be to couple ϕ to T^{00}, but that would not be Lorentz-invariant. Instead, we can use the fact that, for non-relativistic matter, $T^\mu_\mu \simeq -T^{00}$, and make our coupling $g\phi T^\mu_\mu$. However, it is not obvious that this is stable under renormalisation. For example, EM fields have $T^\mu_\mu = 0$, but can make significant contributions to the masses of particles. If we specify the coupling $g\phi T^\mu_\mu$ at some energy scale, does ϕ couple to macroscopic objects in a EP-preserving way?

(c) Suppose that we have a localised system which is periodic in time (for example, EM waves in a cavity). Show that, in flat spacetime,

$$\int d^3x\, T^{00} = -\left\langle \int d^3x\, T^\mu_\mu \right\rangle \tag{2}$$

where the angled brackets denote time averaging.

An analogous argument at the quantum level shows that, after integrating out physics at smaller scales, the couplings of ϕ to larger objects remain weak-EP-preserving. Given that, how can we look for a long-range ϕ with this coupling?

(d) Suppose a pulsar and a black hole form a binary system (for simplicity, we will assume that they have the same mass M, and a circular orbit at separation L). If $L \gg MG$, what is the power emitted in gravitational waves? If, in addition to gravity, these is a very light scalar ϕ with coupling $\frac{\beta}{M_{pl}}\phi T^{\mu}_{\mu}$, what is the approximate power emitted in ϕ waves? (feel free to ignore numerical constants)