Questions on Dark Matter Production and Direct Detection

1. Asymmetric dark matter

Suppose that the dark matter is a Dirac fermion, and there is an asymmetry in the dark matter sector analogous to that in the baryon sector, so the abundance of dark matter and anti-dark-matter is different. In this case, either the depletion of the less abundant component (as for the baryons) *or* the expansion of the universe (as for conventional WIMP dark matter, as we have discussed in lectures) can cut off the annihilation processes that deplete the dark matter density. This idea has gained considerable interest over the last few years, in part because of the apparent coincidence between the amounts of dark and baryonic matter in the universe – if their abundances are set by entirely different processes, why are they only different by a factor of five or so?

In this problem we will work out the freeze-out of annihilations for this scenario.

(a) Let us denote the DM by χ^+ and the anti-DM by χ^-. Without loss of generality, we will assume χ^+ is more abundant than χ^-. Let the average $\chi^+\chi^-$ annihilation cross section be given by $\langle\sigma v\rangle$. Justify (qualitatively, in words) the coupled Boltzmann equations for this system:

$$\frac{dn^\pm}{dt} + 3Hn^\pm = -\langle\sigma v\rangle(n^+n^- - n^+_{\rm eq}n^-_{\rm eq}).$$

Here n describes the number density of the two species and the subscript "eq" refers to an equilibrium value. (We are assuming here that the annihilation reaction is the only number-changing reaction for either DM or anti-DM.)

(b) Let us define $Y^\pm = n^\pm/s$, where s is the entropy density of the universe. You may assume that entropy is conserved, so $s \propto a^{-3}$. Let $\eta = Y^+ - Y^-$. Show that η is conserved by the Boltzmann equations from (a), and explain the physical reason for this behavior.

(c) Define the fractional asymmetry $r = n^-/n^+$. Note by definition $0 \le r \le 1$. What is the annihilation rate in terms of the overall DM density $n = n^+ + n^-$ and r? Comment on the limits $r = 1$ and $r = 0$.

(d) Assume the universe is radiation dominated so $H(T) \propto T^2$ and $T \propto 1/a$; you may assume no significant change in g_* over the time period of greatest interest. Define $r_{\rm eq} = n^-_{\rm eq}/n^+_{\rm eq}$, and $x = m/T$. Show that the Boltzmann equations yield a dynamical equation for r of the form:

$$\frac{dr}{dx} \propto -\eta\langle\sigma v\rangle x^{-2}\left[r - r_{\rm eq}\left(\frac{1-r}{1-r_{\rm eq}}\right)^2\right] \propto -\eta\langle\sigma v\rangle x^{-2}\left[r - \frac{Y^+_{\rm eq}Y^-_{\rm eq}}{\eta^2}\left(1-r^2\right)^2\right].$$

(e) As the DM freezes out, the $Y_{\rm eq}$ terms in the above equation will be exponentially suppressed (by the usual Boltzmann suppression). In the limit where this term approaches zero, solve the resulting differential equation for the late-time value of r. How does it depend on the annihilation cross section? Comment.

2. Astrophysics-independent comparisons in direct detection

As mentioned in lecture, the usual calculation for direct detection makes several assumptions about the relevant particle physics, nuclear physics and astrophysics. In this problem, we will look at one way to factor out the unknown astrophysics, given certain assumptions on the particle physics model.

For elastic scattering, we will write the differential rate with respect to recoil energy in the form:

$$\frac{dR}{dE_R} = \frac{N_T M_T \rho}{2m_\chi \mu^2}\sigma(E_R)\int_{v_{\rm min}}^{\infty} dv \frac{f(v,t)}{v}. \tag{1}$$

For the purposes of this problem, ignore both the motion of the Sun and the motion of the Earth around the Sun, and assume the DM velocity distribution in the Galactic frame is isotropic.

(a) Suppose a candidate WIMP event is detected in a silicon detector with recoil energy 12.3 keV (you may round all atomic masses to the nearest integer), and interpreted as the scattering of a 10 GeV WIMP. What is the smallest possible speed of the WIMP in question, in the lab frame?

(b) Suppose the scattering occurs at that minimum velocity. What is the maximum recoil energy of an identical WIMP (with the same velocity) scattering in a detector with target material: (i) Xenon, (ii) Sodium, (iii) Iodine, (iv) Germanium.

(c) Suppose a differential rate $dR/dE_R = K(E_R)$ is measured at one experiment, with target mass M_T^1, target number N_T^1 and cross section $\sigma_1(E_R)$. By writing $\int_{v_{\min}}^{\infty} dv \frac{f(v,t)}{v}$ in terms of $K(E_R)$, one can immediately predict the differential rate at a different experiment at the energy corresponding to the same $v_{\min}$. Use this idea to write down the predicted differential rate as a function of recoil energy at an experiment with target mass M_T^2, target number N_T^2 and cross section $\sigma_2(E_R)$, eliminating all dependence on the function $f(v)$.

This approach has been used to perform "astrophysics-independent" comparisons between possible signals (and constraints) at different experiments, although it does rely on knowing the kinematics of the interaction.