Fluid Mechanics: Worksheet 1

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Set up

1. Download Pplane8.m for your computer from:
   https://www.mathworks.com/matlabcentral/fileexchange/61636-pplane
2. Run it!

Exercises

We consider the Lotka-Volterra, predator-prey model

$$\begin{cases} \dot{x} &= ax - bxy - cx^2 \\ \dot{y} &= dy + exy - jy^2. \end{cases} \tag{1}$$

For the following set of parameters

- a=2, b=2, c=0, d=-1, e=1, j=0
- a=2, b=2, c=1, d=-1, e=1, j=0
- a=2, b=2, c=1, d=-1, e=2, j=0
- a=2, b=2, c=0, d=-1, e=1, j=1
- a=1, b=8, c=1, d=-1, e=4, j=0

perform the following actions:

1. Identify the steady state of the system. That is to say find all the possible solutions to
$$\begin{cases} 0 &= ax - bxy - cx^2 \\ 0 &= dxy - ey + jy^2. \end{cases} \tag{2}$$
2. Linearize the system around each steady state:
   Given a steady state (x_0, y_0) find
$$A_0 = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix},$$

such that (**??**) behaves like

$$\begin{cases} \dot{\tilde{x}} &= a_{1,1}\tilde{x} + a_{1,2}\tilde{y} \\ \dot{\tilde{y}} &= a_{1,2}\tilde{x} + a_{2,2}\tilde{y}. \end{cases} \tag{3}$$

where $\tilde{x} = x - x_0$ and $\tilde{y} = y - y_0$.

Compute the eigenvalues of the matrix A_0:

Step 1. Input the matrix A_0 into matlab:

First initialize:

$$A0 = zeros[2,2]$$

Then, input:

$$A0[1,1] = a_{1,1}$$
$$A0[1,2] = a_{1,2}$$
$$A0[2,1] = a_{2,1}$$
$$A0[2,2] = a_{2,2}$$

Step 2. Calculate the eigenvalues:

$$eig(A_0)$$

Are they positive, negative, complex? What type of equilibria is (x_0, y_0).

3. Using pplane plot the phase diagrams.
4. Use pplane solutions tab to find the steady states of the system. Do they coincide with the ones you calculated?
5. Use the solutions tab to plot a trajectory close to the steady state. What is the relationship between the eigenvalues of A_0 and the behavior of the trajectories?
6. Use pplane to find the nullclines.
7. Use the solutions tab and plot trajectories with initial conditions on each side of the nullclines.
8. Is any of the species doomed for extinction? Is there initial conditions in which both species become extinct?