Exercises (J. Pekola)

1. (a) Show that electric current through a tunnel barrier between usual metals (constant density of states) is temperature independent.

(b) Derive the corresponding expression for the junction where one of the electrodes is superconducting

$$N_S(E) = \begin{cases} N_0 \dfrac{|E|}{\sqrt{E^2-\Delta^2}}, & |E| > \Delta \\ 0, & |E| < \Delta \end{cases}.$$

(Find an analytic expression when temperature $T \ll \Delta/k_B$, and $|V| \ll \Delta/e$.)

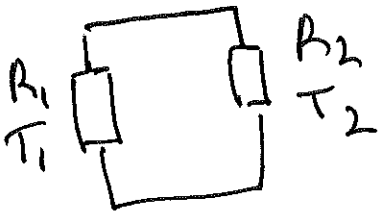
(c) Repeat (a) and (b) for heat currents to the electrodes.

Conclusion: normal system is not a thermometer neither a refrigerator, unlike the normal–superconductor hybrid.

2. Show that the heat current $\dot{Q}$ between two resistors R_1 and R_2 at temperatures T_1 and T_2 is given by

$$\dot{Q} = \frac{4R_1R_2}{(R_1+R_2)^2}\,\frac{\pi k_B^2}{12\hbar}\,(T_1^2 - T_2^2).$$

(Photonic heat current)



Thus for small temperature difference $\Delta T \equiv T_1 - T_2$ and for $R_1 = R_2$ we have

$$\dot{Q} = G_q \Delta T,$$

where

$$G_q = \frac{\pi k_B^2}{6\hbar} T$$

("quantum of heat conductance.")