Construction and Floquet realization of counter-diabatic protocols in quantum and classical systems

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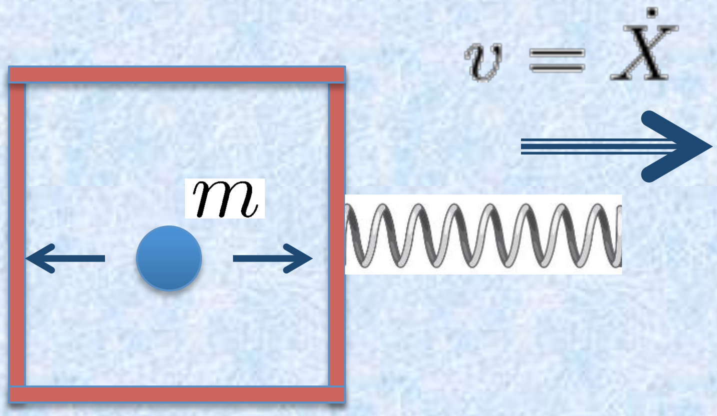
Classical Hamiltonian Dynamics in a moving frame



Lab frame: need to deal with a time dependent potential (hard)

$$H(p, x, t) = \frac{p^2}{2m} + V(x - X(t))$$

Easier way: go to the moving frame (Galilean transformation):

$$x' = x - X(t),\ p' = p, \quad H' = \frac{p'^2}{2m} + V(x') - \dot{X} p'$$

Rotating frame

$$x' = x\cos(\theta(t)) - y\sin(\theta(t)),$$
$$y' = y\cos(\theta(t)) + x\sin(\theta(t))$$

$$H \to H' = H - \dot{\theta} L$$

Moving frame in general: time-dependent canonical transformation

Gauge potentials as generators of canonical transformations.

$$\frac{\partial x_i}{\partial \lambda_a} = -\frac{\partial \mathcal{A}_a}{\partial p_i} = \{\mathcal{A}_a, x_i\}$$

$$\frac{\partial p_i}{\partial \lambda_a} = \frac{\partial \mathcal{A}_a}{\partial x_i} = \{\mathcal{A}_a, p_i\}$$

Translations

$$x = x_0 - X(t),\ p = p_0, \quad \Rightarrow \mathcal{A}_X = p$$

$$\frac{\partial x}{\partial X} = -1 = -\frac{\partial \mathcal{A}_X}{\partial p}, \quad \frac{\partial p}{\partial X} = 0 = \frac{\partial \mathcal{A}_X}{\partial x}$$

Equations of motion in a moving frame

$$\frac{dx_i}{dt} = \frac{\partial x_i}{\partial t} + \frac{\partial x_i}{\partial \lambda_a}\dot{\lambda}_a = \{x_i, H\} - \dot{\lambda}_a\{x_i, \mathcal{A}_a\} = \{x_i, H_m\}$$

$$\frac{dp_i}{dt} = \frac{\partial p_i}{\partial t} + \frac{\partial p_i}{\partial \lambda_a}\dot{\lambda}_a = \{p_i, H\} - \dot{\lambda}_a\{p_i, \mathcal{A}_a\} = \{p_i, H_m\}$$

Moving Hamiltonian $\quad H_m = H - \dot{\lambda}_a \mathcal{A}_a$

Quantum Systems

Need to solve a time dependent Schrodinger equation (hard)

$$i\hbar\partial_t|\psi\rangle = H(\lambda(t))|\psi\rangle$$

Do a time-dependent unitary transformation (= basis rotation)

$$|\psi\rangle = U(\lambda)|\tilde{\psi}\rangle$$

Plug in to the Schrodinger equation

$$i\hbar\partial_t|\tilde{\psi}\rangle = H_m(\lambda)|\tilde{\psi}\rangle$$

$$H_m = U^\dagger(H - \dot{\lambda}\mathcal{A}_\lambda)U, \quad \mathcal{A}_\lambda = i\hbar(\partial_\lambda U)U^\dagger$$

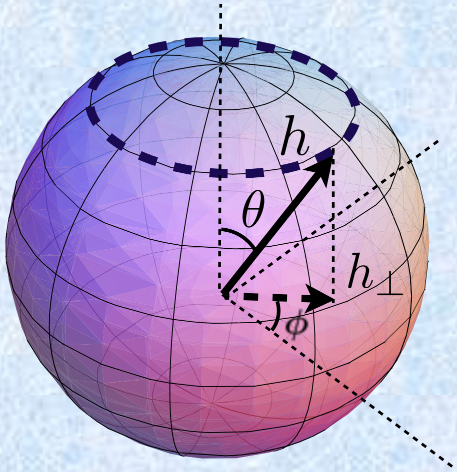
$\mathcal{A}_\lambda$ is the gauge potential (gauge connection)

$$\mathcal{A}_\lambda = i\hbar\partial_\lambda \quad \Leftrightarrow \quad i\hbar\langle n(\lambda)|\partial_\lambda|m(\lambda)\rangle = \langle n(\lambda)|\mathcal{A}_\lambda|m(\lambda)\rangle$$

Special frame: the one which diagonalizes H

$$H'_m = U^\dagger(H - \dot{\lambda}\mathcal{A}_\lambda)U, \quad \mathcal{A}_\lambda = i\hbar(\partial_\lambda U)U^\dagger$$

All dynamics: transitions, dissipation, inertia, Lorentz-Coriolis forces is encoded in the adiabatic gauge potential.



$$H = -h\cos\theta(t)\,\sigma_z - h\sin\theta(t)\,\sigma_x$$

$$U = \begin{pmatrix} \cos(\theta/2) & \sin(\theta/2) \\ \sin(\theta/2) & -\cos(\theta/2) \end{pmatrix}$$

$$H_m = H - \dot{\theta}\mathcal{A}_\theta = H - \dot{\theta}\frac{\hbar\sigma_y}{2}$$

Recover a standard transformation to the rotating frame. Can recover leading non-adiabatic effects from ordinary (adiabatic) perturbation theory.

Adiabatic transformations in quantum systems

$$H(\lambda)|n(\lambda)\rangle = E_n(\lambda)|n(\lambda)\rangle \quad \Rightarrow \quad U^\dagger(\lambda)H(\lambda)U(\lambda) = \sum_n E_n(\lambda)|n_0\rangle\langle n_0|$$

$$|n(\lambda)\rangle = U(\lambda)|n_0\rangle$$

$$\frac{d}{d\lambda}\left(U^\dagger(\lambda)H(\lambda)U(\lambda)\right) = U^\dagger(d_\lambda H + \frac{i}{\hbar}[\mathcal{A}_\lambda, H])U, \quad \mathcal{A}_\lambda = i\hbar(\partial_\lambda U)U^\dagger = \mathcal{A}_\lambda^\dagger$$

$\mathcal{A}_\lambda$ is the gauge potential - generator of adiabatic rotations, also connection

Combine:

$$\boxed{d_\lambda H + \frac{i}{\hbar}[\mathcal{A}_\lambda, H] = \sum_n (\partial_\lambda E_n)|n\rangle\langle n| \quad \Leftrightarrow \quad \left[d_\lambda H + \frac{i}{\hbar}[\mathcal{A}_\lambda, H], H\right] = 0}$$

A generalization of the Wilson-Wegner flow equation

Classical limit

$$\boxed{\{d_\lambda H - \{\mathcal{A}_\lambda, H\}, H\} = 0}$$

Generator of canonical transformations "diagonalizing"=preserving trajectories of a classical Hamiltonian

Three equivalent definitions; set $\hbar \to 1$

$$\mathcal{A}_\lambda = i(\partial_\lambda U)U^\dagger = i\partial_\lambda \quad i\partial_\lambda|n\rangle = \mathcal{A}_\lambda|n\rangle, \quad [d_\lambda H + i[\mathcal{A}_\lambda, H], H] = 0$$

Gauge potentials define adiabatic evolution of eigenstates

Hellmann-Feynman theorem (first order perturbation theory)

$$\langle n|\mathcal{A}_\lambda|m\rangle = i\langle n|\partial_\lambda|m\rangle = i\frac{\langle n|\partial_\lambda H|m\rangle}{E_m - E_n}$$

Adiabatic gauge potential has a problem of small denominators.

$$||\mathcal{A}_\lambda||^2 = \frac{1}{D}\sum_{n=1}^{D}\langle n|\mathcal{A}_\lambda^2|n\rangle_c = \frac{1}{D}\sum_{m\neq n}^{D}\frac{|\langle n|\partial_\lambda H|m\rangle|^2}{(E_n - E_m)^2}$$

Chaotic systems – ETH (RMT)

$$|\langle n|\partial_\lambda H|m\rangle| \sim \exp[-S/2],$$
$$\min|E_m - E_n| \sim \exp[-S]$$

$$||\mathcal{A}_\lambda||^2 \propto \exp[S]$$

In chaotic systems the gauge potential does not exists as a smooth differentiable operator. Classical chaotic systems: C. Jarzynski (1995). Corollary: existence of the gauge potential implies integrability.

$$\boxed{\mathcal{A}_{\lambda,nm} = i\hbar\frac{\langle n|\partial_\lambda H|m\rangle}{E_m - E_n} \quad \leftrightarrow \quad \mathcal{A}_{\lambda,mn} = \lim_{\epsilon\to 0^+}\int_0^\infty dt\, e^{-\epsilon t} e^{-i(E_m-E_n)t}\langle m|\partial_\lambda H|n\rangle}$$

$$\mathcal{A}_\lambda = \lim_{\epsilon\to 0^+}\int_0^\infty dt\mathrm{e}^{-\epsilon t}\left(e^{-iH(\lambda)t}\partial_\lambda H(\lambda)e^{iH(\lambda)t} - M_\lambda\right), \quad M_\lambda = \sum_n \partial_\lambda E_n|n\rangle\langle n|$$

Existence of the gauge potential (i.e. the problem of existence of adiabatic limit) is equivalent to absence of (exponential) operator growth (e.g. V. Khemani, A. Vishwanath, D. A. Huse).

Equivalently locality of adiabatic transformations is tied to the locality of the perturbation $\partial_\lambda H$ in the rotating frame (interaction picture).

One slide detour: gauge potentials and quantum (information) geometry.

Hamiltonian: $H = H(\vec{\lambda})$. Ground state wave-function: $\psi_0 = \psi_0(\vec{\lambda})$.

Consider the following change $\vec{\lambda} \to \vec{\lambda} + \delta\vec{\lambda}$

$$||\psi_0(\vec{\lambda}) - \psi_0(\vec{\lambda} + \delta\vec{\lambda})||^2 \approx 1 - |\langle\psi_0(\vec{\lambda})|\psi_0(\vec{\lambda} + \delta\vec{\lambda})\rangle|^2 = \chi_{\alpha\beta} d\lambda_\alpha d\lambda_\beta$$

$\chi_{\alpha\beta}$ - geometric tensor (Provost, Vallee, 1980)

$$\chi_{\alpha\beta} = \langle\partial_{\lambda_\alpha}\psi_0|\partial_{\lambda_\beta}\psi_0\rangle_c = \langle 0|\mathcal{A}_\alpha\mathcal{A}_\beta|0\rangle - \langle 0|\mathcal{A}_\alpha|0\rangle\langle 0|\mathcal{A}_\beta|0\rangle$$

$$g_{\alpha\beta} = \frac{1}{2}(\langle\partial_\alpha\psi|\partial_\beta\psi\rangle_c + \langle\partial_\beta\psi|\partial_\alpha\psi\rangle_c) = \frac{1}{2}\langle 0|\mathcal{A}_\alpha\mathcal{A}_\beta + \mathcal{A}_\beta\mathcal{A}_\alpha\rangle_c$$

Metric tensor. Defines the Riemannian metric structure, the fidelity susceptibility, the quantum Fisher information.

$$||\mathcal{A}_\lambda||^2 = \frac{1}{D}\sum_{n=1}^{D}\langle n|\mathcal{A}_\lambda^2|n\rangle_c = \overline{g_{\lambda\lambda}}$$

Berry curvature. Defines the effective magnetic field

$$F_{\alpha\beta} = -i(\chi_{\alpha\beta} - \chi_{\beta\alpha}) = -i\langle 0|[\mathcal{A}_\alpha, \mathcal{A}_\beta]|0\rangle$$

Hall response, topological invariants, Coriolis forces, Lorentz forces,…

Counter-diabatic driiving.

(M. Demirplak, S. A. Rice (2003), M. Berry (2009), S. Deffner, A. Del Campo, C. Jarzynski,.. (2010+), vast literature in NMR, fast-forward technique,…).

$$|\psi\rangle = \sum_n \psi_n(t)|n(\lambda)\rangle, \quad |n(\lambda)\rangle = U(\lambda)|n_0\rangle, \quad i\partial_t\psi = (H - \dot\lambda\mathcal{A}_\lambda)\psi$$

Moving frame Hamiltonian $H_m = H - \dot\lambda\mathcal{A}_\lambda$

Idea: introduce counter-diabatic (CD) term

$$H \to H_{\rm CD} = H + \dot\lambda\mathcal{A}_\lambda, \quad H_m^{\rm CD} = H,$$
$$\dot\lambda \to \infty \;\Rightarrow\; t \to \lambda,\ H_{\rm CD} \to \mathcal{A}_\lambda$$

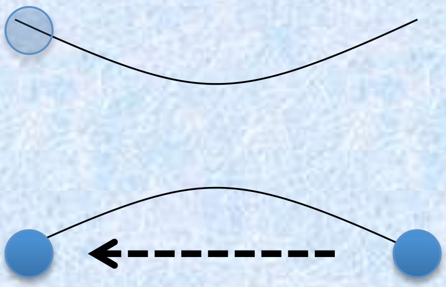
Suppress transitions, fast adiabatic state preparation, suppress dissipation.

A waiter implementing a CD driving protocol to avoid food spillage

No CD term

CD term

Landau Zener Problem (= rotating magnetic field)



$$H_{\mathrm{CD}} = -h_z\sigma_z - \lambda(t)\sigma_x + \frac{\dot{\lambda}h_z}{2(h_z^2+\lambda^2)}\sigma_y$$

Rotate around x-axis to eliminate y-field

$$R = \exp\left[-\frac{i}{2}\varphi(t)\sigma_x\right], \quad \varphi(t) = \arctan\left(\frac{\dot{\lambda}}{2(\lambda^2+h_z^2)}\right)$$

$$H_{\mathrm{FF}} = -h_z\sqrt{1+\frac{\dot{\lambda}^2}{4(h_z^2+\lambda^2)^2}}\sigma_z - (\lambda+\dot{\varphi}/2)\sigma_x = -\sqrt{1+\frac{\dot{\lambda}^2}{4(h_z^2+\lambda^2)^2}}\left[h_z\sigma_z + \frac{\lambda+\dot{\varphi}/2}{\sqrt{1+\frac{\dot{\lambda}^2}{4(h_z^2+\lambda^2)^2}}}\sigma_x\right]$$

Can redefine time to remove overall prefactor

$$dt' = dt\sqrt{1+\frac{\dot{\lambda}^2}{4(h_z^2+\lambda^2)^2}}, \quad H_{\mathrm{FF}} = -h_z\sigma_z - \frac{\lambda+\dot{\varphi}/2}{\sqrt{1+\frac{\dot{\lambda}^2}{4(h_z^2+\lambda^2)^2}}}\sigma_x;$$

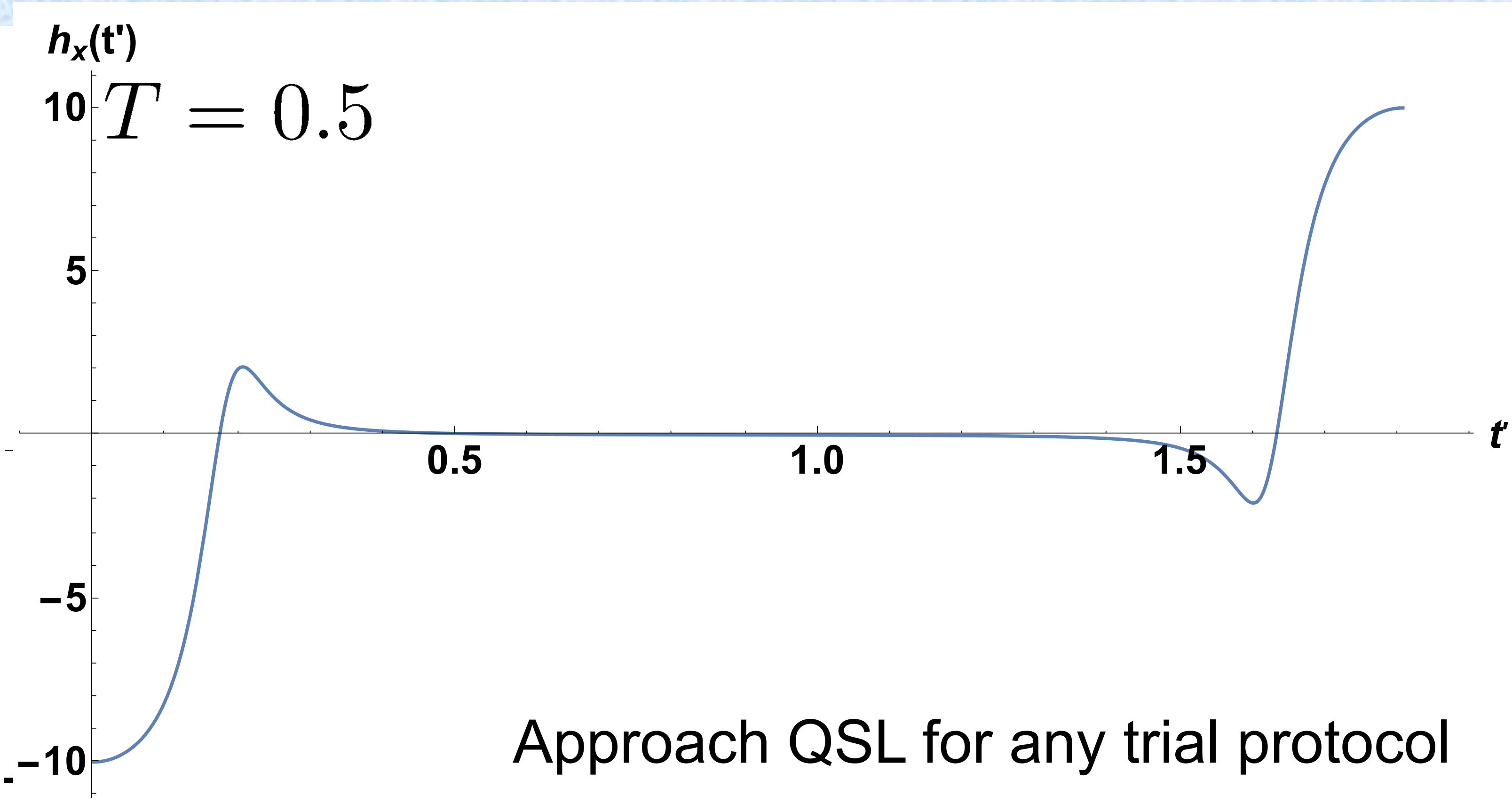
$$\dot{\lambda}\to\infty \quad \Rightarrow \quad H_{\mathrm{FF}}(t') = -h_z\sigma_z + \frac{\pi}{4}(\delta(t)-\delta(T-t))\sigma_x, \quad T = T_{SQL} = \frac{2\delta\theta}{h_z}$$

Can generate many FF protocols (glassy landscape)

$$t'(t) = \int_0^t dt_1 \sqrt{1 + \frac{\dot{\lambda}^2}{(h_z^2 + \lambda^2)^2}}, \quad h_x(t) = \frac{\lambda + \dot{\varphi}/2}{\sqrt{1 + \frac{\dot{\lambda}^2}{4(h_z^2+\lambda^2)^2}}}, \quad H_{\mathrm{FF}} = -h_z\sigma_z - h_x(t)\sigma_x$$

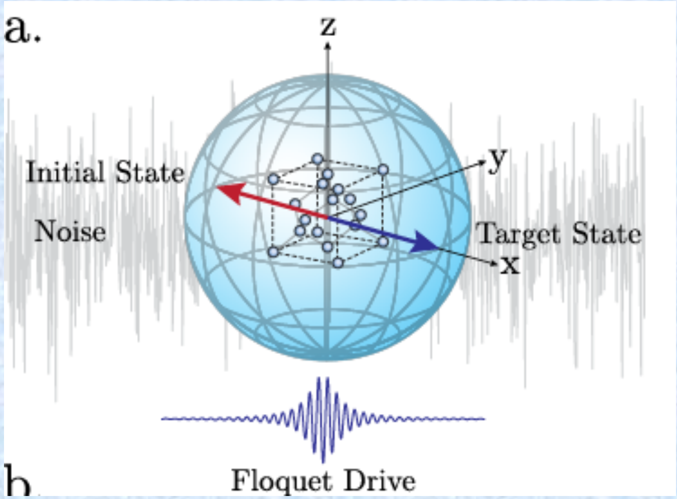
Choose some $\lambda(t)$, e.g. $\lambda(t) = -\lambda_0 + 2\lambda_0 \sin^2\left(\frac{\pi}{2}\sin^2\left(\frac{\pi}{2}\frac{t}{T}\right)\right)$

Find $h_x(t)$, $t'(t)$ and plot $h_x(t')$. $\quad h_z = 1,\ \lambda : -10 \to 10$



Floquet Realization of the FF protocol

$$H_{\rm CD} = -h_z\sigma_z - \lambda(t)\sigma_x + \frac{\dot\lambda h_z}{2(h_z^2+\lambda^2)}\sigma_y$$



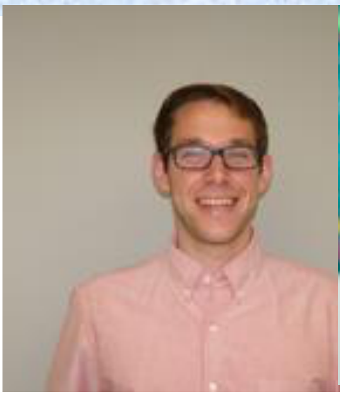
Can engineer y-field, by shaking x and z fields

$$H_{\rm FFE} = -h_z\left(1 - \frac{\mathcal{J}_0(2\kappa)}{2\mathcal{J}_1(2\kappa)}\frac{\dot\lambda\cos\omega t}{(\kappa\mathcal{J}_0(2\kappa))^2+\lambda^2}\right)\sigma_z - (\lambda + \kappa\omega\sin\omega t)\sigma_x$$

In the leading order of thee inverse frequency expansion

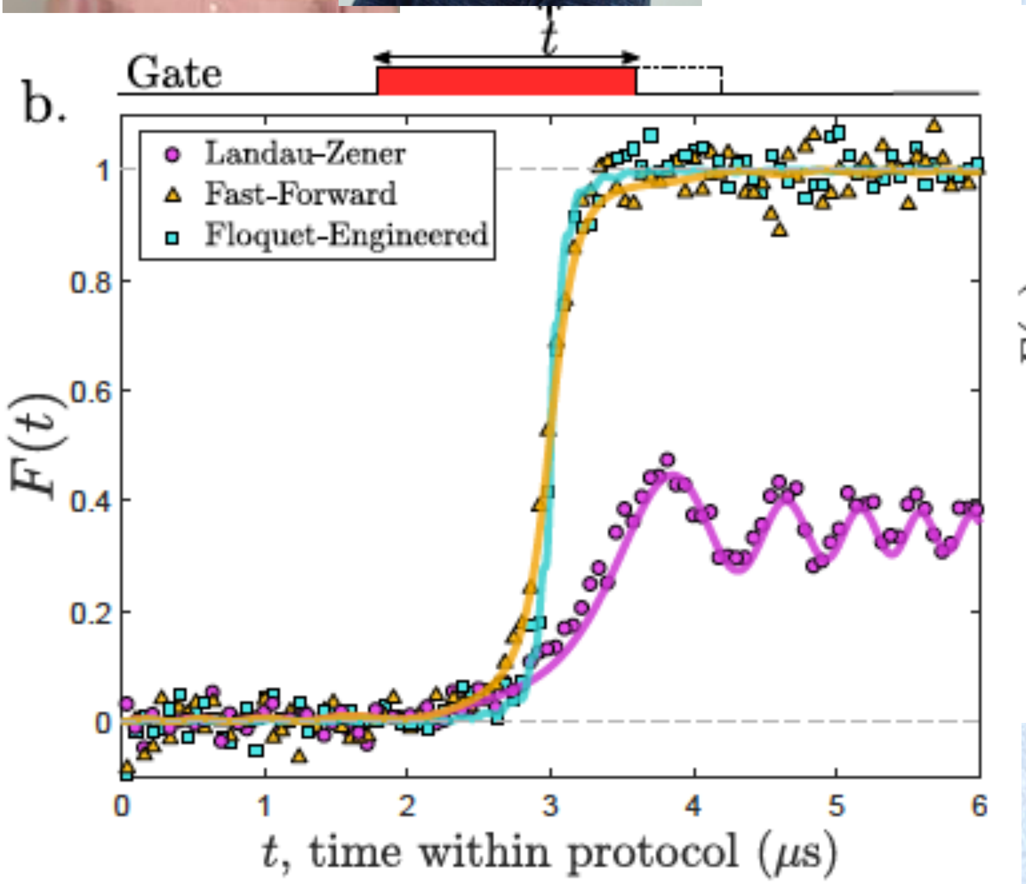
$$H_F = H_{\rm CD} + O(1/\omega)$$

Can use the Floquet engineering to recreate the CD Hamiltonian without introducing new controls.

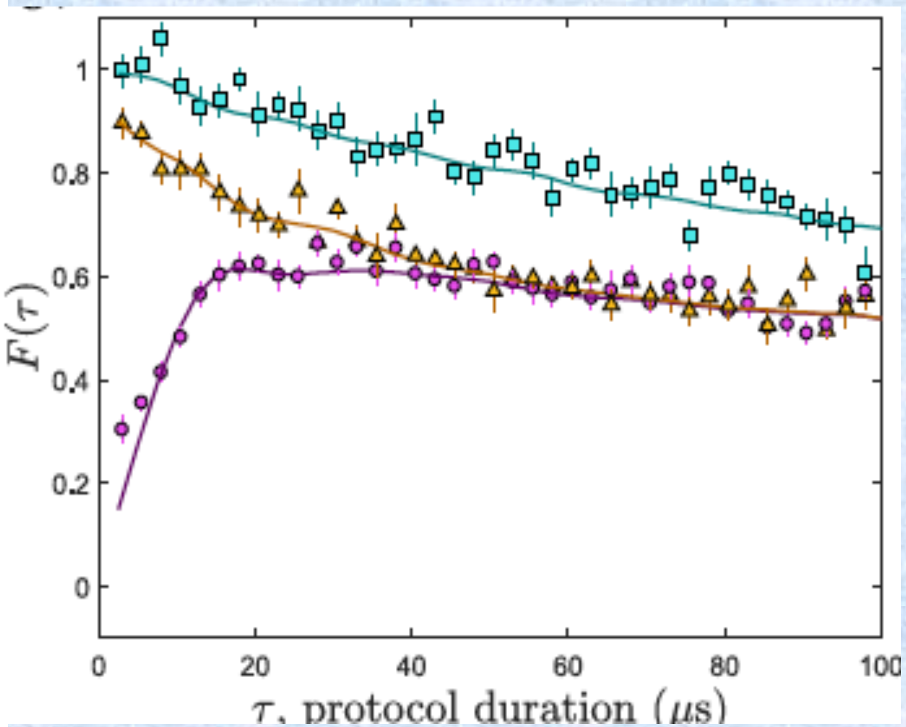


NV center realization

(E. Boyers, ···, A. Sushkov, PRA 2019)



Performance of different protocols



Noise dependence

Floquet protocol offers stability with respect to noise.

Finding Gauge potentials

Need to solve

$$\boxed{[\partial_\lambda H + \frac{i}{\hbar}[\mathcal{A}_\lambda, H], H] = 0 \quad \leftrightarrow \quad \mathcal{A}_{\lambda,nm} = i\hbar \frac{\langle n|\partial_\lambda H|m\rangle}{E_m - E_n}}$$

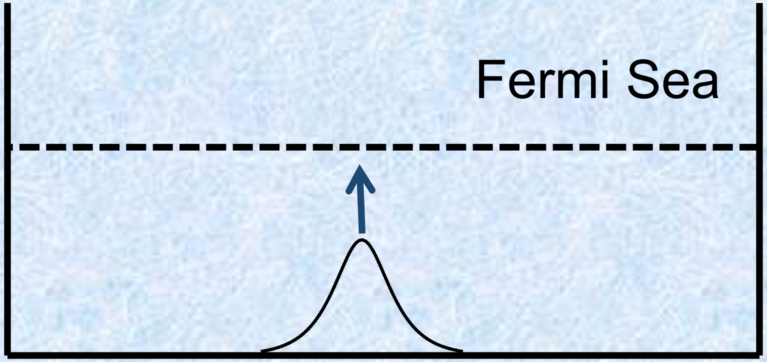
Finding the gauge potential is equivalent to the minimization problem

$$\boxed{[\partial_\lambda H + \frac{i}{\hbar}[\mathcal{A}_\lambda, H], H] = 0 \quad \leftrightarrow \quad \frac{\delta \mathrm{Tr}\left[(\partial_\lambda H + \frac{i}{\hbar}[\mathcal{A}_\lambda, H])^2\right]}{\delta \mathcal{A}_\lambda} = 0}$$

Can develop a variational procedure for finding gauge potentials (D. Sels, A.P., PNAS 2016).

Can use this result to devise a variational procedure to find an approximate (local) gauge potential.

Example: quantum jumper of fighting the Anderson Orthogonality Catastrophy (semi-open system)



$$H = H_0 + \sum_j \lambda v_j c_j^\dagger c_j, \quad \mathcal{A}_\lambda^* = i \sum_j \alpha_j(\lambda)(c_j^\dagger c_{j+1} - c_{j+1}^\dagger c_j)$$

Exact gauge potential will contain arbitrary range hoping terms

Result of the minimization: solution of the Laplace equation

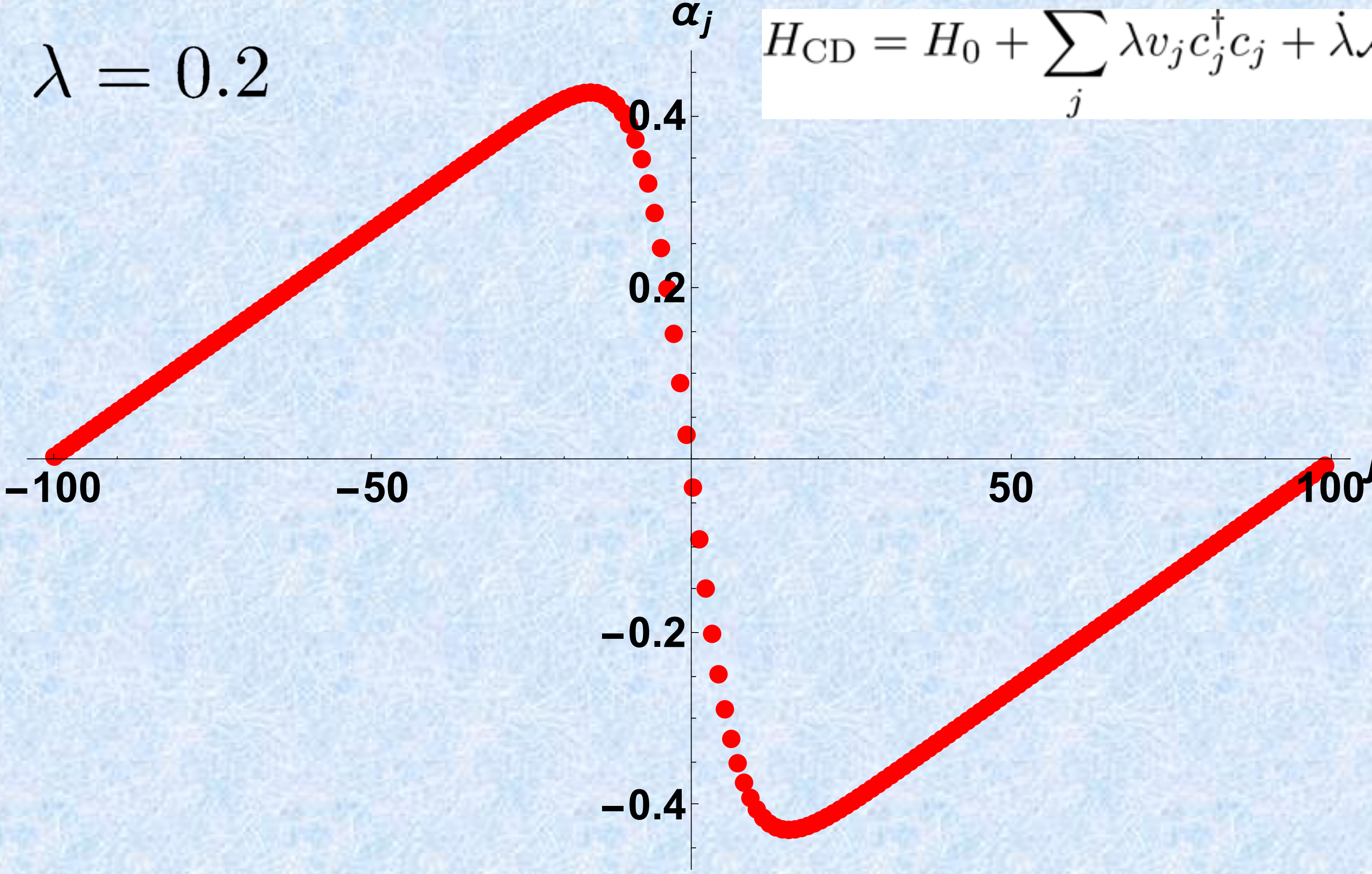
$$-3\Delta\alpha + \lambda^2 (\nabla_j v)^2 \alpha = \lambda \nabla_j v_j$$

$$V(j,\lambda) = \frac{\lambda}{\cosh^2(j/8)}$$

$$\mathcal{A}^*_\lambda = i\sum_j \alpha_j(\lambda)(c^\dagger_j c_{j+1} - c^\dagger_{j+1}c_j)$$

$$H_{\mathrm{CD}} = H_0 + \sum_j \lambda v_j c^\dagger_j c_j + \dot{\lambda}\mathcal{A}^*_\lambda$$

$\lambda = 0.2$



Like with a waiter: doable but difficult. Can map to FF protocol using the Peierls transformation.

$$H_{\rm CD}=H_0+\dot\lambda\mathcal{A}^*_\lambda=-J\sum_j\left[c^\dagger_{j+1}c_j\left(1-i\frac{\alpha_j\dot\lambda}{J}\right)+c^\dagger_jc_{j+1}\left(1+i\frac{\alpha_j\dot\lambda}{J}\right)\right]+\sum_j V_j(\lambda)c^\dagger_jc_j$$

Perform a phase (Peierls) transformation: $c_j\to c_j\mathrm{e}^{-i\varphi_j}$

$$H_{\rm FF}=-\sum_j J_{\rm eff}(j)\left(c^\dagger_{j+1}c_j+c^\dagger_jc_{j+1}\right)+\sum_j V_{\rm eff}(j)c^\dagger_jc_j,$$

$$V_{\rm eff}(j)=V(\lambda,j)-\ddot\lambda\sum_{k=-L}^{j}\frac{\alpha(k)}{1+\dot\lambda^2(\alpha(k))^2},\quad J_{\rm eff}(j)=\sqrt{1+\dot\lambda^2(\alpha(j))^2},$$

The imaginary CD protocol is only sensitive to velocity. Real FF protocol also knows about acceleration

Small velocity: potential renormalization (slowing particles in front)

Large velocity: need to locally renormalize hopping = local time rescaling or the local refraction index (creating a kind of black hole)

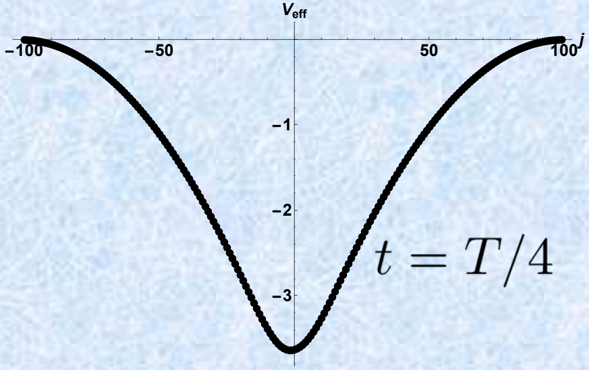
Can use Floquet engineering to design complex hopping

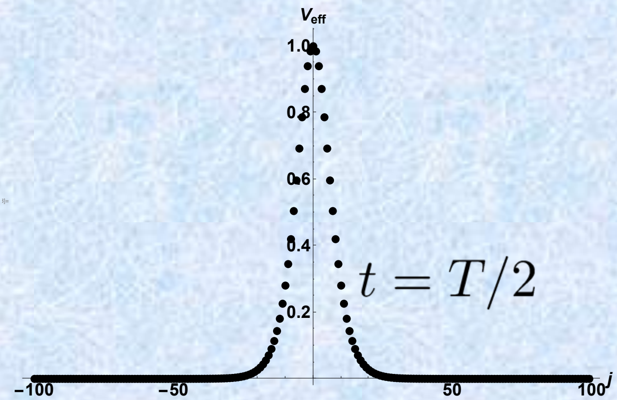
Snapshots of an effective potential and tunneling in FF protocol

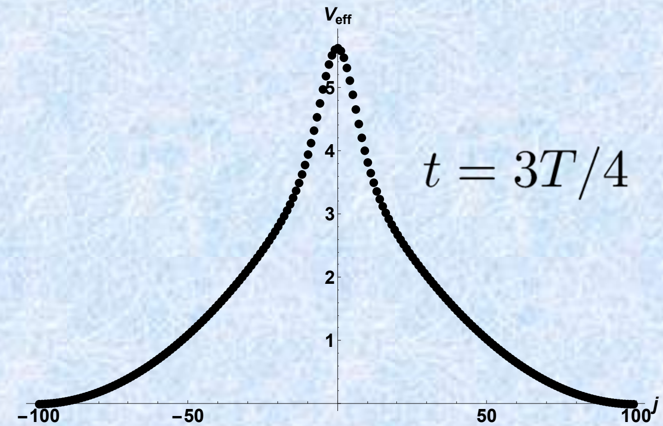
$$V(j,\lambda) = \frac{\lambda}{\cosh^2(j/8)}$$

$$\lambda(t) = 2\sin^2\left(\frac{\pi}{2}\sin^2(\pi t/2T)\right), \quad T = 10;$$

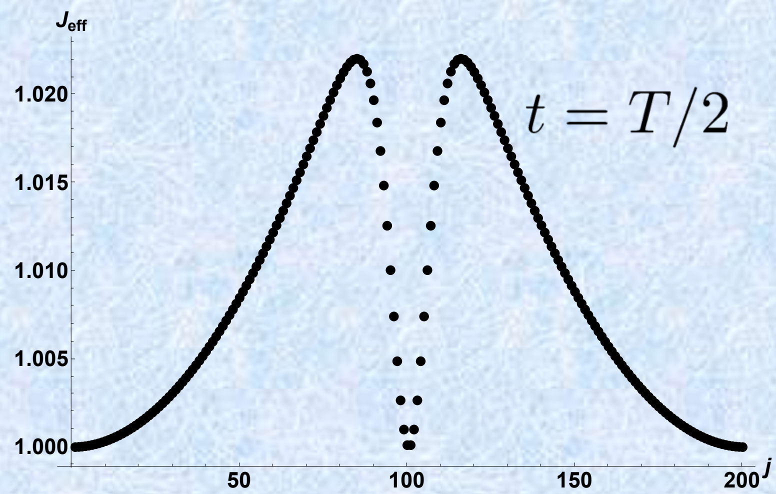
Potential profiles:





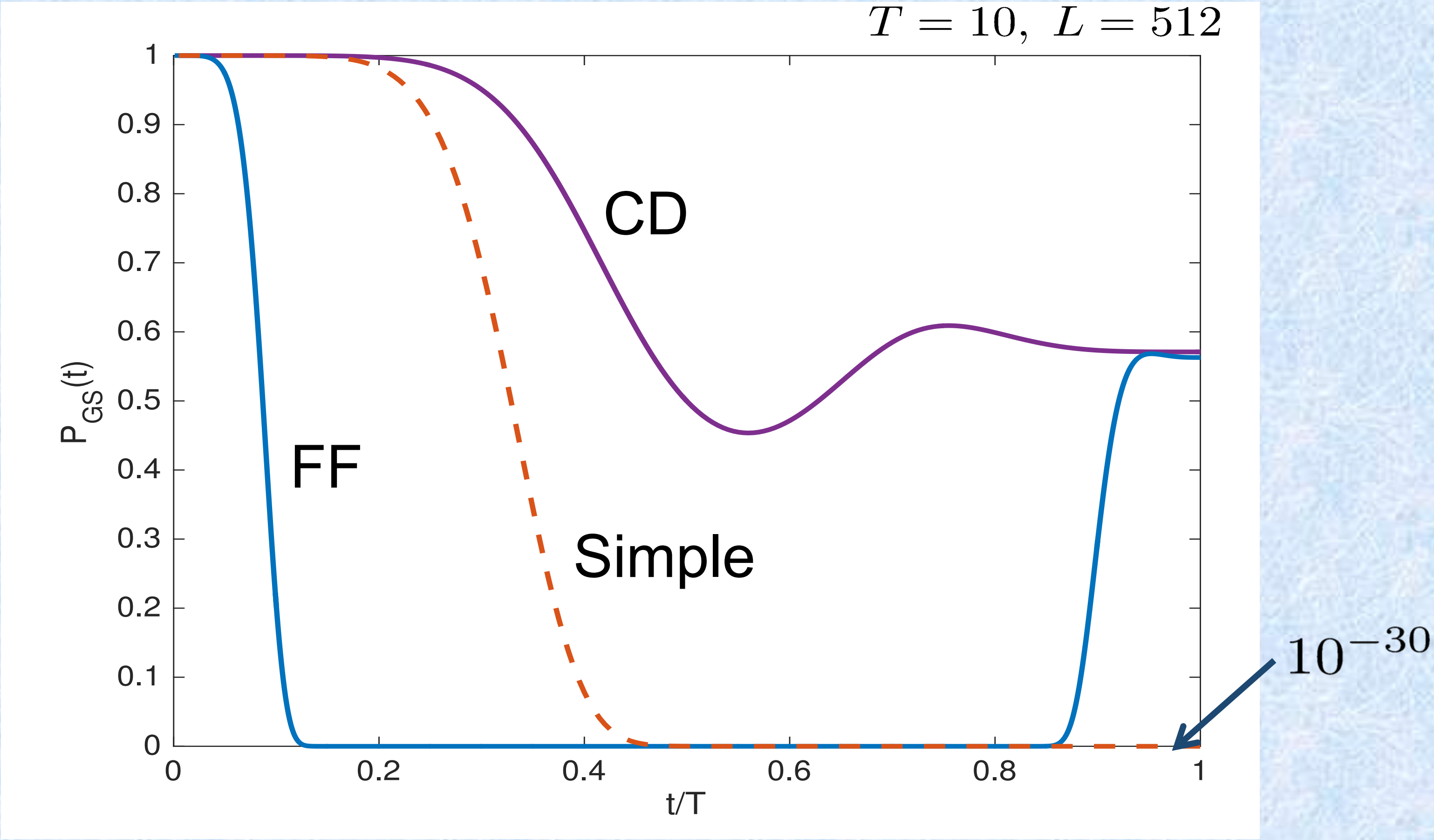


Tunneling profile
(less sensitive to time).



Numerics: half filling, 512 sites

$$V(j,\lambda) = \frac{\lambda(t)}{\cosh^2(j/\xi)} \qquad \lambda(t) = 2\sin^2\left(\frac{\pi}{2}\sin^2(\pi t/2T)\right)$$



Create a very efficient quantum jumper.
Beat Anderson Orthogonality Catastrophe.

Go back to the operator expansion picture

$$\boxed{\mathcal{A}_{\lambda,nm} = i\hbar\frac{\langle n|\partial_\lambda H|m\rangle}{E_m - E_n} \quad \leftrightarrow \quad \mathcal{A}_{\lambda,mn} = \lim_{\epsilon\to 0^+}\int_0^\infty dt\, e^{-\epsilon t} e^{-i(E_m - E_n)t}\langle m|\partial_\lambda H|n\rangle}$$

$$\mathcal{A}_\lambda = \lim_{\epsilon\to 0^+}\int_0^\infty dt e^{-\epsilon t}\left(e^{-iH(\lambda)t}\partial_\lambda H(\lambda)e^{iH(\lambda)t} - M_\lambda\right), \quad M_\lambda = \sum_n \partial_\lambda E_n |n\rangle\langle n|$$

Baker Campbell Housdorff formula:

$$e^{-iHt}\partial_\lambda H e^{iHt} = \sum_{k=0}^\infty \frac{(-it)^k}{k!}\underbrace{[H,[H,\dots[H}_{k},\partial_\lambda H],$$

Even order commutators define the generalized forces M_λ, odd order commutators define the gauge potential. Final ansatz (related ideas M. Hastings 2010)

$$\mathcal{A}_\lambda = i\sum_{k=1}^{k_{\max}} \alpha_k \underbrace{[H,[H,\dots[H}_{2k-1},\partial_\lambda H]],$$

Very few variational parameters. This ansatz reproduces exact gauge potential in all solvable cases. Can be used both to prepare the states and study geometry. Regularizes the locator expansion.

$$\mathcal{A}_\lambda = i \sum_{k=1}^{k_{\max}} \alpha_k \underbrace{[H,[H,\dots[H}_{2k-1},\partial_\lambda H]],$$

Exact gauge potential

$$\mathcal{A}_{\lambda,nm} = i\frac{\langle n|\partial_\lambda H|m\rangle}{E_m - E_n}$$

Variational gauge potential

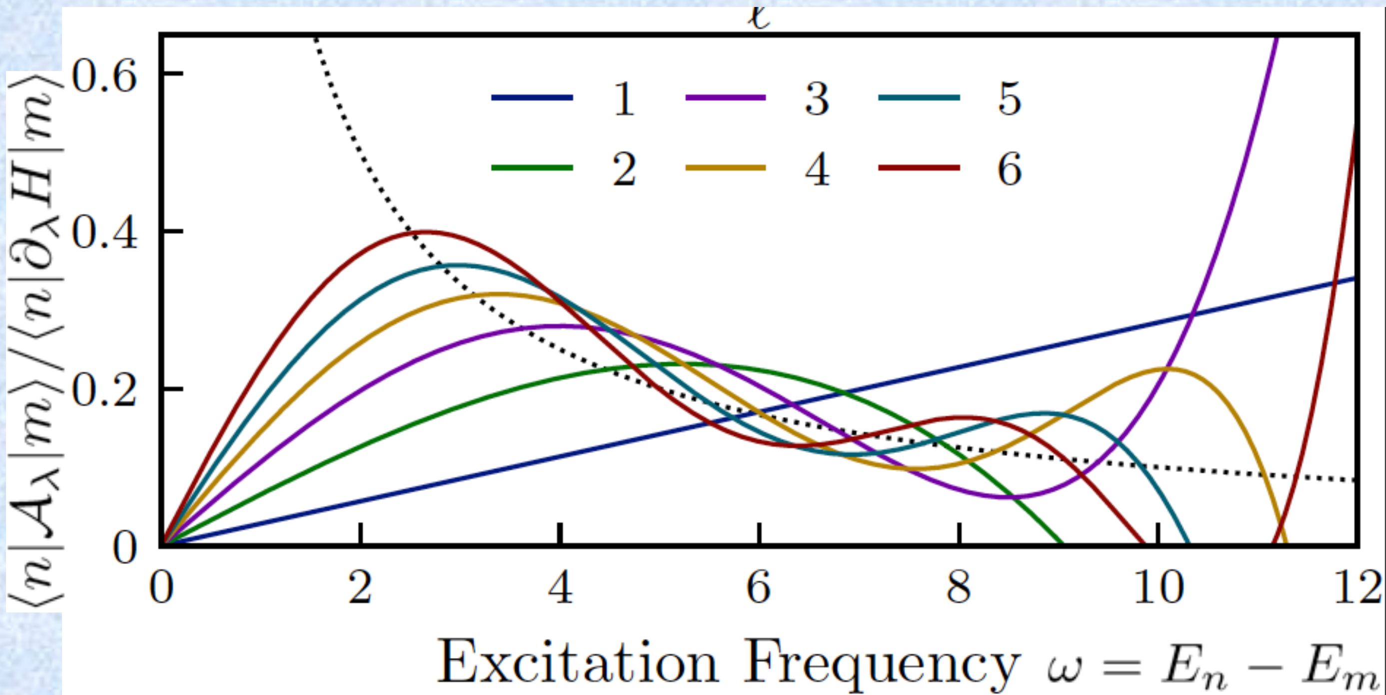
$$\mathcal{A}_{\lambda,nm} = i\sum_{k=1}^{k_{\max}} \alpha_k (E_n - E_m)^{2k-1}\langle n|\partial_\lambda H|m\rangle$$

We are finding best polynomial expansion of the function 1/x. The expansion is almost insensitive to the actual matrix elements

$$\frac{1}{x} \sim \sum_{k=1}^{k_{\max}} \alpha_k x^{2k-1}$$

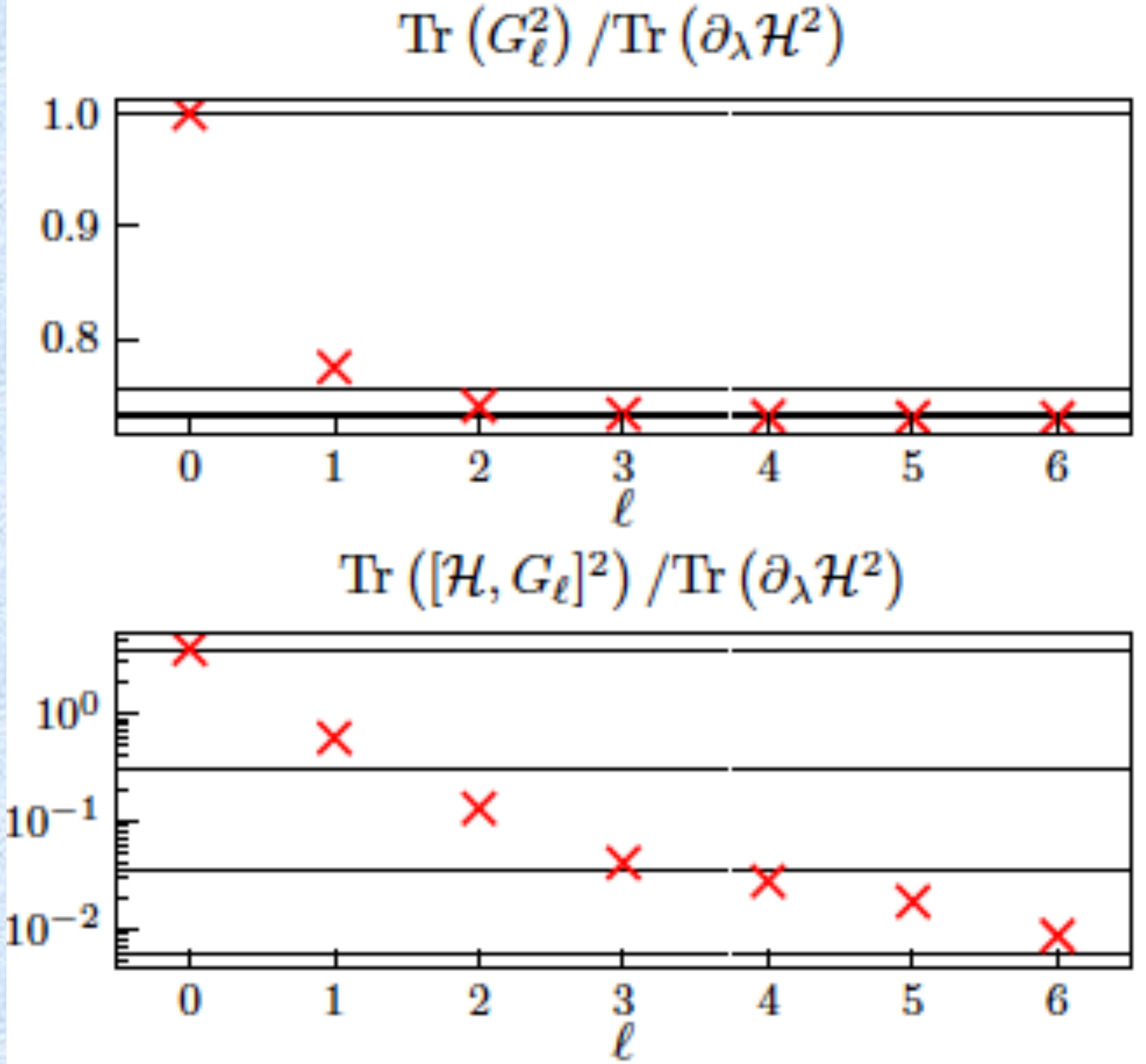
High frequencies – small matrix element, small frequency – expansion order.

Do not really need the variational principle.



$$H = J\sum_{j=1}^{L}\sigma_z^j\sigma_z^{j+1} + \lambda\left(h_z\sum_{j=1}^{L}\sigma_z^j + h_x\sum_{j=1}^{L}\sigma_x^j\right)$$

$$h_x = h_z = 0.3, \quad \lambda = 1$$



Can work in TD limit.

Pretty fast convergence, at least initially.

Easy way to find slowest operators adiabatically connected to the magnetization.

$$H_{\rm CD} = H + \dot\lambda \mathcal{A}_\lambda \qquad \mathcal{A}_\lambda = i\alpha_1[H,\partial_\lambda H] + i\alpha_3[H,[H,[H,\partial_\lambda H]]]$$

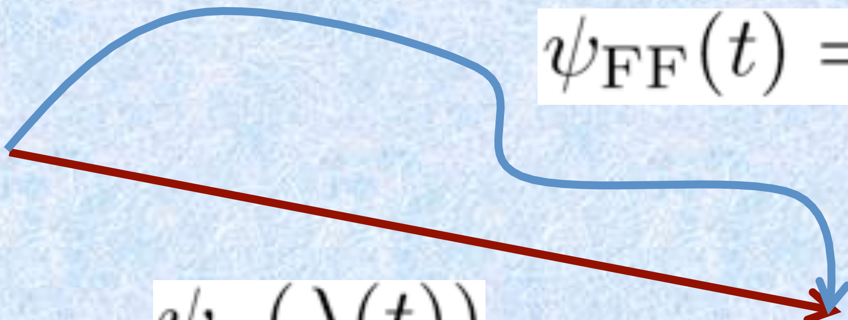
Two general approaches to realize CD driving in the original control space

1. Fast forward driving. Find unitary R

$$H_{\rm FF} = R^\dagger H_{\rm CD} R - i\hbar R^\dagger d_t R, \qquad R(t=0) = R(t=\tau) = I,$$

$H_{\rm FF}$ belongs to the control space

$$\psi_{\rm FF}(t) = R\psi_n(\lambda(t))$$



$$\psi_n(\lambda(t))$$

If rotations are local then evolutions follows eigenstates of a local (rotated) Hamiltonian.

No known general method of constructing R. Most implementations rely on the optimal control.

2. Floquet engineering.

$$H_{\rm CD} = H + \dot\lambda \mathcal{A}_\lambda \quad \mathcal{A}_\lambda = i\alpha_1[H,\partial_\lambda H] + i\alpha_3[H,[H,[H,\partial_\lambda H]]]$$

Exploit the fact that Magnus expansion of the Floquet Hamiltonian is very similar to the gauge potential expansion.

Floquet construction is not unique. Here is one possibility

$$H_{FF}(t) = \left(1 + \frac{\omega}{\omega_0}\cos(\omega t)\right) H(\lambda) + \dot\lambda \sum_{k=1}^{k_{\rm max}} \beta_k \sin((2k-1)\omega t)\partial_\lambda H$$

High frequency limit (like a Kapitza pendulum)

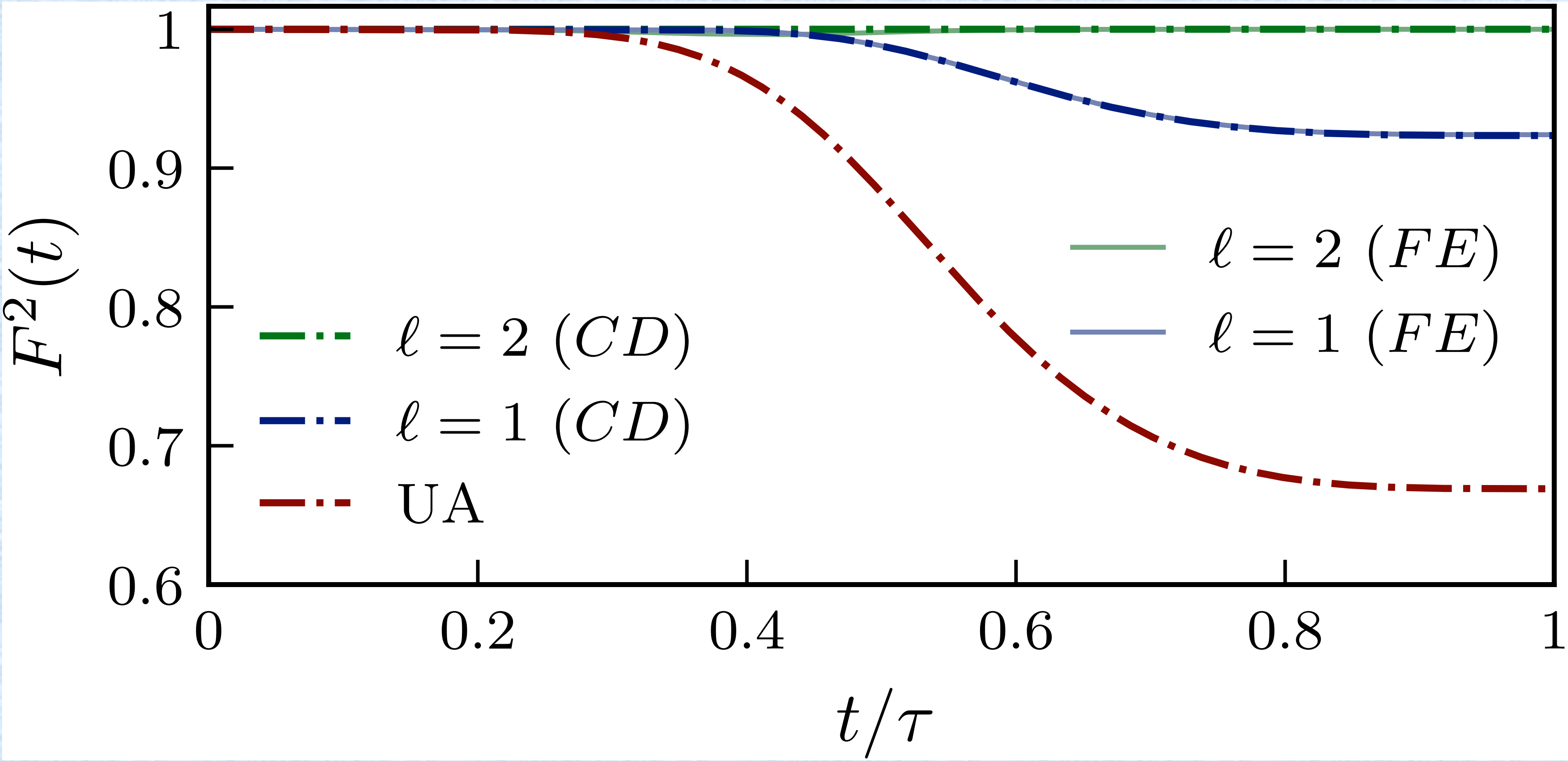
$$H_F^{rot}(t) \approx H + i\dot\lambda\left[\frac{\beta_1}{\omega_0}[H,\partial_\lambda H] + + \frac{1}{12}\frac{\beta_3 - 3\beta_1}{\omega_0^3}[H,[H,[H,\partial_\lambda H]]] + \dots\right]$$

$$H_{\rm CD} = H + \dot\lambda \mathcal{A}_\lambda = H + \dot\lambda(i\alpha_1[H,\partial_\lambda H] + i\alpha_3[H,[H,[H,\partial_\lambda H]]])$$

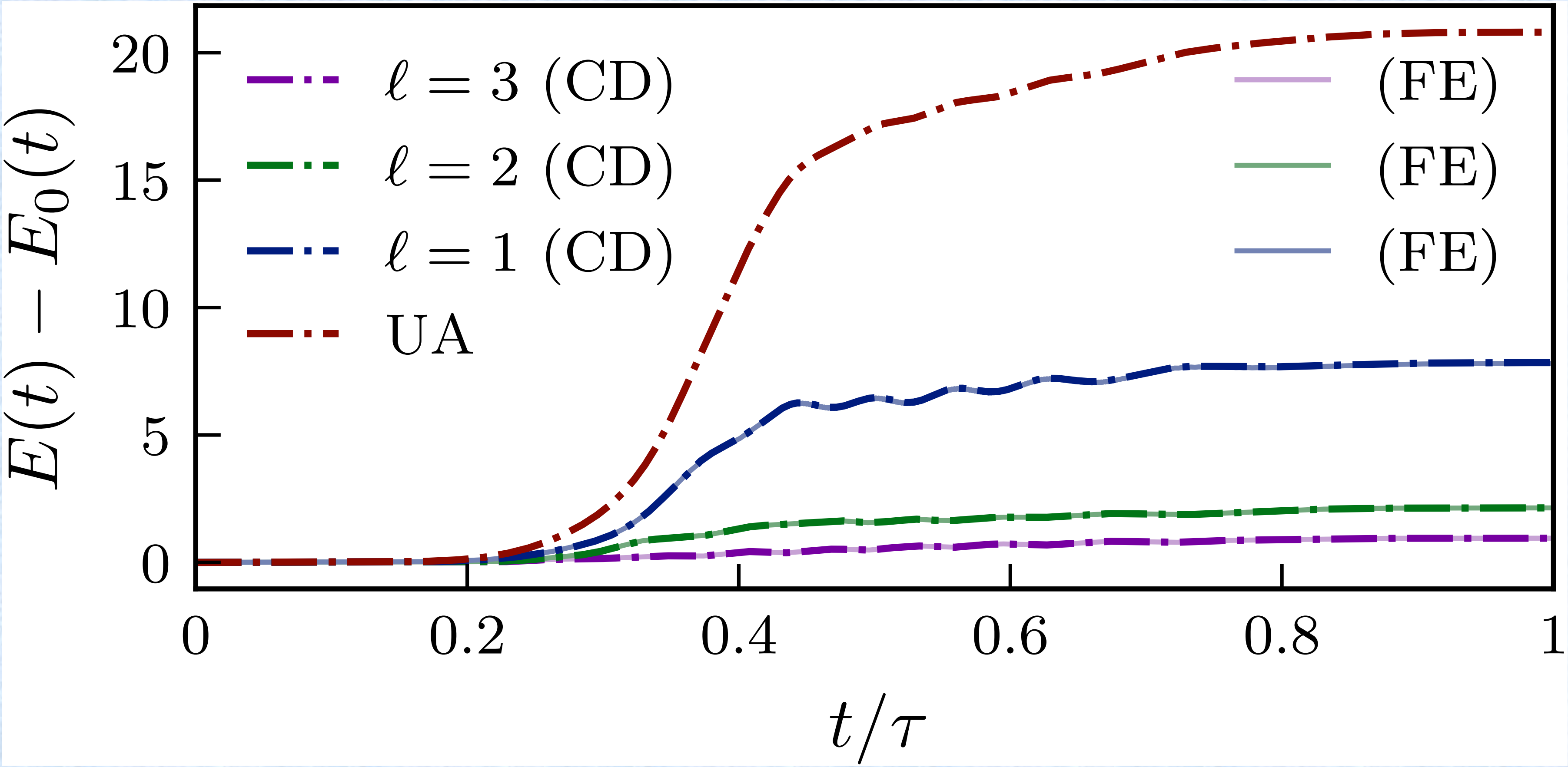
$$\beta_1 = \alpha_1\omega_0, \beta_3 = \alpha_3\omega_0^3 + 3\alpha_1\omega_0, \dots$$

A two Qubit gate

$$H(\lambda) = -2J\sigma_1^z\sigma_2^z - h(\sigma_1^z + \sigma_2^z) + 2h\lambda(\sigma_1^x + \sigma_2^x)$$

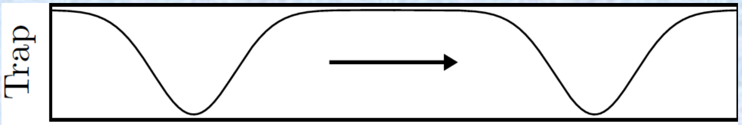


Dissipation by a magnetic tweezer



$$H(\lambda) = H_0 - h_t \sum_{n=1}^{L} \exp\left[-\frac{(n - n(\lambda))^2}{n_t^2}\right] \sigma_n^z,$$

$$H_0 = J \sum_{i=1}^{L} \sigma_i^z \sigma_{i+1}^z + h_z \sum_{i=1}^{L} \sigma_i^z + h_x \sum_{i=1}^{L} \sigma_i^x$$



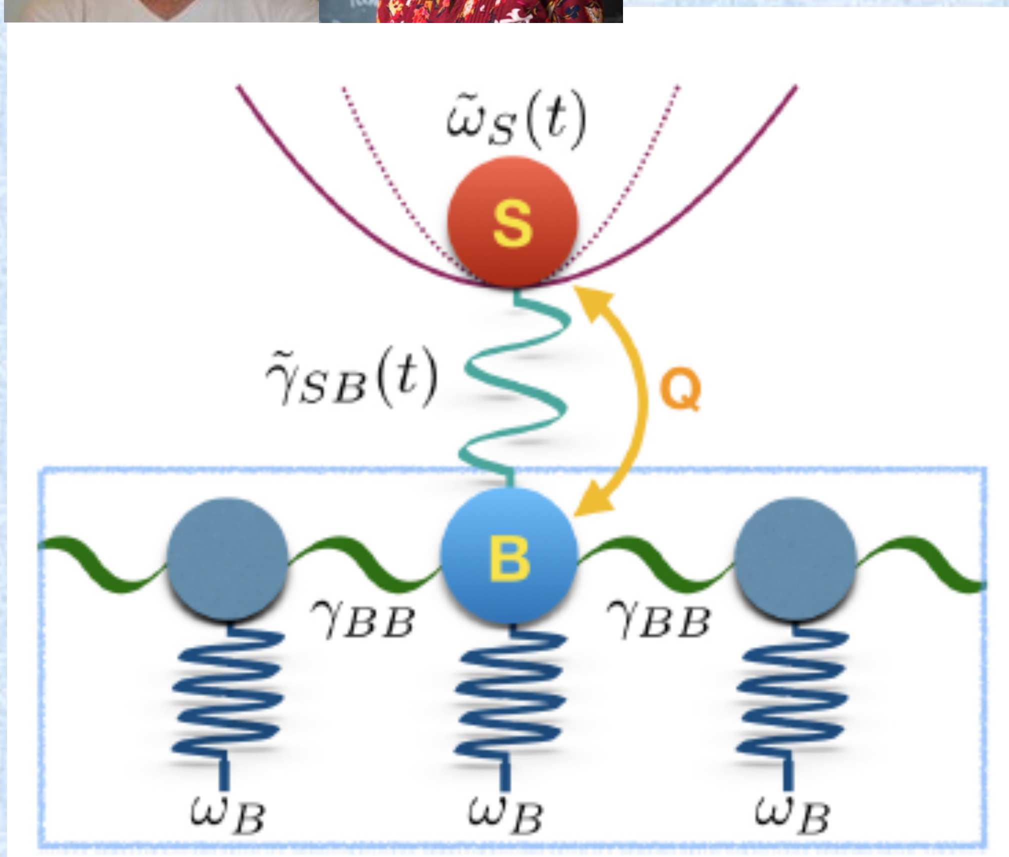
$\tau = 0.5,\ n_0 = 3,\ n_f = 10,$
$L = 12,\ J = -1,\ h_x = 0.8,$
$h_z = 0.9,\ h_t = 8,\ n_t = 1.$

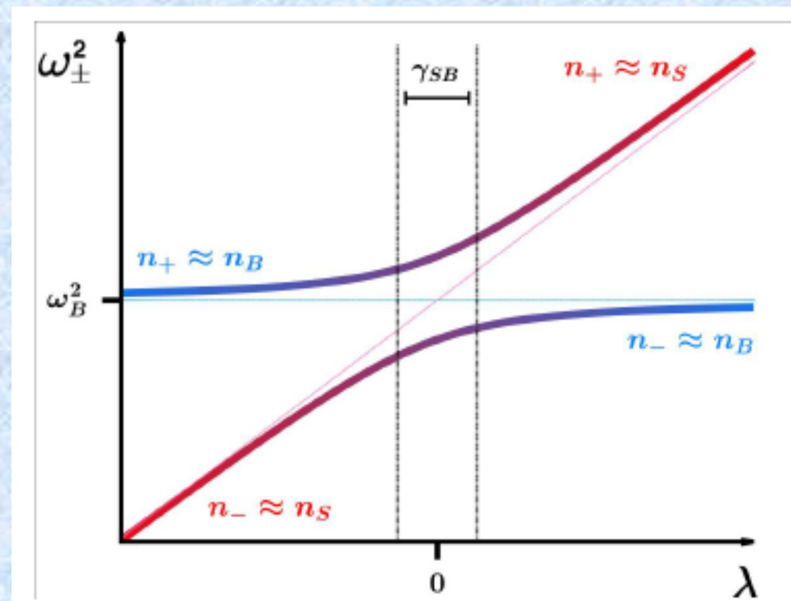
CD driving of open systems: engineering fast dissipative thermalization

$$H_{bath} = \sum_{j=1}^{N}\left[\frac{1}{2}p_j^2 + \frac{1}{2}\omega_B^2 x_j^2 - \gamma_{BB}\,\omega_B^2\, x_j\, x_{j+1}\right]$$

$$H_S(t) = \frac{P^2}{2} + \frac{\omega_S^2}{2}X^2$$

$$H_{\rm SB} = -\tilde{\gamma}_{SB}(t)\omega_B^2 x_B X$$

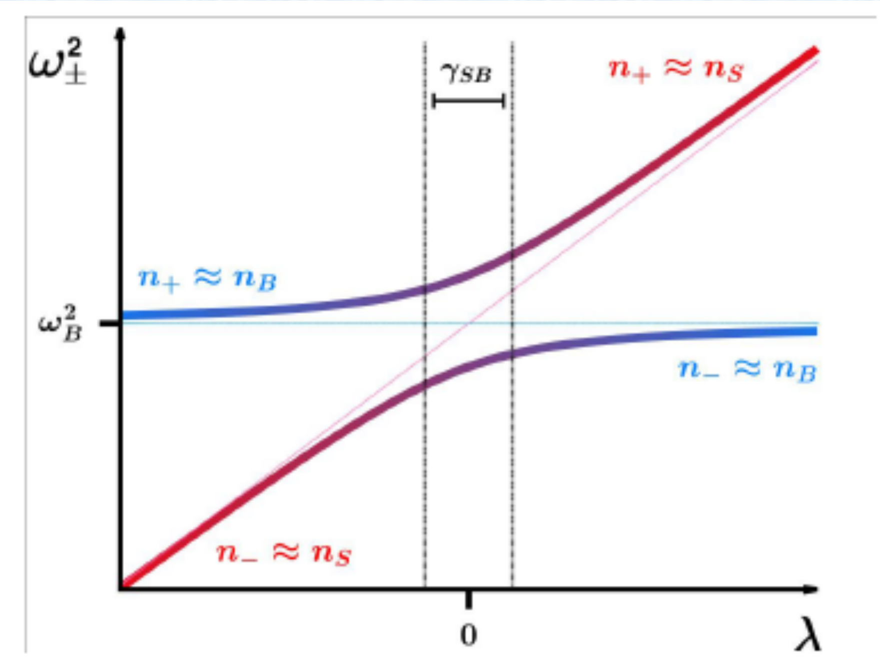




$$\lambda(t) = \frac{\omega_S^2(t) - \omega_B^2}{\omega_B^2}$$

Drive an oscillator through a resonance with an optical phonon bath. Slow limit – isothermal process.

Open systems: adiabatic process for the system + bath = isothermal process for the system



Energy conservation (first law)

$$\Delta W = \Delta E_{\rm sys} + \Delta E_{\rm bath}$$

The bath Hamiltonian does not depend on λ, therefore

$$\Delta E_{\rm bath} = T_{\rm bath} \Delta S_{\rm bath}$$

The condition for adiabaticity assuming the system and the bath are decoupled at the protocol boundaries

$$\Delta S = \Delta S_{\rm sys} + \Delta S_{\rm bath} = 0$$

Combine and find:

$$\Delta W = \Delta E_{\rm sys} - T_{\rm bath} \Delta S_{\rm sys} = \Delta F_{\rm sys}$$

Adiabatic work =minimal work, i.e. the free energy change.

Simpler two oscillator problem.

$$H = \frac{P^2}{2m} + \frac{m\omega_s^2(t)}{2} X^2 + \frac{p_b^2}{2m} + \frac{m\omega_B^2}{2} - \gamma_{\rm SB}\omega_B^2 x_B X$$

Very easy to find the gauge potential and the CD protocol

$$\mathcal{A}_\lambda = -\frac{1}{4(1+\lambda)}(X\,P + P\,X) + \frac{\gamma_{\rm SB}}{\lambda^2 + 4\gamma_{\rm SB}^2}(X\,p_B - x_B\,P) + O(\gamma_{\rm SB}^2), \quad \lambda = \frac{\omega_S^2 - \omega_B^2}{\omega_B^2}$$

Requires coupling to the bath momentum. Singular near the resonance in the weak coupling limit.

The problem simplifies under the rotating wave - phonon number conserving - approximation (RWA)

$$H_{CD} \approx \omega_B \left[\left(1 + \frac{\lambda}{2}\right) a_S^\dagger a_S + a_B^\dagger a_B - \frac{\gamma_{\rm SB}}{2}(a_S^\dagger a_B + a_B^\dagger a_S) + \frac{\dot\lambda}{i\,\omega_B}\frac{\gamma_{\rm SB}}{(\lambda^2 + 4\gamma_{\rm SB}^2)}(a_S^\dagger a_B - a_B^\dagger a_S)\right].$$

$$a_S = \sqrt{\frac{\omega_B}{2}}\left(X + i\frac{P}{\omega_B}\right) \qquad a_B \equiv \sqrt{\frac{\omega_B}{2}}\left(x_B + i\frac{p_B}{\omega_B}\right)$$

Equivalent to the LZ (spin ½) problem if interpret a_S and a_B as Schwinger bosons. Can easily design FF protocol.

Beyond RWA: hard problem. Do a series of canonical (unitary) transformations eliminating unwanted couplings one by one. Similar to a Rubik's cube problem; no unique solution, but finding a solution still hard.

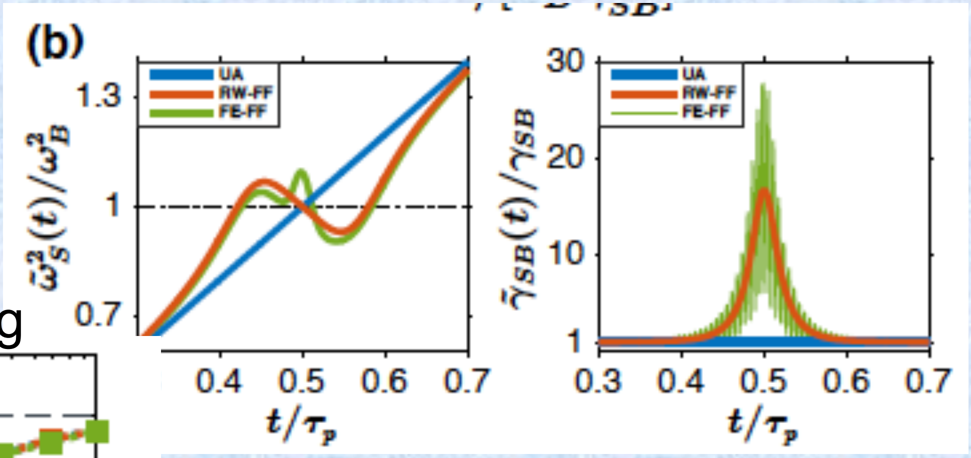
$$H_{FF}^{'} = \frac{P^2}{2} + \frac{\Lambda'(t)\, X^2}{2} + \frac{p_B^2}{2} + \frac{K'(t)\, x_B^2}{2} - C'(t)\, x_B\, X$$

$\Lambda'(t),\ K'(t),\ C'(t)$ are complicated local functionals of $\lambda(t)$

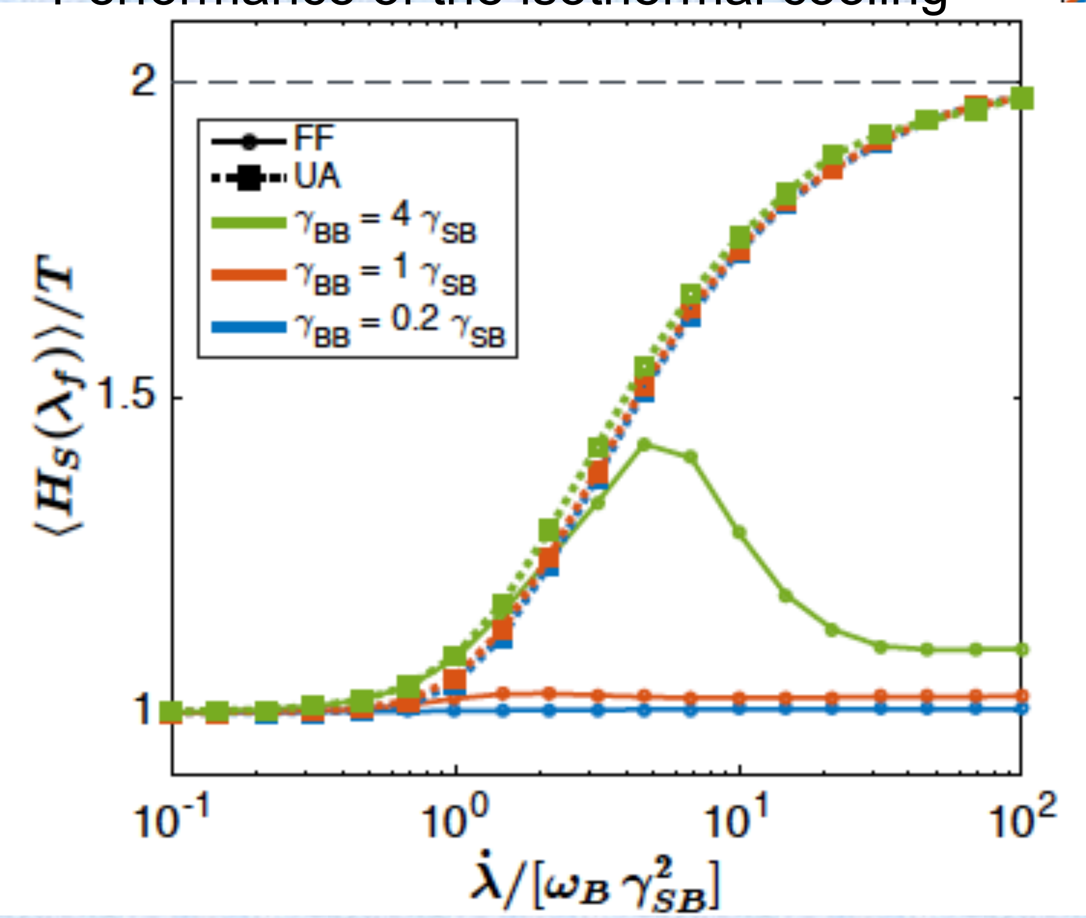
Use the Floquet engineering (shaking of C') to get the desired K'(t)

Slightly different constraints: can not modulate bath degrees of freedom.

Different constraints: can not access bath. Can develop Floquet fast-forward protocol with some efforts



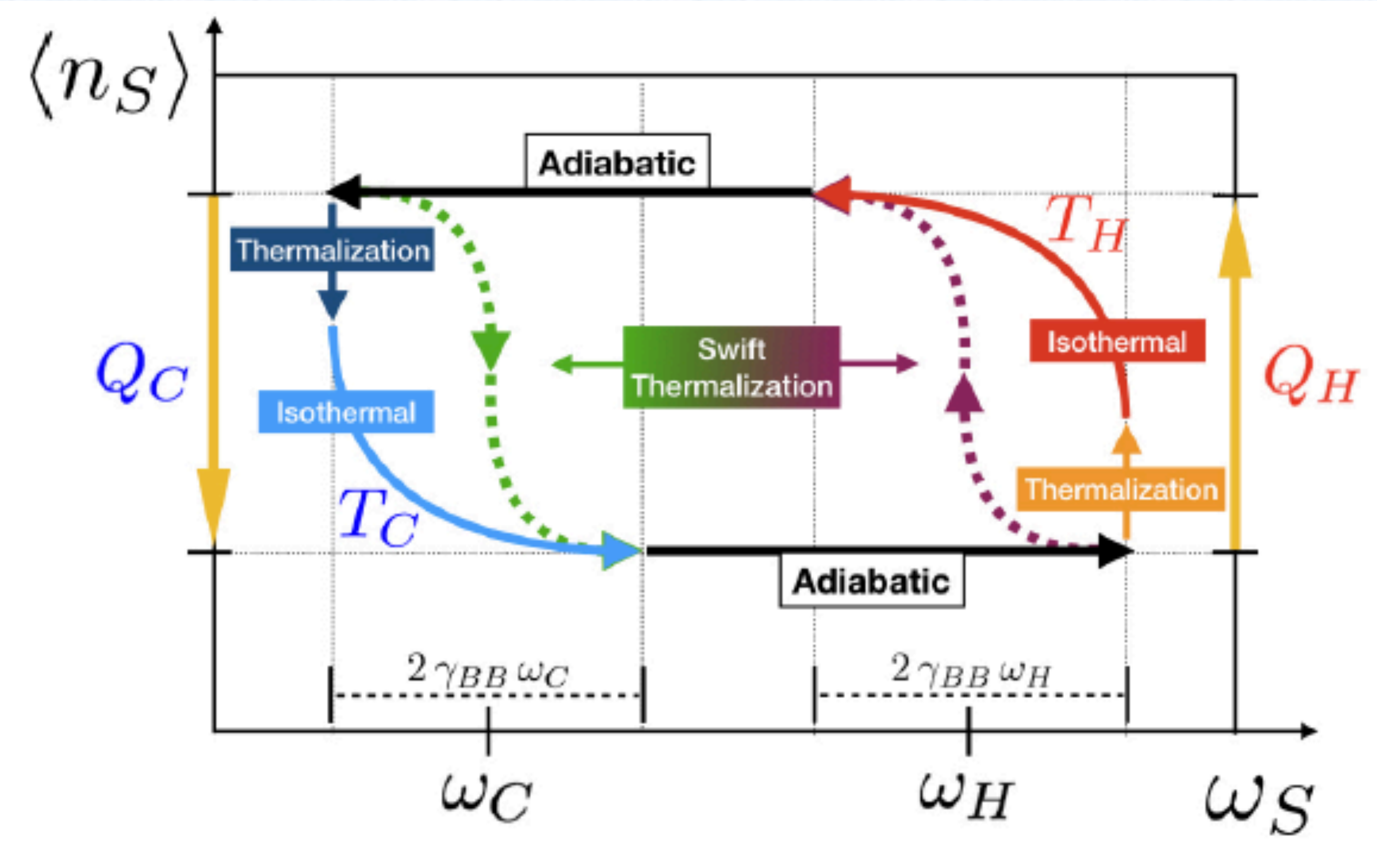
Performance of the isothermal cooling



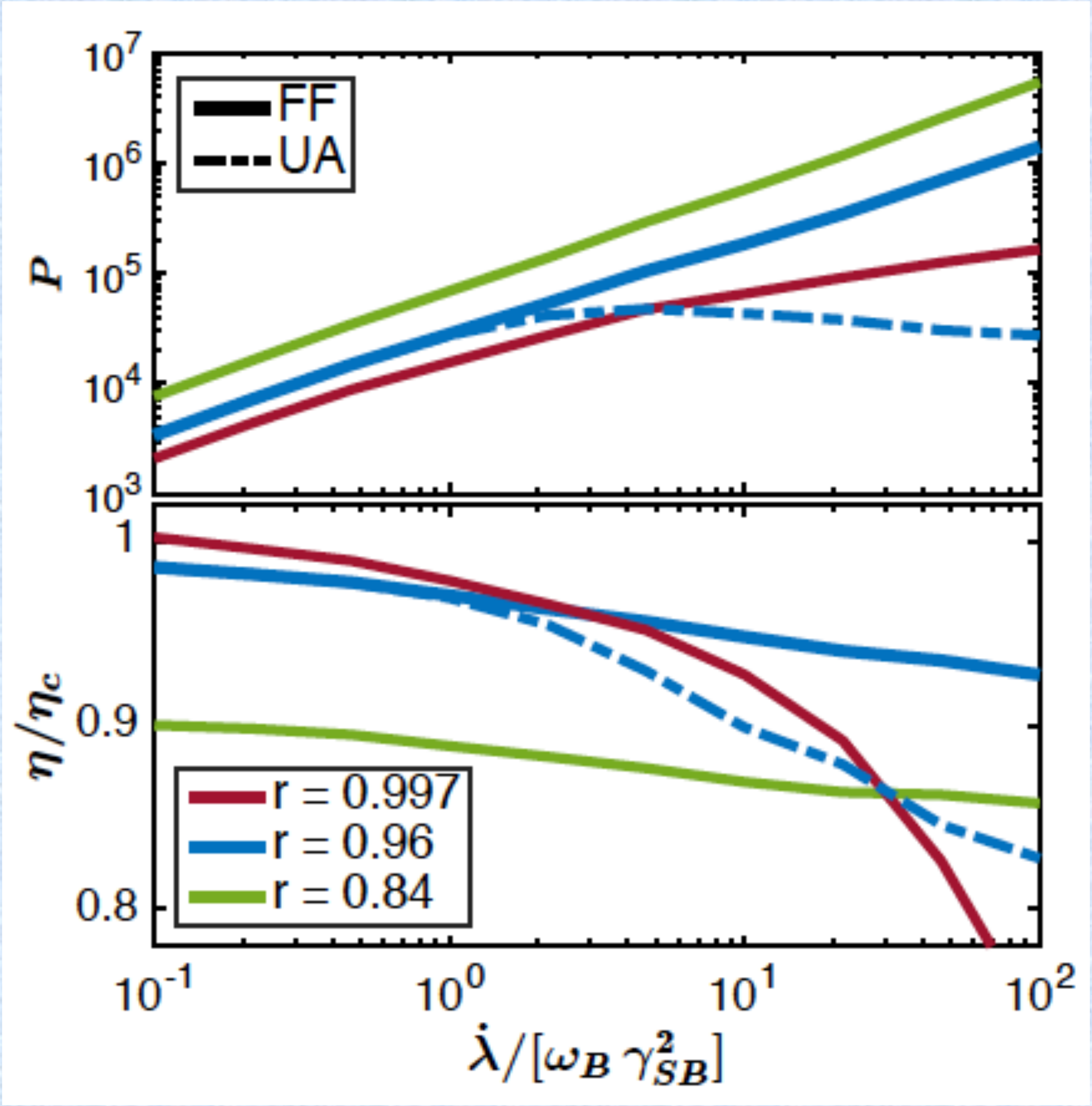
CD protocol only works when The Markovian (Lindblad) approach is not applicable.

It relies on coherence of the bath during the heat exchange.

Can design fast (Quantum) heat engines operating near the Carnot efficiency



Power and efficiency of the Otto engine



$$r = \frac{T_C}{T_H}\frac{\omega_H}{\omega_C}$$

Ideal power and efficiency

$$\eta = 1 - \frac{\omega_C}{\omega_H} \leq \eta_c$$

$$P = \frac{k_B T_H}{\tau}\,\eta\,(1-r)$$

Summary

- Close connections between adiabatic transformations and quantum information geometry, Schrieffer-Wolff transformations, slow operators, chaos and integrability and many more.

- Can use Floquet engineering to design efficient CD protocols for high fidelity state preparation and suppressing dissipation in generic many-body systems.

- Can use this construction in open systems to extract heat, perform a minimal work if protocol times are faster than bath relaxation times, i.e. beyond Lindblad/Markov approximations.