Dynamic Rupture Simulation Methods

Luis A. Dalguer

Consultant at **3Q-Lab GmbH**, Switzerland

Visiting professor at DPRI, Kyoto University, Japan

Adjunct Professor at Aichi Institute of Technology, Japan

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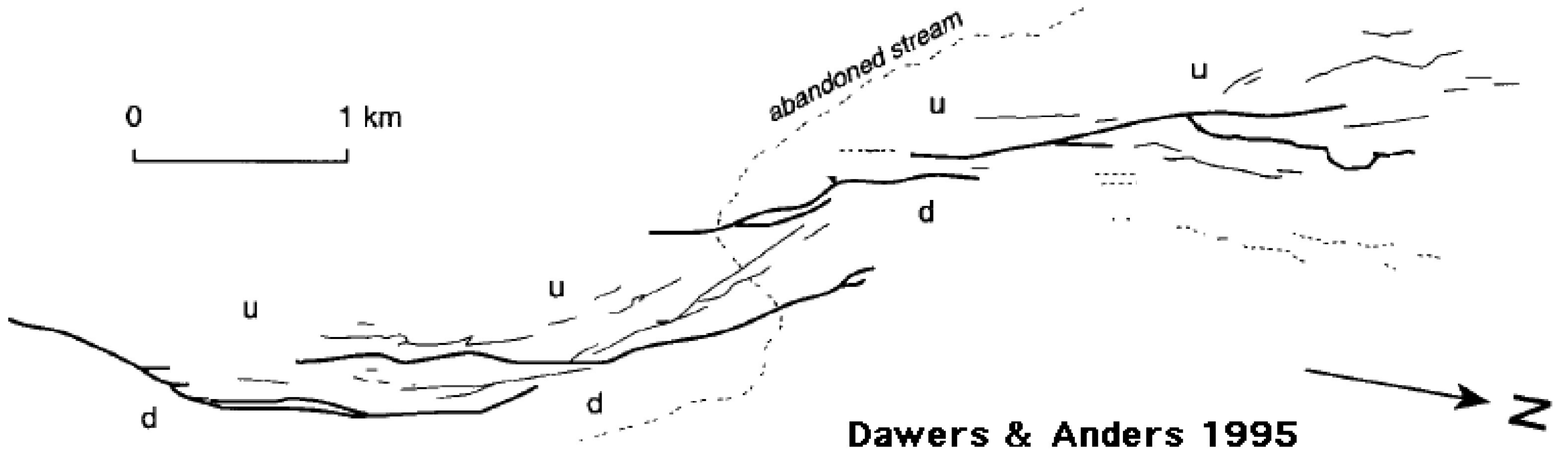
- Introduction: Fault complexity (Earthquakes occur on faults)
- Idealization of faulting for rupture dynamic
- Some types of friction laws
- Problem statement for rupture modeling
- Mathematical representation of earthquake for rupture dynamic
- Geometrical consideration of faults for modeling
- Numerical techniques for rupture dynamic
- Fault representation methods for dynamic rupture simulation
- Numerical resolution to solve rupture dynamic
- Assessment of Fault representation Methods

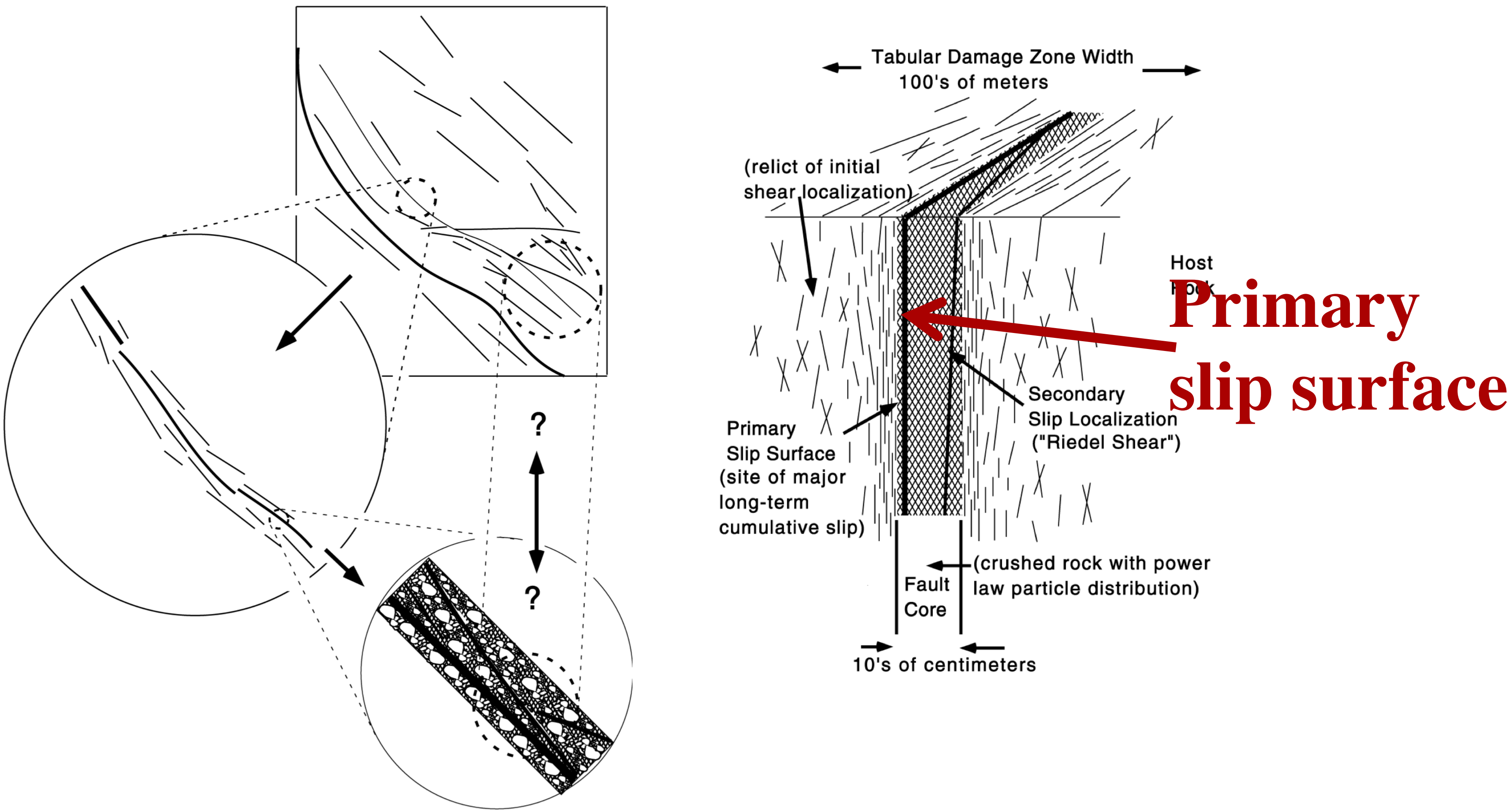
Faults in nature

Earthquakes occur on Faults: But how do they look like?

- Faults are not isolated (segmented and linked, irregular and rough at all scales)



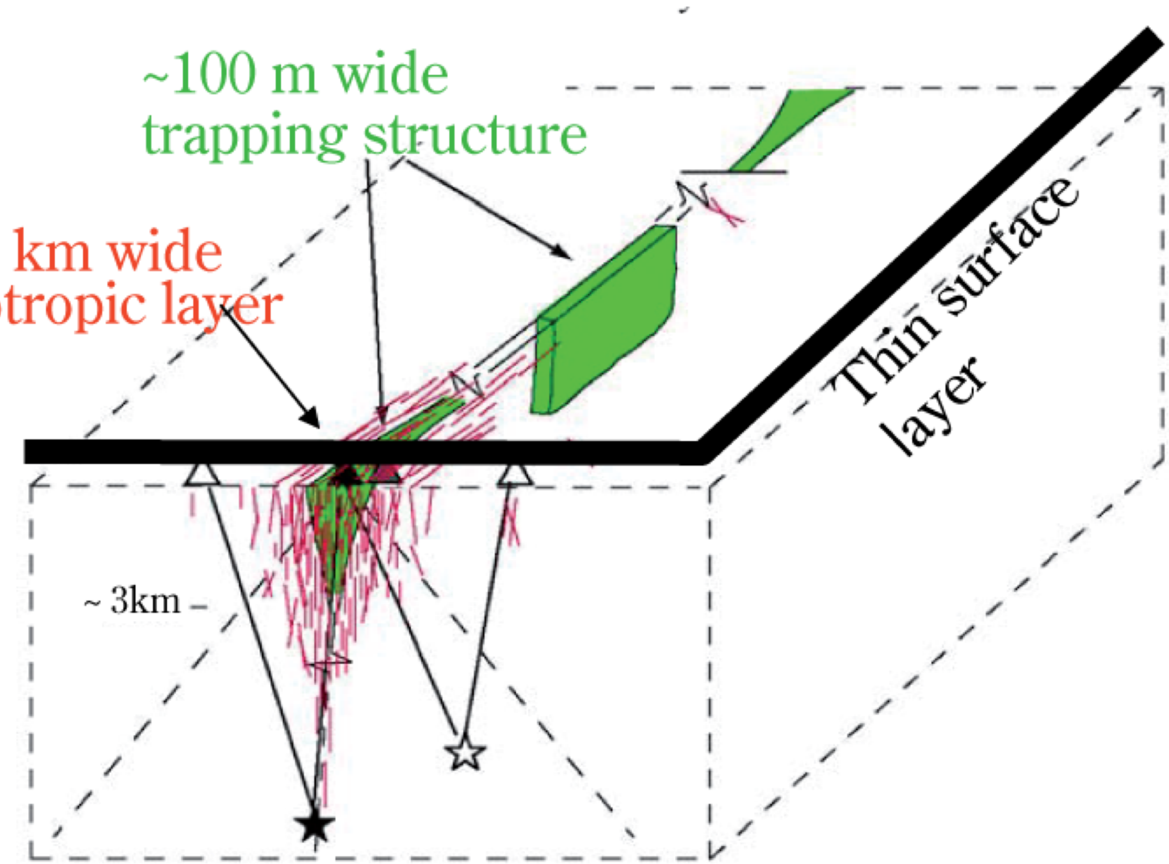
Schematic map views of fault structures at different scales



(Ben-Zion and Sammis, 2003)

Introduction: fault complexity

How faults may looks at depth and shallow?



(Ben-Zion et al, 2007)

Idealization of faulting for rupture dynamic

All is about cracks:

- When active during earthquakes, dominantly operate as dynamically running shear cracks
- Then it is in principle a Fracture Mechanics problem
- Fracture Mechanics: Quantitative description of the mechanical state of a deformable body containing a crack or cracks.
- Then Dynamic Rupture Models have their foundation in Fracture mechanics concepts.
- Dynamic models usually idealize the earthquake rupture as a dynamically running shear crack on a frictional interface embedded in a linearly elastic and/or non linear continuum.
- Incorporation of small scale complexities in numerical simulations requires high resolution models

Idealization of faulting for rupture dynamic

Cohesive zone (Fracture mechanics) and friction model

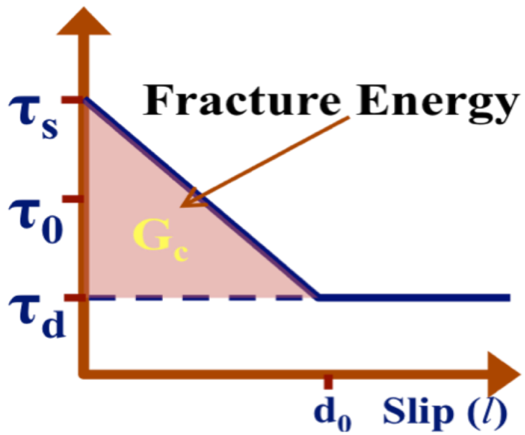
- Models

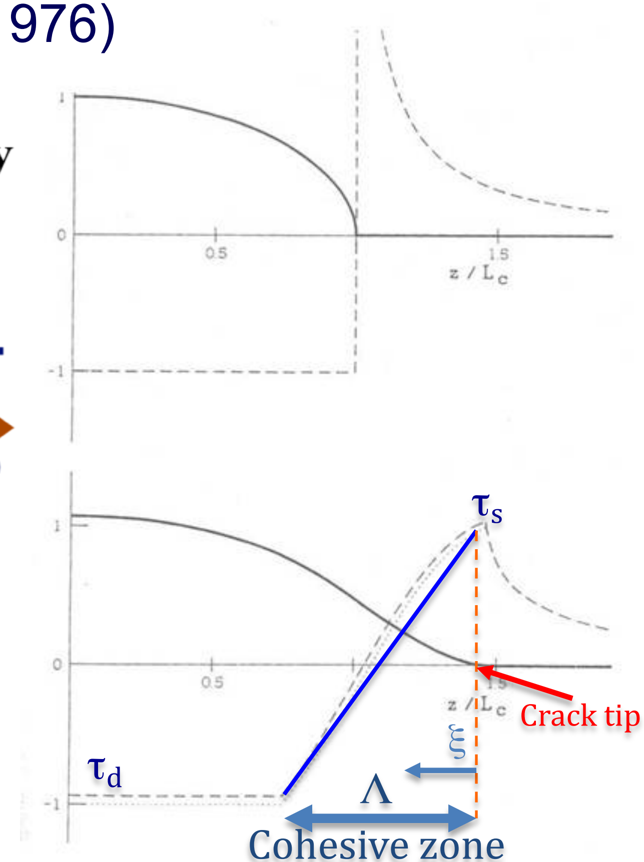
-Constant (Barenblatt, 1959)

-Linearly dependent on distance to crack tip (Palmer and Rice, 1973; Ida, 1973)

-Linearly dependent on slip (Ida, 1973 Andrews; 1976)

$$G_c = \frac{d_0(\tau_s - \tau_d)}{2}$$





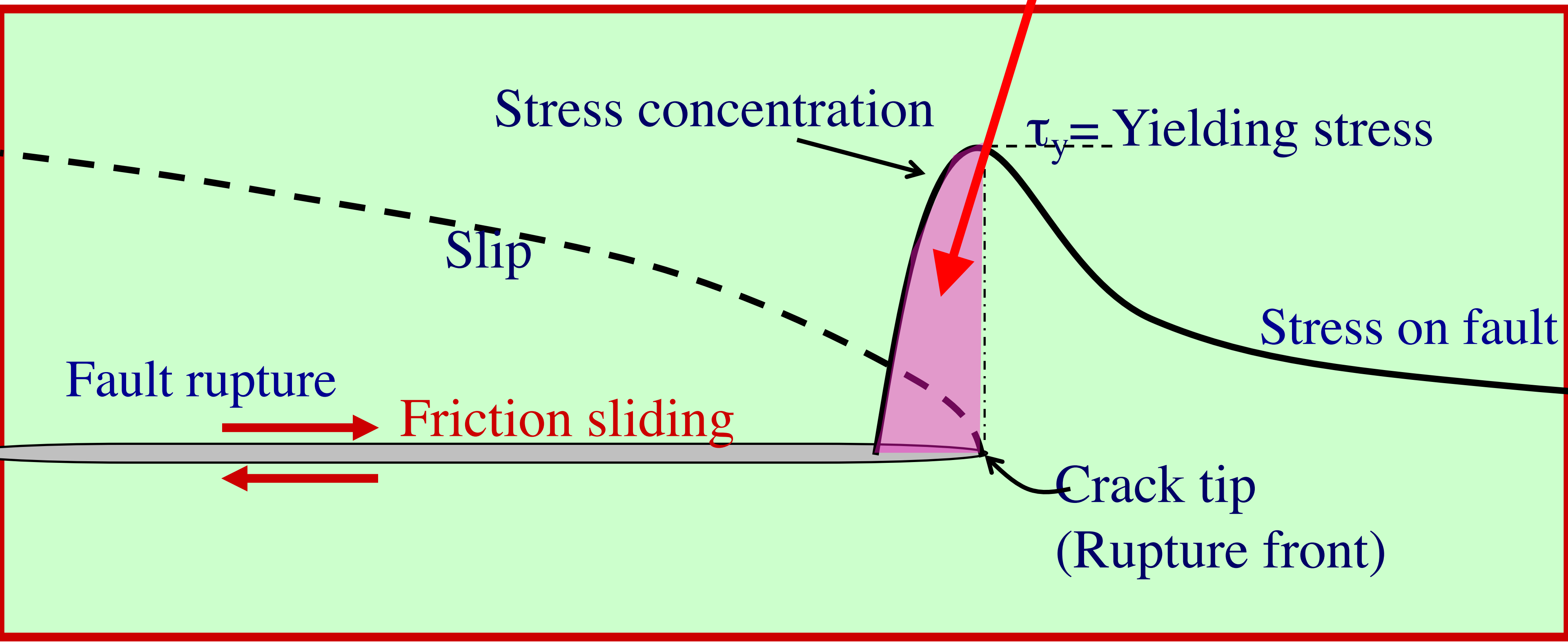
For the scale of earthquake modeling, G_c is a mesoscopic parameter, contains all the dissipative processes in the volume around the crack tip: off-fault yielding, damage, micro-cracking etc.

-They are mapped on the fault plane.

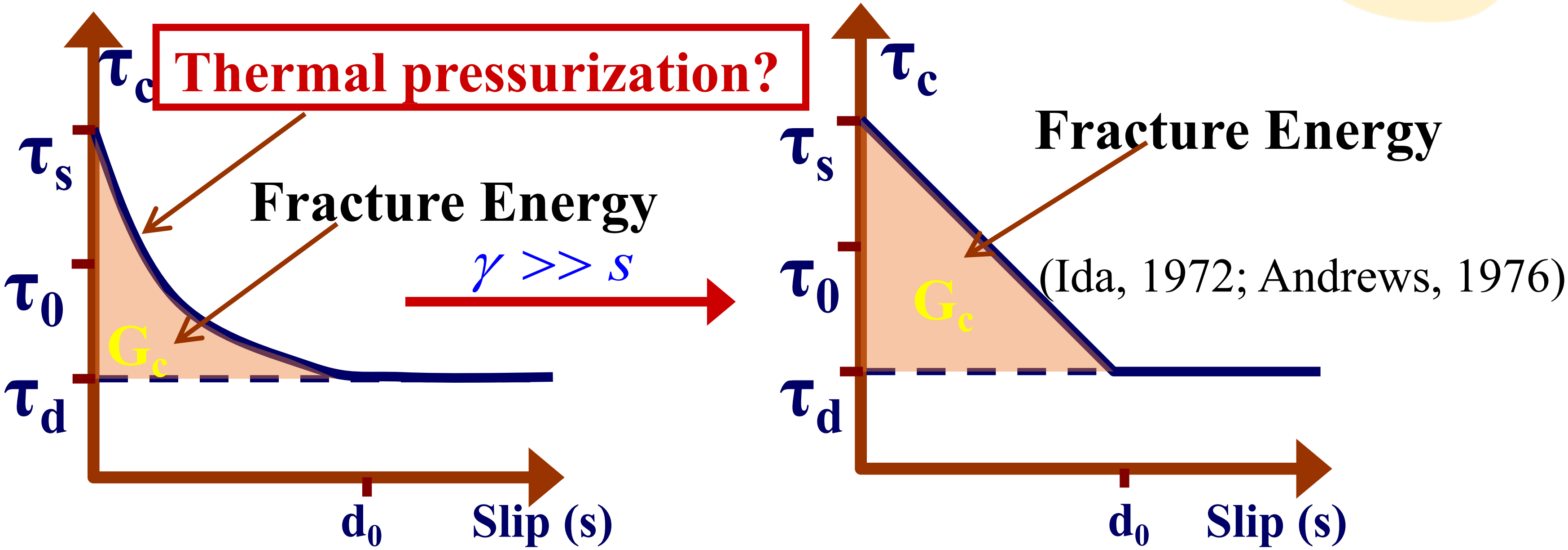
Stress and friction on the fault (crack)

The earthquake rupture can be described as a two-step process: (1) formation of crack and (2) propagation or growth of the crack. The crack tip serves as a

(The cohesive zone: break down process that needs to be accurately solved)



Friction laws: Slip weakening



$$\tau_c = \begin{cases} \dfrac{\gamma\tau_s(d_0 - s)}{d_0(s+\gamma)} + \dfrac{\tau_d s(d_0+\gamma)}{d_0(s+\gamma)} & s < d_0 \\ \tau_d & s \geq d_0 \end{cases} \quad \text{for } \gamma >> s \quad \approx \frac{\tau_s(d_0 - s)}{d_0} + \frac{\tau_d s}{d_0}$$

Input requirement:

$\gamma =$ Define No-linearity $\quad \gamma >> s \quad$ then $\approx$ Linear weakening

$\tau_0 =$ Initial shear stress, $\tau_s =$ Static friction, $\tau_d =$ Dynamic friction

$d_0 =$ Critical slip-weakening

Friction laws: Rate and state

Aging law (Dieterich, 1986; Ruina, 1983)

(its basis in laboratory experiments)

$$\tau_c(V,\theta) = \sigma_n\left[f_0 + a\ln\left(V/V_0\right) + b\ln\left(V_0\theta/L\right)\right]$$

$$\theta = 1 - V\theta/L$$

θ and $\psi =$ State variables

$V =$ Slip rate ($V_0 =$ steady state reference, $V_w =$ weakening)

$f =$ Friction coeficient ($f_{ss} =$ steady state, $f_0 =$ at steady state V_0 , $f_w =$ weakening)

$a, b =$ Friction parameters

Friction laws: Rate and state

Slip law

$$\tau_c(V,\theta) = \sigma_n\left[a\ln\left(V/V_0\right)+\psi\right]$$

$$\psi = -\frac{-V}{L}\left[f - f_{ss}(V)\right]$$

$$f_{ss}(V) = f_0 - (b-a)\ln\left(V/V_0\right)$$

Strong velocity weakening (Flash heating): same as slip law, but

Motivated by high-speed rock sliding experiments (e.g., Tsutsumi and Shimamoto, 1997; Di Toro et al., 2004; Han et al., 2010; Goldsby and Tullis, 2011)

$$f_{ss}(V) = \begin{cases} f_{ss(aging)} = f_0 - (b-a)\ln\left(V/V_0\right) & if \quad V < V_w \\ f_w + \left[f_{ss(aging)} - f_w\right]V_w/V & if \quad V > V_w \end{cases}$$

θ and $\psi =$ State variables

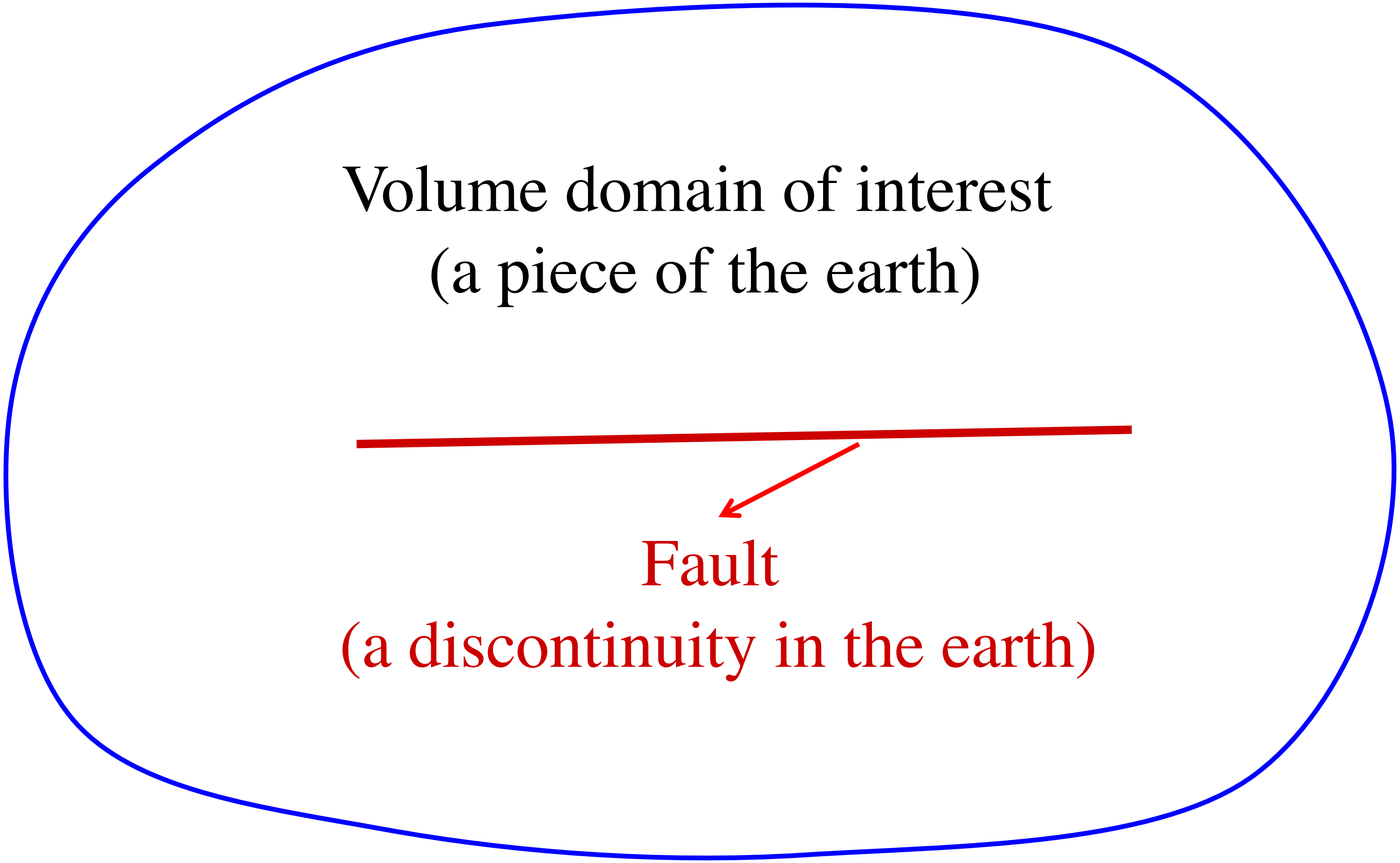
$V =$ Slip rate ($V_0 =$ steady state reference, $V_w =$ weakening)

$f =$ Friction coeficient ($f_{ss} =$ steady state, $f_0 =$ at steady state V_0 , $f_w =$ weakening)

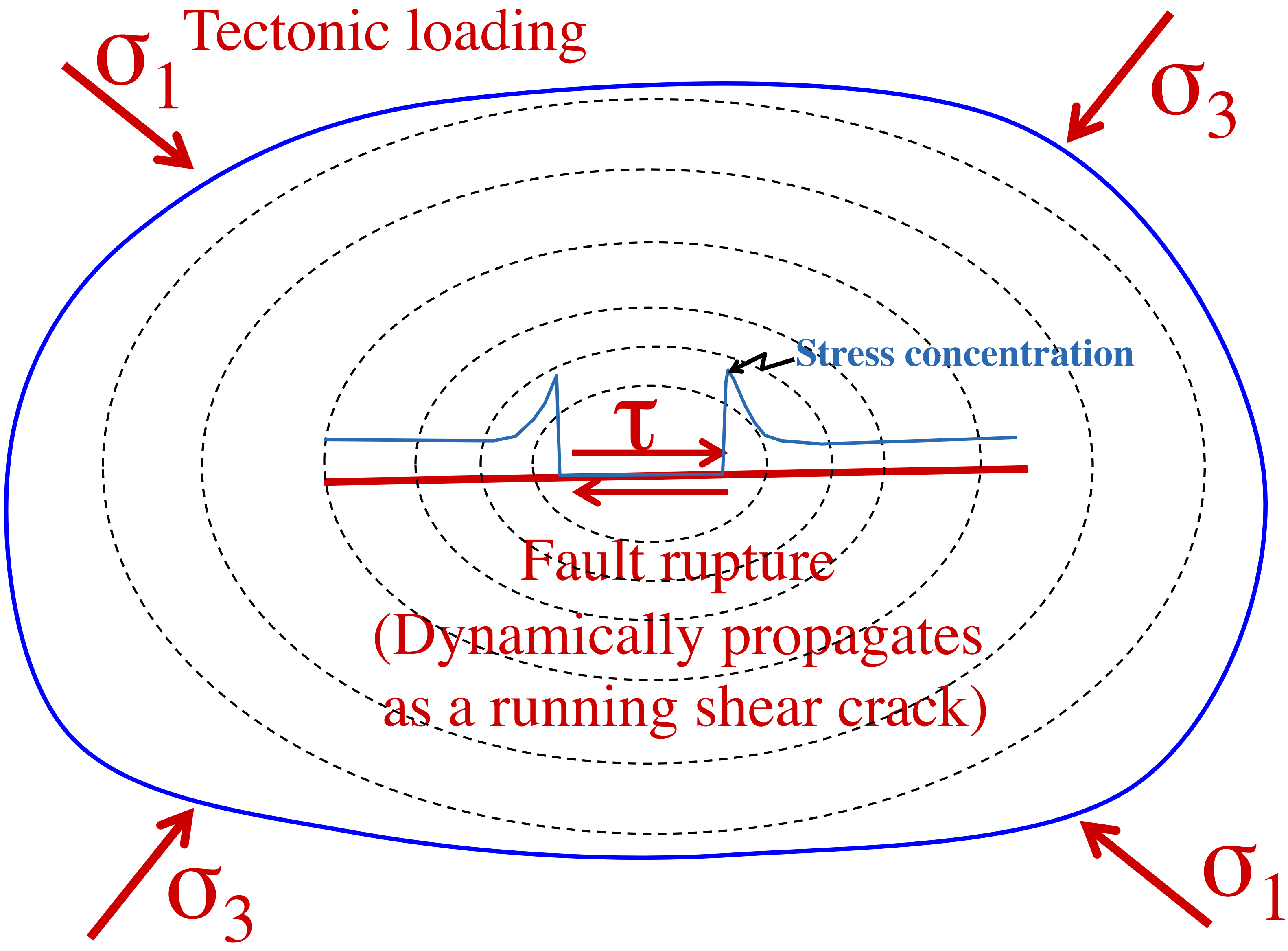
$a,b =$ Friction parameters

(La Pusta et al., 2000; Noda et al., 2009; Rojas et al., 2009; Dunham et al., 2011; Shi and Day, 2013)

Problem statement for rupture modeling



Problem statement for rupture modeling



Mathematical representation

Elastodynamic coupled to frictional sliding
(Highly non-linear problem)

$$\rho \dot{v}_i = \partial_j \sigma_{ij}$$

$$\dot{\sigma}_{ij} = C_{ijpq} \partial_p v_q$$

$$\tau \leq \tau_c$$

$$0 \leq \tau_c = f(\sigma_n, s, \dot{s}, \psi_1, \psi_2 \ldots$$

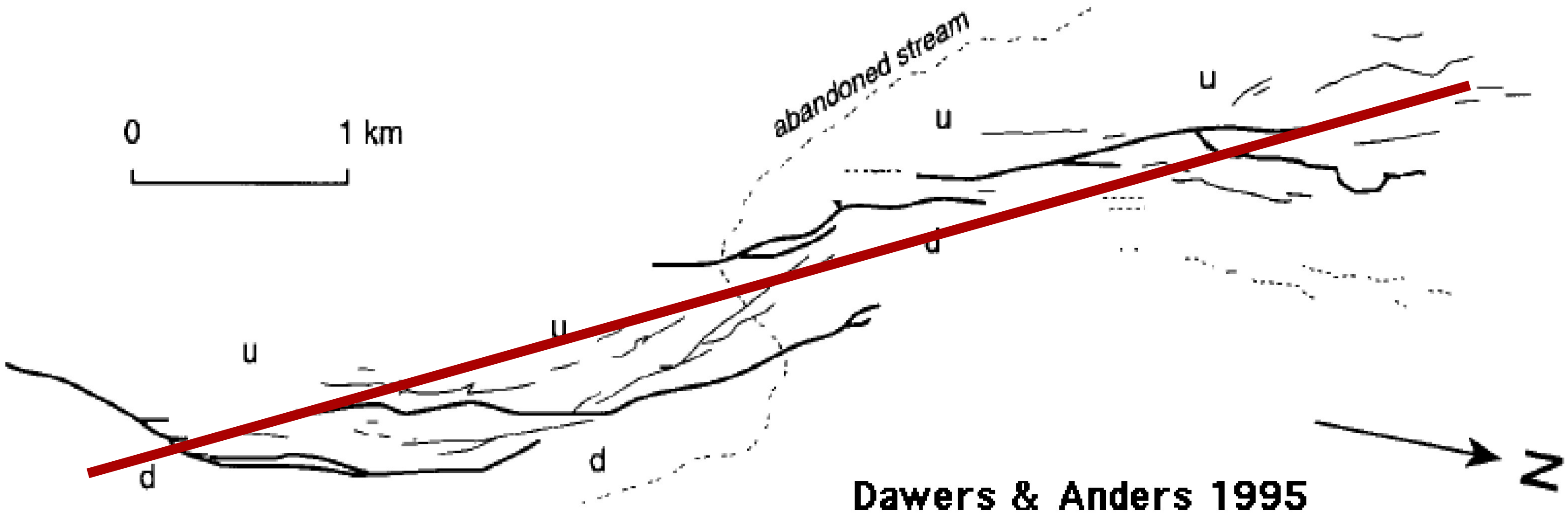
Friction constitutive equation

Fault-surface boundary conditions

For shear (nonlinear)	For opening (nonlinear)
$\tau - \tau_c \leq 0$	$\sigma_n \geq 0$ $U_n \geq 0$ $\sigma_n U_n = 0$

Geometrical representation of faults for modeling

Simplification of fault geometry for earthquake dynamic (depending of numerical method: FDM, FEM, SEM,DG,BIEM



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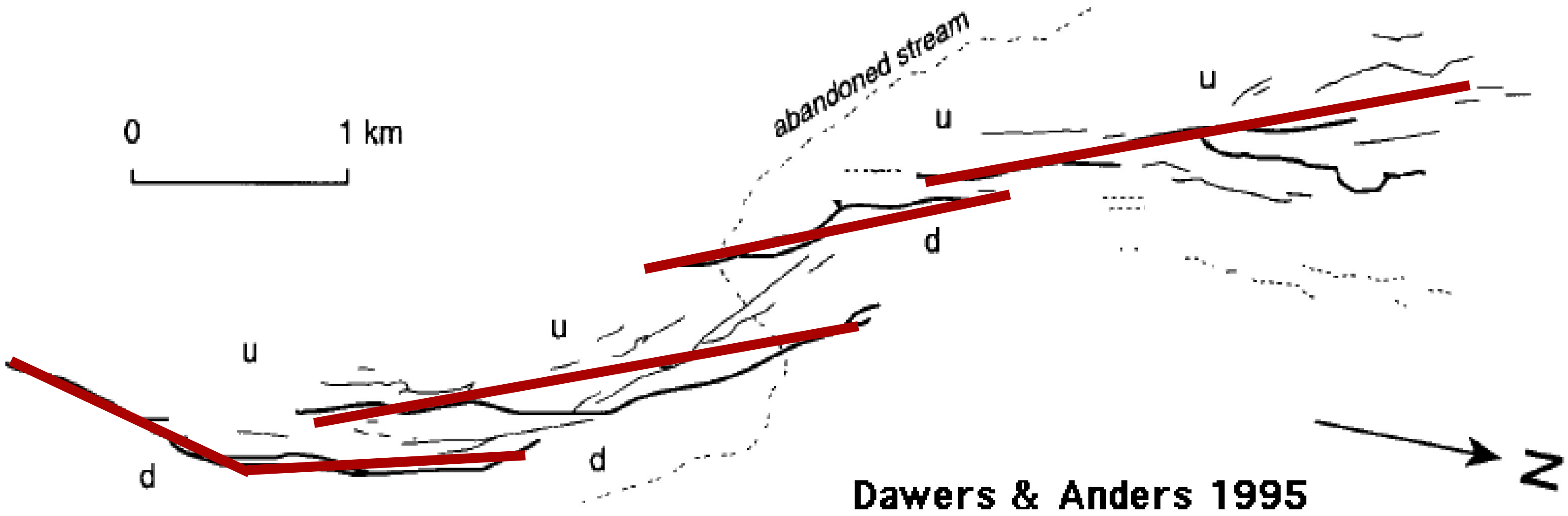

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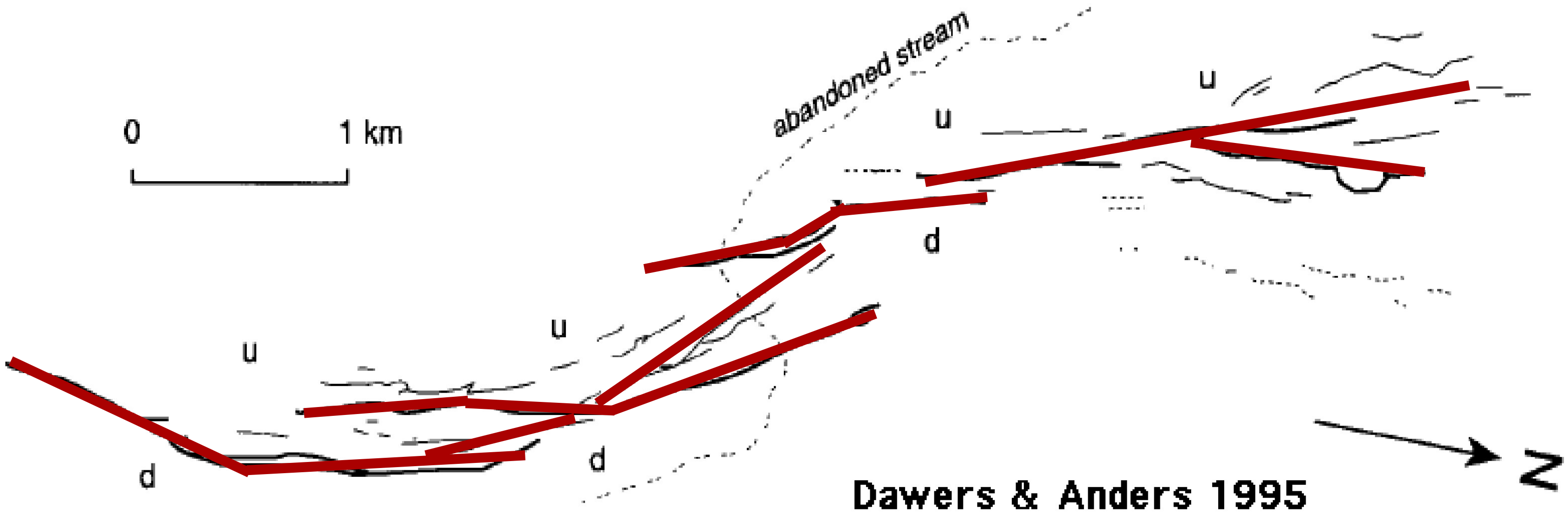


Numerical techniques for rupture dynamic

scecdata.usc.edu/cvws

The SCEC/USGS Spontaneous Rupture Code Verification Project

Benchmark Comparison Tool

Benchmark Descriptions

Newest Benchmarks

Metrics (Version 1)

Code Descriptions

Workshop Presentations

Downloads

Participants

References

Credit Page

Numerical techniques for rupture dynamic

http://scecdata.usc.edu/cvws/code_descriptions.html

Spontaneous Rupture Code Descriptions

(There are also other codes outside of this project)

Aagaard - Finite Element Code (EqSim)

Aagaard et al. - Finite Element Code (PyLith)

Ampuero/Kaneko/Lapusta - Spectral Element Code (SPECFEM3D)

Andrews - 2D Code (SCOOT)

Andrews/Song - 3D Code (Dynelf)

Barall - Finite Element Code (FaultMod)

Chen/Zhang/Zhang - Curved-Grid Finite-Difference (CG-FDM)

Cruz-Atienza - Finite Volume Code

Day - Finite Difference Code (DFM)

Duan - Finite Element Code (EQdyna)

Duru/Dunham/Bydlon - Finite-Difference Quake and Wave (FD-Q-Wavel

Dunham - MultiDimensional Spectral Boundary Integral Code (MDSBI)

Ely - Support Operator Code (SORD)

Kase - Finite Difference Code

Kozdon - AMR code (Tetemeko)

Liu/Lapusta - Spectral Boundary Integral Code

Ma - Finite Element Code (MAFE)

Oglesby - Finite Element Code (DYNA3D)

Olsen - Finite Difference Code (AWM)

Pelties - 3D Discontinuous Galerkin Code (ADER-DG)

Pitarka - Finite Difference Code (FDMSPLIT)

Tago/Cruz-Atienza - 3D Discontinuous Galerkin Code (DGCrack)

Templeton - Finite Element Code (ABAQUS)

- Traction at Split-node method

  Fault Discontinuity explicitly incorporated

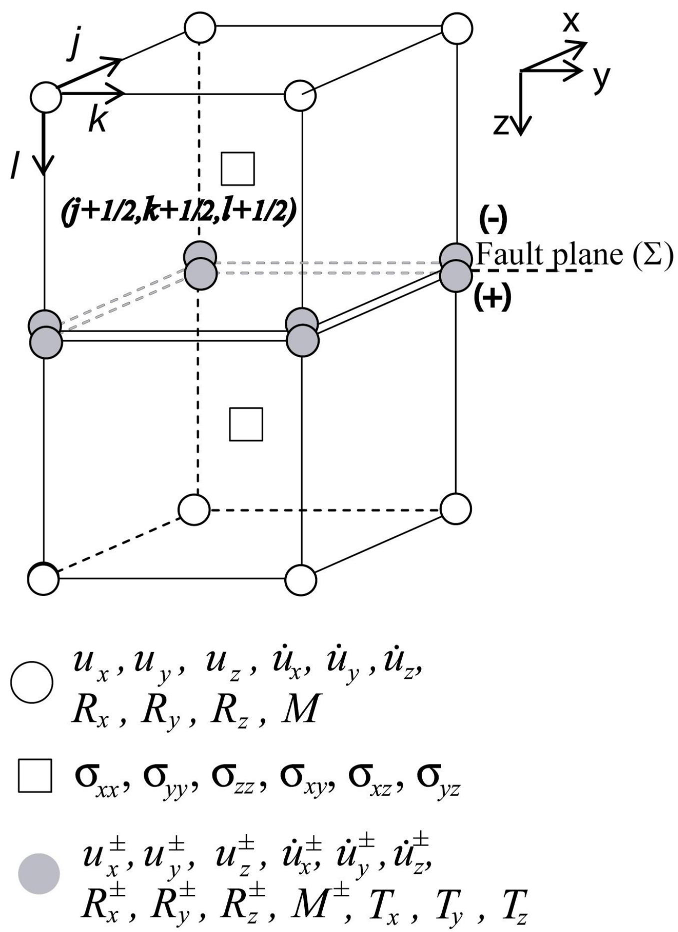
  (Andrews, 1973; DFM model: Day, 1977, 1982; SGSN model, Dalguer and Day, 2007)

- "Inelastic-zone" methods:

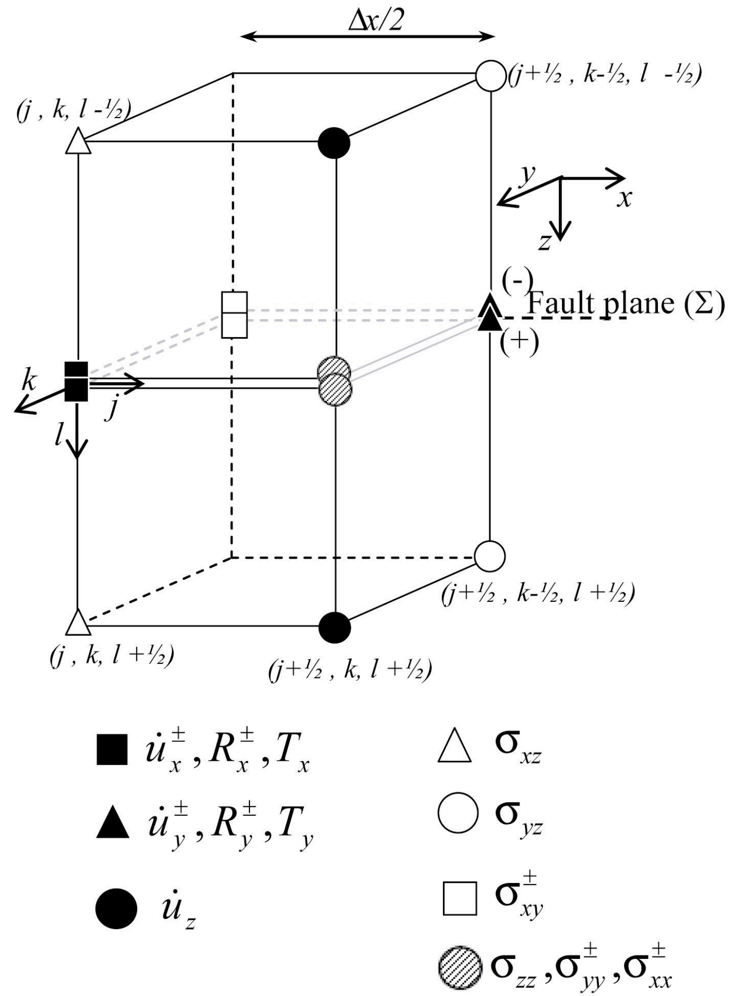
  Fault Discontinuity not explicitly incorporated

  - Thick-fault method (TF) (Madariaga et al., 1998)
  - Stress-glut (SG) method (Andrews 1976, 1999)

Traction at Split-Node method



For partially Staggered Grid
(e.g, model DFM
Day, 1982; Day et al, 2005)



For Staggered Grid
Staggered-Grid Split-Node Method (SGSN)
(Dalguer and Day 2007, JGR)

Fault representation methods for numerical simulation

$\dot{\vec{u}}^{\pm}$ = split-node velocities (+,- side of fault, respectively)

$\vec{R}^{\pm}$ = stress divergence terms from FD eqns (+,- side)

$M^{\pm}$ = nodal mass factors from FD eqns (+,- side)

$\vec{T}$ = split-node traction vector (no jump)

a = interface area of split node

Central Differencing in time (representation of equation of motion on fault

$$\dot{u}_{\nu}^{\pm}(t+\Delta t/2) = \dot{u}_{\nu}^{\pm}(t-\Delta t/2) + \Delta t (M^{\pm})^{-1}\left\{R_{\nu}^{\pm}(t) \mp a\left[T_{\nu}(t) - T_{\nu}^{0}\right]\right\}$$

$$\dot{s}_{\nu} = \dot{u}_{\nu}^{+}(t+\Delta t/2) - \dot{u}_{\nu}^{-}(t+\Delta t/2)$$ (Slip velocity)

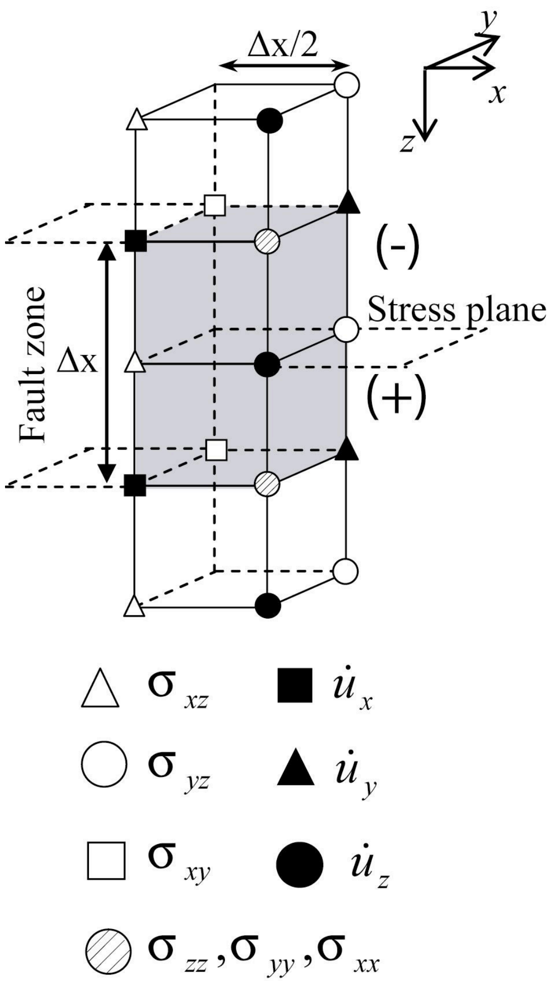
$$T_{\nu} = \begin{cases} \tilde{T}_{\nu} & \text{for} \quad \left[(\tilde{T}_x)^2 + (\tilde{T}_y)^2\right]^{1/2} \le \tau_c \\ \tau_c \dfrac{\tilde{T}_{\nu}}{\left[(\tilde{T}_x)^2 + (\tilde{T}_y)^2\right]^{1/2}} & \text{for} \quad \left[(\tilde{T}_x)^2 + (\tilde{T}_y)^2\right]^{1/2} > \tau_c \end{cases}$$

Compute "trial" traction $\tilde{T}_{\nu}$ (enforces continuity of tangential velocity and continuity of normal displacement.

Then actual nodal traction T_{ν} (tangential components ν=x,y)

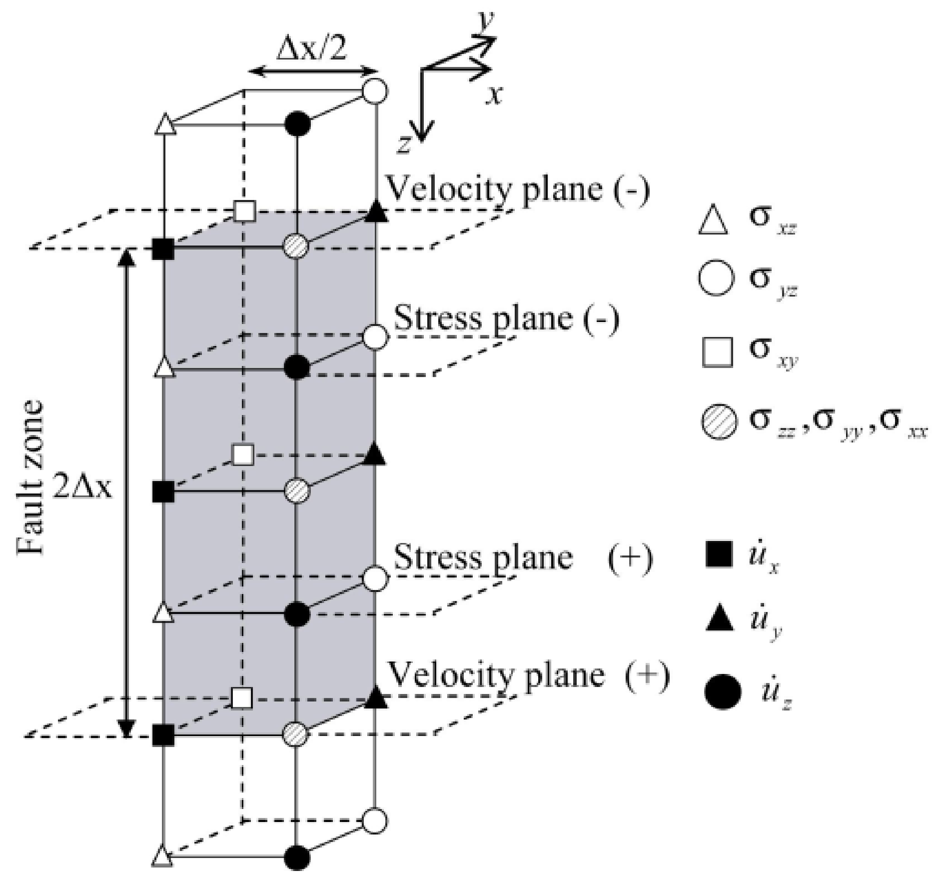
"Inelastic-zone" Fault models (in Staggered Grid FDM)

(Dalguer and Day, 2006, BSSA)



Stress-glut method (SG)

(Andrews 1976, 1999)



Thick-fault method (TF)

(Madariaga et al., 1998)

"Inelastic-zone" Fault models

Nodal Stress by Central Differencing in time gives (example σ_{xz})

$$\sigma_{xz}(t) = \sigma_{xz}(t-\Delta t) + \Delta t 2\mu \dot{\varepsilon}_{xz}(t - \Delta t/2)$$

addition of an inelastic component to the total strain rate $(T_x = \sigma_{xz})$

$$\sigma_{xz} = T_x(t) = T_x(t-\Delta t) + \Delta t 2\mu \left[\dot{\varepsilon}_{xz}(t-\Delta t/2) - \dot{\varepsilon}^{p}_{xz}(t-\Delta t/2)\right]$$

Compute "trial" traction $\tilde{T}_x$ setting $\dot{\varepsilon}^{p}_{xz}(t-\Delta t/2) = 0$

$$\tilde{T}_x(t) = T_x(t-\Delta t) + \Delta t 2\mu \dot{\varepsilon}_{xz}(t-\Delta t/2)$$

Then set the fault plane traction to

$$T_x(t) = \begin{cases} \tilde{T}_x(t) & \text{if } \tilde{T}_x(t) \le \tau_c \\ \tau_c & \text{if } \tilde{T}_x(t) > \tau_c \end{cases}$$

Fault representation methods for numerical simulation

Stress-glut method (SG) (Andrews 1976, 1999)

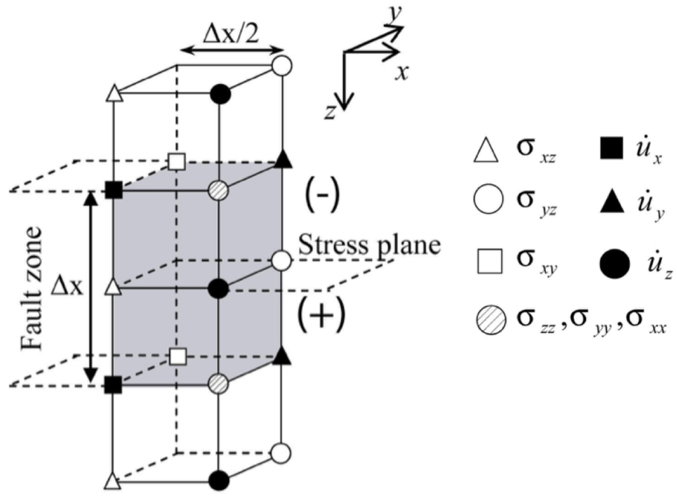
Frictional bound enforced on one plane of traction nodes

Calculate inelastic component $\dot{\varepsilon}^{p}_{xz}$

$$\dot{\varepsilon}^{p}_{xz}(t-\Delta t/2)=\frac{\tilde{T}_x(t)-T_x(t)}{2\mu\Delta t}$$

Calculate the total slip rate by integrating $\dot{\varepsilon}^{p}_{xz}$ over the spatial step Δx

$$\dot{s}_x\left(t-\Delta t/2\right)=2\Delta x\dot{\varepsilon}^{p}_{xz}\left(t-\Delta t/2\right)$$

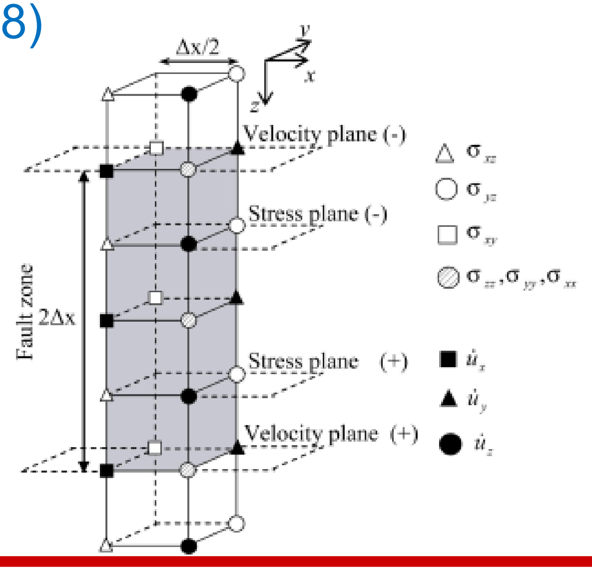


Thick-fault method (TF) (Madariaga et al, 1998)

-Frictional bound enforced on 2 planes of traction nodes

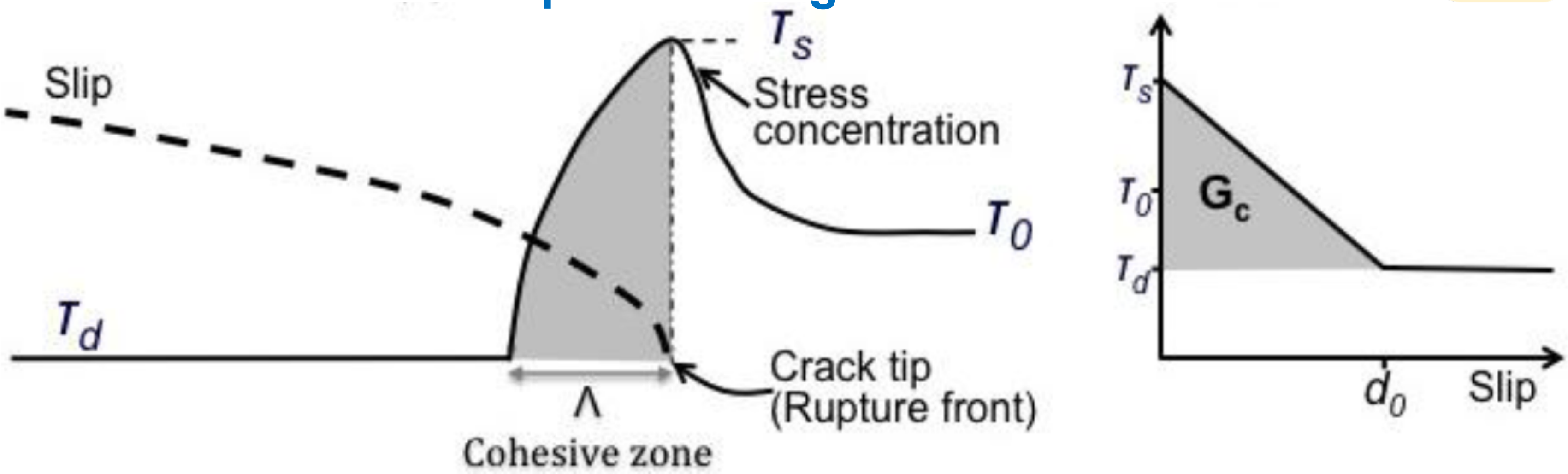
-Slip-velocity given by velocity difference across 2 unit-cell wide zone

$$\dot{s}_x\left(t-\Delta t/2\right)=\dot{u}_x^{(+)}\left(t-\Delta t/2\right)-\dot{u}_x^{(-)}\left(t-\Delta t/2\right)$$



Numerical resolution to solve rupture dynamic

The cohesive zone for a slip-weakening crack



T_s=Static yielding stress; T_d=Dynamic yielding stress
T_0=Initial shear stress; d_0=Critical slip distance
$\Delta\tau = T_0 - T_d$ = Stress drop

- At the scale of natural earthquakes, the cohesive zone examines the crack tip phenomena at a level of observation, in which the fracture energy *Gc* is a mesoscopic parameter which contains all the dissipative processes in the volume around the crack tip, such as off-fault yielding, damage, micro-cracking, etc.
- In the cohesive zone, shear stress and slip rate vary significantly and proper numerical resolution of those changes is crucial for capturing the maximum slip rates and the rupture propagation time and speeds. Therefore, the cohesive zone developed during rupture propagation need to be accurately solved to obtain reliable solution of the problem.

Numerical resolution to solve rupture dynamic

Approximate estimation of the cohesive zone width

From linear fracture mechanics for 2 dimensional cases:

The zero-speed cohesive zone width:

$$\Lambda_0 = \frac{9\pi}{32}\frac{\mu_m d_0}{(\tau_s - \tau_d)}$$

for *m = II, III*, respectively mode II and mode III rupture

$\mu_{II} = \mu$; $\mu_{II} = \mu/(1\text{-}\nu)$; μ =shear module; ν= Poisson's ratio

Cohesive zone width at large propagation distances (for mode III crack problems):

$$\Lambda = \frac{9}{16}\left(\frac{\mu d_0}{\Delta\tau}\right)^2 L^{-1} \quad for\ L \gg L_0$$

L = propagation distance.

L_0=half of critical crack length

$$L_0 = \frac{\mu d_0(\tau_s - \tau_d)}{\pi\Delta\tau^2}$$

Dimensionless ratio Nc (number of grids in the cohesive zone)

$$N_c = \frac{\Lambda}{\Delta x}$$

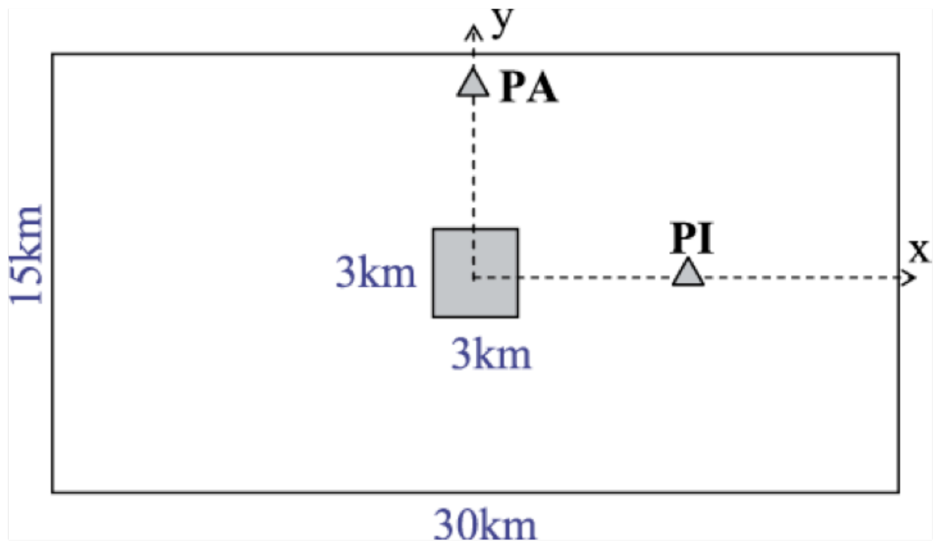
$\Delta\boldsymbol{x}$ = grid size (grid interval)

- This is good initial guidance to define the spatial resolution needed for the test problem.
- Both Λ_0 and Λ should give good initial guidance as to what kind of spatial resolution will be needed in dynamic rupture propagation problems.
- An appropriate **Nc** would depend on the numerical technique and type of fault representation method. It can be determined after a convergence analysis of the solution. Recommended at least Nc≥2

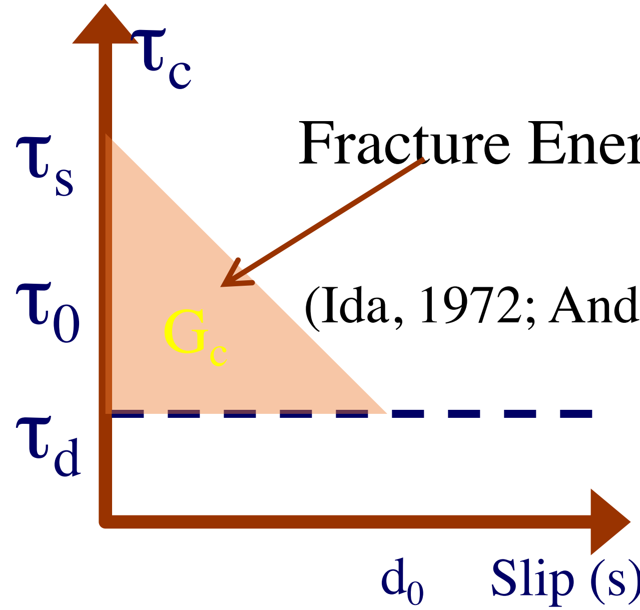
Assessment of Fault representation Methods

SCEC 3D Rupture Dynamics Code Validation Project
(coordinators Ruth Harris, Ralph Archuleta)

Fault model
(Test Problem Version 3, TPV3)



Slip Weakening Friction model



Numerical resolution measured by

Λ = cohesive-zone width (normal to rupture front)

Δx = spatial step size (in numerical solution)

(Dalguer and Day, 2006, BSSA)

Assessment of Fault representation Methods

Parameters for SCEC test problem 3

Parameters	Within Fault Area of 30 km x 15km		Outside Fault Area
	Nucleation	Outside nucleation	
Initial shear stress (τ_0), MPa	81.6	70.0	70.0
Initial normal stress (σ_n), MPa	120.0	120.0	120.0
Static friction coefficient (μ_s)	0.677	0.677	infinite
Dynamic friction coefficient (μ_d)	0.525	0.525	0.525
Static yielding stress ($\tau_s = \mu_s \sigma_n$), MPa	81.24	81.24	infinite
Dynamic yielding stress ($\tau_d = \mu_d \sigma_n$), MPa	63.0	63.0	63.0
Dynamic stress drop ($\Delta\tau = \tau_0 - \tau_d$), MPa	18.6	7.0	7.0
Strength excess ($\tau_s - \tau_0$), MPa	-0.36	11.24	infinite
Critical slip distance, d_0 , m	0.40	0.40	0.40

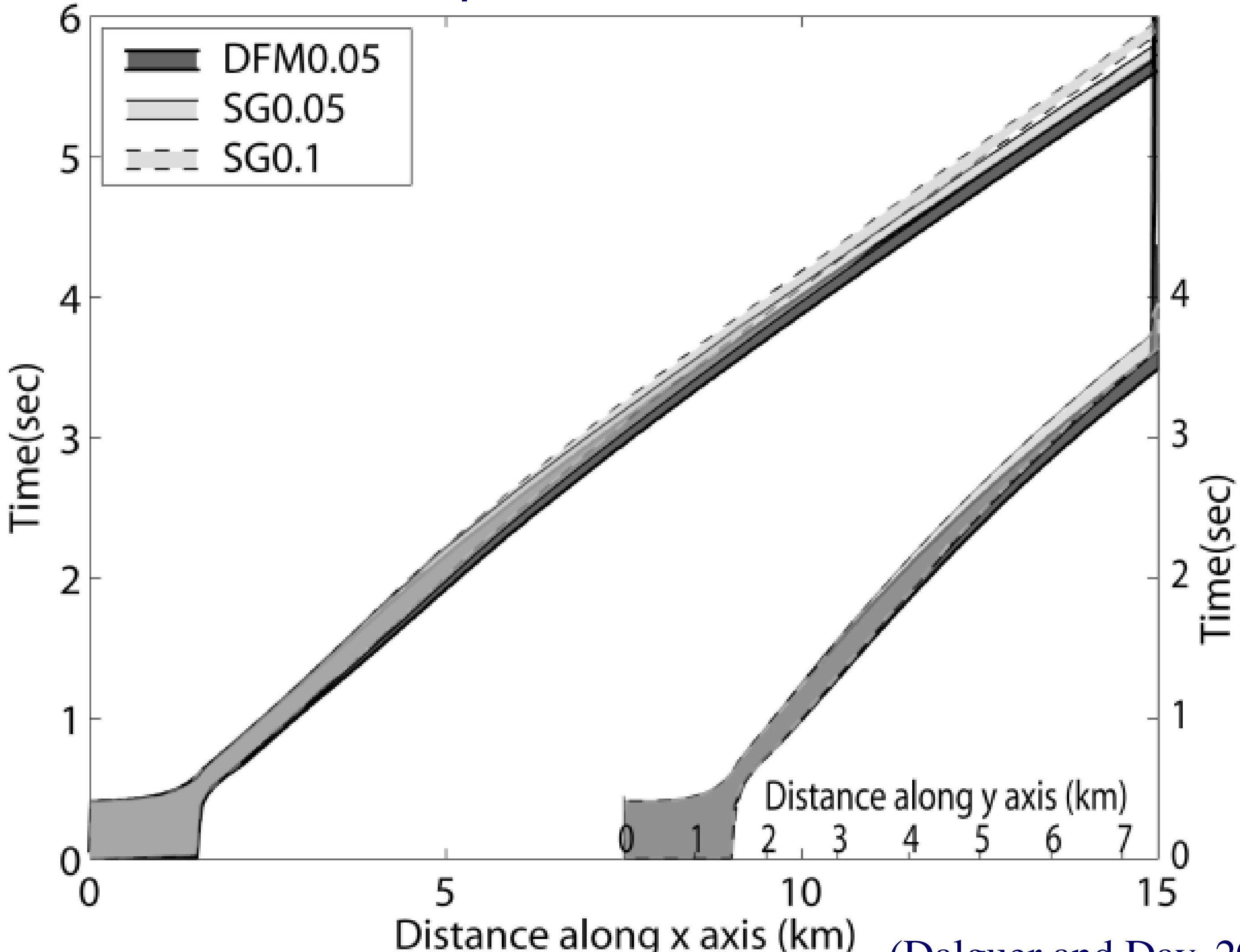
Homogeneous medium:
P wave velocity=6000 m/s
S wave velocity=3464 m/s
Density =2670 kg/m3.

- **Nucleation size:** L_0=1.516km (half of critical crack length), then assumed 3km x 3km
- **Zero-speed cohesive zone** Λ_0 = 620m for mode III, and Λ_0 = 827m for mode II. They can be considered as the upper bound of the problem
- ***Λ* at the maximum propagation distance** L=7.5km along the mode III =251m.
- Assuming a grid size Δx=100m, Nc = 6 to 8 for the upper bound, and 2.5 for the propagation distance.
- Then a good spatial resolution for the problem requires **$\Delta x \leq$100m.**
- The accuracy reached by this resolution will depend on the method used to model the fault as well as the numerical technique.

(Dalguer and Day, 2006, BSSA)

SG inelastic zone - vs - Split-node models

Cohesive zone development

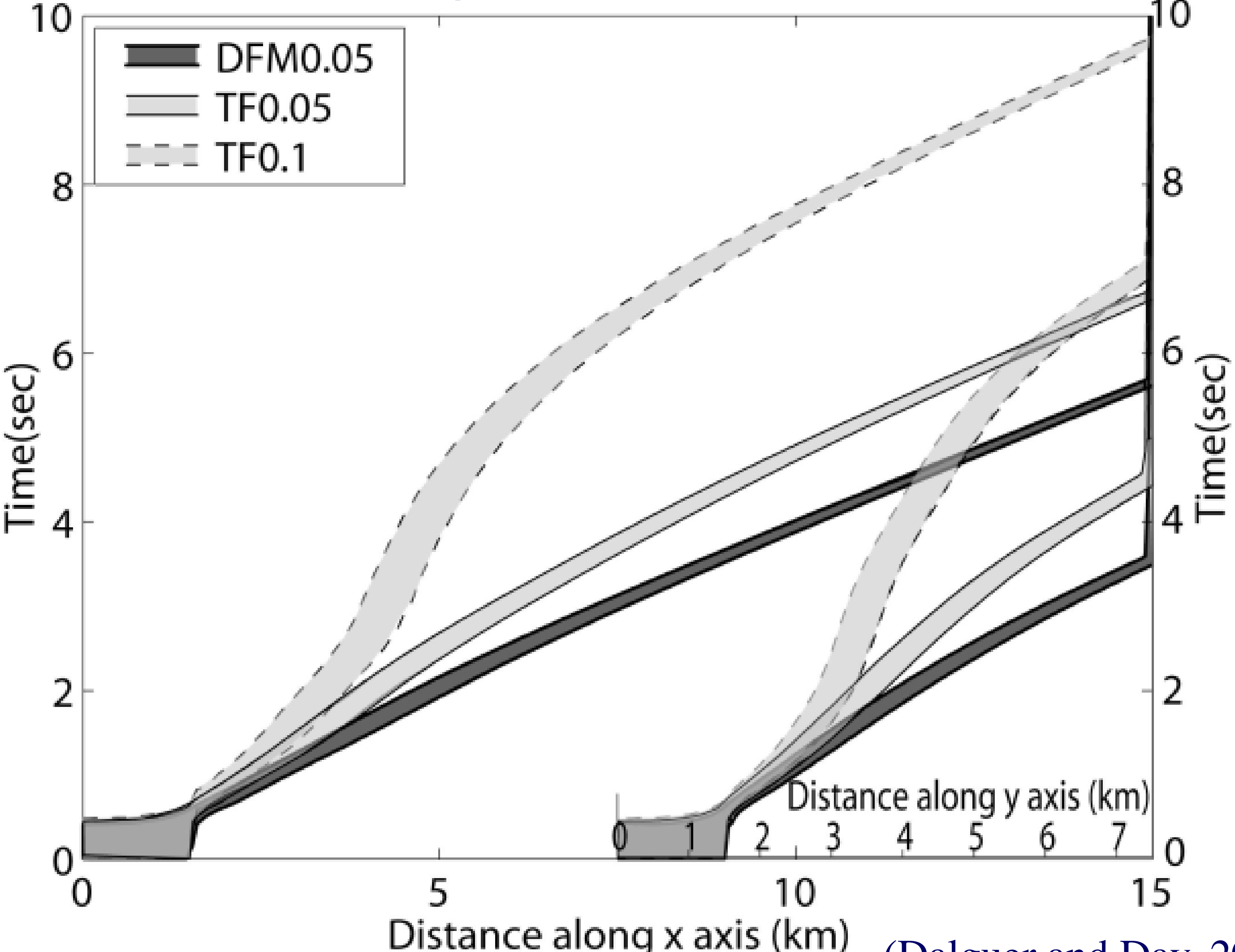


(Dalguer and Day, 2006, BSSA)

Assessment of Fault representation Methods

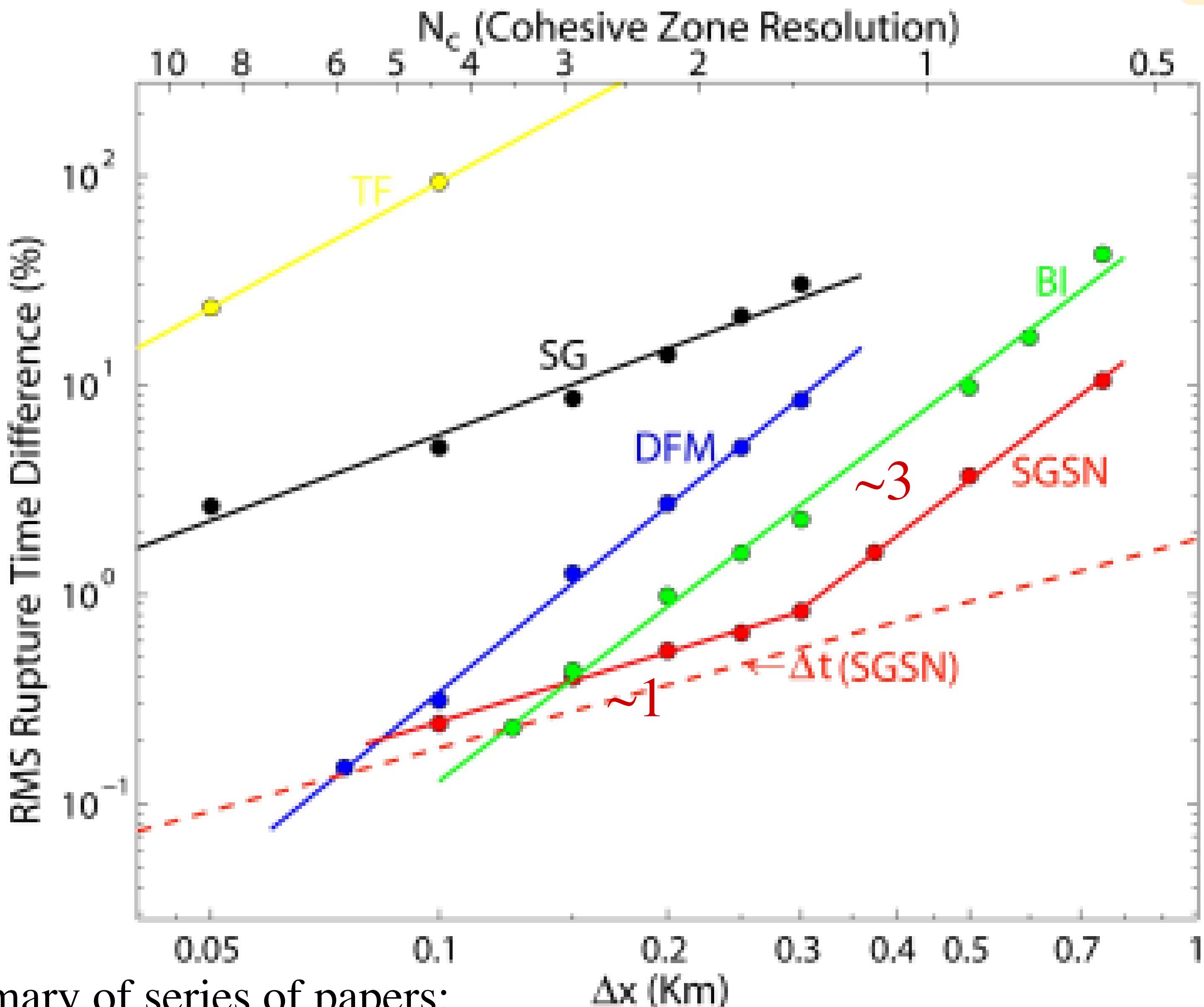
TF inelastic zone - vs - Split-node models

Cohesive zone development



(Dalguer and Day, 2006, BSSA)

Assessment of Fault representation Methods

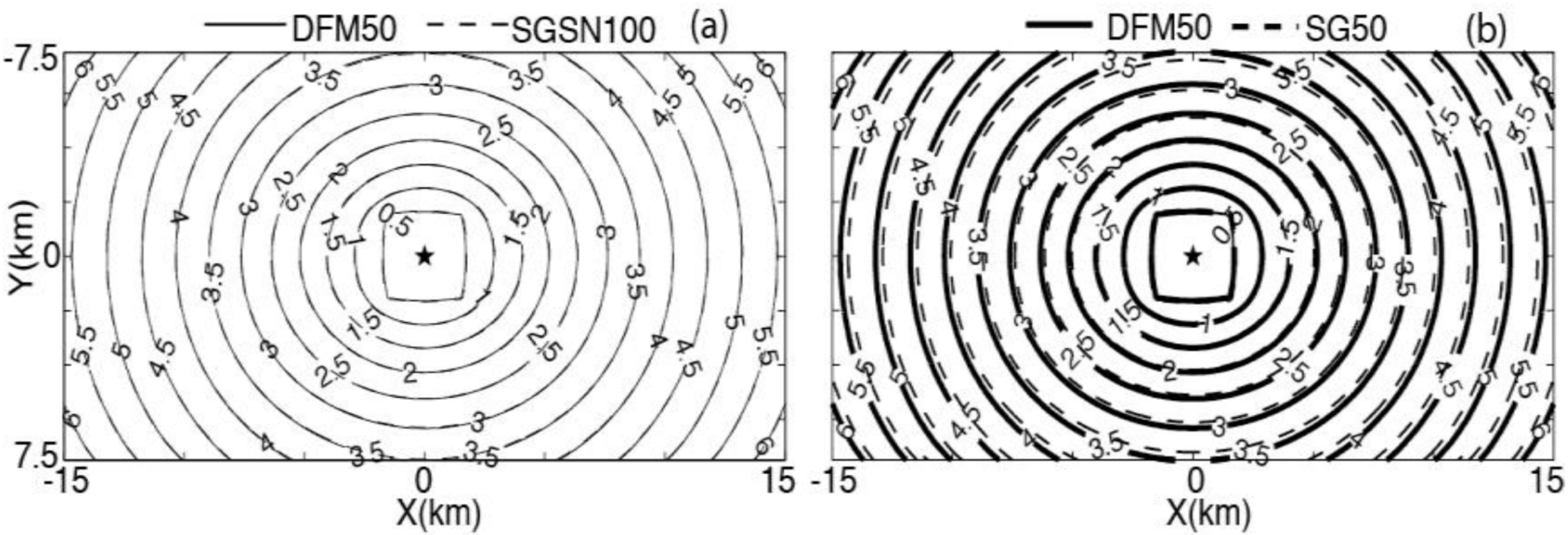


Summary of series of papers:

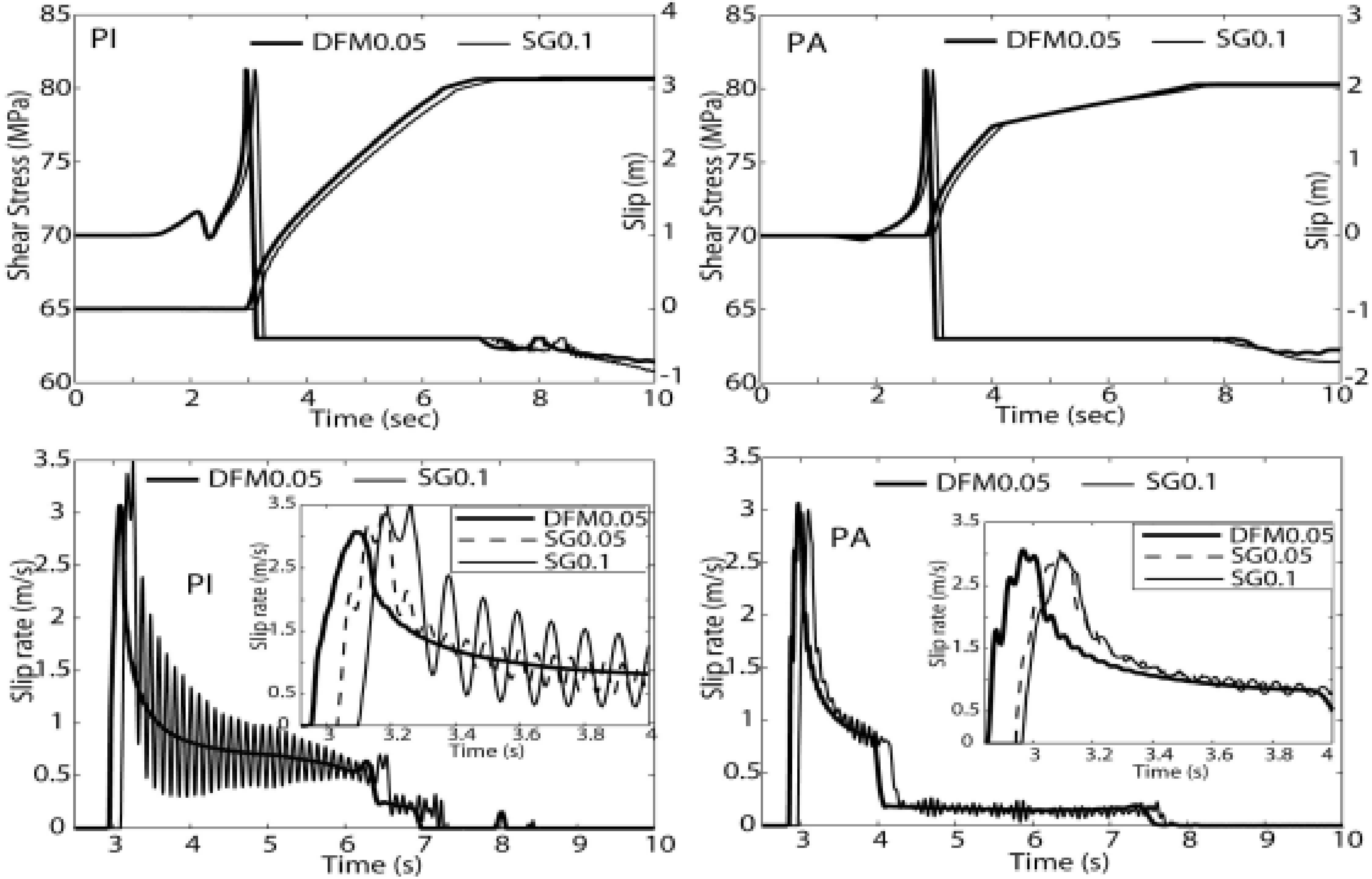
(Day, Dalguer, et al, 2005, JGR; Dalguer and Day, 2006, BSSA; 2007, JGR)

Assessment of Fault representation Methods

Contour plot of the rupture front for the dynamic rupture test problem



SG inelastic zone - vs - Split-node models

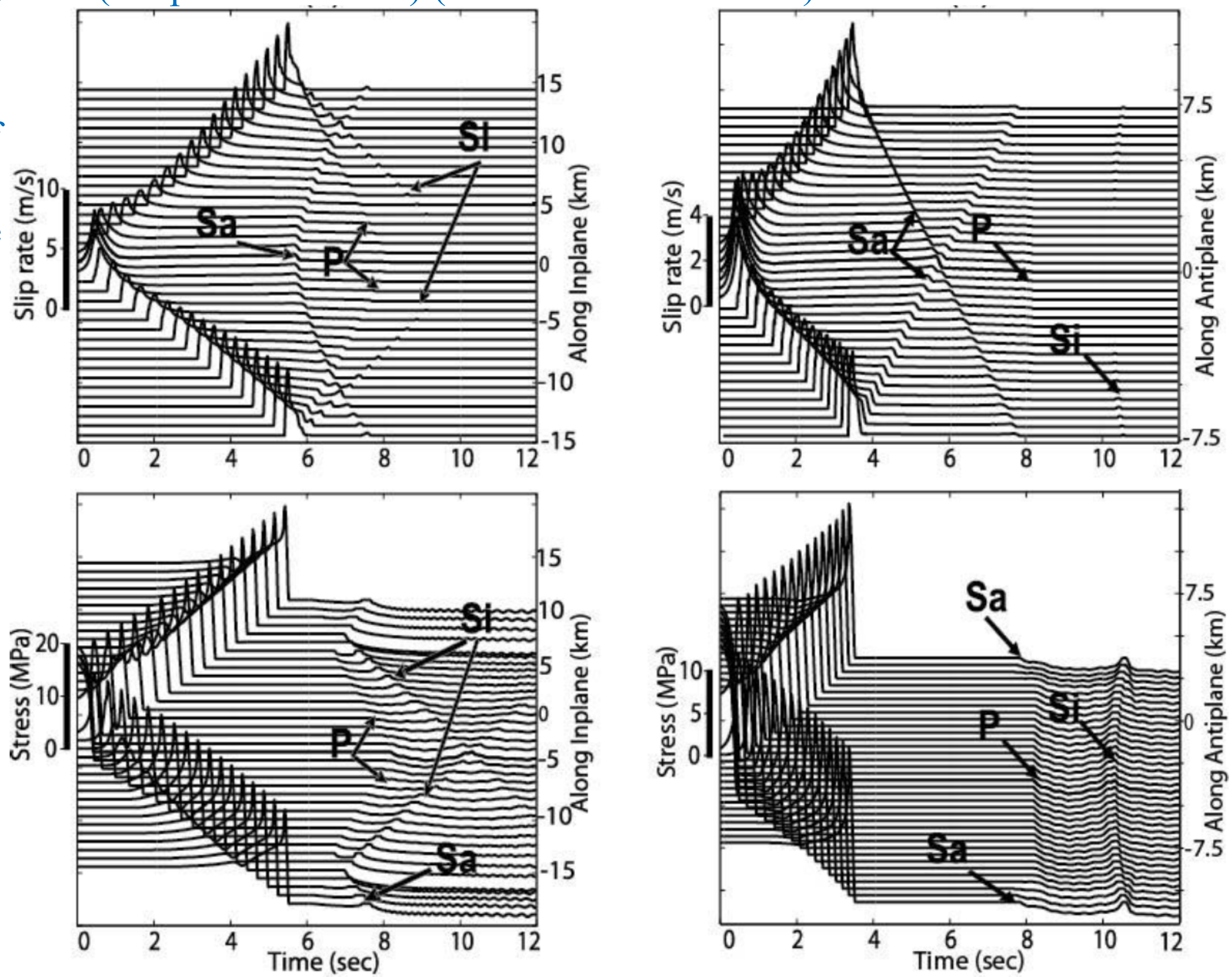


(Dalguer and Day, 2006, BSSA)

Assessment of Fault representation Methods

Slip rate and shear stress time history profiles along the *x* axis (in-plane direction) and the *y* axis (antiplane direction) (results for the DFM50)

A very insightful nature of this kind of dynamic rupture models is the rupture evolution that involves: initiation, evolution and stopping of the slip, and the evolution of the stress after the slipping ceases.



Remarks

- Here we have described the numerical algorithms of two well known methods to represent fault discontinuity for spontaneous rupture dynamic calculation: the so-called traction at split-node (TSN) scheme and the inelastic-zone stress methods that are mainly used for FEM and FDM techniques.
- There are other developments of fault representation and wave propagation techniques, such us those used in Finite Volumes (FV) methods (e.g. Benjemaa et al., 2009) and high order discontinues Galerkin (DG) methods (e.g. de la Puente et al., 2009; Pelties et al., 2012). The nature of the fault representation in these methods is different than the TSN and fault zone method described here. The VF and DG incorporate formulations of fluxes to exchange information between the two surfaces of contact by solving the Riemann problem (e.g. LeVeque, 2002).

- References and additional description of what have been presented here can be found in: **Dalguer, L. A. (2012),** Numerical Algorithms for Earthquake Rupture Dynamic Modeling. Chapter 4 *In "The mechanics of faulting: From Laboratory to Real Earthquakes*", Research Signpost, 93-124, ISBN 978-81-308-0502-3, Editors A. Bizzarri and H Bath. This chapter-paper is included in the material of this lecture.

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