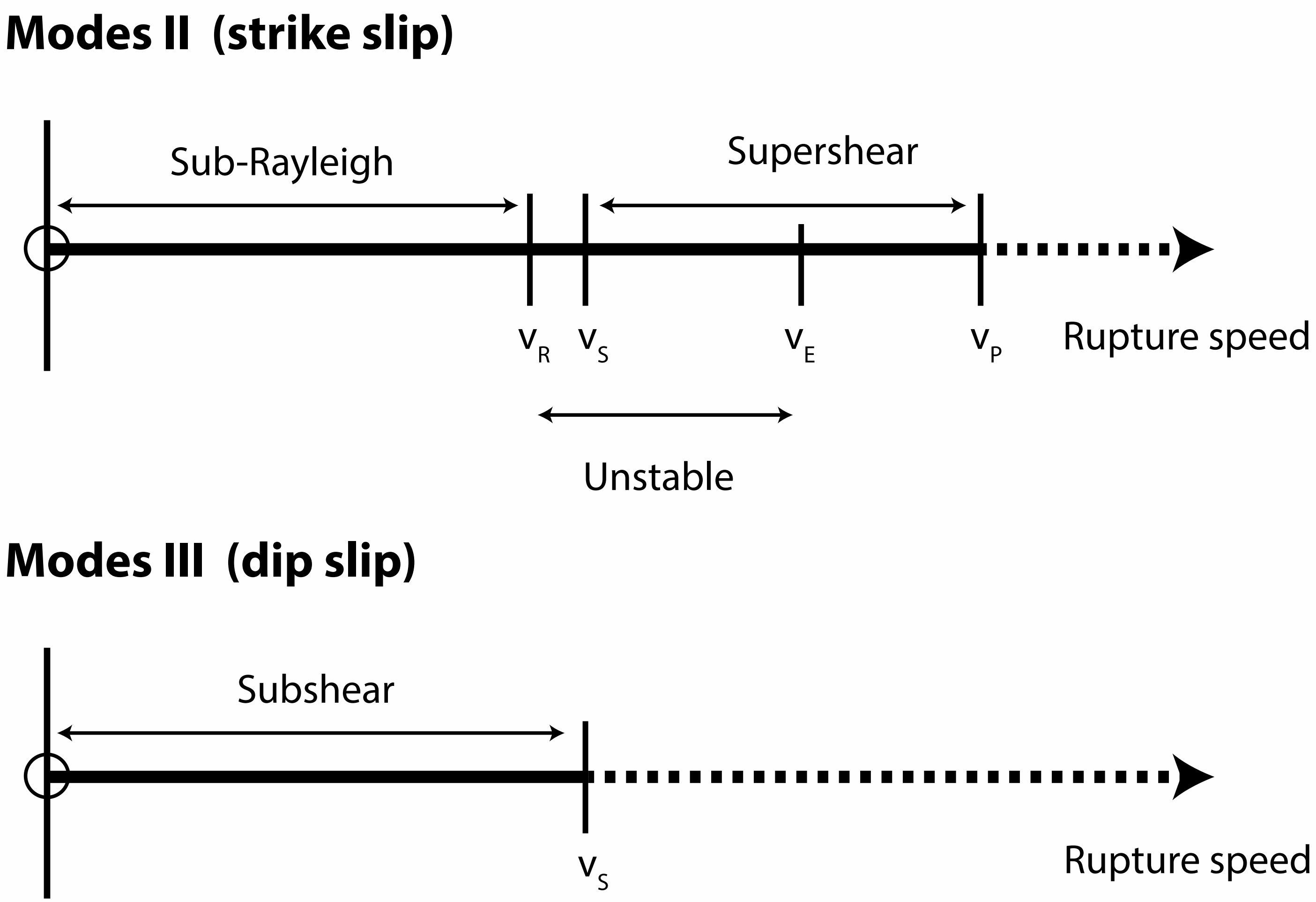
3D rupture effects (seismogenic depth)

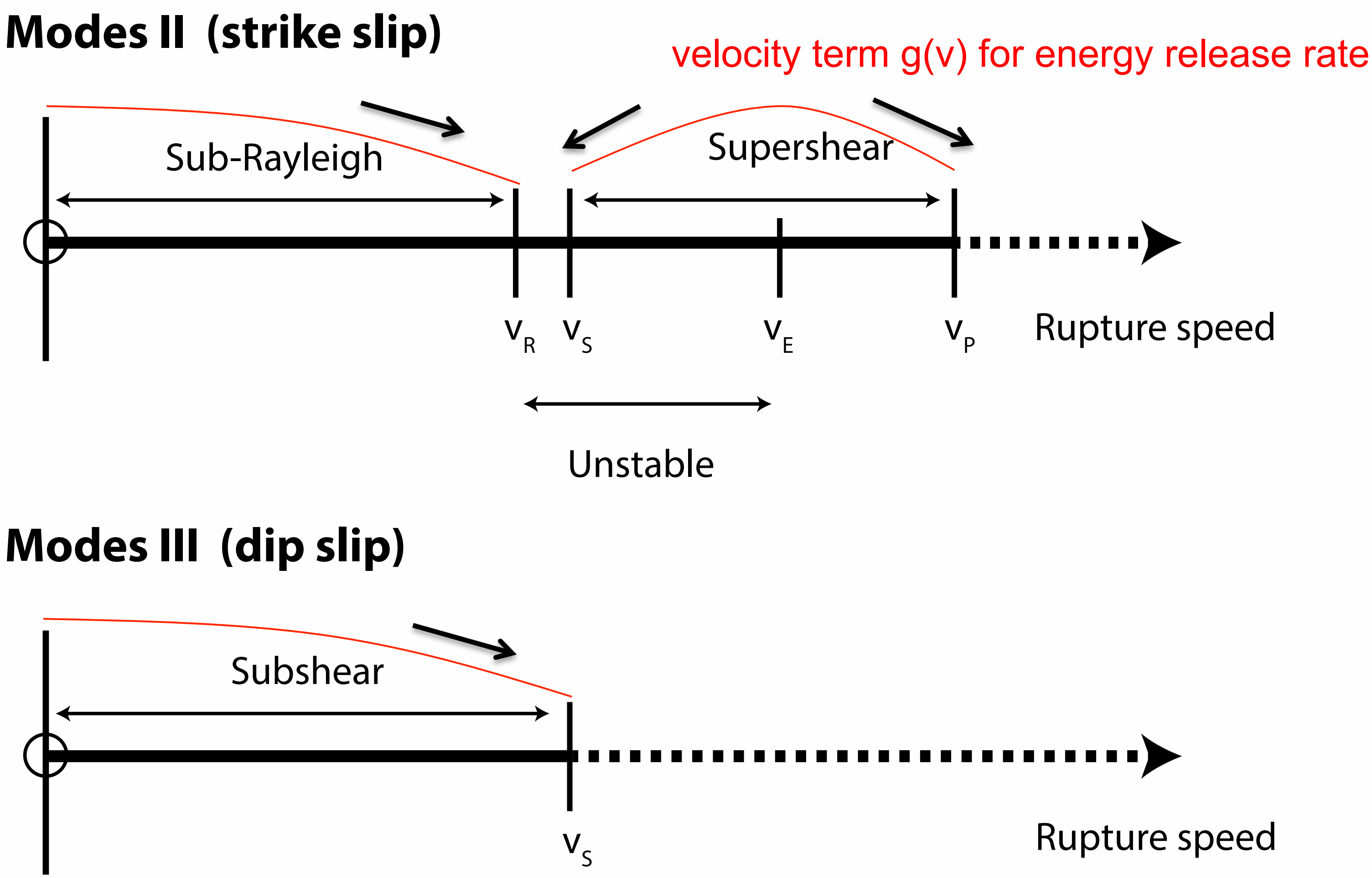
Huihui Weng
Jean-Paul Ampuero

ICTP, Trieste, Italy, 2-14 Sep.

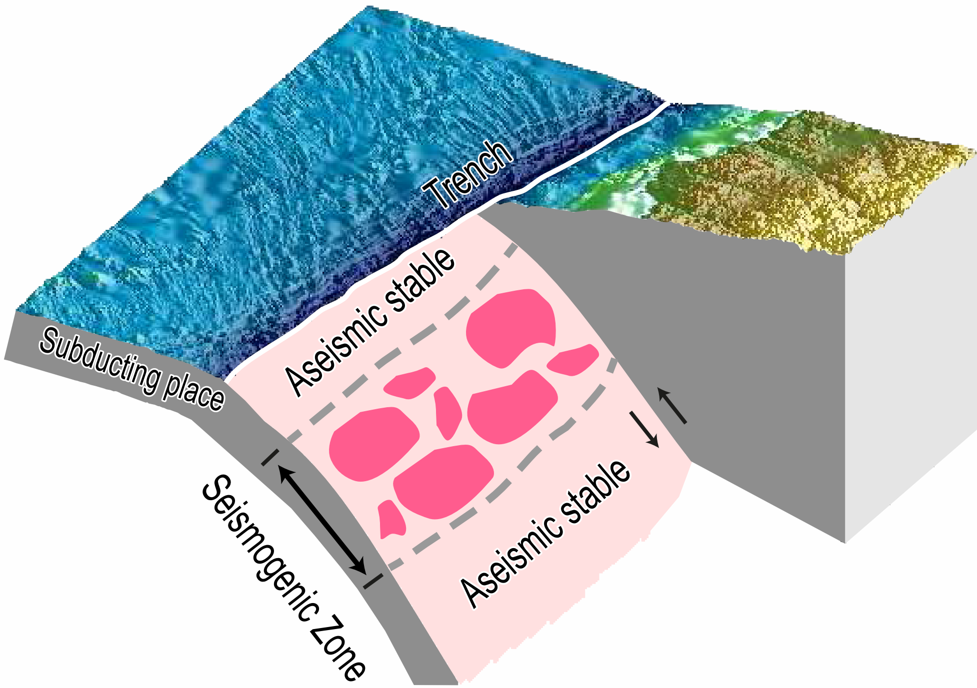
Rupture speeds in 2D



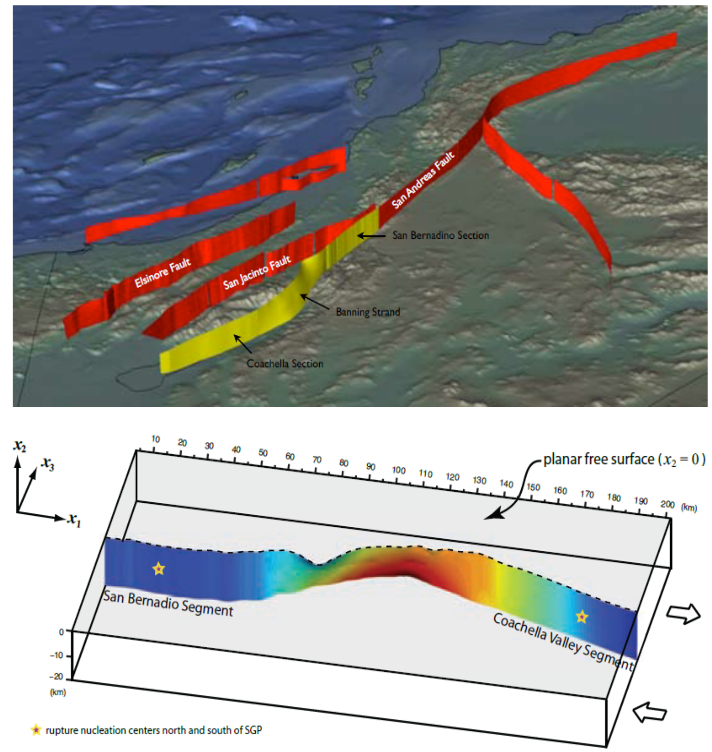
Stable speeds in 2D



Finite seismogenic width



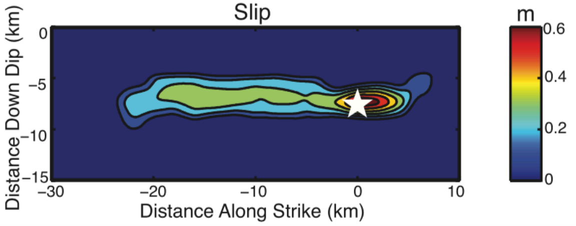
Weng and Ampuero, 2019



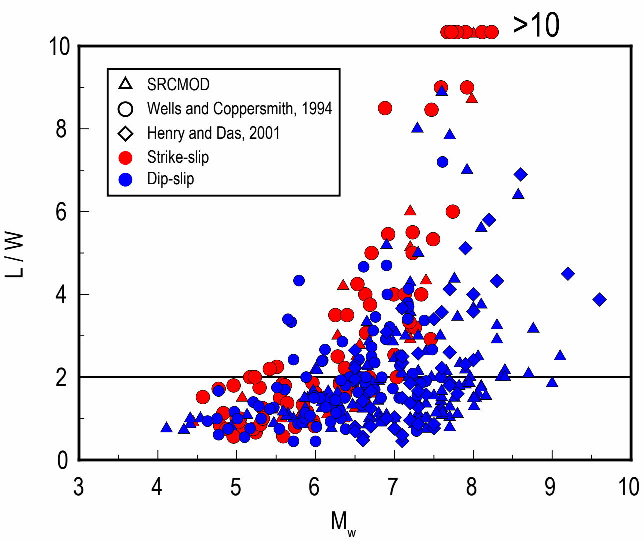
Fault and Rock Mechanics (FARM)

Elongated earthquake ruptures

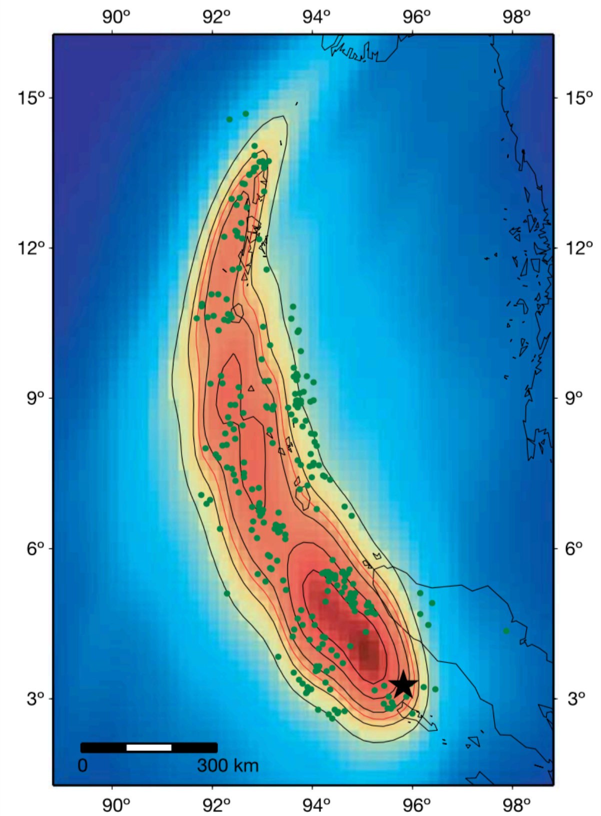
2004 Mw 6 Parkfield



Ma et al 2008

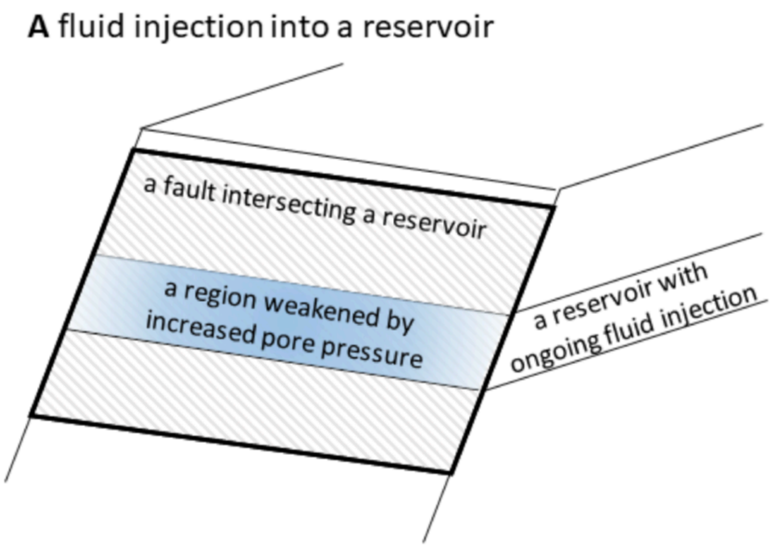


2004 Mw 9.3 Sumatra



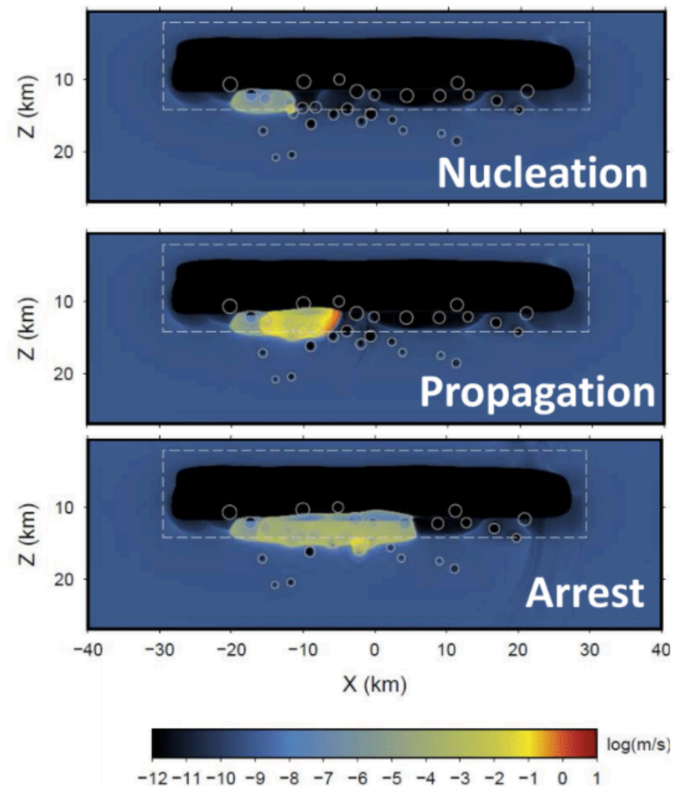
Ishii et al 2005

Elongated earthquake ruptures



Galis et al 2018

Rupture unzipping the lower edge of the seismogenic zone
(simulation by Junle Jiang)



overview

- Equation of motion for mode III in 3D
- Equation of motion for mode II in 3D

  -- Subshear
  -- Supershear
- Ruptures of mixture of modes II and III

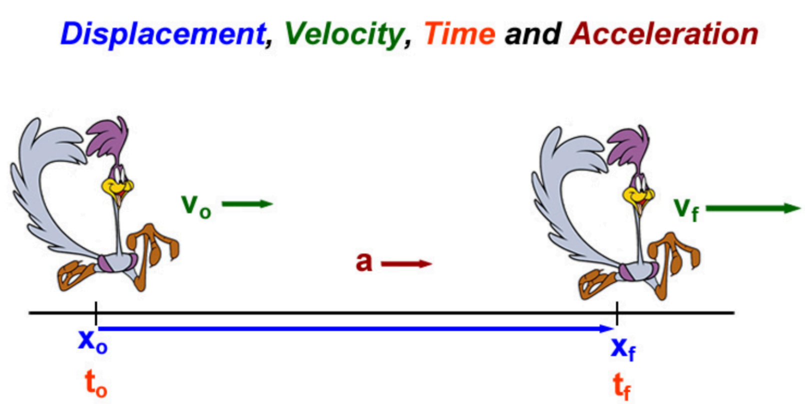
overview

- Equation of motion for mode III in 3D
- Equation of motion for mode II in 3D

  -- Subshear
  -- Supershear
- Ruptures of mixture of modes II and III

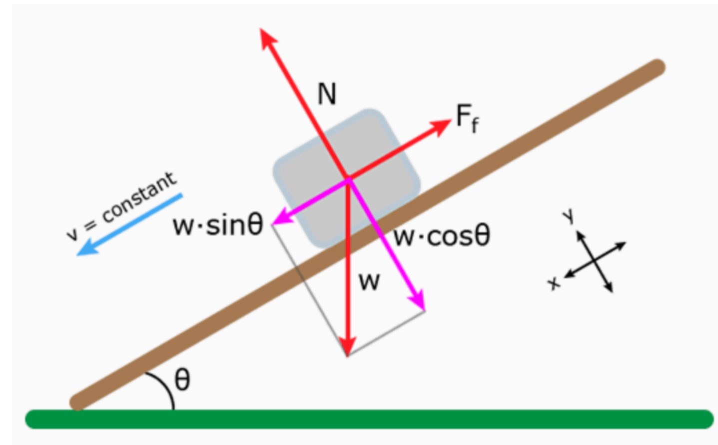
Warm-ups

Kinematics



http://sdsu-physics.org

Dynamics

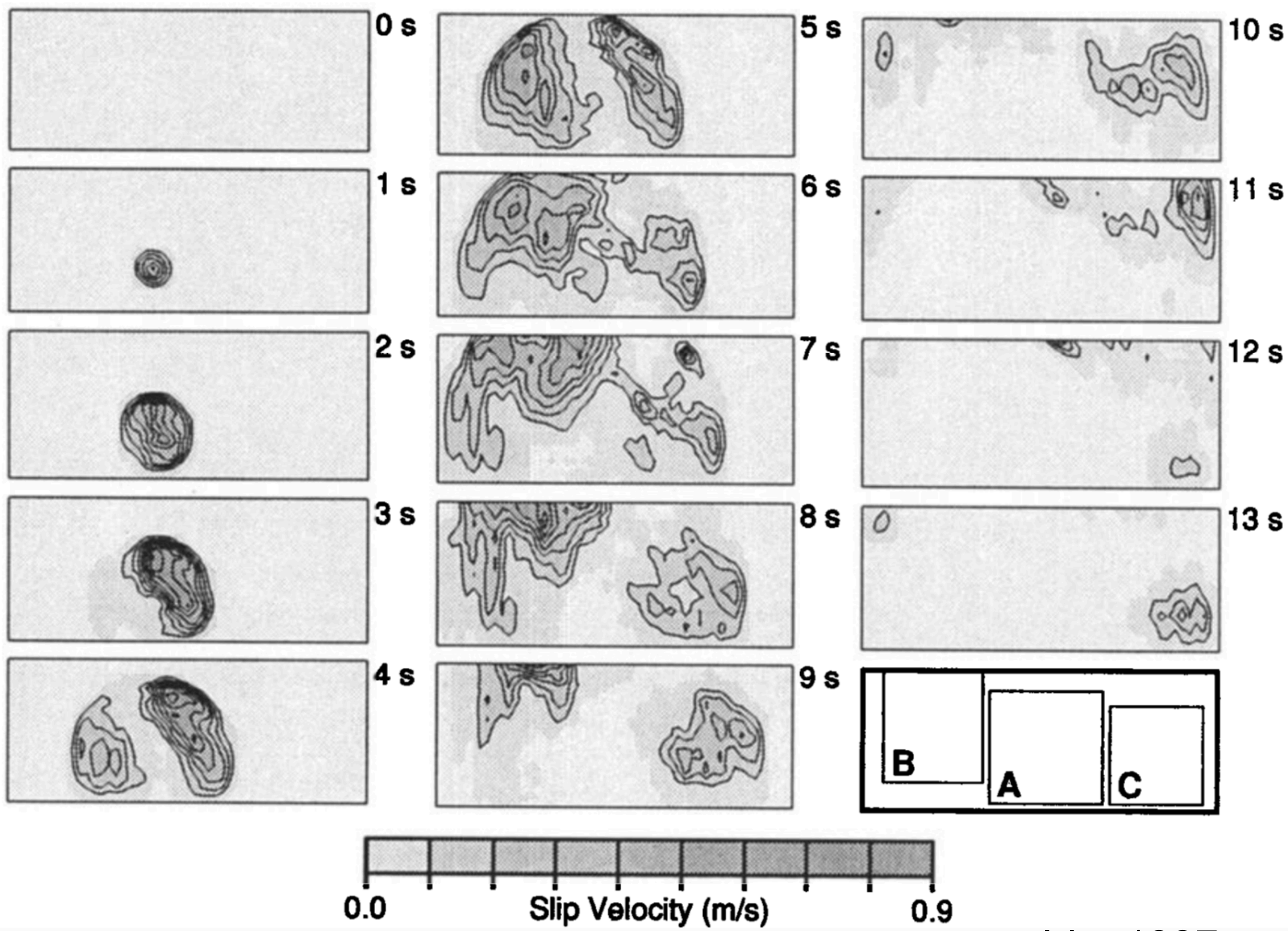


sciencenotes.org

$$F = ma$$

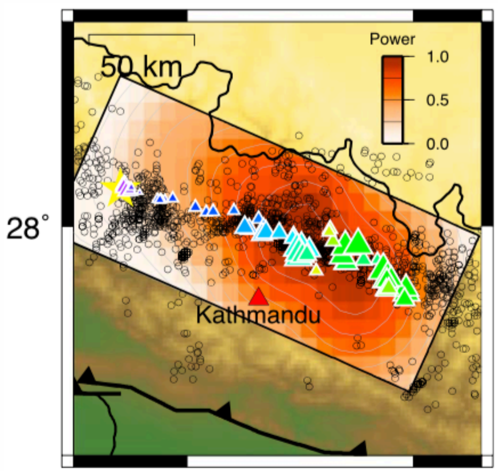
Newton's second law

Slip inversion method

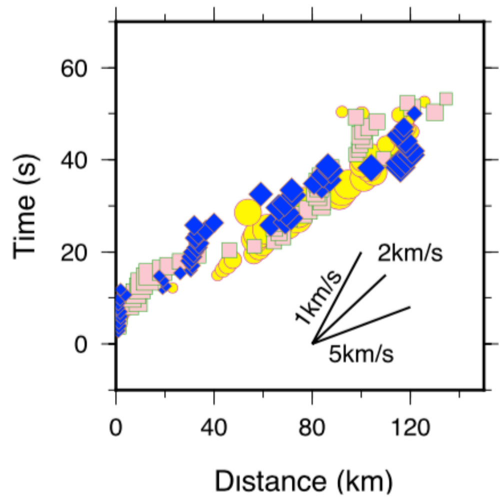


Ide, 1997

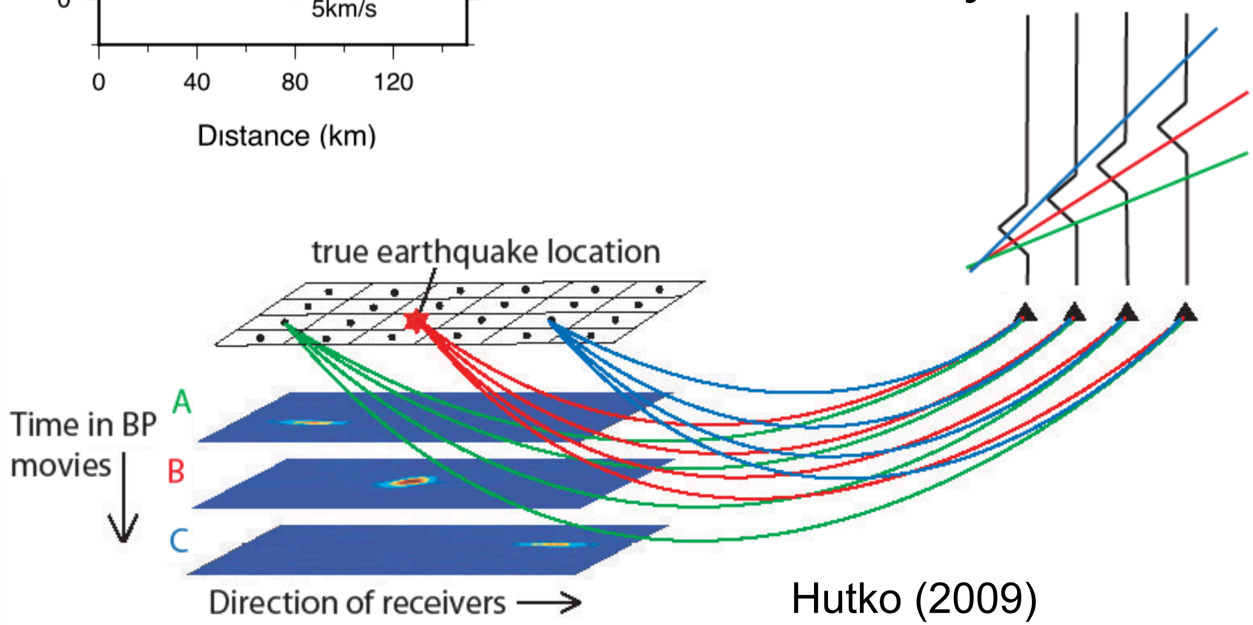
Imaged by back-projection



Meng et al. (2016)



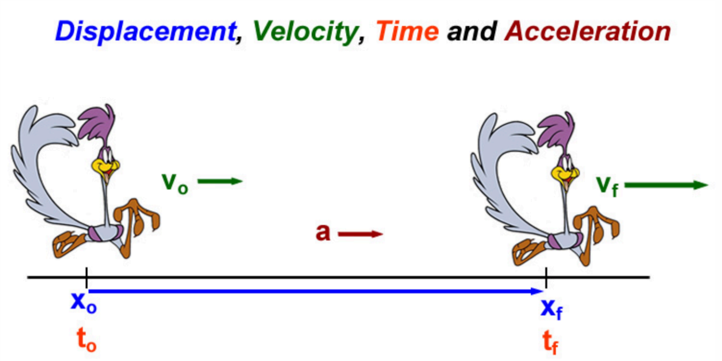
- Track rupture speed
- High frequency data
- Seismic array



Hutko (2009)

Questions

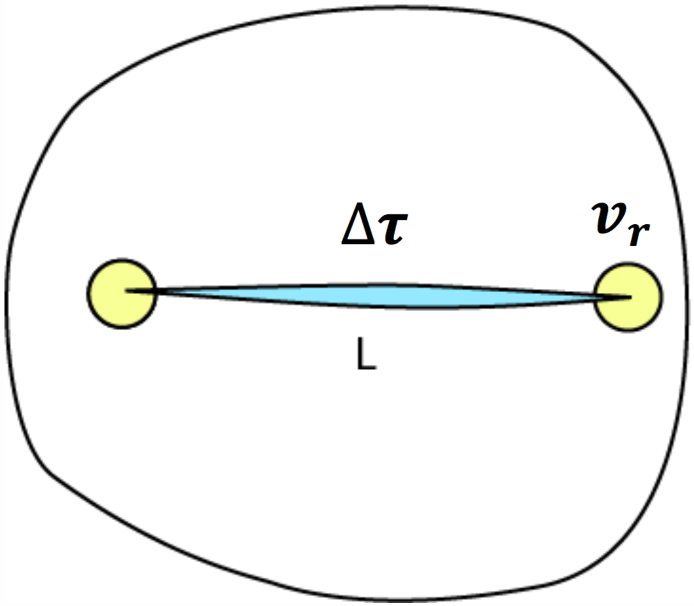
- How to explain observed source kinematics?
- What is the intrinsic earthquake physics?
- How to link kinematics and dynamics of earthquakes?



Fracture mechanics:

-- connection between kinematics and dynamics

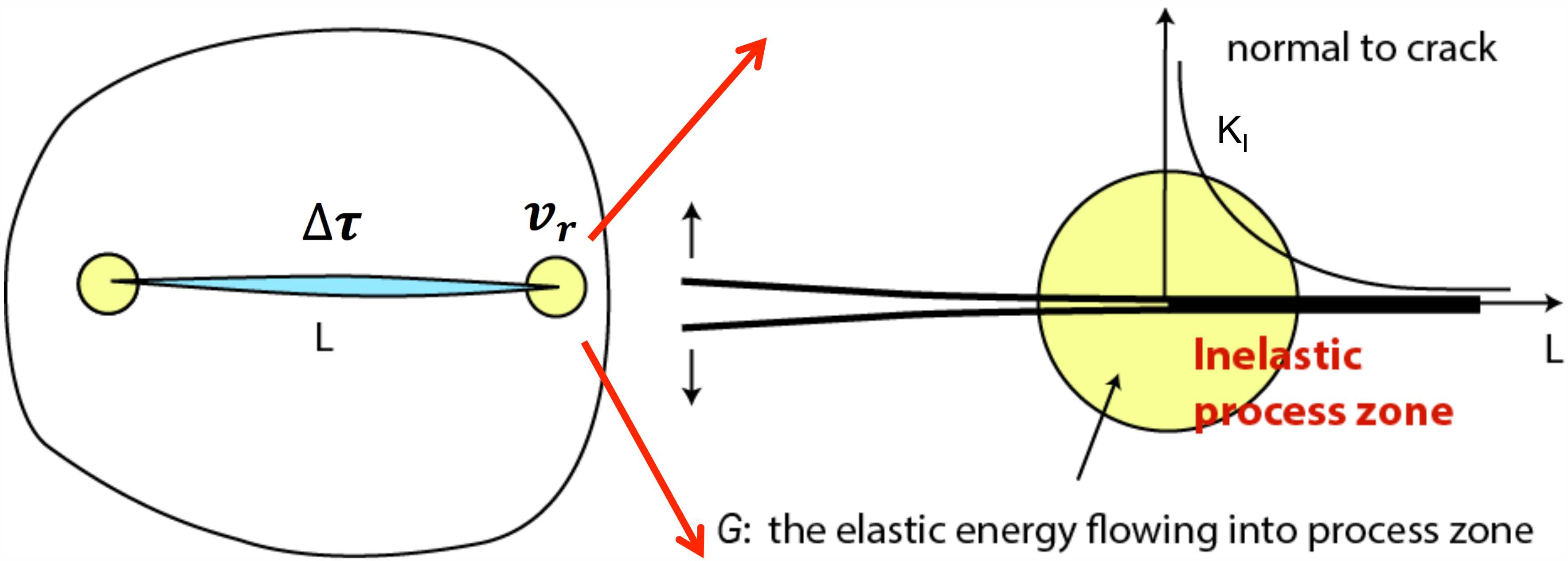
Linear elastic fracture mechanics



- Energy balance between energy release rate and fracture energy
- Rupture speed as a function of distance

Kostrov, Freund, Andrews (60-70s)

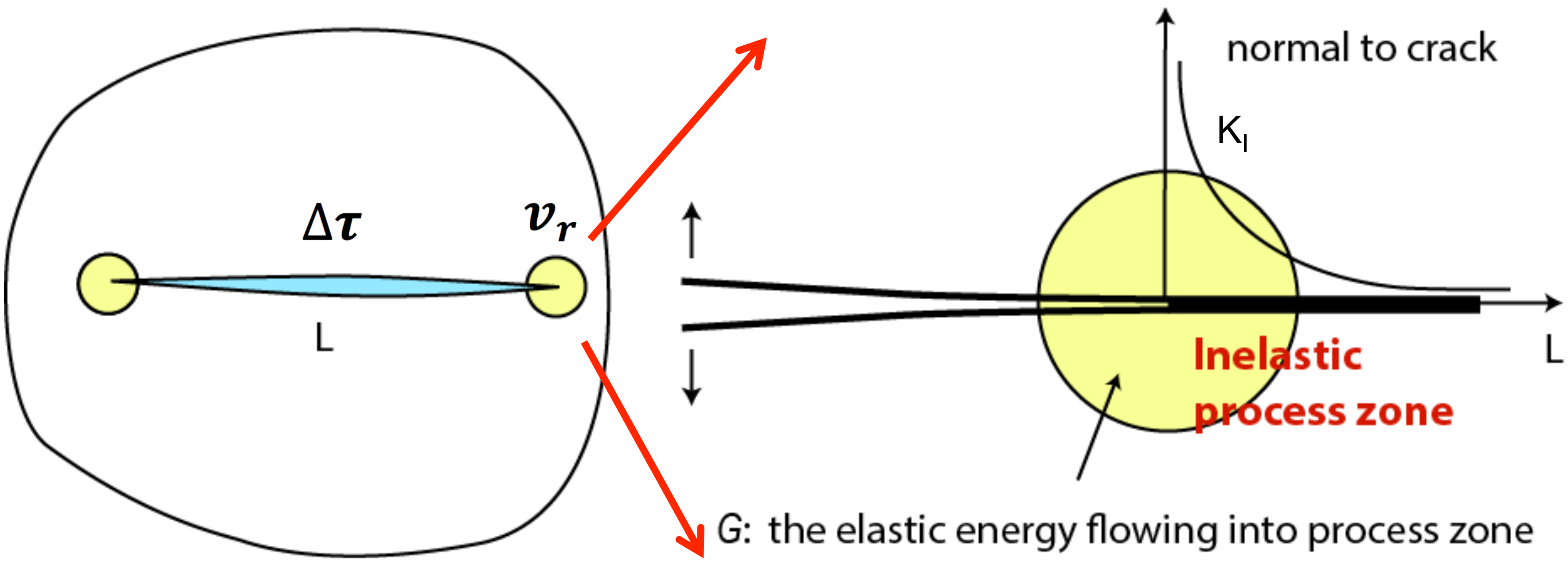
Linear elastic fracture mechanics



$$G(v_r, L, \Delta\tau) = \frac{1}{2\mu} g(v_r) K_I^2(v_r, L, \Delta\tau)$$

Kostrov, Freund, Andrews (60-70s)

Linear elastic fracture mechanics



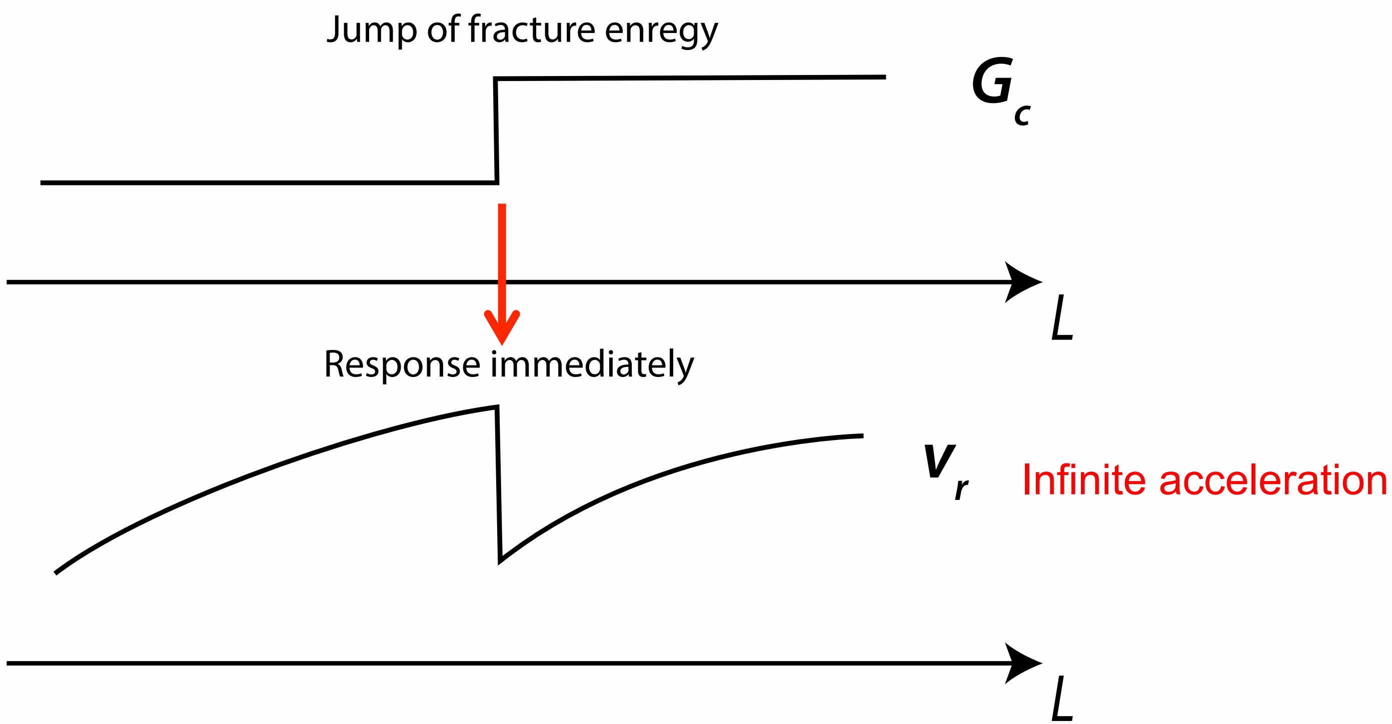
$$G(v_r, L, \Delta\tau) = \frac{1}{2\mu} g(v_r) K_I^2(v_r, L, \Delta\tau)$$

$$\boldsymbol{G_c = G(v_r, L, \Delta\tau)}$$

Kostrov, Freund, Andrews (60-70s)

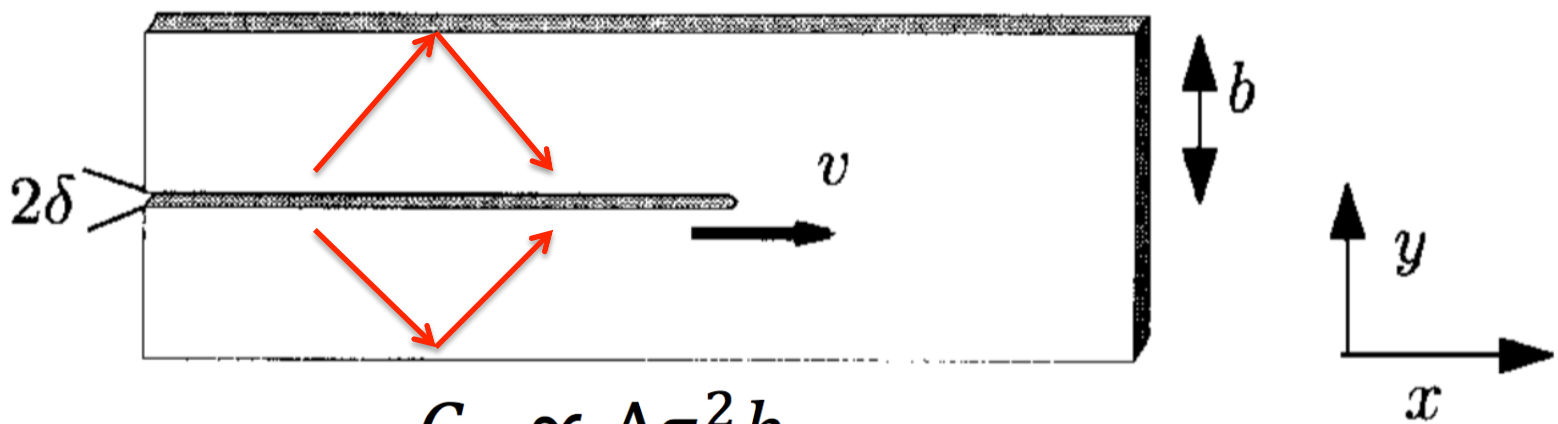
Linear elastic fracture mechanics

- Classical LEFM is not "inertial" $G_c = G(v_r, L, \Delta\tau)$
- Speed is independent of acceleration



Crack in bounded media

Waves are reflected back



$$G_0 \propto \Delta\tau^2 b$$

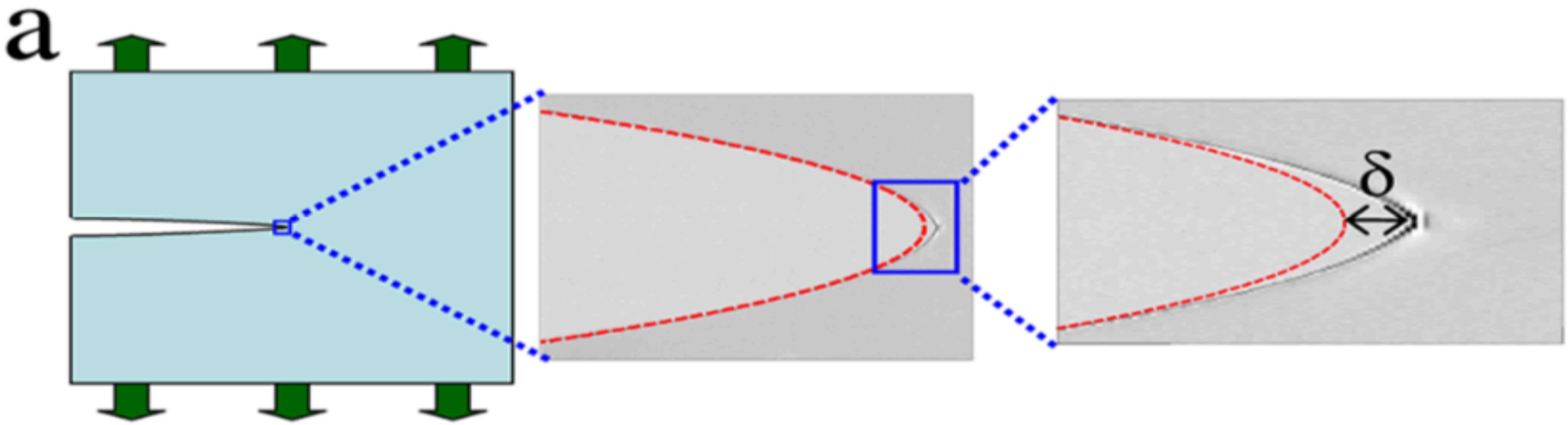
Marder (1998)

- Release elastic energy is linearly proportional to width of strip
- $$G_c = G_0\left(1 - \frac{\dot{v}_r b}{v_s^2}\frac{1}{(1-(v_r/v_s)^2)^2}\right)$$

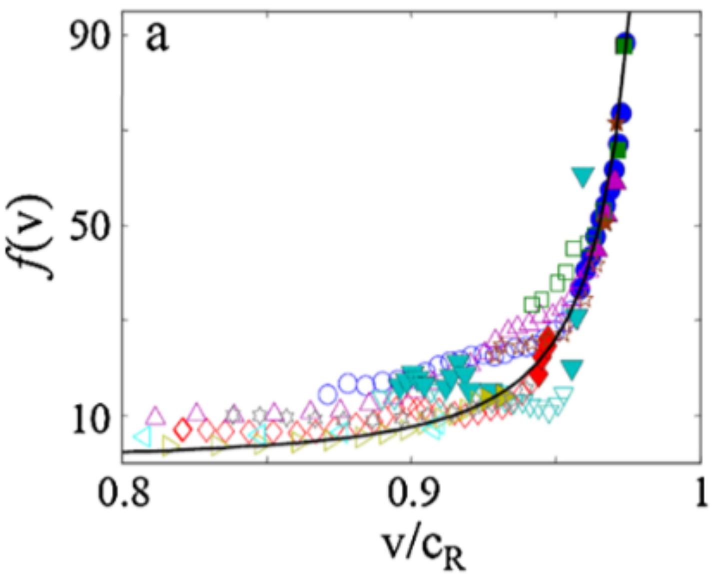
LEFM

$$\boldsymbol{G_c = G(v_r, L, \Delta\tau)}$$

Strip experiments



Livne et al 2008



$$G = W\left(1 - \frac{b\dot{v}}{c_l^2} f(v)\right) \approx W\left(1 - \frac{b\dot{v}}{c_l^2} \frac{1}{(1 - (\frac{v}{c_R})^2)^2}\right).$$

Goldman et al 2010

Inertial equation of motion

$$G_c = G_0\left(1 - \frac{\dot{v}_r b}{v_s^2}\frac{1}{(1-(v_r/v_s)^2)^2}\right)$$

↓

$$(1 - G_c/G_0) = \frac{W}{v_s^2 A \alpha_s^P} \cdot \dot{v}_r$$

Inertial equation of motion

$$G_c = G_0\left(1 - \frac{\dot{v}_r b}{v_s^2}\frac{1}{(1-(v_r/v_s)^2)^2}\right)$$

$$(1 - G_c/G_0) = \frac{W}{v_s^2 A \alpha_s^P}\cdot \dot{v}_r$$

Force? Apparent mess? Acceleration

F = ma

Key points

1 Classical LEFM links kinematics and dynamics of 2D infinite media, which is not “inertial”

$$G_c = G(v_r, L, \Delta\tau)$$

2 The crack-tip-equation-of-motion for 2D strip media is “inertial”

$$G_c = G_0\left(1 - \frac{\dot{v}_r b}{v_s^2}\frac{1}{(1-(v_r/v_s)^2)^2}\right)$$

Which may control ruptures on 3D bounded fault?
1 or 2 ?

The dynamics of elongated ruptures:

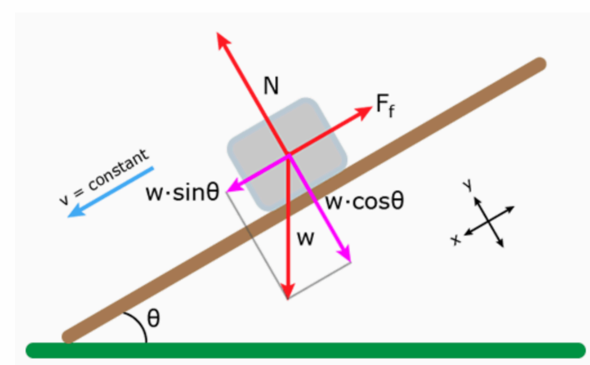
-- Rupture acceleration (how rupture begins?)

-- Rupture deceleration (how rupture stops?)

Analytical model

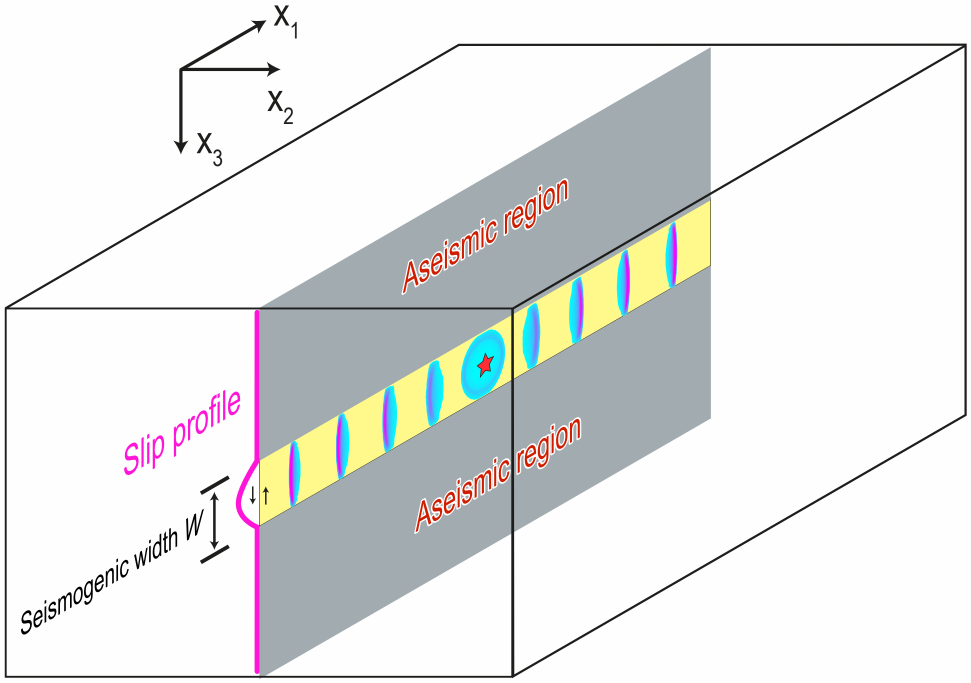
Ingredients

- Anti-plane fault embed in 3D full-space
- Uniform elastic properties
- Uniform fault parameters
- Uniform seismogenic width
- Steady-state speed



Analytical model

$$\sigma_{ij,j} = \rho \ddot{u}_i$$ (3 equations)

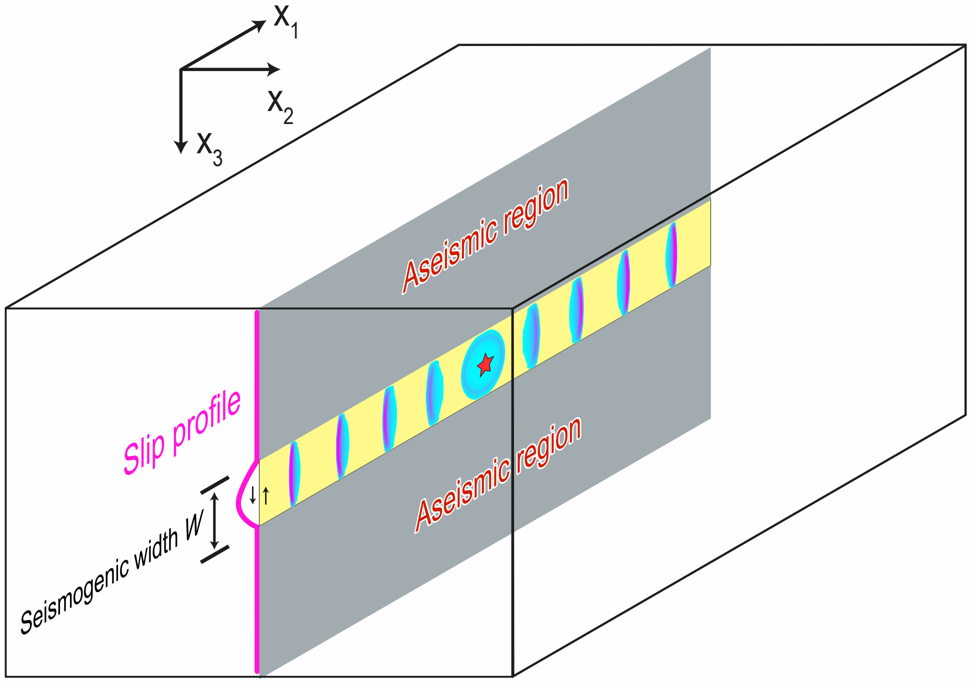


Analytical model

$$\sigma_{ij,j} = \rho \ddot{u}_i$$ (3 equations)

↓ Reduce to 1 equation

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \frac{\partial^2 u}{\partial x_3^2} = \frac{1}{v_s^2}\frac{\partial^2 u}{\partial t^2}$$



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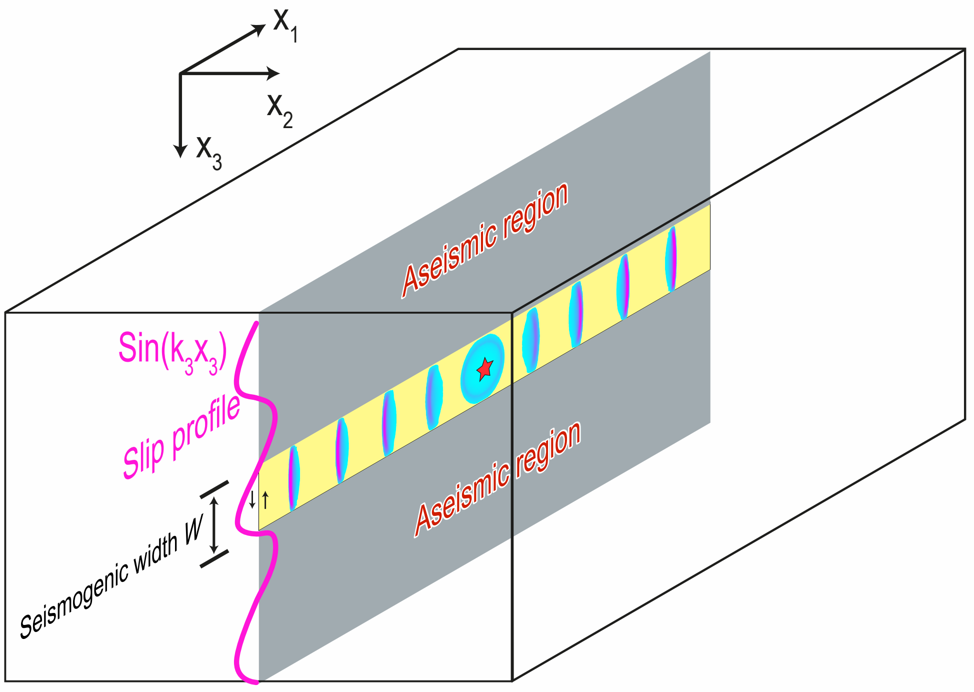
Slip approximation

$$u(x_1, x_2, x_3) = u(x_1, x_2, t)e^{ik_3x_3}$$

$$k_3 = \pi/W$$



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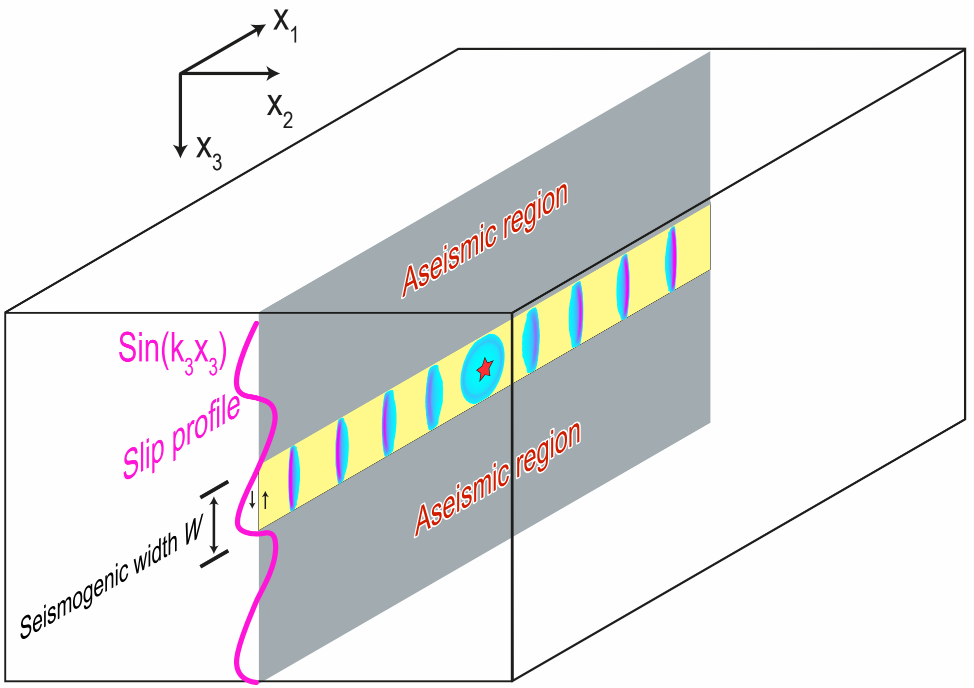
$$k_3 = \pi / W$$

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} - {k_3}^2 u = \frac{1}{v_s^2}\frac{\partial^2 u}{\partial t^2}$$

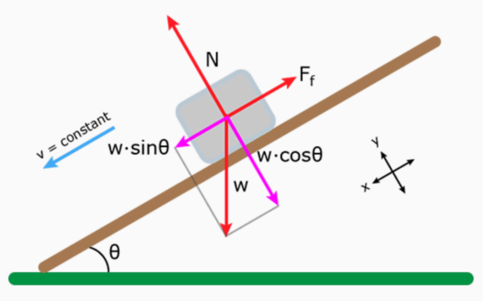


Energy release rate

- ✓ Energy release rate G is a constant independent of rupture speed and distance, i.e., $G = G_0$
- ✓ $G_c = G_0$ → propagate at any speed

$$G_0 = \frac{\Delta\tau^2 W}{\pi\mu}$$

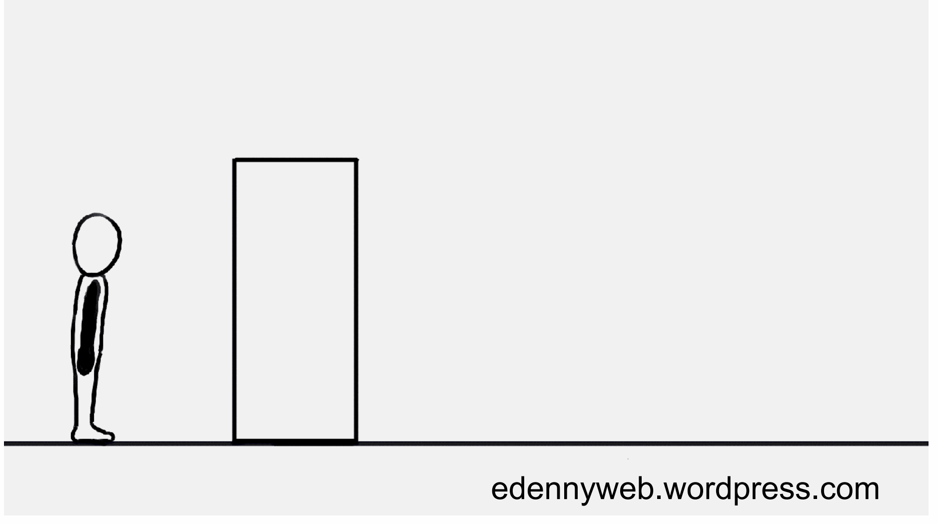
$G_c \neq G_0$?



Energy balance at rupture tip

Intuitive physical process

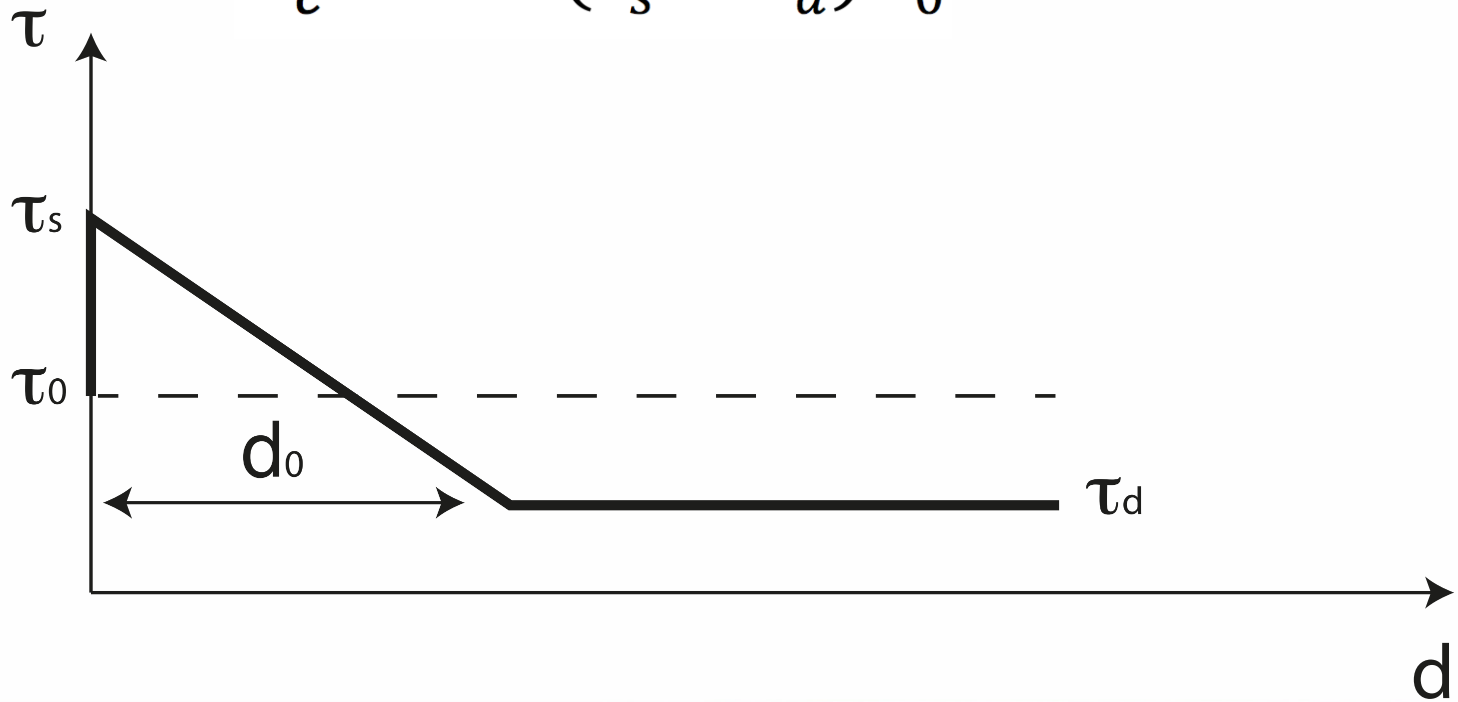
- $G_0 = G_c$ → ruptures propagate steadily
- $G_0 > G_c$ → ruptures accelerate ↑
- $G_0 < G_c$ → ruptures decelerate ↓



Validation from numerical simulations

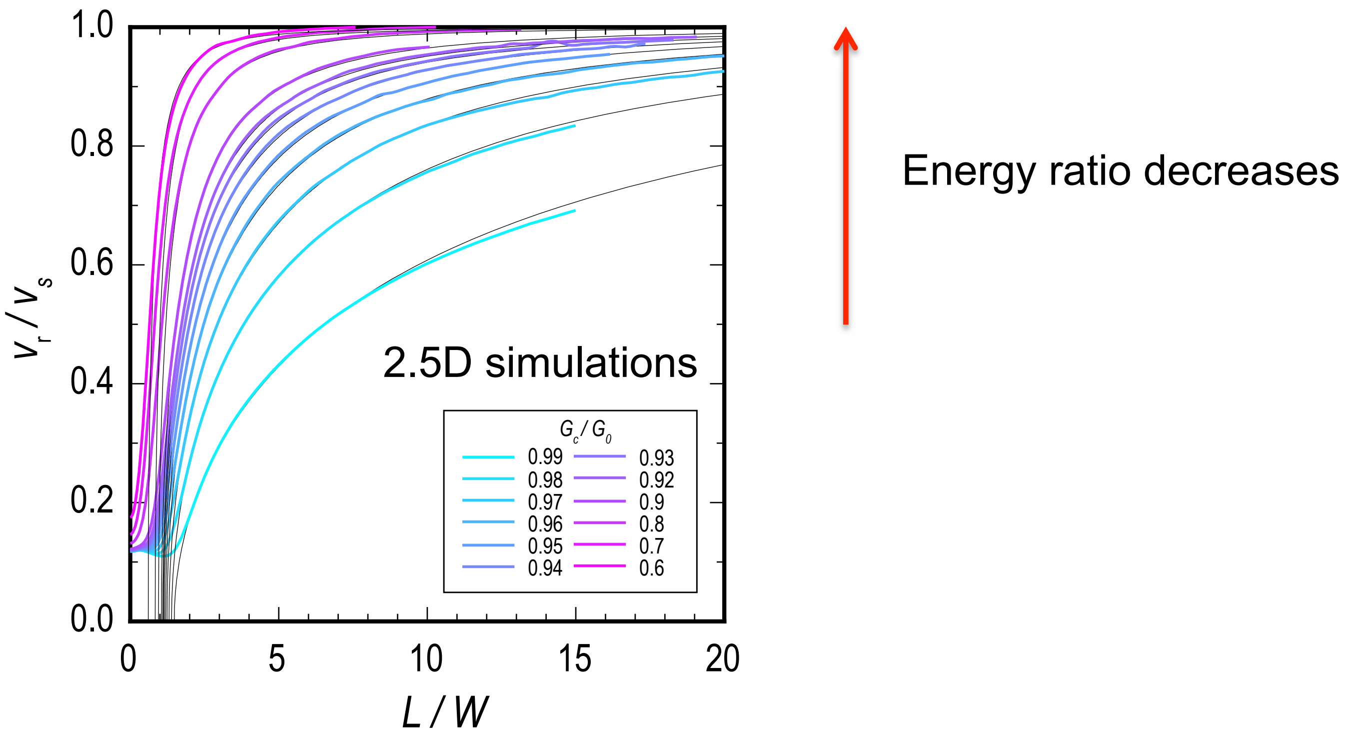
$$G_0 = \frac{\Delta\tau^2 W}{\pi\mu}$$

$$G_c = 0.5(\tau_s - \tau_d)d_0$$



Rupture acceleration

- $G_0 > G_c$ → ruptures accelerate ↑
- G_c / G_0 plays an important role in controlling rupture speed



Weng and Ampuero, 2019

Rupture acceleration

- $G_0 > G_c$ → ruptures accelerate ↑
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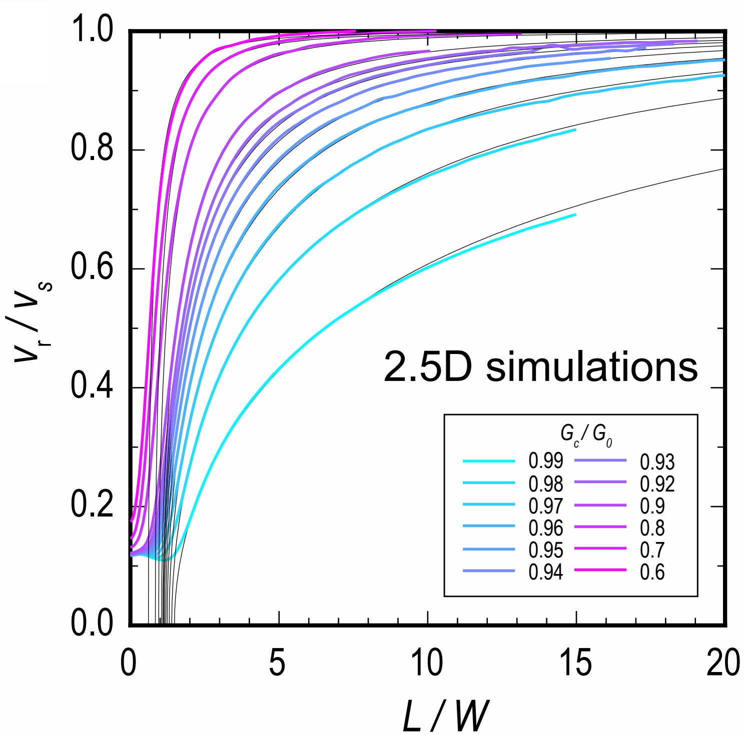
$$G_c = G_0 \left(1 - \frac{\dot{v}_r W}{v_s^2} \frac{1}{A \alpha_s^P}\right)$$

$$\alpha_s = \sqrt{1 - (v_r / v_s)^2}$$



Energy ratio decreases

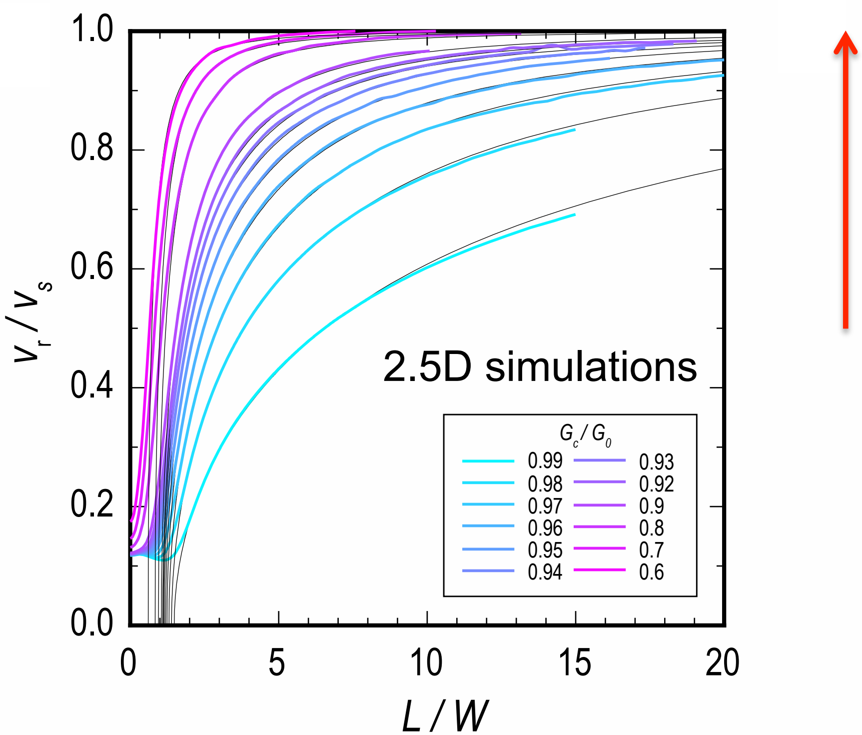
Weng and Ampuero, 2019

Rupture acceleration

- $G_0 > G_c$ → ruptures accelerate ↑
- G_c / G_0 plays an important role in controlling rupture speed

$$\frac{\dot{v}_r W}{v_s^2 (1 - G_c/G_0)} = A\alpha_s^P$$

$$\alpha_s = \sqrt{1 - (v_r/v_s)^2}$$



Energy ratio decreases

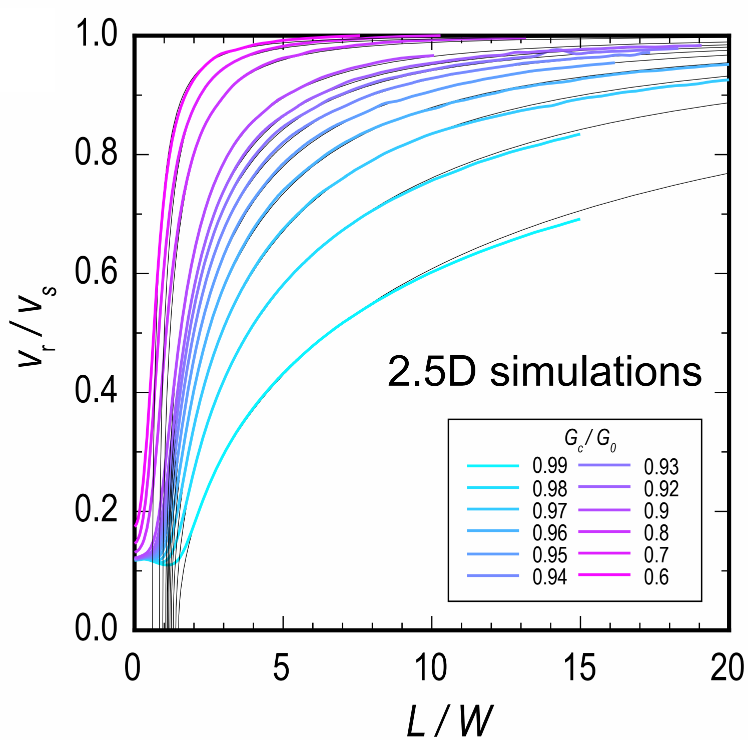
Weng and Ampuero, 2019

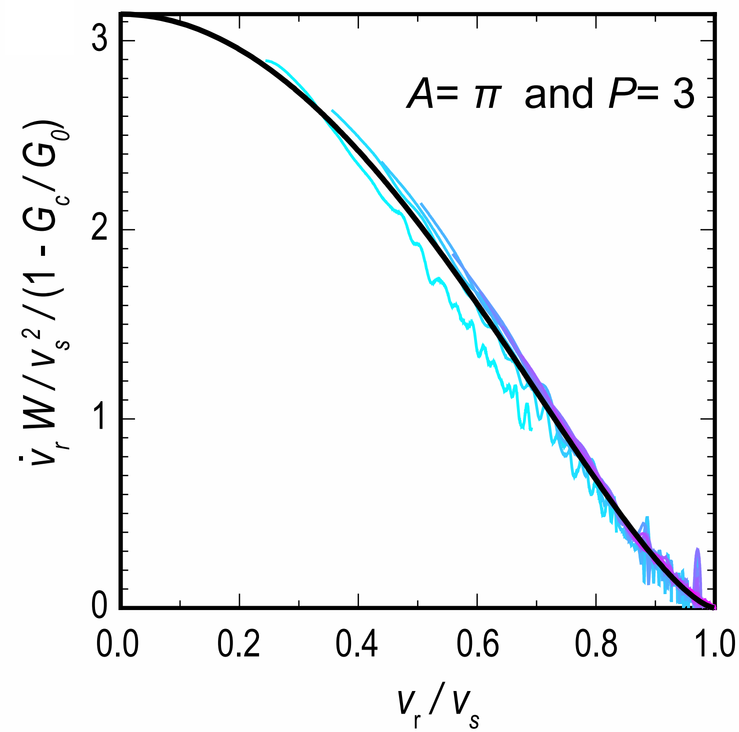
Rupture acceleration

- $G_0 > G_c$ → ruptures accelerate ↑
- G_c/G_0 plays an important role in controlling rupture speed

$$\frac{\dot{v}_r W}{v_s^2(1 - G_c/G_0)} = \pi\alpha_s^3$$

$$\alpha_s = \sqrt{1 - (v_r/v_s)^2}$$

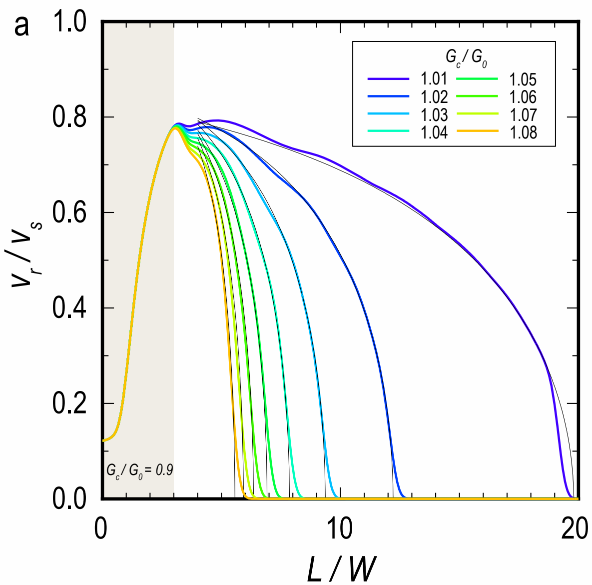




Weng and Ampuero, 2019

Rupture deceleration

- $G_0 < G_c$ → ruptures decelerate ↓
- Starting speed also plays a role
- Larger rupture speed lead to longer distance



2.5D simulations

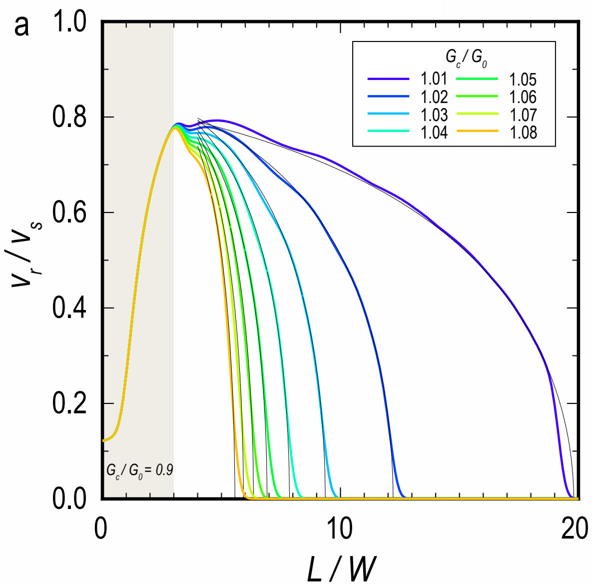
Weng and Ampuero, 2019

Rupture deceleration

- $G_0 < G_c$ → ruptures decelerate ↓
- Starting speed also plays a role
- Larger rupture speed lead to longer distance

$$\frac{\dot{v}_r W}{v_s^2 (1 - G_c/G_0)} = A\alpha_s^P$$

$$\alpha_s = \sqrt{1 - (v_r/v_s)^2}$$



2.5D simulations

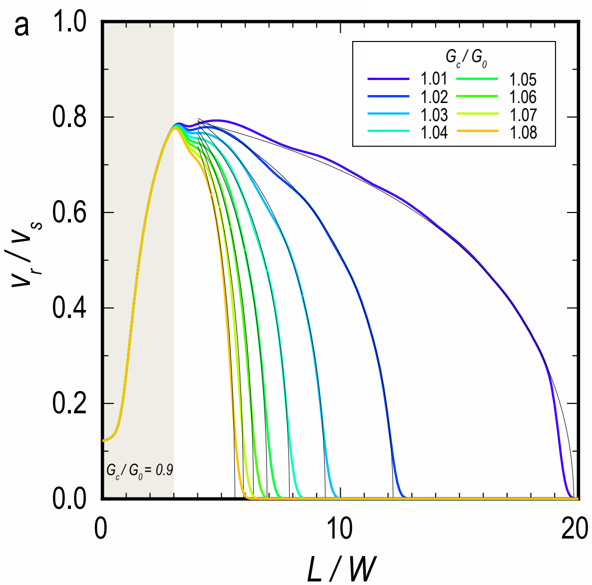
Weng and Ampuero, 2019

Rupture deceleration

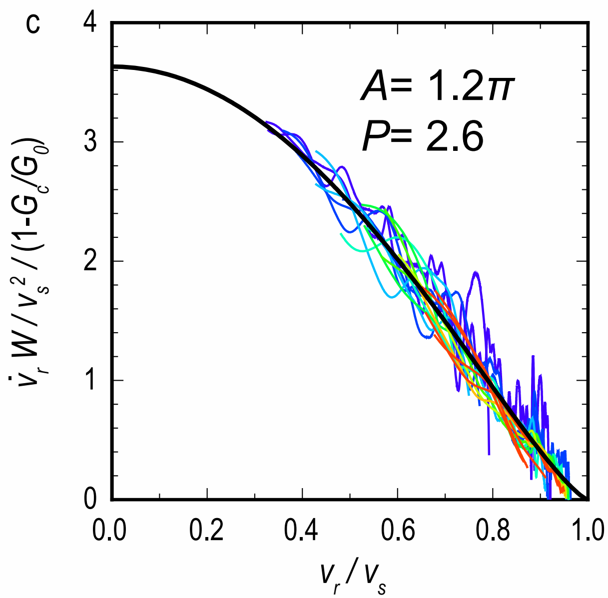
- $G_0 < G_c$ → ruptures decelerate ↓
- Starting speed also plays a role
- Larger rupture speed lead to longer distance

$$\frac{\dot{v}_r W}{v_s^2(1 - G_c/G_0)} = 1.2\pi\alpha_s^{2.6}$$

$$\alpha_s = \sqrt{1 - (v_r/v_s)^2}$$

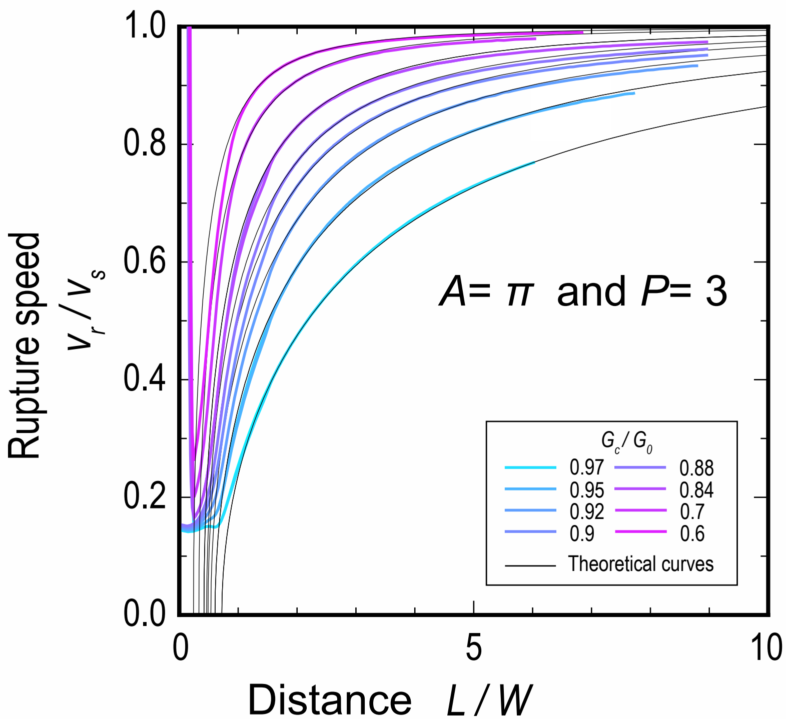


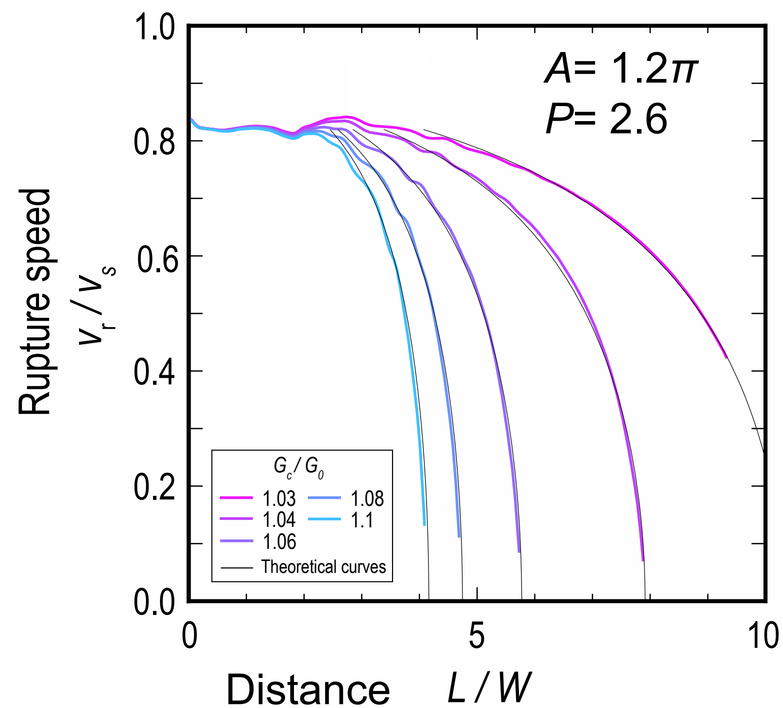
2.5D simulations



Weng and Ampuero, 2019

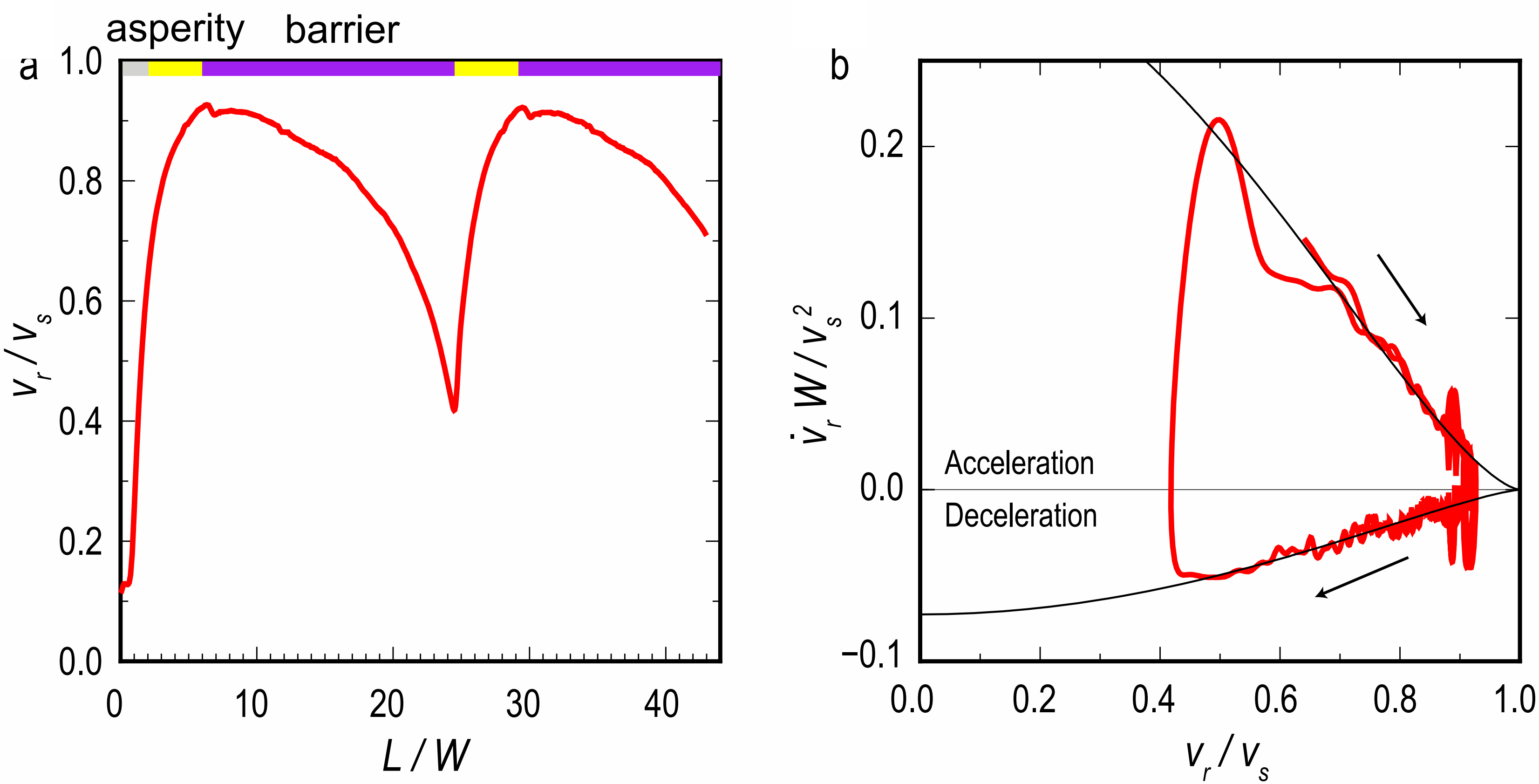
Validation from 3D simulations





Weng and Ampuero, 2019

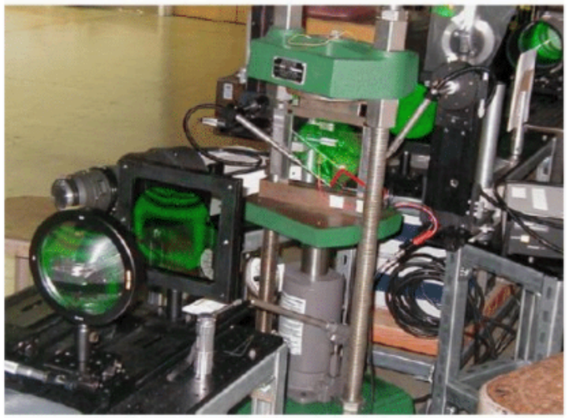
“Inertial” rupture



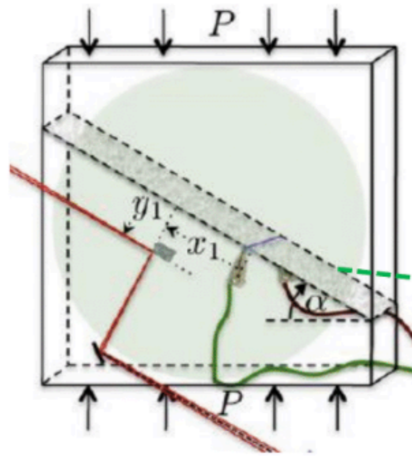
- Rupture evolution predicted by rupture-tip-equation-of-motion
- Rupture is also “inertial”

Weng and Ampuero, 2019

Elongated ruptures in the lab



Laboratory earthquake experiment



Mello et al (2014)
in Rosakis lab (Caltech)

$$G_0 \approx \frac{\Delta\tau^2 W}{\pi\mu}$$

Key points

1 Closed-form energy release rate on 3D bounded fault is a constant:

$$G_0 = \frac{\Delta\tau^2 W}{\pi\mu}$$

2 Ruptures on 3D bounded fault are controlled by the theoretical equation for very long ruptures:

$$G_c = G_0\left(1 - \frac{\dot{v}_r W}{v_s^2}\frac{1}{A\alpha_s^P}\right)$$

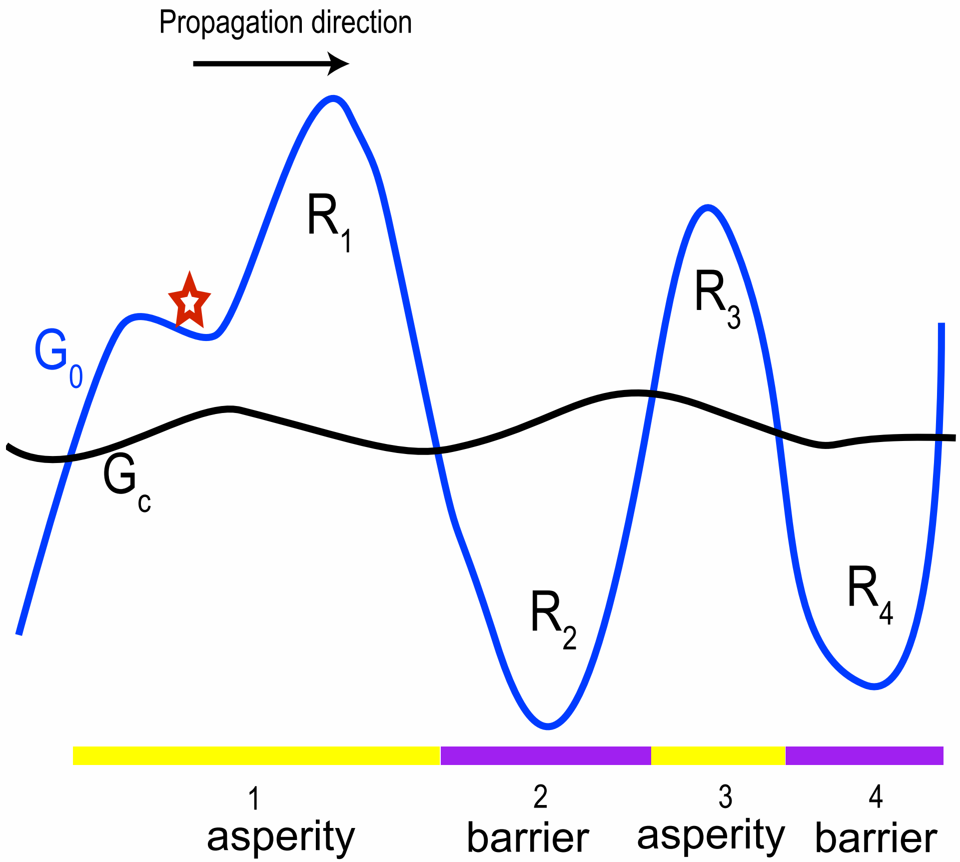
$$\alpha_s = \sqrt{1 - (v_r/v_s)^2}$$

Implications:

-- Rupture potential and final earthquake size

-- Super-cycle

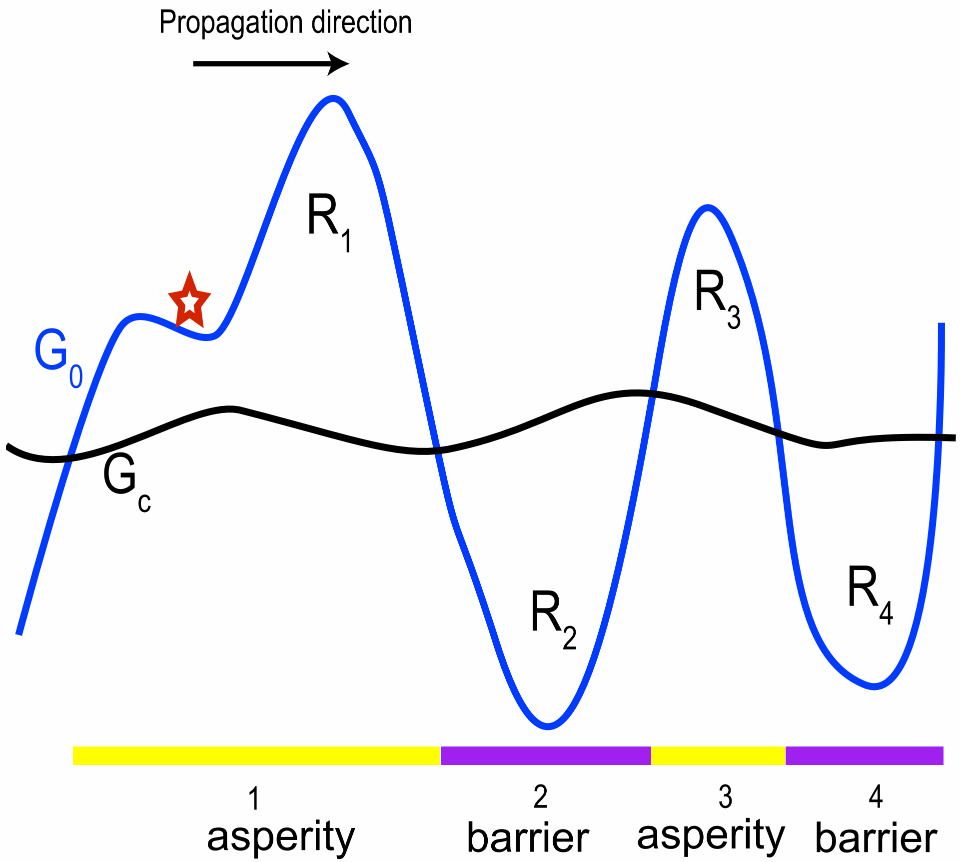
Rupture potential



$$\frac{\dot{v}_r W}{v_s^2(1 - G_c/G_0)} = A\alpha_s^P$$

$$\frac{v_r d v_r}{v_s^2 \alpha_s^P} = A(1 - G_c/G_0)dx/W$$

Rupture potential



$$\frac{\dot{v}_r W}{v_s^2(1 - G_c/G_0)} = A\alpha_s^P$$

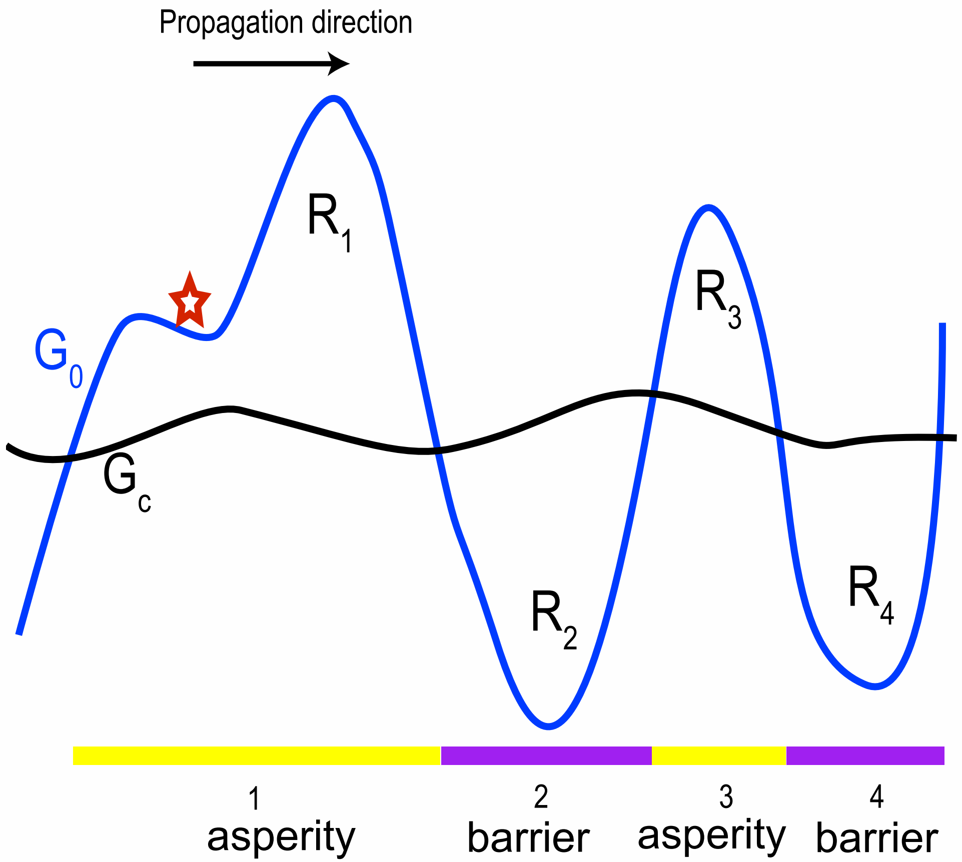
↓

$$\frac{v_r dv_r}{v_s^2 \alpha_s^P} = A(1 - G_c/G_0)dx/W$$

"Kinetic" energy? ↓ "Potential" energy?

$$\frac{1}{P-2}(\alpha_s^{2-P} - 1)\Big|_{v_{r1}}^{v_{r2}} = \int_{L_1}^{L_2} A(1 - G_c/G_0)dx/W$$

Rupture potential



$$\frac{\dot{v}_r W}{v_s^2(1 - G_c/G_0)} = A\alpha_s^P$$

$$\frac{v_r\,dv_r}{v_s^2\,\alpha_s^P} = A(1 - G_c/G_0)dx/W$$

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Rupture potential

$$\varphi(L) = \int_0^L A(1 - G_c/G_0)dx/W$$

Determine earthquake size

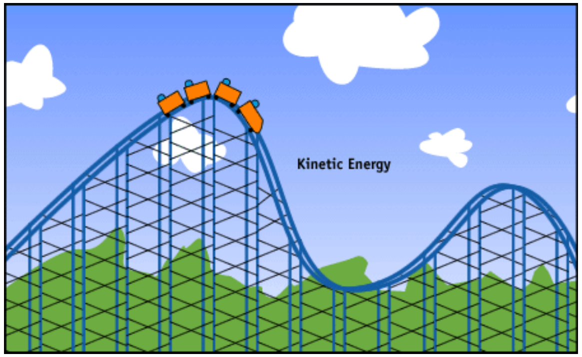
$$\frac{\dot{v}_r W}{v_s^2(1-G_c/G_0)} = A\alpha_s^P$$

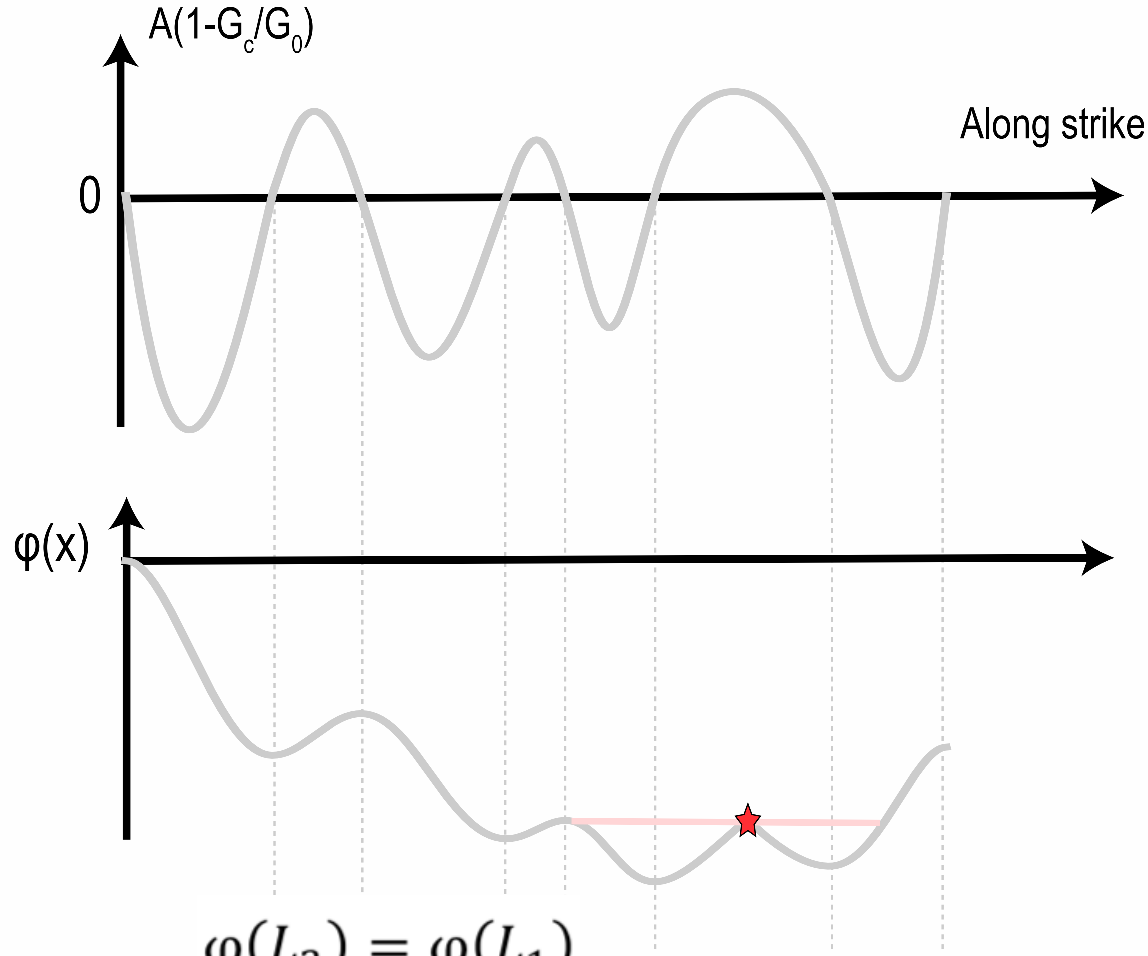
↓

$$\varphi(L) = \int_0^L A(1-\ G_c/G_0)dx/W$$

Rupture potential

Gravity potential





$$\varphi(L_2) = \varphi(L_1)$$

www.thinglink.com

Weng and Ampuero, JGR, in revision

Determine earthquake size

$$\frac{\dot{v}_r W}{v_s^2(1 - G_c/G_0)} = A\alpha_s^P$$

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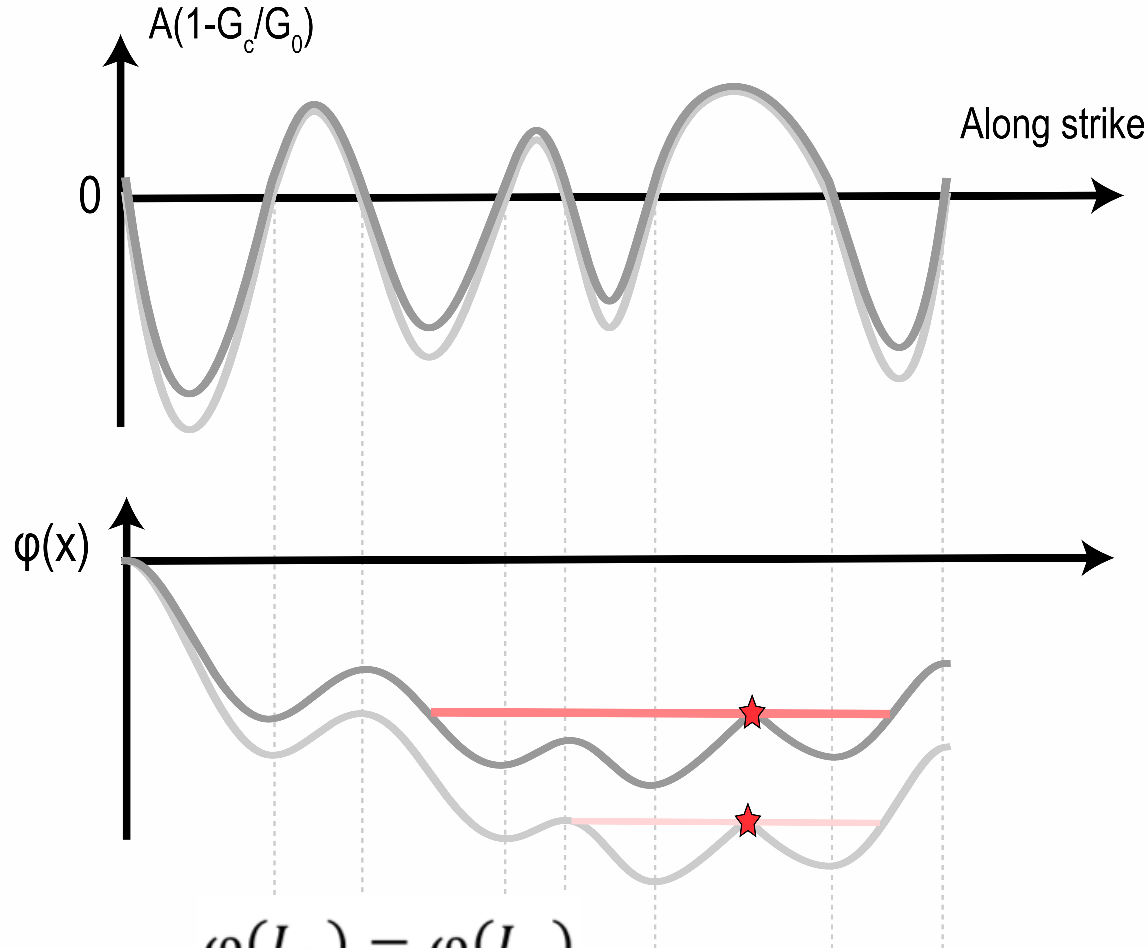

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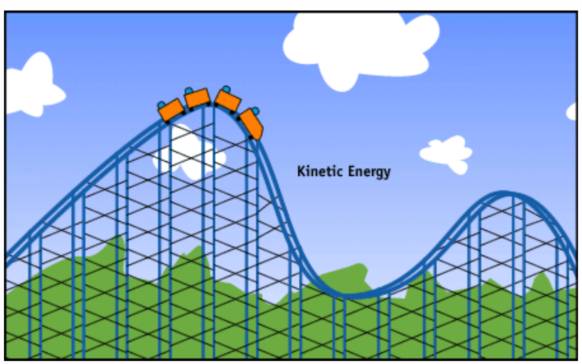
Gravity potential



www.thinglink.com

Weng and Ampuero, 2019

Determine earthquake size

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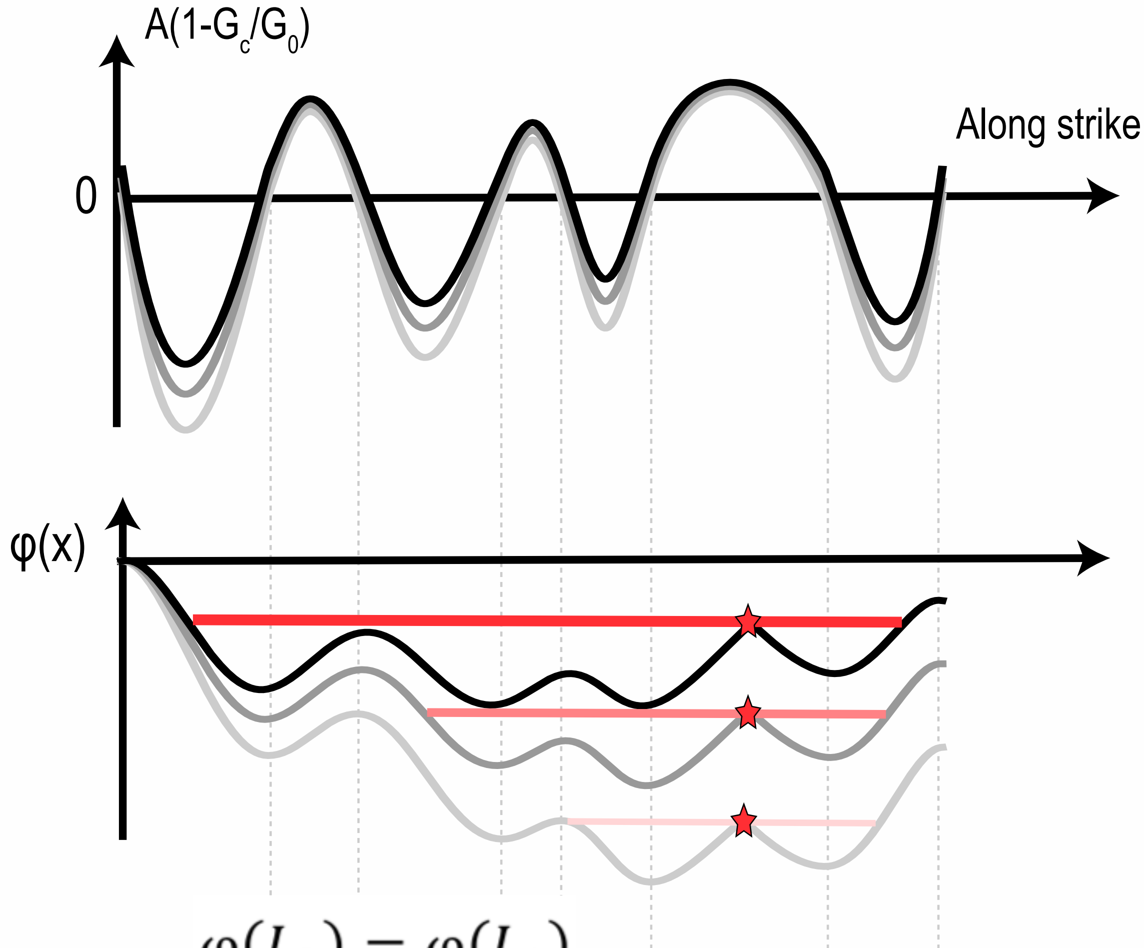

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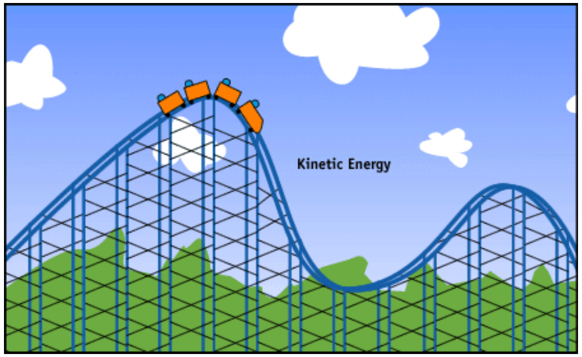
Weng and Ampuero, 2019

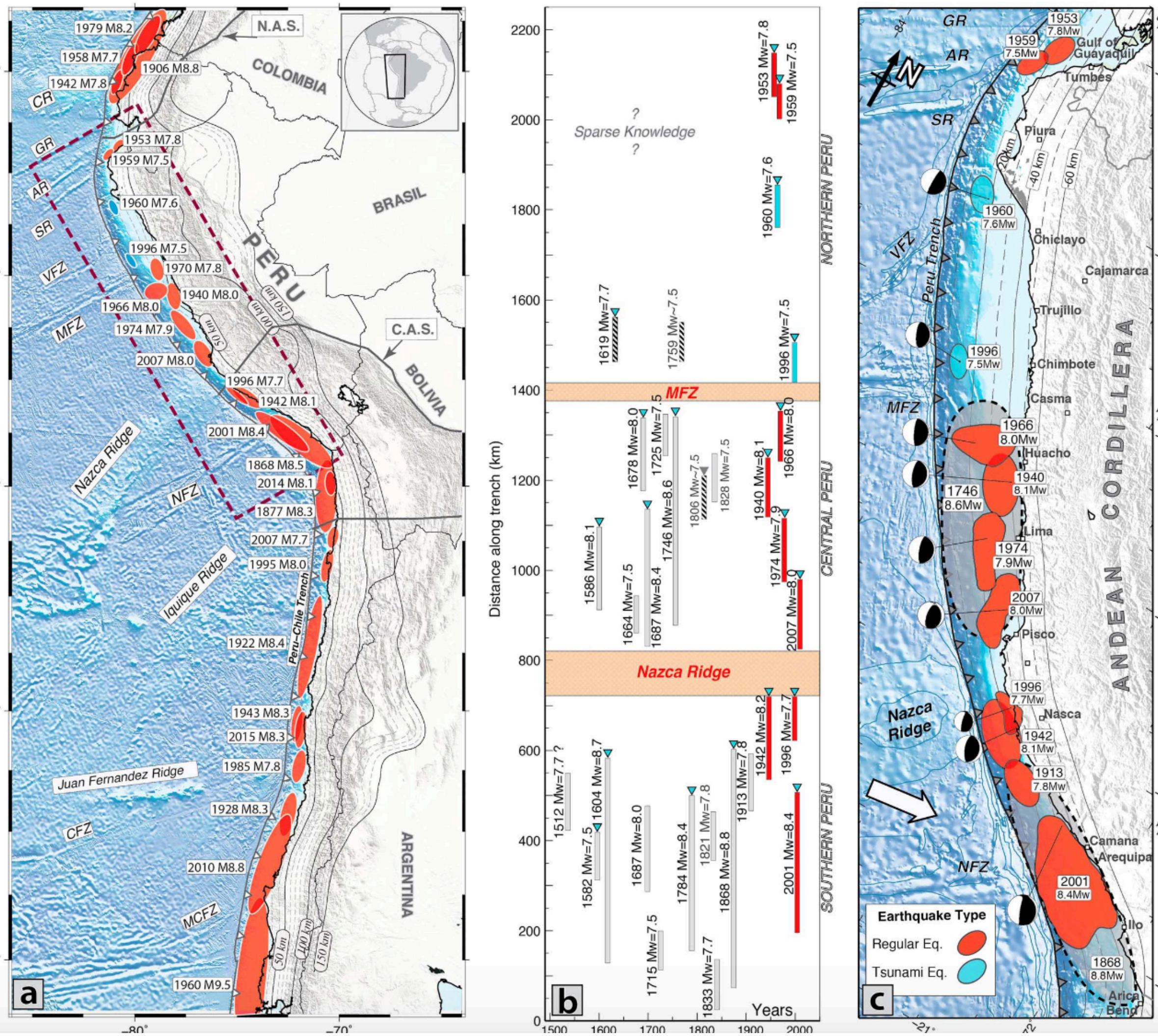
Gravity potential



www.thinglink.com

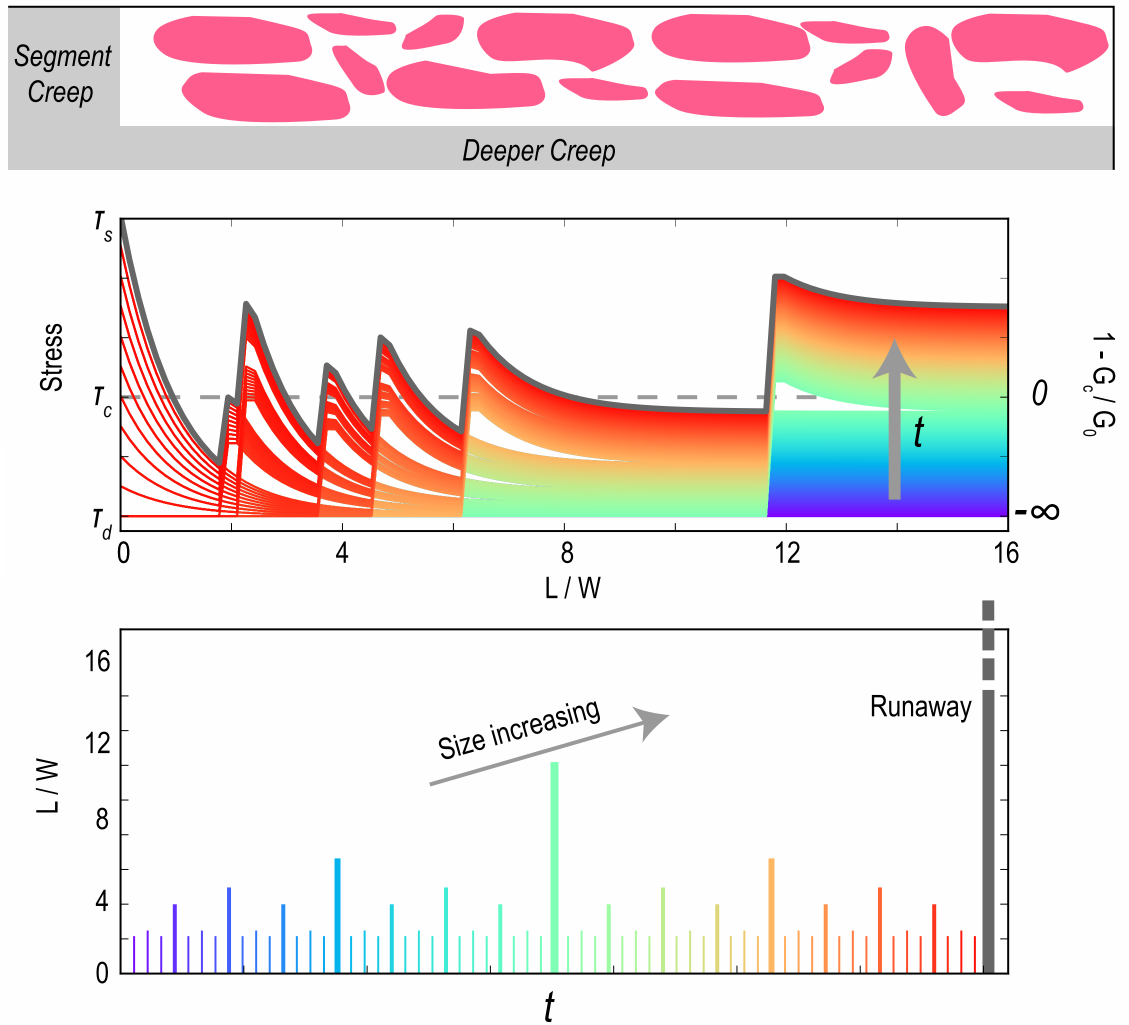
Super earthquake cycles?

- Fault segmentation
- Maximum magnitude?



Villegas-Lanza et al., (2016)

Super cycles



Stressing rate:

$$\dot{\tau}(L) = \gamma_l \exp(-L/W) + \gamma_b$$

Assumption:

$$G_c/G_0 = B\Delta\tau^{n-2}$$

Weng and Ampuero, 2019

overview

- Equation of motion for mode III in 3D
- Equation of motion for mode II in 3D

  -- Subshear
  -- Supershear
- Ruptures of mixture of modes II and III

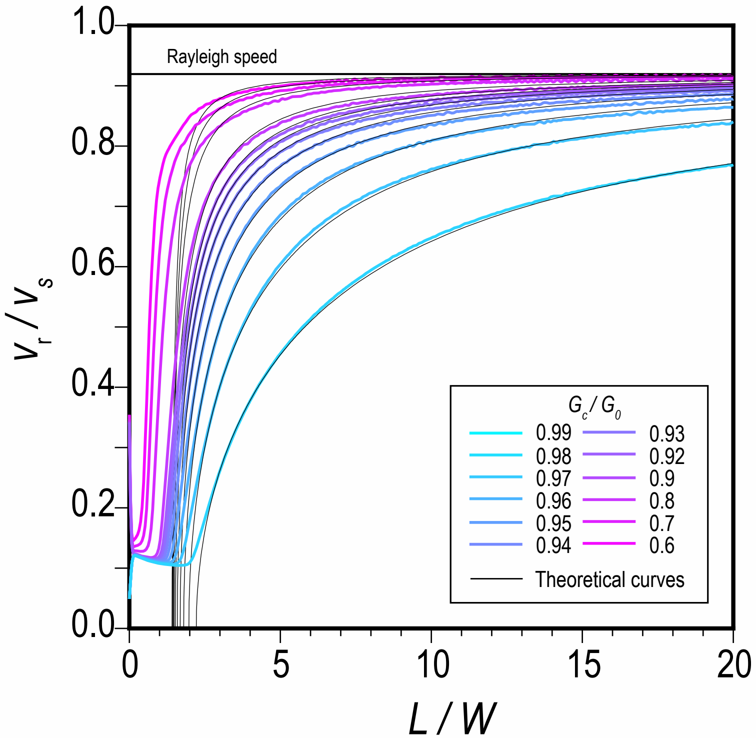
In-plane sub-shear

Analytic result (similar as mode III):

$$G_0 = \lambda \frac{\Delta\tau^2 W}{\mu}$$

$$\frac{\dot{v}_r W}{v_s^2(1 - G_c/G_0)} = A\alpha_R^P$$

$$\alpha_R = \sqrt{1 - (v_r/v_R)^2}$$



Weng and Ampuero, In prep.

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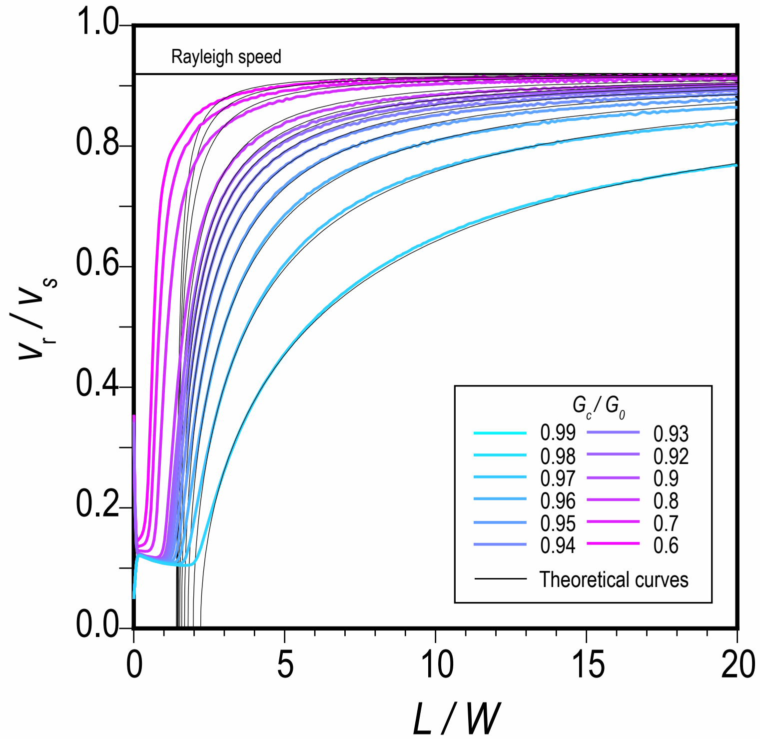
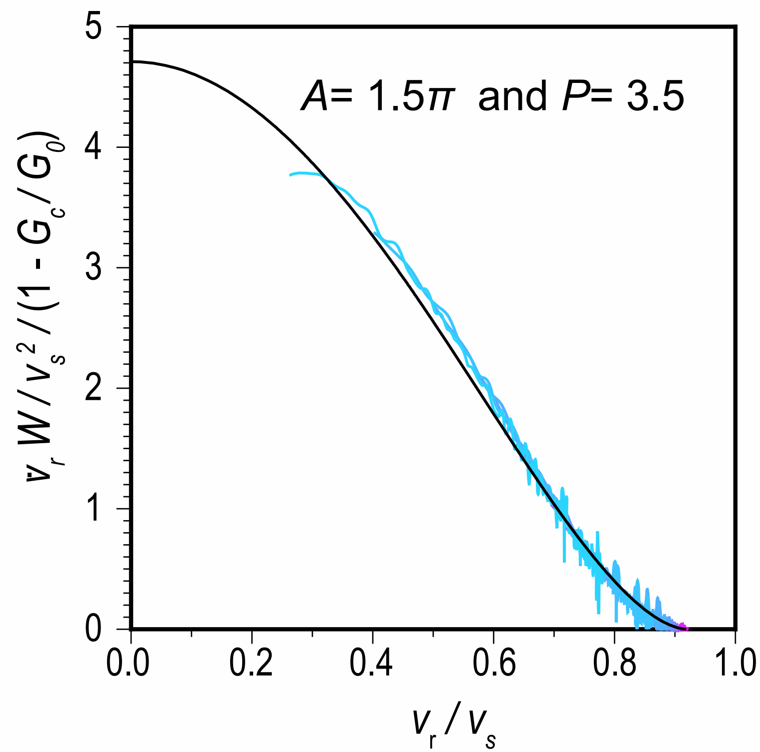




Weng and Ampuero, In prep.

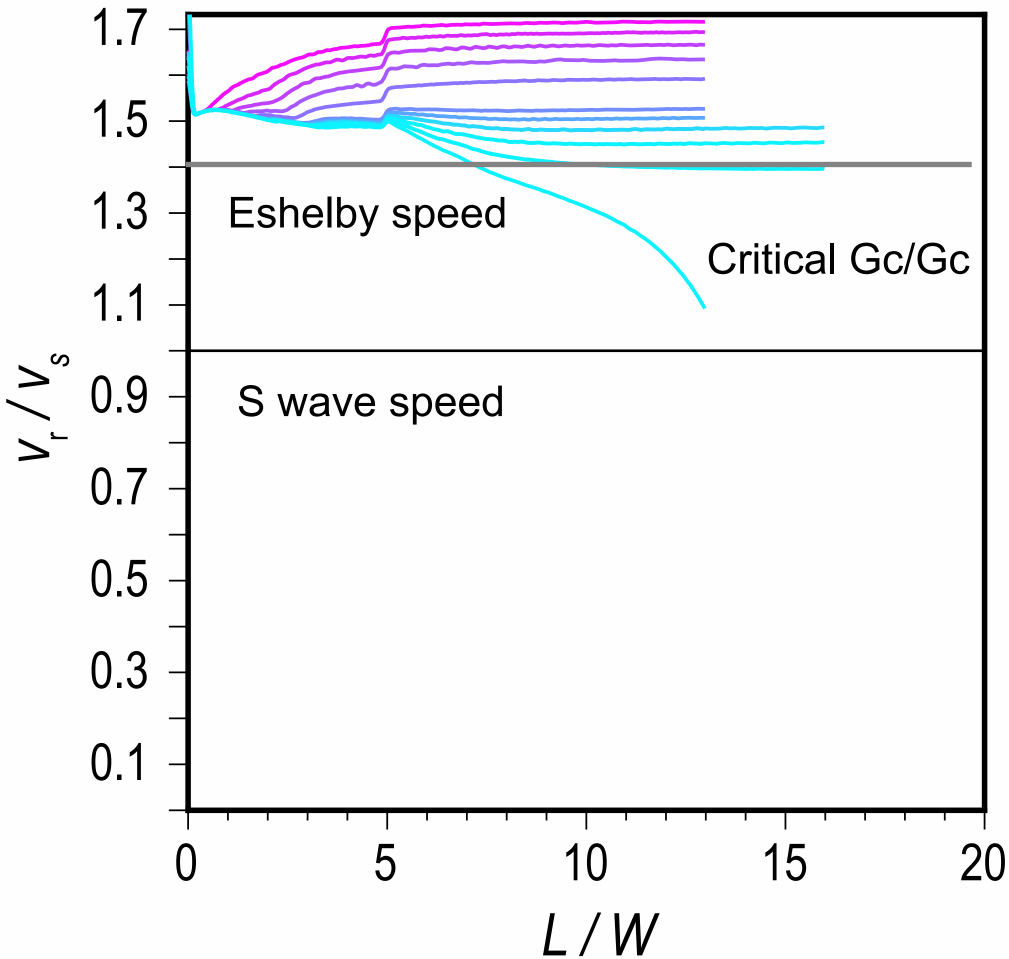
overview

- Equation of motion for mode III in 3D
- Equation of motion for mode II in 3D
  -- Subshear
  -- Supershear
- Ruptures of mixture of modes II and III

Dynamics of supershear ruptures

- Steady-state supershear
- G_c/G_0 controls supershear speed
- Critical value of G_c/G_0 for supershear

3D numerical simulations



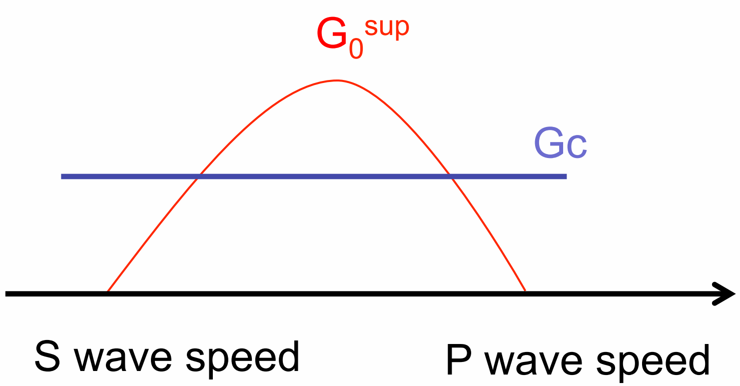
Weng and Ampuero, In manuscript.

Dynamics of supershear ruptures

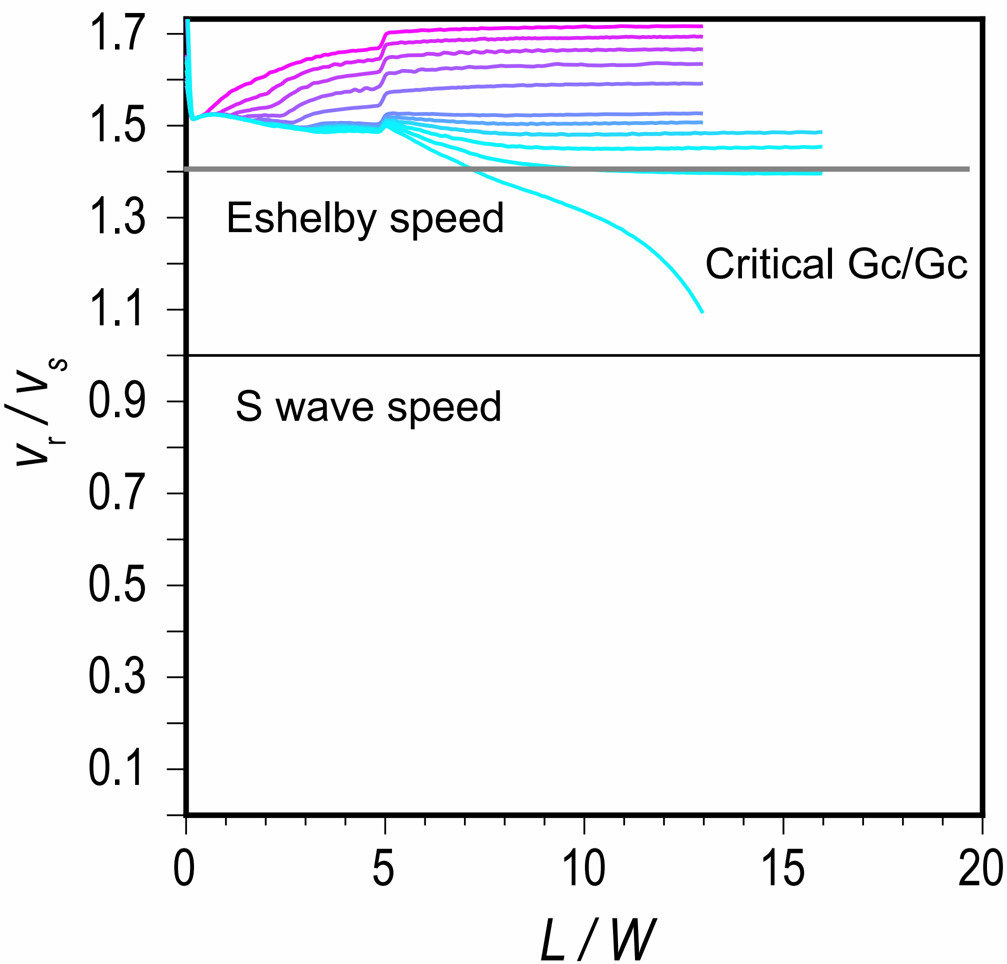
Analytical work:

$$G_0^{sup} = g(v_r) G_0 \left(\frac{\Lambda_0}{W}\right)^{p(v_r)}$$

$$G_0^{sup} = G_c$$



3D numerical simulations



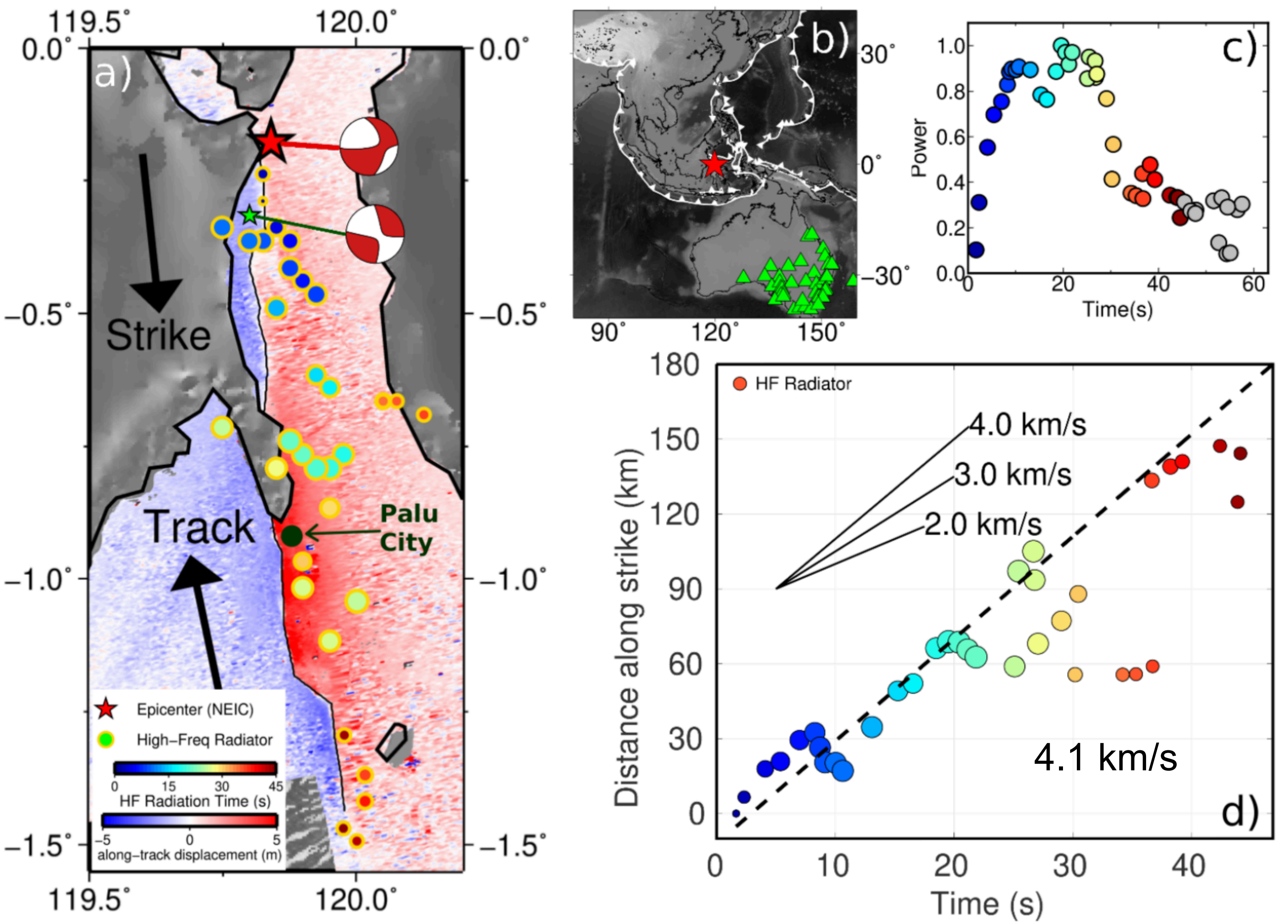
Weng and Ampuero, In manuscript.

overview

- Equation of motion for mode III
- Equation of motion for mode II

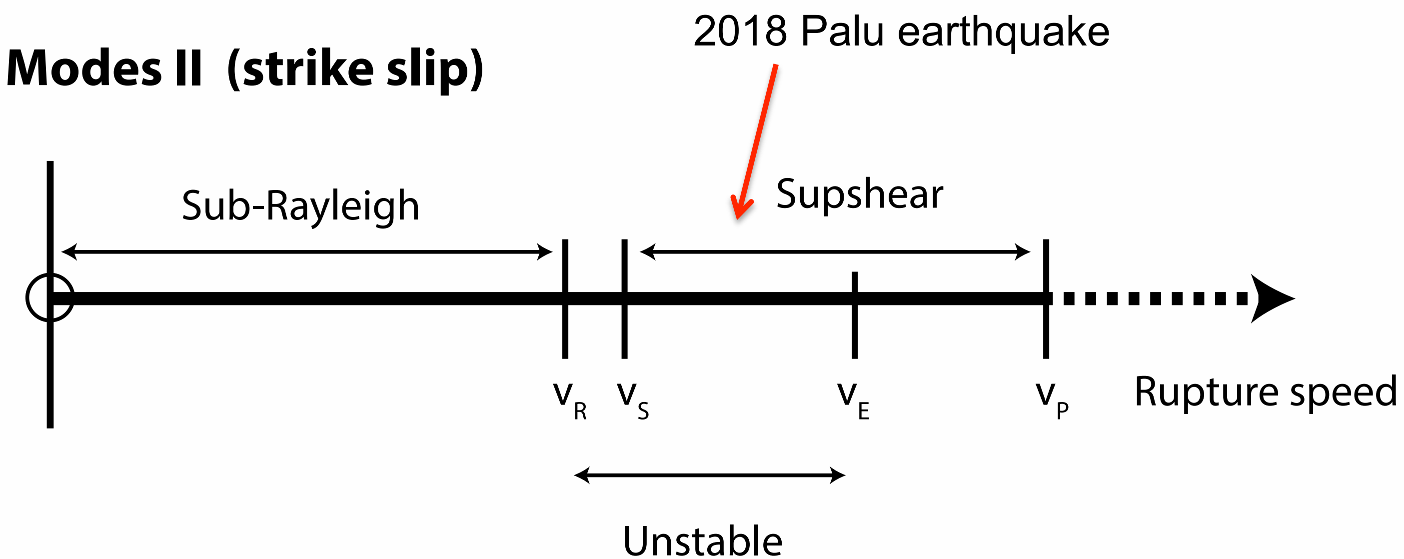
  -- Subshear
  -- Supershear
- Ruptures of mixture of modes II and III

2018 Mw7.5 Palu earthquake

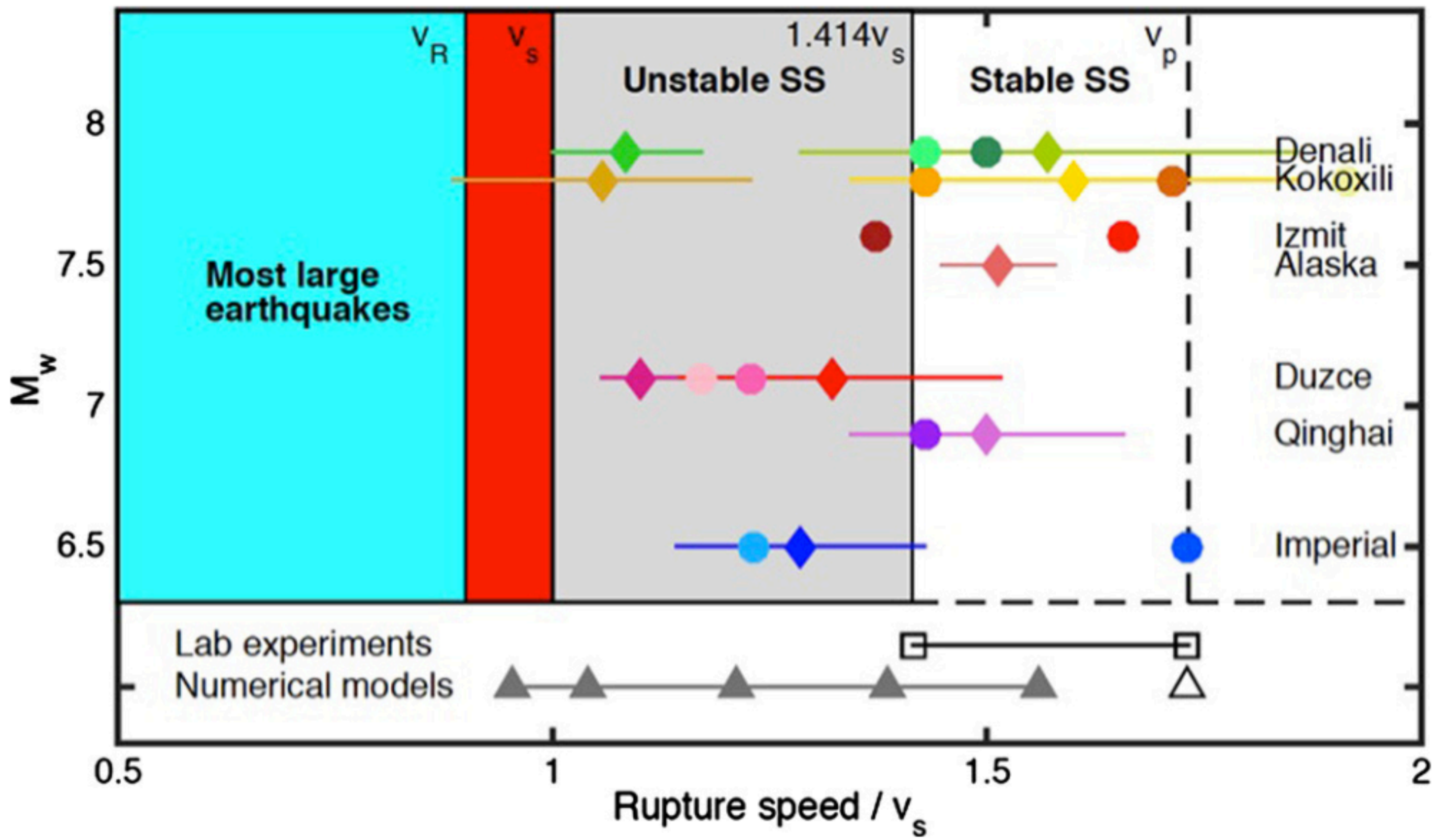


Bao et al, 2019

Slow supershear (sub-Eshelby)



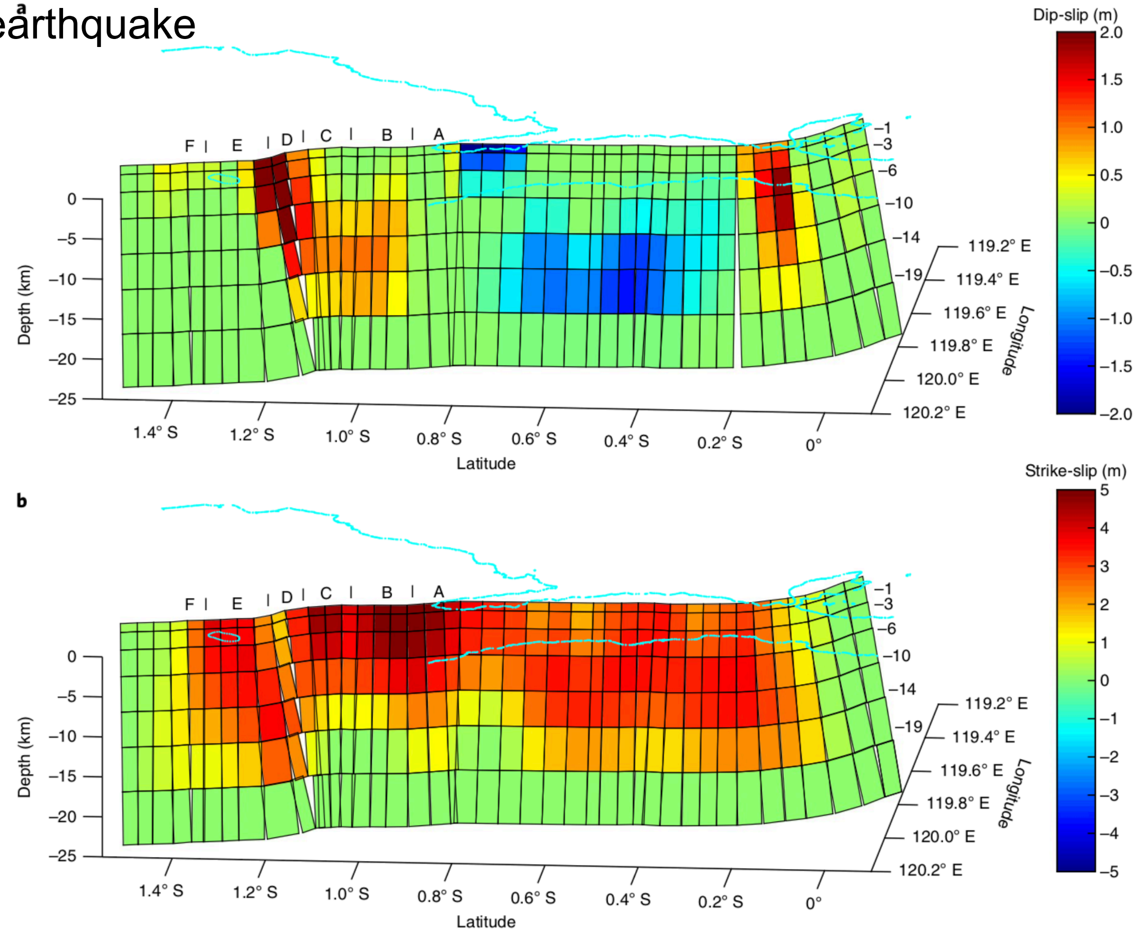
Slow supershear (sub-Eshelby)



Huang et al, 2016

Non-pure strike slip

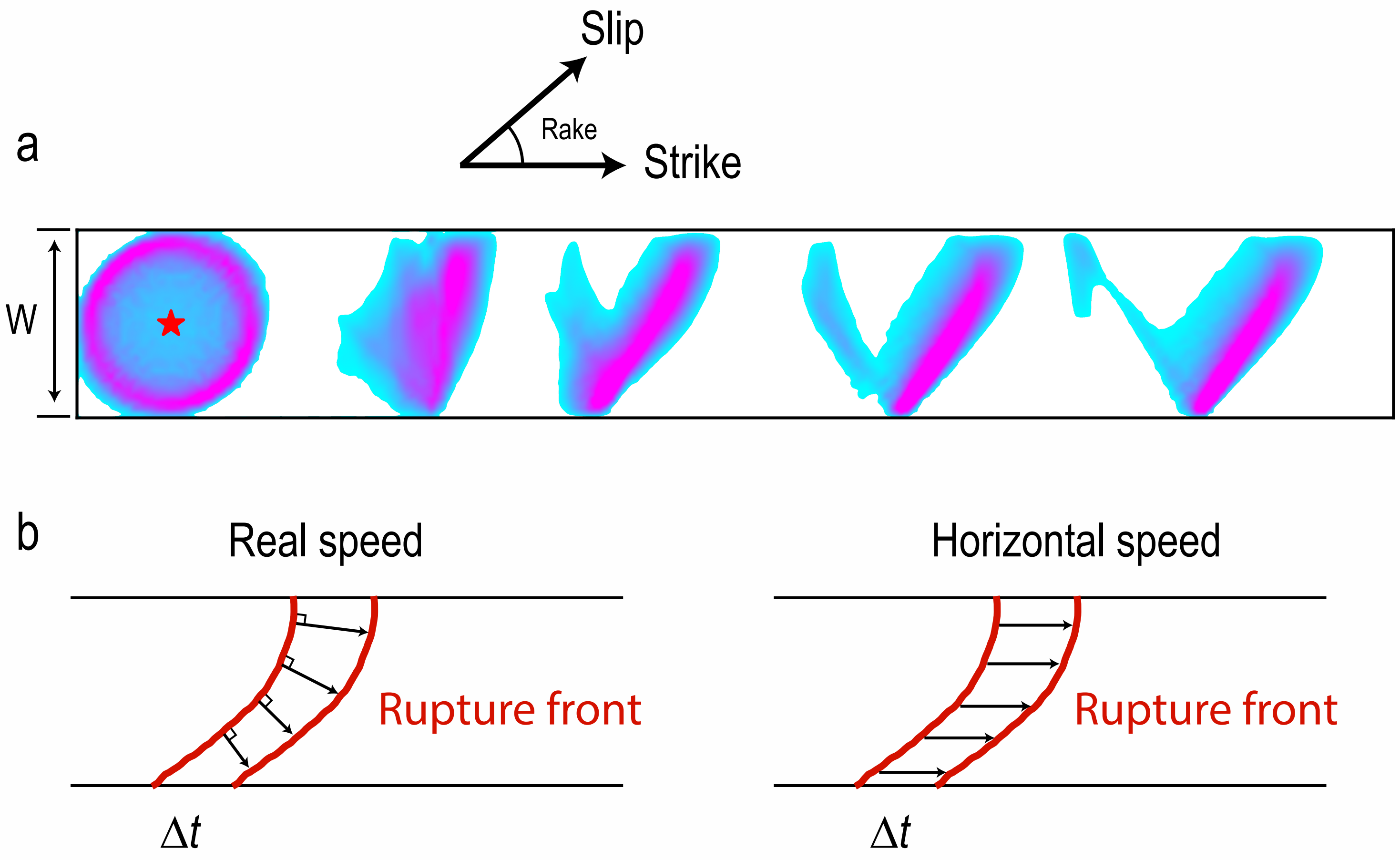
2018 Palu eårthquake



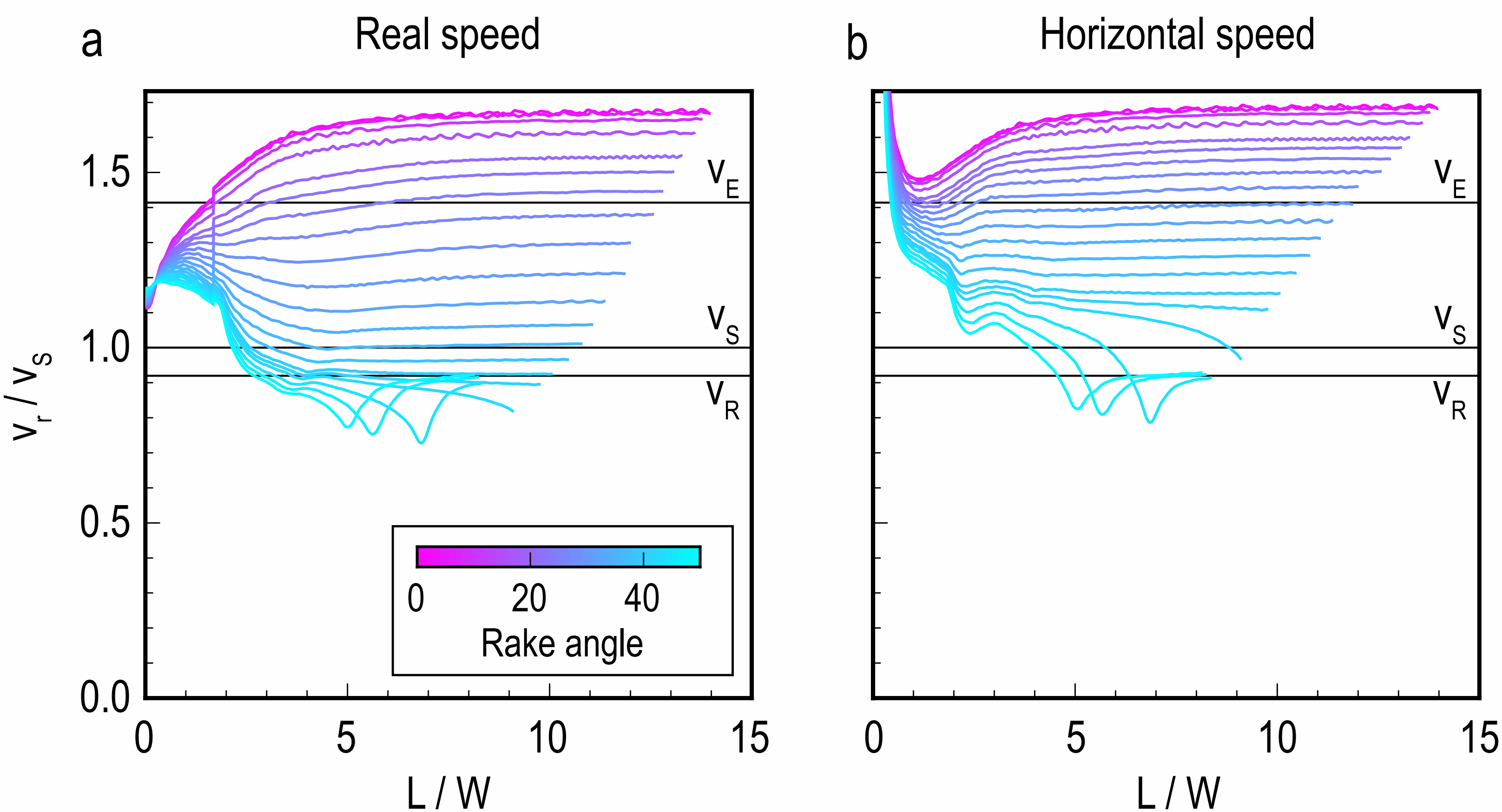
Socquet et al (2019)

- How to explain the observed slow supershear earthquakes?
- What is the effects of rake angle (mixture of modes II and III) on dynamic ruptures?

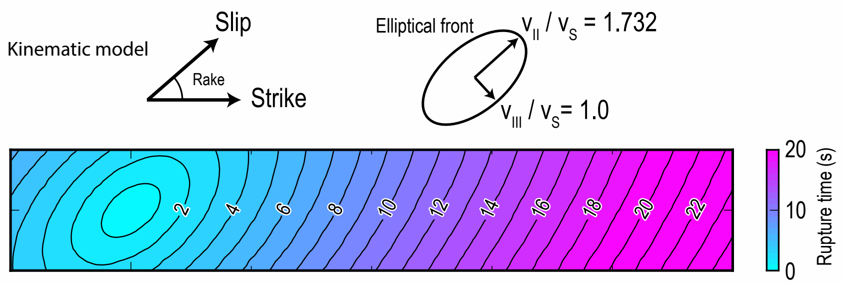
Mixture of modes II and III



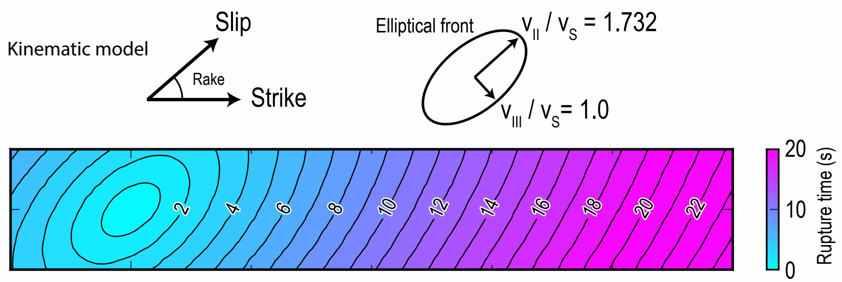
Slow supershear (sub-Eshelby)

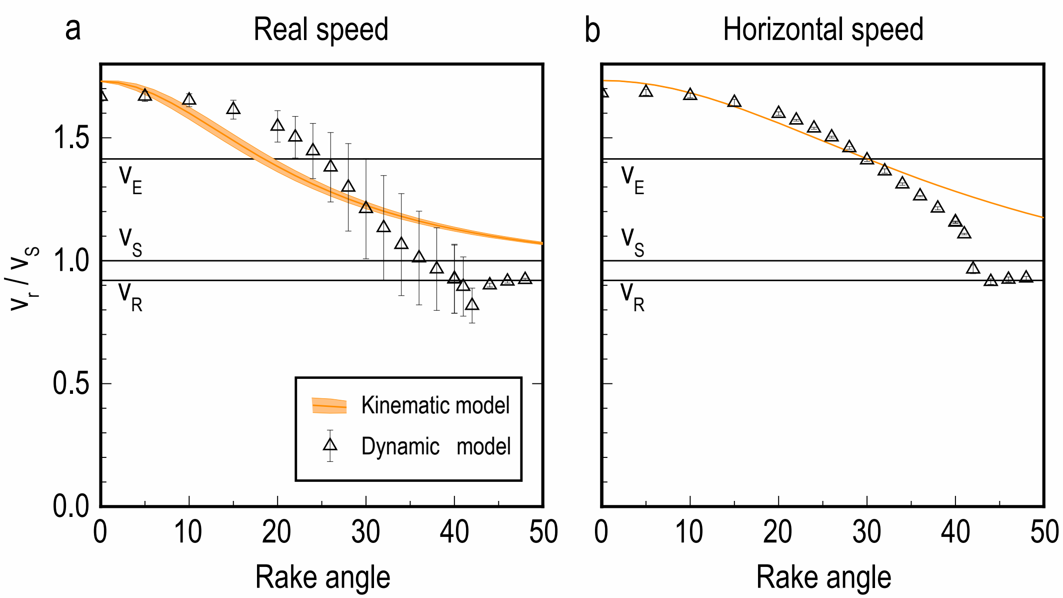


Geometrical effects?

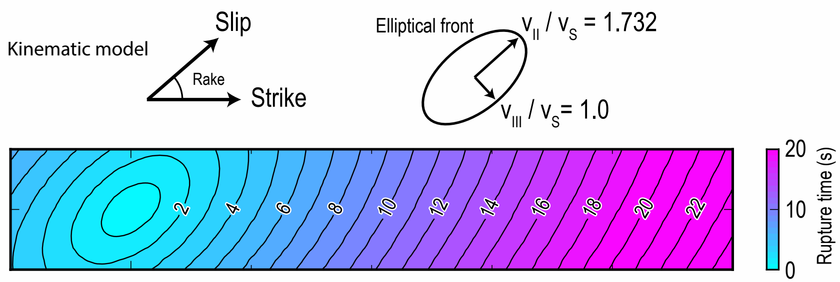


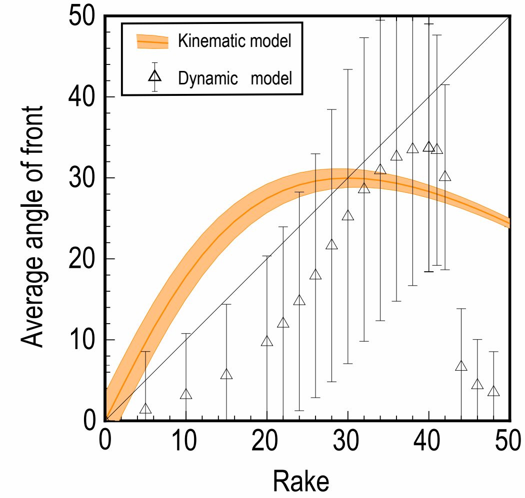
Geometrical effects?

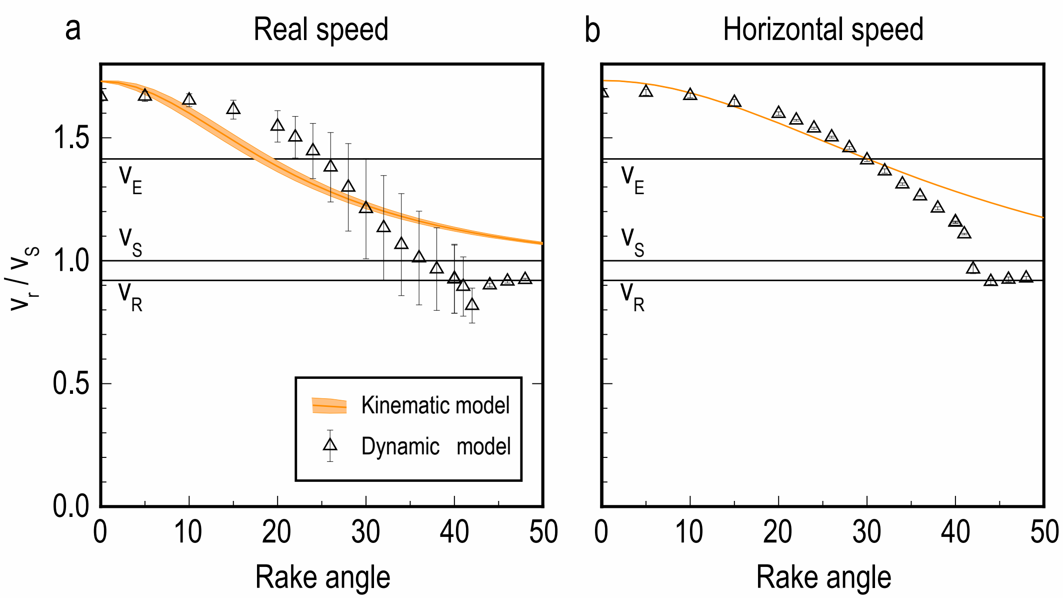




Geometrical effects?

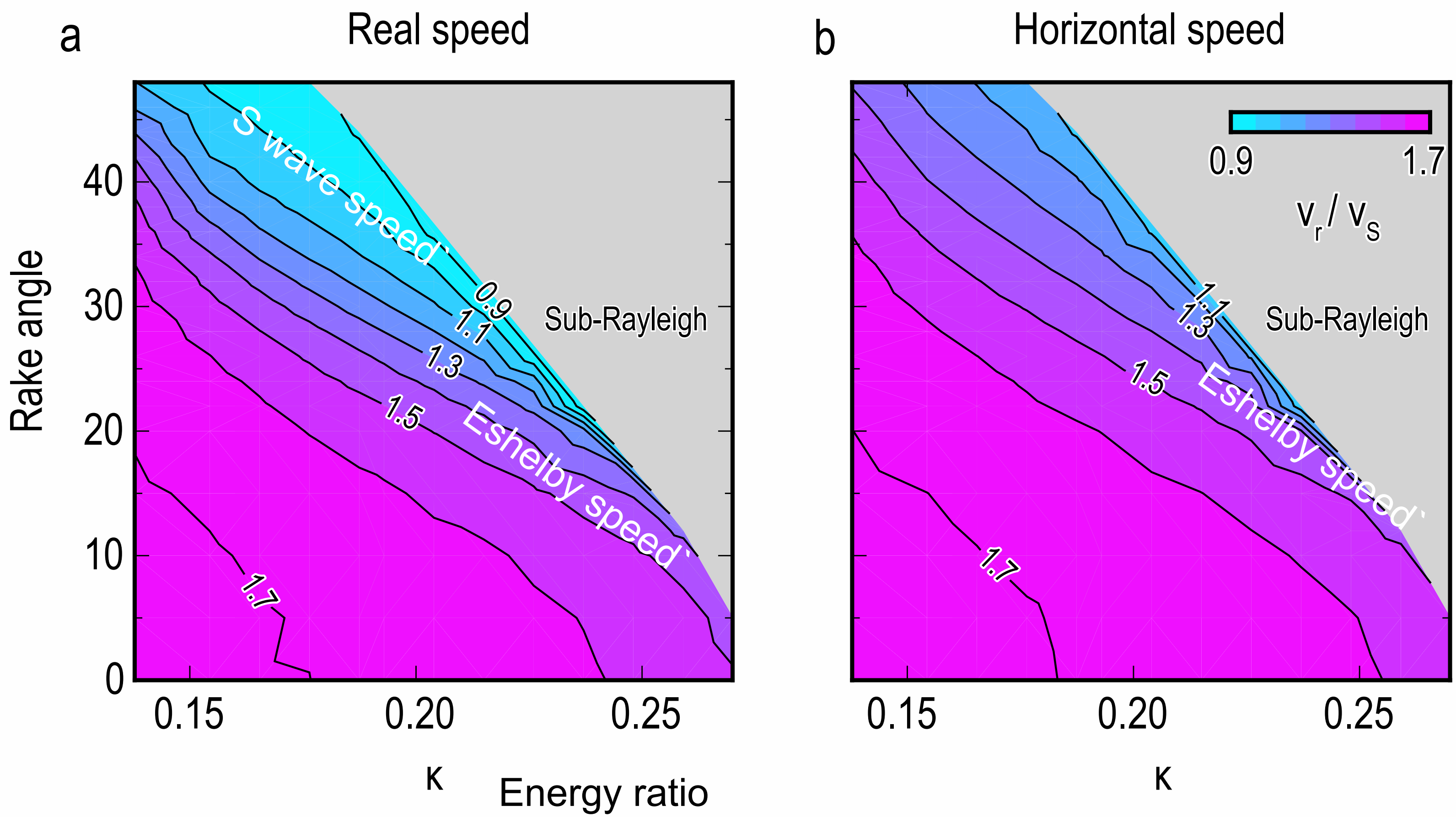




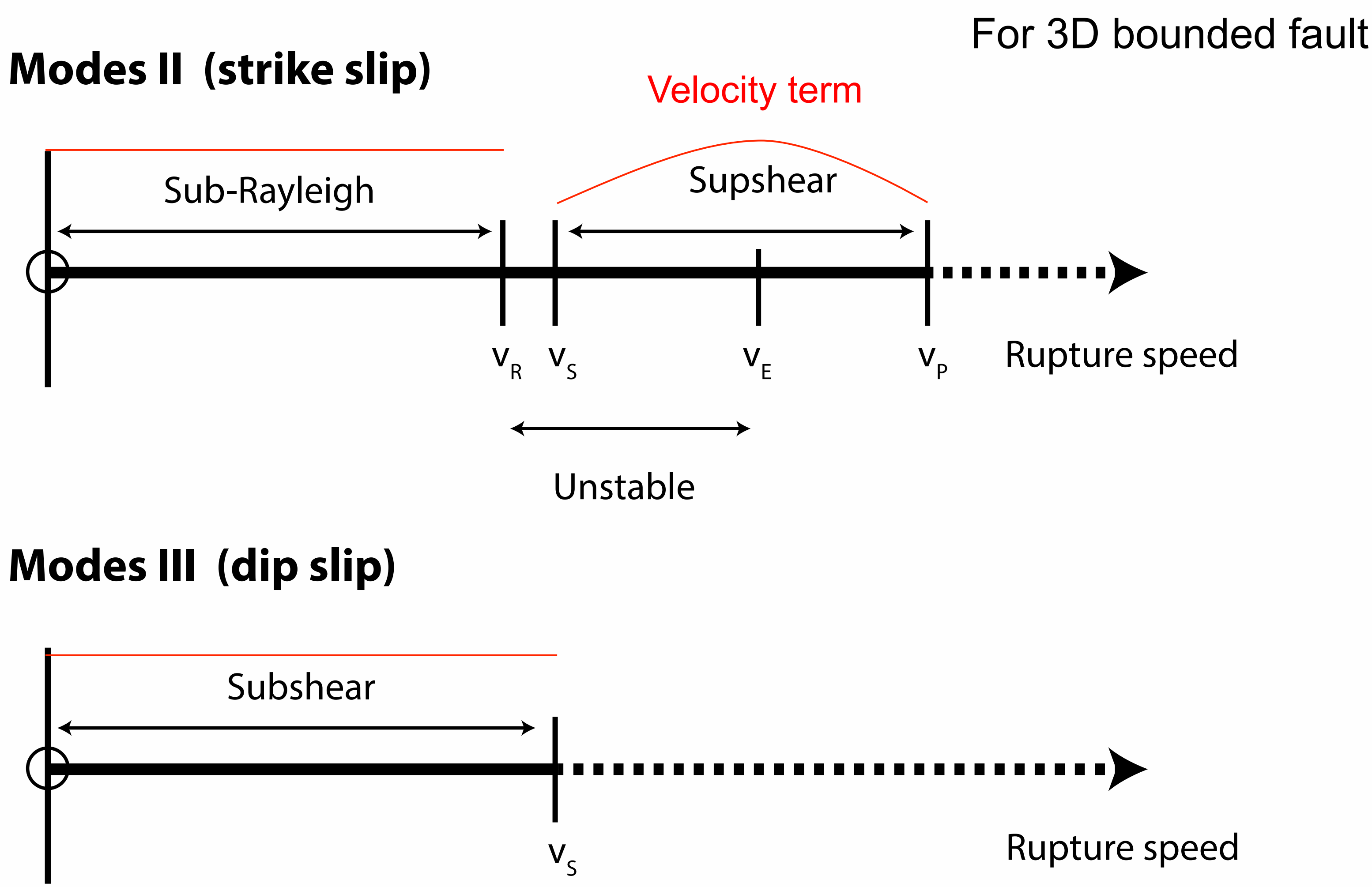


- Slow supershear can partly be explained by geometrical effect

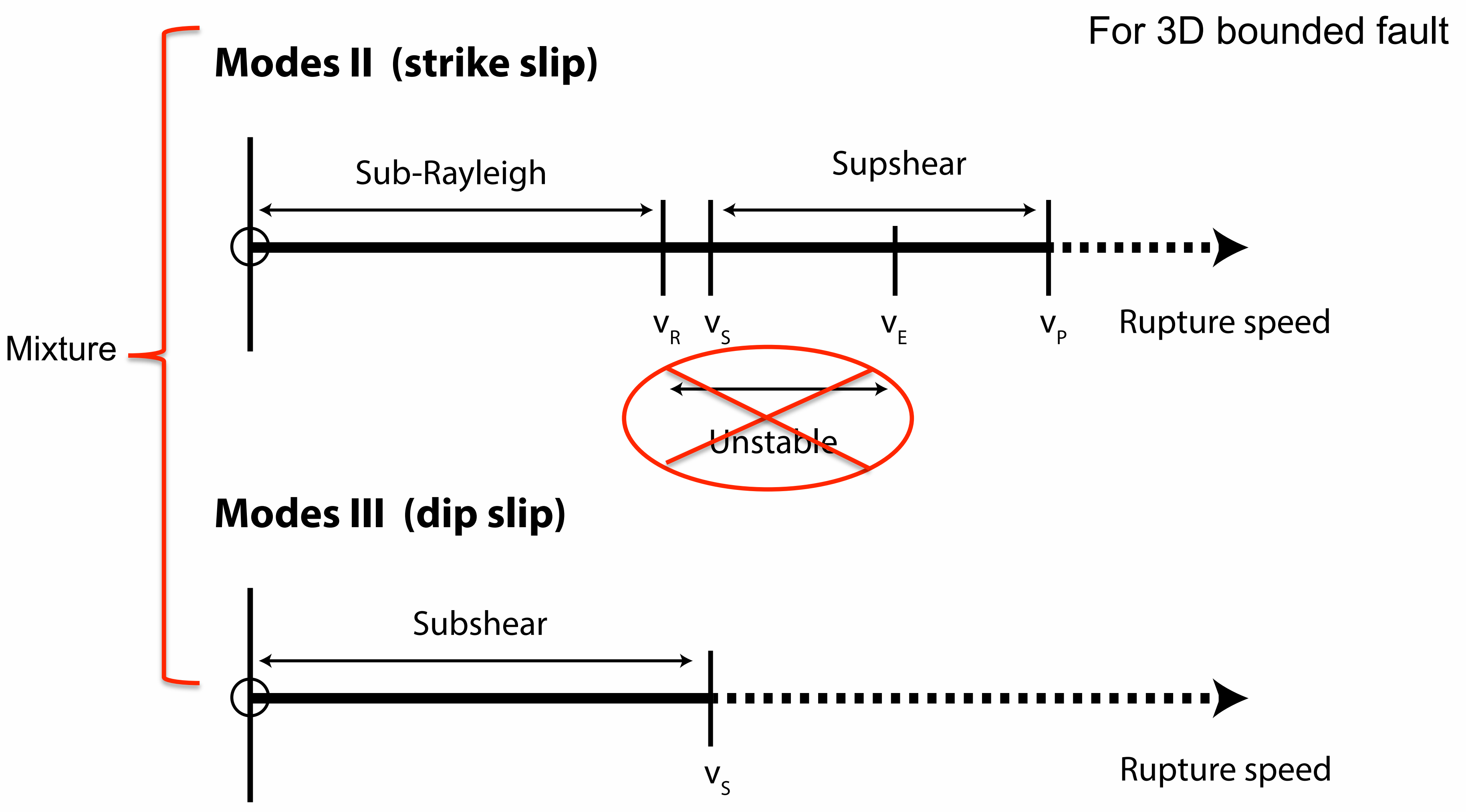
Insight for seismology



Summary


For 3D bounded fault
Modes II (strike slip)
Velocity term
Sub-Rayleigh
Supshear
v_R
v_S
v_E
v_P
Rupture speed
Unstable
Modes III (dip slip)
Subshear
v_S
Rupture speed

Summary



For 3D bounded fault
Modes II (strike slip)
Sub-Rayleigh
Supshear
v_R v_S v_E v_P
Rupture speed
Mixture
Unstable
Modes III (dip slip)
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