

Advanced Workshop on
Earthquake Fault Mechanics:
Theory, Simulation and Observations

ICTP, Trieste, Sept 2-14 2019

Lecture 6: macroscopic source properties

Jean Paul Ampuero (IRD/UCA Geoazur)

Overview

Macroscopic source properties constrained by seismology:

- seismic moment
- source time function
- corner frequency
- radiated energy

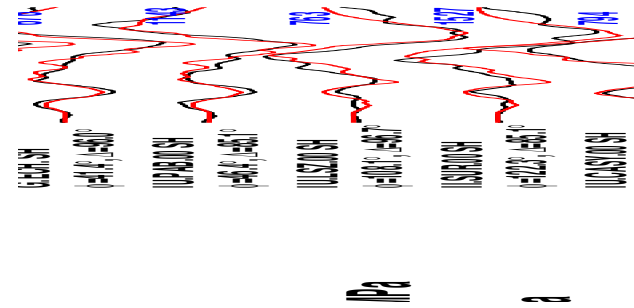
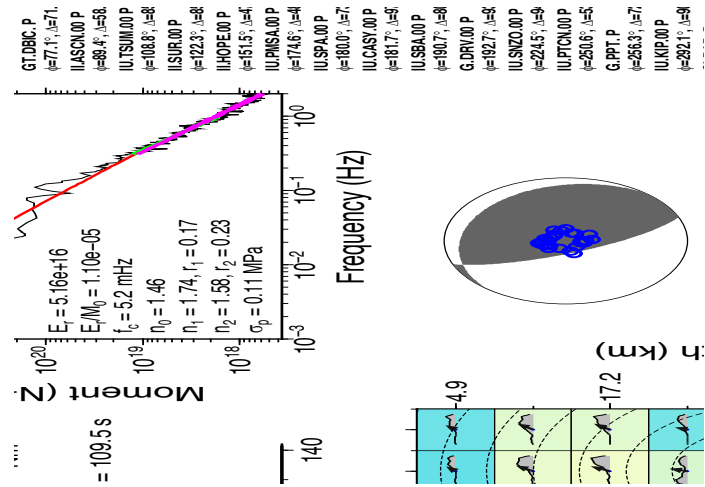
→ stress drop, rupture speed, rupture size

Detailed source parameters

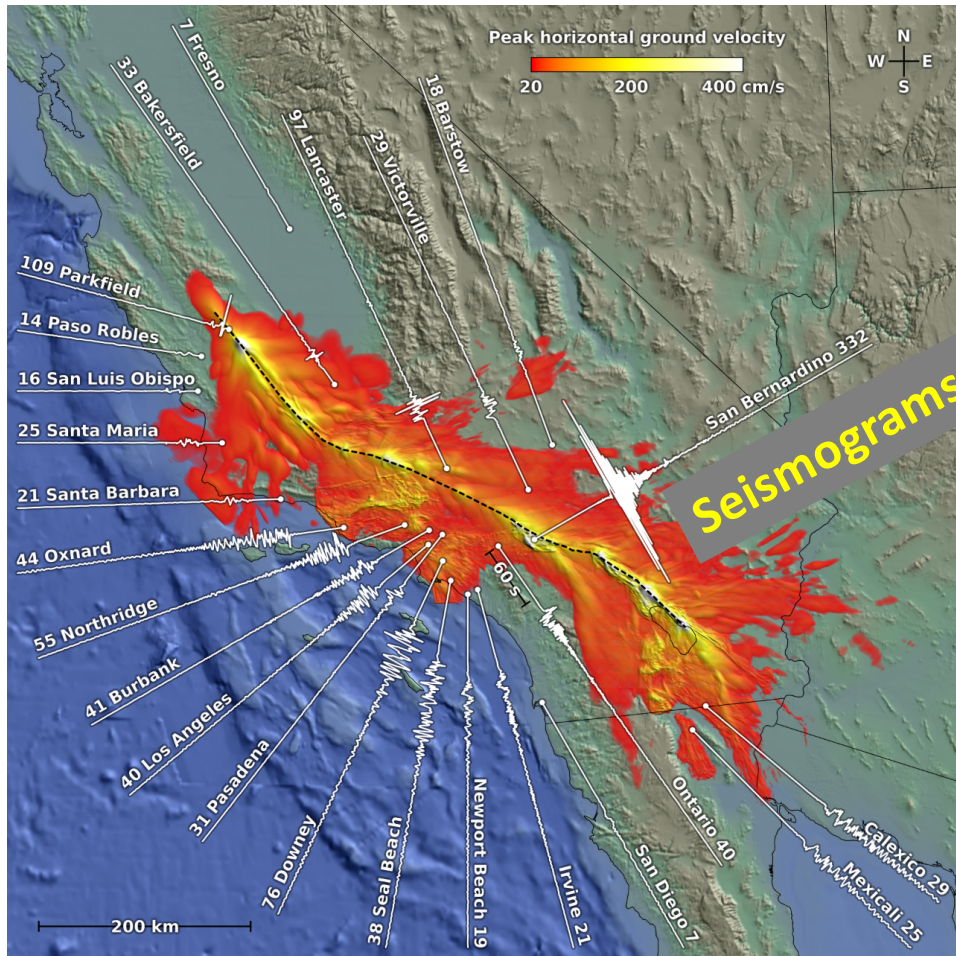
Outputs of dynamic rupture models:

Detailed space-time distribution of slip on the fault

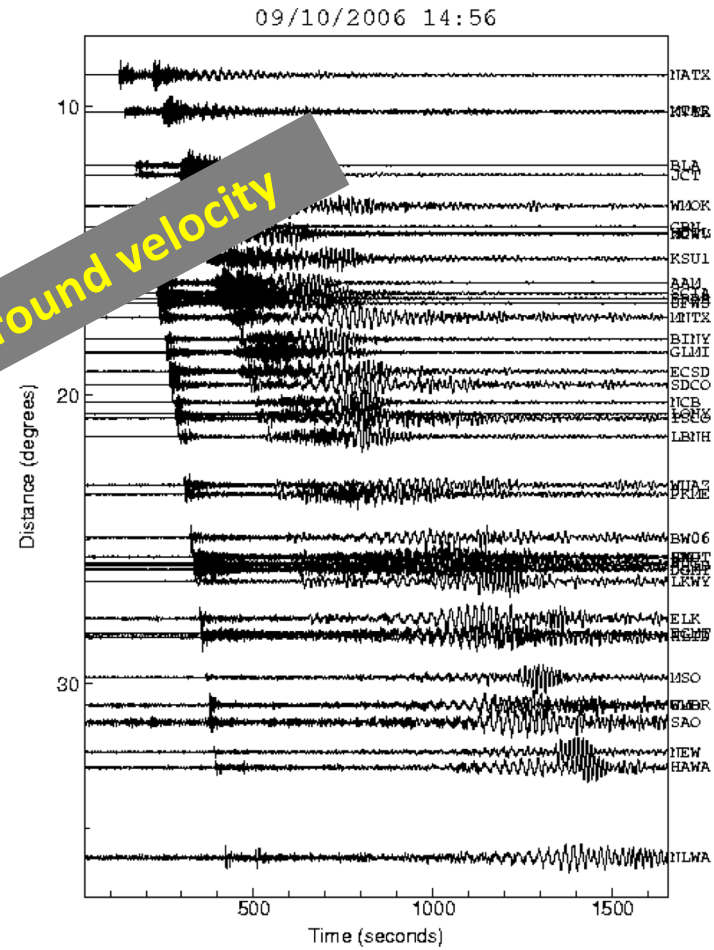
+ seismograms & ground displacements



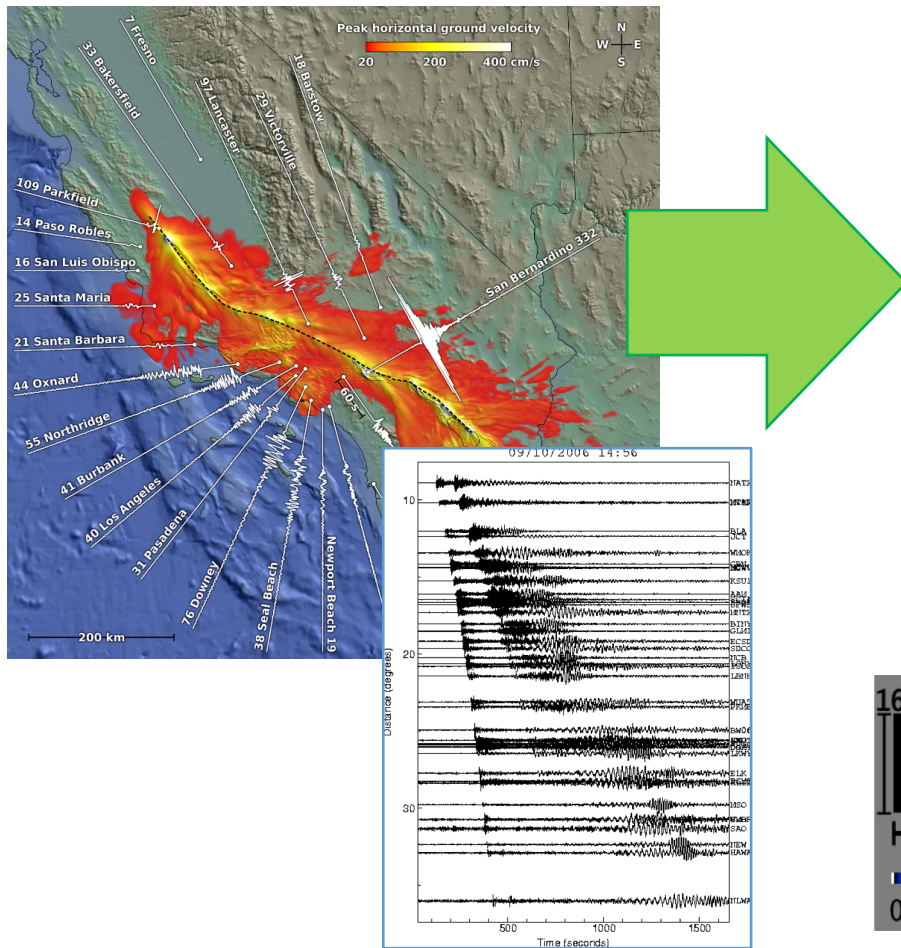
From ground motion recordings ...



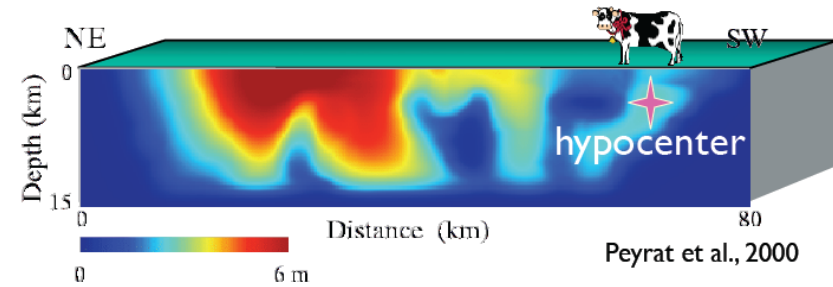
Seismograms = ground velocity



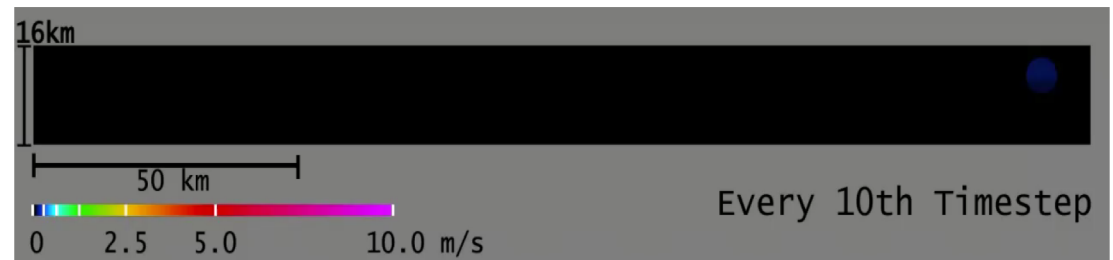
From ground motion recordings to the rupture process



How much did the **fault** slip?



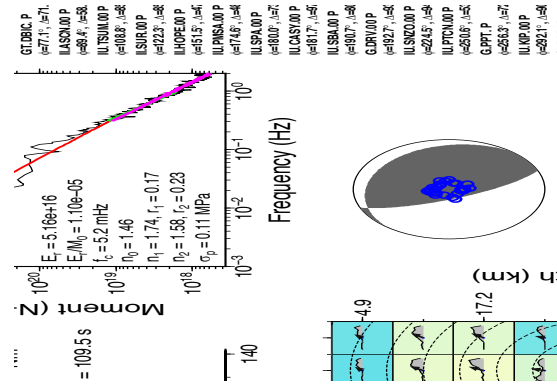
How did it slip?
Fast/slow? Smooth/tortuous? Loud/silent?



Fine vs. coarse source parameters

Outputs of dynamic rupture models:

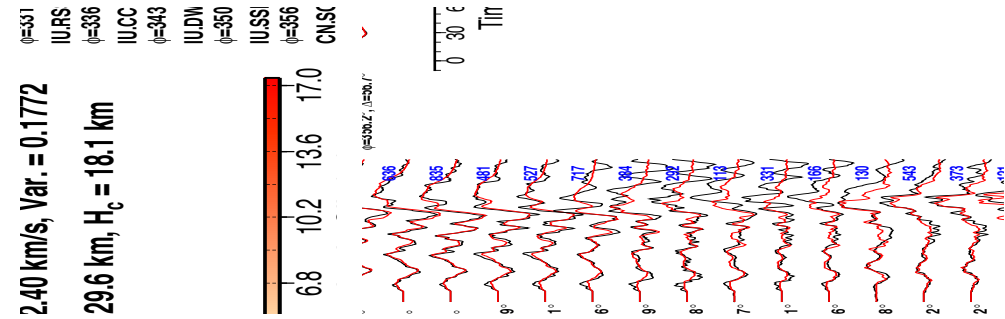
Detailed space-time distribution of slip on the fault



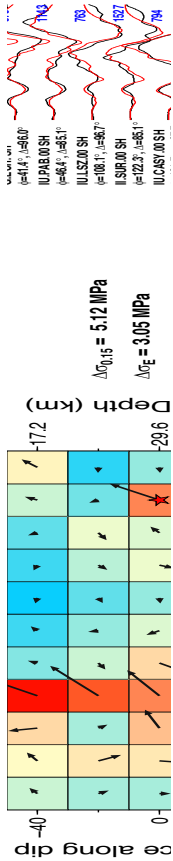
+ seismograms & ground displacements

Macroscopic source parameters:

- Seismic moment
- Seismic moment rate (source time function)
- Rupture size
- Rupture duration
- Average rupture speed



Trade-offs in earthquake source studies



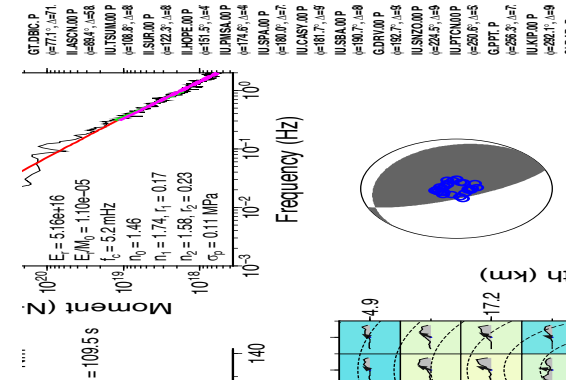
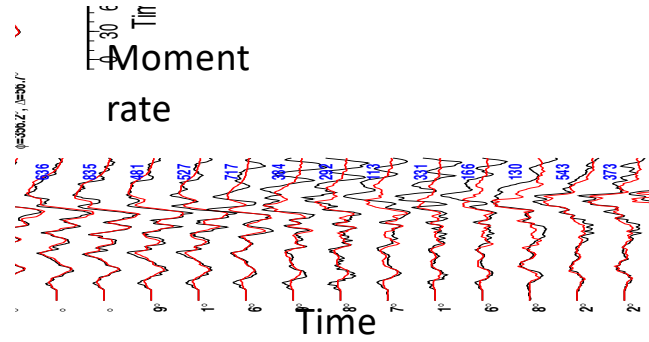
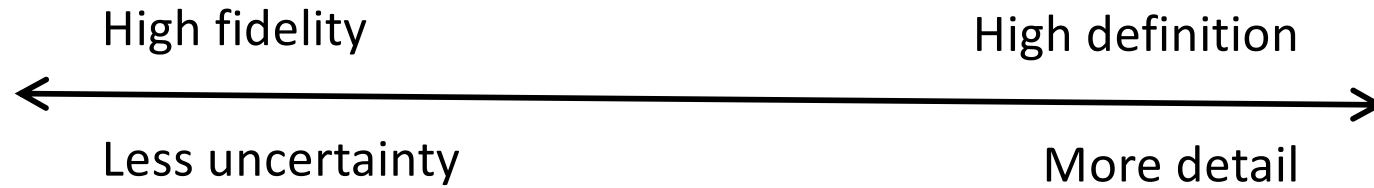
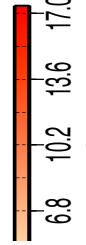
$\Delta\sigma_{0.15} = 5.12 \text{ MPa}$
 $\Delta\sigma_E = 3.05 \text{ MPa}$

II.PAB.00 SH $\phi=64^\circ, \lambda=85.1^\circ$
 II.SZC.00 SH $\phi=106.1^\circ, \lambda=86.7^\circ$
 II.SRQ.00 SH $\phi=122.2^\circ, \lambda=85.1^\circ$
 II.CAS.00 SH $\phi=114^\circ, \lambda=86.0^\circ$

2.40 km/s, Var. = 0.1772

29.6 km, $H_c = 18.1 \text{ km}$

II.RS $\phi=331$
 II.CC $\phi=336$
 II.DW $\phi=343$
 II.SS $\phi=350$
 II.SS $\phi=356$
 CNSK $\phi=356$

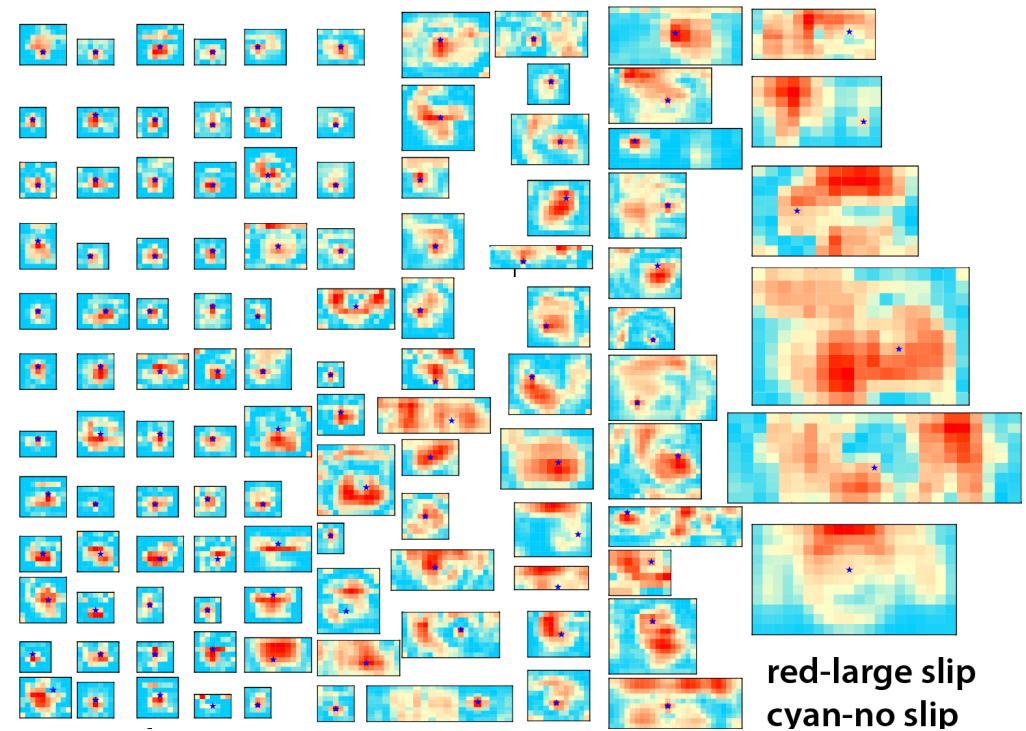
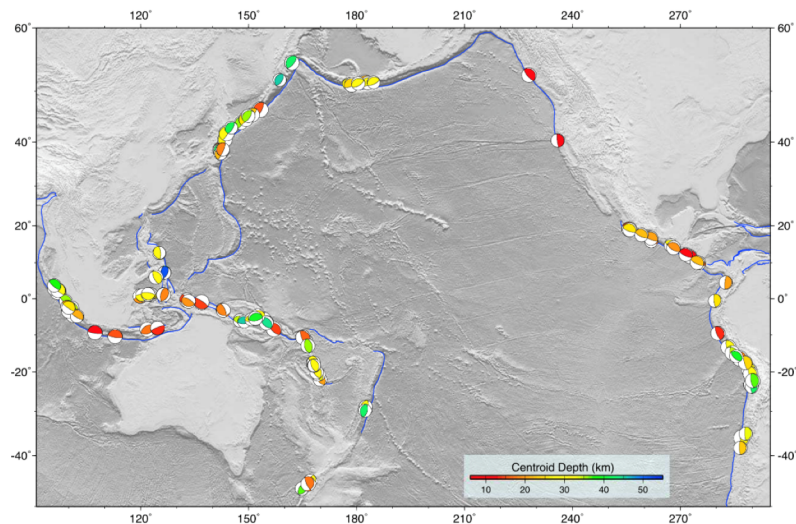


Source time functions

Global source studies

Ye et al (JGR 2016)

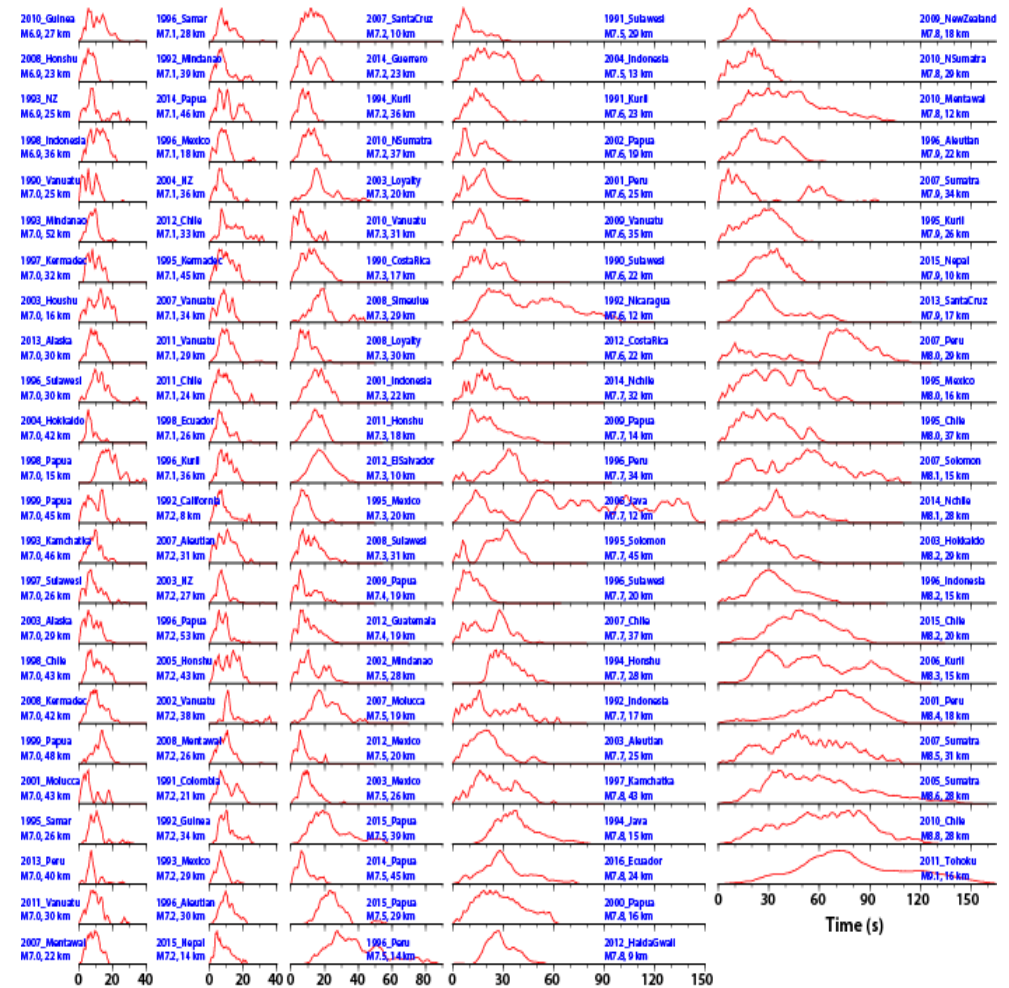
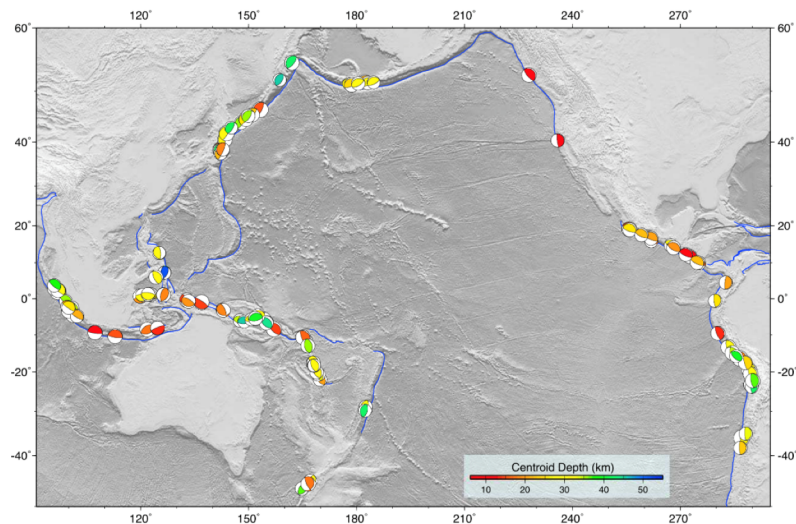
- 116 **M7+** shallow **subduction zone** thrust earthquakes
- finite source **inversions with teleseismic data**, 0.005-0.9 Hz
- Robust source time functions (STF, **moment rate**)
- Uniform method and careful manual analysis



Global source studies

Ye et al (JGR 2016)

- 116 **M7+** shallow **subduction zone** thrust earthquakes
- finite source **inversions with teleseismic data**, 0.005-0.9 Hz
- Robust source time functions (STF, **moment rate**)
- Uniform method and careful manual analysis



STF from deconvolution

seismogram = (Green's function)*(Source Time Function)

$$d_i(t) = G_i(t) * \dot{M}_0(t)$$

* means convolution

G can be synthetic or empirical

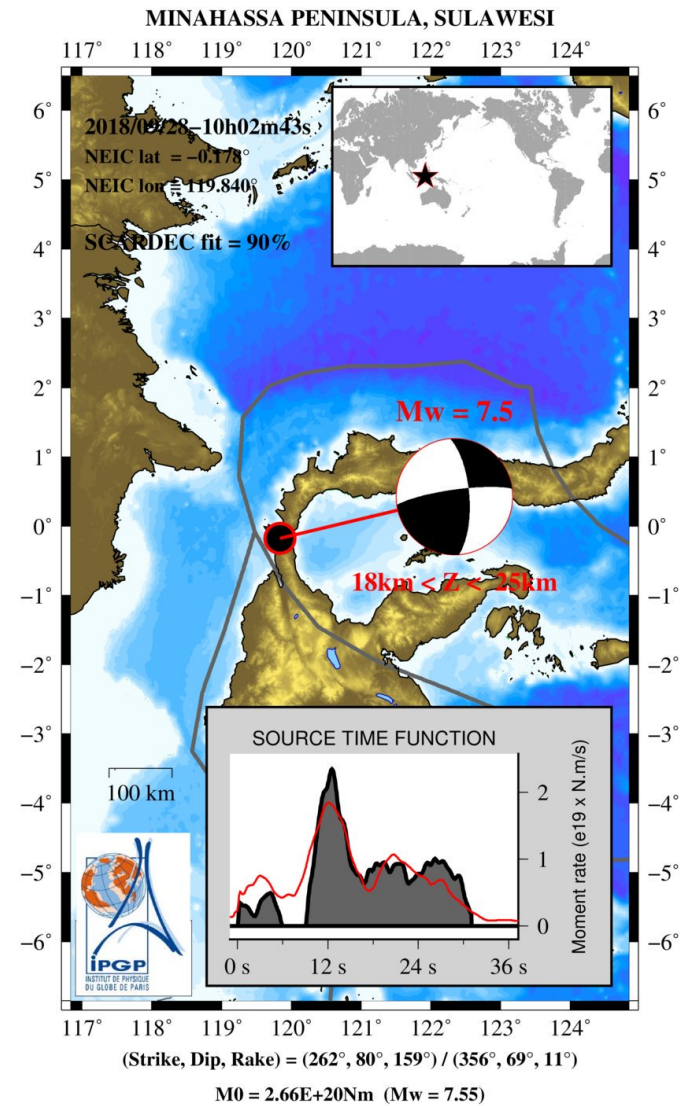
Deconvolution: infer $\dot{M}_0(t)$ from $d_i(t)$

SCARDEC (by Martin Vallée, IPGP):

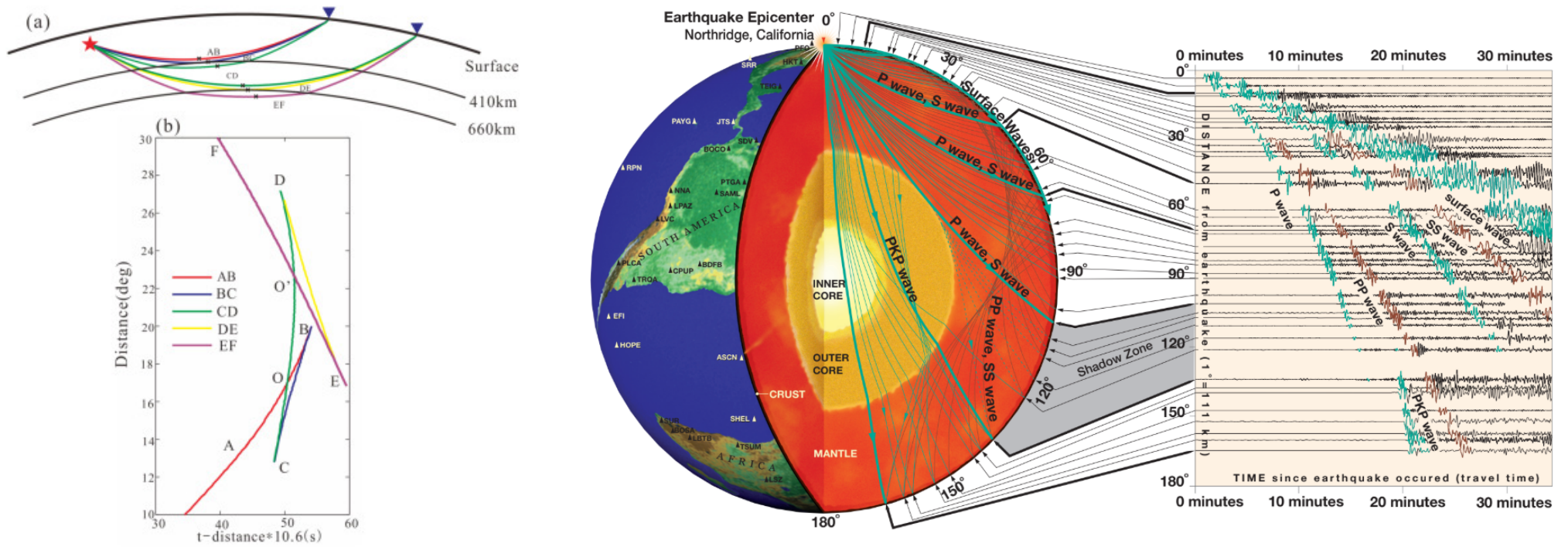
real-time STF from teleseismic data

large catalog of past events

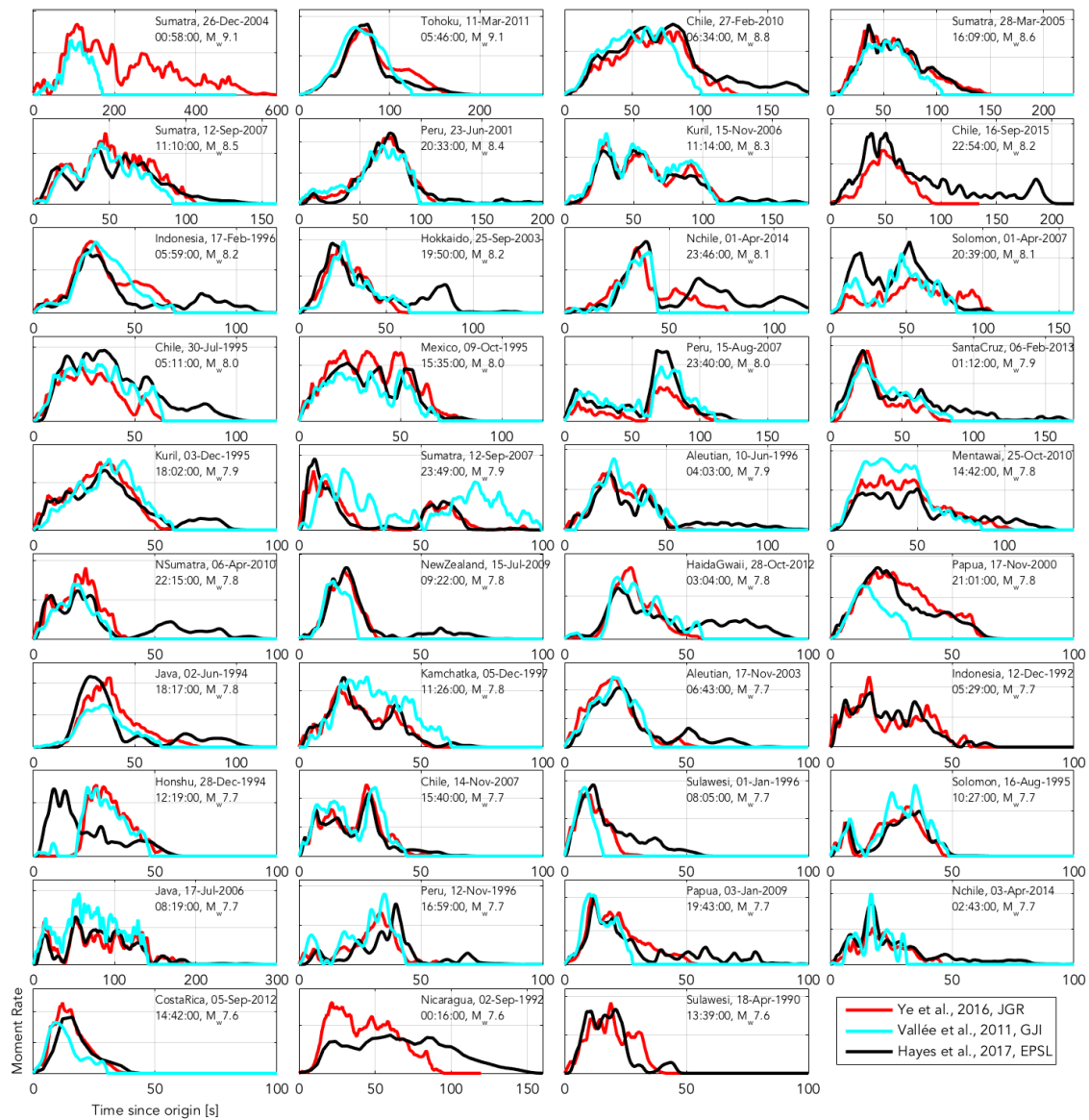
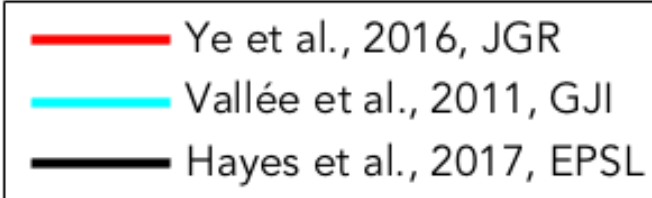
new events posted rapidly on Twitter by @geoscope_ipgp

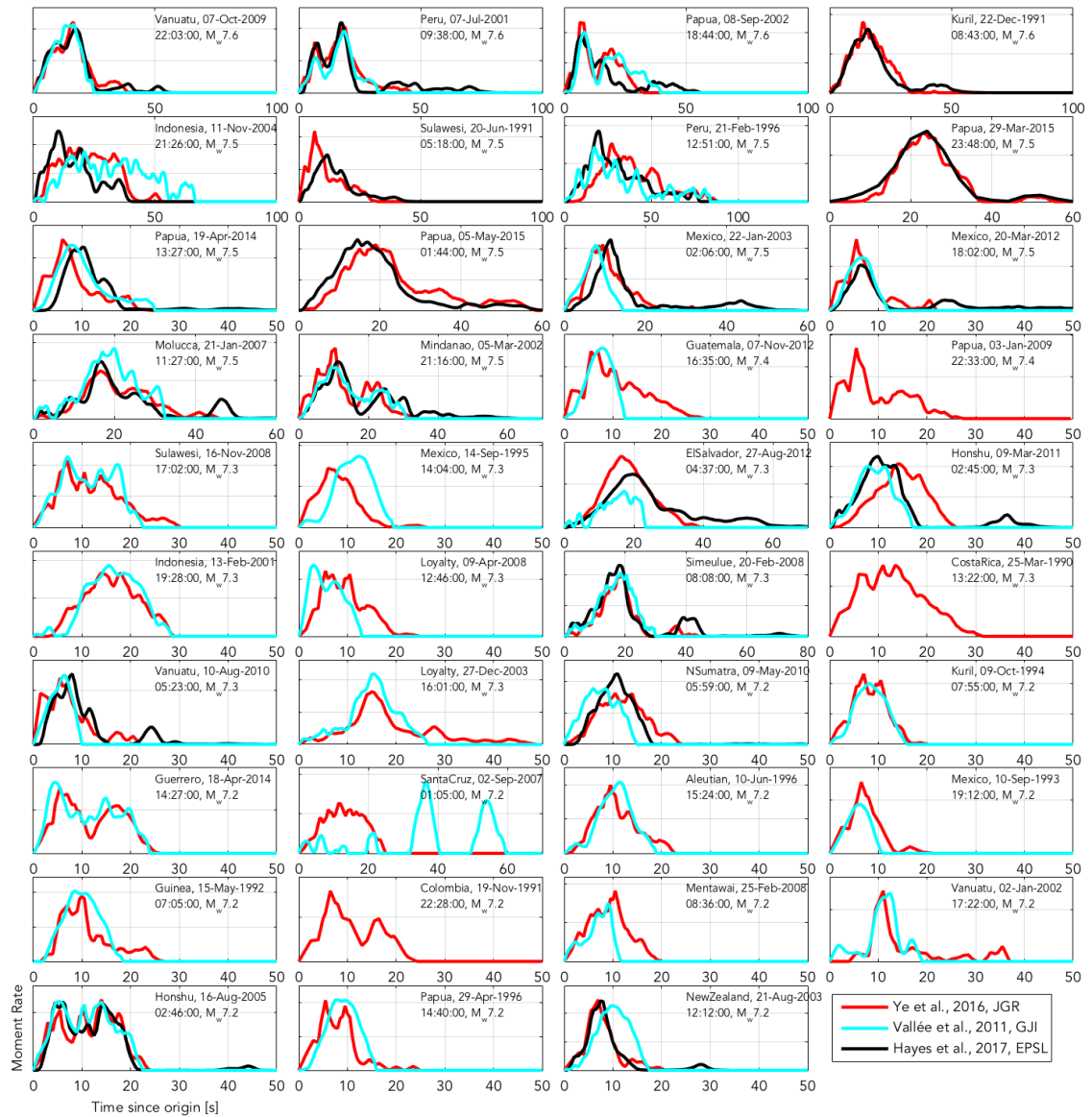
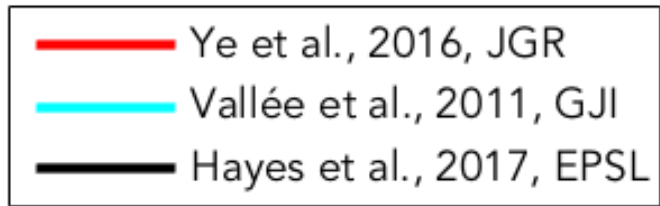


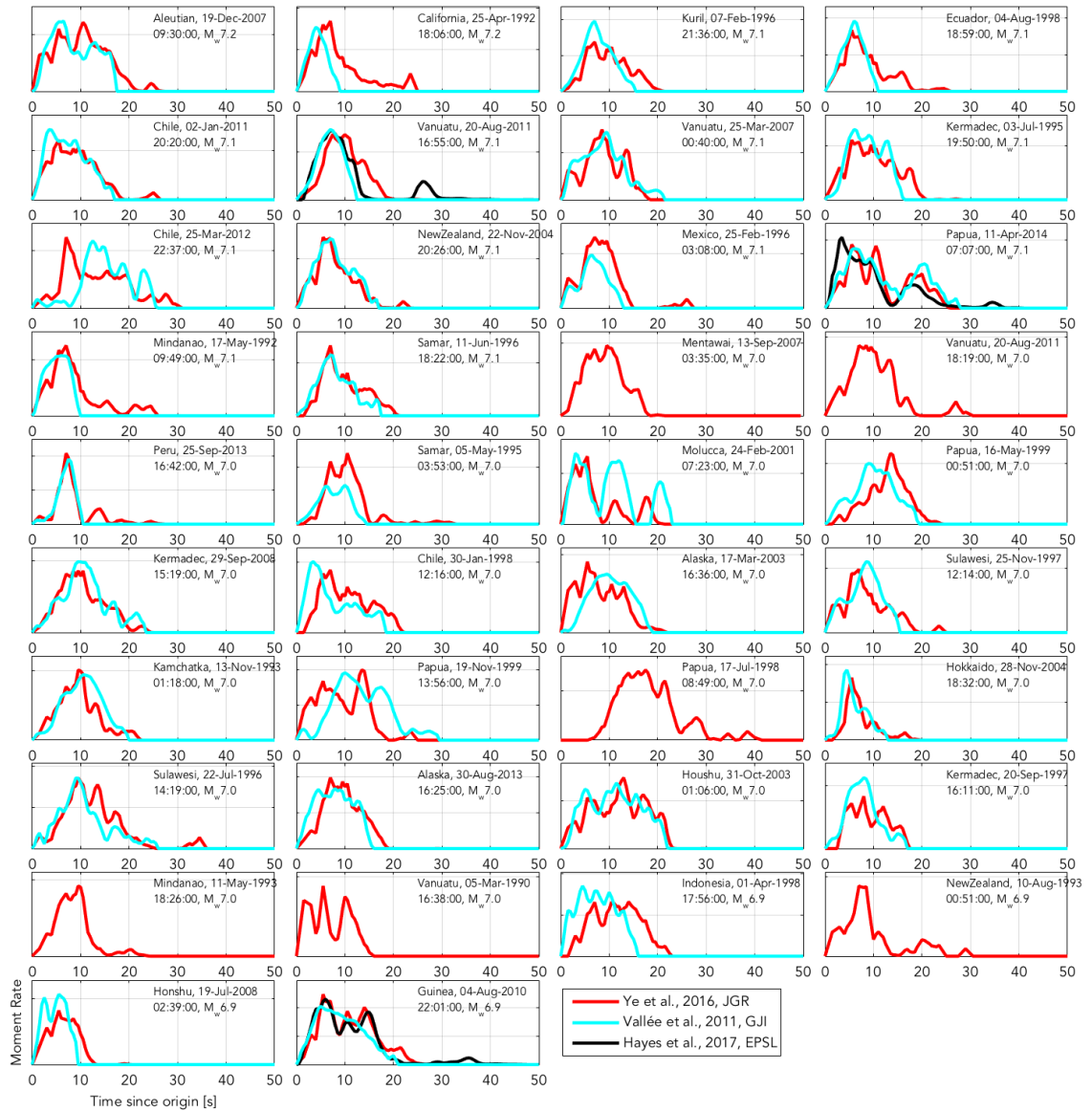
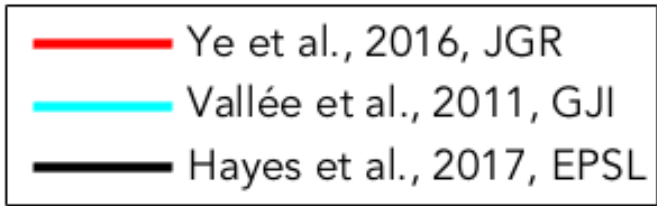
Teleseismic waves



<https://www.iris.edu/hq/inclass/fact-sheet/>







Questions to address:

- What are the common features of earthquakes?
 - Do small and large earthquakes start equal?
 - Are earthquakes self-similar at all magnitudes?
- How are earthquakes different from each other?
 - Is there such a thing as a freak event?
- What do those similarities and differences tell us about earthquake dynamics?



Meier *et al.*, *Science* **357**, 1277–1281 (2017) 22 September 2017

The hidden simplicity of subduction megathrust earthquakes

M.-A. Meier,* J. P. Ampuero, T. H. Heaton

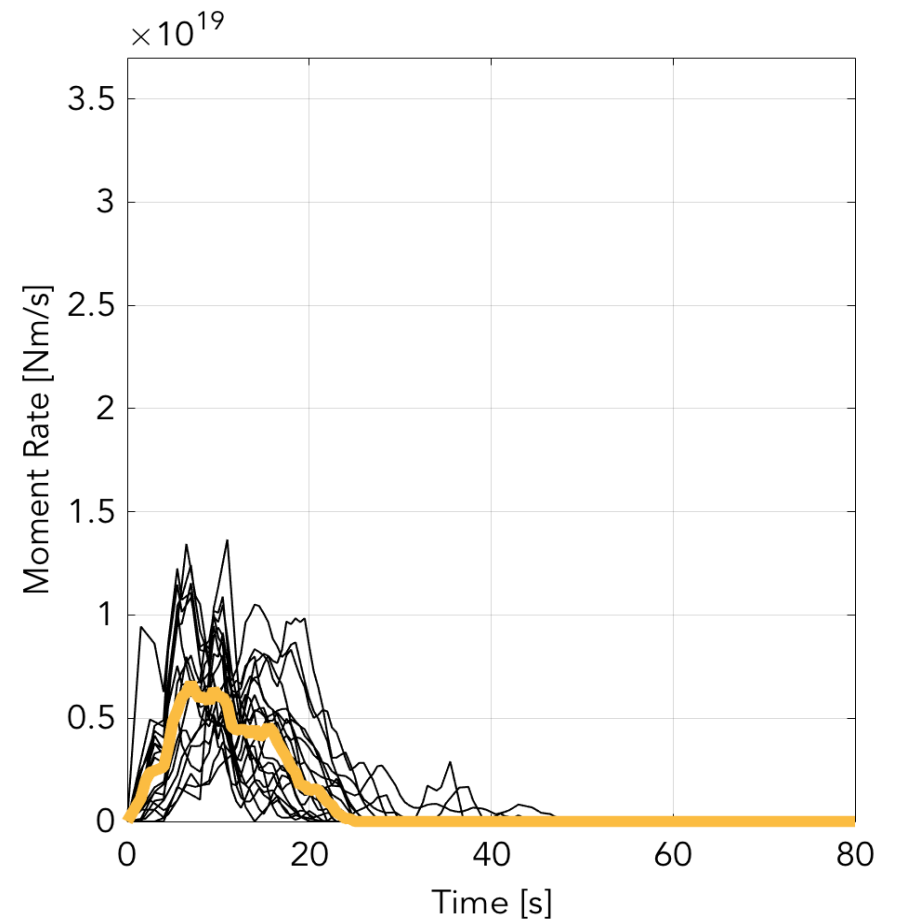
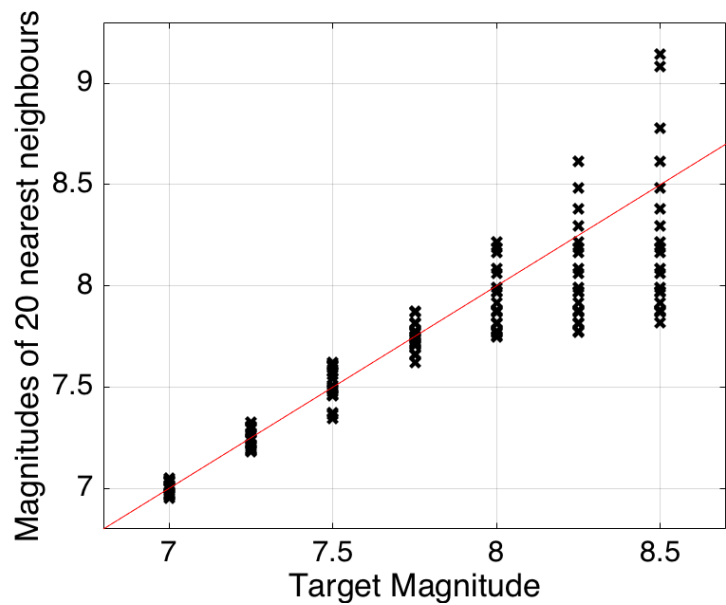
Enabled by global earthquake source products by
Lingling Ye (Caltech), Martin Vallée (IPGP) and Gavin Hayes (USGS)

Caltech

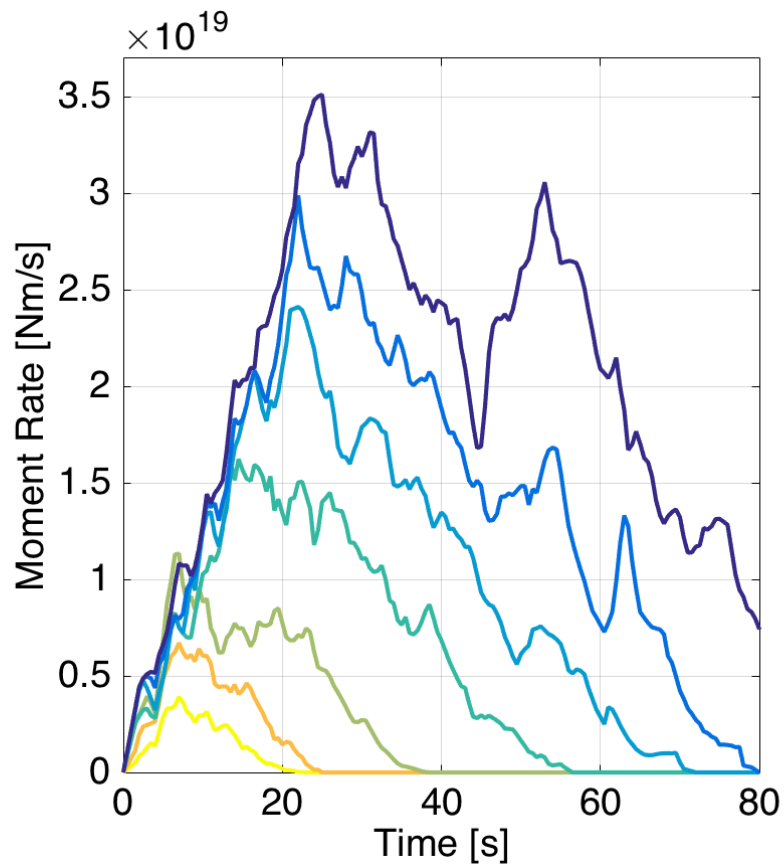


General patterns of Source Time Functions

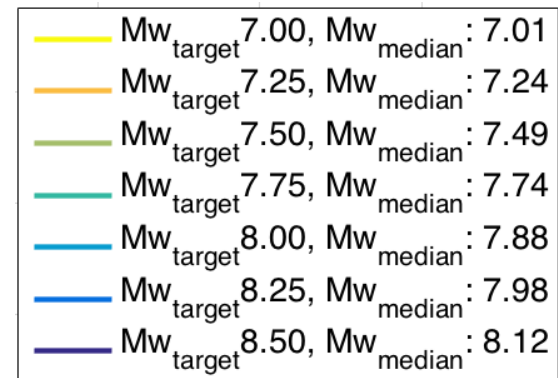
- Bin STFs by magnitude, 20 **nearest neighbours**
- In each bin, at each point in time, compute **median STF**



General patterns of Source Time Functions



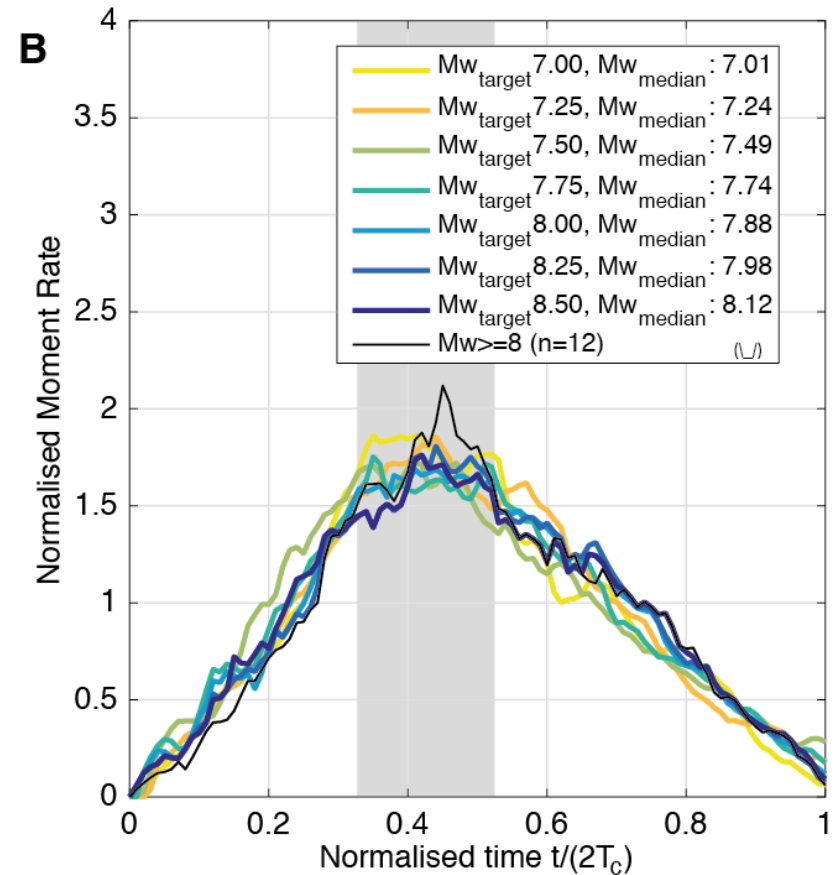
Median STFs have **linear onset**,
same for all magnitudes $M_w > 7.2$

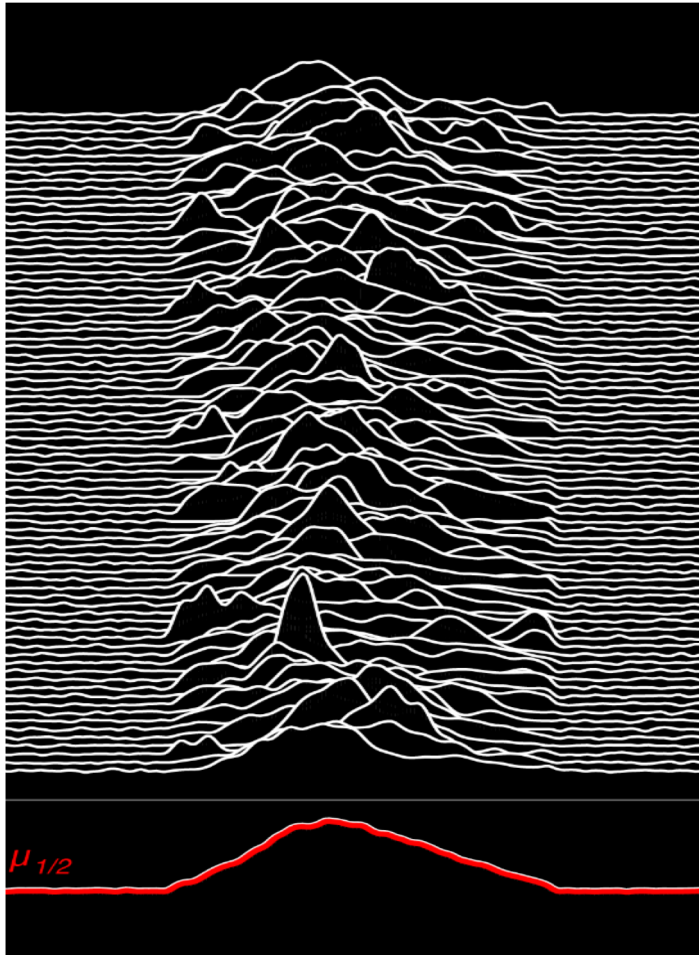


General patterns of STF

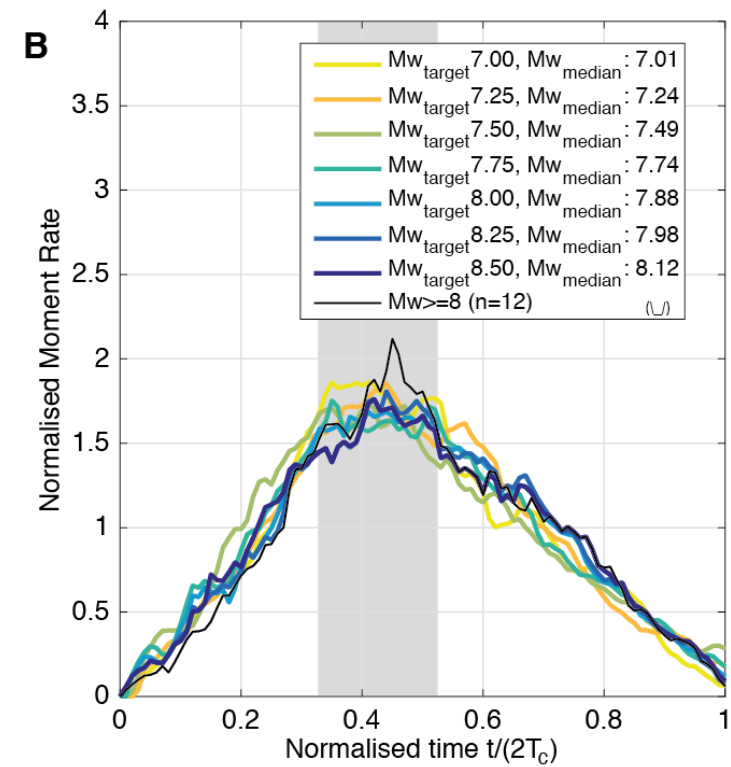
- Normalize each STF by its duration
- Scale them such that they integrate to 1
- Compute median of normalized STF

On average, all STF
can be scaled to a very simple,
quasi-triangular shape

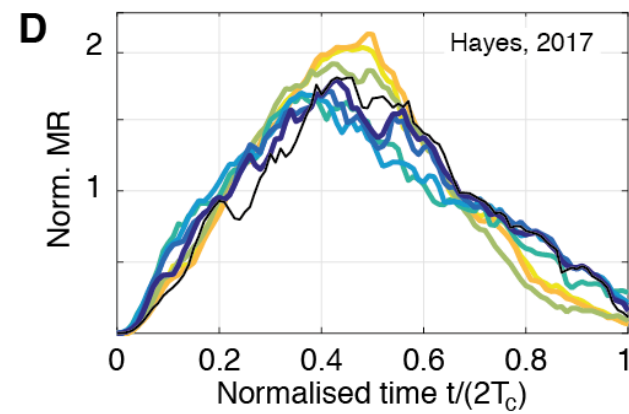
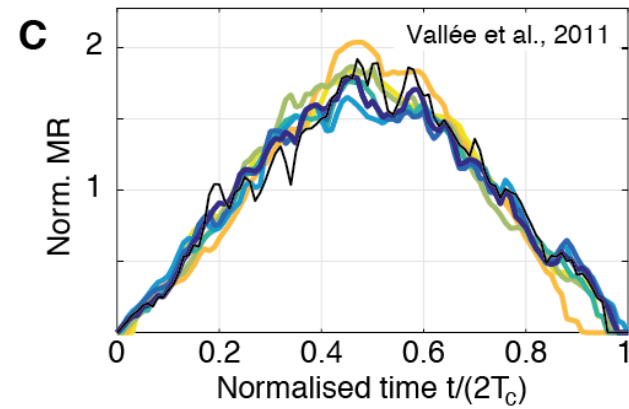
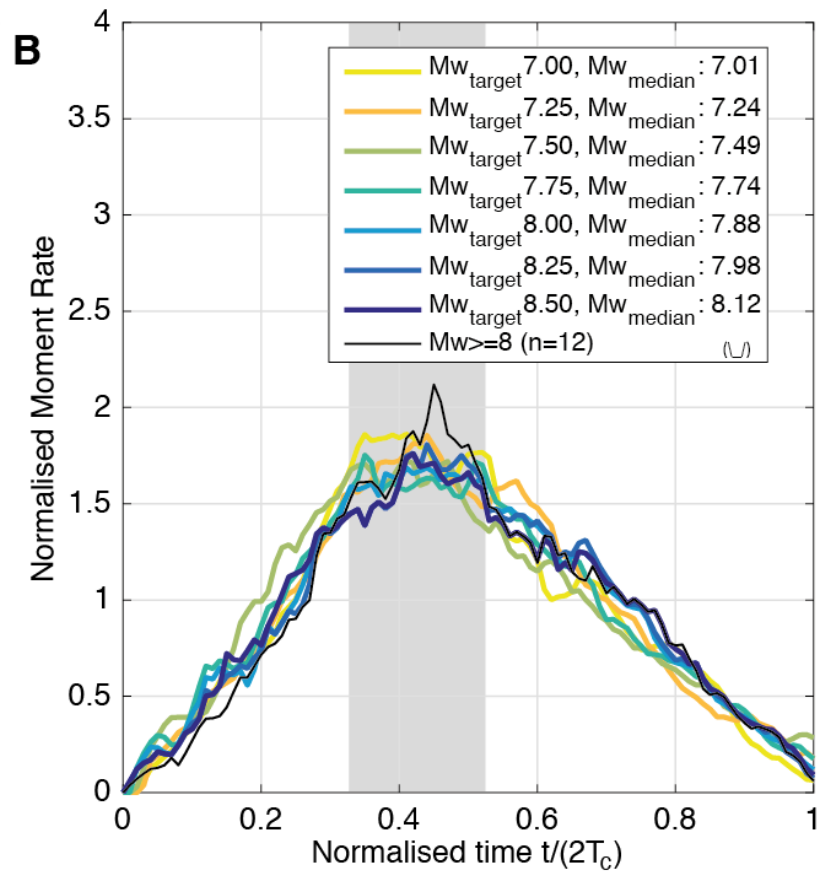




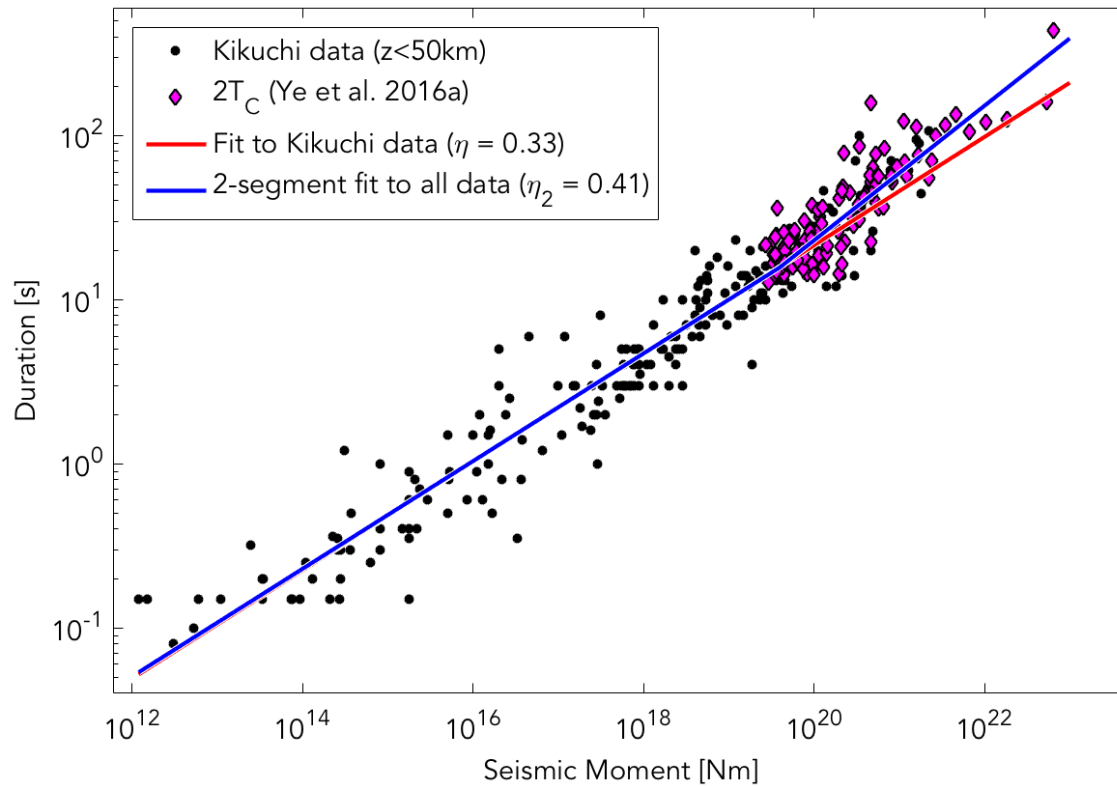
On average (median), all STFs can be scaled to a very simple, quasi-triangular shape



Meier, Ampuero and Heaton (2017)



Implications for moment / duration scaling



Linear growth suggests
 $M_0 \sim T^2$ scaling

In contrast to the widely
reported $M_0 \sim T^3$ scaling

→ scaling break !

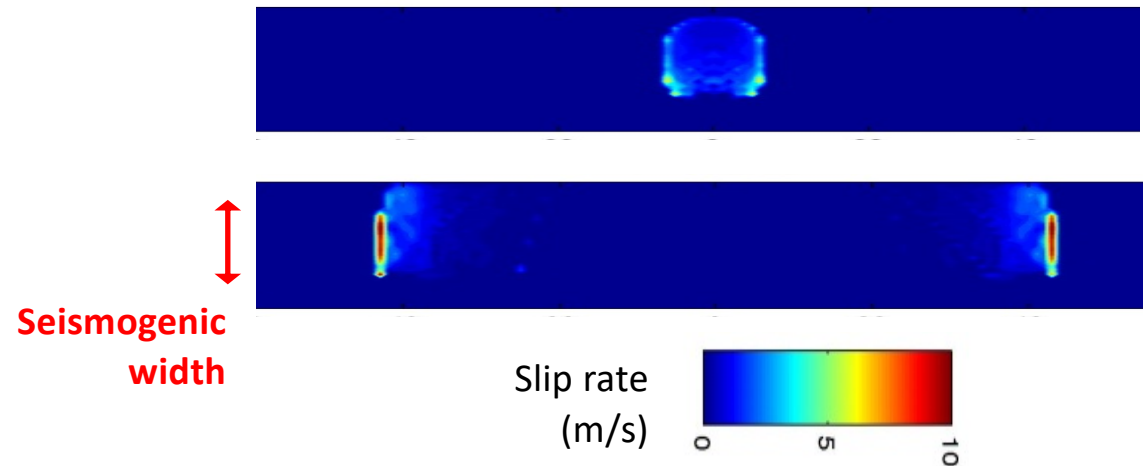
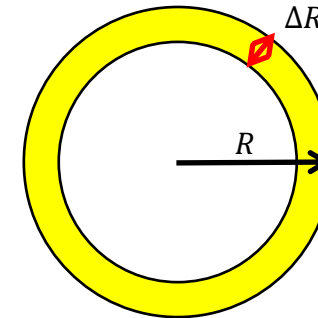
Why is linear moment rate growth surprising?

Self-similar model for small earthquakes:
Circular rupture with constant stress drop and
constant rupture speed

$$\dot{M}_0 \propto t^2$$

Ruptures become **elongated** after they break
the whole **seismogenic width**:
moment grows slower than quadratic

But the linear trend ($M_0 \sim t$) is
observed after ~ 5 s,
before rupture saturates the
seismogenic width



Implications for Rupture Growth Scaling

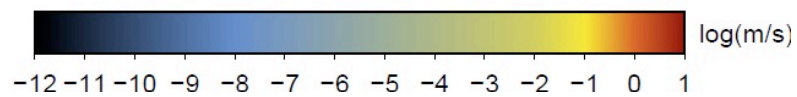
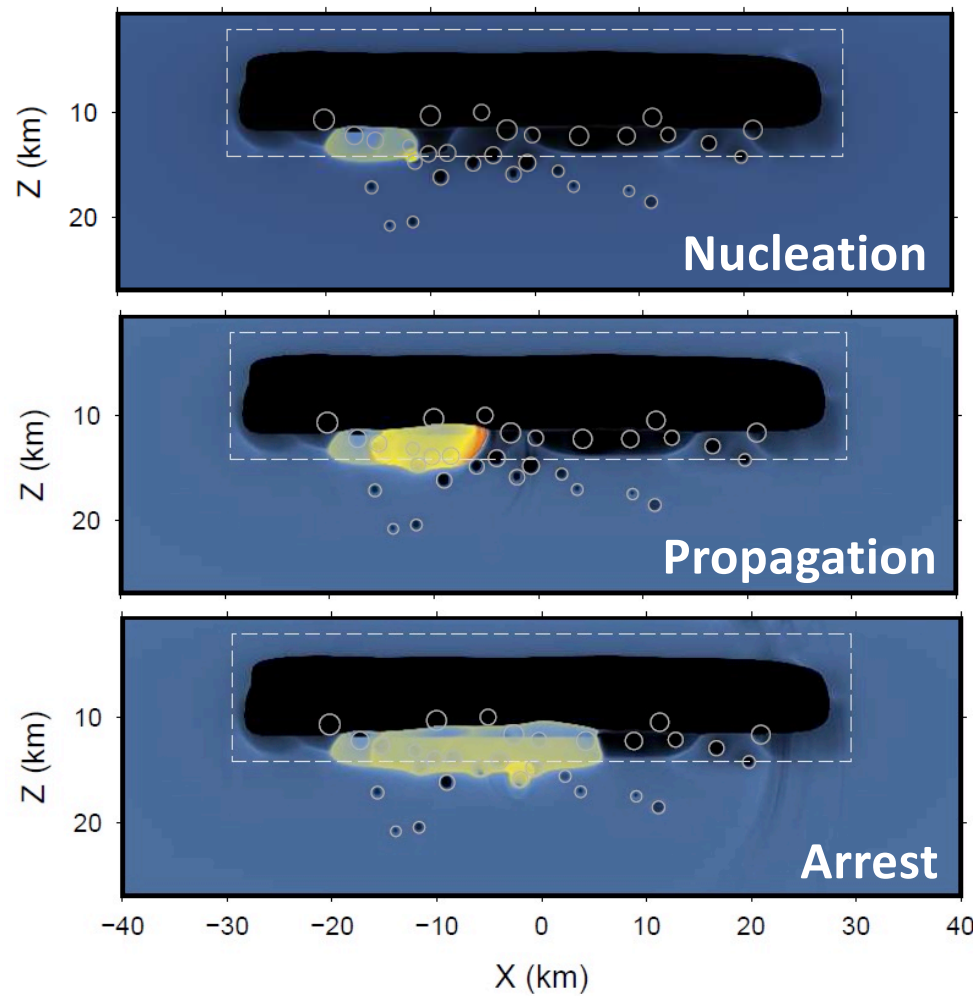
- Observed **STF growth is linear** $STF \propto t^1$
- If rupturing **area** grows as $A(t) \propto t^\alpha$
- ... and **average slip** grows as $D(t) \propto t^\beta$
- Seismic **moment** $M_0(t) \propto A(t)D(t) \propto t^{\alpha+\beta}$
- Moment rate **exponent** $\eta = \alpha + \beta - 1$
- Since we observe linear growth $\eta^{obs} \sim 1 \rightarrow \alpha + \beta \sim \mathbf{2}$

- Self-similar pulse or crack $\eta^{ss} = 2 + 1 = 3$

→ **How can we lower the moment rate**

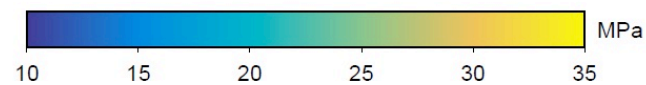
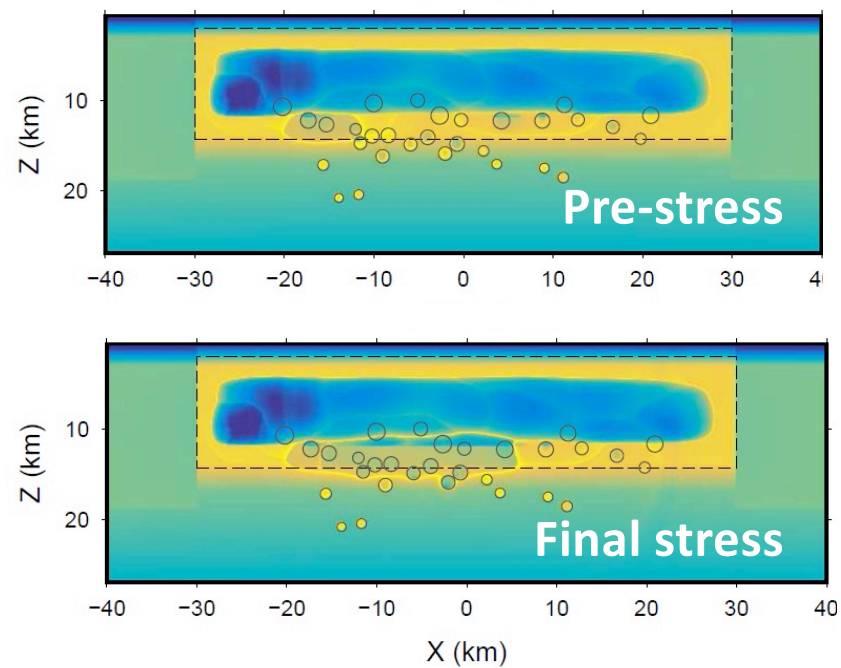
growth? Lower alpha, lower beta, or combination of both?

- Pulse-like rupture with areas of systematic slip deficits?

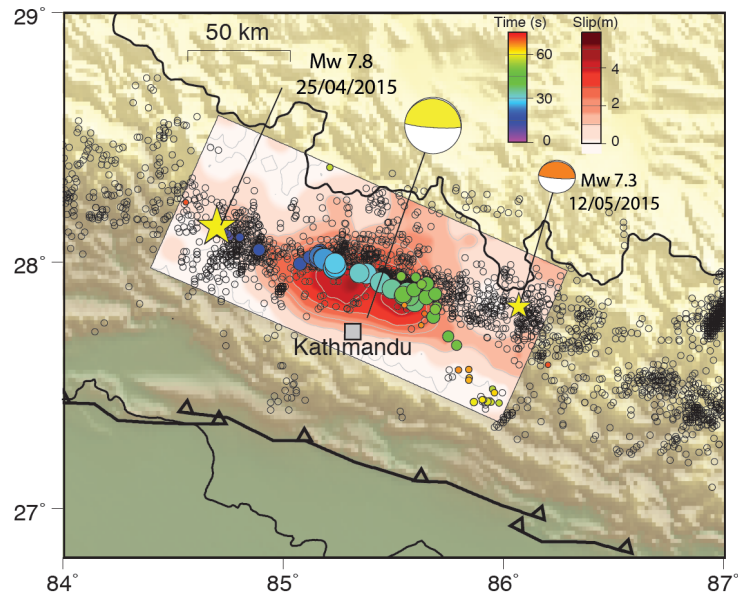


Intermediate-size event unzipping part of the lower edge of the coupled zone

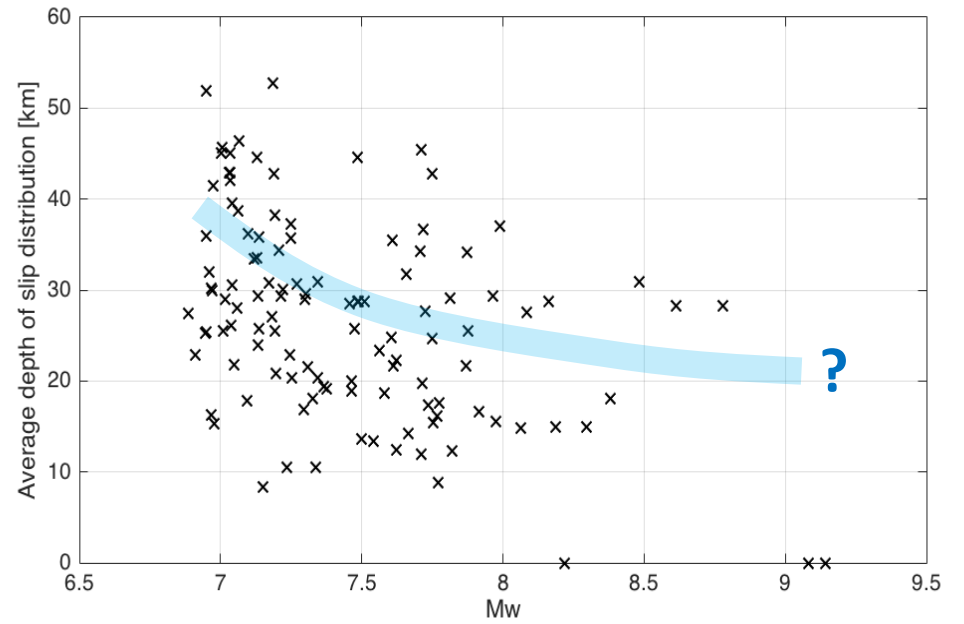
(Junle Jiang, Caltech)



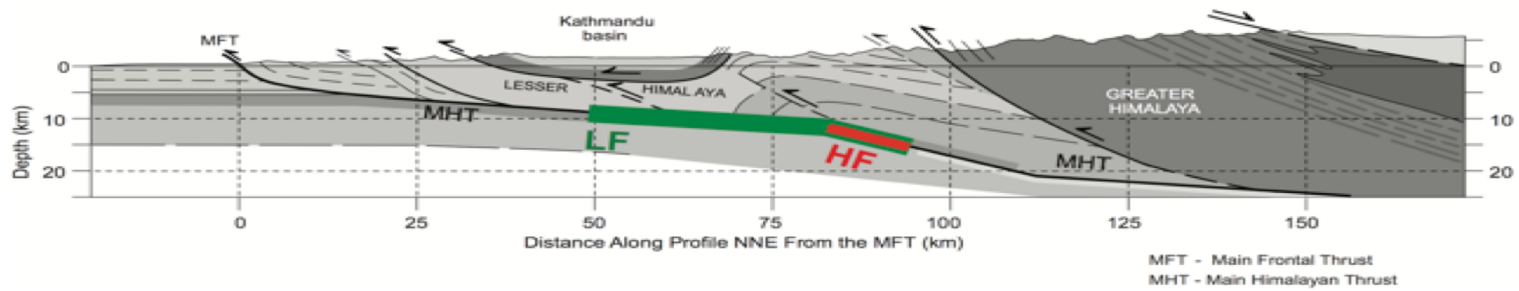
2015 Mw 7.8 Gorkha, Nepal earthquake



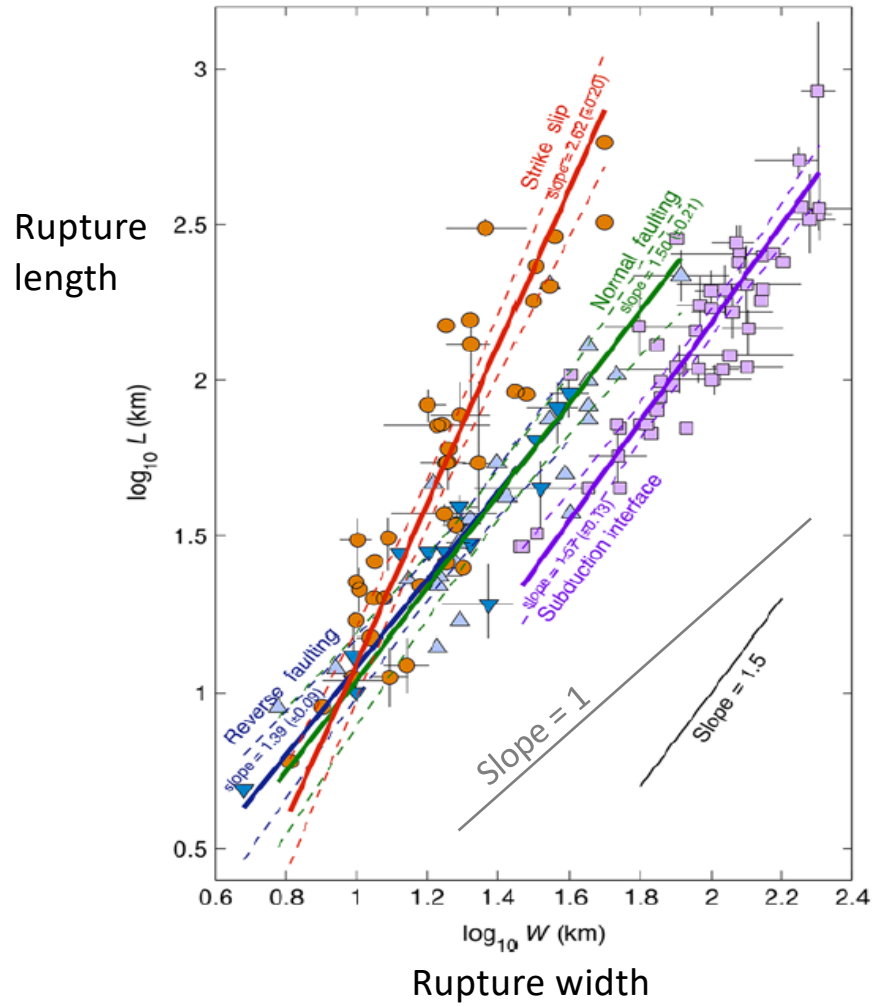
All M>7 subduction earthquakes



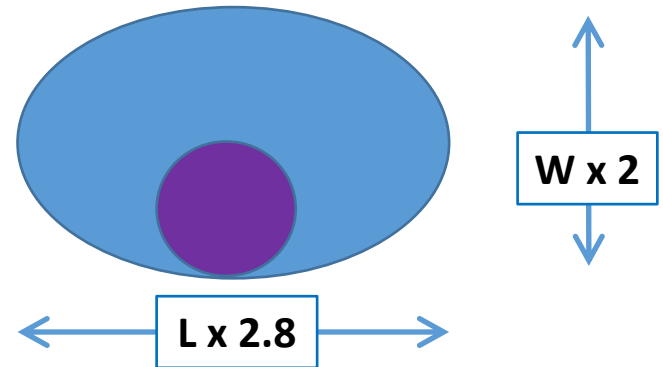
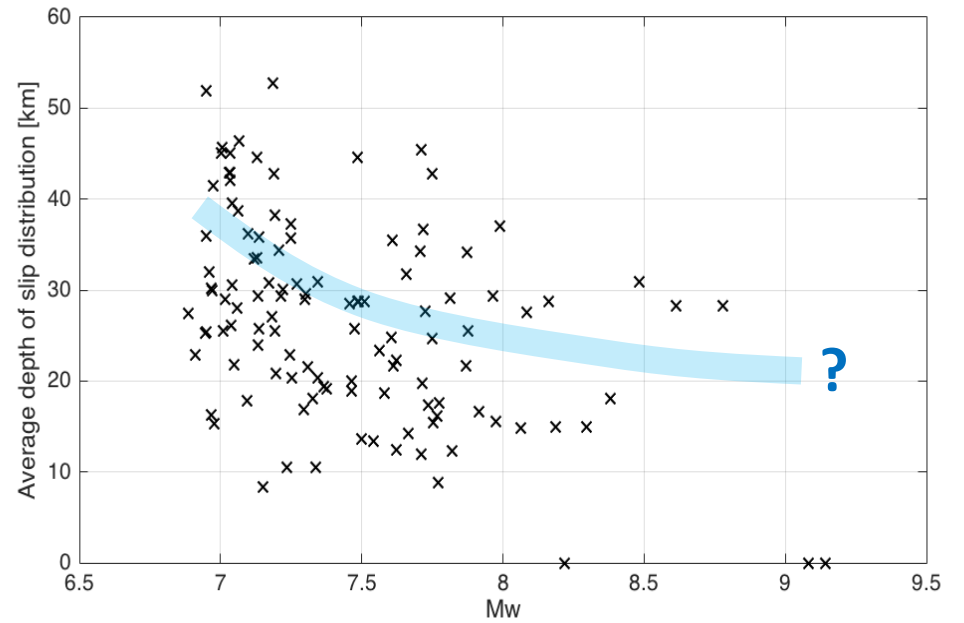
Avouac et al (Nat Geo, 2015)



Thingbaijam et al (2017)



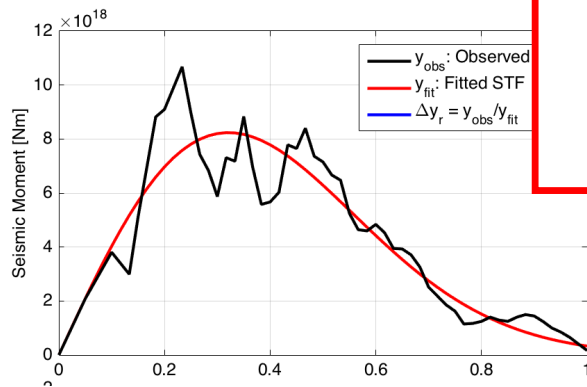
All $M > 7$ subduction earthquakes



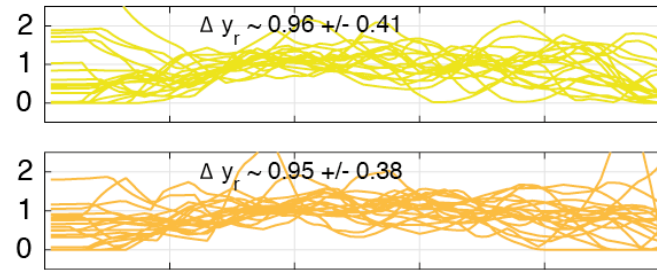
Fluctuations around the median STF

Fit a function to STFs:

$$y = \mu t \exp[-1/2 * (\lambda * t)^\eta]$$

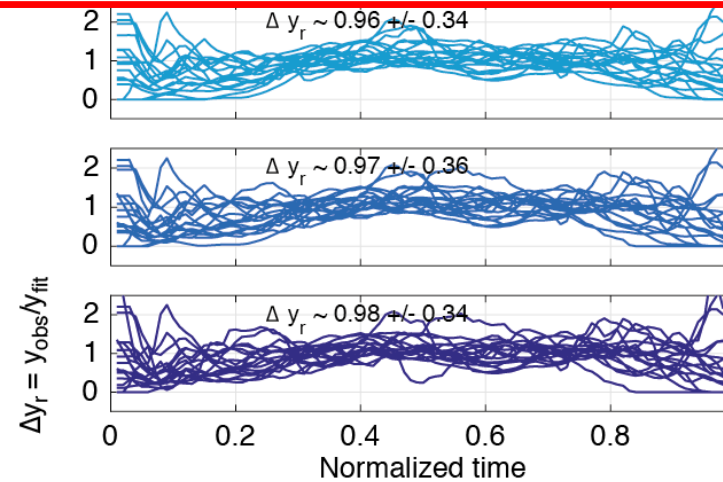


STF residuals



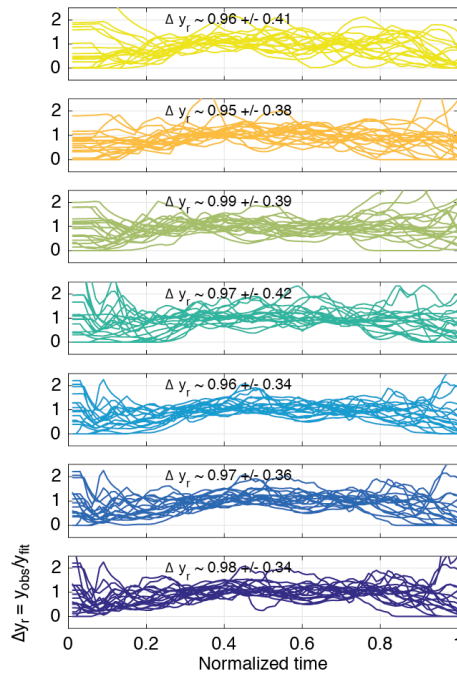
Multiplicative noise

$$y_{\text{obs}}(t') = y_{\text{fit}}(t') \times [1 + \epsilon(t')]$$

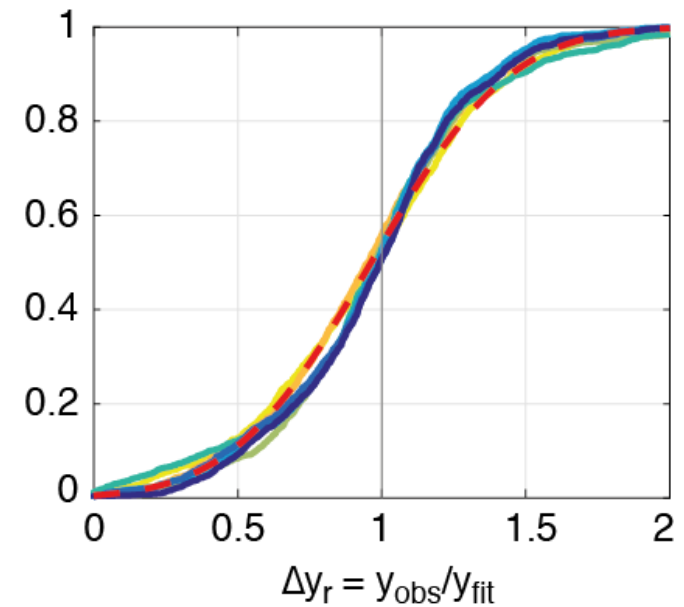
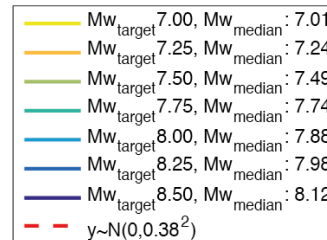


—	Mw _{target} 7.00, Mw _{median} : 7.01
—	Mw _{target} 7.25, Mw _{median} : 7.24
—	Mw _{target} 7.50, Mw _{median} : 7.49
—	Mw _{target} 7.75, Mw _{median} : 7.74
—	Mw _{target} 8.00, Mw _{median} : 7.88
—	Mw _{target} 8.25, Mw _{median} : 7.98
—	Mw _{target} 8.50, Mw _{median} : 8.12
—	y ~ N(0, 0.38 ²)

STF fluctuations are multiplicative and Gaussian

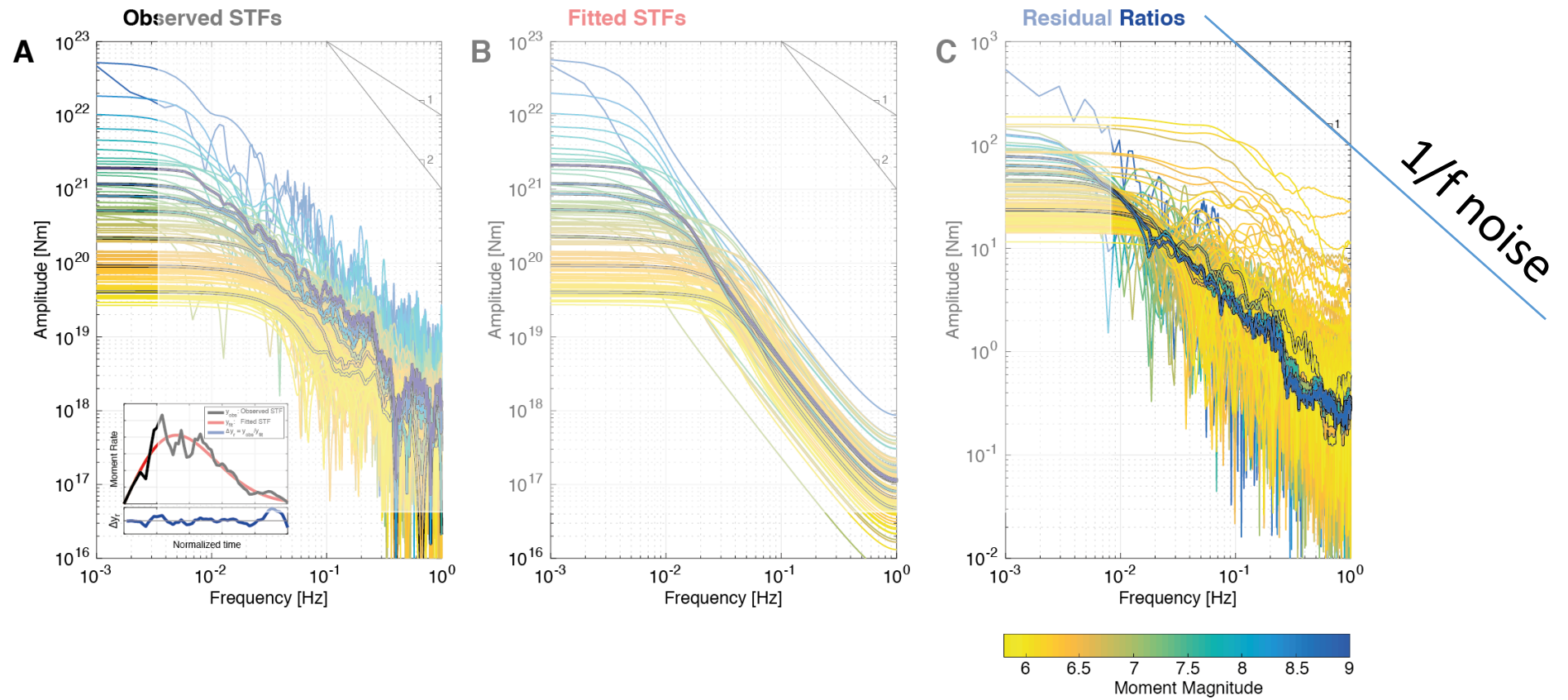


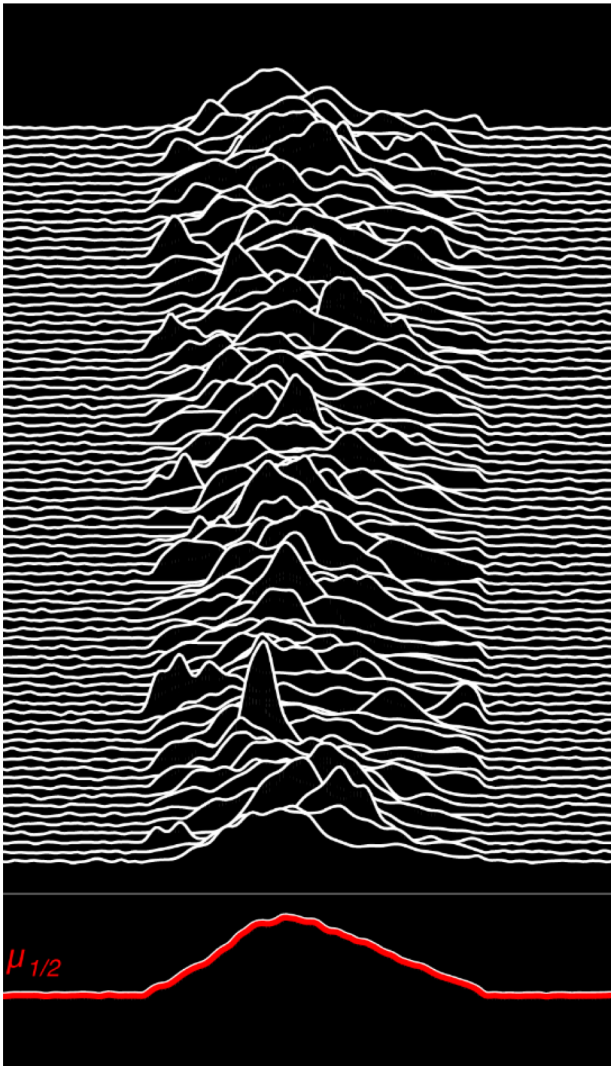
Empirical cumulative distribution of STF residuals



$$y_{\text{obs}}(t') = y_{\text{fit}}(t') \times [1 + \varepsilon(t')], \text{ where } \varepsilon \sim N(0, 0.38^2)$$

STF fluctuations are multiplicative, Gaussian and Brownian



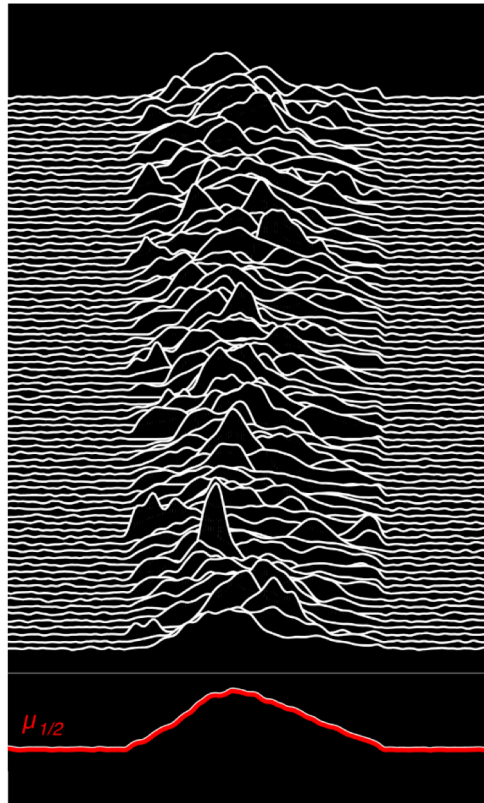


Summary of observed STF characteristics, $M_w > 7$

- All STFs can be scaled to a common, quasi-triangular shape
- Onsets are linear and the same for all
- Fluctuations are multiplicative, Gaussian and Brownian

CONCLUSIONS

Today we have **enough data** to uncover **general patterns** of earthquake rupture
Focusing on **temporal evolution** facilitates testing conceptual rupture models



A few things are certain

- ∴ Large earthquakes are small earthquakes that did not stop
(**all earthquakes start the same**)
- . Earthquakes have large variability, but on average they follow a **simple pattern**
- . The pattern **deviates from standard models after few seconds**
- . Rupture evolution is **weakly predictable**

More questions than answers

- ∴ **Physical origin** of the pattern?
- . What **dynamical models** can explain the linear STF growth?
- . What causes **break of self-similarity** at ~ 1 s?
- . Transition to **elongated ruptures** at the bottom of seismogenic zone?

What to do next?

- Analysis of strike-slip ruptures
- Source studies with uncertainty quantification
- Develop methods for systematic analysis across the magnitude range of scaling transition from M6 to M8+
 - break of self-similarity, scaling of rupture aspect ratio
- Develop dynamic rupture models consistent with these observations

Corner frequency

Earthquake spectra

seismogram

= (Green's function)*(Source Time Function)

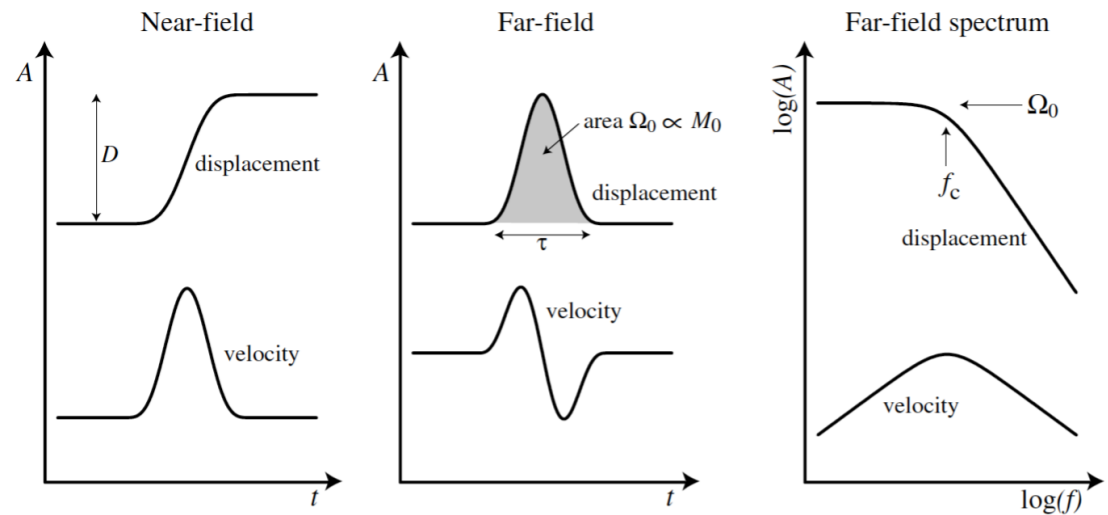
$$d(t) = G(t) * \dot{M}_0(t)$$

In the far-field, $G(t) \propto \delta(t - r/c)$

$$d(t) \propto \dot{M}_0(t - r/c)$$

→ Far-field displacements are proportional to Source Time Function

→ Far-field spectrum $d(f)$ proportional to moment rate spectrum $\dot{M}_0(f)$



Earthquake spectra

seismogram

= (Green's function)*(Source Time Function)

$$d(t) = G(t) * \dot{M}_0(t)$$

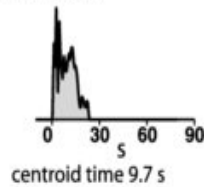
In the far-field, $G(t) \propto \delta(t - r/c)$

$$d(t) \propto \dot{M}_0(t - r/c)$$

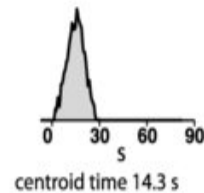
→ Far-field displacements are proportional to Source Time Function

→ Far-field spectrum $d(f)$ proportional to moment rate spectrum $\dot{M}_0(f)$

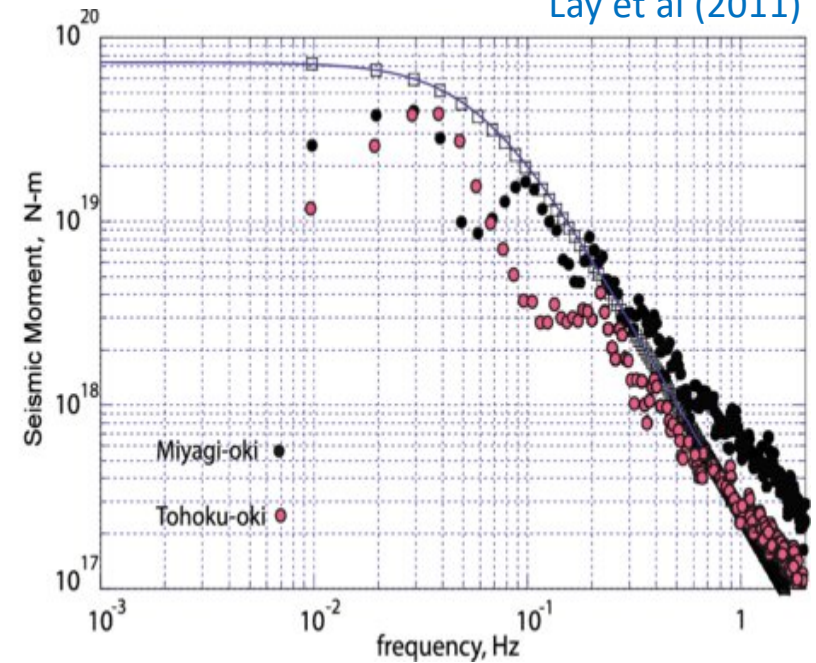
16 August 2005 Miyagi-oki
 $M_0 = 0.9 \times 10^{20}$ Nm ($M_w = 7.2$)
 Depth 36 km



9 March 2011 Tohoku-oki
 $M_0 = 1.9 \times 10^{20}$ Nm ($M_w = 7.5$)
 Depth 14 km

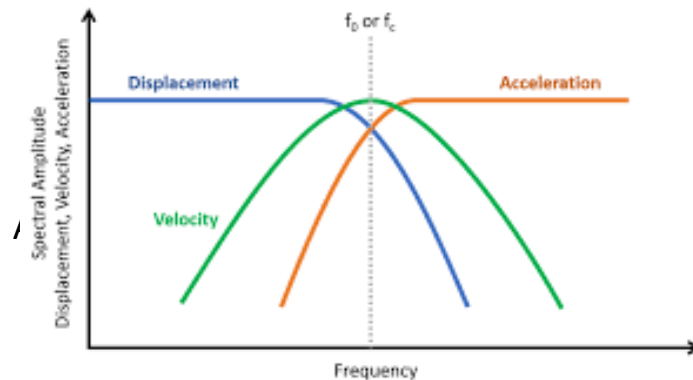


Lay et al (2011)



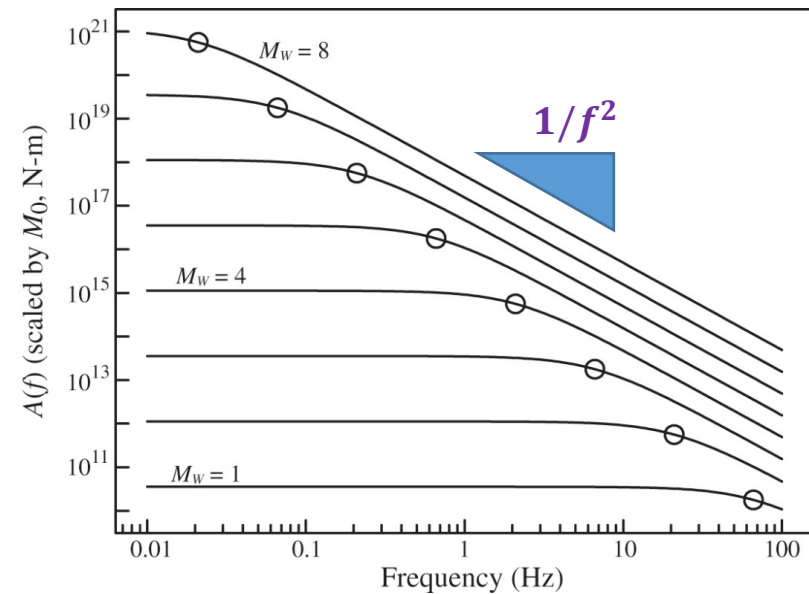
ω^2 model of earthquake spectrum

One corner frequency, f_c , that separates flat spectrum at low- f and $1/\omega^2$ at high- f :



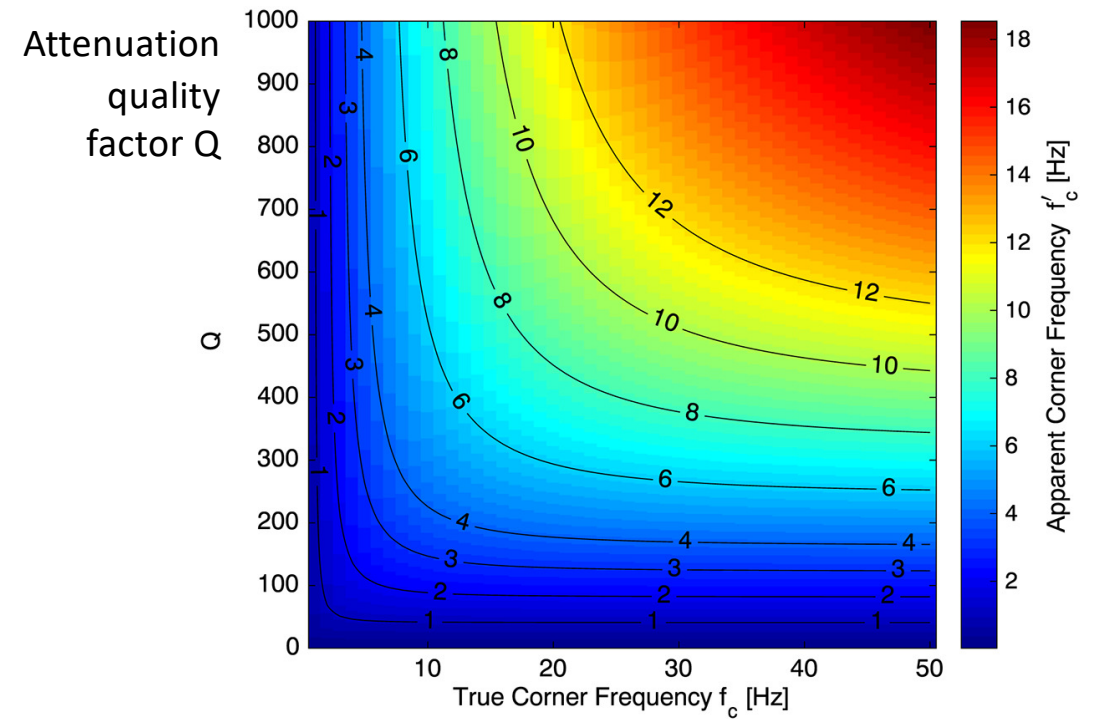
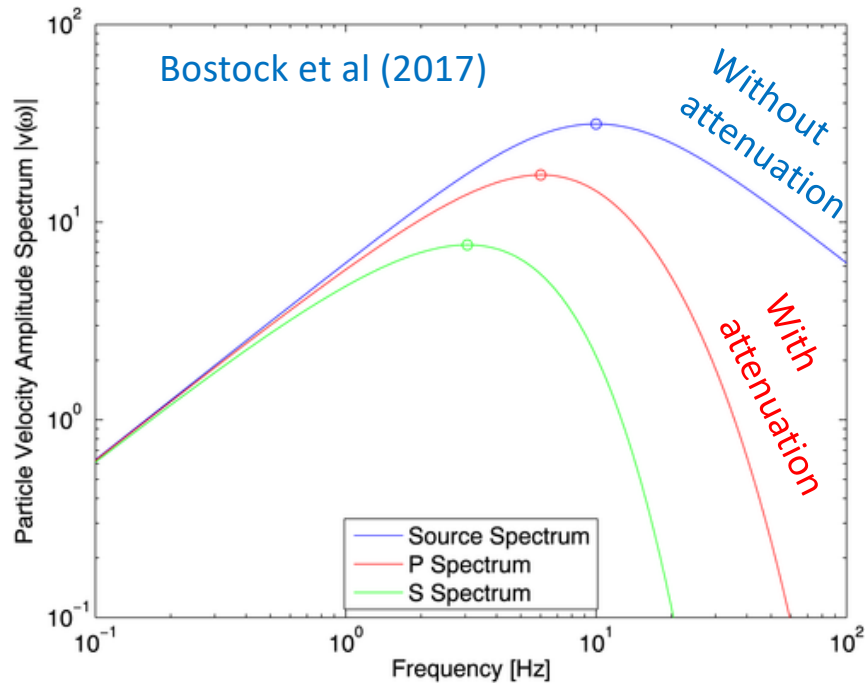
At high frequencies ($f \gg f_c$):

$$\dot{M}_0(f) \approx M_0 f_c^2 / f^2$$



$$d(f) \propto \dot{M}_0(f) \rightarrow a(f) \propto f^2 \dot{M}_0(f) \approx M_0 f_c^2$$

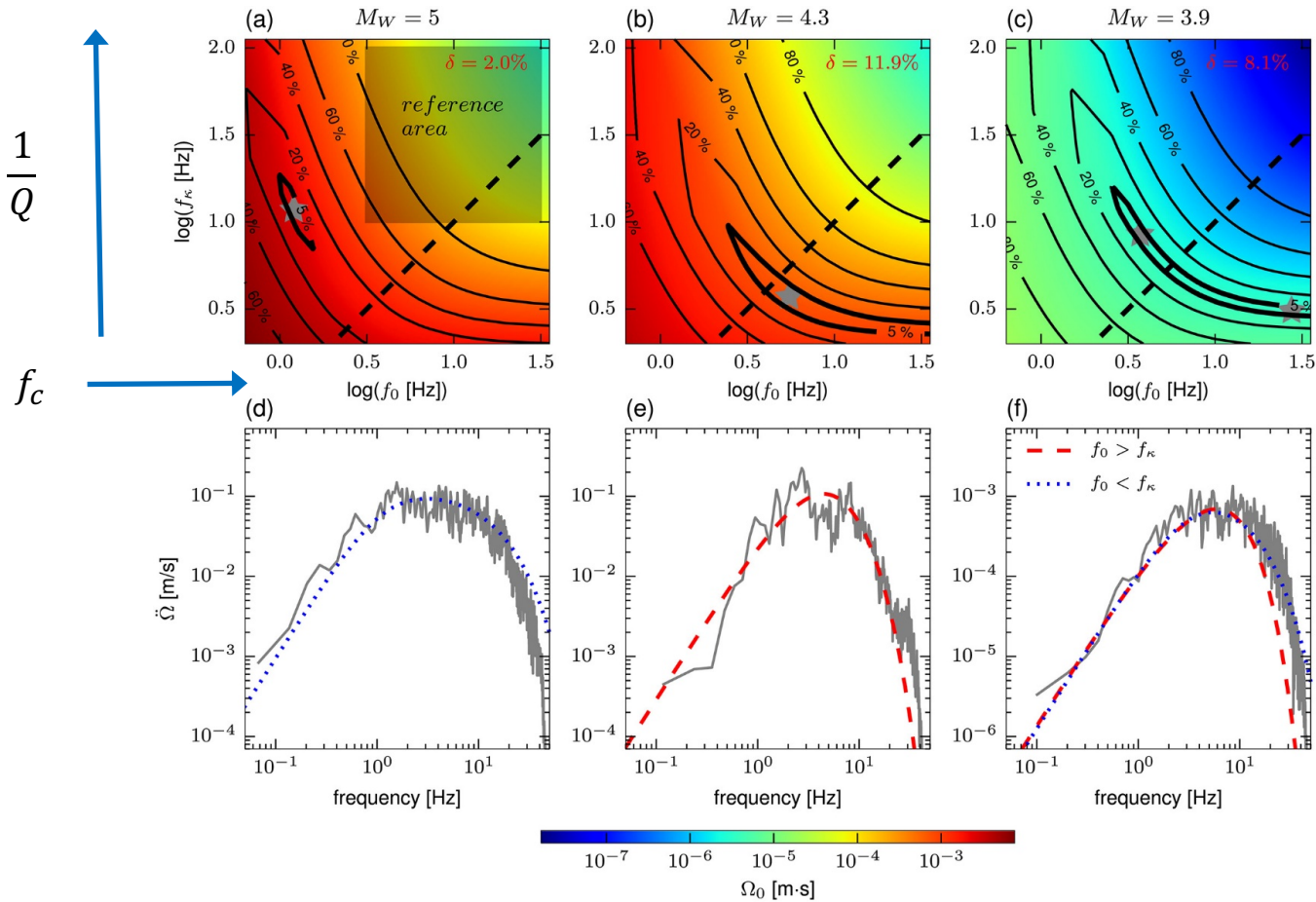
Attenuation vs corner frequency



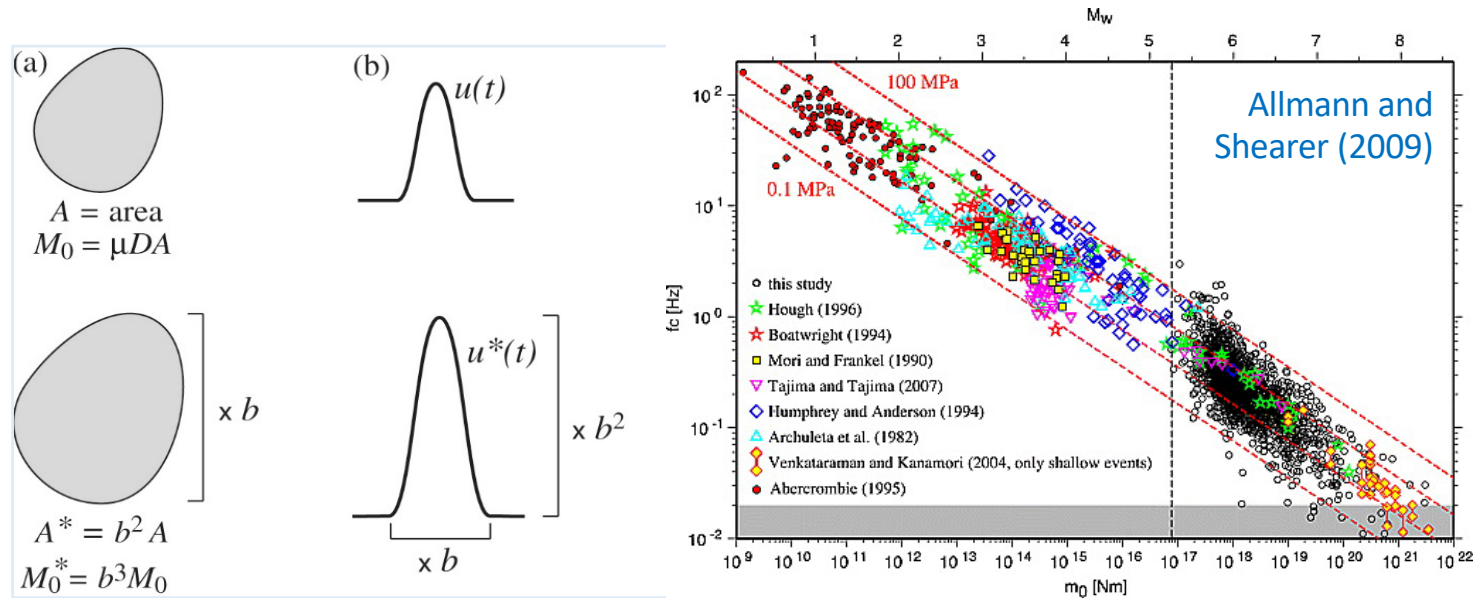
Attenuation (low Q) multiplies the spectrum by $\exp(-\pi f r / c Q)$
It reduces the apparent corner frequency

Trade-off between attenuation and corner frequency

$$f_{\kappa} = \frac{1}{\pi\kappa} \sim \frac{1}{Q}$$



Self-similar source model



Corner frequency $f_c \sim 1/(\text{source duration})$

$$\Rightarrow f_c \sim M_0^{-1/3}$$

Circular rupture model

Circular rupture, constant rupture speed, final radius R .

Only one characteristic time-scale: source duration

$$T \approx 2R/v_r$$

- corner frequency

$$f_c = \frac{kv_r}{R} \approx 1/T$$

where k is a factor of order 1 that depends mildly on rupture speed ($k = 0.44$ for $v_r = 0.9c_s$).

- relation between stress drop and slip from elasticity theory:

$$\Delta\sigma = \frac{7\pi}{16} \mu \frac{D}{R}$$

- definition of seismic moment $M_0 = \mu D \pi r^2$

→ Corner frequency scaling $f_c = kv_r \left(\frac{16 \Delta\sigma}{7 M_0} \right)^{1/3}$

→ estimate of stress drop: $\Delta\sigma = \frac{7}{16} \left(\frac{f_c}{kv_r} \right)^3 M_0$

Rectangular rupture model

Rupture length $L \gg$ width W .

- Rupture duration controlled by the longest rupture dimension: $f_c \sim 1/T$ with $T = L/v_r$.
- Elastic stiffness controlled by the shortest rupture dimension: $\Delta\tau \sim \frac{\mu}{W} D$
- Seismic moment: $M_0 = \mu D W L \sim \Delta\tau W^2 L$

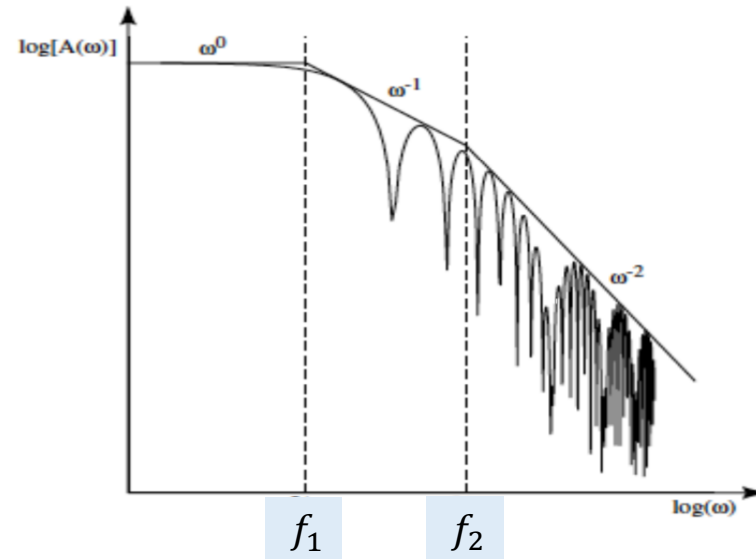
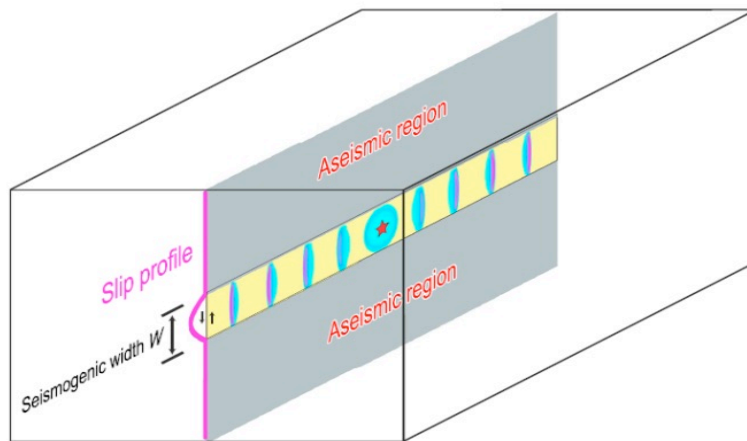
→ corner frequency scaling $f_c \sim \Delta\tau W^2 v_r \times M_0^{-1}$

→ estimate of stress drop $\Delta\tau \sim M_0 f_c / W^2 v_r$

A dependence of rupture aspect ratio on magnitude can break self-similarity and affect estimates of stress drop.

Spectra with two corner frequencies

Haskell's model, unilateral pulse-like rupture



Two time scales:

Total rupture duration $T_{rup} = L/v_r$

Local slip duration, rise time T_{ris}

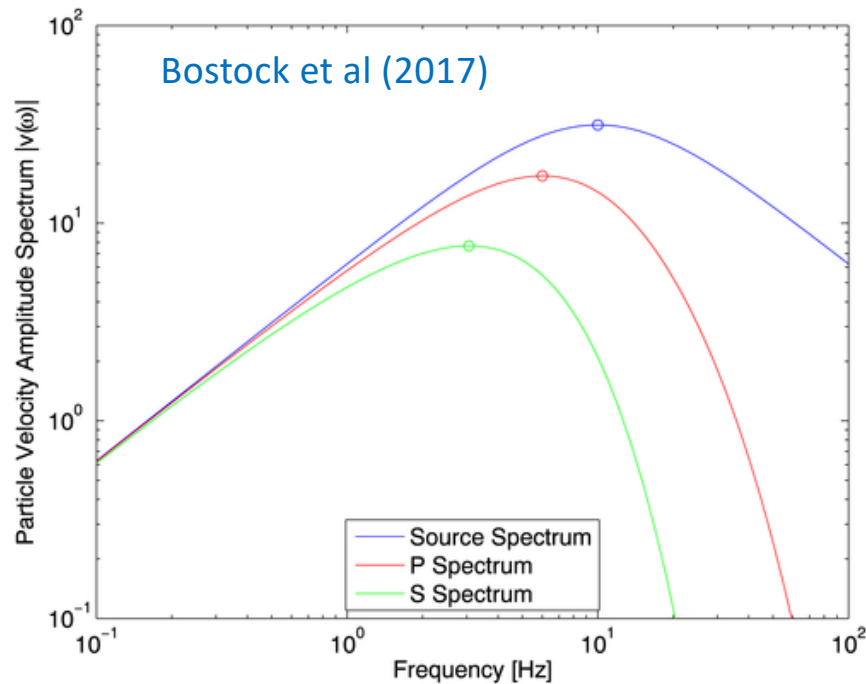
→ two corner frequencies:

$f_1 = 1/T_{rup}$

$f_2 = 1/T_{ris}$

Radiated energy

Radiated energy



Energy radiated to the far-field:

$$E \propto \int \dot{u}(t)^2 dt$$

$$E \propto \int \dot{u}(f)^2 df$$

... integrated over a far-field sphere

Observational challenges:

- attenuation
- incomplete coverage of take-off angles
- interference between depth phases

Radiated energy

Energy radiated to the far-field:

$$E \propto \int \dot{u}^2 dt \propto \dot{u}^2 T$$

Self-similar model: $\Delta\sigma$ and v_r do not depend on earthquake size R

$$M_0 \propto R^3$$

$$T \propto R$$

Far-field displacement and velocity:

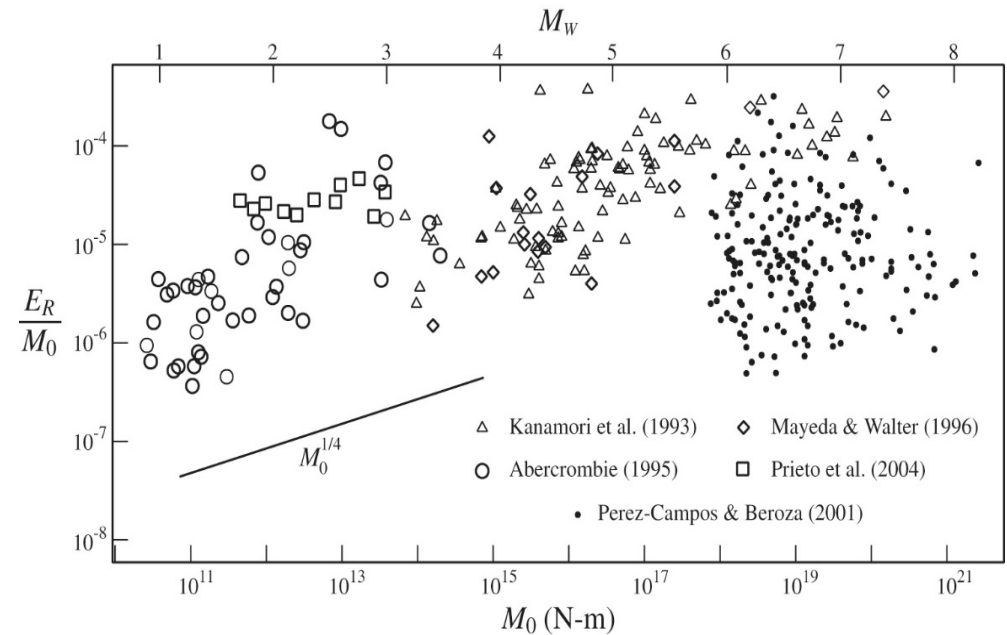
$$u \propto \dot{M}_0 \sim M_0/T \propto R^2 \propto M_0^{2/3}$$

$$\dot{u} \propto \ddot{M}_0 \sim M_0/T^2 \propto R \propto M_0^{1/3}$$

→ Energy radiated to the far-field:

$$E \propto \dot{u}^2 T \propto M_0$$

$$\log_{10} E = \log_{10} M_0 + f(\Delta\tau, v_r) + \dots$$



Earthquake energy balance

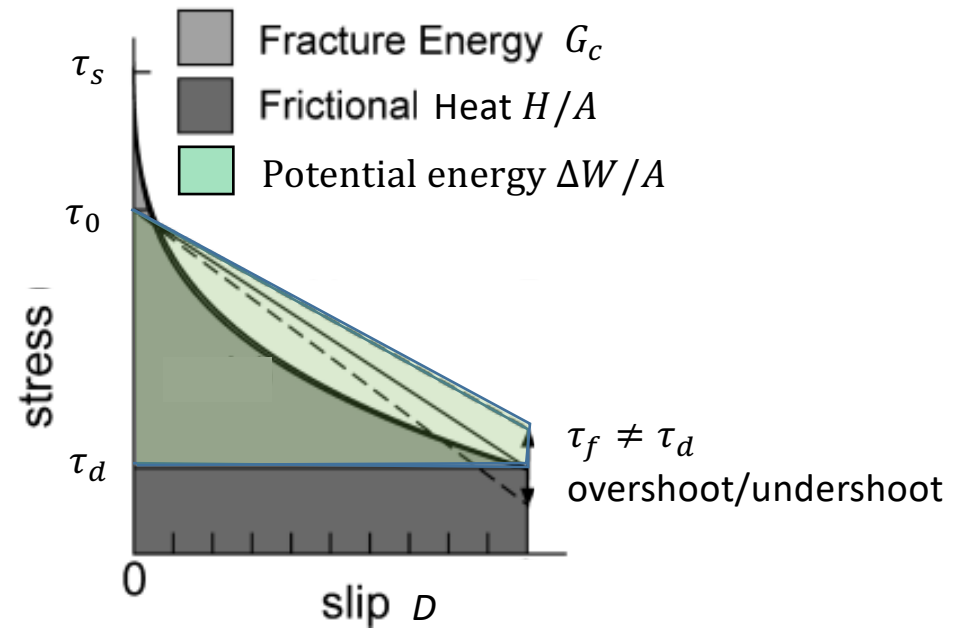
Potential energy change =
fracture energy + heat + radiated energy

$$\Delta W = G_c A + H + E_r$$

Per unit of fault surface:

$$\frac{1}{2}(\tau_0 + \tau_f)D = G_c + \tau_d D + E_r/A$$

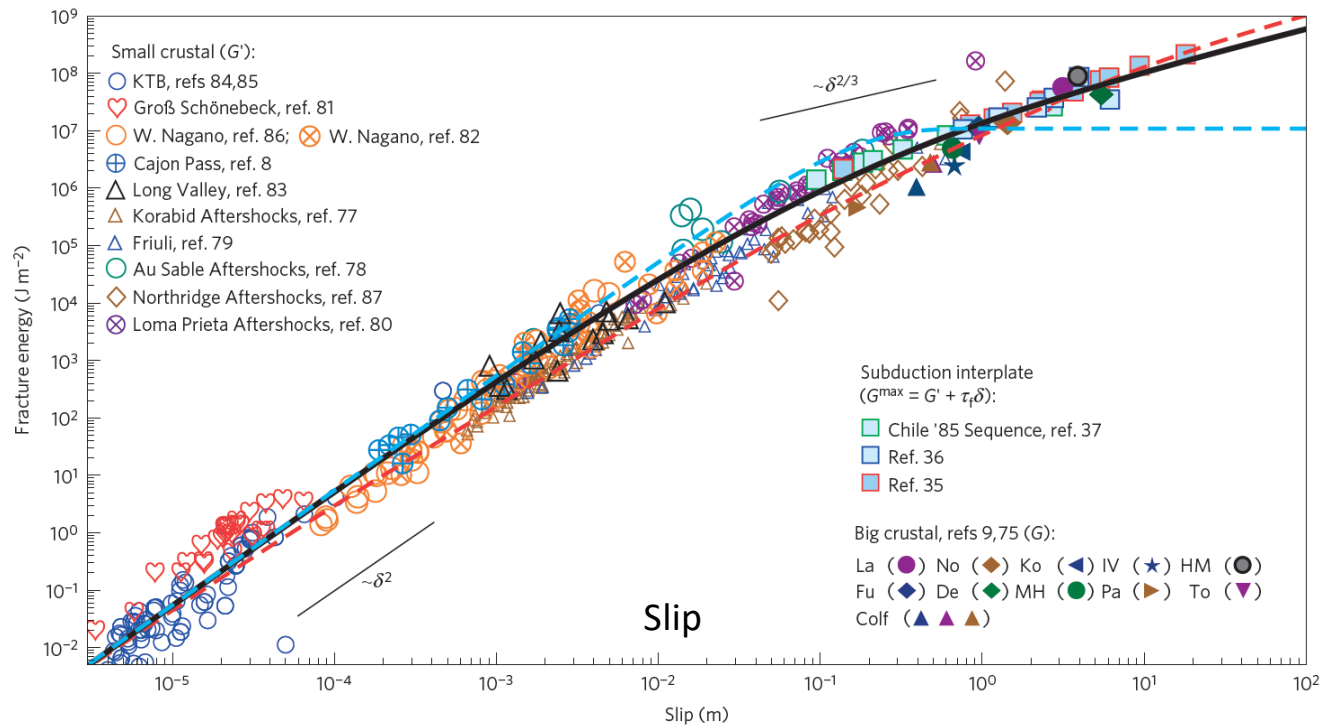
$$G_c = \frac{1}{2}(\tau_0 - \tau_f)D - \frac{E_r}{A} + (\tau_f - \tau_d)D$$



Seismological constraints

G_c inferred from radiated energy, seismic moment and corner frequency, or from finite source inversion

Fracture energy



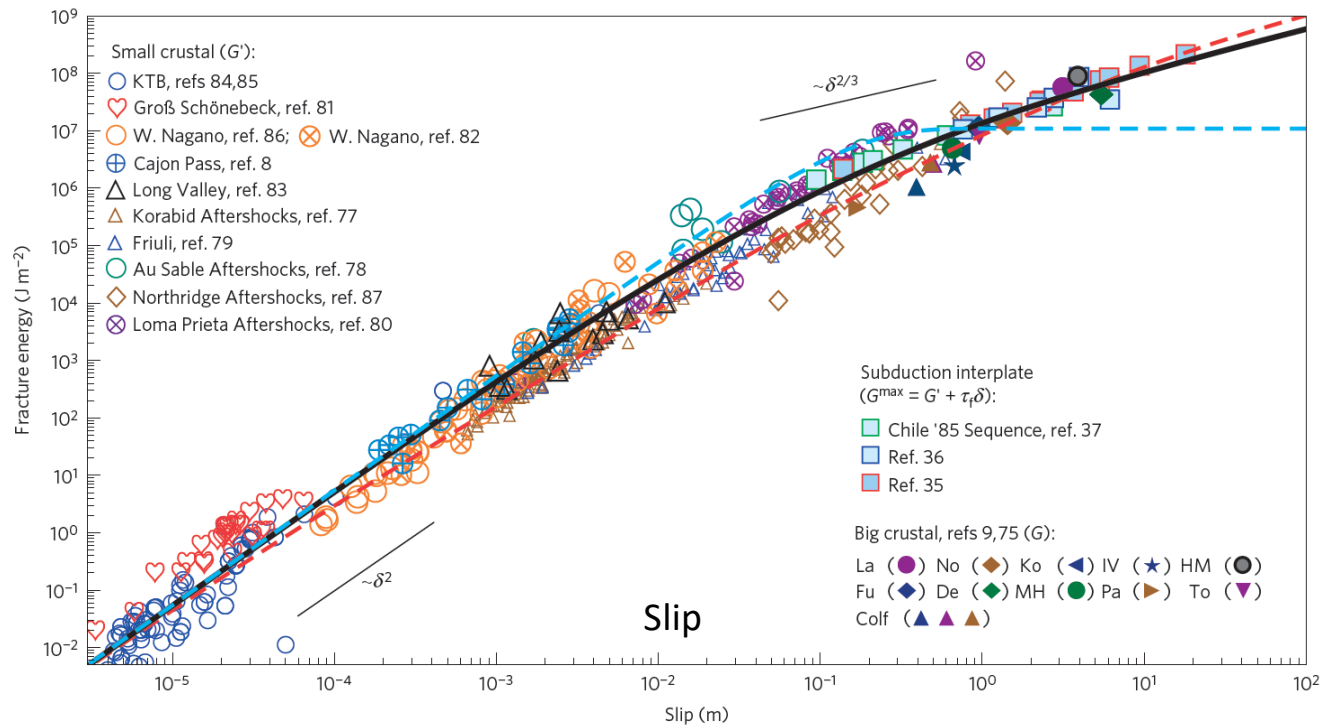
Viesca and Garagash (2015)

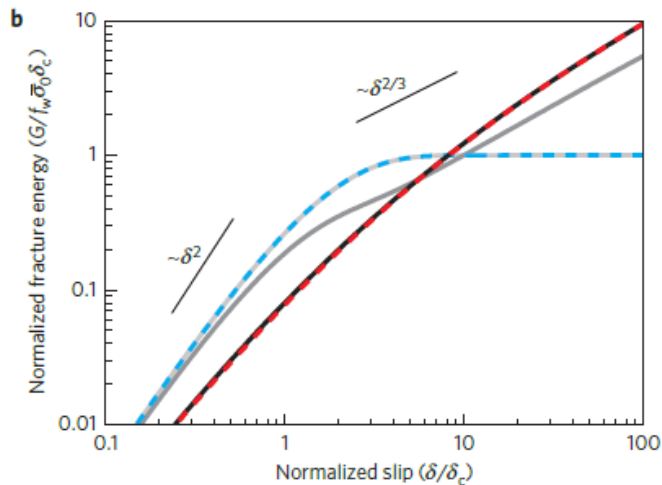
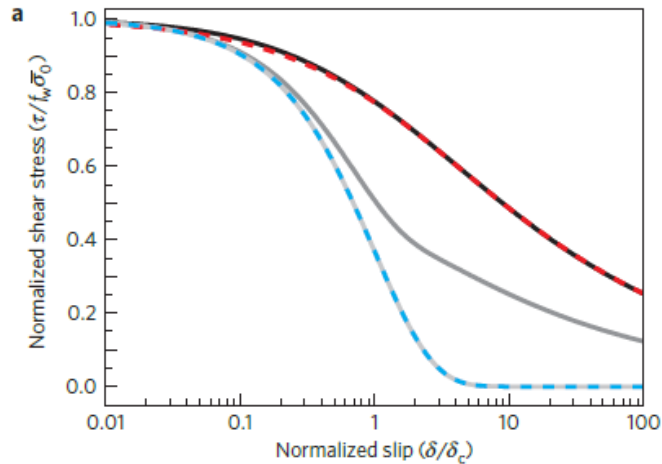
Seismological constraints

G_c inferred from radiated energy, seismic moment and corner frequency, or from finite source inversion

Consistent with **thermal pressurization weakening** $\tau = \mu(\sigma - P)$

Fracture energy





Velocity-and-slip-dependent friction law:

$$\mu(V, \delta) \sim \mu_{FH} \left(\frac{V}{V_c} \right) \mu_{UA} \left(\frac{\delta}{\delta_c} \right) \mu_{HD} \left(\frac{\delta}{\delta_d} \right)$$

Flash heating:

$$\mu_{FH}(x) = \mu_d + \frac{(\mu_s - \mu_d)}{1 + x}$$

Undrained-adiabatic thermal pressurization:

$$\mu_{UA}(x) = \exp(-x)$$

Diffusion-dominated thermal pressurization:

$$\mu_{HD}(x) = 1/(1 + x^{1/3})$$

Overview

Macroscopic source properties constrained by seismology:

- seismic moment
- source time function
- corner frequency
- radiated energy

→ stress drop, rupture speed, rupture size