Advanced Workshop on Earthquake Fault Mechanics: Theory, Simulation and Observations

ICTP, Trieste, Sept 2-14 2019

Lecture 9: earthquake cycle modeling

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Earthquake cycle modeling: definition

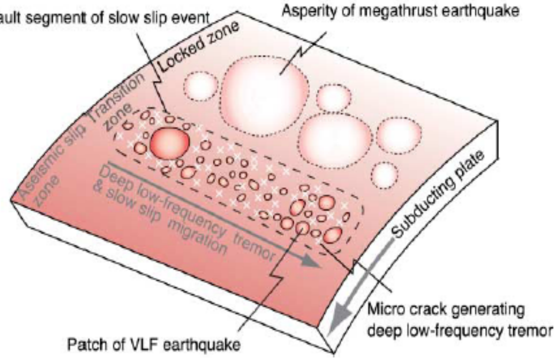
Scope:

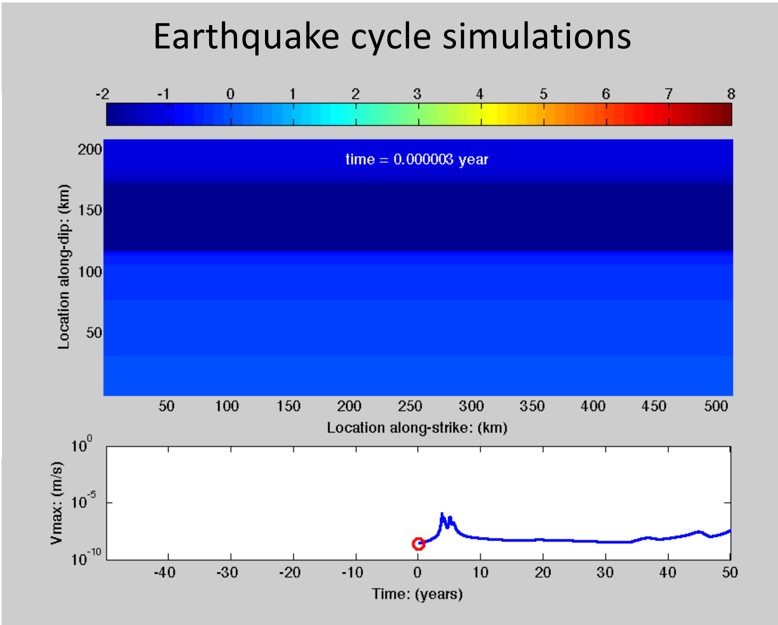
Fault slip and deformation processes at time scales spanning several major earthquakes on a given fault zone

Multi-scale modeling: includes seismic and aseismic processes (earthquake rupture, aftershocks, postseismic slip, background seismicity, interseismic loading, foreshocks, nucleation)

+ other aseismic fault processes: slow slip events

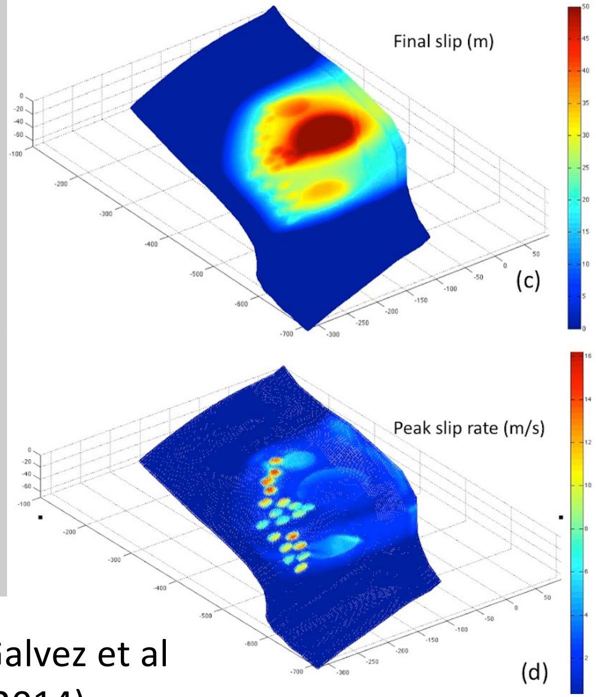
Earthquake cycle modeling





Luo and Ampuero

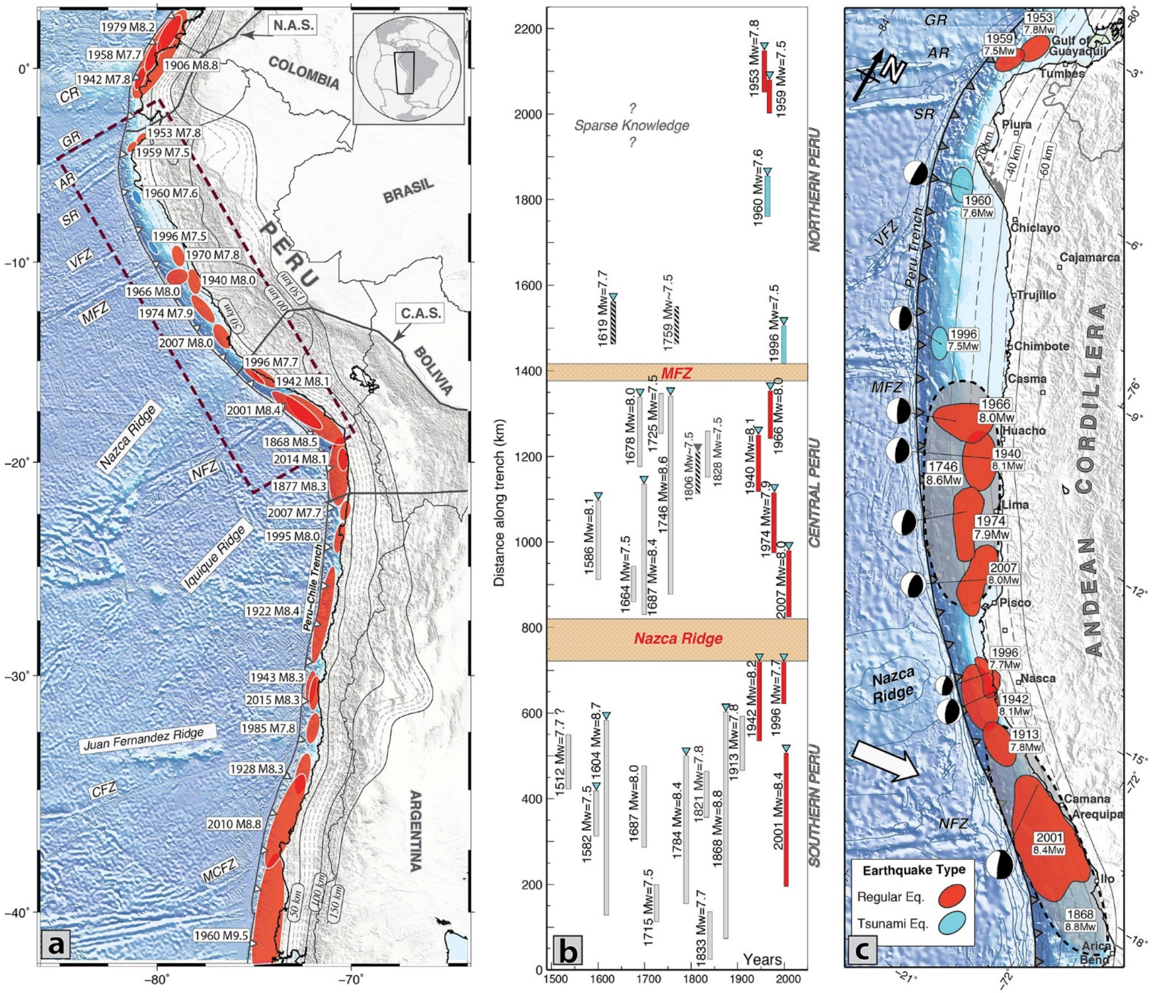
Earthquake Dynamic rupture simulation of The 2011 Tohoku earthquake.



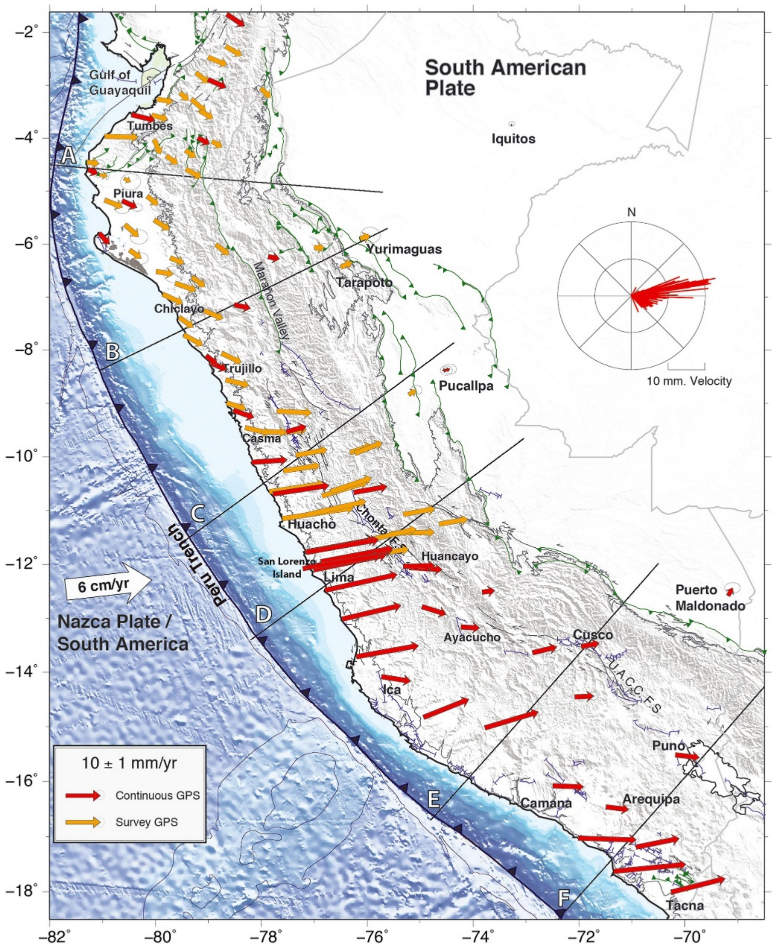
Galvez et al (2014)

Historical seismicity in the Peru subduction zone

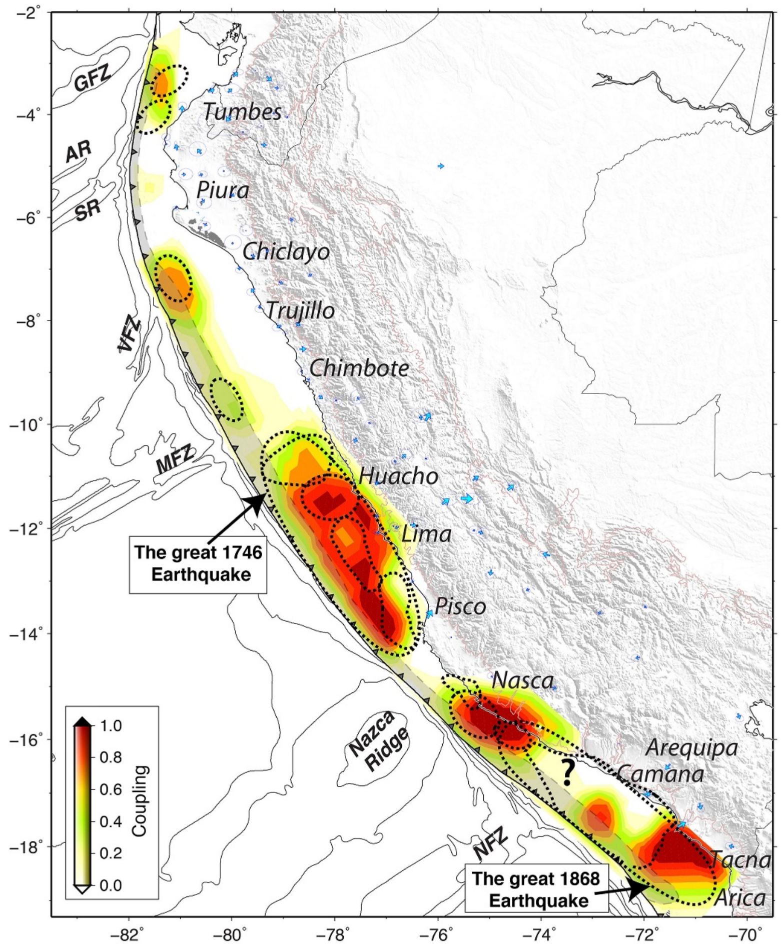
(Villegas-Lanza et al 2016)



Geodetic data (GPS)



Inferred seismic coupling



Villegas-Lanza et al (2016)

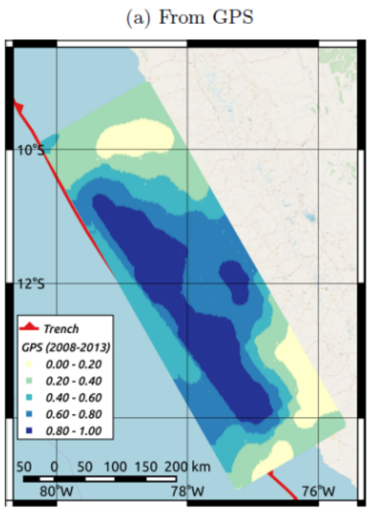
Why model the earthquake cycle?

- To study the earthquake cycle:
  - Interpret geodetic observations in the framework of current friction laws
  - Infer friction properties from geodetic observations
  - Develop implications of new friction laws
  - Add physics-based constraints on seismic hazard assessment

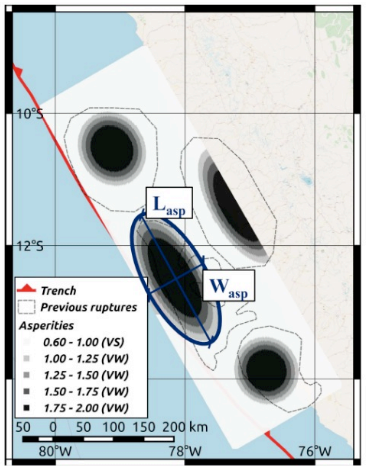
Why model the earthquake cycle?

Example: infer friction properties from geodetic observations (Ceferino et al 2019)

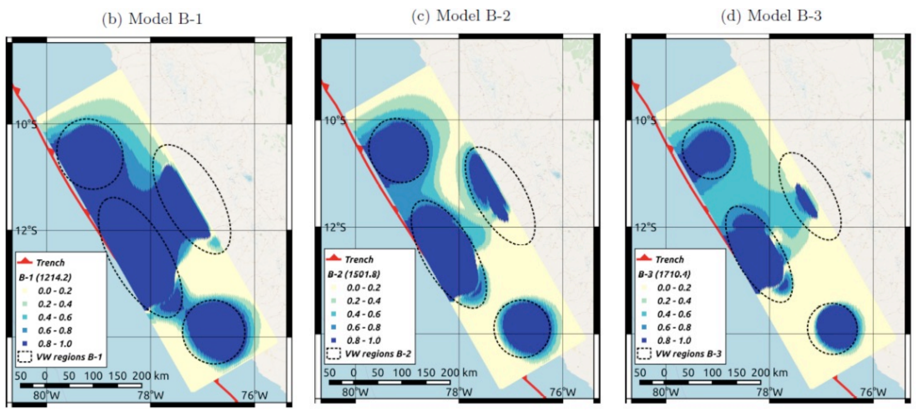
Observational constraint: seismic coupling map inferred from GPS data



Rate-and-state earthquake cycle modeling



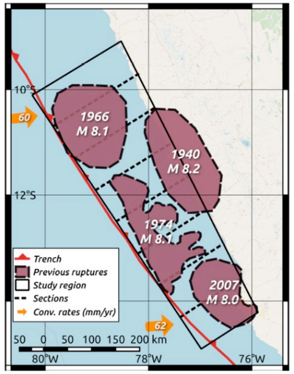
Tuning friction parameters → Family of plausible models



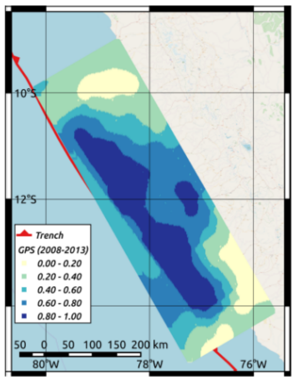
Why model the earthquake cycle?

Example: Add physics-based constraints on seismic hazard assessment (Ceferino et al 2019)

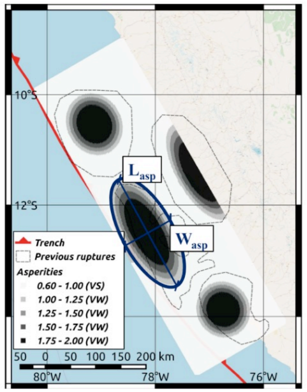
Observations



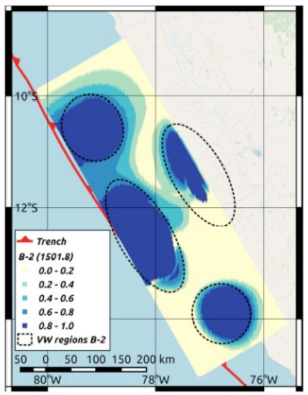
(a) From GPS



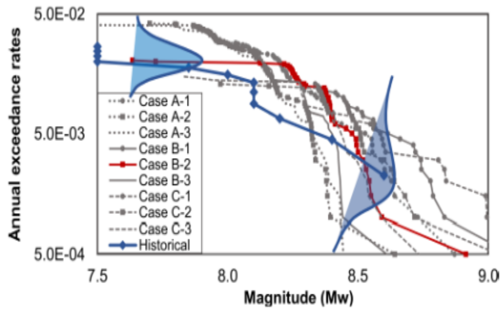
Models constrained by observations



(c) Model B-2

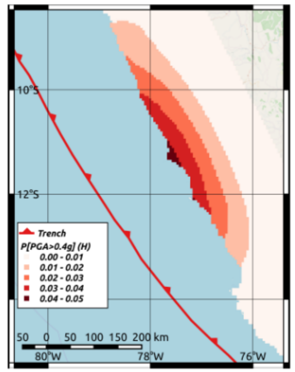


Model statistics

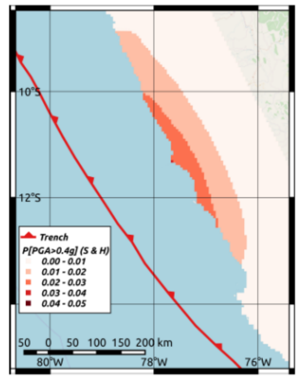


Effect on hazard map

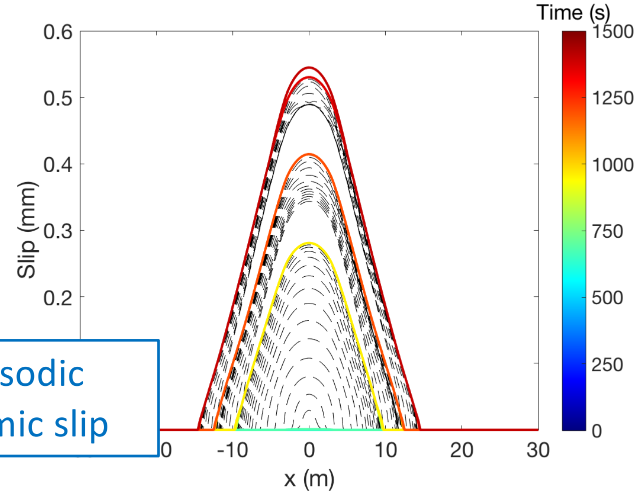
(a) Only historical Data



(b) Synthetic & historical Data

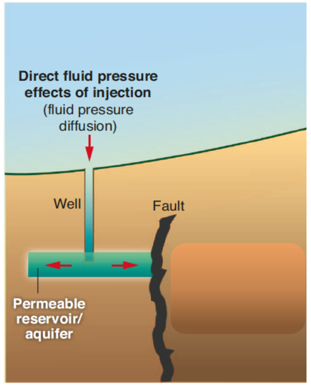


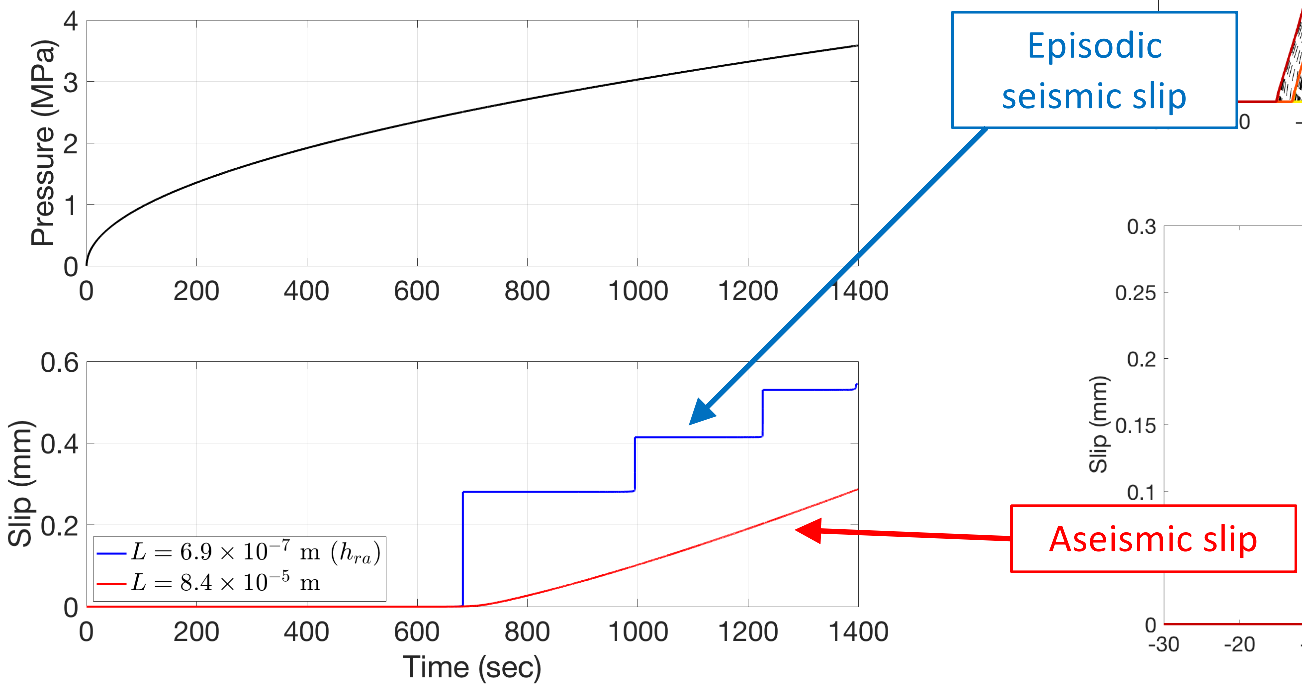
Why model aseismic slip?

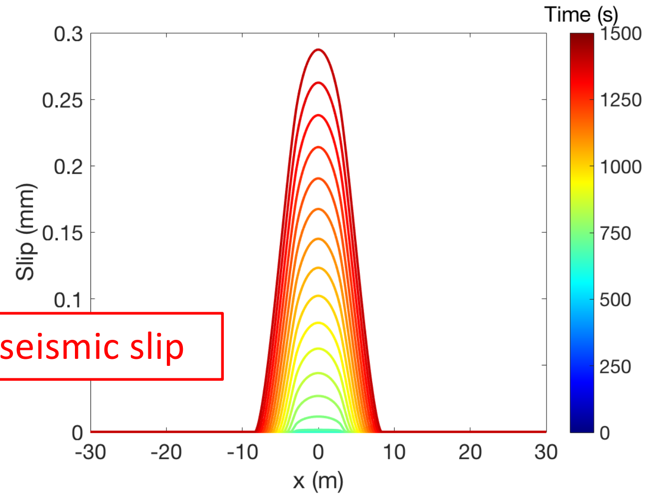


Fault slip induced by fluid injection

Rate-and-state friction + pressure diffusion modeling
(Larochelle et al 2019 in prep.)



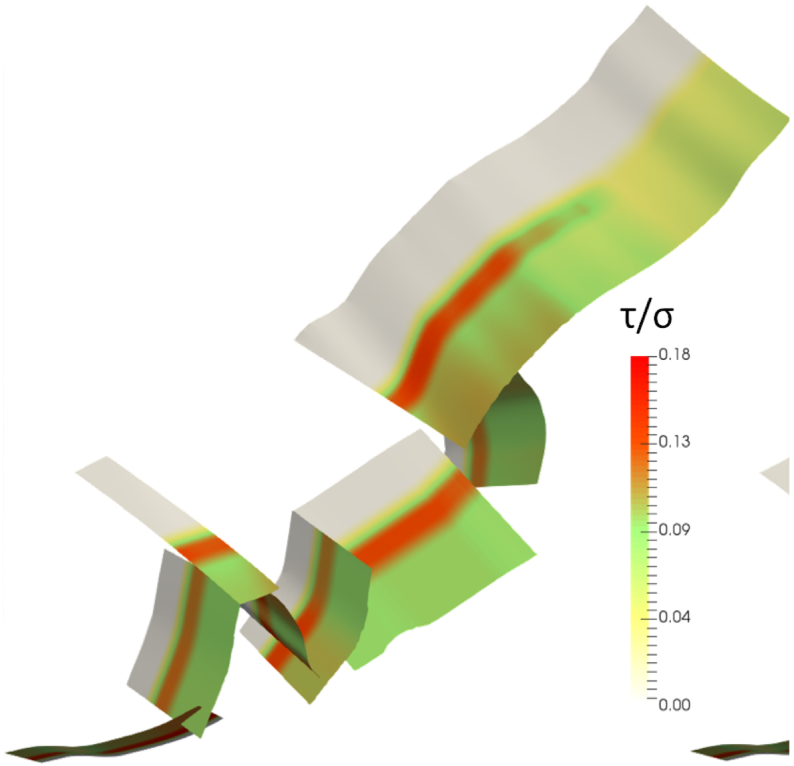


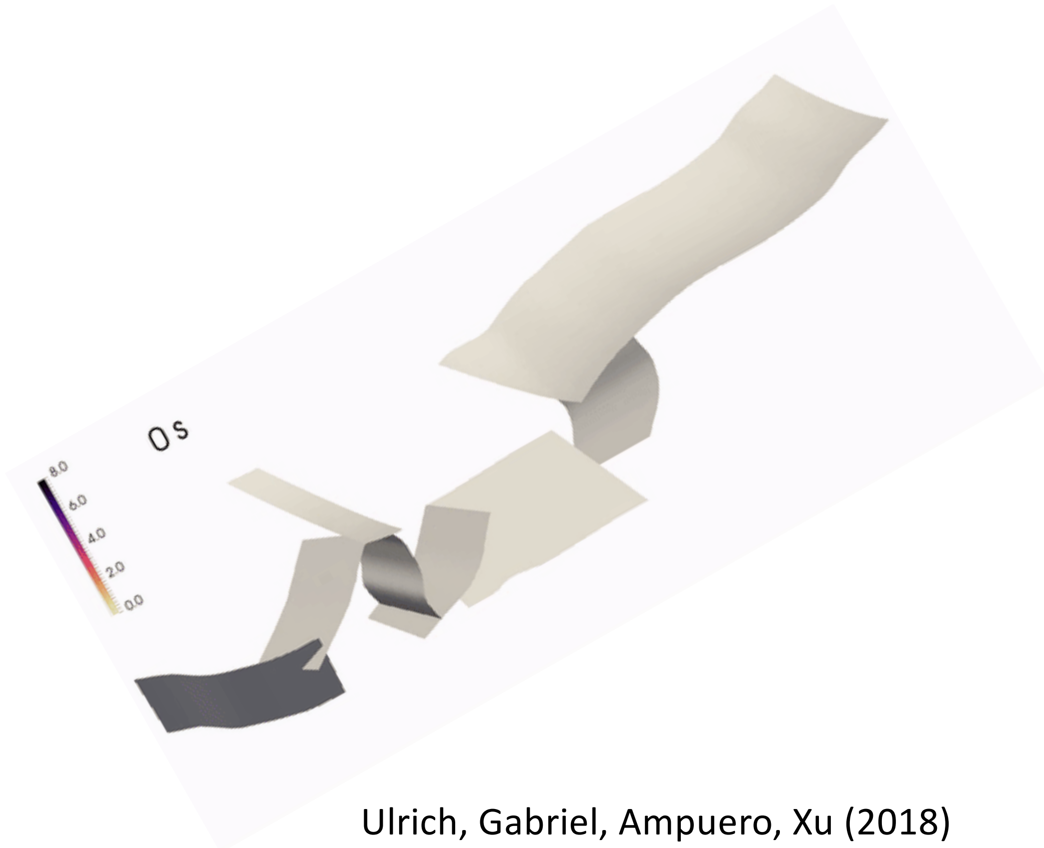


Why model the earthquake cycle?

- To get “initial stresses” for earthquake simulations that are mechanically consistent with long-term processes
  - Dynamic rupture simulations of single earthquakes (previous lectures) assume initial stresses arbitrarily
  - Earthquake cycle models provide stresses organized spontaneously throughout the long-term activity of the fault (multiple earthquakes)

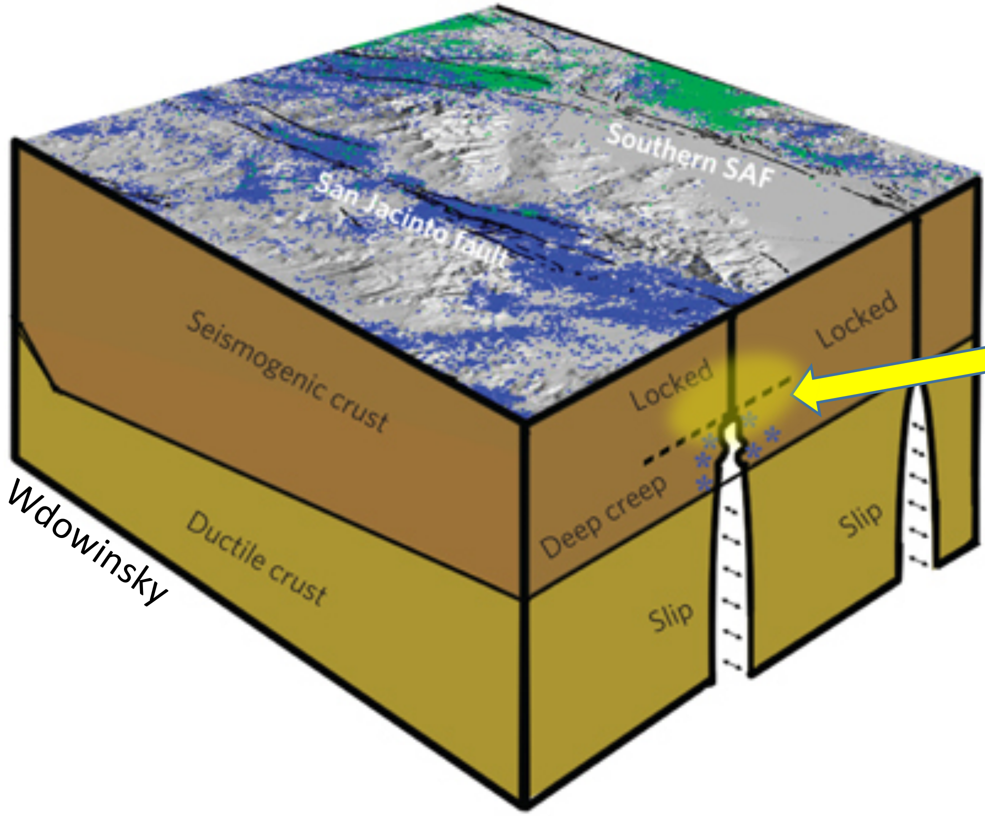
Dynamic model of the 2016 Mw 7.8 Kaikoura earthquake
A rupture cascade on weak faults





Ulrich, Gabriel, Ampuero, Xu (2018)

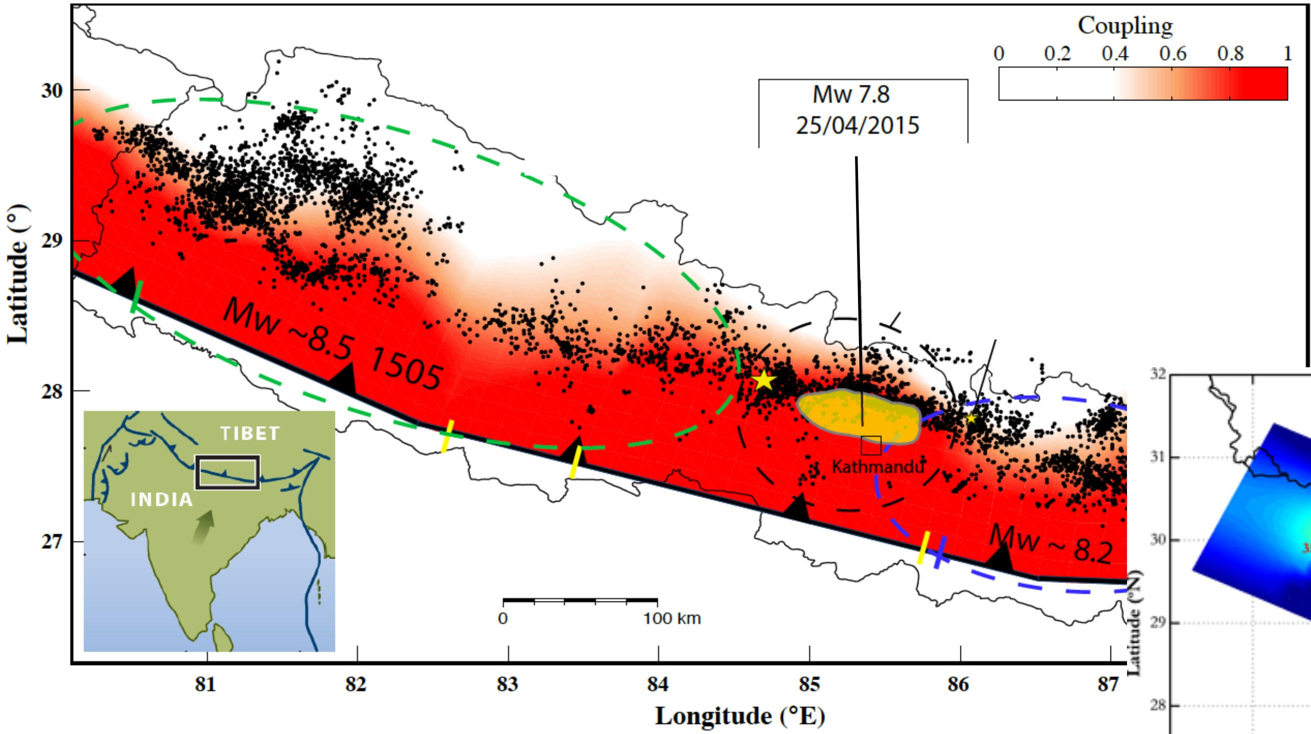
Loading of natural faults



Fault loaded by deep creep

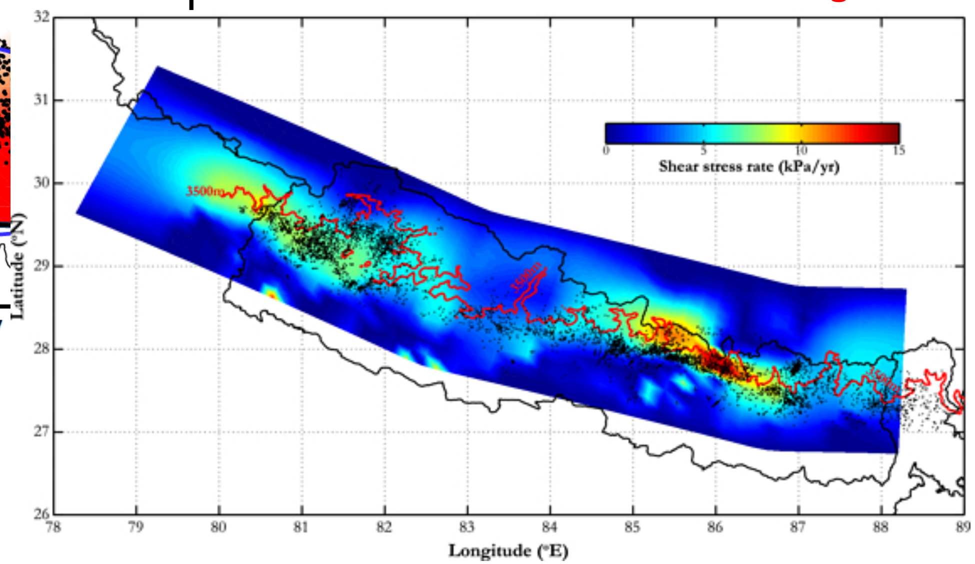
Stress concentration

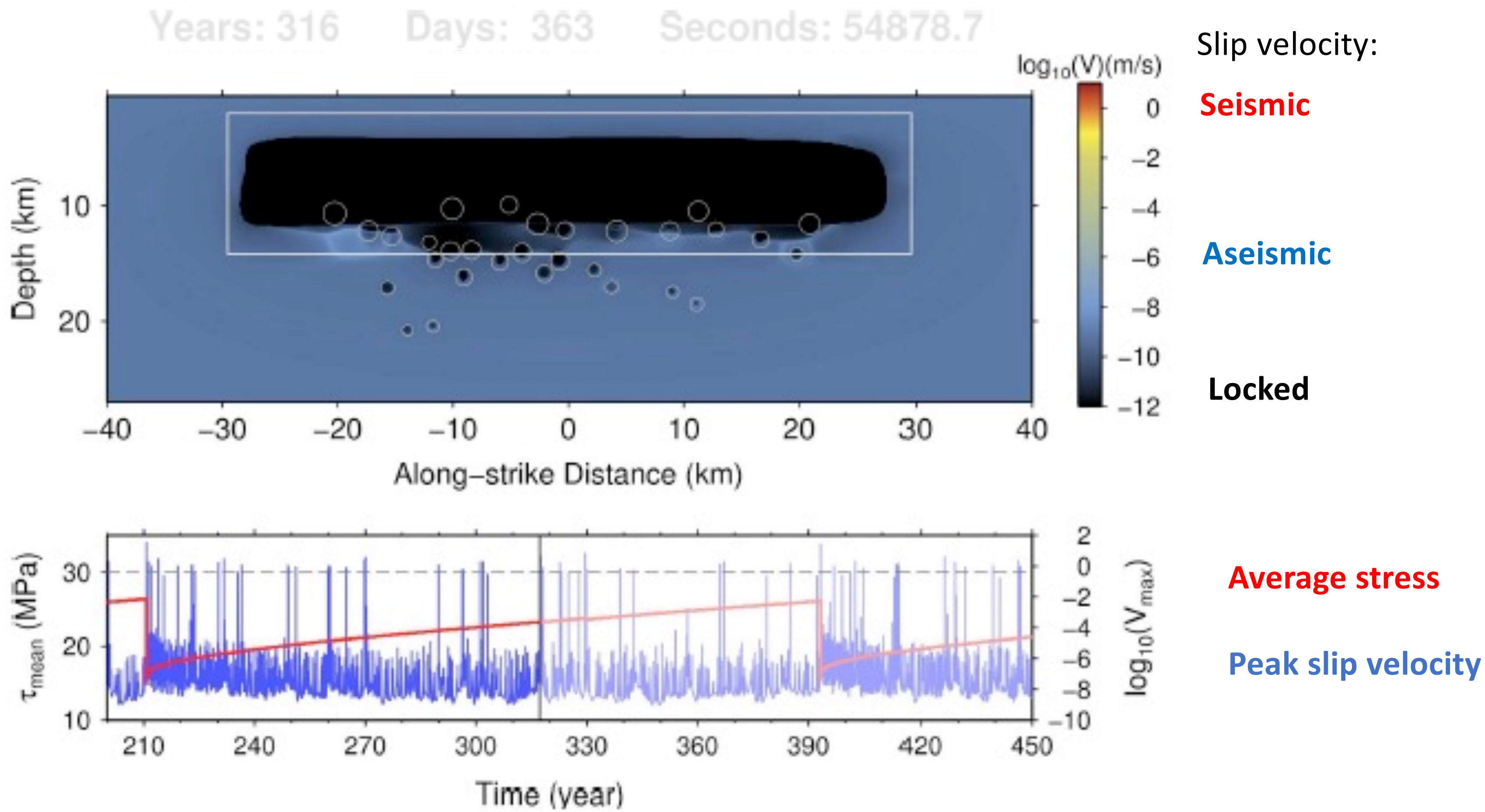
2015, Mw 7.8 Gorkha, Nepal earthquake



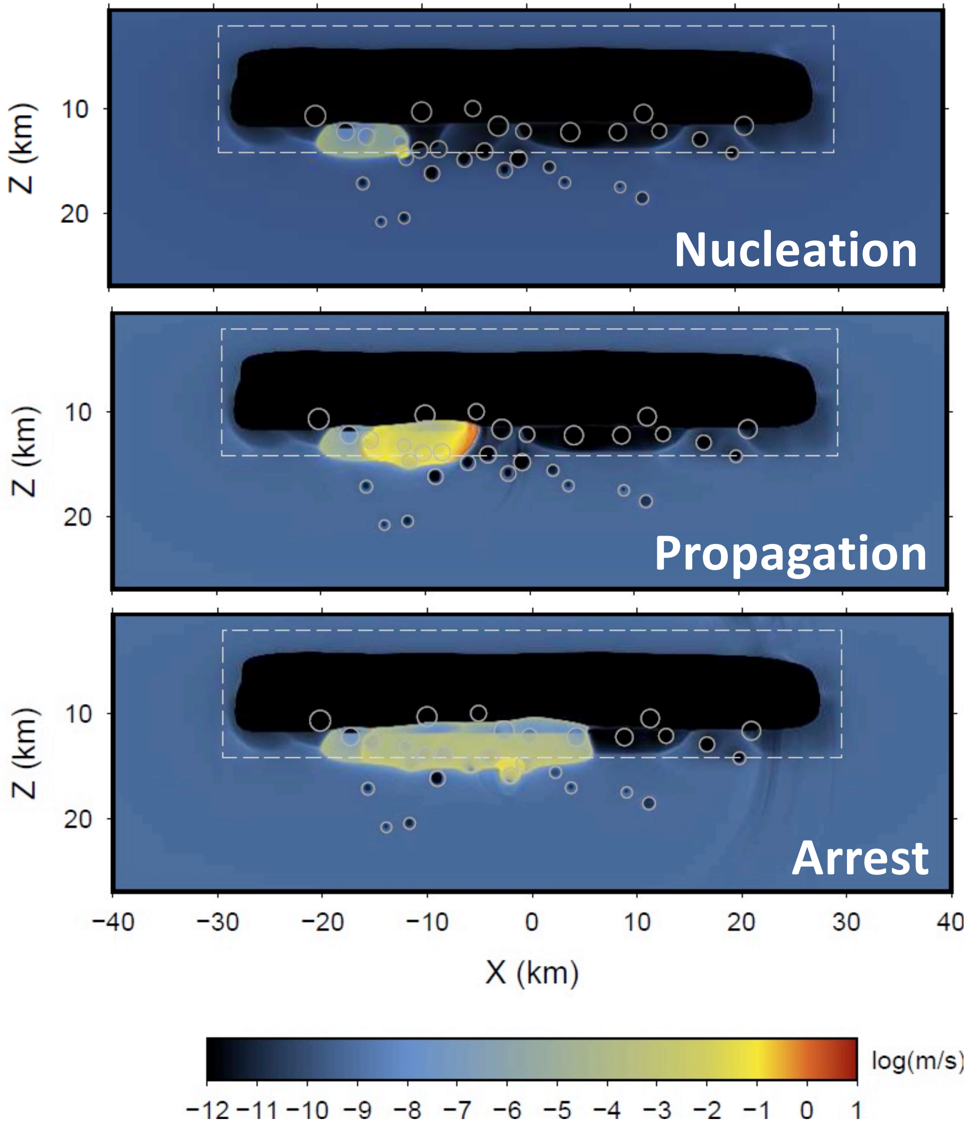
Seismic coupling
inferred from 20 years of GPS data
Ader et al (2012)

Stressing rate



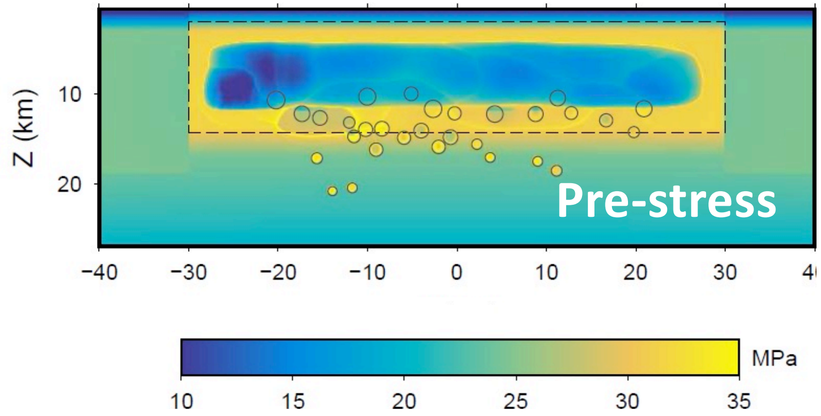


Extracted from Jiang and Lapusta's dynamic earthquake cycle simulations.



Intermediate-size event unzipping part of the lower edge of the coupled zone

(Junle Jiang, Caltech)



Basic earthquake cycle problem

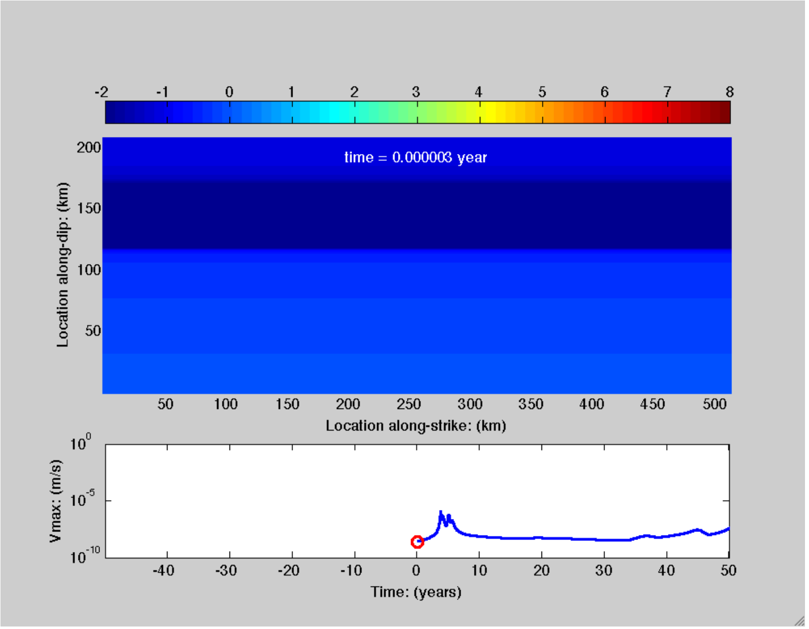
- Ingredients:
  - Fault embedded in an elastic crust
  - Fault zone is thin: slip on a pre-existing surface
  - Fault geometry is prescribed and fixed
  - The relation between fault stress and slip is governed by a friction law
  - Initial state
  - Tectonic loading (remote or creep) + other transient loading
- Mathematical formulation:
  - Linear elasticity equations
  - Non-linear boundary conditions (friction)
  - Initial conditions
- Outputs:
  - Spatio-temporal evolution of slip (on each fault point, at each time) over time scales that span several earthquake cycles
  - Seismicity patterns
  - Surface deformation

Example questions addressed by earthquake multi-cycle modeling

- Earthquake nucleation:
  - How much precursory aseismic slip is expected?
  - Where do earthquakes tend to nucleate?
  - How does a fault respond to external stimuli (tides, waves, fluids)?
- Earthquake rupture:
  - Is this fault seismic or aseismic?
  - How fast does a slow slip event migrate?
  - How does slip and rupture duration scale with earthquake size?
  - How to start a single-earthquake dynamic rupture simulation?
- Seismicity patterns:
  - How does seismicity organize in a fault network?
  - How do tremors migrate?
  - How are foreshocks related to aseismic slip?

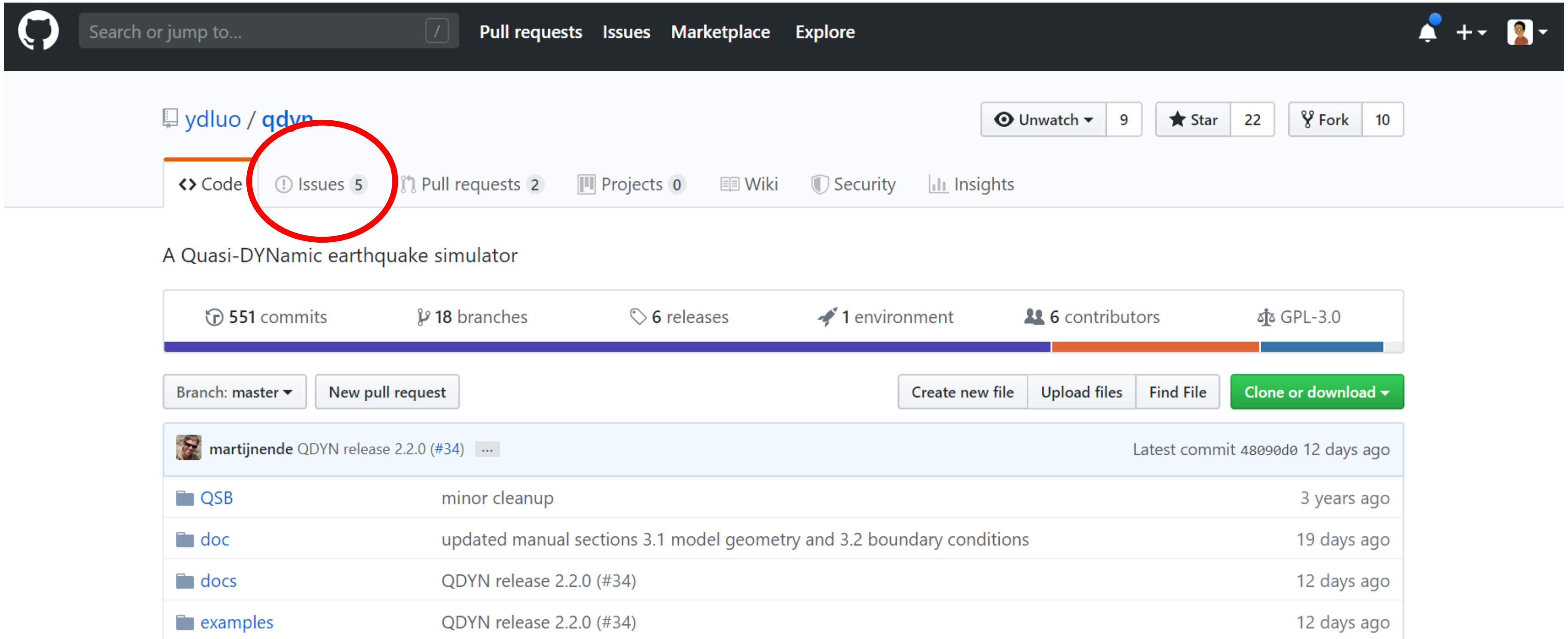
Quasi-**DYN**amic earthquake simulator

https://github.com/ydluo/qdyn



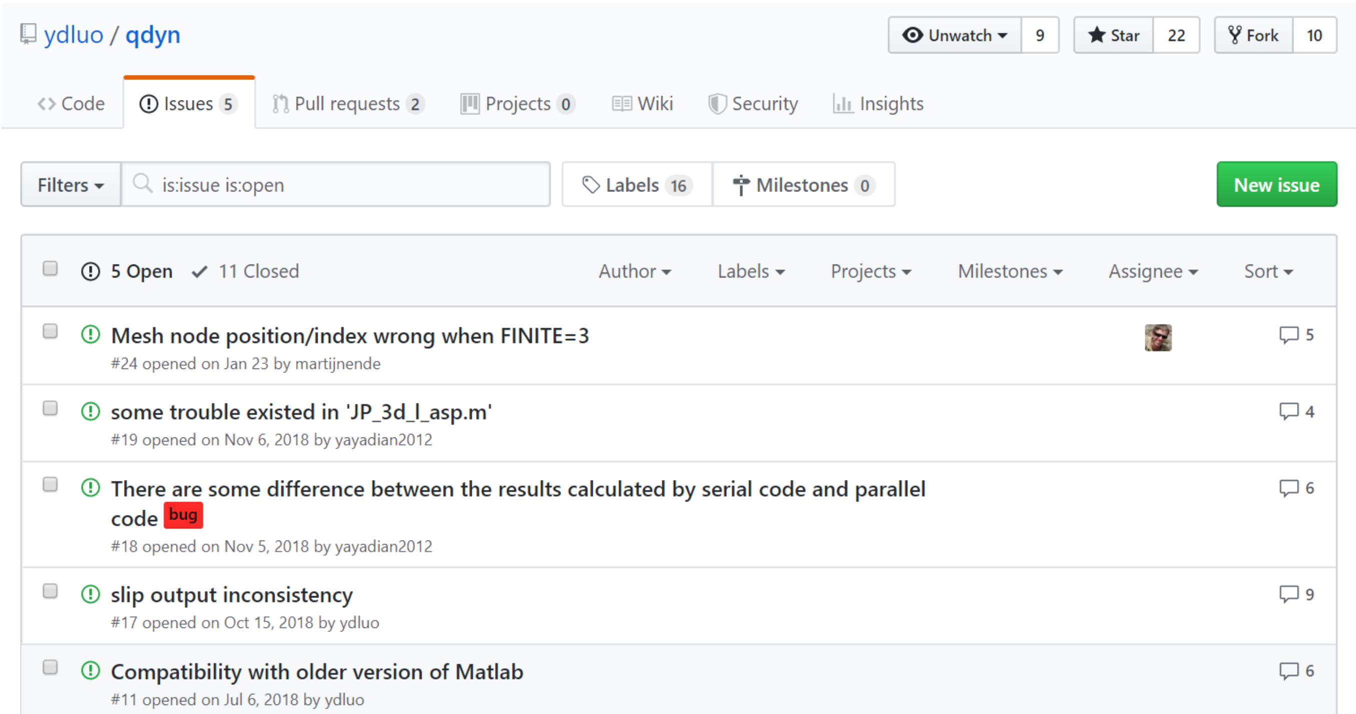
QDYN is an open-source software for earthquake cycle modeling

Hosted in Github https://github.com/ydluo/qdyn



We welcome your feedback!

User support: post an “issue” on Github, the team will address it



QDYN Documentation

Overview

QDYN is a boundary element software to simulate earthquake cycles (seismic and aseismic slip on tectonic faults) under the quasi-dynamic approximation (quasi-static elasticity combined with radiation damping) on faults governed by rate-and-state friction and embedded in elastic media.

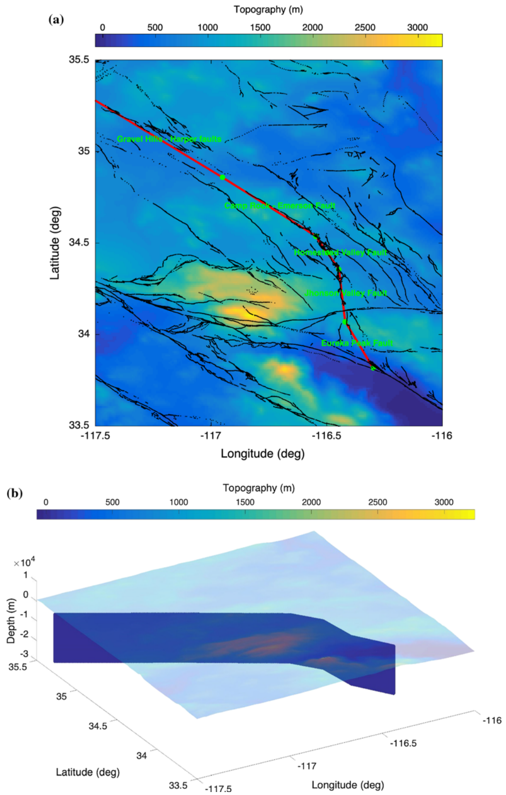
QDYN includes various forms of rate-and-state friction and state evolution laws, and handles non-planar fault geometry in 3D and 2D media, as well as spring-block simulations. Loading is controlled by remote displacement, steady creep or oscillatory load. In 3D it handles free surface effects in a half-space, including normal

Model assumptions: rheology of the crust

- Linear elastic half-space
- Uniform elastic properties or a low rigidity layer around the fault
- Thermal/fluid diffusion within the fault zone
- Missing: heterogeneous media, viscosity, plasticity/damage

Model assumptions: fault geometry

- Slip on pre-existing surfaces: inelastic deformation localized in infinitely thin fault planes
- Currently in QDYN: single fault with prescribed depth-dependent dip, fixed rake
- Future version: arbitrary fault geometry (non-planar faults, network of multiple faults)



Galvez et al (PAGEOPH 2019)

Model assumptions: Quasi-dynamic approximation

- Fault embedded in an elastic crust

  → linear elastodynamics equations (F=ma & Hooke's law)

- Quasi-dynamic approximation: includes only dynamic stress changes due to waves radiated in the direction normal to the fault plane ("radiation damping", Rice 1993)

$$\Delta\tau = -\frac{\mu}{2c_s}V$$

- Convenient: lower computational cost and program complexity

  → simulation of multiple earthquake cycles with many fault cells

- Generally adequate approximation.
  - Quantitative differences: smaller stress drop, rupture speed and slip velocity than fully dynamic simulations.
  - Qualitative differences if friction has severe velocity-weakening (Thomas et al 2014)

Radiation damping: derivation

A plane S wave with particle displacement

$$u(t - x/c_s)$$

propagating in the direction x normal to a fault plane carries the following dynamic shear stress change (Hooke's law + chain rule):

$$\tau = \mu \frac{\partial u}{\partial x} = -\frac{\mu}{c_s} \frac{\partial u}{\partial t}$$

Next to the fault, displacement = half slip :

$$\frac{\partial u}{\partial t} = \frac{V}{2}$$

Hence,

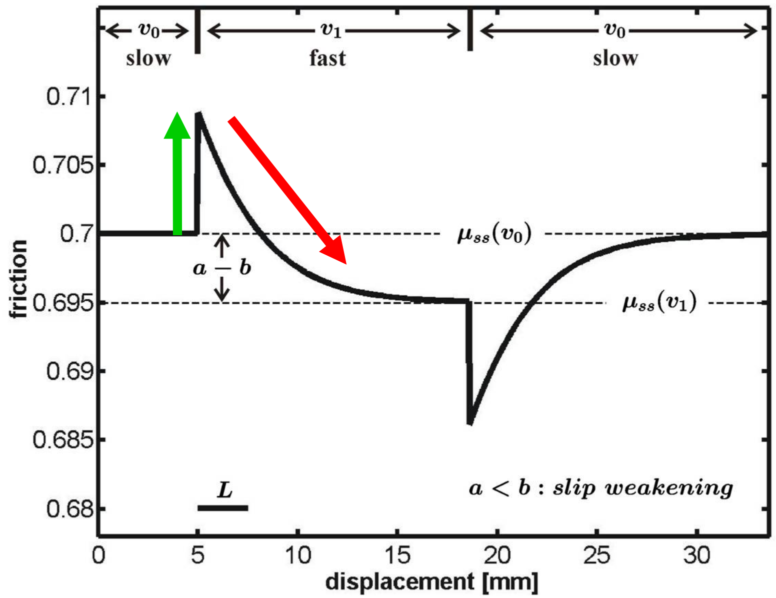
$$\boldsymbol{\Delta\tau = -\frac{\mu}{2c_s} V}$$

More generally, for SH waves radiated at an angle θ from the fault normal:

$$\Delta\tau = -\frac{\mu}{2c_s} V \cos(\theta)$$

Model assumptions: rate-and-state friction

Evolution of friction coefficient
during velocity step experiment



Phenomenological friction law developed from lab experiments at low velocity

$$\frac{\tau}{\sigma} = f(V,\theta) = f^* + a\log\left(\frac{V}{V^*}\right) + b\log\left(\frac{V^*\theta}{L}\right)$$

non-linear viscosity + **evolution effect**

State evolution law, several flavors:

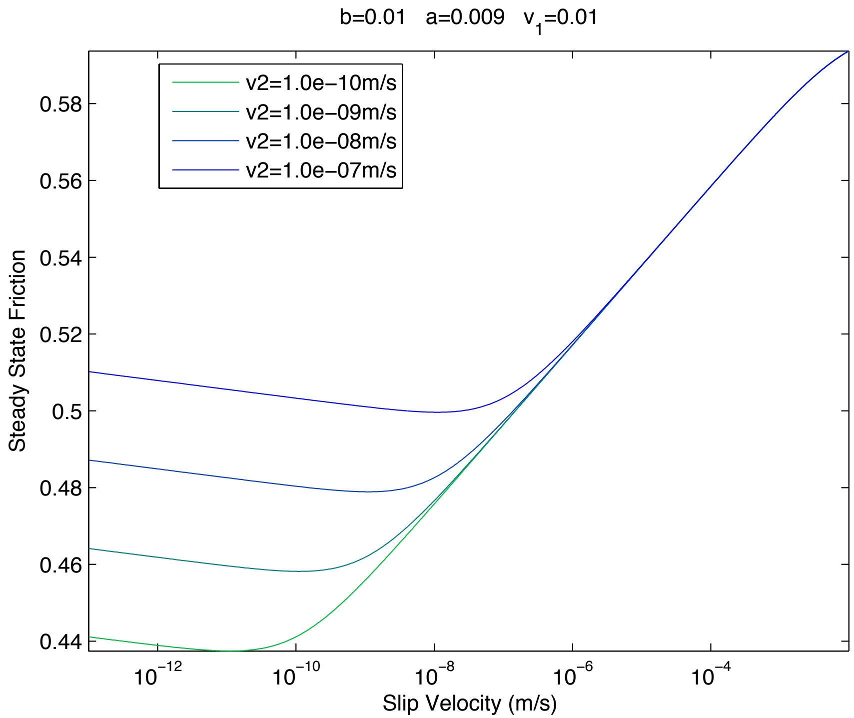
Ageing law $\dot{\theta} = 1 - \frac{V\theta}{L}$

Slip law $\dot{\theta} = -\frac{V\theta}{L}\log\left(\frac{V\theta}{L}\right)$

Stability of slip depends on the sign of (a-b):

- $a - b > 0$: velocity strengthening, stable
- $a - b < 0$: velocity weakening, potentially unstable

Model assumptions: rate-and-state friction



Variant with two velocity cut-offs V_1 and V_2:

$$f(V,\theta) = f^* - a \log\left(1 + \frac{V_1}{V}\right) + b \log\left(1 + \frac{V_2\theta}{L}\right)$$

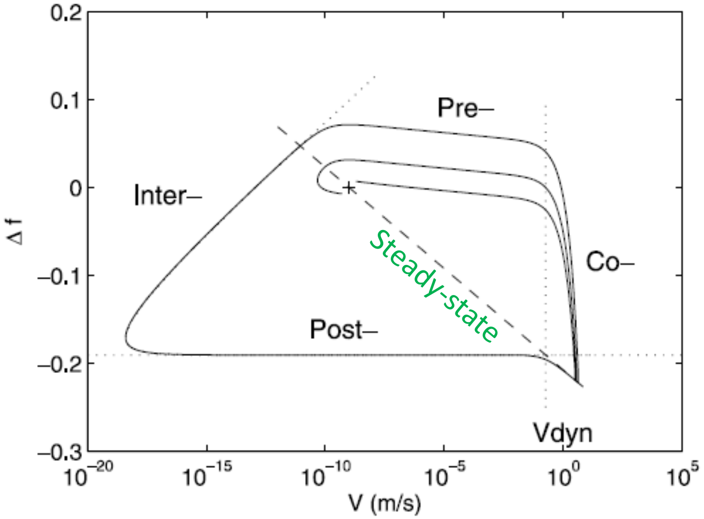
Apparent a−b $= \frac{\partial f_{ss}}{\partial logV}$ **is not constant, it depends on** $\boldsymbol{V}$

Weakening at low slip rate $V \ll V_2$

Strengthening at intermediate slip rate $V_1 \gg V \gg V_2$

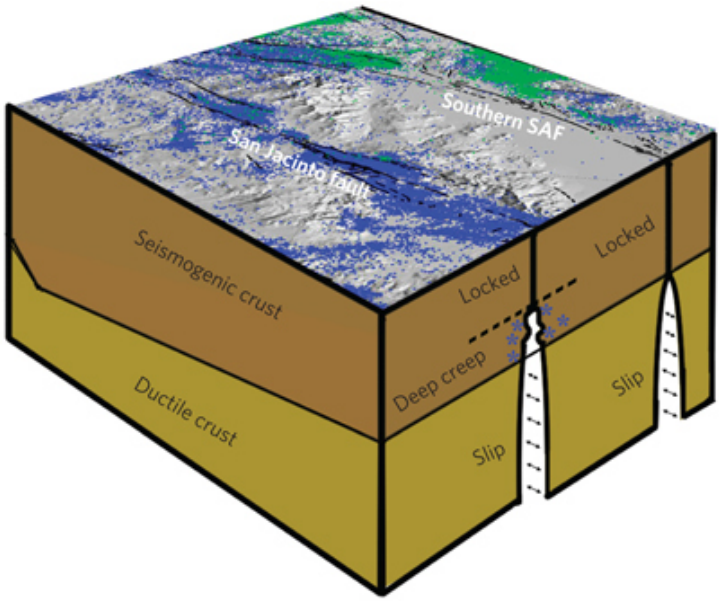
→slow slip events

Model assumptions: initial conditions



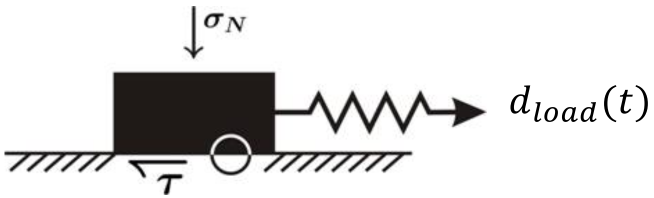
- Need to prescribe slip velocity V and state variable θ at t=0
- The long-term behavior of the fault does not strongly depend on this initial condition
- Usual procedure:
  1. Give an initial “kick” to the system such that $\frac{V(0)\theta(0)}{L} > 1$
  2. Run several “warm-up cycles” to erase the effect of the arbitrary initial conditions

Model assumptions: tectonic loading



- Fault extends infinitely beyond the seismogenic zone
- Fault is driven by
  - steady creep (constant slip velocity) on its deeper extension
  - + imposed displacements far from the fault
  - + arbitrary external loads, e.g. oscillatory loading induced by tides, fluid injection

Formulation: spring-block system



- Equation of motion:

$$\tau(t) = -\eta v(t) - K\big(d(t) - d_{load}(t)\big) \qquad \textbf{Eq. 1}$$

where

τ =shear stress at the base of the block,

d, v = displacement and velocity of the block

d_{load} = loading point displacement

$\eta = \mu/2c_s$= impedance

K = stiffness of the spring

- Friction:

$$\tau(t) = \sigma f(v, \theta) \qquad \textbf{Eq. 2}$$

$$\dot{\theta} = g(v, \theta) \qquad \textbf{Eq. 3}$$

Formulation: time integration

Reduction to a system of **Ordinary Differential Equations**

Set Eq. 1 = Eq. 2 and take the time derivative:

$$\dot{v} = -\frac{\sigma g(v,\theta) + K(v - v_{load})}{\sigma \frac{\partial f}{\partial v}(v,\theta) + \eta} \qquad \textbf{Eq. 4}$$

+ equation 3 $\dot{\theta} = g(v,\theta))$

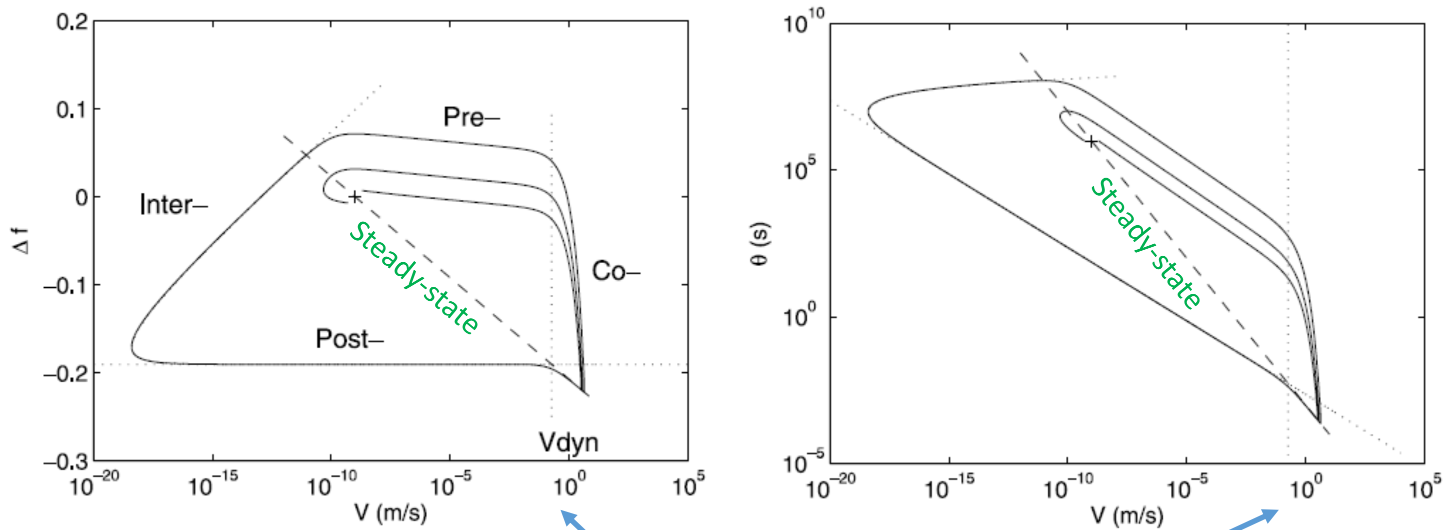
Standard ODE form:

$$\dot{X} = f(X) \qquad \text{where } X = (v,\theta)$$

Given initial conditions $v(0)$ and $\theta(0)$, solve the ODE system to get $v(t)$ and $\theta(t)$.

Standard ODE solvers, e.g. Runge-Kutta with **adaptive time step**

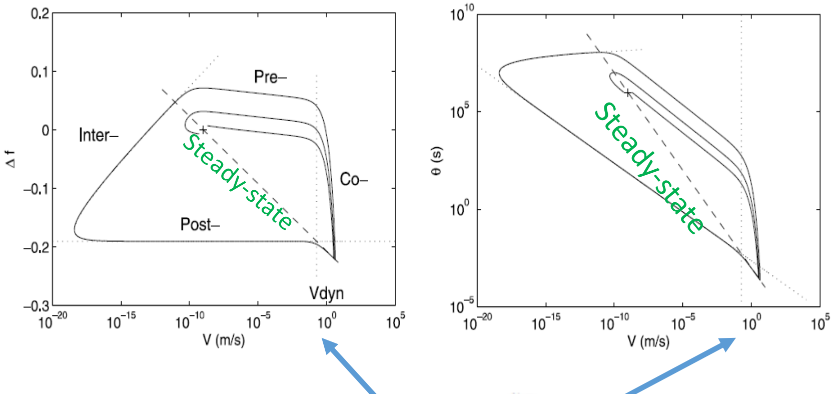
Typical spring-block cycle



$$V_{\text{dyn}} = 2\frac{a\sigma}{\mu}c_s$$

Rubin and Ampuero (2005)

Typical spring-block cycle



$$V_{\text{dyn}} = 2\frac{a\sigma}{\mu}c_s$$

$$\dot{v} = -\frac{\sigma g(v,\theta) + K(v - v_{load})}{\sigma \frac{\partial f}{\partial v}(v,\theta) + \eta}$$

Denominator = direct effect + radiation damping

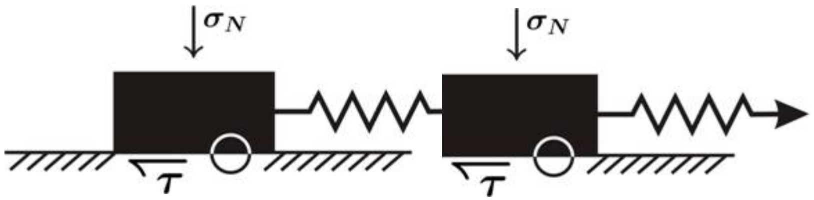
$$= \frac{a\sigma}{V} + \frac{\mu}{2c_s}$$

The two effects are comparable if

$$V \approx 2\frac{a\sigma}{\mu}c_s$$

Rubin and Ampuero (2005)

Formulation: two spring-blocks system



- System of equations of motion:

$$\tau_1 = \eta v_1 - K_{11}(d_1 - d_{load}) - K_{12}(d_2 - d_{load})$$
$$\tau_2 = \eta v_2 - K_{22}(d_2 - d_{load}) - K_{21}(d_1 - d_{load})$$

elastic coupling between blocks

- Define $X = (v_1, \theta_1, v_2, \theta_2)$
- The rest is the same …

Formulation: continuum fault

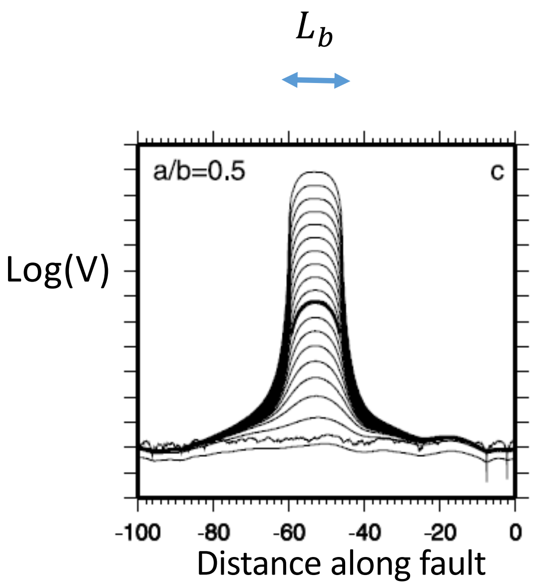
- Boundary element method: fault discretized by a grid of N rectangular cells
- System of N equations of motion:

$$\tau_i = -\eta v_i - \boxed{\sum_j K_{ij}(d_j - d_{load})}$$

elastic coupling (all to all)

- K_{ij} is the stress on cell *i* due to a unit slip on cell *j*
- The matrix K is computed analytically (Okada's formulas)
- The rest is the same
- The matrix multiplication Kd dominates the computational cost
- Speed-up of Kd computation by FFT in regular grids, by H-matrix in non-regular grids

Resolution length



Smallest length of the problem: minimum slip localization length and the **size of the process zone at the rupture tip**.

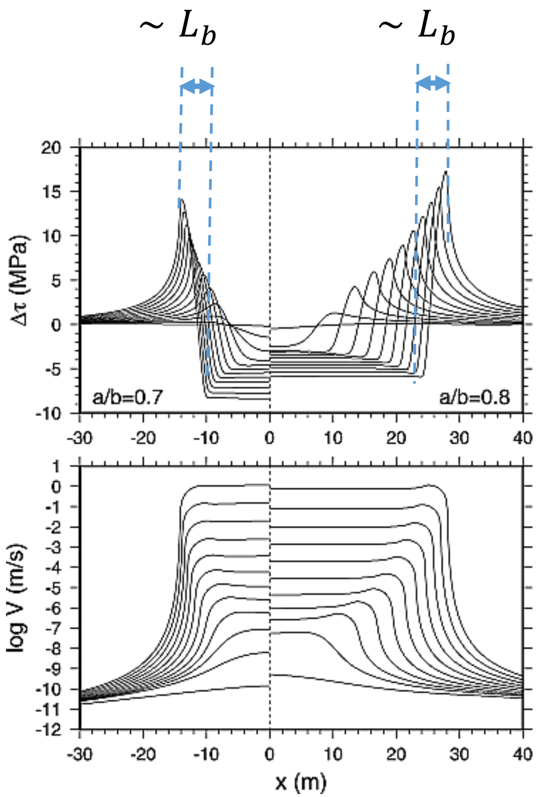
For the ageing law:

$$L_b = \mu D_c / b\sigma$$

To guarantee good numerical resolution the cell size Δx must be much smaller than L_b

Rubin and Ampuero (2005)

Resolution length



Distance along fault

Smallest length of the problem: minimum slip localization length and the **size of the process zone at the rupture tip**.

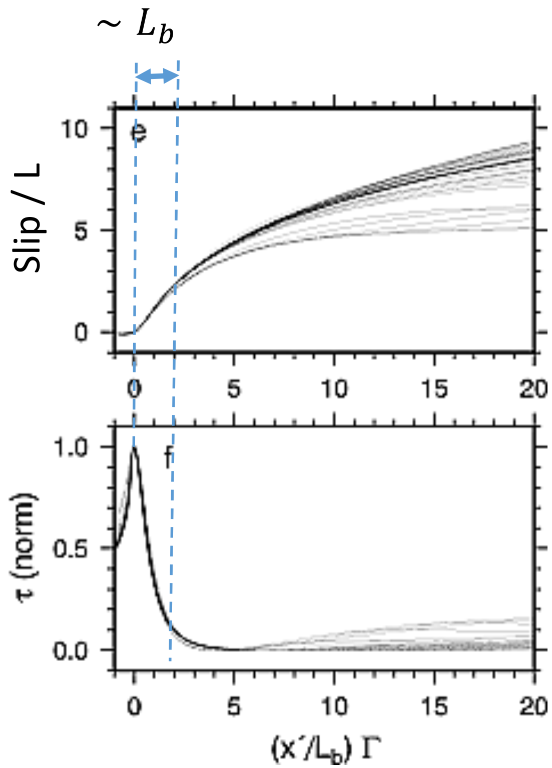
For the ageing law:

$$L_b = \mu D_c / b\sigma$$

To guarantee good numerical resolution the cell size Δx must be much smaller than L_b

Rubin and Ampuero (2005)

Resolution length



Distance from the rupture tip normalized by L_b

Smallest length of the problem: minimum slip localization length and the **size of the process zone at the rupture tip**.

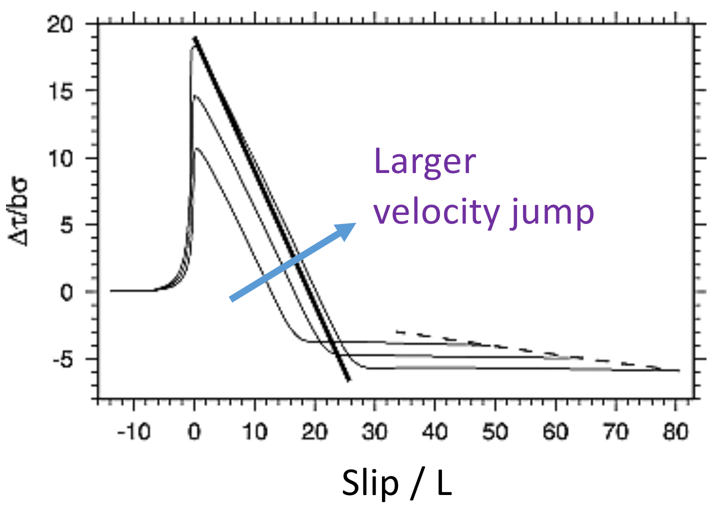
For the ageing law:

$$L_b = \mu D_c / b\sigma$$

To guarantee good numerical resolution the cell size Δx must be much smaller than L_b

Rubin and Ampuero (2005)

Process zone in rate-and-state friction



Rubin and Ampuero (2005)

From lecture 3: process zone size

$$\Lambda_0 \approx 2\mu G_c/(\tau_s - \tau_d)^2$$

Rate-and-state behaves as slip-weakening near the rupture front, with equivalent properties:

$$D_c = L\ln(V/V^*) \approx 20\ L$$

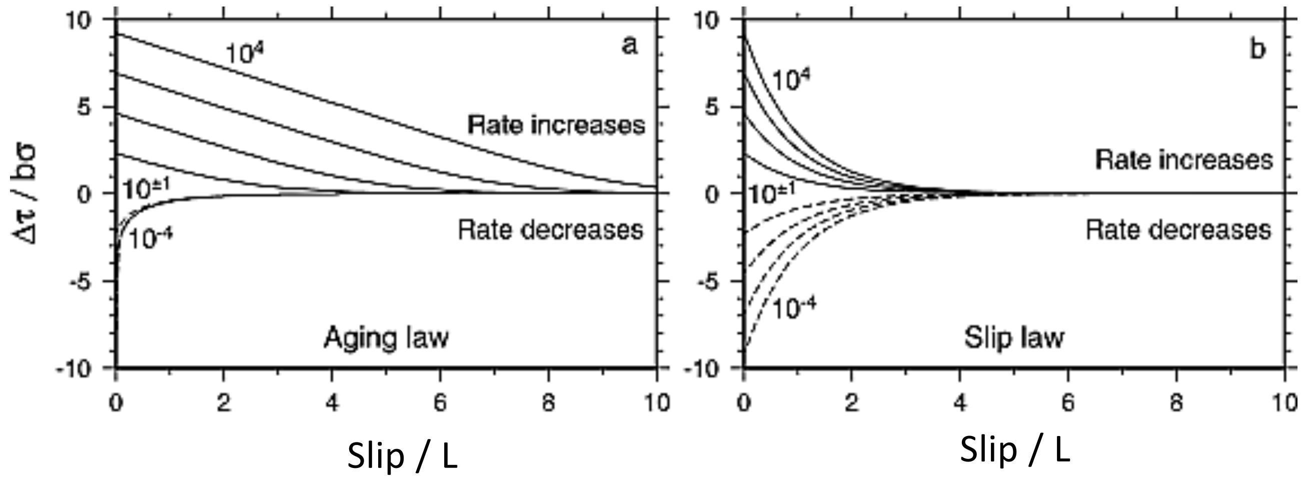
$$\tau_s - \tau_d \approx b\sigma\ln(V/V^*)$$

$$G_c \approx \frac{1}{2} b\sigma L \ln\left(\frac{V}{V^*}\right)^2$$

→ Process zone size:

$$\Lambda_0 \approx \frac{\mu L}{b\sigma} = L_b$$

Two flavors of rate-and-state friction



Ageing law: $\dot{\theta} = 1 - V\theta/L$

$$D_c = L\ln(V/V^*)$$

$$\Lambda_0 \approx \frac{\mu L}{b\sigma} = L_b$$

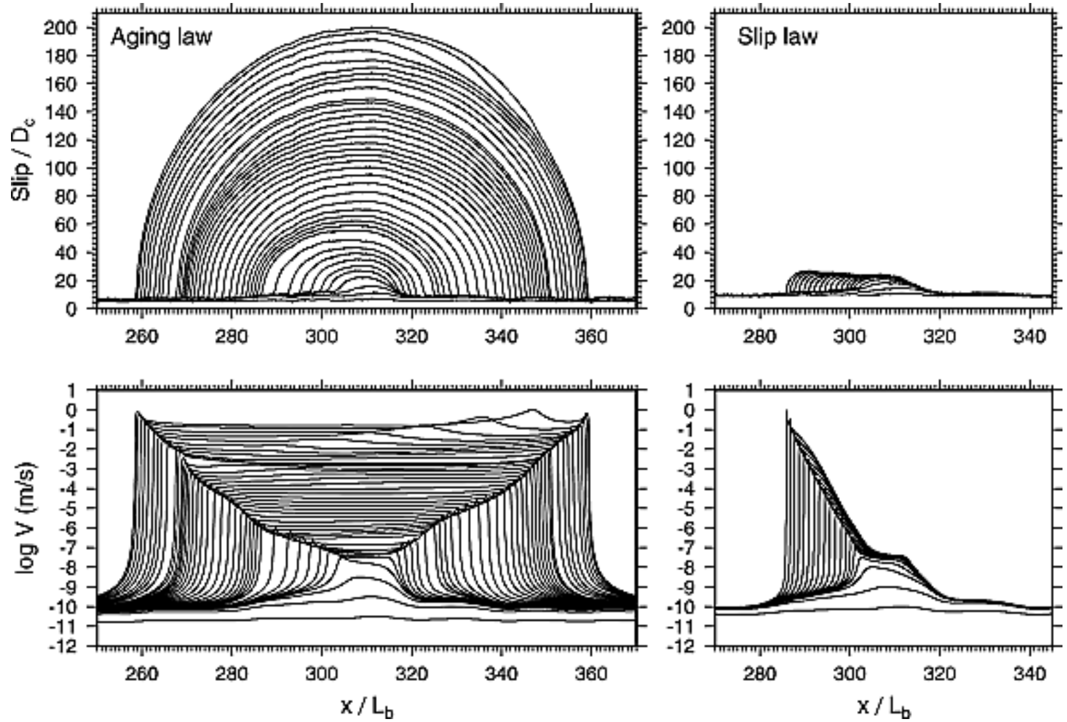
Slip law: $\dot{\theta} = -V\theta/L\,\log(V\theta/L)$

$$D_c \approx L$$

$$\Lambda_0 \approx L_b/\log\left(\frac{V}{V^*}\right)$$

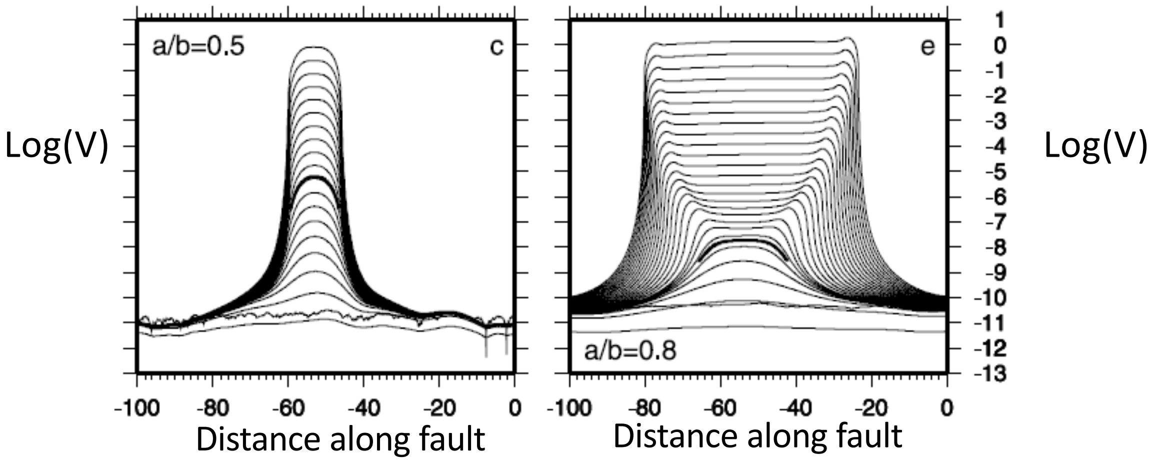
It shrinks → more challenging to resolve

Two flavors of rate-and-state friction



Ageing and slip laws predict radically different nucleation processes

Other important lengths



$$L_c = \frac{\mu L}{(b-a)\sigma} = \frac{b}{b-a} L_b$$ (Rice, 1993, also called h^*)

$$L_\infty = \mu L \frac{b}{(b-a)^2\sigma} = \left(\frac{b}{b-a}\right)^2 L_b$$ (Rubin and Ampuero, 2005)

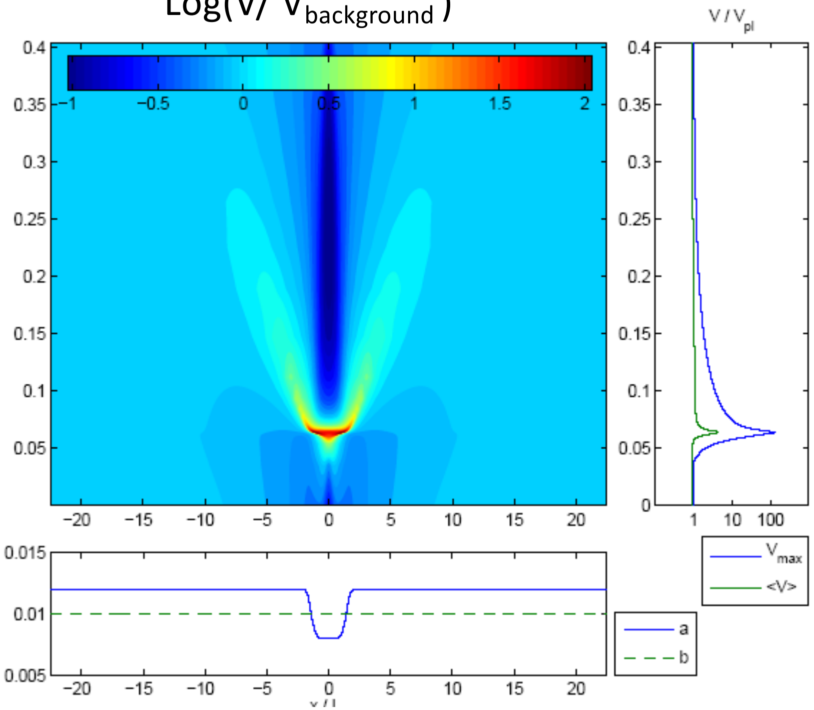
Do not confuse process zone with the other characteristic sizes, they are larger!

Example: brittle asperity isolated in a creeping fault zone

An isolated brittle asperity (v-weakening) within a creeping fault (v-strengthening).
Constant slip velocity $V_{background}$ imposed far from the asperity.

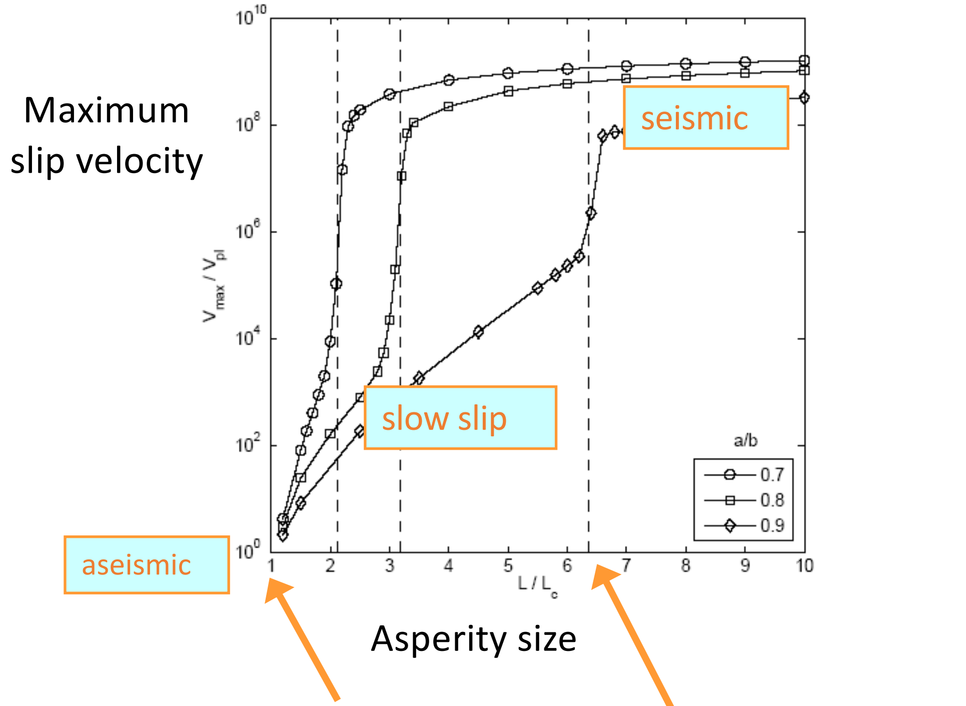
Log(V/ $V_{background}$)

Time normalized by Dc/ $V_{background}$



Position along-strike

Example: brittle asperity isolated in a creeping fault zone



$$L_b \doteq \frac{\mu D_c}{b\sigma}$$

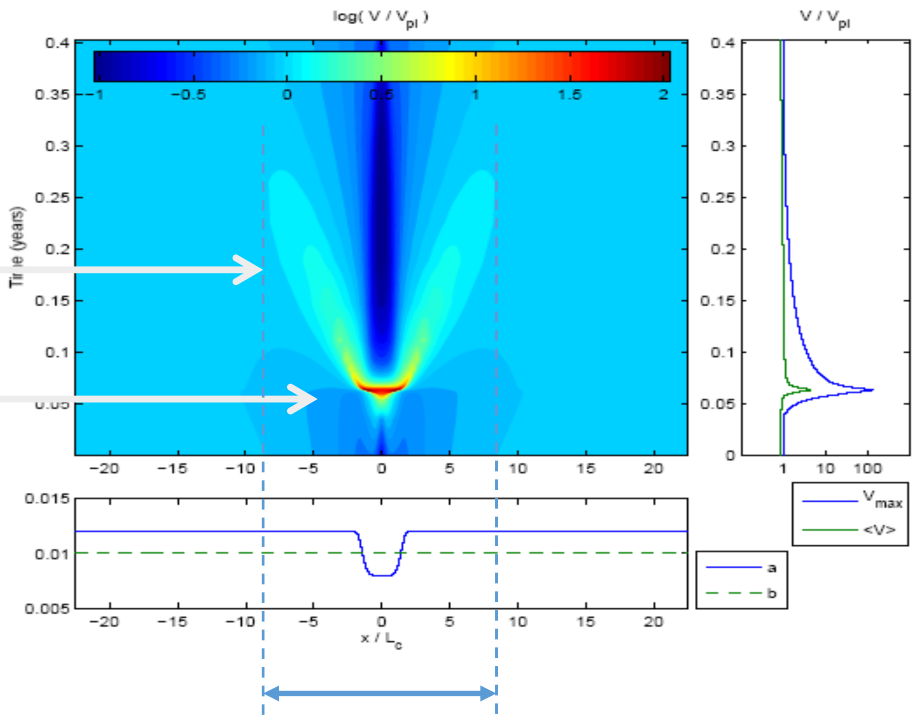
$$L_c \doteq \frac{\mu D_c}{(b-a)\sigma}$$

$$L_\infty = \frac{1}{\pi}\left(\frac{b}{b-a}\right)^2 L_b$$

Migrating swarms: asperity interactions mediated by creep transients

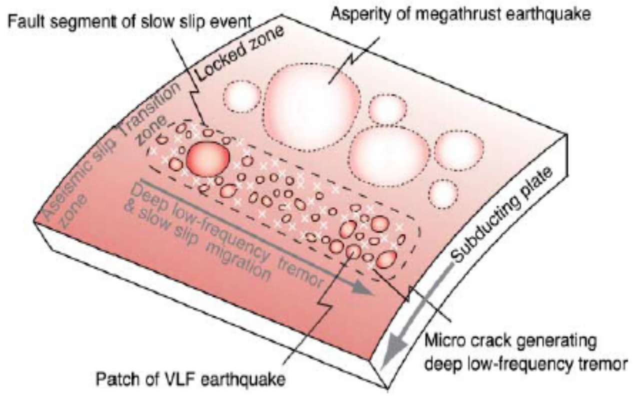
It triggers a migrating aseismic transient

The asperity breaks



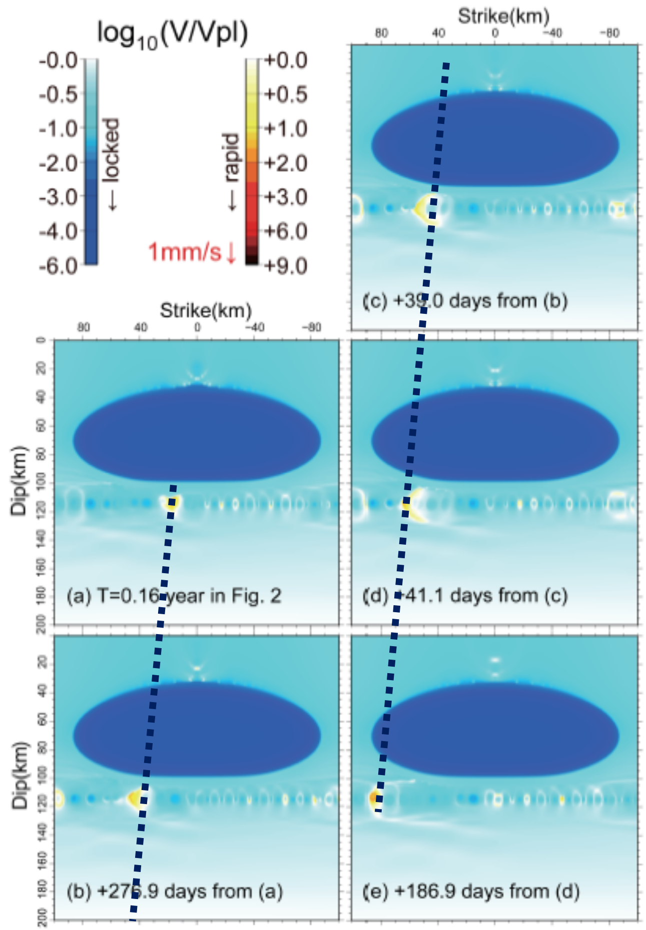
Influence radius

Migrating swarms: asperity interactions mediated by creep transients



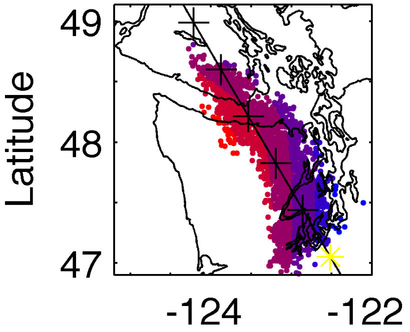
Cascading failure of a population of brittle asperities

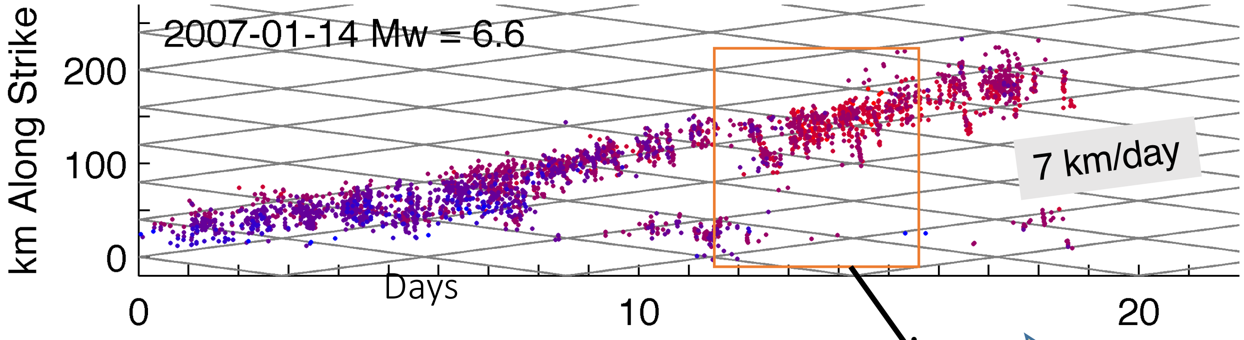
→ Tremor swarm

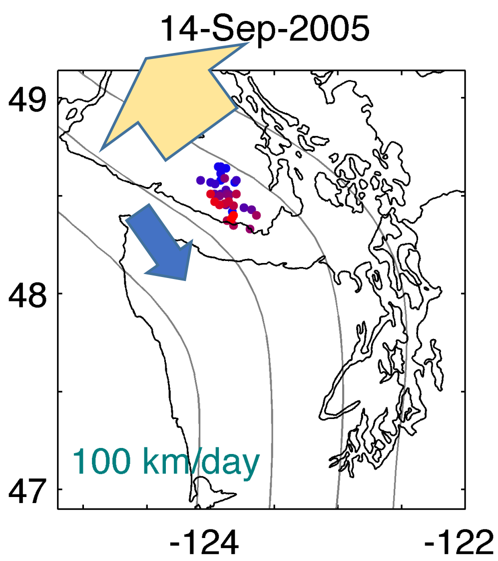


Quasi-dynamic 3D simulations with K. Ariyoshi (JAMSTEC)

Slow slip and tremor migration patterns



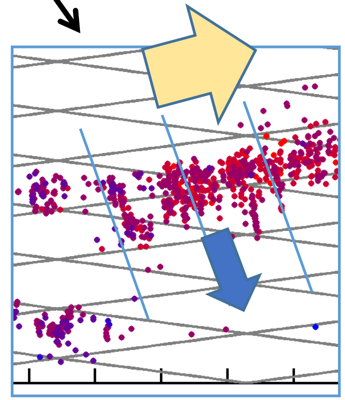




Non-volcanic tremor migration patterns in Cascadia, USA

Tremor migrates slowly along strike (~10 km/day) tracking the front of the slow slip event

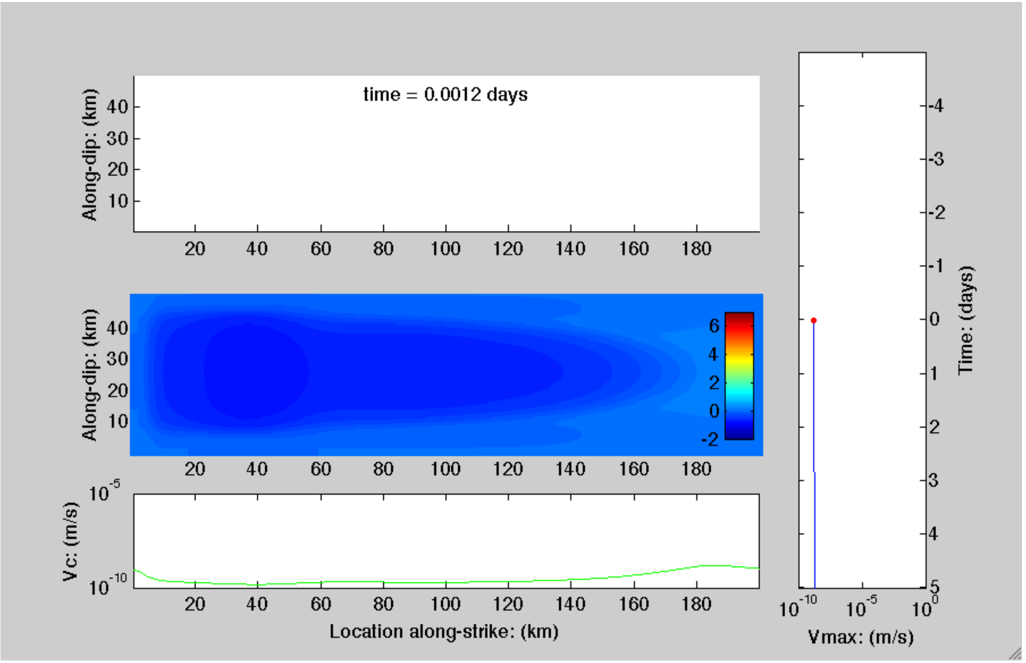
Episodic tremor swarms propagate backwards, faster (~ 100 km/day)



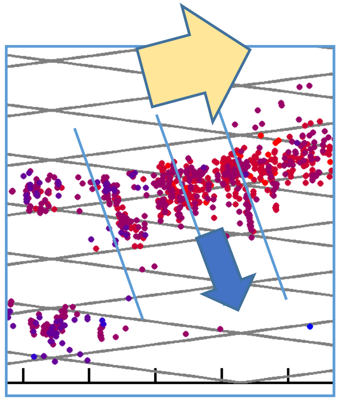
Houston et al (2010)

Simulations of slow slip and tremor

QDYN model of slow slip and tremor

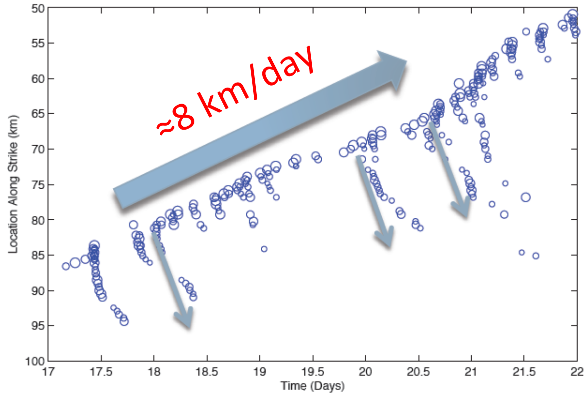


Luo and Ampuero



Rapidal Tremor Reversals observed in Cascadia

Houston et al (2010)



Model