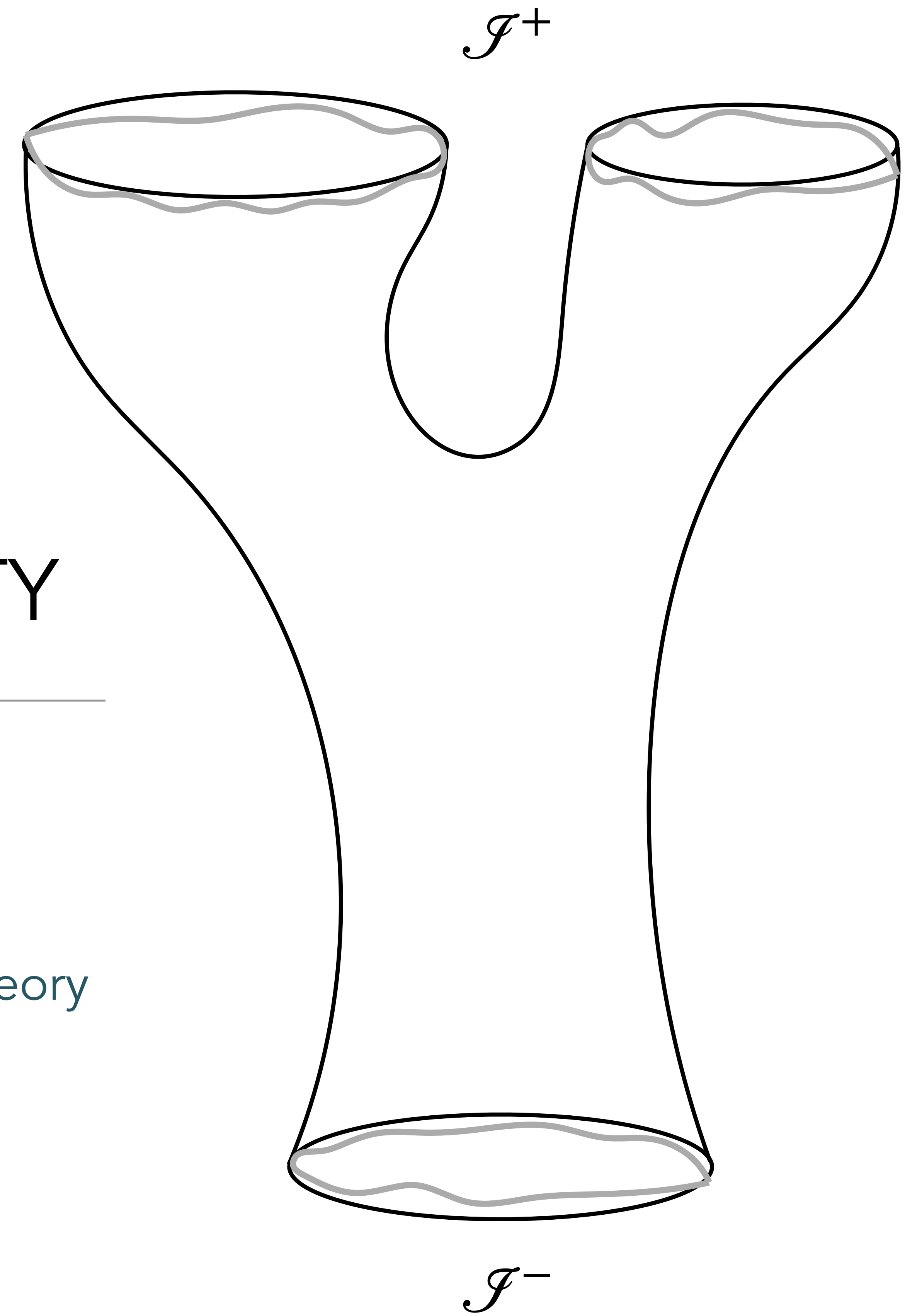
NEARLY DE SITTER GRAVITY

arXiv:1905.03780 (Cotler, KJ, Maloney)

see also arXiv:1904.01911 (Maldacena, Turiaci, Yang)

Non-Perturbative Methods in Quantum Field Theory

ICTP Trieste, 3 September 2019

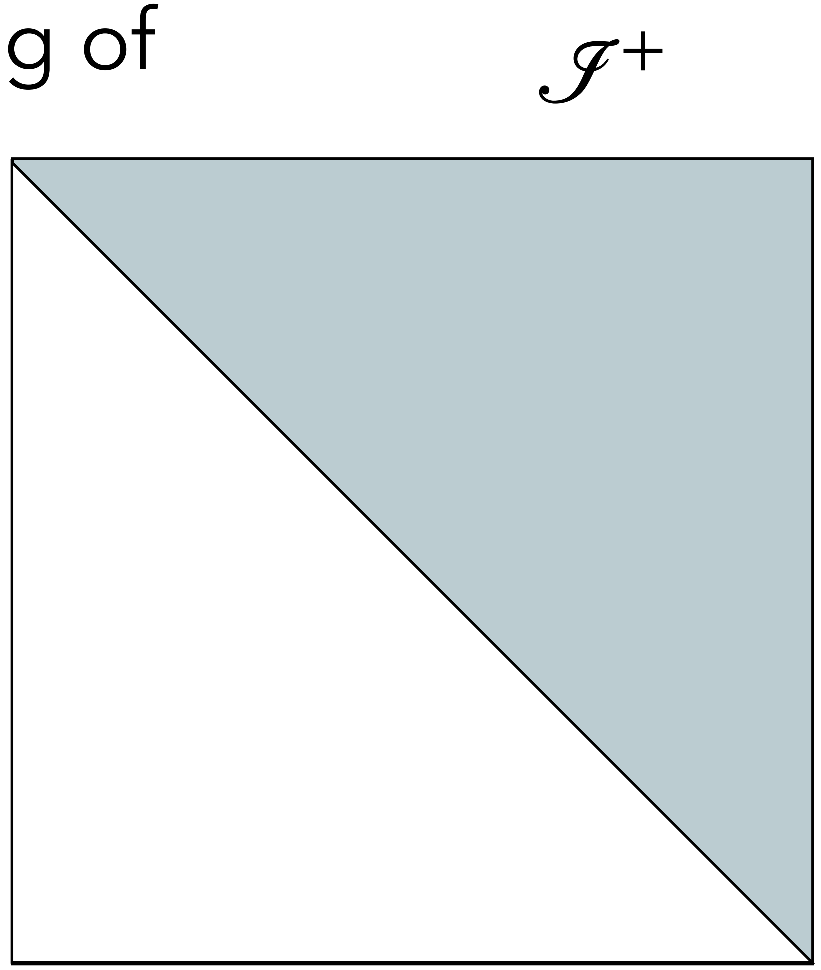


DE SITTER HOLOGRAPHY?

By now we have a fairly good understanding of AdS holography: defined by a dual CFT.

What about de Sitter?

"dS/CFT": a non-unitary CFT dual
to an inflating patch.



But basic observables of dS are transition amplitudes between infinite past and future ("metaobservables"). Is there a dual "CFT" which computes them?

DE SITTER HOLOGRAPHY?

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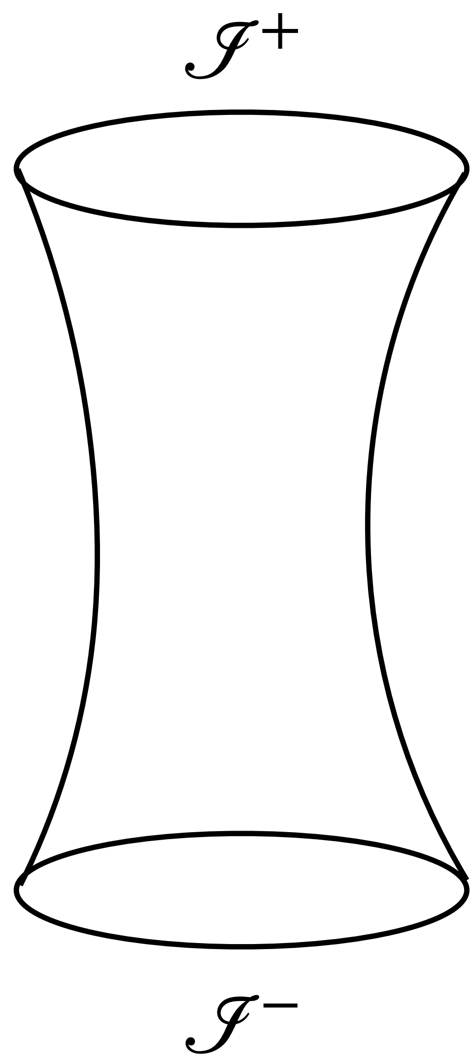


$$= \langle g^+ | \mathscr{U} | g^- \rangle \overset{?}{=} \int [d\xi^+][d\xi^-] e^{-S_{\mathrm{CFT}}[\xi^+,\xi^-]}$$

Immediate problem:

There are nonzero correlations between $\mathscr{I}^+$ and $\mathscr{I}^-$ without local interactions which couple the boundaries.

JACKIW-TEITELBOIM GRAVITY

Enter "JT" gravity, a toy model for 2d quantum gravity.

AdS version: [KJ] [Maldacena, Stanford, Yang] [Engelsöy, Mertens, Verlinde]

$$\boxed{S_{\mathrm{JT}} = -S_0\chi - \frac{1}{16\pi G}\int d^2x\sqrt{g}\,\varphi\left(R + \frac{2}{L^2}\right)}$$

Usual Euler term familiar from worldsheet string theory.

"Dilaton"

No bulk dof; however there is a boundary reparameterization mode. Loops can sometimes be summed to all orders in G. [Stanford, Witten]

JACKIW-TEITELBOIM GRAVITY

$$\boxed{S_{\mathrm{JT}} = S_0\chi + \frac{1}{16\pi G}\int d^2x\sqrt{-g}\,\varphi\left(R - \frac{2}{L^2}\right)}$$

There is also a version with positive cosmological constant.

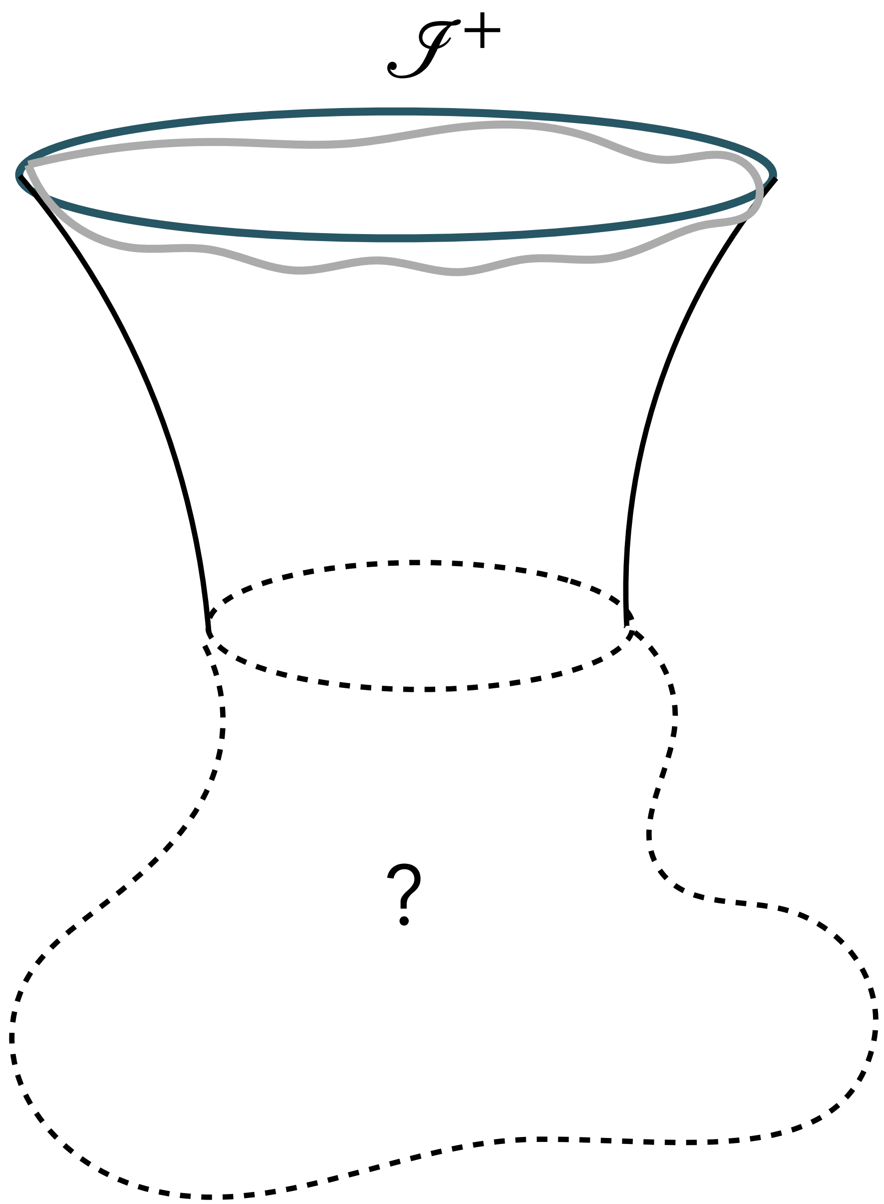
"Nearly dS_2" solutions:
$$\begin{cases} ds^2 = -\,dt^2 + \cosh^2\left(\frac{t}{L}\right)dx^2 \\ \varphi = \frac{\sinh t}{\ell} \end{cases}$$

Gives us a theoretical laboratory to study dS quantum gravity.

GOAL: QUANTUM COSMOLOGY DONE RIGHT

NO-BOUNDARY WAVEFUNCTION

Consider the "disk" partition function: $Z_{\mathrm{HH}} \approx \langle \beta | \mathscr{U} | \varnothing \rangle$



Boundary conditions:

Smoothly caps off in the past

Near $\mathscr{I}^+$ we have a cutoff slice ε,

$$\begin{cases} dS^2 \approx \dfrac{dx^2}{\varepsilon^2}, \quad x \sim x + \beta, \\ \varphi \approx \dfrac{1}{J\varepsilon}, \end{cases}$$

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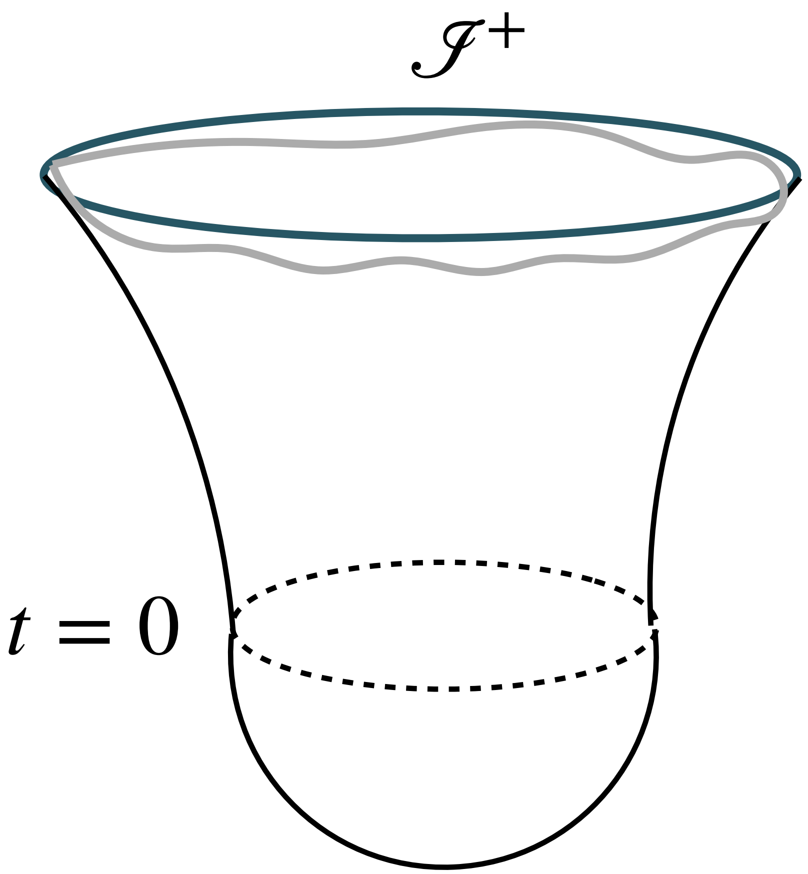

Classical solution is complex:

$$t \geq 0 : \begin{cases} ds^2 = -\,dt^2 + \cosh^2 t\, d\theta^2\,, \\ \varphi = \frac{2\pi}{\beta J} \sinh t\,, \end{cases}$$

$$\tau \in [0,\pi/2] : \begin{cases} ds^2 = d\tau^2 + \cos^2 \tau\, d\theta^2\,, \\ \varphi = -\,\frac{2\pi i}{\beta J} \sin \tau\,, \end{cases}$$

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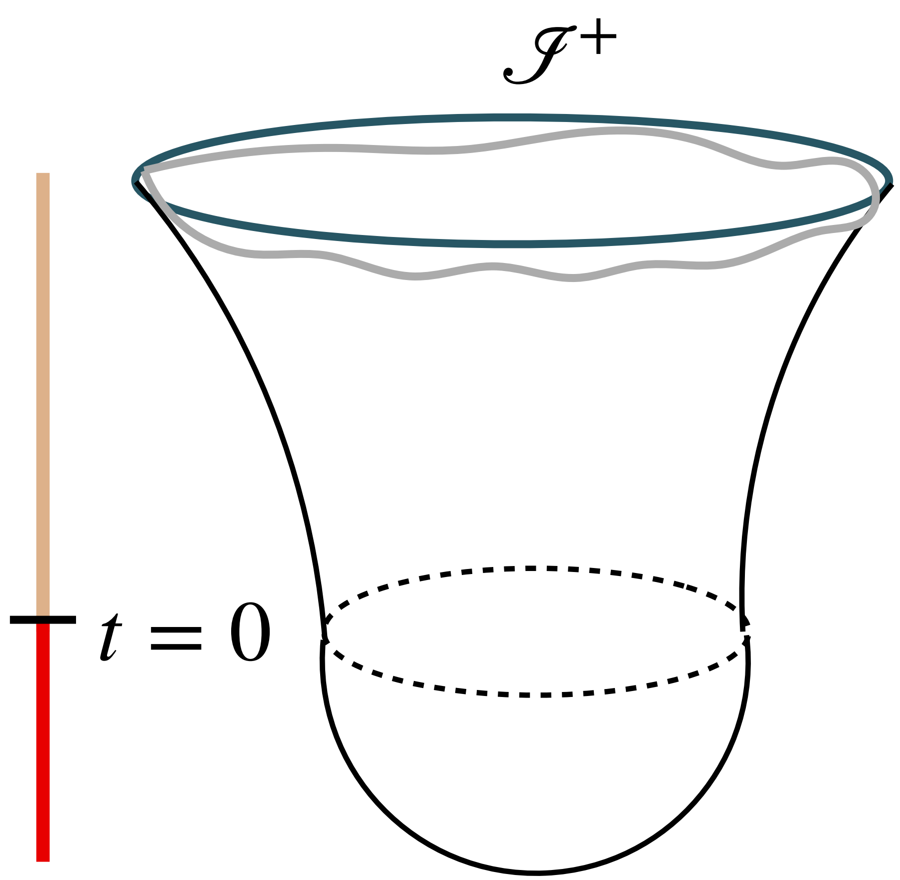

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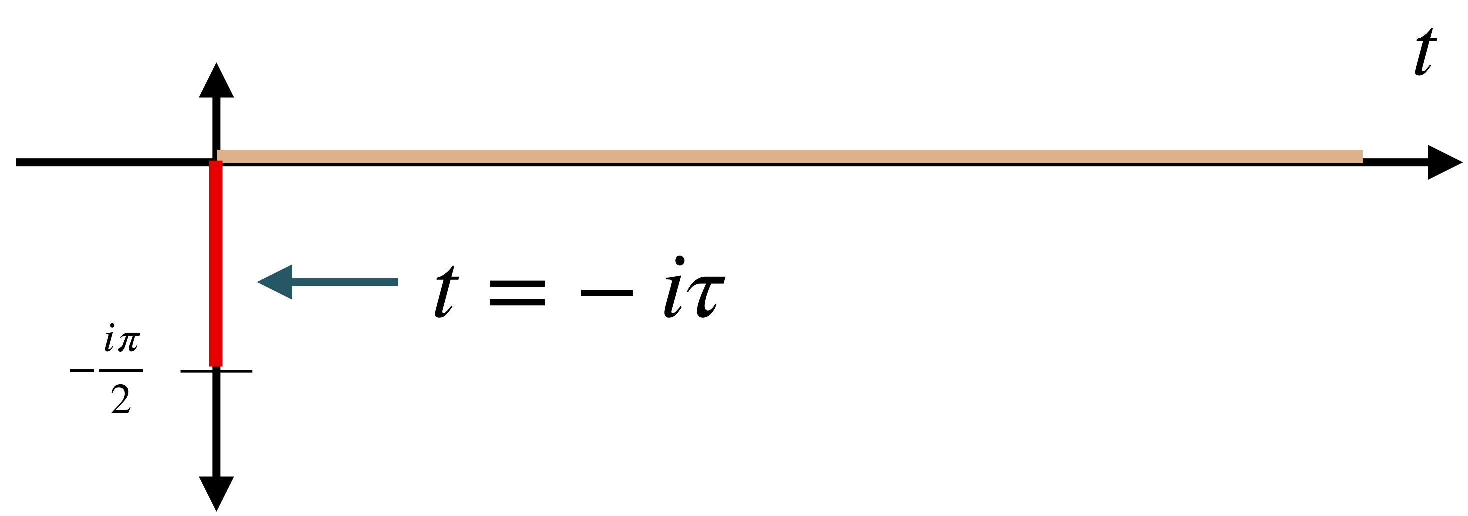

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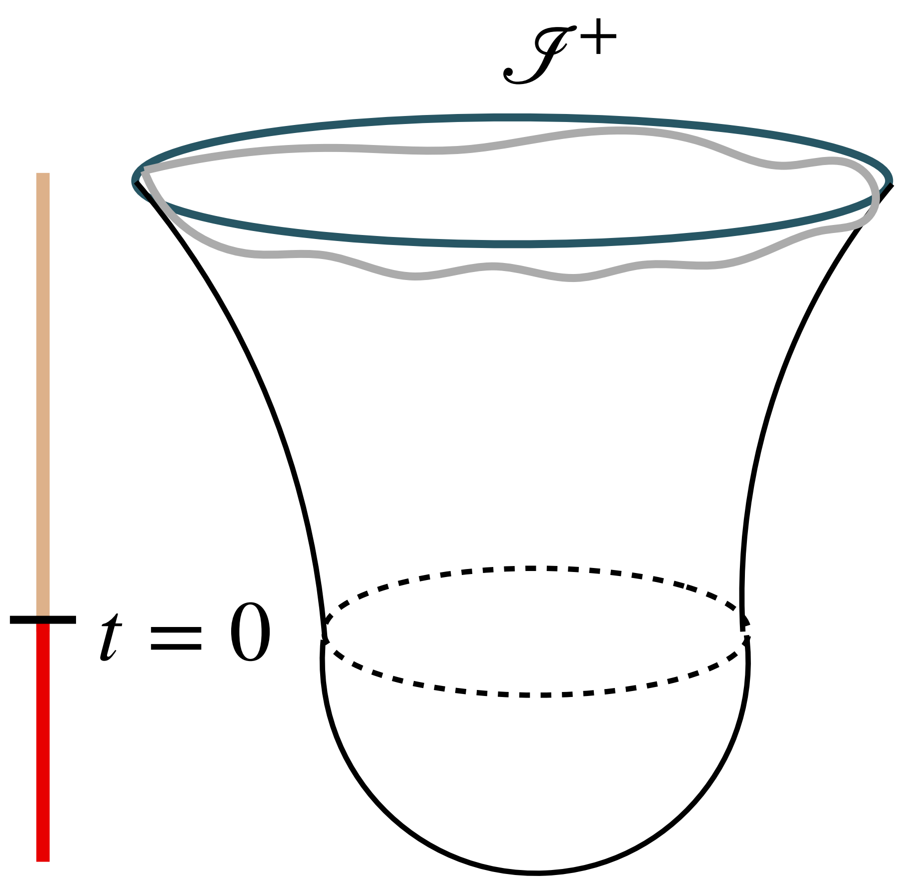

$$S_{\mathrm{JT}} = S_0 \chi + \frac{1}{16\pi G} \int d^2x \sqrt{-g}\, \varphi\, (R-2) + \frac{1}{8\pi G} \int d\theta \sqrt{h}\, \varphi (K-1)$$

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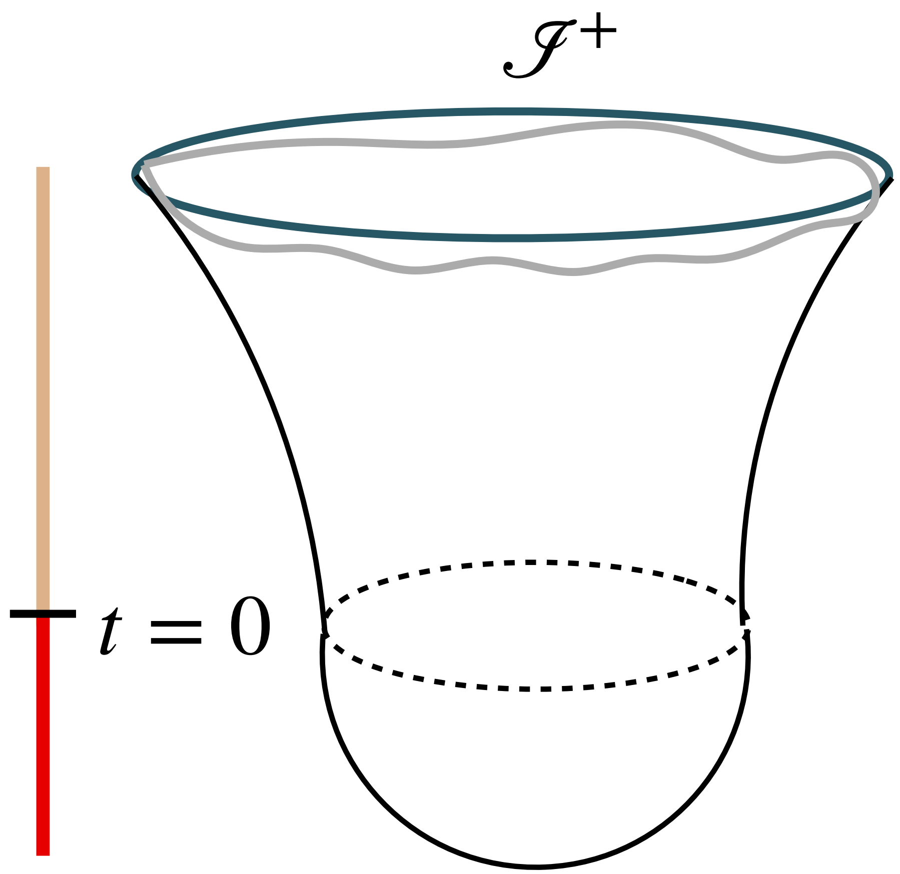

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$-iS_0$

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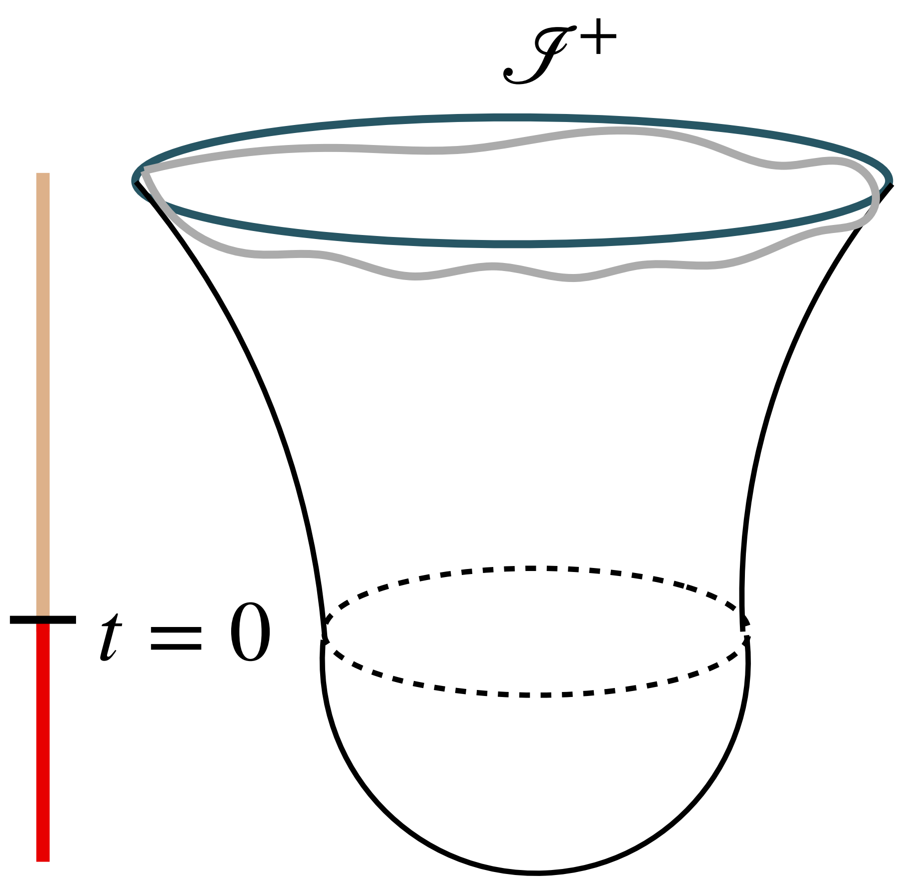

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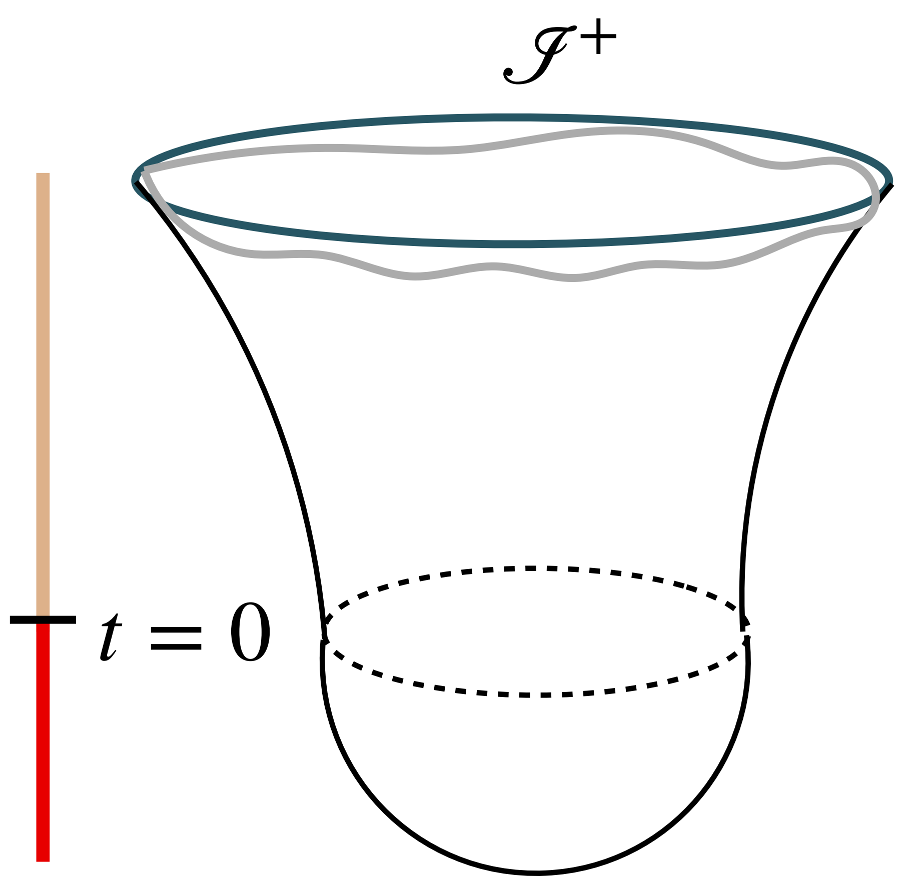

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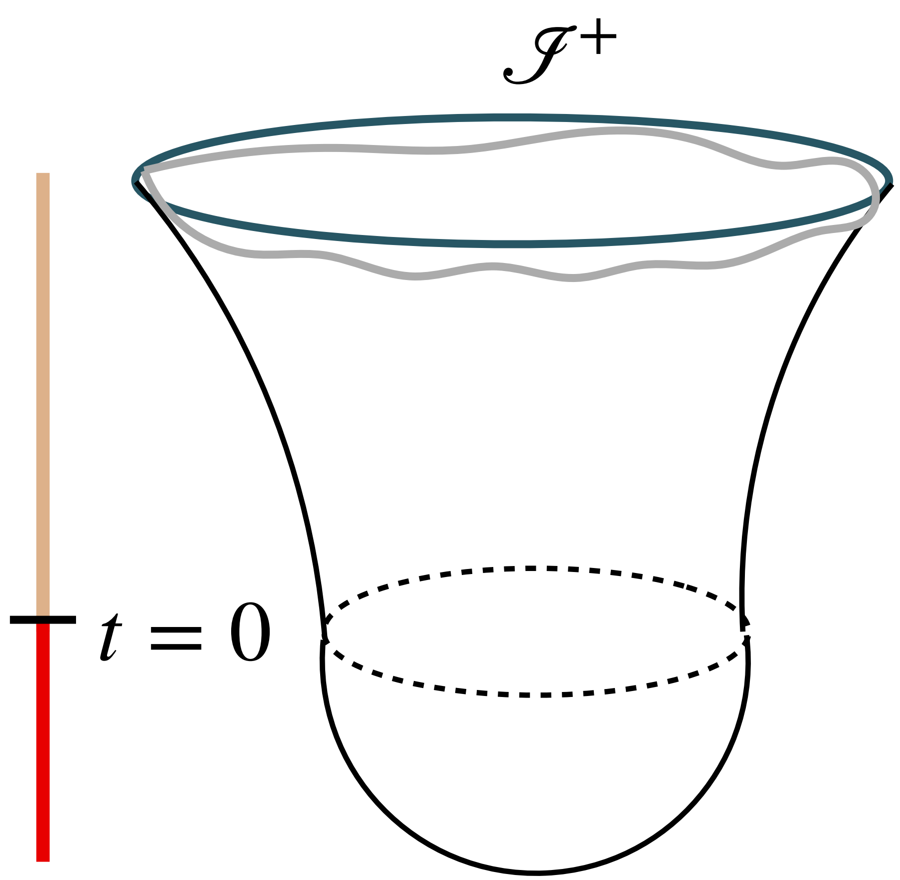

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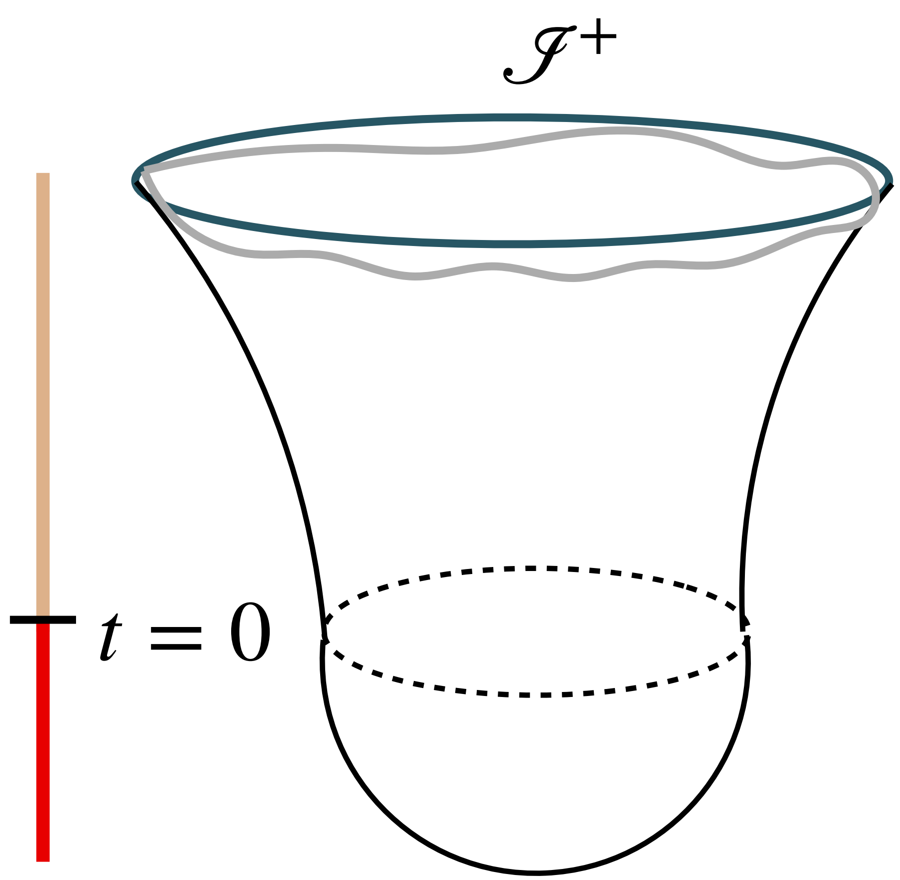

$$S_{\mathrm{JT}} = -\,iS_0 + \frac{1}{8\pi G}\int d\theta \sqrt{h}\,\varphi(K - 1)$$

gives Schwarzian action

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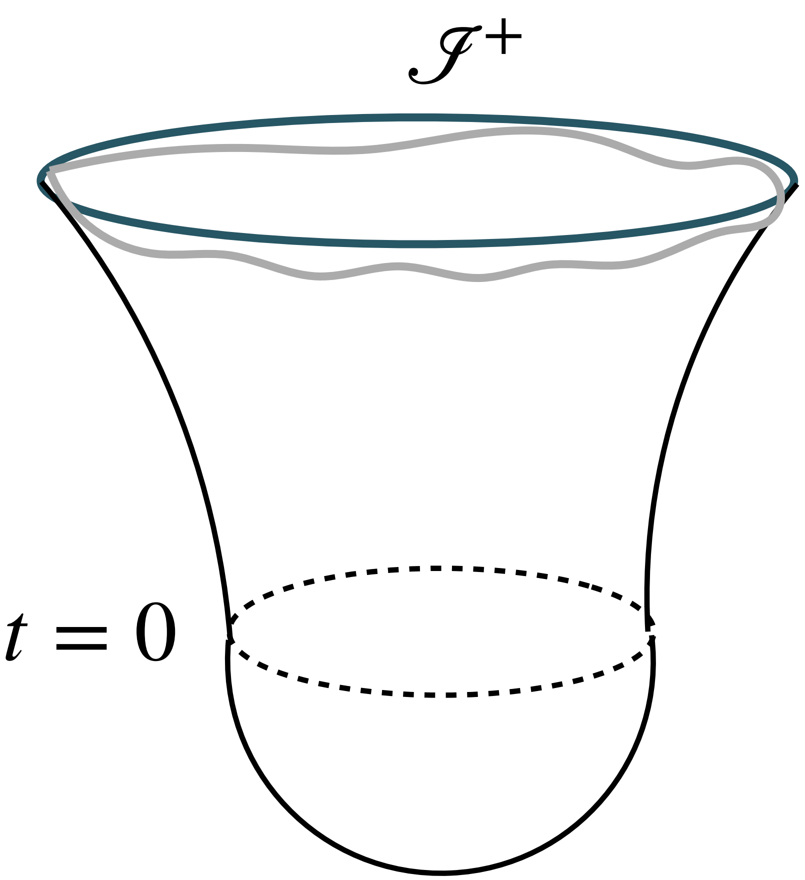

$$Z_{\mathrm{HH}} = e^{S_0} \int [Df]\, e^{iS[f]}$$

$$S[f] = \frac{1}{4G\beta J} \int_0^{2\pi} d\theta \left(\{f(\theta), \theta\} + \frac{1}{2} f'(\theta)^2 \right)$$

$$\{f(\theta), \theta\} = \frac{f'''(\theta)}{f'(\theta)} - \frac{3}{2} \left(\frac{f''(\theta)}{f'(\theta)} \right)^2$$ = Schwarzian derivative

$$f(\theta + 2\pi) = f(\theta) + 2\pi$$

n.b.

$$\tan\left(\frac{f}{2}\right) \sim \frac{a \tan\left(\frac{f}{2}\right) + b}{c \tan\left(\frac{f}{2}\right) + d}, \quad ad - bc = 1 \quad \Rightarrow \quad f \in \mathrm{Diff}(\mathbb{S}^1)/PSL(2;\mathbb{R})$$

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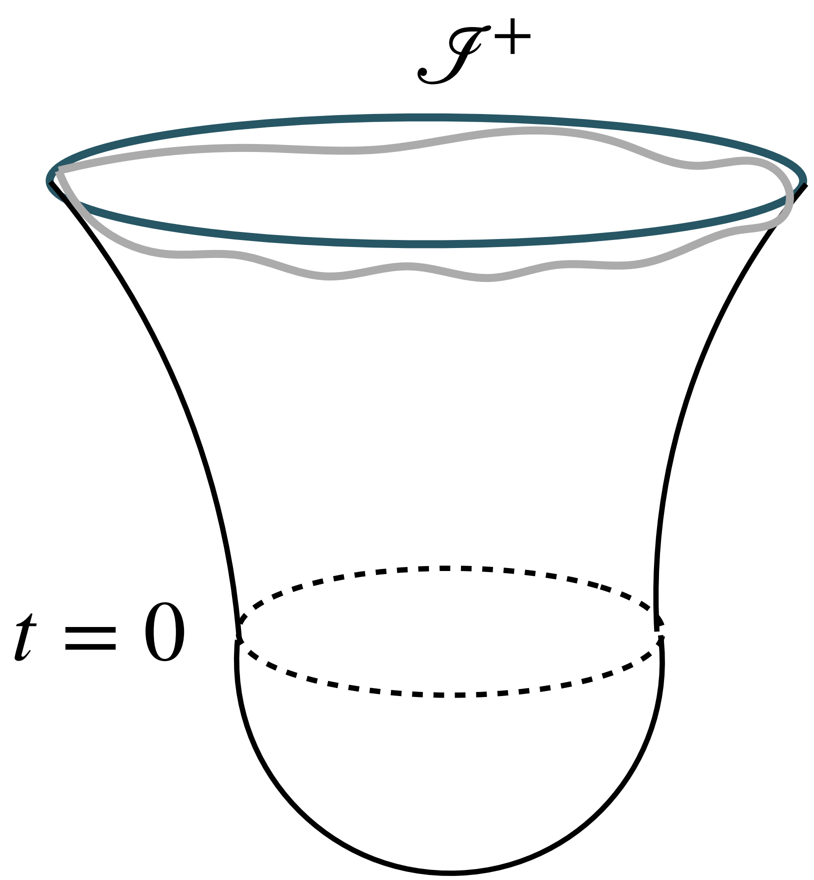

$$Z_{\text{HH}} = e^{S_0} \int [Df]\, e^{iS[f]}$$

$$\boxed{= \frac{1}{\sqrt{2\pi}(-2i\beta J)^{3/2}} e^{S_0 + \frac{\pi i}{4G\beta J}}}$$

exact to all orders in G!

Not clear how to normalize $|\beta\rangle$ or $|\varnothing\rangle$,
but we do see relative suppression to nucleate
at large β, i.e. large universes.

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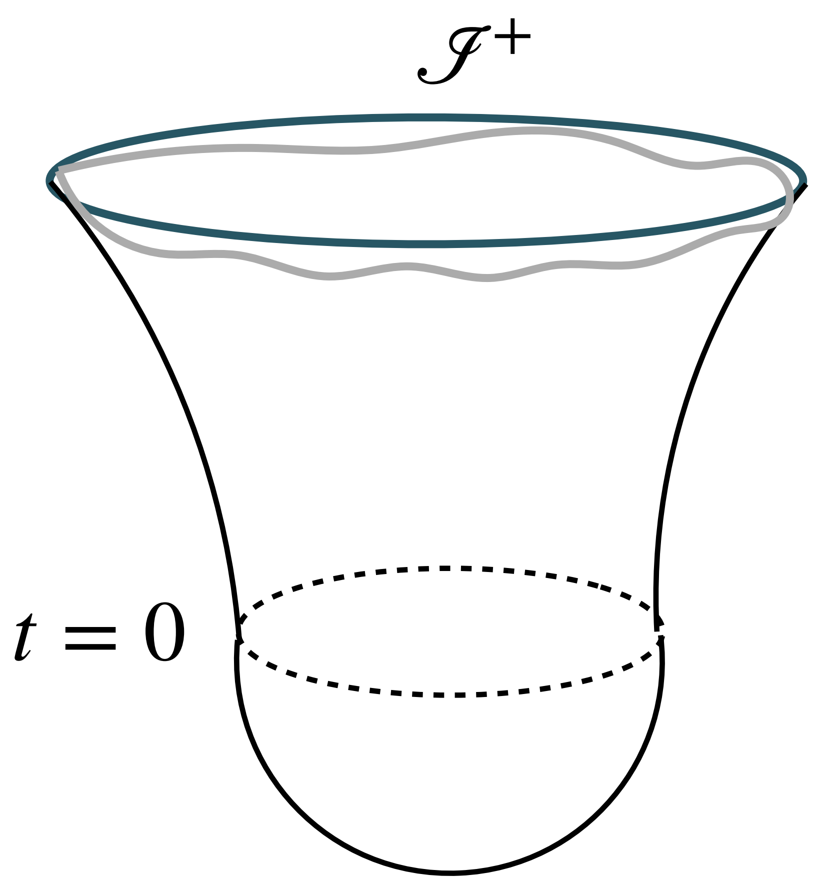
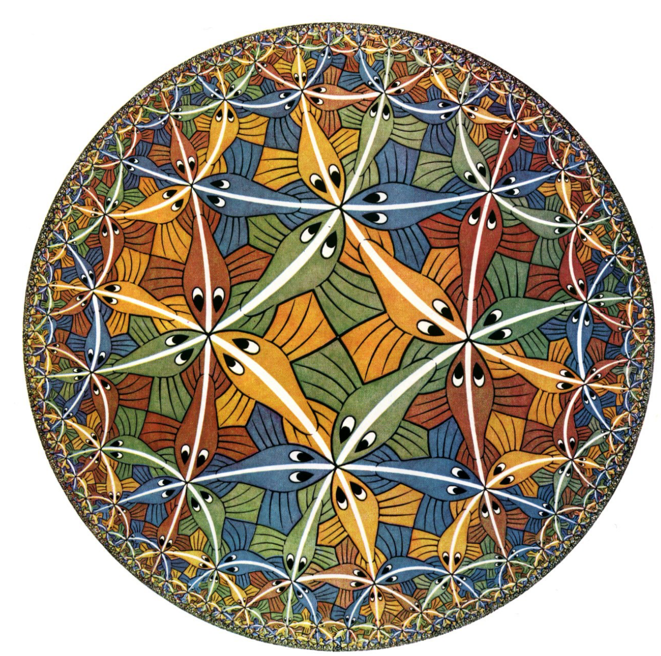


$$Z_{\mathrm{HH}} = \frac{1}{\sqrt{2\pi}(-2i\beta J)^{3/2}} e^{S_0 + \frac{\pi i}{4G\beta J}}$$
$$= Z_{\mathrm{disc}}(-i\beta J)$$

continuation of Euclidean AdS_2 result!



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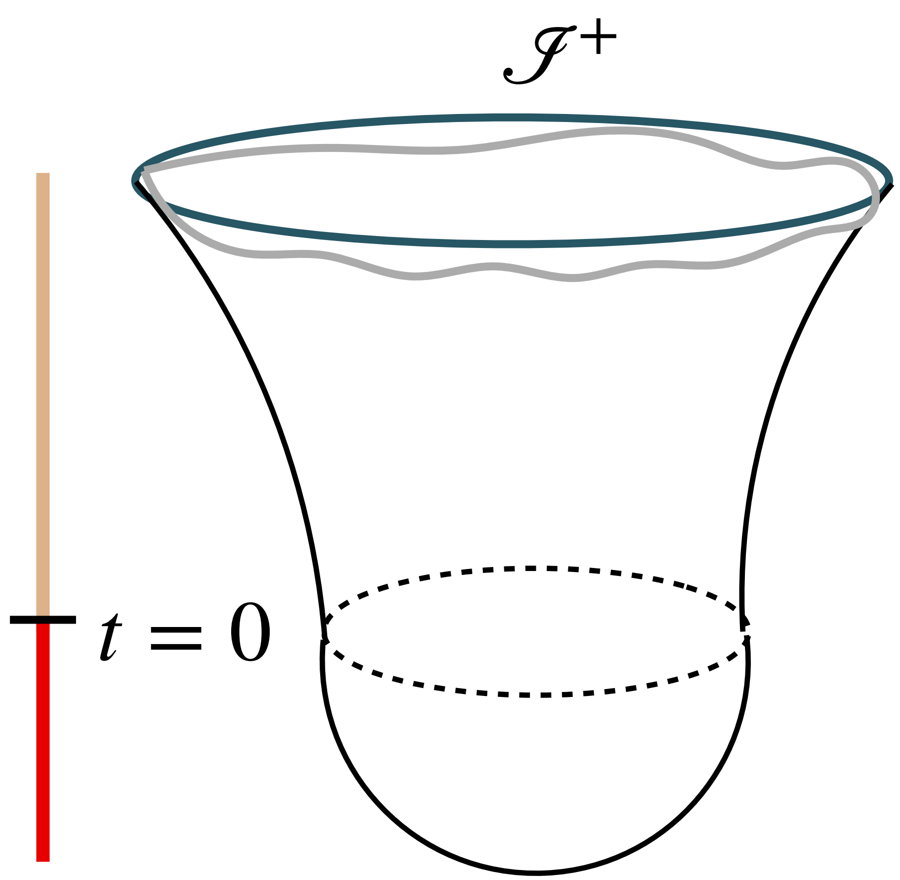

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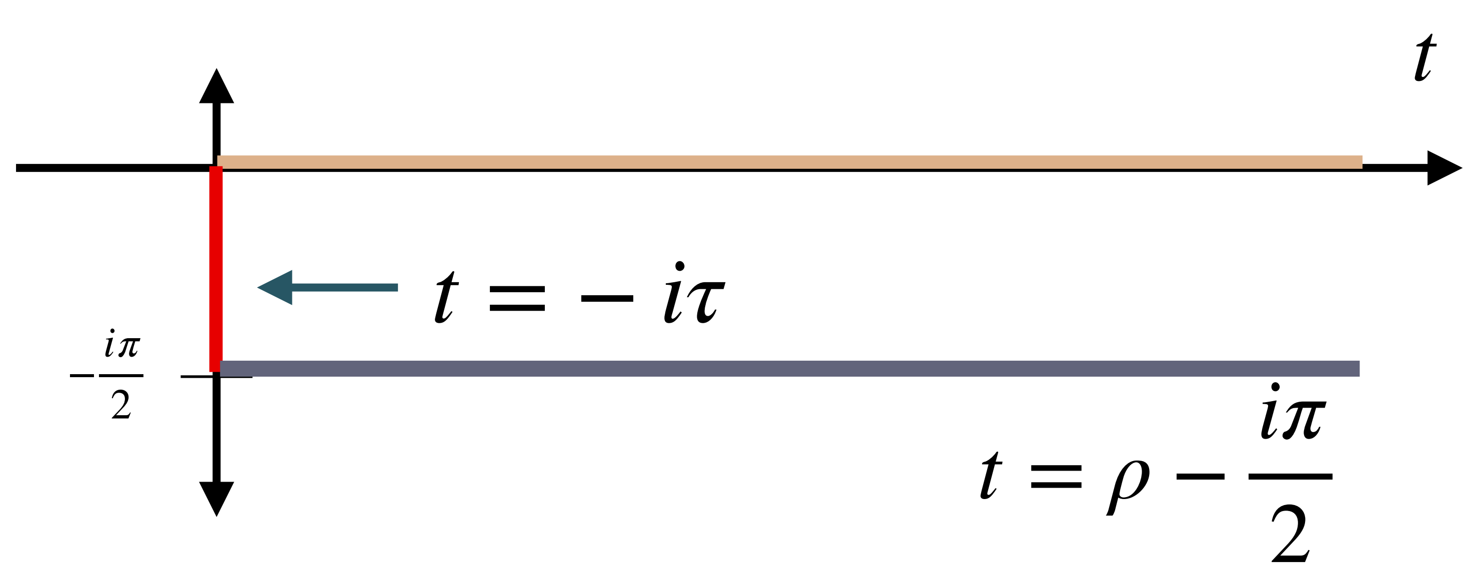

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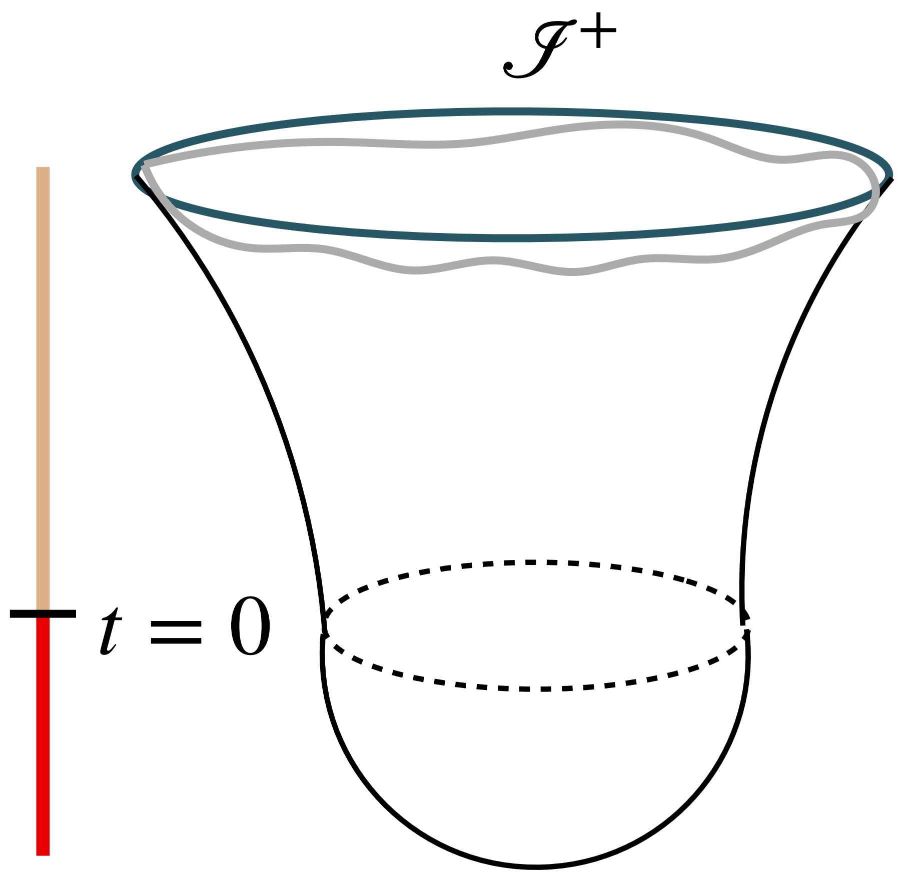
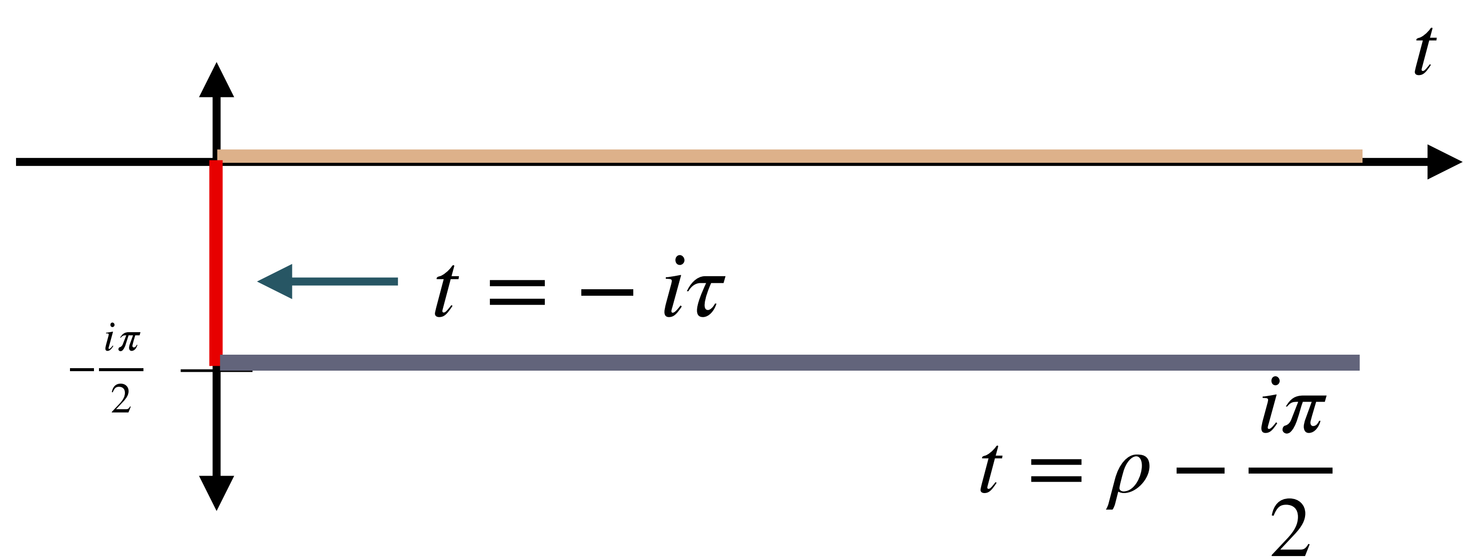


Classical solution is complex:

$$\begin{cases} ds^2 = -\left(d\rho^2 + \sinh^2\rho\, d\theta^2\right) \,, \\ \varphi = -\dfrac{2\pi i}{\beta J} \cosh\rho \,, \end{cases}$$



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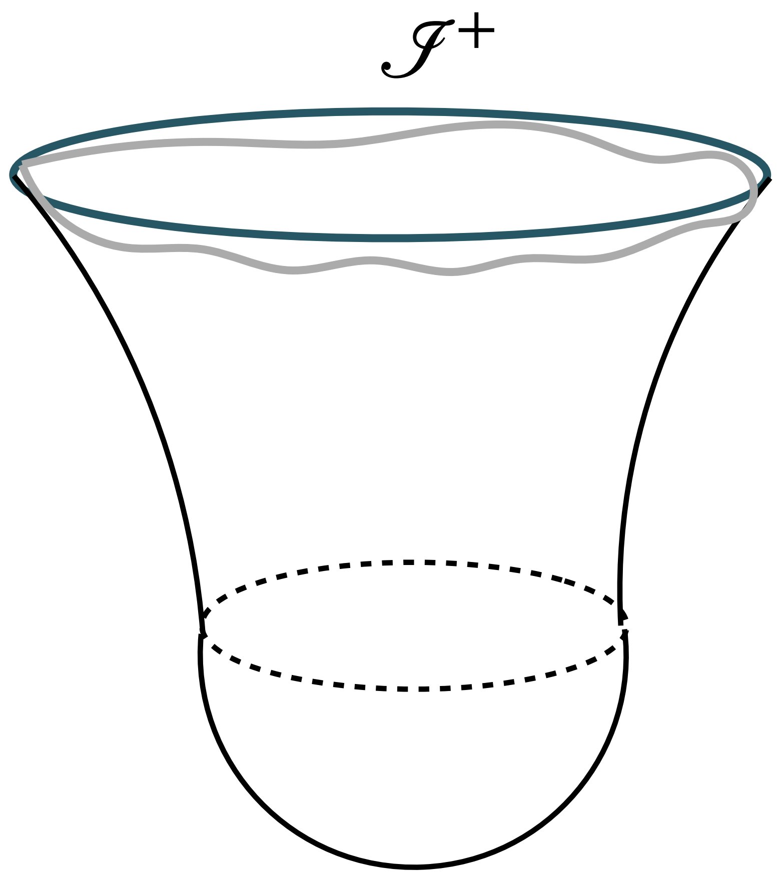
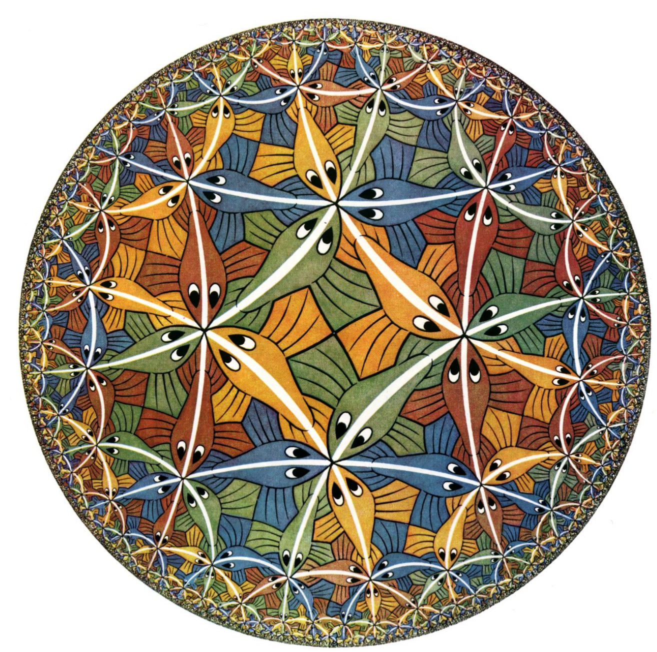


$$\begin{cases} ds^2 = -\left(d\rho^2 + \sinh^2\rho\, d\theta^2\right)\,, \\ \varphi = -\dfrac{2\pi i}{\beta J}\cosh\rho\,, \end{cases}$$

Hyperbolic disc in (-,-) signature.

Continuation from dS to EAdS

[Maldacena, '10]



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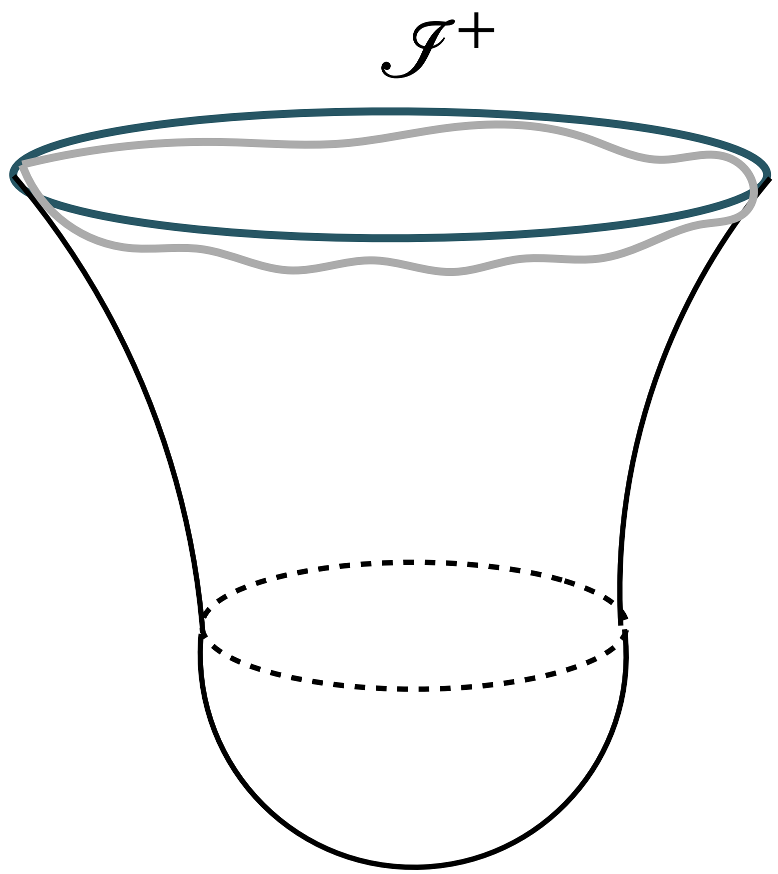
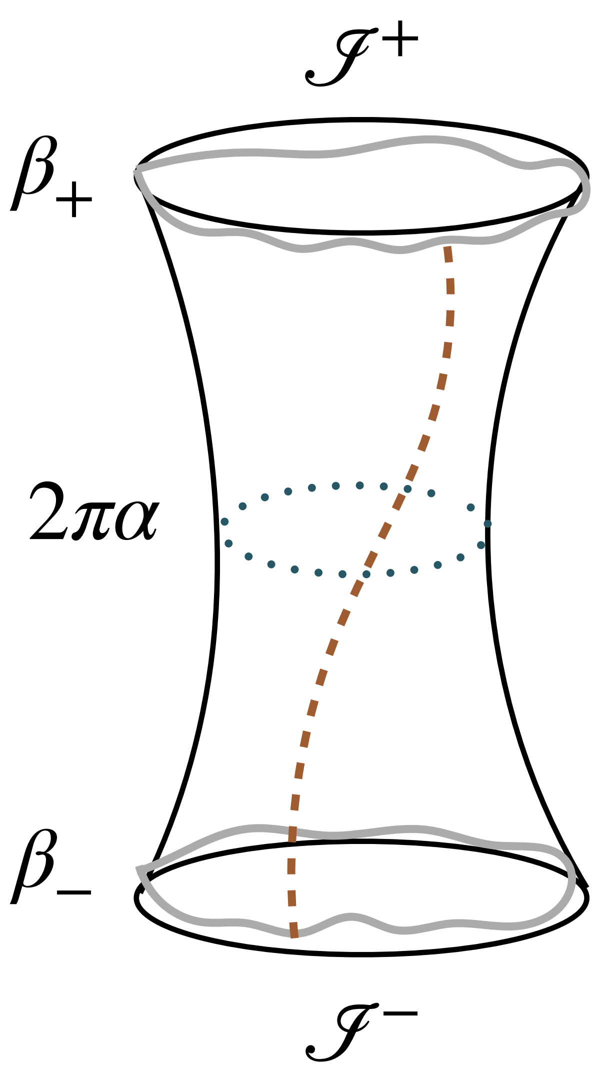


$$Z_{\mathrm{HH}} = \int_0^\infty dE\, \rho(E) e^{i\beta E} \sim \mathrm{tr}(e^{i\beta H})$$

$$\rho(E) = \frac{e^{S_0}\sqrt{G}}{2\pi^{3/2} J} \sinh\left(\sqrt{\frac{\pi E}{GJ}}\right)$$

GLOBAL NEARLY DS$_2$

Now the annulus partition function: $Z_{\text{global}} \approx \langle \beta_+ | \mathcal{U} | \beta_- \rangle$



Classical solutions (for $\beta_+ = \beta_- = \beta$):

$$ds^2 = -dt^2 + \alpha^2 \cosh^2 t \, d\Psi^2 \,,$$

$$\varphi = \frac{2\pi\alpha}{\beta J} \sinh t$$

$$\Psi = \theta + \gamma \Theta(t)$$

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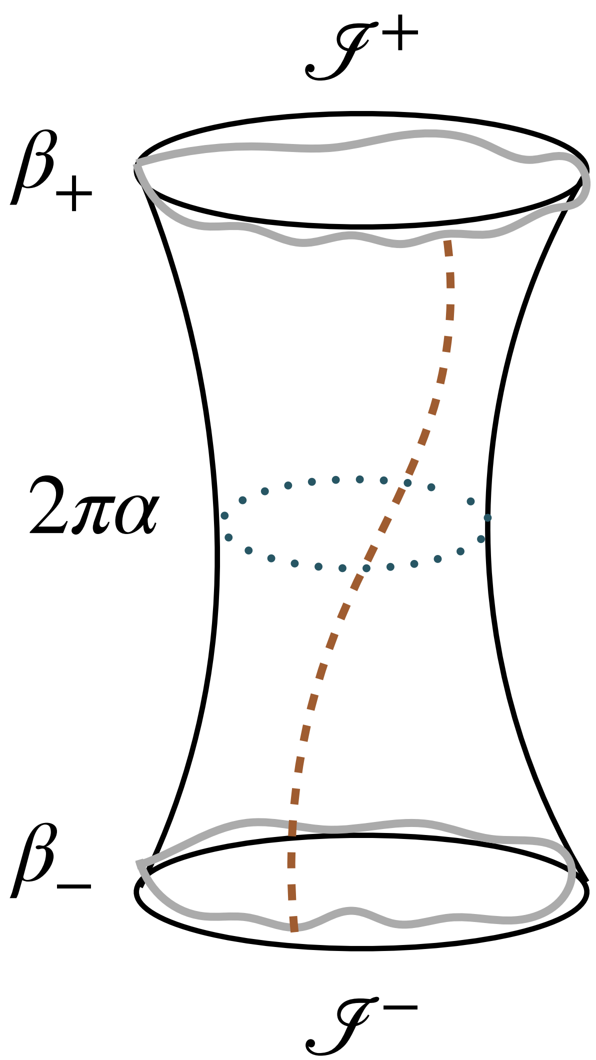

$$Z_{\text{global}} = -\int_0^{\alpha} \frac{d\alpha\,\alpha}{2G} \int_0^{2\pi} d\gamma \int [Df_+][Df_-] e^{iS[f_+,f_-]}$$

$$S[f_+, f_-] = \frac{1}{4G\beta_+ J} \int_0^{2\pi} d\theta \left(\{f_+(\theta), \theta\} + \frac{\alpha^2}{2} f_+'(\theta)^2 \right) - (+ \to -)$$

$$Z_{\text{global}} = -2\pi \int_0^{\infty} \frac{d\alpha\,\alpha}{2G} Z_T(\beta_+ J, \alpha) Z_T^*(\beta_- J, \alpha)$$

$$Z_T(\beta J, \alpha) = \frac{1}{\sqrt{2\pi}(-2i\beta J)^{1/2}} e^{\frac{\pi i \alpha^2}{4G\beta J}}$$

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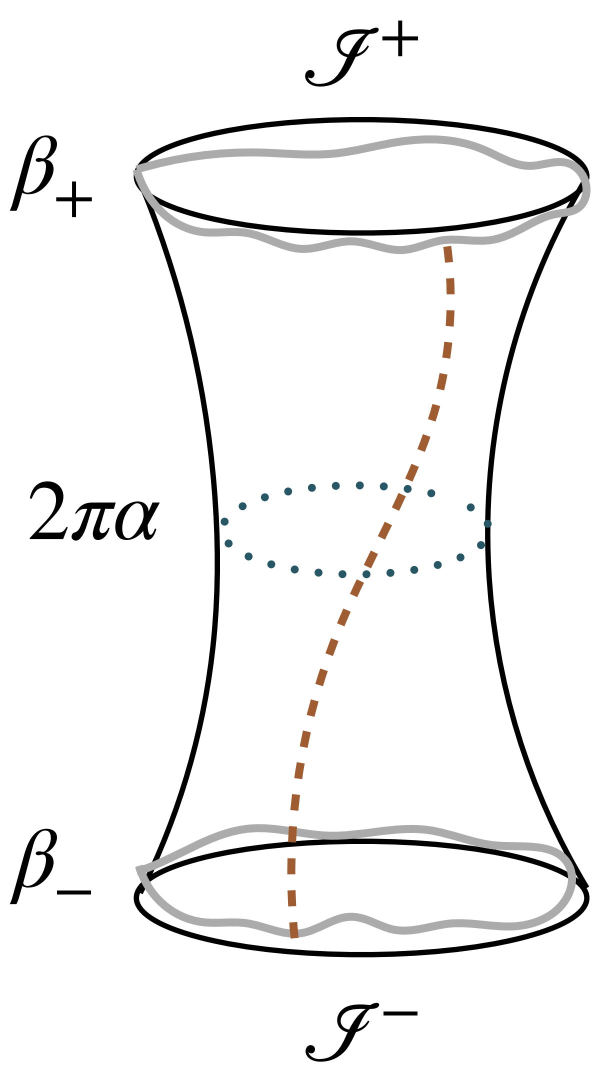

$$Z_{\text{global}} = -\int_0^{\alpha} \frac{d\alpha\,\alpha}{2G} \int_0^{2\pi} d\gamma \int [Df_+][Df_-] e^{iS[f_+,f_-]}$$

$$\boxed{= \frac{i}{2\pi} \frac{\sqrt{\beta_+\beta_-}}{\beta_+ - \beta_-}}$$

Again exact to all orders in G.

Can interpret as propagator for the universe.

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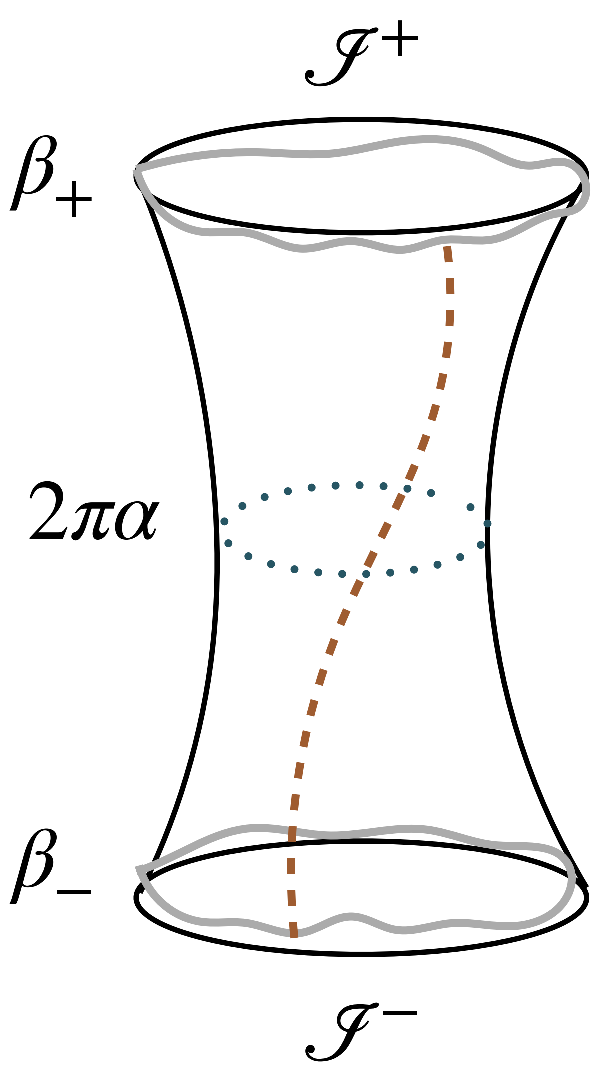


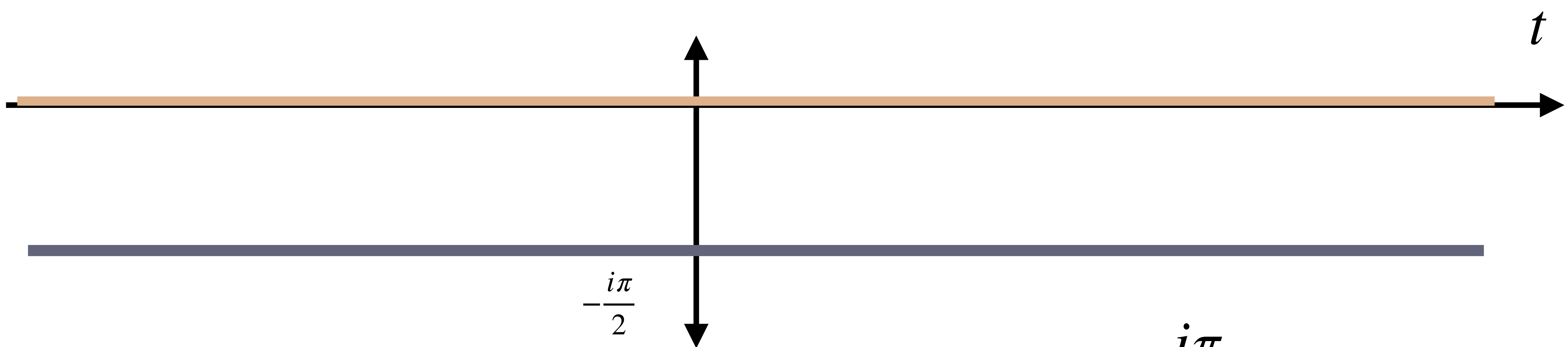
$$Z_{\text{global}} = \frac{i}{2\pi} \frac{\sqrt{\beta_+ \beta_-}}{\beta_+ - \beta_-}$$
$$= Z_{0,2}(\beta_1 J \to -i\beta_+ J, \beta_2 J \to i\beta_- J)$$

Continuation of annulus Z of $EAdS_2$

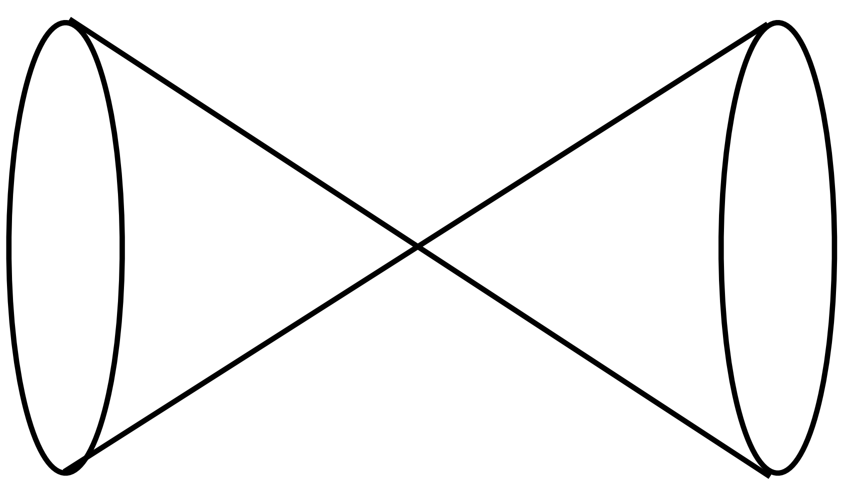
[Saad, Shenker, Stanford]

GLOBAL NEARLY DS_2

$$ds^2 = -\,dt^2 + \alpha^2 \cosh^2 t \, d\theta^2$$



$$t = \rho - \frac{i\pi}{2}$$



$$ds^2 = -\,(d\rho^2 + \alpha^2 \sinh^2 \rho \, d\theta^2)$$

TOPOLOGICAL GAUGE THEORY

Another way of thinking about it:

JT gravity in dS is equivalent to a $PSL(2;\mathbb{R})$ BF theory.

So is JT in Euclidean AdS!

For the annulus partition function of BF one integrates over Wilson loops around the circle.

Integral over α = integral over elliptic monodromies of $PSL(2;\mathbb{R})$.

HIGHER TOPOLOGIES

This viewpoint is ideally situated to tackle more complicated topologies, and so the genus expansion of JT dS gravity.

There are no non-singular Lorentzian R=2 geometries beyond the annulus. However we can *define* the gravity on more complicated topologies by integrating over smooth, flat gauge configurations. After some work (assuming a conjecture [Do '11]), the genus expansion coefficients are the continuation from those recently obtained for Euclidean AdS.

MATRIX INTEGRAL INTERPRETATION

Let us return to the question of de Sitter *holography.*

What dual structure can compute the various amplitudes?

[Saad, Shenker, Stanford] recently showed that the genus expansion of EAdS JT gravity coincides with the genus expansion of an appropriate double scaled one Hermitian-matrix integral

$$Z_{\mathrm{MM}} = \int dH \, \exp\left(-L \mathrm{tr}(V(H))\right)$$

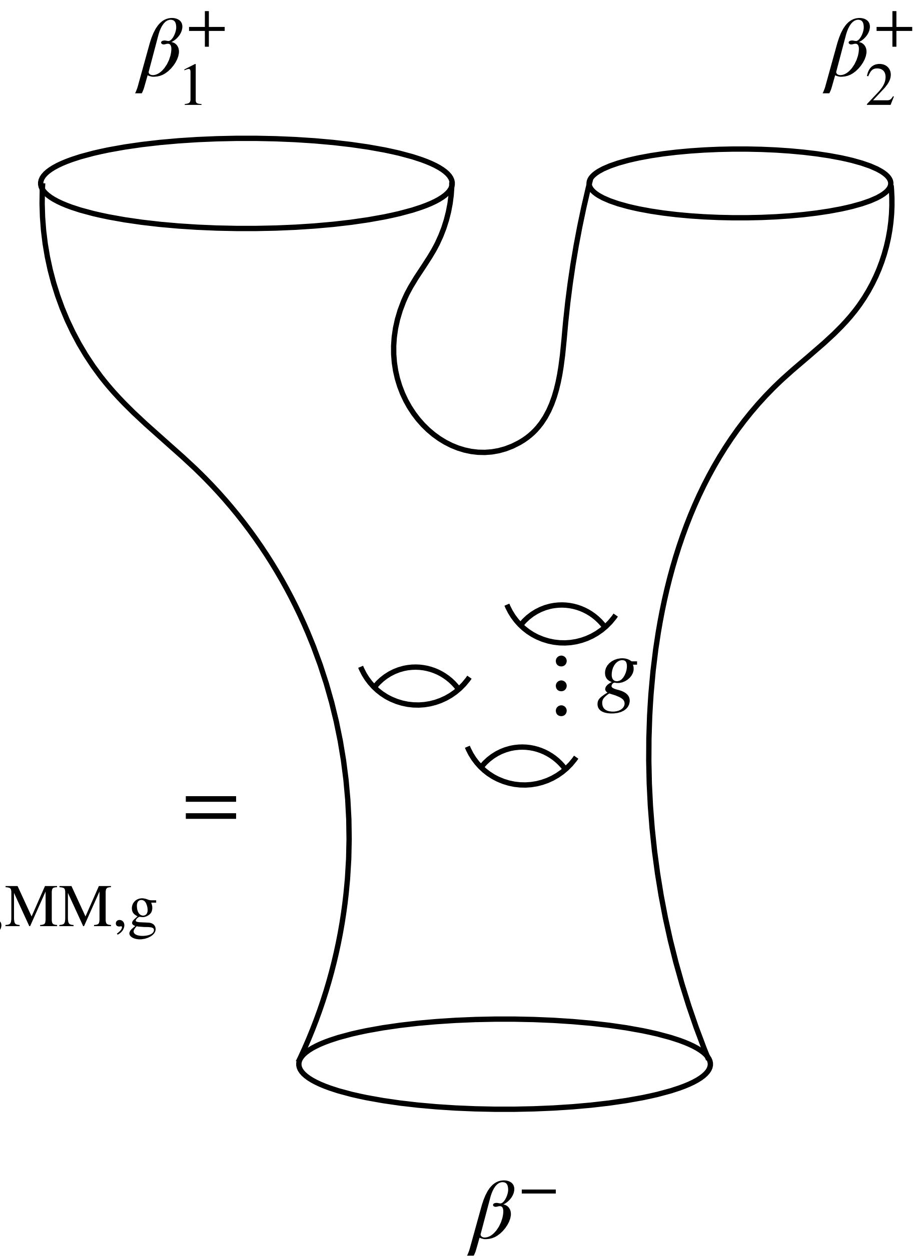
(whose leading density of states coincides with that of the Schwarzian theory).

MATRIX INTEGRAL INTERPRETATION

Our result implies that the genus expansion of JT dS gravity is encoded in the *same* ensemble.

An example of the dictionary:

$$\left\langle \mathrm{tr}\left(e^{i\beta_1^+ H}\right) \mathrm{tr}\left(e^{i\beta_2^+ H}\right) \mathrm{tr}\left(e^{-i\beta^- H}\right) \right\rangle_{\text{conn,MM,g}} =$$



BECAUSE THERE ARE RESURGICISTS HERE

The genus expansion is asymptotic, breaking down when

$$g = \beta J G \sim \exp\left(\frac{2S_0}{3}\right) .$$

The non-perturbative completion is non-unique.

A basic example of a non-perturbative effect is that the exact density of states is non-perturbatively small below the cut.

UNITARITY?

Is time evolution in this toy model unitary? [Cotler, KJ, unpublished]

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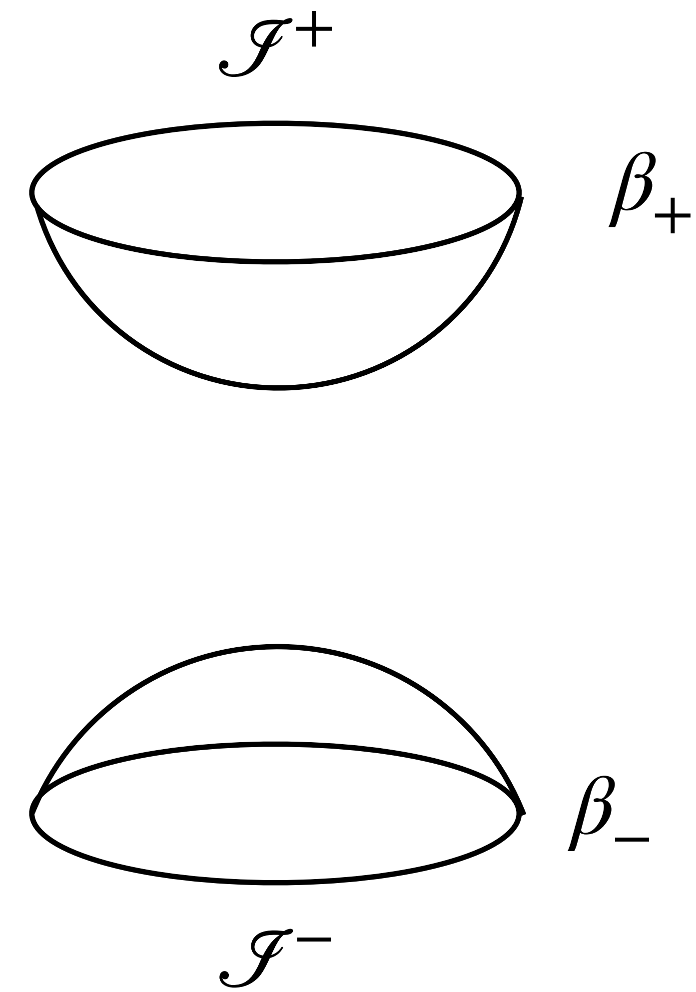
Is time evolution in this toy model unitary? [Cotler, KJ, unpublished]

Naively no:

The dominant process
is the creation/annihilation of
baby universes, e.g.



(enhanced by $\sim e^{2S_0}$ relative to annulus)

UNITARITY?

Then $\langle \beta_1 | \mathscr{U}^\dagger \mathscr{U} | \beta_2 \rangle \approx \langle \beta_1 | \mathscr{U}^\dagger | \varnothing \rangle \langle \varnothing | \mathscr{U} | \beta_2 \rangle$ is completely uncorrelated between β_1 and β_2.

However:

Need to account for normalization of $|\varnothing\rangle$!

Depends on $Z_{\text{sphere}} \sim e^{2S_0}$.

If we can discard $|\varnothing\rangle$*, then the "propagator" we found from annulus is consistent with approximate unitary evolution at large e^{S_0}, with a measure on $|\beta\rangle$ of the form $\dfrac{d\beta}{\beta}$.

$$Z_{\text{global}} = \frac{i}{2\pi} \frac{\sqrt{\beta_+ \beta_-}}{\beta_+ - \beta_-}$$

*I know of no principled reason to do this.

CONCLUSIONS

1. JT gravity as a toy model for quantum cosmology.

2. Partition functions related by continuation from Euclidean AdS JT gravity.

3. By virtue of [Saad, Shenker, Stanford], genus expansion of dS JT coincides with that of a double-scaled matrix integral.

4. Approximate bulk unitarity..? [WIP]

THANK YOU!