Flux Tube S-matrix bootstrap

Non-Perturbative Methods in Quantum Field Theory
ICTP — 9/2019

Joan Elias Miró

talk based on hep-th/1906.08098
w/ A. Guerrieri, A. Hebbar, J. Penedones, P. Vieira

1.- Flux tube effecive action

2.- Observables:

2.1 S-matrix of branons, bounds on Wilson coefficients),

2.2 Finite volume Energy spectrum.

3.- **Phenomenology** of flux tubes and YM data.

Set up: a QFT_D, gapped, with string like states.

for instance:

- Yang Mills Fluxe Tubes,
- Nielsen-Abrikosov stirngs,
- Domain walls in 3D Ising.

Bulk Poincaré is spontaneously broken,

$$ISO(1, D-1) \to ISO(1,1) \otimes O(D-2)$$

Goldstone modes

$$X^{\mu} = (\sigma^{\alpha}, X^{i}(\sigma))$$

We build the effective action out of

$$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

$$K^\mu_{\alpha\beta} = \nabla_\alpha \partial_\beta X^\mu = \partial_\alpha \partial_\beta X^\mu + \cdots$$

Bulk Poincaré is spontane

$$ISO(1, D -$$

Goldstone modes

$$X^\mu = (\sigma^\alpha, X^i(\sigma))$$

$$A = \int d^2\sigma \sqrt{-h} \left[\ell_s^{-2} + \mathcal{R} + K^2 + a\, \ell_s^2 (K^i_{\alpha\beta} K_i^{\alpha\beta})^2 + b\, \ell_s^2 (K^i_{\alpha\beta} K^{j\,\alpha\beta})^2 + \cdots \right]$$

We build the effective action out of

$$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

$$K^\mu_{\alpha\beta} = \nabla_\alpha \partial_\beta X^\mu = \partial_\alpha \partial_\beta X^\mu + \cdots$$

Bulk Poincaré is spontane

$$ISO(1, D -$$

Goldstone modes

$$X^\mu = (\sigma^\alpha, X^i(\sigma))$$

$$A = \int d^2\sigma \sqrt{-h} \left[\ell_s^{-2} + \mathcal{R} + K^2 + a\, \ell_s^2 (K^i_{\alpha\beta} K_i^{\alpha\beta})^2 + b\, \ell_s^2 (K^i_{\alpha\beta} K^{j\,\alpha\beta})^2 + \cdots \right]$$

S-matrix

We scatter two massless vectors of O(D-2),

$$\mathbb{S}(s) = \sigma_1(s)\delta_a^b\delta_c^d + \sigma_2(s)\delta_a^c\delta_b^d + \sigma_3(s)\delta_a^d\delta_b^c$$

$$= S_{\rm sing}(s)\mathbb{P}_{\rm sing} + S_{\rm sym}(s)\mathbb{P}_{\rm sym} + S_{\rm asym}(s)\mathbb{P}_{\rm asym}$$

It is convenient to use phase-shifts

$$S_{\rm rep}(s) \equiv e^{2i\delta_{\rm rep}(s)}$$

Unitarity: for $s > 0$

$$|S_{\text{sing}}|^2 \equiv |(D-2)\sigma_1 + \sigma_2 + \sigma_3| \leq 1$$

$$|S_{\text{asym}}|^2 \equiv |\sigma_2 - \sigma_3| \leq 1$$

$$|S_{\text{sym}}|^2 \equiv |\sigma_2 + \sigma_3| \leq 1$$

$$\begin{aligned} \mathbb{S}(s) &= \sigma_1(s)\delta_a^b\delta_c^d + \sigma_2(s)\delta_a^c\delta_b^d + \sigma_3(s)\delta_a^d\delta_b^c \\ &= S_{\text{sing}}(s)\mathbb{P}_{\text{sing}} + S_{\text{sym}}(s)\mathbb{P}_{\text{sym}} + S_{\text{asym}}(s)\mathbb{P}_{\text{asym}} \end{aligned}$$

It is convenient to use phase-shifts

$$S_{\text{rep}}(s) \equiv e^{2i\delta_{\text{rep}}(s)}$$

Phase-shfits, units $\ell_s = 1$ target Lorentz implies $\alpha_2 = \frac{D-26}{384\pi}$

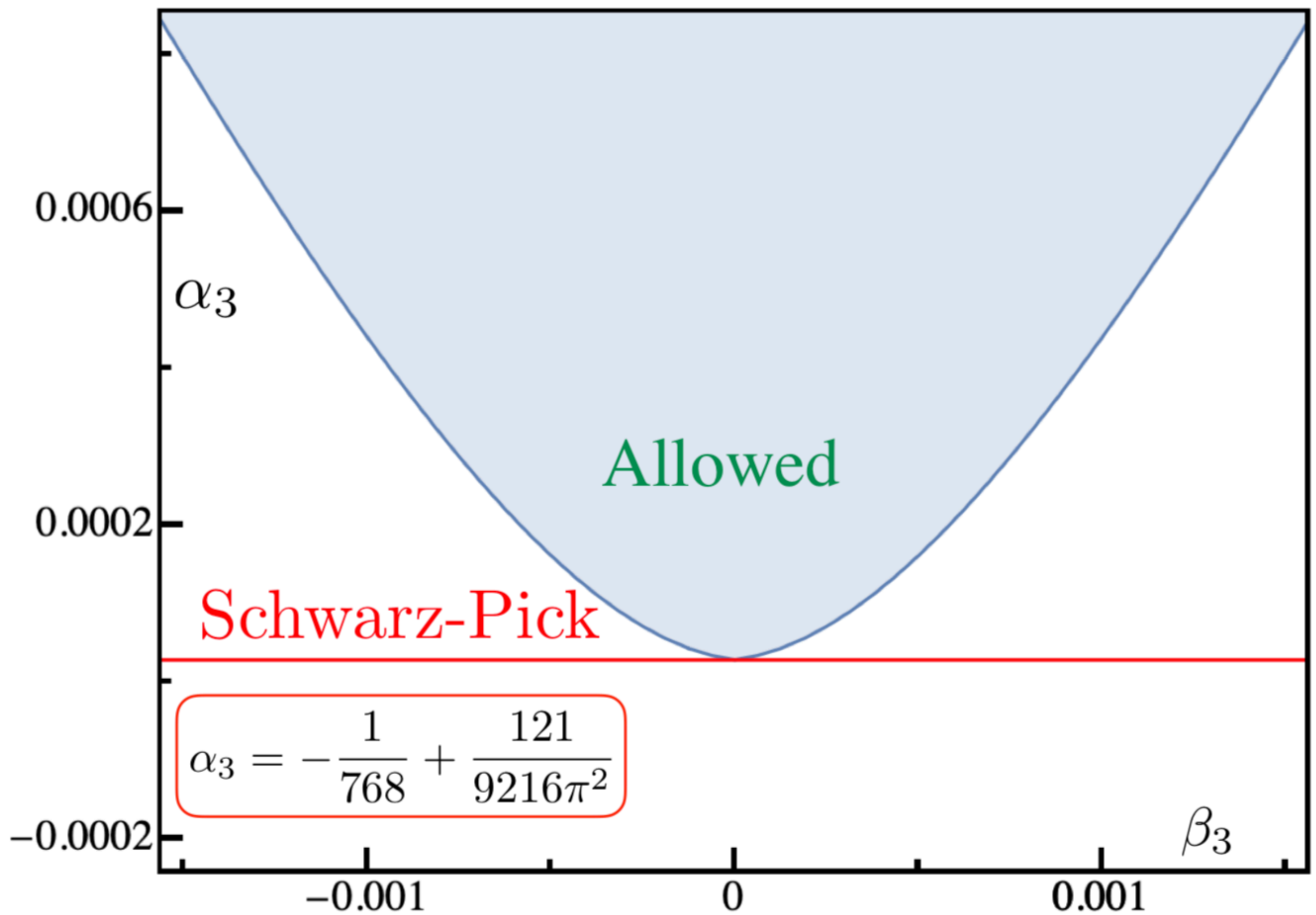
$$2\delta_{sym} = \frac{s}{4} + \alpha_2 s^2 + \alpha_3 s^3 + O(s^4)$$

$$2\delta_{anti} = \frac{s}{4} - \alpha_2 s^2 + (\alpha_3 + 2\beta_3) s^3 + O(s^4)$$

$$2\delta_{sing} = \frac{s}{4} - (D-3)\alpha_2 s^2 + (\alpha_3 - (D-2)\beta_3) s^3 + O(s^4)$$

(rotation of non univ. ops.)

$(\alpha_3, \beta_3) = M.(a, b)$



Phase-shfits, units $\ell_s = 1$ target Lorentz implies $\alpha_2 = \frac{D-26}{384\pi}$

$$2\delta_{sym} = \frac{s}{4} + \alpha_2 s^2 + \alpha_3 s^3 + O(s^4)$$

(rotation of non univ. ops.)

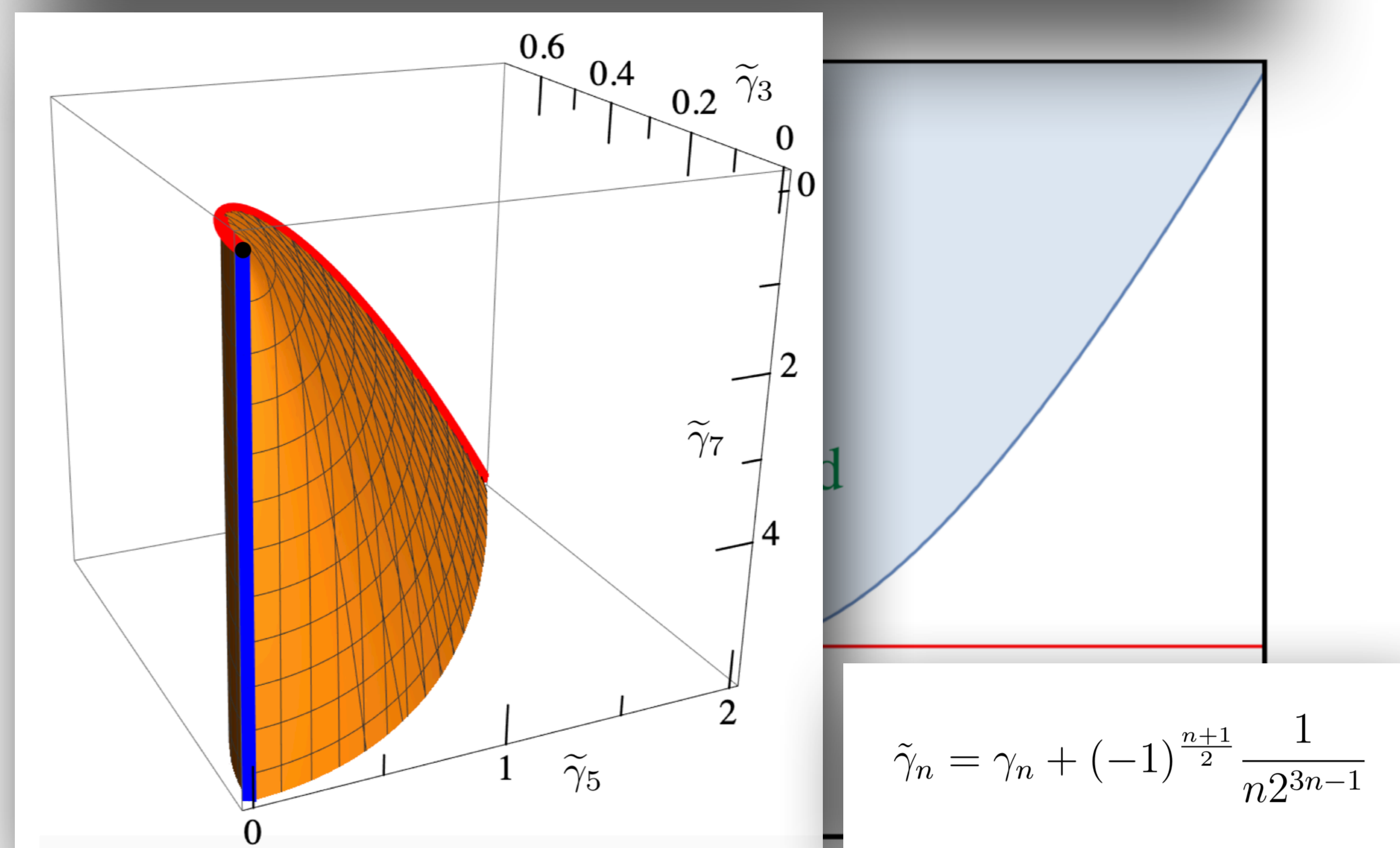
$$2\delta_{anti} = \frac{s}{4} - \alpha_2 s^2 + (\alpha_3 + 2\beta_3) s^3 + O(s^4)$$

$$(\alpha_3, \beta_3) = M.(a, b)$$

$$2\delta_{sing} = \frac{s}{4} - (D-3)\alpha_2 s^2 + (\alpha_3 - (D-2)\beta_3) s^3 + O(s^4)$$

D=3

$$2\delta(s) = \frac{s}{4} + \gamma_3 s^3 + \gamma_5 s^5 + \gamma_7 s^7 + i\gamma_8 s^8 + O(s^9)$$

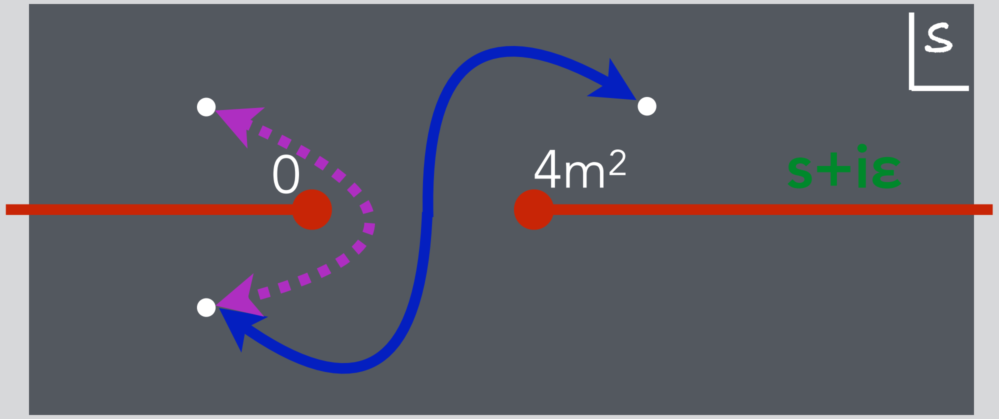


$$\tilde{\gamma}_n = \gamma_n + (-1)^{\frac{n+1}{2}} \frac{1}{n 2^{3n-1}}$$

$$\text{Reality } S^*(s) = S(s^*)$$

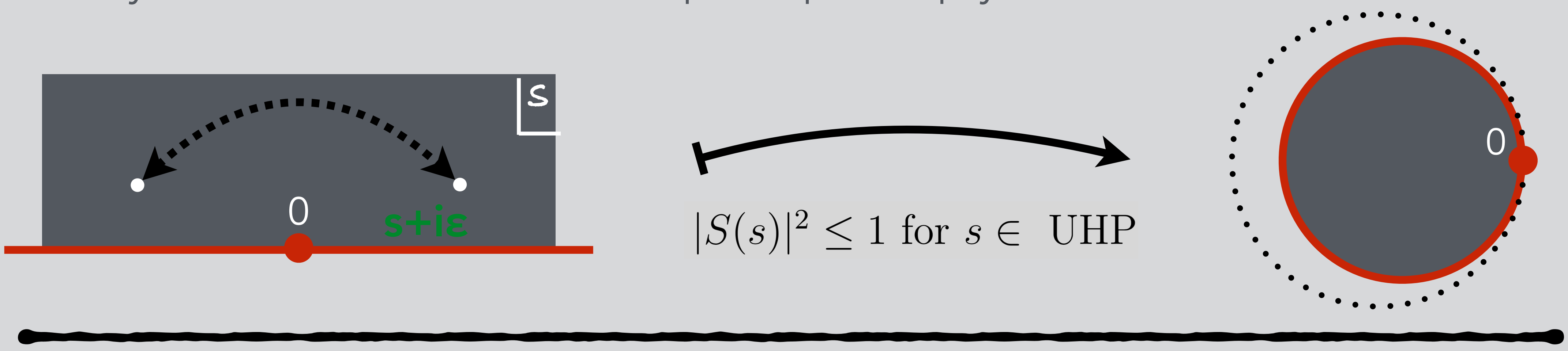
$$\text{Crossing } S(4m^2 - s) = S(s)$$

$$|S(s+i\epsilon)|^2 \leq 1 \text{ for } s > 4m^2$$



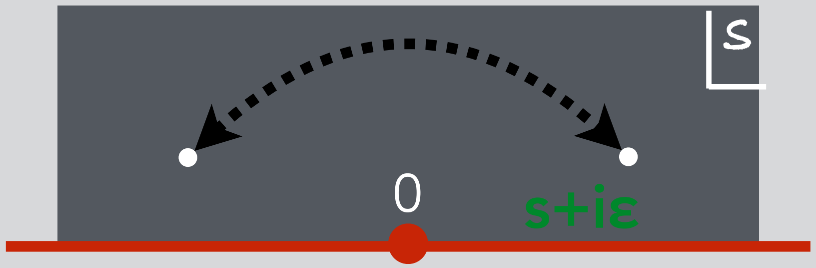
In the massless limit points in the UHP are related by $S(-s^*) = [S(s)]^*$

Unitarity and maximum modulus principle imply

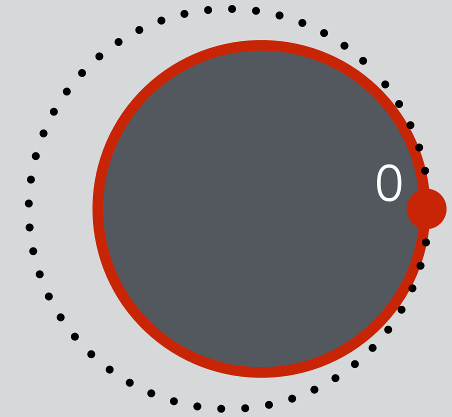


–0.001 0 0.001

Unitarity and maximum modulus principle imply



$$|S(s)|^2 \leq 1 \text{ for } s \in \text{ UHP}$$



Trick!

$$S^{(1)}(z|w) \equiv \frac{S(z)-S(w)}{1-S(z)\overline{S(w)}} \Big/ \frac{z-w}{z-\overline{w}}$$

Again, unitarity and maximum modulus principle imply

$$|S^{(1)}(z|w)|_{\mathrm{Im}z\geq 0} \leq 1$$

Schwarz-Pick thm.

Expansion around threshold leads to

$$S^{(1)}(ix|iy) = -1 + \left(\tfrac{1}{96} + 8\gamma_3\right) xy + \cdots \geq -1 \quad , \quad \gamma_3 \geq -\frac{1}{768}$$

Generalisation to multiple points

$$\{S^{(1)}[S,w](s),\ S^{(2)}[S,w_1,w_2](s) = S^{(1)}[S^{(1)}[S,w],w_2](s), \cdots\}$$

−0.001 0 0.001

Phase-shfits, units $\ell_s = 1$ target Lorentz implies $\alpha_2 = \frac{D-26}{384\pi}$

$$2\delta_{sym} = \frac{s}{4} + \alpha_2 s^2 + \alpha_3 s^3 + O(s^4)$$

$$2\delta_{anti} = \frac{s}{4} - \alpha_2 s^2 + (\alpha_3 + 2\beta_3) s^3 + O(s^4)$$

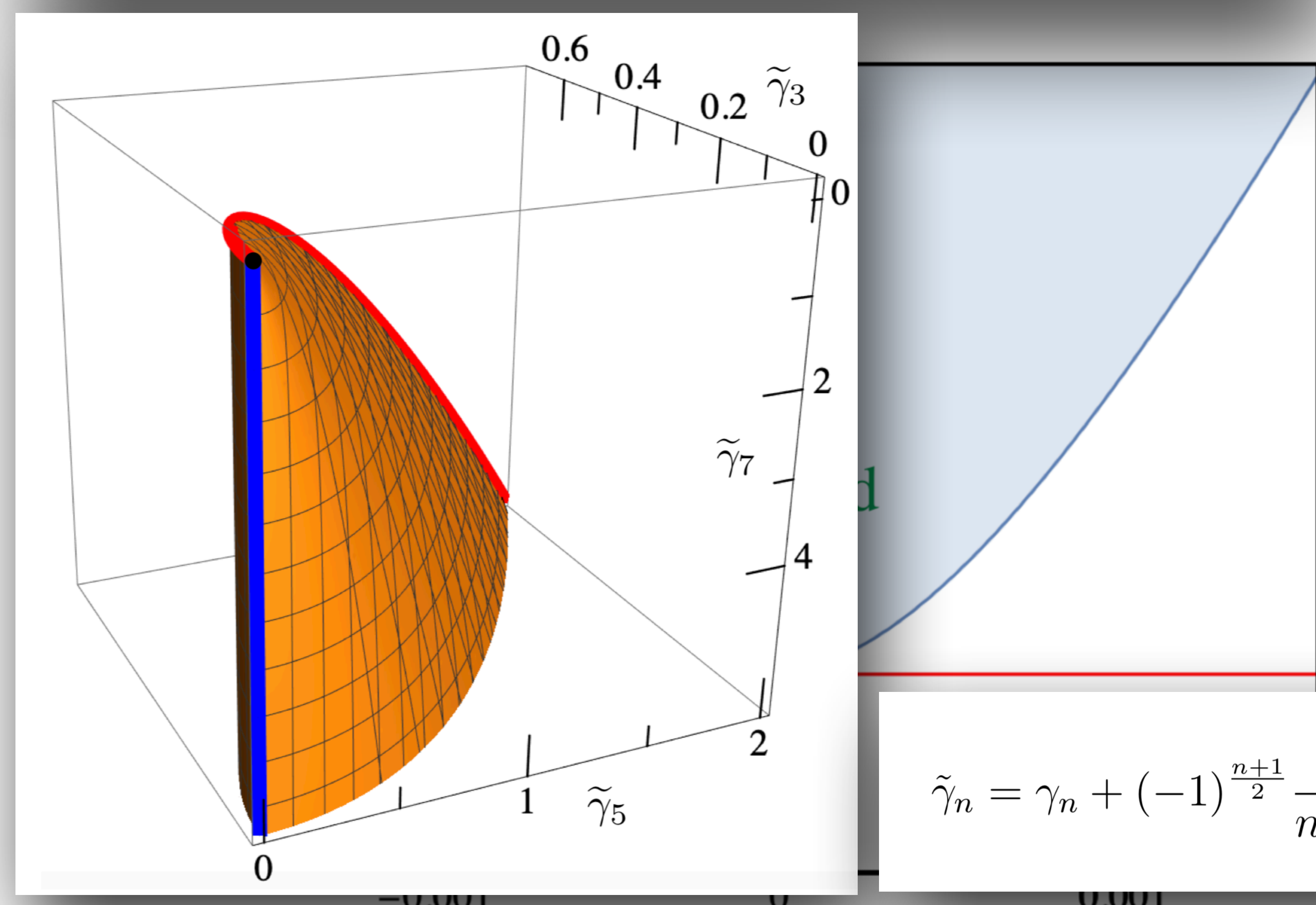
$$2\delta_{sing} = \frac{s}{4} - (D-3)\alpha_2 s^2 + (\alpha_3 - (D-2)\beta_3) s^3 + O(s^4)$$

(rotation of non univ. ops.)

$(\alpha_3, \beta_3) = M.(a, b)$

D=3

$$2\delta(s) = \frac{s}{4} + \gamma_3 s^3 + \gamma_5 s^5 + \gamma_7 s^7 + i\gamma_8 s^8 + O(s^9)$$



$$\tilde{\gamma}_n = \gamma_n + (-1)^{\frac{n+1}{2}} \frac{1}{n 2^{3n-1}}$$

Phase-shfits, units $\ell_s = 1$ target Lorentz implies $\alpha_2 = \frac{D-26}{384\pi}$

$$2\delta_{sym} = \frac{s}{4} + \alpha_2 s^2 + \alpha_3 s^3 + O(s^4)$$

$$2\delta_{anti} = \frac{s}{4} - \alpha_2 s^2 + (\alpha_3 + 2\beta_3) s^3 + O(s^4)$$

$$2\delta_{sing} = \frac{s}{4} - (D-3)\alpha_2 s^2 + (\alpha_3 - (D-2)\beta_3) s^3 + O(s^4)$$

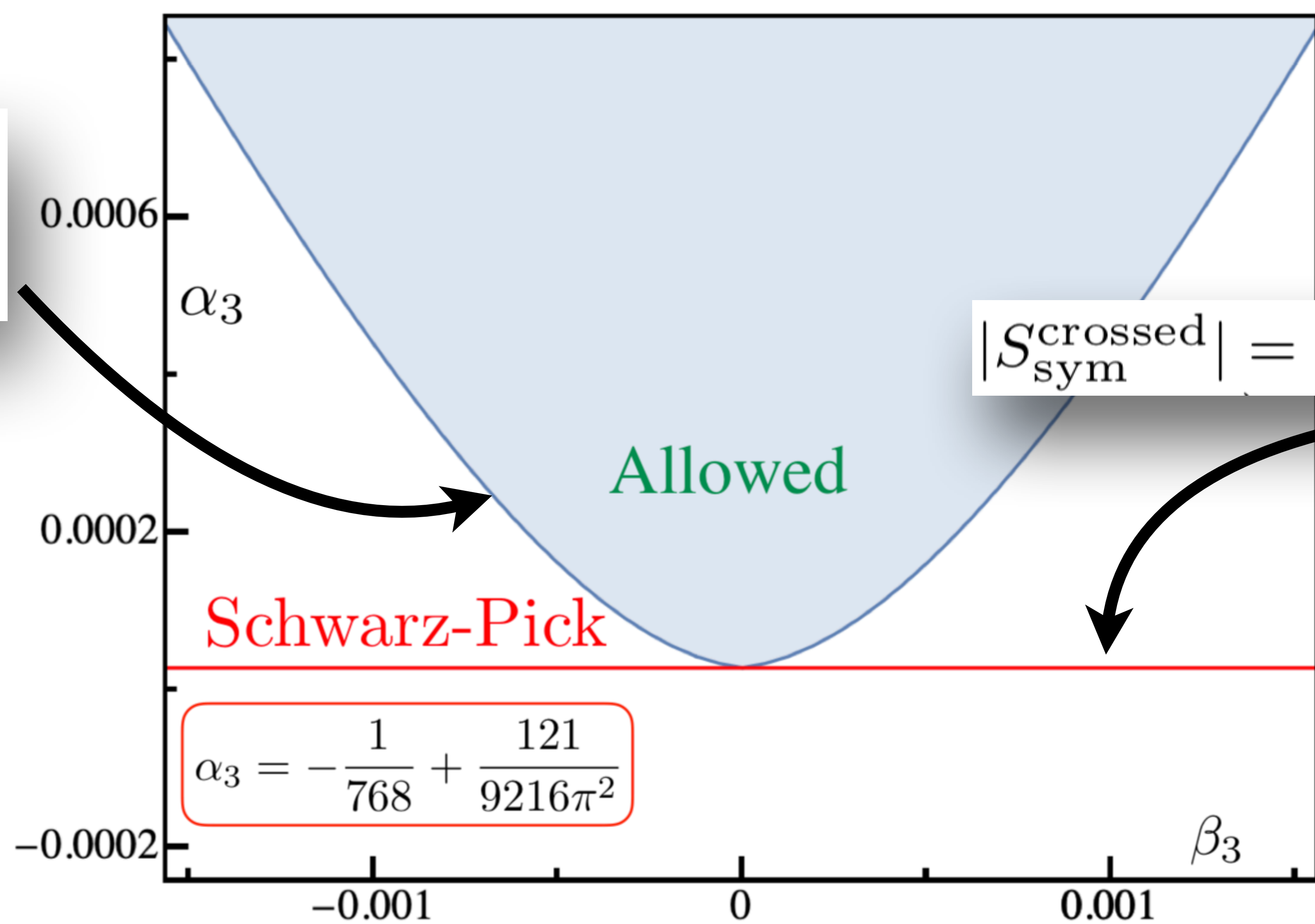
(rotation of non univ. ops.)

$(\alpha_3, \beta_3) = M.(a, b)$

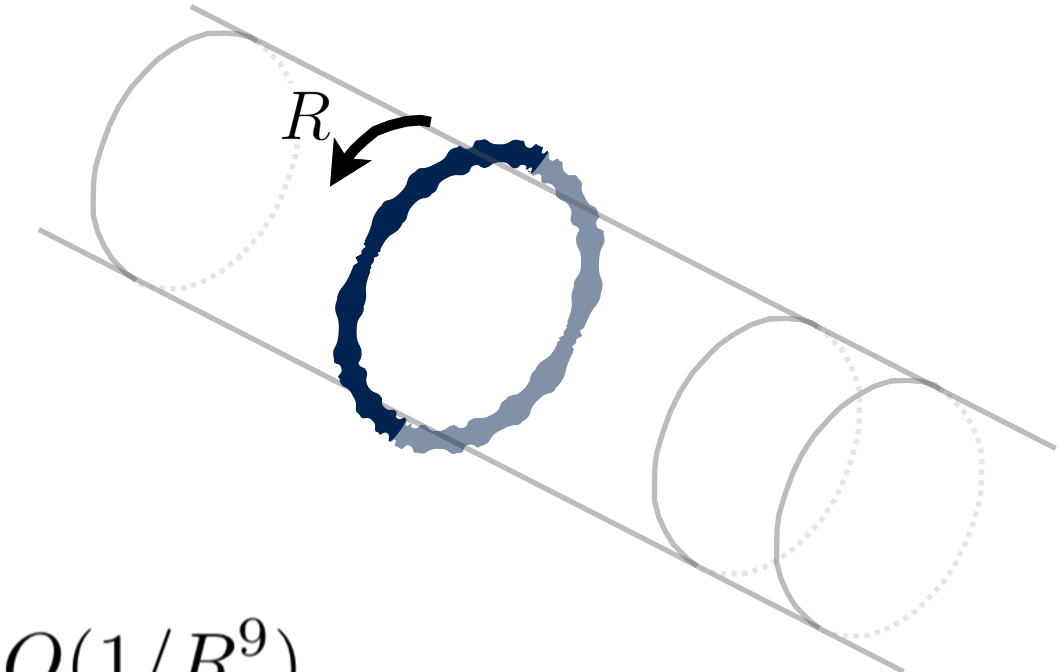
Numerics

$$S_{\text{ansatz}} = \sum_{n=0}^{N_{max}} a_n \chi^n$$

$$|S_{\text{sym}}^{\text{crossed}}| = \frac{1}{2} |S_{\text{sing}} + S_{\text{anti}}| \leq 1$$



Finite volume energy levels



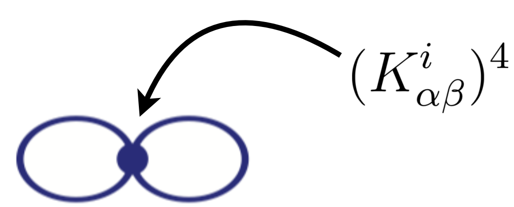
$$E_0(R) = \sqrt{R^2 - \tfrac{\pi}{3}(D-2)} + \frac{\delta(D)}{R^7} + O(1/R^9)$$

$$\delta(D) = \frac{32\pi^6(2-D)((D-2)\alpha_3+(D-4)\beta_3)}{225}$$

$$\delta(4) = -\frac{128\pi^6\alpha_3}{225} \leq \frac{\pi^6}{1350} - \frac{121\pi^4}{16200}$$

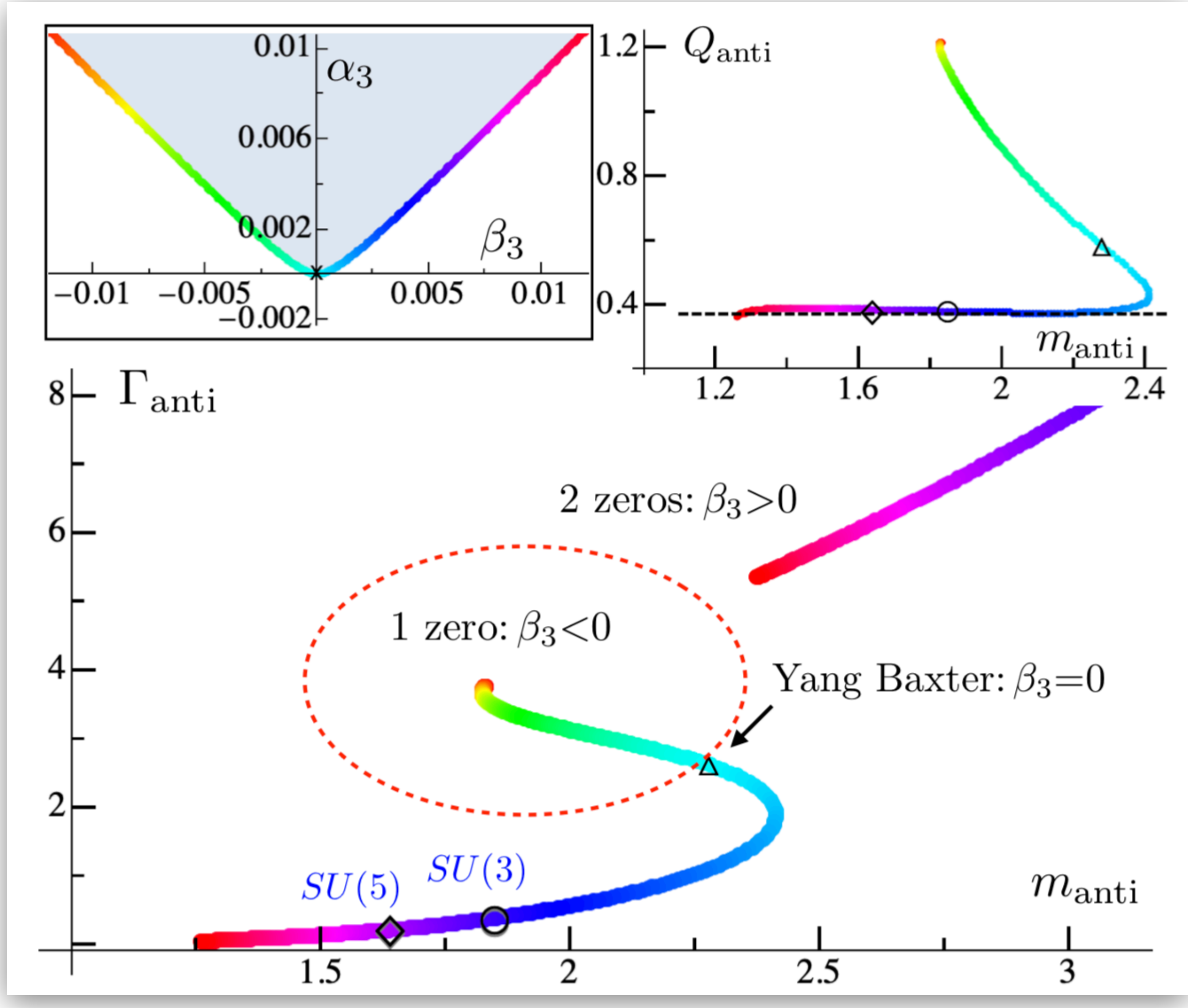
$$\delta(3) = -\frac{32\pi^6\gamma_3}{225} \leq \frac{\pi^6}{5400}$$

This high order calculation is possible thanks to a trick combining

TBA + 

also splitting of excited energy levels is sensitive to $(K^i_{\alpha\beta})^4$

Flux Tube Phenomenology



spectrum $[m,\Gamma]$	$SU(3)$	$SU(5)$
axion	[1.85, 0.39]	[1.64, 0.22]
axion*	[3.25, 8.84]	[2.83, 7.02]
symmetron	[2.36, 4.99]	[2.34, 4.54]
dilaton	[1.88, 3.37]	[1.84, 3.52]

Summary and outlook

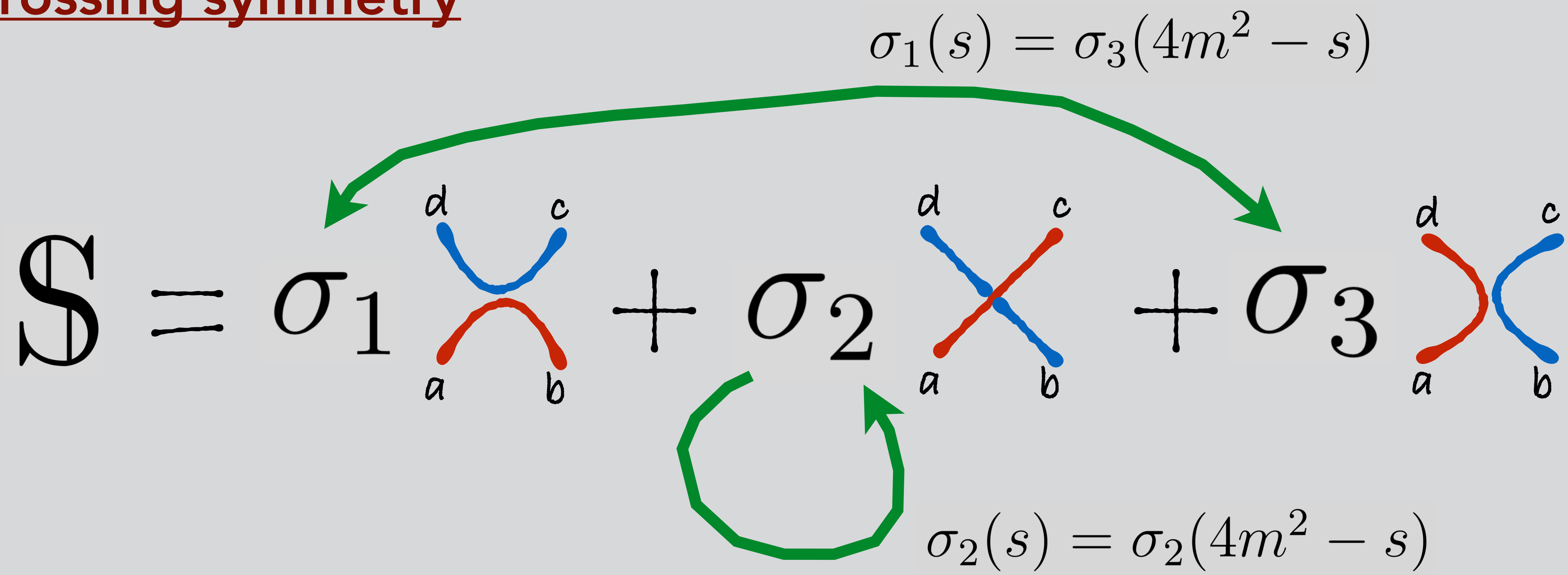
- First time optimal bounds on Wilson coefficients are derived.
- Would be nice to apply similar ideas to 4D EFTs.

On the branon scattering

- Derive the D=4 Flux tube line analytically, maybe some theorem for vector valued holomorphic functions?
- Take into account what is known about universal inelasticity.
- Understand better the high energy regime.
- It would be nice to fully pin down the Yang-Mills flux tube EFT :-)
- …

Backup slides

Crossing symmetry



$$
\begin{aligned}
\mathbb{S}(s) &= \sigma_1(s)\,\delta_a^b\delta_c^d + \sigma_2(s)\,\delta_a^c\delta_b^d + \sigma_1(s)\,\delta_a^d\delta_b^c \\
&= S_{\text{sing}}(s)\,\mathbb{P}_{\text{sing}} + S_{\text{sym}}(s)\,\mathbb{P}_{\text{sym}} + S_{\text{asym}}(s)\,\mathbb{P}_{\text{sym}}
\end{aligned}
$$

Unitarity: for $s > 0$

$$|S_{\text{sing}}|^2 \equiv |(D-2)\sigma_1 + \sigma_2 + \sigma_3| \leq 1$$

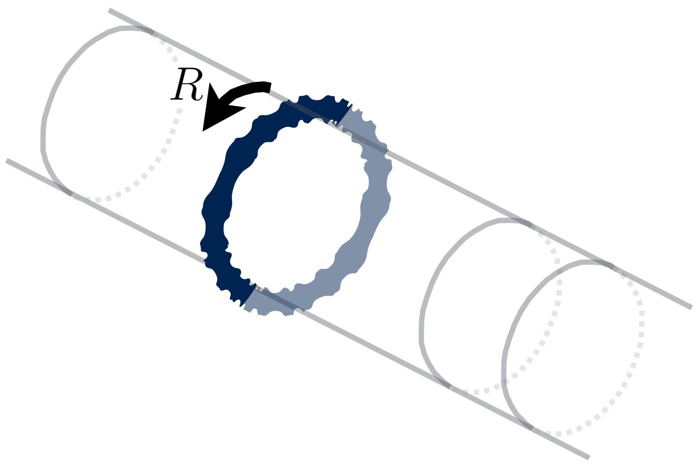
$$|S_{\text{asym}}|^2 \equiv |\sigma_2 - \sigma_3| \leq 1$$

$$|S_{\text{sym}}|^2 \equiv |\sigma_2 + \sigma_3| \leq 1$$

$$\begin{aligned} \mathbb{S}(s) &= \sigma_1(s)\delta_a^b\delta_c^d + \sigma_2(s)\delta_a^c\delta_b^d + \sigma_3(s)\delta_a^d\delta_b^c \\ &= S_{\text{sing}}(s)\mathbb{P}_{\text{sing}} + S_{\text{sym}}(s)\mathbb{P}_{\text{sym}} + S_{\text{asym}}(s)\mathbb{P}_{\text{asym}} \end{aligned}$$

It is convenient to use phase-shifts

$$S_{\text{rep}}(s) \equiv e^{2i\delta_{\text{rep}}(s)}$$



 $= \dfrac{32\pi^6(2-D)((D-2)\alpha_3+(D-4)\beta_3)}{225R^8}$

$$a\,\ell_s^2(K^i_{\alpha\beta}K_i^{\alpha\beta})^2 + b\,\ell_s^2(K^i_{\alpha\beta}K^{j\,\alpha\beta})^2$$

$$A = A_{\text{int}} + A_{\cancel{int}}$$

$$(\partial_\mu\partial_\nu X^i)^2\left[(\partial_\rho X^j)^4 - \frac{1}{2}\partial_\rho X^j\partial_\sigma X^j\partial^\rho X^k\partial_\sigma X^k\right]$$

 $= \partial_\nu\partial_\alpha\partial_\beta\Delta_R(0)\,\partial_\nu\partial_\gamma\partial_\beta\Delta_R(0)\,\partial_\alpha\partial_\gamma\Delta_R(0) = 0$

Large R TBA w/

$$2\delta_{sym} = \frac{s}{4} + \alpha_2 s^2 + \alpha_3 s^3 + O(s^4)$$
$$2\delta_{anti} = \frac{s}{4} - \alpha_2 s^2 + (\alpha_3+2\beta_3)s^3 + O(s^4)$$
$$2\delta_{sing} = \frac{s}{4} - (D-3)\alpha_2 s^2 + (\alpha_3-(D-2)\beta_3)s^3 + O(s^4)$$