

A SUSY RG-Flow Using Hamiltonian Methods

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True for: Yukawa in 3D or fermion mass in 4D

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There's no dependence of observables on the volume

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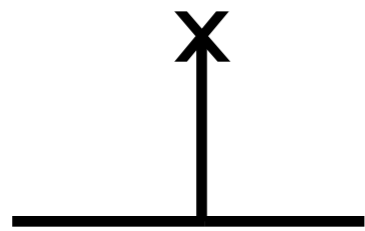
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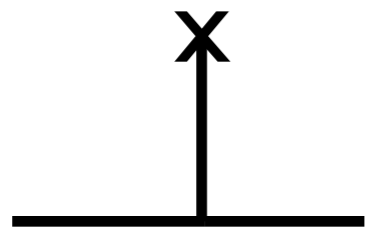
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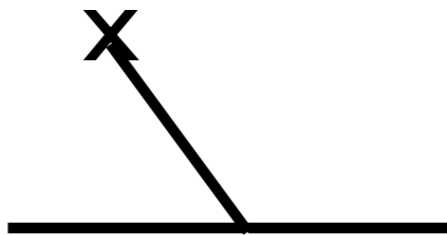
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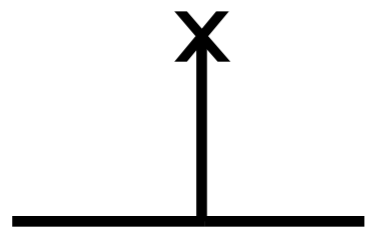
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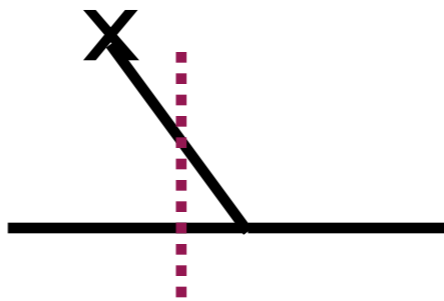


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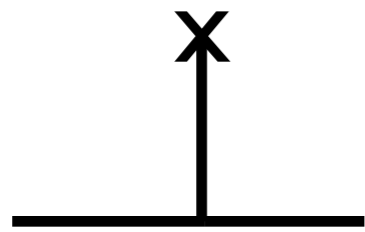
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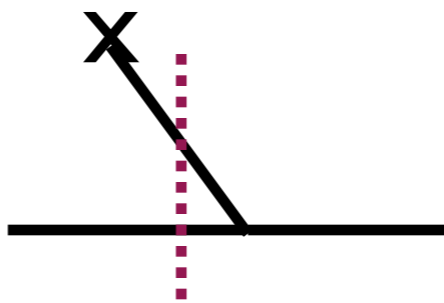


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LC lacks appropriate intermediate states!

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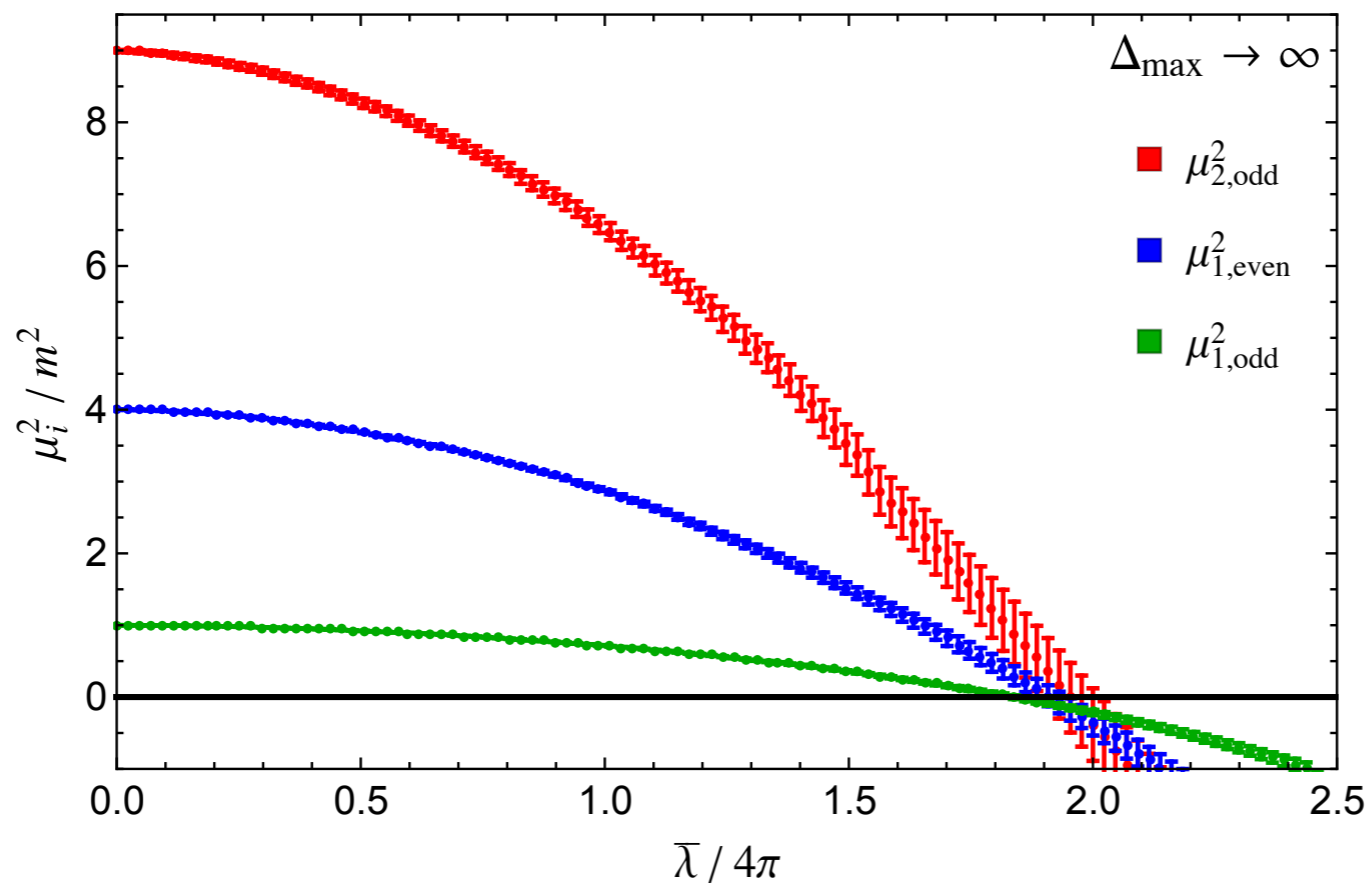
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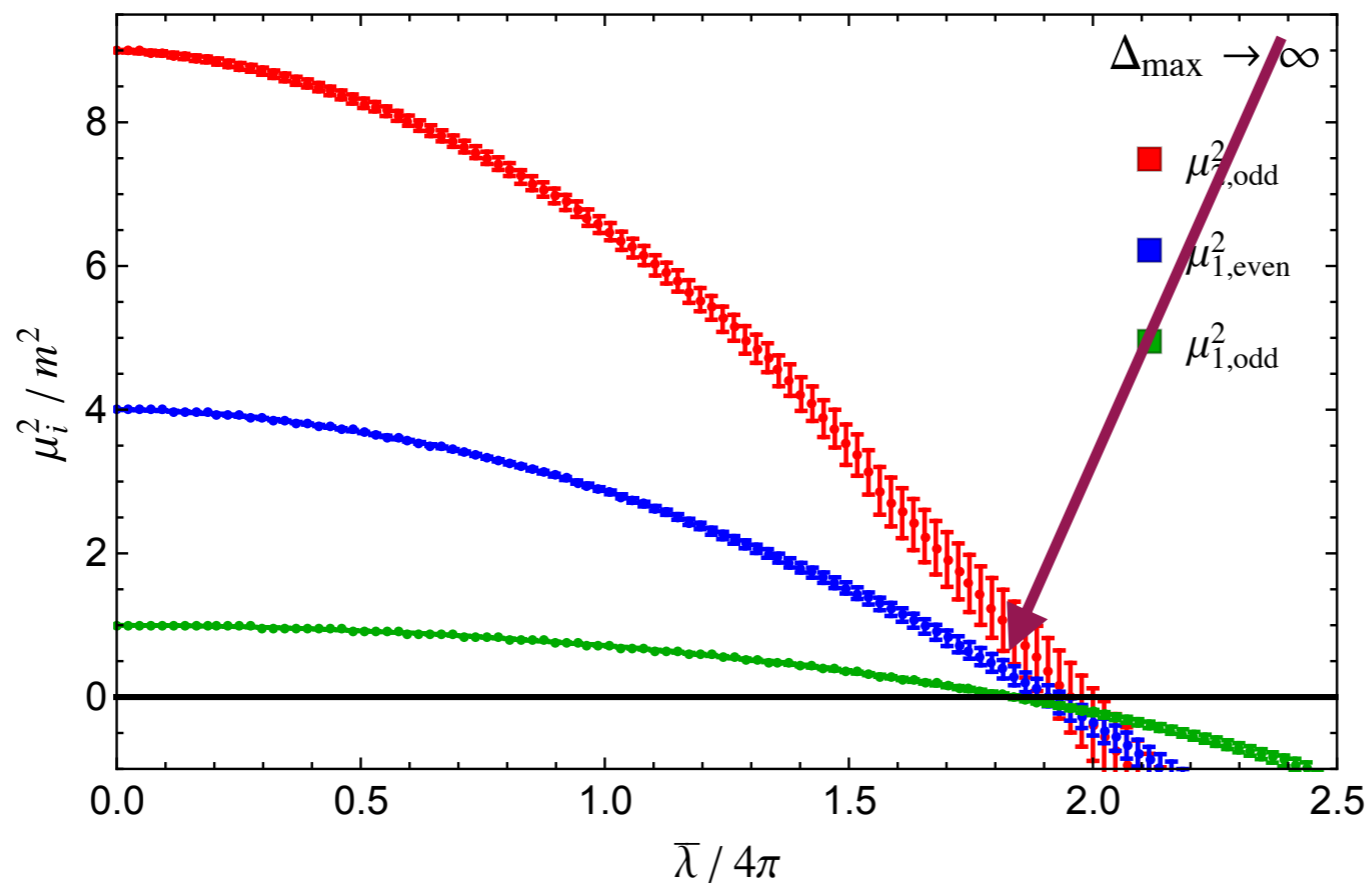


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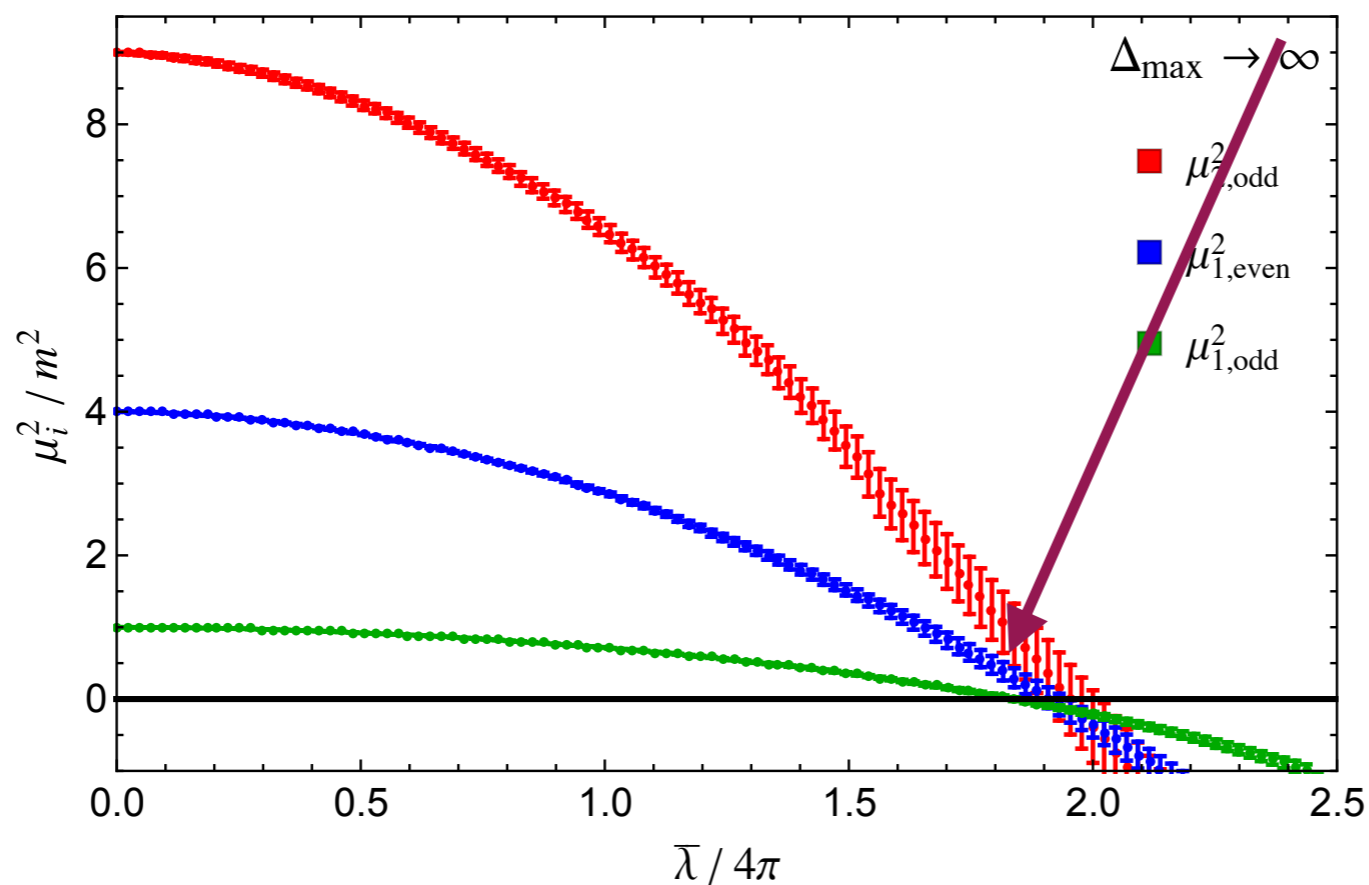


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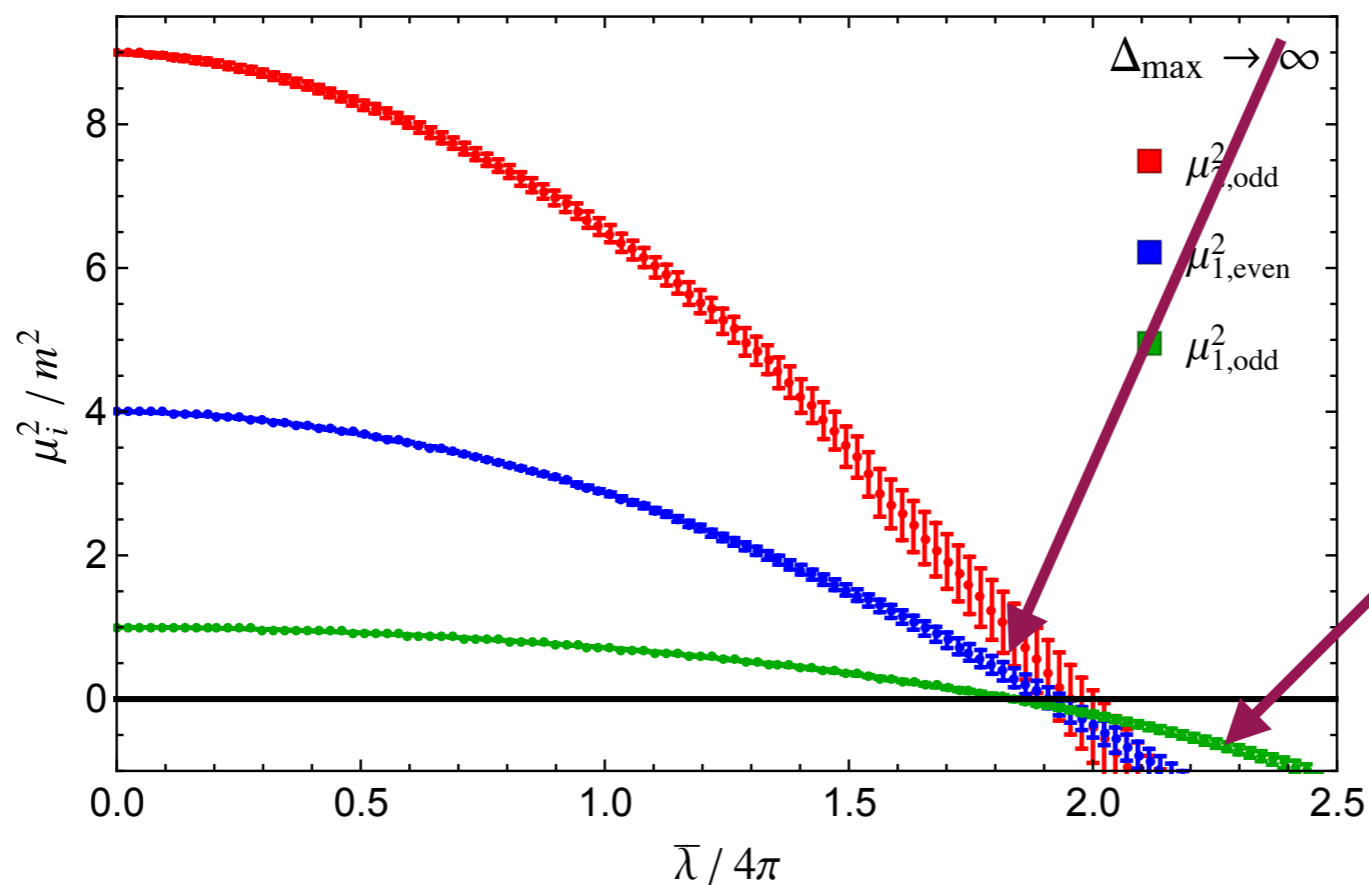
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NEC violation!

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I.e. “Zero-Modes” have large energies and need to be integrated out properly!

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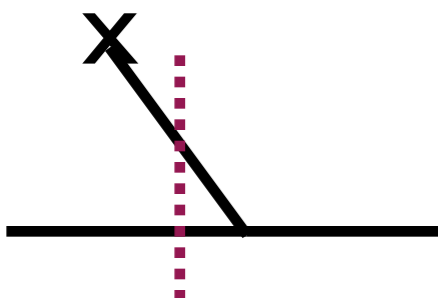
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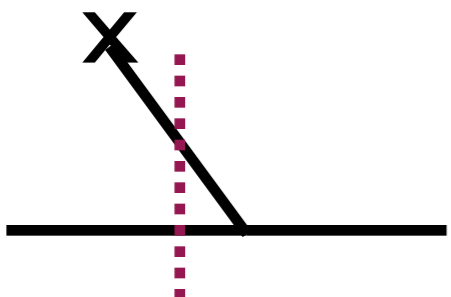
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Non-Analytic in q_0

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Prescription reproduces missing vevs!

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Happens when 4-pt fcn loses
spectral decomposition on the LC

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Happens when 4-pt fcn loses
spectral decomposition on the LC

Only possible if some 3pt-functions vanish on the LC:

$$\Delta' = \Delta + \Delta_R + 2n$$

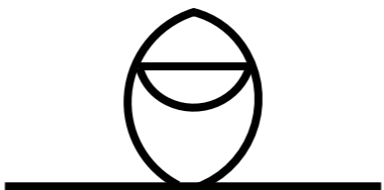
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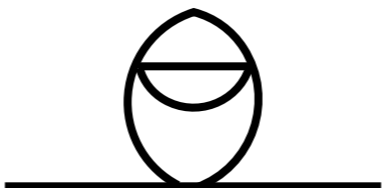
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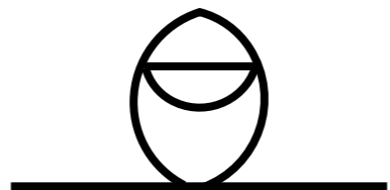
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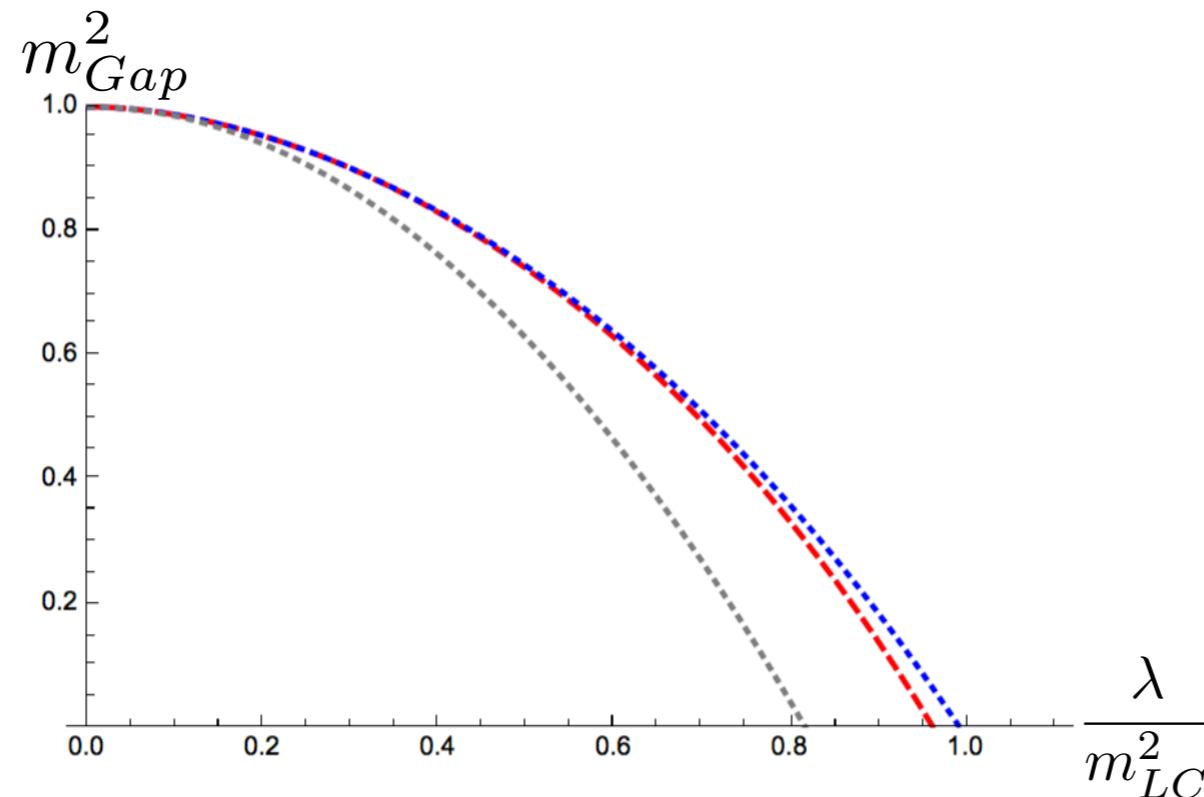
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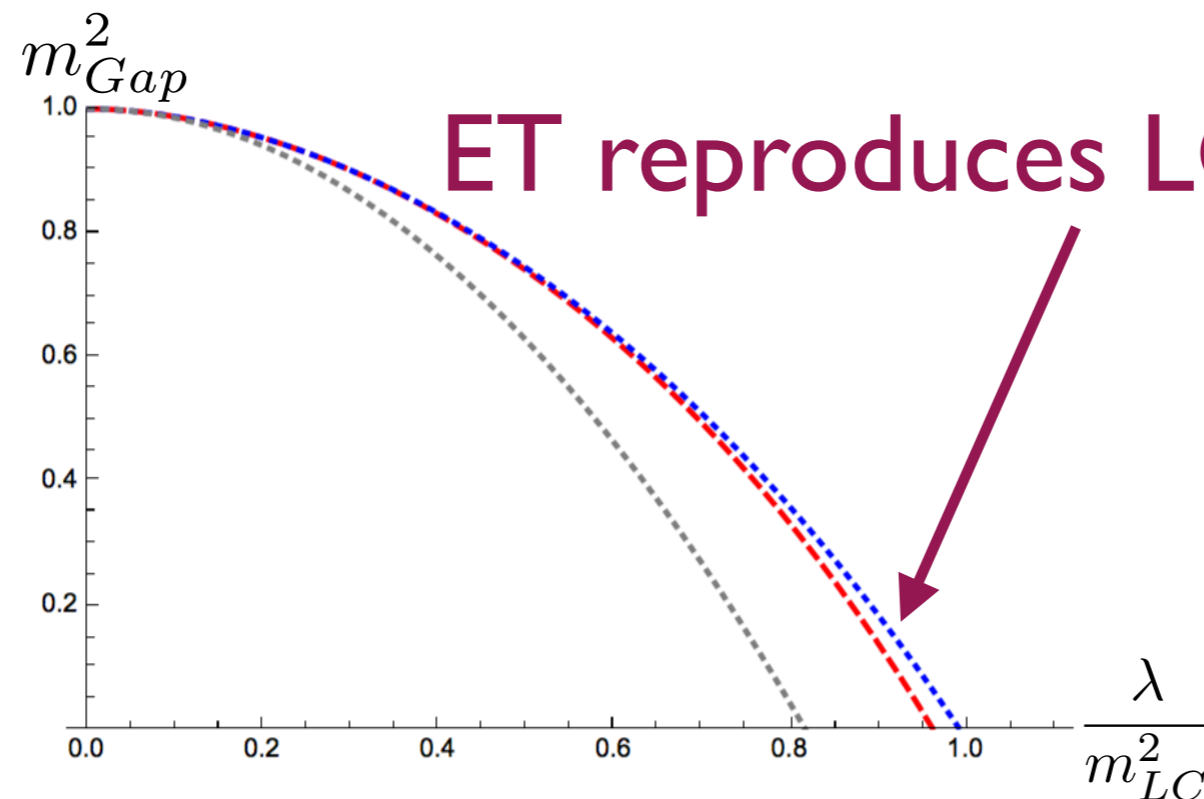
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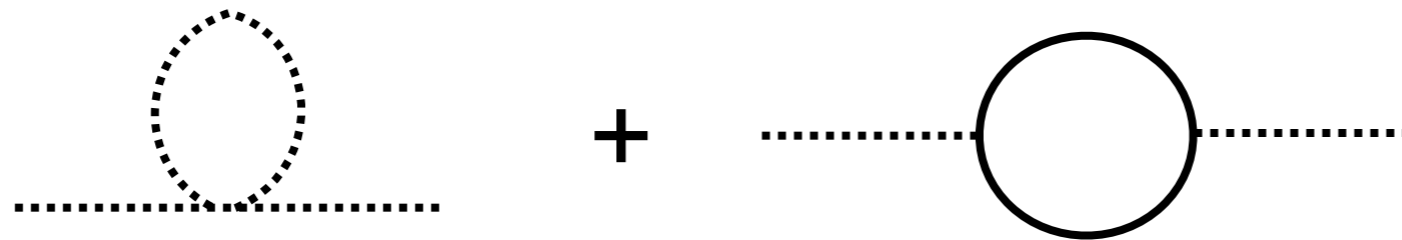
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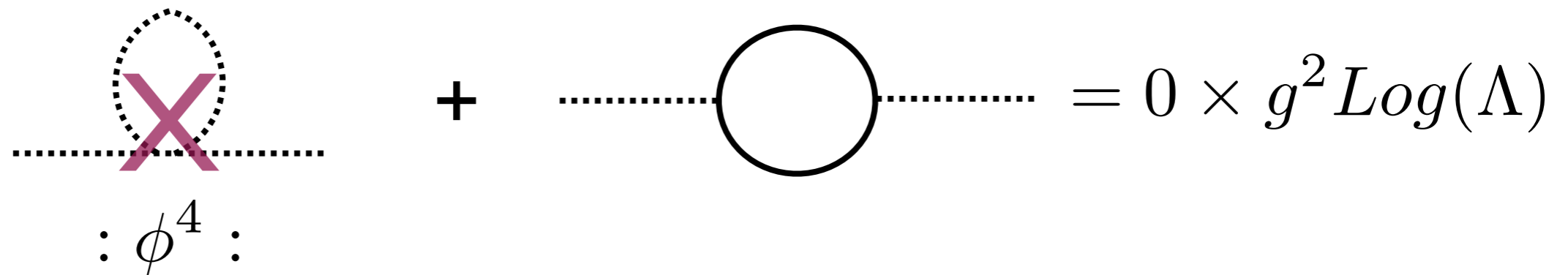
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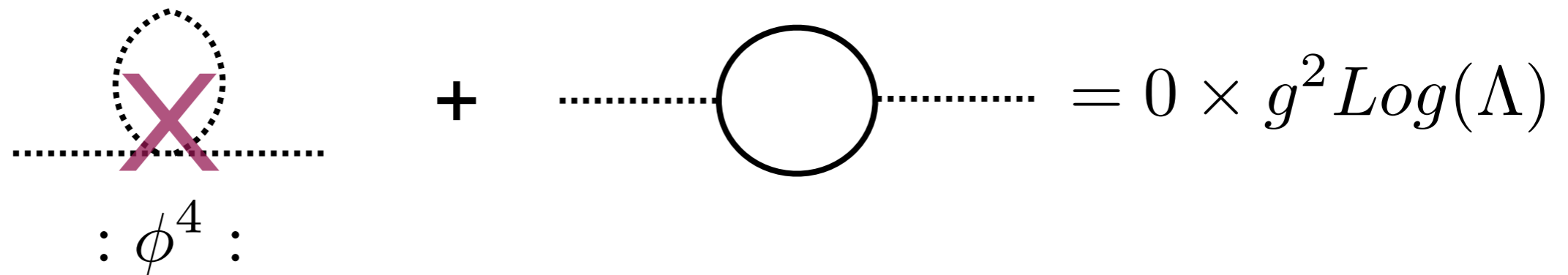
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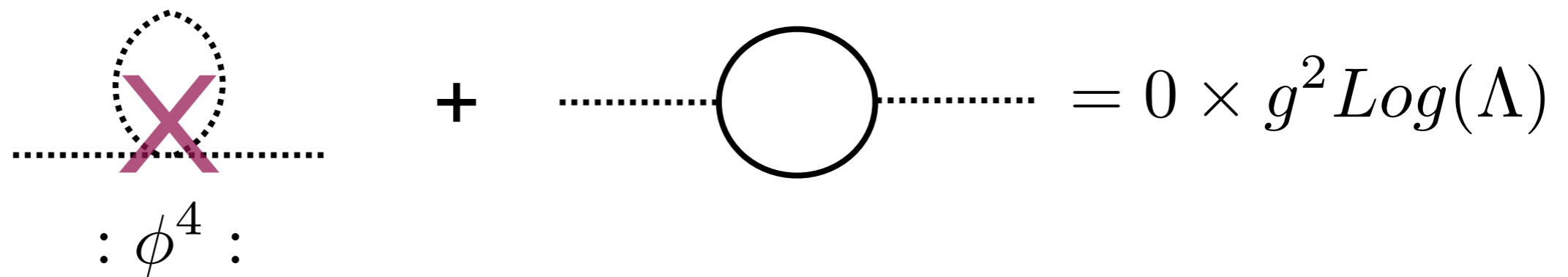
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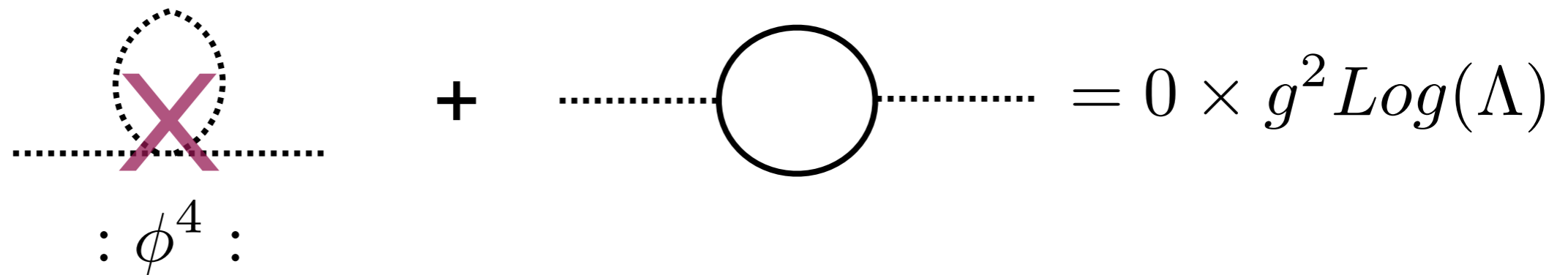
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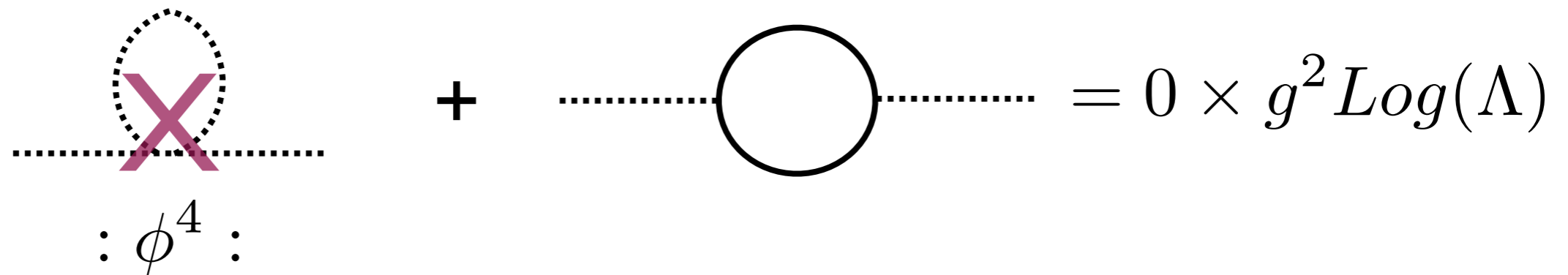
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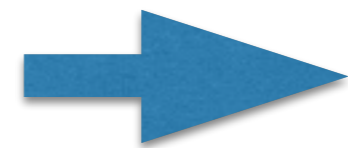
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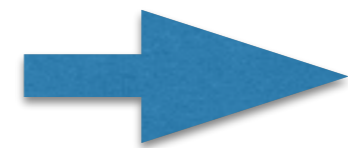
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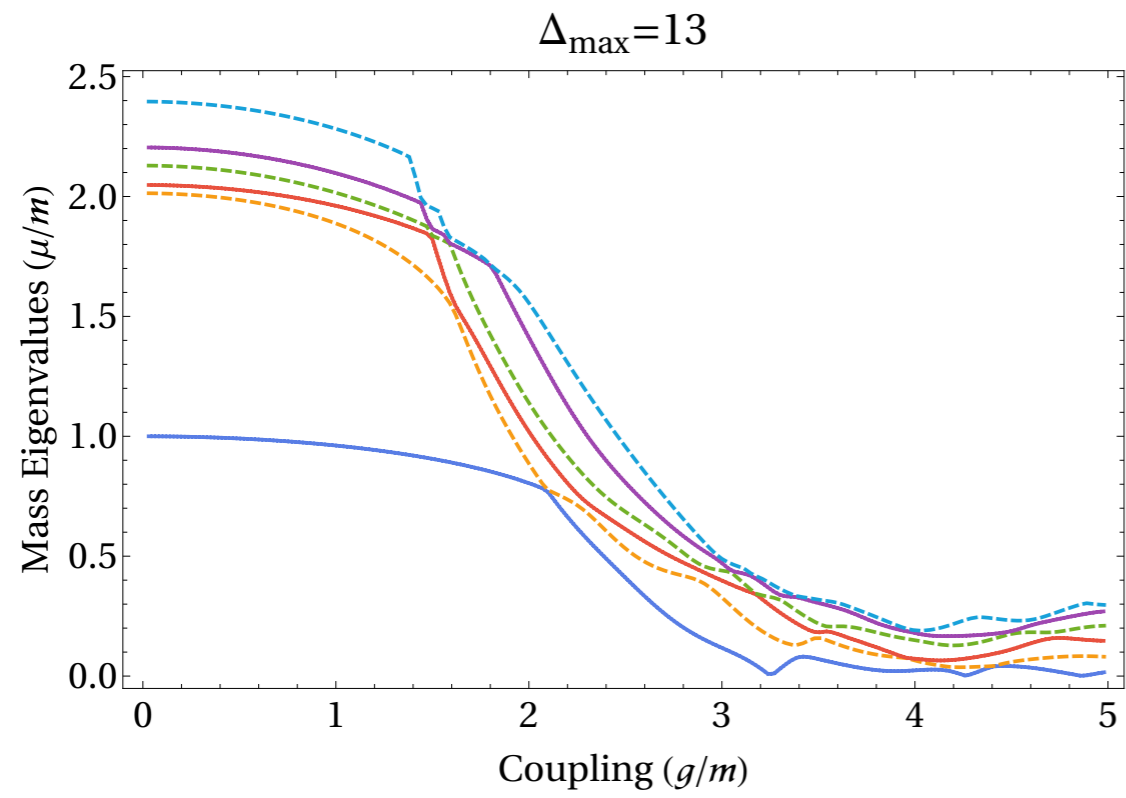
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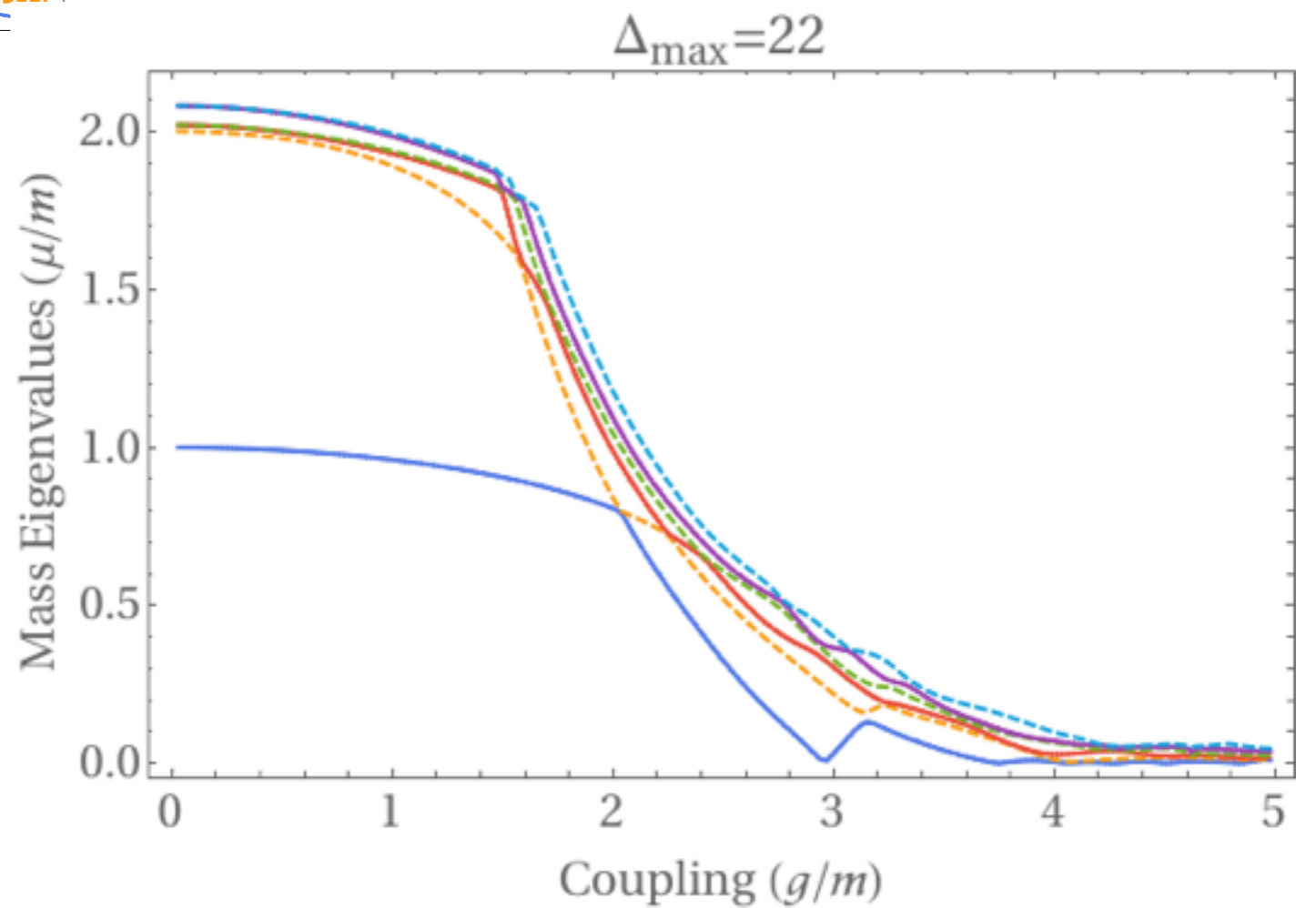
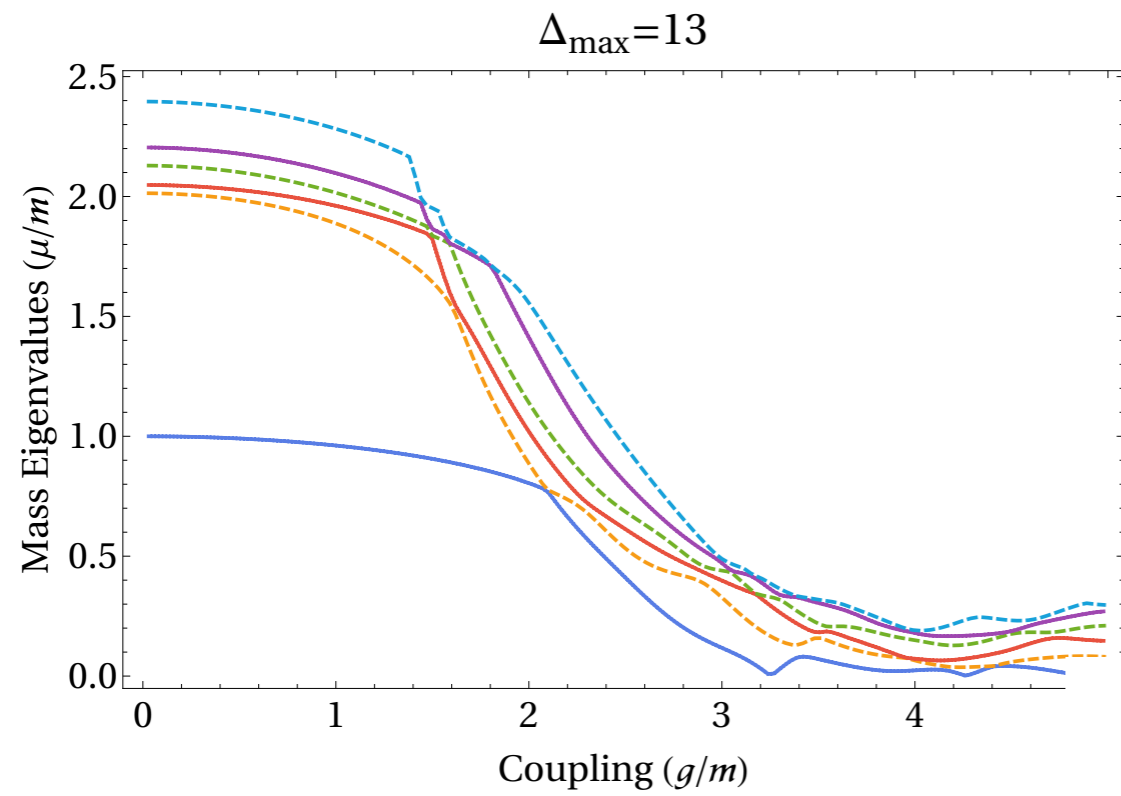
$$P_+ = Q_+^2 > 0 \quad \text{(NEC)}$$

The Spectrum

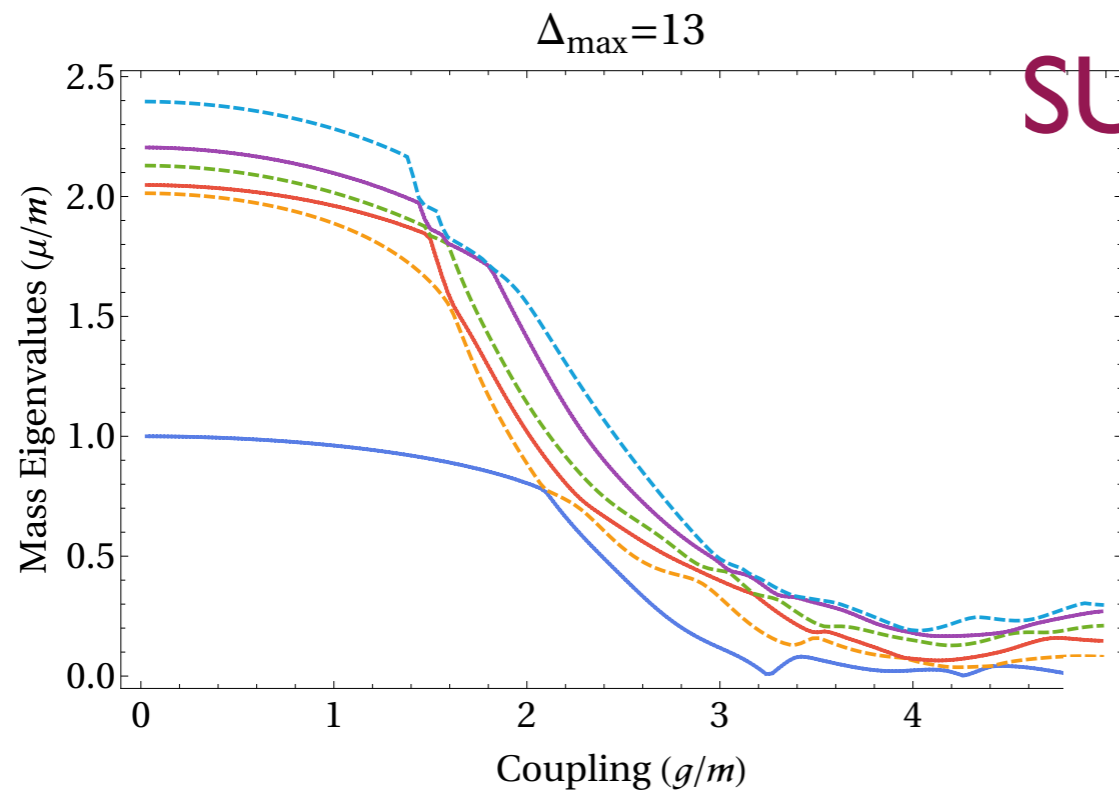
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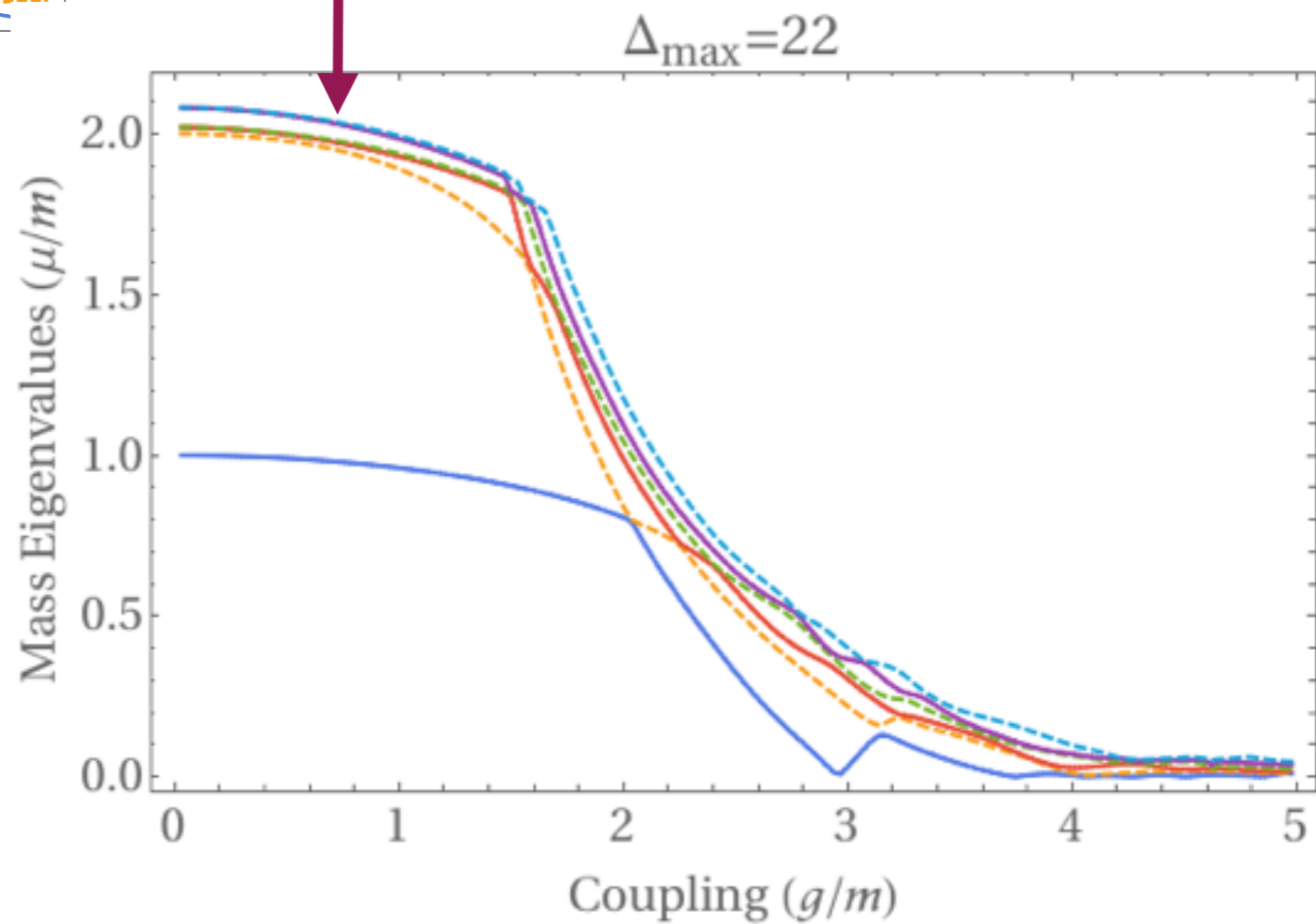
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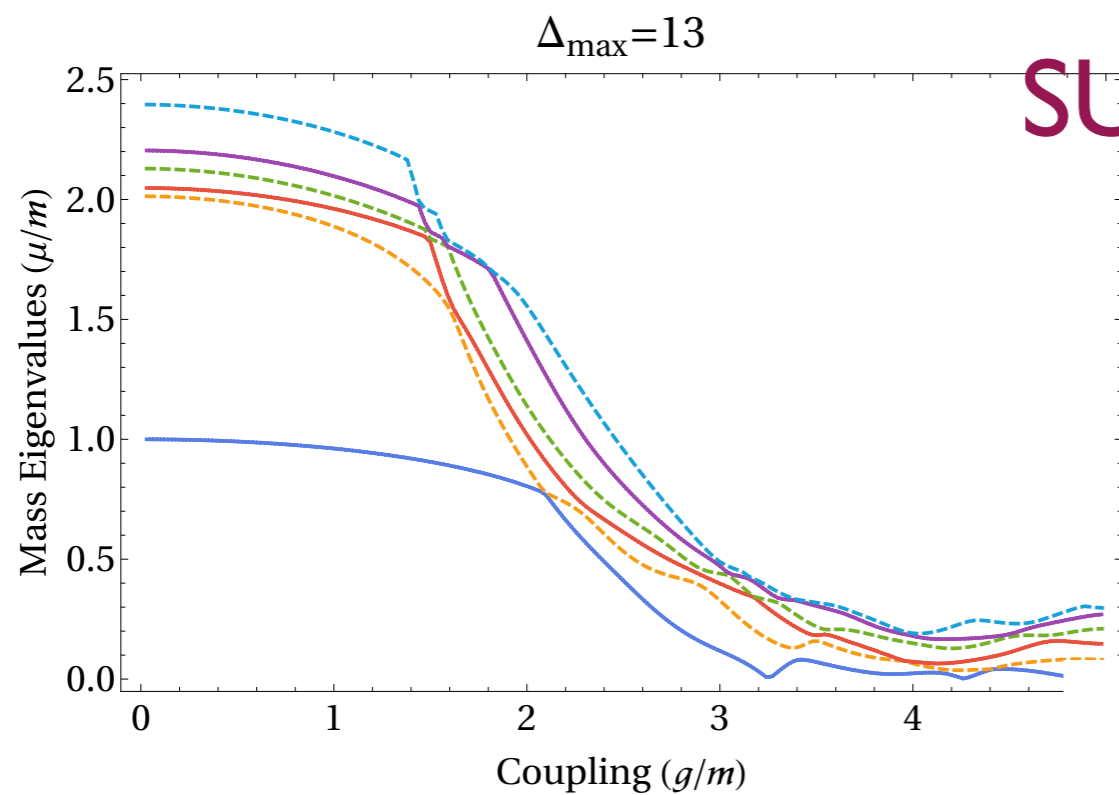
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SUSY degeneracy
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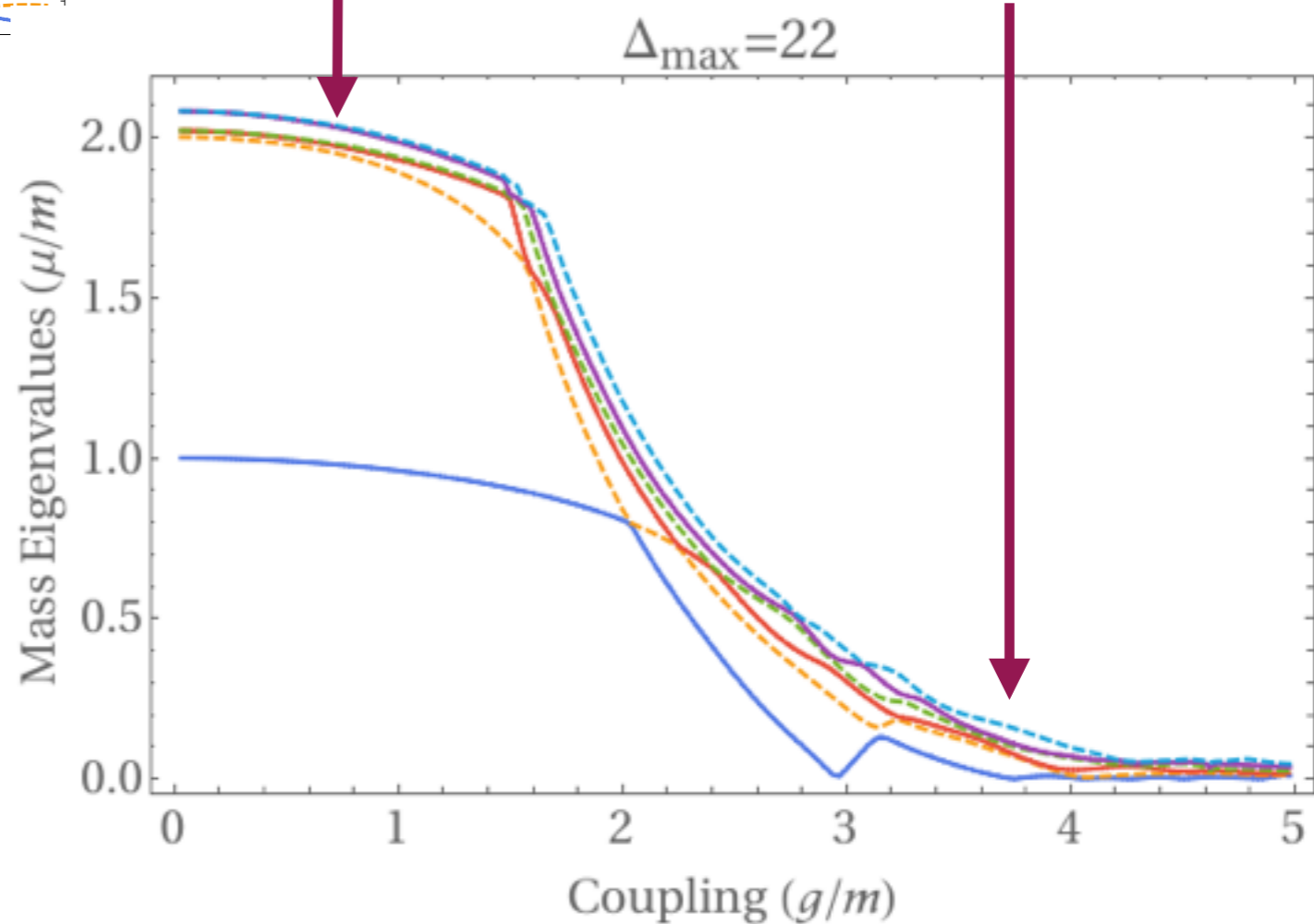


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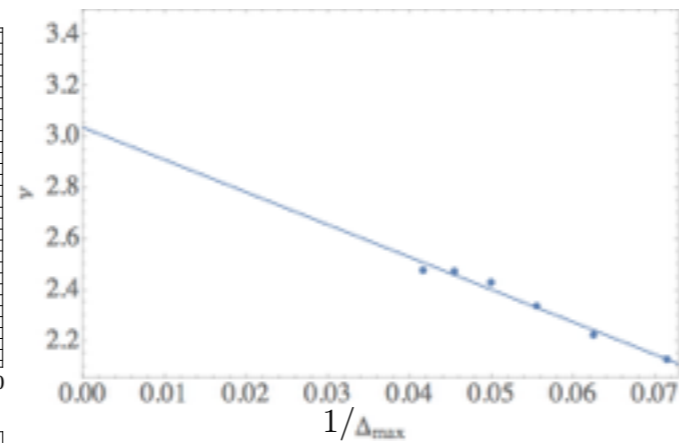
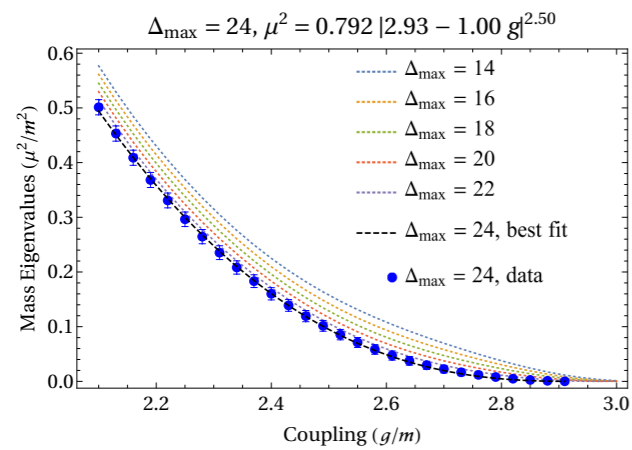
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Evidence for
Ising gapless phase

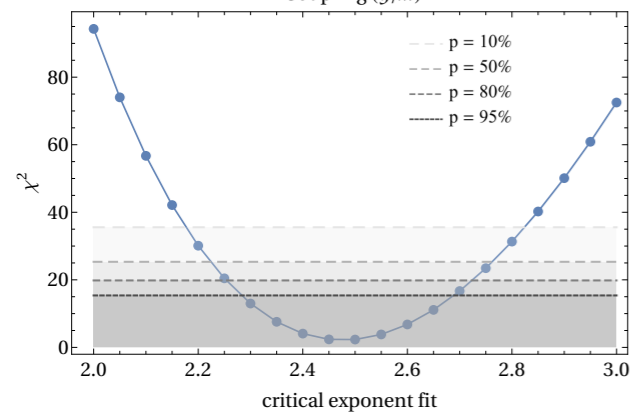


The Critical Exponent

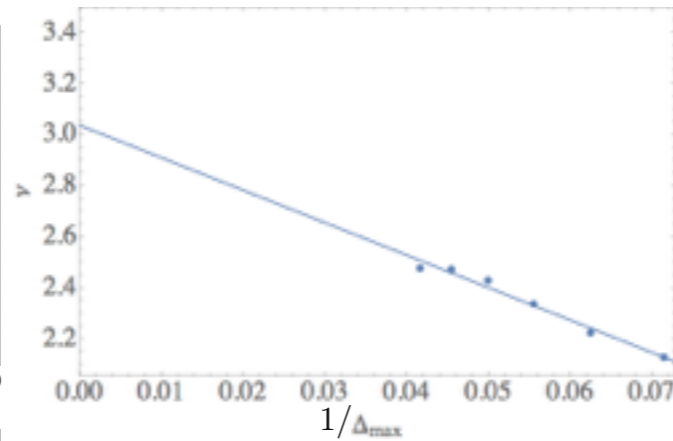
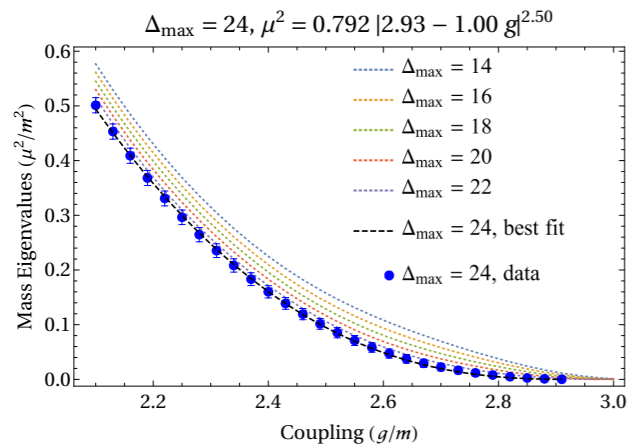
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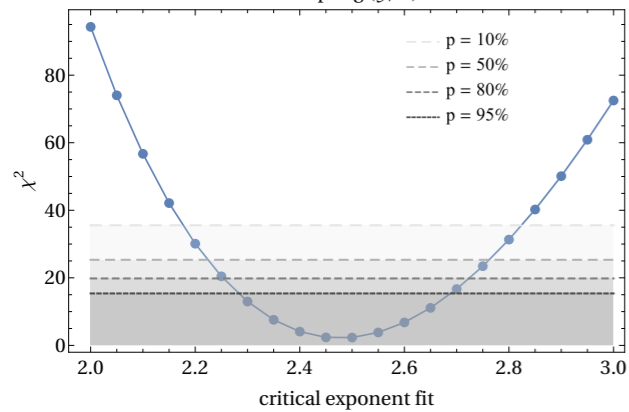
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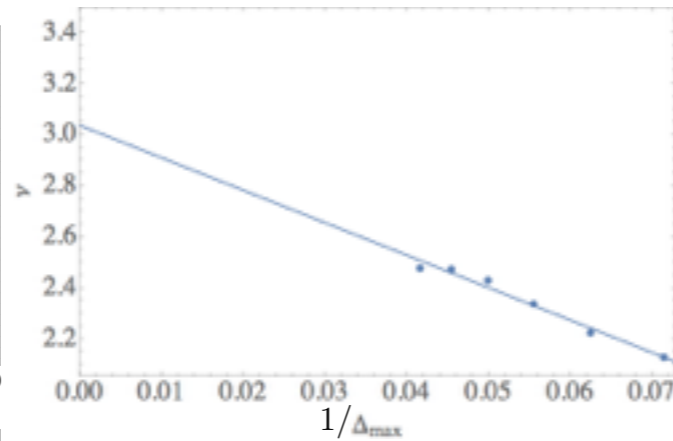
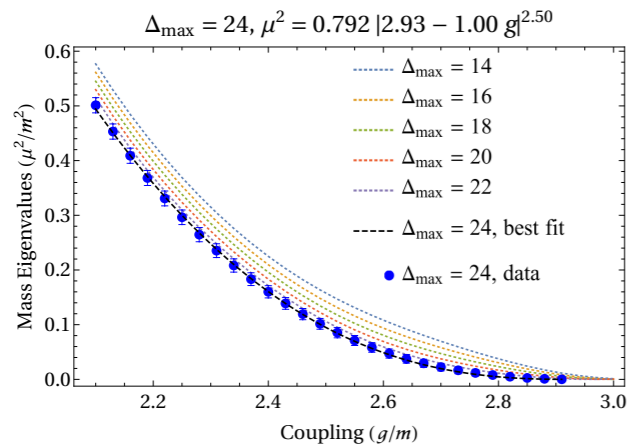
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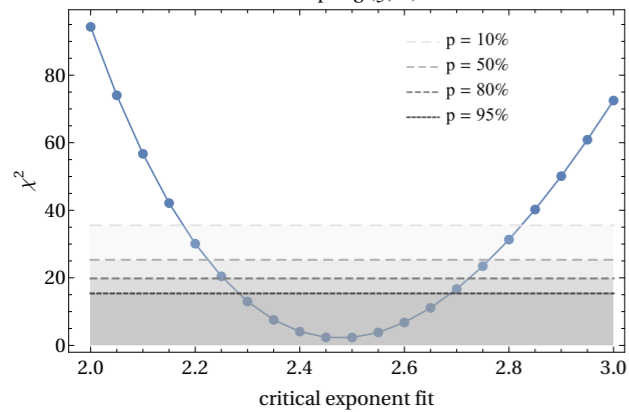
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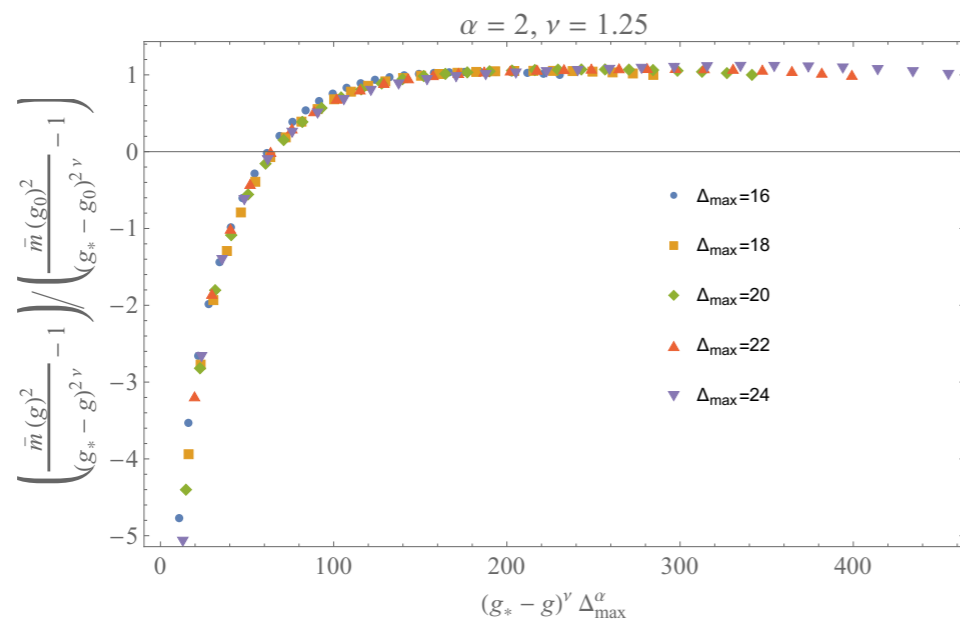


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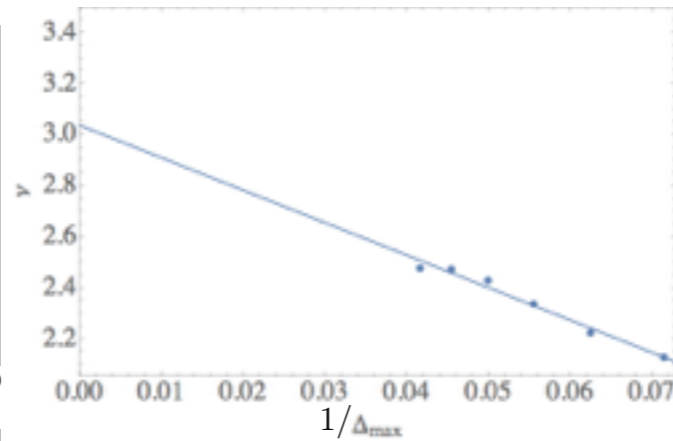
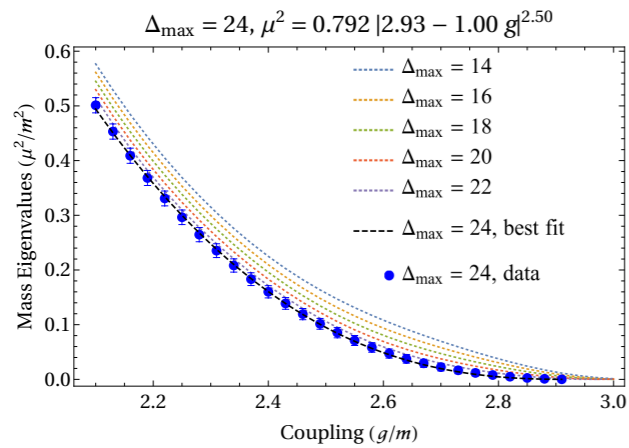


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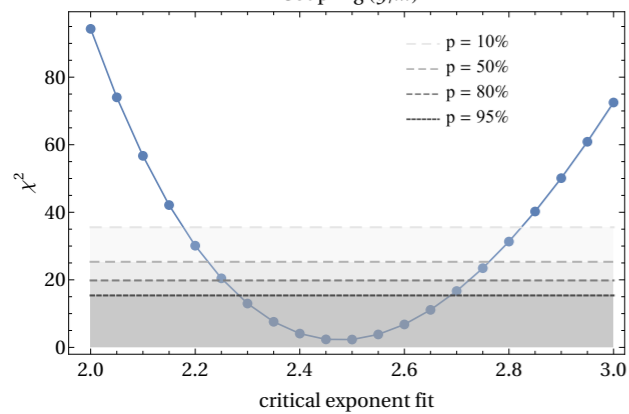
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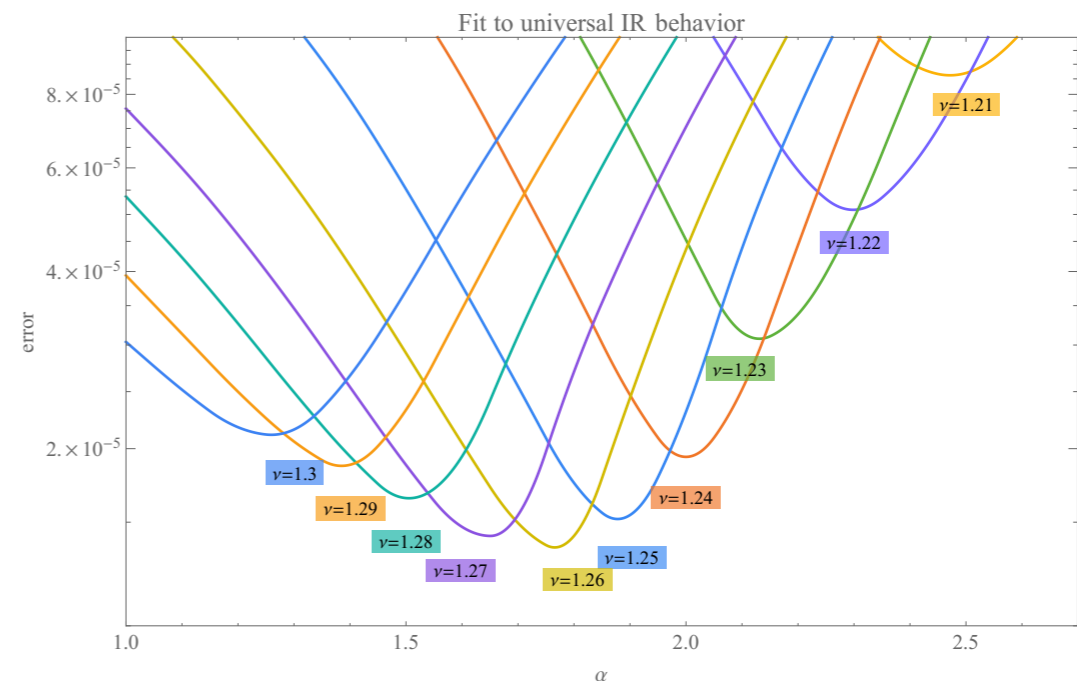
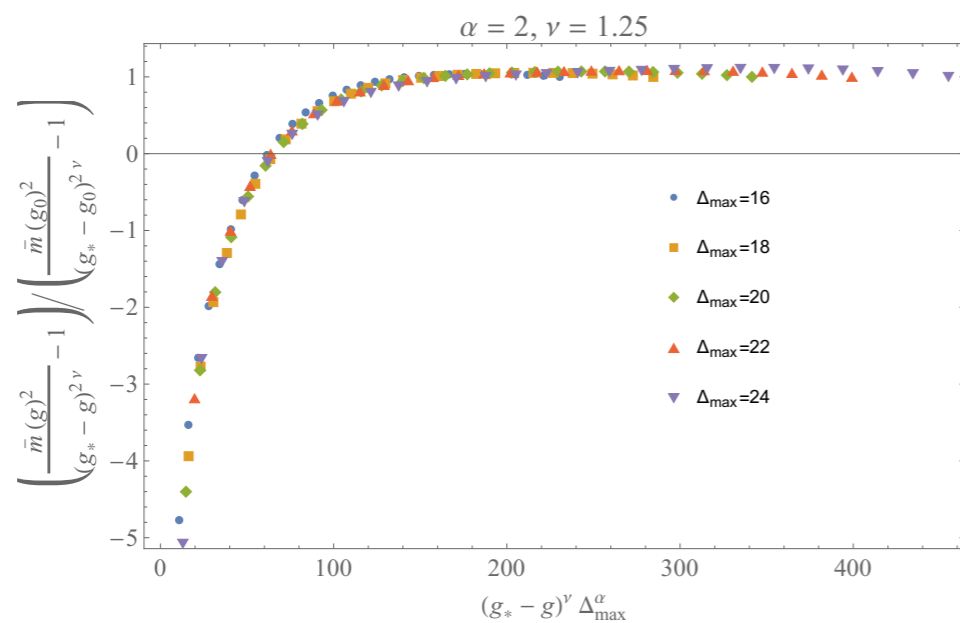


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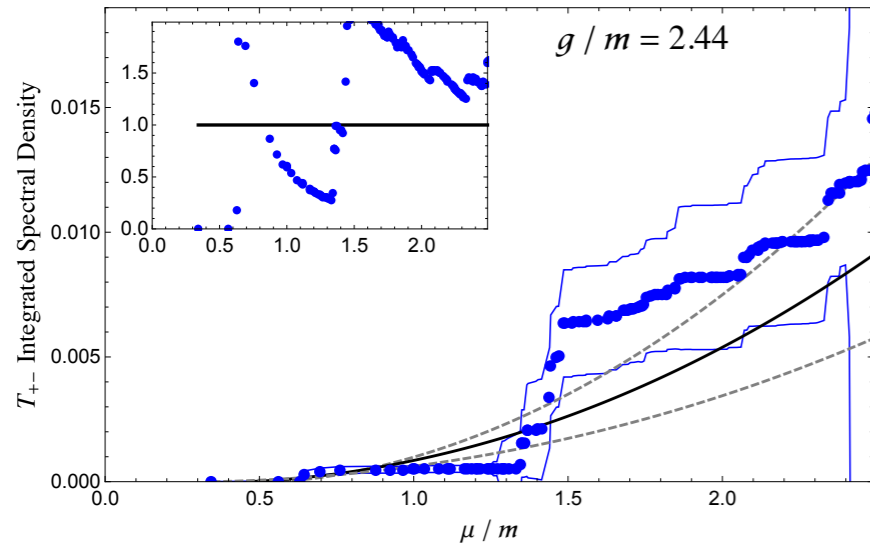
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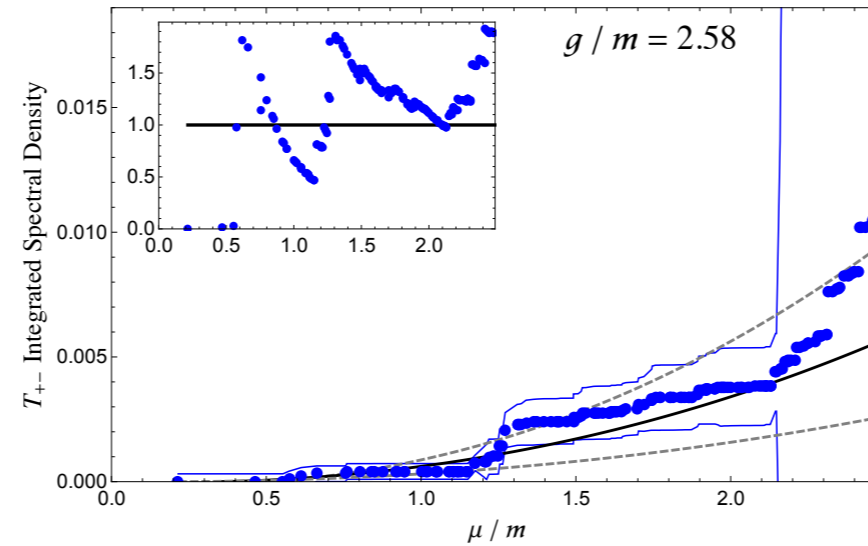
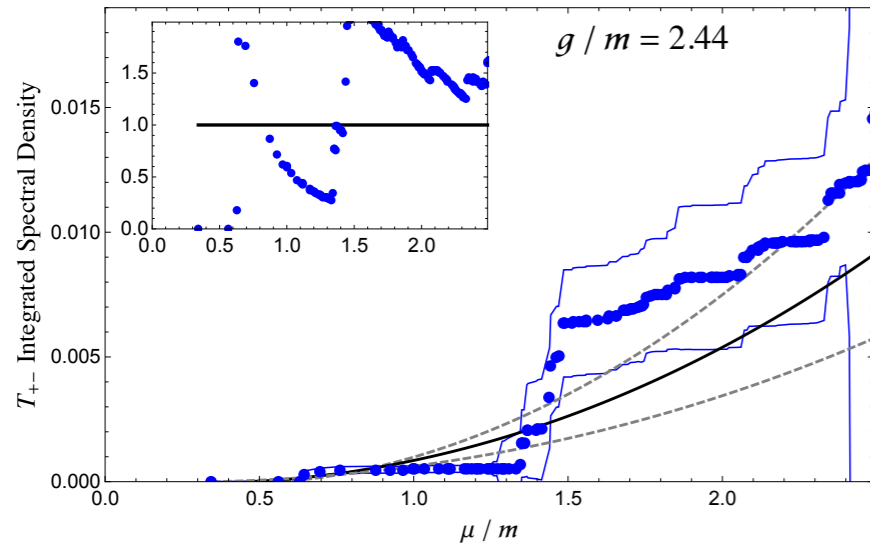
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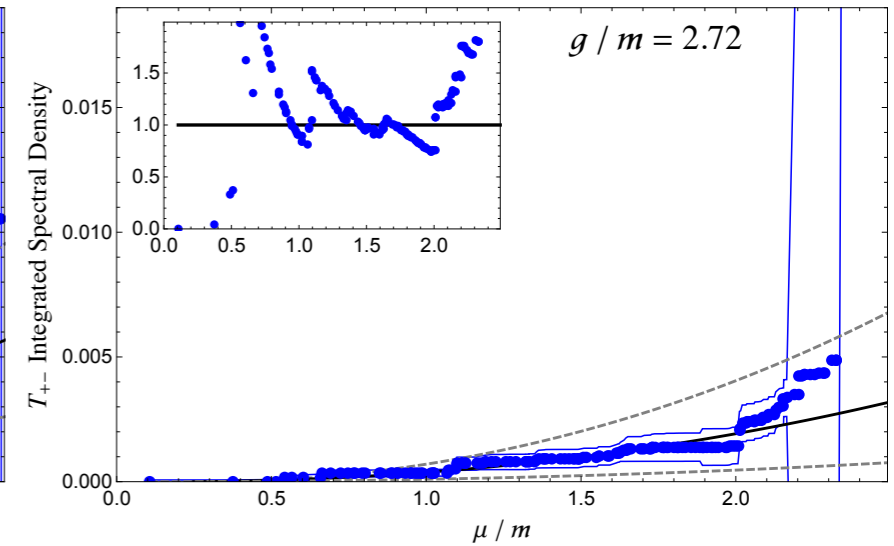
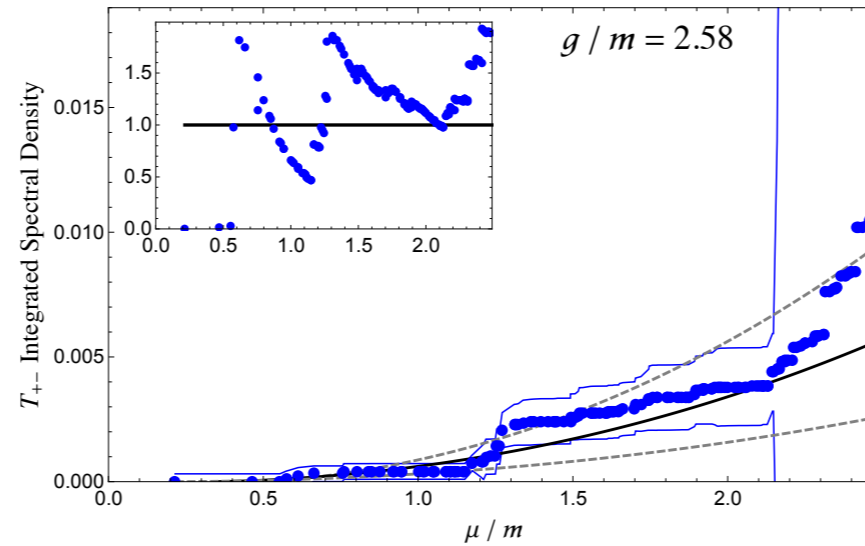
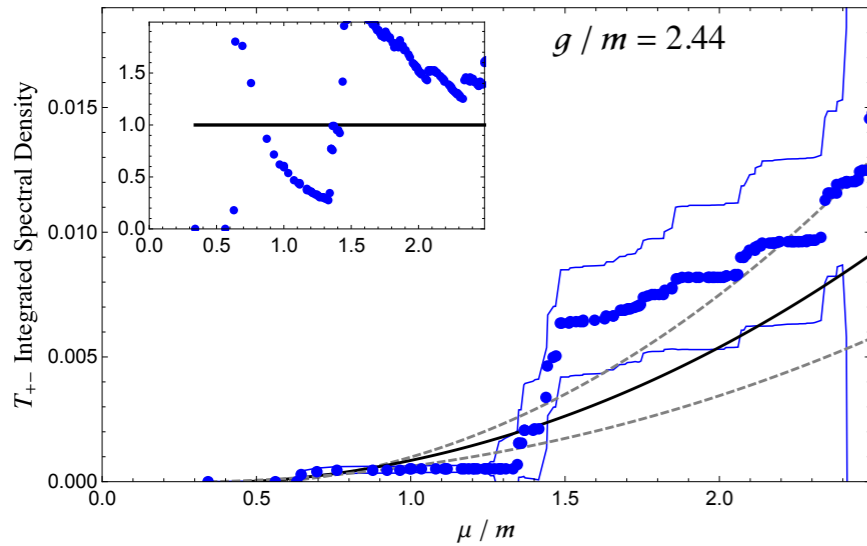
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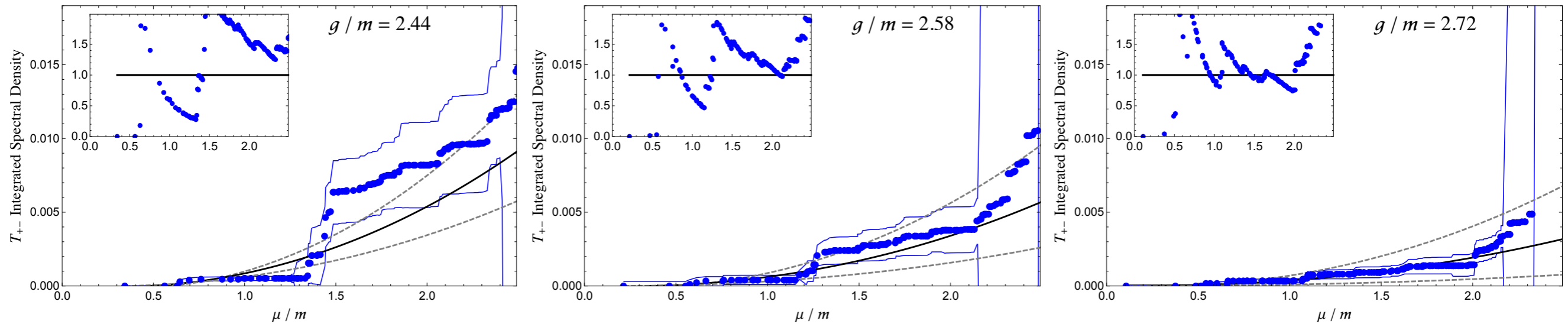
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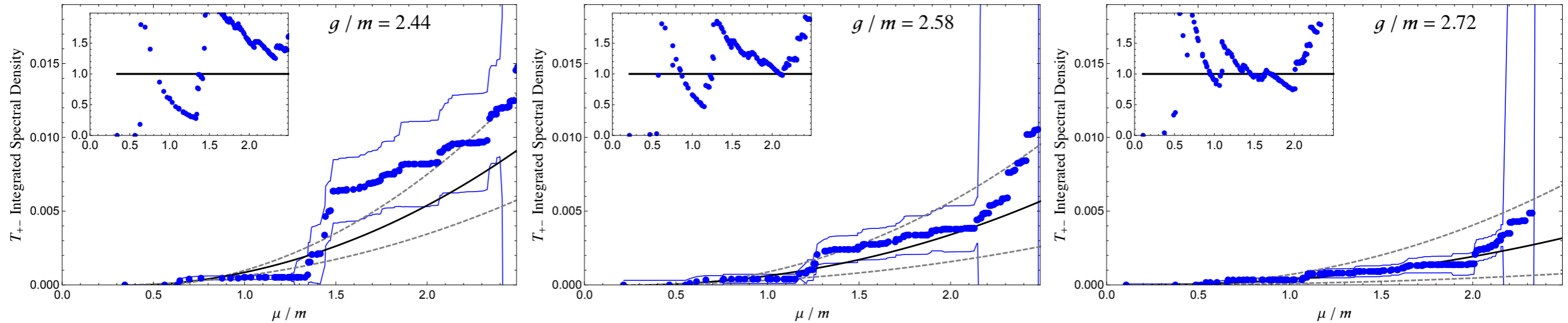
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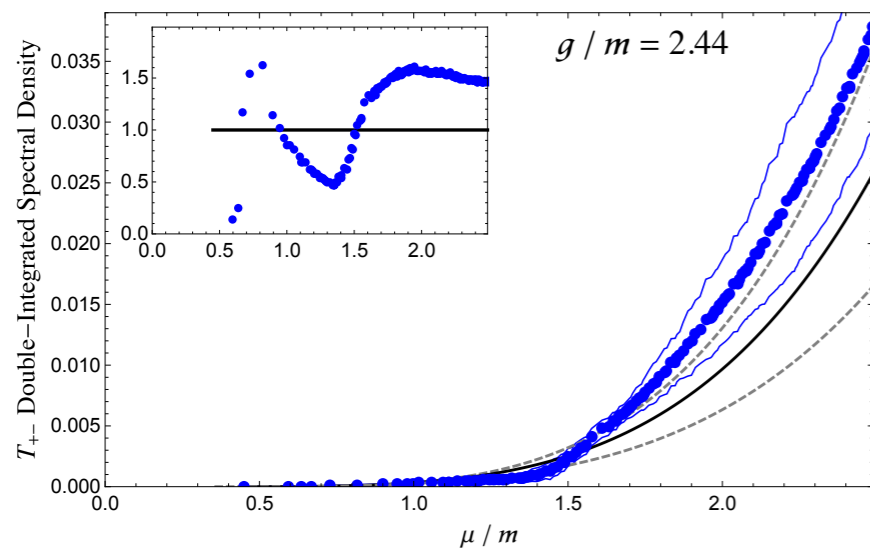
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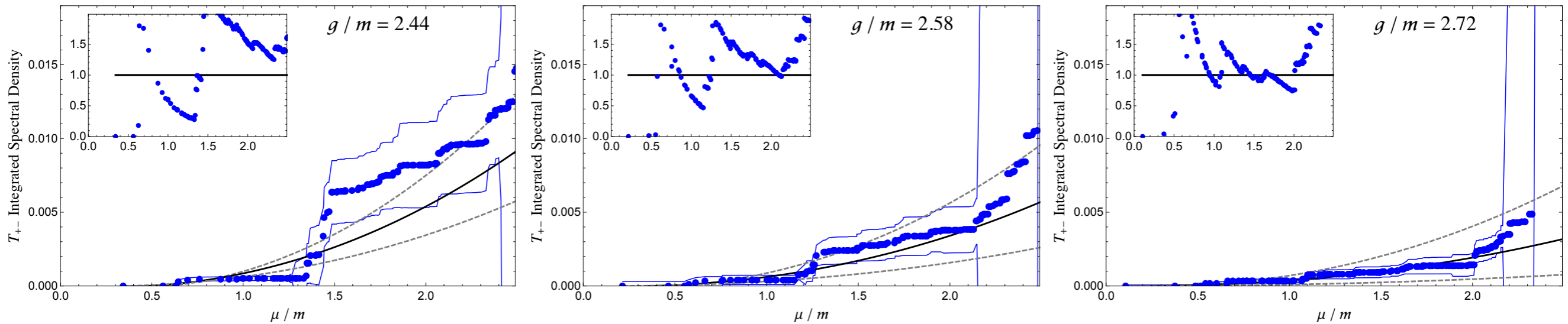


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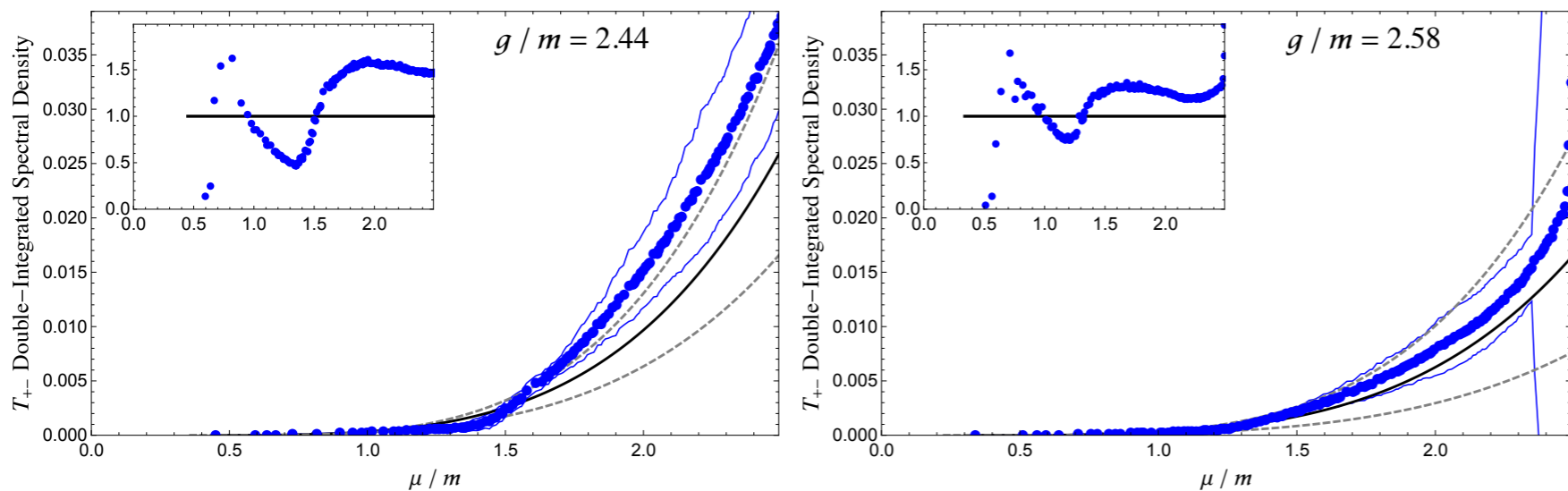


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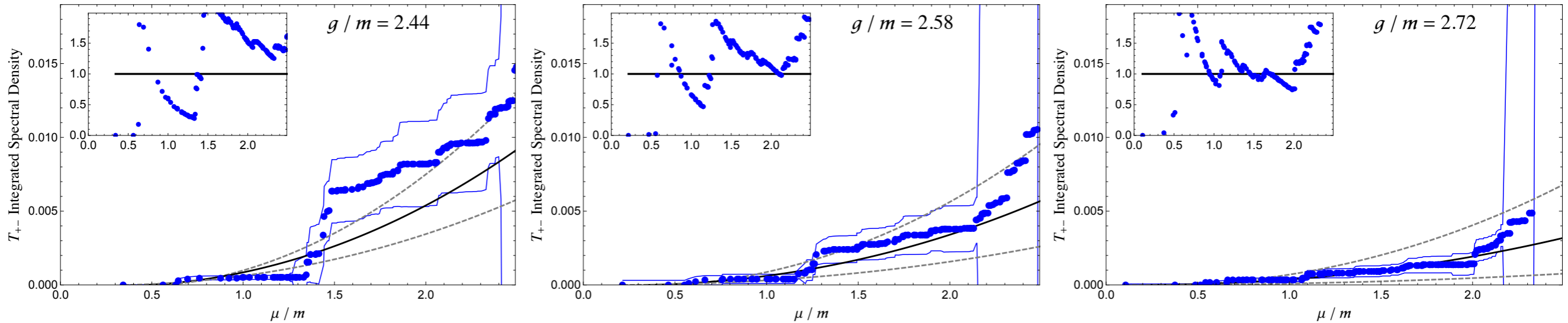


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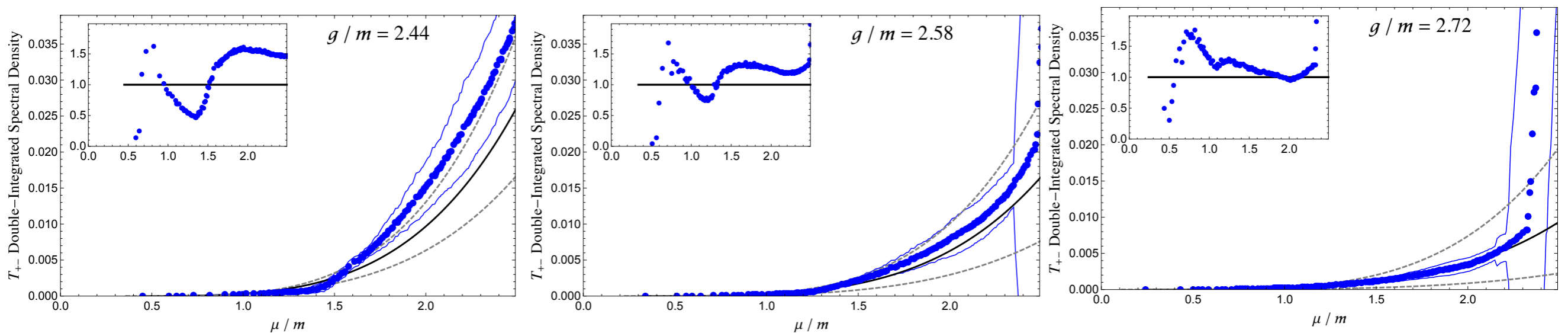


The spectral density of: $T_{\mu}^{\mu} = 2T_{+-}$

Known TIM behavior in the IR: $T_{+-} = m_{\text{kin}k}^{\frac{4}{5}} \epsilon' + \dots$



For a smoother taste, let's integrate again:



A Dream

A Dream

Large-N RG-flow:

$$\mathcal{N} = 4 \text{ SYM} \rightarrow \mathcal{N} = 2 \text{ SYM}$$

A Dream

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A Dream

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SUSY Non-Renorm Thms protect
naive LC from “zero-modes”!