String Field Theory and its Applications

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Trieste, June 2019

PLAN

1. Why do we need string field theory?

2. Formulation of string field theory

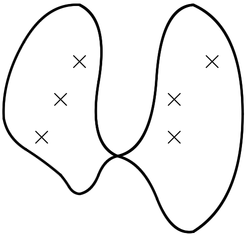
In the conventional world-sheet approach to string theory, the scattering amplitudes with n external states take the form:

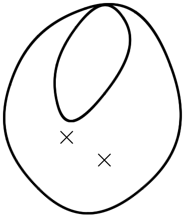
$$\sum_{g\geq 0} (g_s)^{2g} \int_{M_{g,n}} I_{g,n}$$

$M_{g,n}$: Moduli space of genus g Riemann surface with n punctures

$I_{g,n}$: an appropriate correlation function of vertex operators and other operators (ghosts, PCOs) on a genus g Riemann surface.

– typically a divergent integral near the boundaries of $M_{g,n}$.





Example: Consider N tachyon amplitude in bosonic string theory

$$\propto \int \prod_{i=4}^{N} d^2 z_i \prod_{i<j} |z_i - z_j|^{p_i.p_j}$$

This integral diverges if $p_i.p_j < -2$ for any pair (i,j).

Conventional approach is to define this using analytic continuation

– not useful for numerical evaluation or putting bounds.

There are more severe divergences in higher genus that cannot be treated with analytic continuation.

String field theory provides a way to systematically remove all these divergences.

Mathematical description: Sewing at a pair of punctures

– either on different surfaces or on the same surface

1. Take local complex coordinates w_1 and w_2 around the punctures.

2. Sew them using the relation:

$$w_1 w_2 = q \equiv e^{-s-i\theta}, \quad s \geq 0,\ 0 \leq \theta < 2\pi$$

– sews $|w_1| = |q|^{1/2}$ with $|w_2| = |q|^{1/2}$.

As $w_1 \to 0$ we emerge at large w_2.

Region of divergence: $q \to 0$, i.e. $s \to \infty$

String field theory is a quantum field theory with infinite number of fields in which perturbative amplitudes are computed by summing over Feynman diagrams.

Each Feynman diagram can be formally represented as an integral over the moduli space of a Riemann surface with

– the correct integrand $I_{g,n}$ (as in world-sheet description)

– but only a limited range of integration.

Sum over all Feynman diagrams reproduces the integration over the whole moduli space $M_{g,n}$.

Contribution to n-string amplitude from elementary n-string interaction vertex has the form

$$\sum_{g\geq 0} (g_s)^{2g} \int_{R_{g,n}} I_{g,n}$$

$R_{g,n}$: A subspace of $M_{g,n}$ that excludes regions around all boundaries

– does not suffer from any divergences.

Propagator: Generalization of $N/(k^2+m^2)$ for some numerator factor N:

$$N\,(L_0^+)^{-1}\,\delta_{L_0^-,0}, \quad L_0^\pm \equiv (L_0 \pm \bar{L}_0)$$

We could (formally) represent this as

$$N\,\frac{1}{2\pi}\int_0^\infty ds\,e^{-sL_0^+}\int_0^{2\pi} d\theta\,e^{-i\theta\,L_0^-}$$

Now consider a general Feynman diagram containing propagators and vertices

– contains integrals coming from lower order vertices, and two integrals coming from each propagator (momentum integrals already performed)

Together they have interpretation of integration over $M_{g,n}$ with the correct integrand.

Sum of all Feynman diagrams:

$$\sum_{g\geq 0}(g_s)^{2g}\int_{M_{g,n}} I_{g,n}$$

Boundaries of $M_{g,n}$ correspond to $s\to\infty$ limit in one or more propagators

– sources of divergence

$$\frac{1}{L_0^+}\delta_{L_0^-} = \frac{1}{2\pi}\int_0^\infty ds\, e^{-sL_0^+}\int_0^{2\pi} d\theta\, e^{-i\theta\, L_0^-}$$

1. For $L_0^+ < 0$ the left hand side is finite but the right hand side is divergent (as $s \to \infty$)

2. For $L_0^+ = 0$ both sides are divergent.

All divergences appearing in the world-sheet description have their origin in one of these two cases

– divergences appearing at the boundary of $M_{g,n}$ where the Riemann surface degenerates

$$\frac{1}{L_0^+}\delta_{L_0^-} = \frac{1}{2\pi}\int_0^\infty ds\, e^{-sL_0^+}\int_0^{2\pi} d\theta\, e^{-i\theta\, L_0^-}$$

Divergences coming for $L_0^+ < 0$ are fake

– resolved in string field theory by using the left hand side instead of the right hand side.

The divergences we encountered in the Koba-Nielsen amplitude are of this kind.

$$\frac{1}{L_0^+}\delta_{L_0^-} = \frac{1}{2\pi}\int_0^\infty ds\, e^{-sL_0^+}\int_0^{2\pi} d\theta\, e^{-i\theta\, L_0^-}$$

Divergences coming from $L_0^+ = 0$ are genuine since both sides diverge

– divergences associated with poles of propagators in QFT.

In string field theory, we can use the usual understanding of such divergences in QFT to remove these divergences

– can be used used to understand both the origin and resolution of these divergences.

Examples:

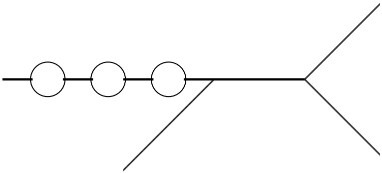
Pius, Rudra, A.S.; A.S.

1. **Mass renormalization**

2. **Vacuum shift**

Mass renormalization:

In a quantum field theory, self energy insertions on external legs need special treatment.



The internal propagators, being on-shell, diverge.

Steps required:

1. Separate graphs with self-energy insertions on external lines

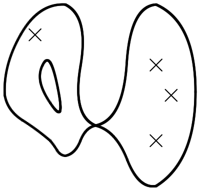
2. Resum to compute off-shell 2-point function

3. Look for pole positions to find renormalized mass

4. Use LSZ prescription to compute S-matrix

In the usual world-sheet approach we do not do any of this.

Result: integration over $M_{g,n}$ diverges from the separating type degeneration.



For a given amplitude, the usual world-sheet description of string perturbation theory gives one term at every loop order

– usually considered an advantage, but this does not allow us to separate the self-energy graphs and resum.

String field theory deals with this problem exactly as in ordinary quantum field theory.

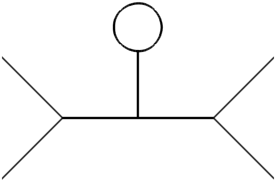
Vacuum shift:

Suppose we have massless ϕ^3 theory in which one loop correction generates a term linear in ϕ:

$$\mathsf{V} = \mathsf{A}\, \mathsf{g_s}^{-2}\, \phi^3 - \mathsf{B}\, \phi$$

A,B: constants, $\mathsf{g_s}$: coupling constant

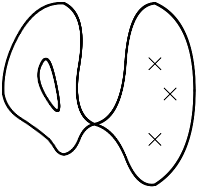
Naive perturbation theory diverges.



Correct procedure: Expand the effective action around the minimum at $\phi = \mathsf{g_s}\sqrt{\mathsf{B}/3\mathsf{A}}$ and derive new Feynman rules.

Not possible in usual string perturbation theory since we do not have separate tadpole graphs.

Result: Tadpole divergence in integration over $M_{g,n}$.



In contrast, in string field theory we can deal with this situation by following the standard procedure in quantum field theory.

Review: arXiv:1703.06410: Closed heterotic and type II strings

Corinne de Lacroix, Harold Erbin, Sitender Pratap Kashyap, A.S., Mritunjay Verma

Field theory of open and closed superstrings: to appear

Faroogh Moosavian, A.S., Mritunjay Verma

General structure of string field theory

Begin with classical closed bosonic string field theory

Saadi, Zwiebach; Kugo, Suehiro; Sonoda, Zwiebach; Zwiebach; $\cdots$

A string field ψ is an element of some vector space $\mathcal{H}$.

$\mathcal{H}$ is a subspace of the full Hilbert space of matter and ghost world-sheet CFT, defined by the constraints:

$$b_0^-|\psi\rangle = 0, \quad L_0^-|\psi\rangle = 0, \quad n_g|\psi\rangle = 2|\psi\rangle$$

$$b_0^\pm = b_0 \pm \bar{b}_0, \quad L_0^\pm = L_0 \pm \bar{L}_0, \quad c_0^\pm = \frac{1}{2}(c_0 \pm \bar{c}_0)$$

$$n_g = \text{ghost number}$$

Matter CFT: Any CFT with c=26.

Note: No physical state constraint on $|\psi\rangle$

If $\{|\phi_{\mathbf{r}}\rangle\}$ is a basis in $\mathcal{H}$, then we can expand $|\psi\rangle$ as

$$|\psi\rangle = \sum_{\mathbf{r}} \psi_{\mathbf{r}} |\phi_{\mathbf{r}}\rangle$$

ψ_r are the dynamical degrees of freedom

– path integral $\equiv$ integration over the ψ_r's

$\sum_{\mathbf{r}}$ includes integration over momenta along non-compact directions

$\Rightarrow$ makes $\psi_{\mathbf{r}}$ into fields (in momentum space)

Classical action (setting $g_s = 1$):

$$S = \frac{1}{2}\langle\psi|c_0^- Q_B|\psi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}$$

Q_B: BRST charge

For $|A_i\rangle \in \mathcal{H}$, $\{A_1 \cdots A_n\}$ is constructed from correlation functions of the vertex operators A_i on the sphere, integrated over a subspace $R_{0,n}$ of the moduli space $M_{0,n}$.

1. Since A_i's are off-shell, the correlation function depends on the choice of world-sheet metric, or equivalently the choice of local coordinate system z in which the metric = $|dz|^2$ locally.

2. The subspace $R_{0,n}$ avoids all degenerations, and its choice is correlated with the choice of local coordinates in step 1.

Different choices (z, $R_{0,n}$) give equivalent string field theories related by field redefinition

$$\mathbf{S} = \frac{1}{2}\langle\psi|\mathbf{c_0^-}\mathbf{Q_B}|\psi\rangle + \sum_{\mathbf{n}} \frac{1}{\mathbf{n}!}\{\psi^{\mathbf{n}}\}$$

This action has infinite parameter gauge invariance of the form

$$\delta|\psi\rangle = \mathbf{Q_B}|\lambda\rangle + \cdots$$

$|\lambda\rangle$ **represents gauge transformation parameter.**

This theory can be quantized using Batalin-Vilkovisky (BV) formalism

– introduces ghosts and anti-fields

Net result: Relax the constraint on the ghost number of $|\psi\rangle$.

The action has similar structure:

$$S_{BV} = \frac{1}{2}\langle\psi|c_0^- Q_B|\psi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}$$

But now $\{A_1 \cdots A_n\}$ contains contribution from integrals over subspaces of $M_{g,n}$ for all g

The higher genus contributions are needed to cancel gauge non-invariance of the path integral measure.

Note: We shall continue to use the symbols

$\mathcal{H}$ for this extended Hilbert space carrying arbitrary n_g

$|\psi\rangle$ for the extended string field $\in \mathcal{H}$

$\{A_1 \cdots A_n\}$ for the new, quantum corrected product.

In Siegel gauge $b_0^+|\psi\rangle = 0$, the action takes the form:

$$S_{gf} = \frac{1}{2}\langle\psi|c_0^- c_0^+ L_0^+|\psi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}$$

Propagator:

$$b_0^+ \, b_0^- \, \frac{1}{L_0^+}\delta_{L_0^-} = b_0^+ \, b_0^- \, \frac{1}{2\pi} \int_0^\infty ds \, e^{-sL_0^+} \int_0^{2\pi} d\theta e^{-i\theta \, L_0^-}$$

Second step is valid only for $L_0^+ > 0$.

Once we have the propagator we can compute amplitudes using Feynman diagrams.

Each Feynman diagram has vertices and propagators.

We have some integrals from the vertices (integration over subspaces of $M_{g',n'}$).

g', n' refer to individual vertices

We also have two integrals from each propagator (s, θ)

Together the total set of integrals can be interpreted as integral over a subspace of $M_{g,n}$ with the correct integrand

(g,n) refer to the full amplitude

Sum over all Feynman diagrams generate integration over the full moduli space $M_{g,n}$ with the correct integrand

Instead of summing over all Feynman diagrams, one could sum over only one particle irreducible (1PI) diagrams

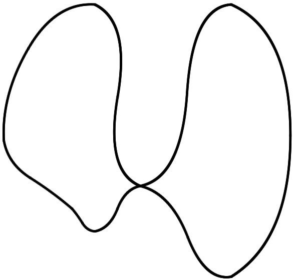
– gives 1PI effective action

$$S_{1PI} = \frac{1}{2}\langle\psi|c_0^- Q_B|\psi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}_{1PI}$$

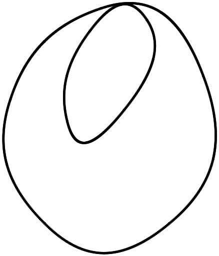
The definition of $\{A_1 \cdots A_n\}_{1PI}$ remains similar to that of $\{A_1 \cdots A_n\}$, except that the subspace of $M_{g,n}$ that we integrate over is larger

– includes boundaries of the moduli space that are non-separating type

(degenerating cycle that does not split the Riemann surface into two disconnected parts.)



Separating



Non-separating

Note: For bosonic string theory, the 1PI effective action is a formal object due to tachyons propagating in the loop.

But there will be no such problem in heterotic and type II theories.

Heterotic string theory:

World-sheet theory contains β, γ ghosts and associated ξ, η, ϕ system after bosonization

$$\beta = \partial\xi \, \mathbf{e}^{-\phi}, \qquad \gamma = \eta \, \mathbf{e}^{\phi}$$

Hilbert space $\mathcal{H}$ splits into direct sum $\oplus_{\mathbf{n}} \mathcal{H}_{\mathbf{n}}$

n: picture number

– integer for NS sector, integer + 1/2 for R sector

Picture changing operator (PCO) Friedan, Martinec, Shenker; Knizhnik

$$\mathcal{X}(\mathbf{z}) = \{\mathbf{Q_B}, \xi(\mathbf{z})\}$$

Heterotic string field theory: A.S.

Introduce a pair of string fields

$$|\psi\rangle \in \mathcal{H}_{-1} + \mathcal{H}_{-1/2}, \quad |\phi\rangle \in \mathcal{H}_{-1} + \mathcal{H}_{-3/2}$$

Action

$$S = \langle\phi|c_0^- Q_B|\psi\rangle - \frac{1}{2}\langle\phi|c_0^- Q_B G|\phi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}$$

G=Identity in NS sector, $G = \mathcal{X}_0 \equiv \oint dz\, z^{-1}\, \mathcal{X}(z)$ in R sector

$$S = \langle \phi | c_0^- Q_B | \psi \rangle - \frac{1}{2} \langle \phi | c_0^- Q_B G | \phi \rangle + \sum_n \frac{1}{n!} \{ \psi^n \}$$

$\{A_1 \cdots A_n\}$ is defined as in bosonic string theory, with the extra ingredient that we have to insert certain number of PCO's to conserve picture number

Total picture no: (2g-2) on a genus g Riemann surface

Different string field theory actions, associated with different choices of PCO locations, are related by field redefinition.

$$\langle\phi|\mathbf{c_0^-}\mathbf{Q_B}|\psi\rangle - \frac{1}{2}\langle\phi|\mathbf{c_0^-}\mathbf{Q_B}\mathbf{G}|\phi\rangle + \sum_{\mathbf{n}}\frac{1}{\mathbf{n}!}\{\psi^{\mathbf{n}}\}$$

Note: We have doubled the number of degrees of freedom ($|\phi\rangle$ and $|\psi\rangle$)

However since $|\phi\rangle$ enters the action at most quadratically, it describes free field degrees of freedom

– completely decouples from the interacting part of the theory described by $|\psi\rangle$

– has no observable effects.

Quantization of this theory proceeds in the same way as in bosonic string theory.

For type II string theory the structure of the theory is similar.

$$|\psi\rangle \in \mathcal{H}_{-1,-1} \oplus \mathcal{H}_{-1,-1/2} \oplus \mathcal{H}_{-1/2,-1} \oplus \mathcal{H}_{-1/2,-1/2}$$

$$|\phi\rangle \in \mathcal{H}_{-1,-1} \oplus \mathcal{H}_{-1,-3/2} \oplus \mathcal{H}_{-3/2,-1} \oplus \mathcal{H}_{-3/2,-3/2}$$

$$S = \langle\phi|c_0^- Q_B|\psi\rangle - \frac{1}{2}\langle\phi|c_0^- Q_B G|\phi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}$$

G: identity in NSNS sector, $\mathcal{X}_0$ in NSR sector,

$\bar{\mathcal{X}}_0$ in RNS sector, $\mathcal{X}_0\bar{\mathcal{X}}_0$ in RR sector

The tree level ψ-ψ propagator has standard form in the 'Siegel gauge'

$$(L_0 + \bar{L}_0)^{-1}\, b_0^+\, b_0^-\, G\, \delta_{L_0, \bar{L}_0}$$

We could (formally) represent this as

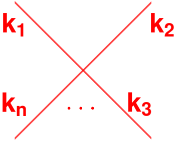
$$b_0^+\, b_0^-\, G\, \frac{1}{2\pi} \int_0^\infty ds\, e^{-sL_0^+} \int_0^{2\pi} d\theta\, e^{-i\theta\, L_0^-}$$

and (formally) recover the usual representation of amplitudes as integrals over $M_{g,n}$.

But we could also regard string field theory as a field theory with infinite number of fields and momentum space propagator

$$(k^2 + M^2)^{-1} \times \text{polynomial in momentum}$$

The polynomial comes from matrix element of $b_0^+\, b_0^-\, G$.



Vertices are accompanied by a suppression factor of

$$\exp\left[-\frac{A}{2}\sum_i (k_i^2 + m_i^2)\right]$$

A: a positive constant whose precise value depends on the choice of coordinate system used to define the off-shell vertex.

Hata, Zwiebach

This makes

– momentum integrals UV finite (almost)

– sum over intermediate states converge

Momentum dependence of vertex includes

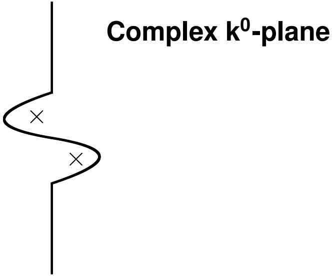
$$\exp\left[-\frac{A}{2}\sum_i (k_i^2 + m_i^2)\right] = \exp\left[-\frac{A}{2}\sum_i (\vec{k}_i^{\,2} + m_i^2) + \frac{A}{2}(k_i^0)^2\right]$$

Integration over $\vec{k}_i$ converges for large $|\vec{k}_i|$, but integration over k_i^0 diverges at large $|k_i^0|$.

The spatial components of loop momenta can be integrated along the real axis, but we have to treat integration over loop energies more carefully.

Resolution: Need to have the ends of loop energy integrals approach $\pm i\infty$.

In the interior the contour may have to be deformed away from the imaginary axis to avoid poles from the propagators.



We shall now describe how to choose the loop energy integration contour.

General procedure:

1. Begin with a configuration of off-shell external momenta where all energies are imaginary and all spatial momenta are real.

2. In this case we can take all loop energy contours to lie along the imaginary axis without encountering any singularity.

3. Now deform the energies to real values (Wick rotation)

4. If some pole of a propagator approaches the loop energy integration contours, deform the contours away from the pole, keeping their ends at $\pm i\infty$.

Result: Such deformations are always possible

– the loop energy contours do not get pinched by poles from two sides during this deformation.

Applications

Consider N tachyon amplitude in bosonic string theory

$$\propto \int \prod_{i=4}^{N} d^2 z_i \prod_{i<j} |z_i - z_j|^{p_i . p_j}$$

This integral diverges if $p_i . p_j < -2$ for any pair (i,j).

In string field theory these divergences can be associated with $L_0^+ < 0$ states propagating in the internal line.

We need to represent the propagator by $1/L_0^+$ instead of using Schwinger parametrization.

This suggests a specific procedure.

1. Remove all regions where 2 or more z_i's approach each other

$|q_k| \geq \epsilon_k$ for fome small positive constants ϵ_k

– gives a finite integral over some region R with boundary B_k corresponding to $|q_k| = \epsilon_k$.

2. The missing regions are compensated for by adding boundary terms.

Also need additional boundary terms where two boundaries intersect etc.

$$\int_R I + \int_{B_k} I_k + \int_{B_k \cap B_\ell} I_{k\ell} + \cdots$$

– an alternative to analytic continuation.

If I denotes the bulk integrand, then the contribution on the k-th boundary B_k is given by

$$\int_{B_k} I_k, \qquad dI_k = -I \quad \text{near } B_k$$

String field theory also gives a specific procedure for constructing I_k.

If the boundary is at $q_k = 0$ and if I behaves near $q_k = 0$ as

$$I = \sum_n a_n\, q_k^{b_n}\, \bar{q}_k^{c_n}\, dq_k \wedge d\bar{q}_k \wedge dV$$

then we can take

$$I_k = -\sum_n a_n\, (b_n+1)^{-1}\, q_k^{b_n+1}\, \bar{q}_k^{c_n}\, d\bar{q}_k \wedge dV$$

On the intersection $B_k \cap B_\ell$ we have to add

$$\int_{B_k \cap B_\ell} I_{k\ell}, \qquad dI_{k\ell} = I_\ell - I_k \quad \text{near } B_k \cap B_\ell$$

etc.

The boundary terms correspond to Feynman diagrams with one or more internal propagators.

The bulk term corresponds to the elementary N-string vertex.

Any ambiguity in determining l_k by solving $dl_k = -l$ etc. cancels in the final expression.

List of applications made so far:

1. Proof of unitarity — Pius, A.S.

2. Finding the domain of analyticity of the S-matrix — De Lacroix, Erbin, A.S.

3. Systematic procedure for dealing with vacuum shift and mass renormalization

4. String theory in RR background — Cho, Collier, Yin

– begin with string theory in NSNS background described by a world-sheet CFT, and then study the effect of RR background by giving vev to RR fields