AdS4 Black hole, hyperbolic 3-manifold, and twisted analytic torsion

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A Magnetically charged AdS4 Black hole

Classically

$$ds^2/L^2 = -\left(\rho - \frac{1}{2\rho}\right)^2 dt^2 + \left(\rho - \frac{1}{2\rho}\right)^{-2} d\rho^2 + \rho^2 ds^2(\Sigma_g)$$

$$F = \frac{vol(\Sigma_g)}{L^2} \quad \text{(Magnetic flux for U(1) gauge field)}$$

Determined by

- G_4 **(4d Newton Const) ,**
- L **(AdS4 radius)**
- $g > 1$ **(genus number)**

- BPS Solution of **4D $\mathcal{N}$ =2 minimal gauged supergravity**

$$I = \frac{1}{16\pi G_4} \int d^4x\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{L^2}{4}F^2\right) + \text{(fermions)}$$

- **Near horizon** $\left(\rho = \frac{1}{2^{1/2}}\right)$: $AdS_2 \times \Sigma_g$,

  Asymptotically $(\rho \to \infty)$: AdS_4 with asymptotic boundary $\boldsymbol{R}_t \times \Sigma_g$

- In terms of **AdS/CFT**, the BH solution describes

  RG : (**3D** $\mathcal{N}$ =2 SCFT on $\boldsymbol{R}_t \times \Sigma_g$) ⟶ (**1D** SCQM on $\boldsymbol{R}_t$)

  topological twisting : $\left(A^{(b,g)}\right)_R = \omega(\Sigma_g)$ **Superconformal R-symmetry : *"universal twist"***

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From semiclassical analysis [Bekenstein, Hawking]

$$S_{\rm BH} = \frac{A}{4G_4} = \frac{(g-1)L^2\pi}{2G_4} + (\text{subleadings in } G_4 \text{ and } 1/L)$$

If the BH solution (AdS4 supergravity) can be embedded into an UV complete Quantum Gravity,

we may give a non-perturbative definition of d_{micro}(# of micorstates of BH), which should satisfy

1) $d_{micro}\,(G_4,\ L, g)$ is a non-negative **integer** (after including all corrections)

➡ **AdS4** SUGRA with **generic** $(G_4,\ L)$ in `**swampland**'? only discrete choices of $(G_4,\ L)$ in **QG**?

2) $S_{\rm BH} = \log d_{micro}\,(G_4,\ L, g) = \dfrac{(g-1)L^2\pi}{2G_4} + (\text{subleadings in } G_4 \text{ and } 1/L)$

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In this talk,

First, embed the BH into **M-theory**

N M5-branes on $\boldsymbol{R} \times \Sigma_g \times M_3 \subset \boldsymbol{R} \times (T^*\Sigma_g) \times (T^*M_3)$

$N \to \infty$

The BH in AdS_4 xM_3x$\tilde{S}^4$

- M_3 (hyperbolic 3-manif
- N (number of M5s)
- $g > 1$ (genus number)

$$L^2 / G_4 = \frac{2N^3 vol(M_3)}{3\pi^2}$$

$$L/L_{planck} \sim N^{1/3}$$

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$\subset \boldsymbol{R} \times (T^*\Sigma_g) \times (T^*M_3)$

$\xrightarrow{N \to \infty}$ **The BH in** $AdS_4 \times M_3 \times \tilde{S}^4$

- $\boldsymbol{M_3}$ **(hyperbolic 3-manif**
- ***N*** **(number of M5s)**
- $g > 1$ **(genus number)**

Second, give non-perturbative def $\boldsymbol{d_{micro}(M_3, N, g)}$ using AdS4/CFT3

$$L^2/G_4 = \frac{2N^3 vol(M_3)}{3\pi^2}$$

AdS4/CFT3 : (***N*** M5-branes compactified on $\boldsymbol{M_3}$) = (**M theory on** $AdS_4 \times M_3 \times \tilde{S}^4$)

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N M5-branes on $\mathbb{R} \times \Sigma_g \times M_3 \subset \mathbb{R} \times (T^*\Sigma_g) \times (T^*M_3)$ $\xrightarrow{N \to \infty}$ **The BH in $AdS_4 \times M_3 \times \widetilde{S}^4$**

- M_3 (hyperbolic 3-manif
- N (number of M5s)
- $g > 1$ (genus number)

Second, give non-perturbative def $d_{micro}(M_3, N, g)$ using AdS4/CFT3

AdS4/CFT3 : (N M5-branes compactified on M_3) = (**M theory on $AdS_4 \times M_3 \times \widetilde{S}^4$**)

$$L^2/G_4 = \frac{2N^3 vol(M_3)}{3\pi^2}$$

$$L/L_{\text{planck}} \sim N^{1/3}$$

Finally, we will check 1) d_{micro} is an **integer** (after including all corrections)

2) $S_{BH} = \log d_{micro} = \frac{(g-1)L^2\pi}{2G_4} = \frac{(g-1)vol(M_3)}{3\pi} N^3 + (\text{subleadings in } 1/N)$.

Two classes of AdS4/CFT3 using M-theory

BH solution with asymptotically AdS4 ⟹ Can be studied using AdS4/CFT3

Two classes of well-established AdS4/CFT3 using M-theory

AdS4/CFT3 from M2-branes	AdS4/CFT3 from M5-branes
$\boldsymbol{R}^{1,2} \times \mathrm{Cone}(Y_7)$ ($\boldsymbol{Y_7}$: SE 7-manifold) with $\boldsymbol{N}$ M2-branes on $R^{1,2}$ ⟶ $\boldsymbol{T_N[Y_7]}$ 3D $\boldsymbol{\mathcal{N}}$=2 SCFT with global $U(1)_R \subset G = \mathrm{ISO}(Y_7)$	$\boldsymbol{R}^{1,2}$x (T^*M_3)x $\boldsymbol{R}^2$ ($\boldsymbol{M_3}$: **Closed hyperbolic 3-manifold**) with $\boldsymbol{N}$ M5 branes on $R^{1,2}$xM_3 ⟶ $\boldsymbol{T_N[M_3]}$ 3D $\boldsymbol{\mathcal{N}}$=2 SCFT, with global $G = U(1)_R$
M-theory on $\boldsymbol{AdS_4}$ xY_7 ↓ $(G_4/L^2 = \sqrt{\frac{27}{8N^3\pi^4}\mathrm{Vol}(Y_7)}\,)$ 4D $\boldsymbol{\mathcal{N}}$=2 gauged supergravity with G = ISO(Y_7) $S_{\mathrm{BH}} = \frac{(g-1)L^2\pi}{2G_4} = (g-1)\sqrt{\frac{2}{27\mathrm{Vol}(Y_7)}}\boldsymbol{N^{3/2}}\pi^3$	M-Theory on warped $\boldsymbol{AdS_4}$ **x**$\boldsymbol{M_3}$**x**$\boldsymbol{\tilde{S}^4}$ [Pernici ;'85] [Gauntlet-Kim-Waldram;00] ↓ $(G_4/L^2 = \frac{3\pi^2}{2N^3 vol(M)})$ 4D $\boldsymbol{\mathcal{N}}$ =2 gauged supergravity with G $=U(1)_R$ $S_{\mathrm{BH}} = \frac{(g-1)L^2\pi}{2G_4} = \frac{(g-1)vol(M_3)}{3\pi}\boldsymbol{N^3}$
Field theoretic description of $T_N[Y_7]$ **[HLLLP;08][ABJM;08][ABJ;08]........** e.g) $T_N[S^7/Z_k]$=ABJM model	Field theoretic description of $T_N[M_3]$ **[Dimofte-Gukov-Gaiotto;11][Dimofte-Gabella-Goncharov;14][DG-Yonekura;18]** e.g) $T_{N=2}$[ **5**$_2$]= (U(1) + Φ with k=-7/2)

Non-perturbative definition of $d_{\rm micro}$ using AdS4/CFT3

Question : Which quantity in CFT3 corresponds to the $\mathrm{d}_{\rm micro}$ of the BH ?

Hints:

BH : Asymptotic AdS_4 with $\partial(AdS_4) = \boldsymbol{R}_t \times \Sigma_g$	→	Near horizon $AdS_2 \times \Sigma_g$,
RG : (3D $\boldsymbol{N}$ =2 SCFT on $\boldsymbol{R}_t \times \Sigma_g$)	→	(1D SCQM on $\boldsymbol{R}_t$)

Natural Answer : the number of ground states of 3d SCFT on Σ_g

$d_{\rm micro}$ = # of supersymmetric ground states of (3D $\boldsymbol{N}$ =2 SCFT on Σ_g) **"Difficult to compute"**

$cf)$ $d^{\text{SUSY}}_{\rm micro} := \mathrm{Tr}_{\,H^{E=0}(3D\ \boldsymbol{N}=2\ \text{SCFT}\ on\ \Sigma_g)}\,(-1)^R$ **"Twisted index"**

Recently, people found that [Benini-Hristov-Zaffaroni ;'15][Hosseini-Zaffaroni ;'16].......

$$\mathrm{Log}(d^{\text{SUSY}}_{\rm micro}(T_N[Y_7],g)) \xrightarrow{\ \boldsymbol{N}\to\infty\ } \frac{(g-1)L^2\pi}{2G_4} = (g-1)\sqrt{\frac{2}{27\mathrm{Vol}(Y_7)}}\,N^{3/2}\pi^3 + \text{sub-leadings}$$

$d^{\text{SUSY}}_{\rm micro}(T_N[Y_7],g) = d_{\rm micro}(T_N[Y_7],g)$?? **Maybe no!**

Twisted index $d^{\text{SUSY}}_{\text{micro}}(g) = \text{Tr}_{H(3\text{D}\ \mathcal{N}=2\ \text{SCFT}\ on\ \Sigma_g)}(-1)^R$

For $g = 1$ $(\Sigma_g = T^2)$ case : It is just usual **Witten index** [Kim-Kim ;'10] [Seiberg-Intrilligator ;'12]

For $g = 0$ (S^2) case [Benini-Zaffaroni ;'15]

For general g [Benini-Zaffaroni ;'16] [Closset-Kim ;'16]

For general $3D$ $\mathcal{N} = 2$ theory with gauge G,
the index can be written as **finite sum** over so called \`**Bethe vacua**' [Nekrasov-Shatasivilli] [Gukov-Pei;'15] [Benini-Hristov-Zaffaroni ;'15]

$$\boldsymbol{d^{\text{SUSY}}_{micro}(g) = \sum_{\alpha:\text{Bethe}-\text{vacua}} (H^{\alpha})^{g-1}},$$

Bethe vacua: solutions of $\exp\left(2\pi i z_i \dfrac{\partial W}{\partial z_i}\right) = 1, \quad$ for $i = 1,..,\text{rank}(G)$

$W(z_1,...,z_{\text{rank}(G)})$: Twisted superpotential for $2d$ (2,2) theory obtained by S^1 reduction keeping all infinity KK−modes

Chiral field : $\delta W = \text{Li}_2\left(\prod z_i^{-Q_i}\right)$, CS term $\delta W = k_{ij}\text{Log}[z_i]\text{Log}[z_j]$

$H^{\alpha}(z_1,...,z_{\text{rank}(G)})$: \`handle gluing operator' ,

$$\text{Log}[H] = -\log \text{Det}[\partial_{\log[z_i]}\partial_{\log[z_j]}\text{Log}[W]] + \sum_{\text{Chiral}} \text{Li}_1(z_i^{-Q_i})$$

Most recent studies on AdS4 **BH** in **M-theory** are about BHs from **M2-branes**

$$\text{Log}(d^{\text{SUSY}}_{\text{micro}}(T_N[Y_7],\text{g})) \xrightarrow{N\to\infty} (g-1)\sqrt{\frac{2}{27\text{Vol}(Y_7)}}N^{3/2}\pi^3 + \text{sub-leadings}$$

Good : Gauge theory description is simple → Matrix model technique

Flavor symmetry other than U(1) R-symmetry → Rich SUSY BHs

Bad : Improperly quantized superconformal R-charge : generically no universal BH

Computation of sub-leading seems to be challenging

AdS4 **BHs** from **M5-branes?**

$$\text{Log}(d^{\text{SUSY}}_{\text{micro}}(T_N[M_3],\text{g})) \xrightarrow{N\to\infty} (g-1)\frac{N^3\text{vol}(M_3)}{3\pi} \;??$$

Bad : UV Gauge theory description is very ugly, no matrix model ($u(1)^{N^3}$ gauge group)

Good : we can use the power of **3d-3d relation**

(***full perturbative sub-leadings*** are computable)

$d^{SUSY}_{micro}(T_N[M_3]$,g) from 3d-3d relation

3d-3d relation : $(T_N[M_3]$ on **S**$) \sim (\mathrm{PSL}(N,\mathbb{C})$ Chern Simons theory on $M_3)$, **not duality but a relation**

[Yamazaki-Terashima ;'11][Dimoft-Gukov-Gaiotto;11]
[Yagi;13][Lee-Yamazaki;13][Cordova-Jafferis;13]

M-theoretic derivation : $6dA_{N-1}(2,0)$ theory on **S** × M_3

$M_3 \to 0$: $(T_N[M_3]$ on **S**$)$

S $\to 0$: $(\mathrm{PSL}(N,\mathbb{C})$ Chern Simons theory on $M_3)$

S = $S^3, S^1\mathrm{x}S^2, S^3/Z_k$,….

$d^{SUSY}_{micro}(T_N[M_3], g)$ from 3d-3d relation

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$\mathbf{S} = S^3, S^1 \mathrm{x} S^2, S^3/Z_k, \ldots$

Dictionary :

[Dimofte-Gukov-Holland ;'10]
[DG-Kim-Pando Zayas;'19]

$T_N[M_3]$	$PSL(N,\mathbb{C})$ Chern Simons theory on M_3
Bethe vacuum α	$PSL(N,\mathbb{C})$ irreducible flat connection A^α
Handle gluing operator H^α	$N\ \mathrm{Exp}[-2S^\alpha(1)]$

$$dA^\alpha + A^\alpha \wedge A^\alpha = 0$$

$$CS[A] = \int_M \mathrm{tr}\left(AdA + \frac{2}{3}A^2\right)$$

Recall that $d^{SUSY}_{micro}(g) = \sum_{\alpha:\mathrm{Bethe-vacua}} (H^\alpha)^{g-1}$, $\quad (for\ M_3\ with\ \mathbf{H_1}(M_3, Z_N) = 0)$

$S^\alpha(n)$: n loop perturbative expansion coefficient around a flat connection A^α

$$\log \int \frac{[d(\delta A)]}{(\mathrm{gauge})} \mathrm{Exp}\left[\frac{1}{2\hbar} CS[A^\alpha + \delta A]\right] \longrightarrow \frac{1}{\hbar} S^\alpha(0) + S^\alpha(1) + ..\hbar^{n-1} S^\alpha(n) + \ldots$$

$$\mathbf{S}^\alpha(\mathbf{1}) = \frac{1}{4} \mathbf{Log}\left[\frac{\left(\mathbf{det}' \Delta_0^{(\alpha)}\right)^3}{\left(\mathbf{det}' \Delta_1^{(\alpha)}\right)}\right]$$ (**Analyic Ray-singer torsion**)

$\Delta_n^{(\alpha)} = * d_A * d_A + d_A * d_A *$, $\quad d_A = d + A^\alpha \wedge$
(Laplacian acting on n-form twisted by A^α)

$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$ from 3d-3d relation

3d-3d relation : $(T_N[M_3]$ on $\mathbf{S}) \sim (\text{PSL}(N,\mathbb{C})$ Chern Simons theory on $M_3)$, **not duality but a relation**

M-theoretic derivation : $6dA_{N-1}(2,0)$ theory on $\mathbf{S} \times M_3$

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$M_3 \to 0$: $(T_N[M_3]$ on $\mathbf{S})$

$\mathbf{S} \to 0$: $(\text{PSL}(N,\mathbb{C})$ Chern Simons theory on $M_3)$

$\mathbf{S} = S^3, S^1\text{x}S^2, S^3/Z_k, \ldots$

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$$dA^\alpha + A^\alpha \wedge A^\alpha = 0$$

$$CS[A] = \int_M \text{tr}\left(AdA + \frac{2}{3}A^2\right)$$

Derivation : From $M_3 = \left(\bigcup_{i=1}^k \Delta_i \bigcup_i^m S_i\right)/\sim$, Δ: (*ideal tetrahedron*), S: *solid torus*

[Dimofte-Garoufalidis;12]
+other mathematical facts

Field theoretic description of $T_N[M_3]$

[Dimofte-Gaiotto-Gukov;11]
[Dimofte-Gabella-Goncharov;14]
[DG-Yonekura;18]

(Localization)

$\text{N Exp}[-2S^\alpha(1)]$ = H^α

M-theoretic derivation is missing

$d^{SUSY}_{micro}(T_N[M_3],\text{g})$ from 3d-3d relation

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$$S^\alpha(1) = \frac{1}{4}\mathrm{Log}\left[\frac{\left(\det{}'\Delta_0{}^{(\alpha)}\right)^3}{\left(\det{}'\Delta_1{}^{(\alpha)}\right)}\right]$$ (**Twisted analyic Ray-Singer torsion**)

$$d^{SUSY}_{micro}(T_N[M_3],\text{g}) = N^{g-1}\sum_\alpha \exp(-2S^\alpha(1)[M_3\ ;N]\ )^{g-1}$$

$(for\ M_3\ with\ \boldsymbol{H_1}(M_3, Z_N) = 0)$

We reduce the microstates counting of BH to a mathematical problem !!
Use mathematical results to study BH entropy !!

$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$ from 3d-3d relation

3d-3d relation : $(T_N[M_3]) \sim (\text{PSL}(N,C)$ Chern Simons theory on $M_3)$, not duality but a relation

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$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = N^{g-1}\sum_{\alpha}\exp(-2S^{\alpha}(1)[M_3\ ;N])^{g-1}$$

$(for\ M_3\ with\ \boldsymbol{H_1}(M_3, Z_N) = 0)$

We reduce the microstates counting of BH to a mathematical problem !!
Use mathematical results to study BH entropy !!

Using the expression, Let us check followings

1) $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$ is an integer (after including all corrections)

2) $S_{\text{BH}} = \log d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = \frac{(g-1)vol(M_3)}{3\pi}N^3 + (\text{subleadings in } 1/N).$

Integrality of $d_{micro}^{\text{SUSY}}(T_N[M_3],\text{g})$

Irreducible flat connection : $dA^\alpha + A^\alpha \wedge A^\alpha = 0$

$$S^\alpha(1)[\mathrm{M};\mathrm{N}] = \frac{1}{4}\mathrm{Log}\left[\frac{\left(\det{}'\Delta_0{}^{(\alpha)}\right)^3}{\left(\det{}'\Delta_1{}^{(\alpha)}\right)}\right] \text{ (Ray-singer torsion)}$$

$$d_{micro}^{\text{SUSY}}(T_N[M_3],\text{g}) = N^{g-1}\sum_\alpha \exp(-2S^\alpha(1)[M_3\ ;N]\ )^{g-1}$$

$(for\ M_3\ with\ \boldsymbol{H_1}(M_3, Z_N) = 0)$

Conjecture

$d_{micro}^{\text{SUSY}}(T_N[M_3],\text{g}) \in \boldsymbol{Z}$

Integrality of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

Irreducible flat connection : $dA^\alpha + A^\alpha \wedge A^\alpha = 0$

$$S^\alpha(1)[M;N] = \frac{1}{4}\text{Log}\left[\frac{\left(\det{}'\Delta_0{}^{(\alpha)}\right)^3}{\left(\det{}'\Delta_1{}^{(\alpha)}\right)}\right] \text{ (Ray-singer torsion)}$$

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = N^{g-1}\sum_\alpha \exp(-2S^\alpha(1)[M_3\,;N]\,)^{g-1}$$

$(for\ M_3\ with\ \boldsymbol{H_1}(M_3, Z_N) = 0)$

Conjecture

$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) \in \boldsymbol{Z}$

e.g) $M_3 =$  $_5$, $N=2$

[Computable using tools developed by mathematicians]

$$\{\exp(-2S^\alpha(1)[M_3;N]\,)\}_{\alpha=1,2,3,4}$$

$=\{-1.90538-0.568995\,i, -1.90538+0.568995\,i, 1.73992, 2.57085\}$ $\quad (\,x^4 - 1/2\,x^3 - 8x^2 + 283/16 = 0\,)$

$\longrightarrow$ $\{d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})\}_{g=0,1,2,..}$ = { 0, 4, 1, 65, 97, 1045,…}

Conjecture

$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g=0}) = 0$

No ground states for **g=0**

Large N of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = N^{g-1}\sum_\alpha \exp(-2S^\alpha(1)[M_3\ ;N]\,)^{g-1}$$

$(for\ M_3\ with\ \boldsymbol{H_1}(M_3, Z_N) = 0)$

Two canonical irreducible flat connections A^{hyp} *and* $A^{\overline{hyp}}$

$A^{hyp} = \rho_N[\omega + ie]$, $A^{\overline{hyp}} = \rho_N[\omega - ie]$

ρ_N: su(2) → su(N), N − dimensional irred representation

ω: spin connection
e: vielbein

for unique hyperbolic metric on M satisfying $R_{\mu\nu} = -2g_{\mu\nu}$

Both of them can be locally considered as so(3) valued 1 forms

$\omega \pm ie$: *sl(2, C) valued 1 form satisfying flat connection equation* $dA + A \wedge A$

These two give dominant contributions to the d^{SUSY}_{micro} in large N (g>1)

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = \left(H^{\text{hyp}}\right)^{g-1} + (c.c)$$

+exponentially small in $1/N$

$$H^\alpha = N\exp(-2S^\alpha(1)[M_3\ ;N]\,)$$

Large N of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

Two canonical flat connections give dominant contributions

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = (H^{\text{hyp}})^{g-1} + (c.c) + \text{exponentially small in } 1/N \qquad H^\alpha = N\exp(-2S^\alpha(1)[M_3\,;N])$$

Mathematicians studied [Muller;14] [Park;17] using **Selberg's trace formula**

$$2\text{Re}[S^{\text{hyp}}(1)[M_3;N]] \longrightarrow -\frac{(N^3-N)\text{vol}(M_3)}{3\pi} - \frac{\text{vol}(M_3)}{6\pi}(N-1) - \text{a}(M_3) - \text{b}(M_3;N)$$

$$a(M_3) = -\boldsymbol{Re}\sum_\gamma \frac{1}{s}\left(\frac{e^{-sl_{\mathbb{C}}(\gamma)}}{1-e^{-sl_{\mathbb{C}}(\gamma)}}\right)^2, \qquad \text{b}(M_3;N) = \boldsymbol{Re}\sum_\gamma\sum_{s=1}\frac{1}{s}\left(\frac{e^{-\frac{(N+1)sl_{\mathbb{C}}(\gamma)}{2}}}{1-e^{-sl_{\mathbb{C}}(\gamma)}}\right)^2$$

γ : primitive conjugacy class of $\pi_1(M_3)$

$l_{\mathbb{C}}(\gamma)$: complex length of γ

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = 2\text{Cos}[\theta_N[\boldsymbol{M_3}]]\exp\left((g-1)\left(\frac{2N^3-N-1}{6\pi}\text{vol}(M_3) + \log N + a(M_3) + \text{b}(M_3;N)\right)\right) + (\dots)$$

(Relative phase)

$(for\ M_3\ with\ \boldsymbol{H_1}(M_3,Z_N) = 0)$

Large N of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = 2\text{Cos}[\theta_N[M_3]]\exp\left((g-1)\left(\frac{2N^3-N-1}{6\pi}\text{vol}(M_3)+\log N+a(M_3)+\text{b}(M_3;N)\right)\right)+(\dots)$$

(Relative phase) *(Bekenstein-Hawking)* (Logarithmic correction $3(g-1)\log L$)

$(for\ M_3\ with\ \boldsymbol{H_1}(M_3,Z_N)=0)$

$$a(M_3) = -\boldsymbol{Re}\sum_{\gamma}\frac{1}{s}\left(\frac{e^{-sl_{\mathbb{C}}(\gamma)}}{1-e^{-sl_{\mathbb{C}}(\gamma)}}\right)^2, \quad \text{b}(M_3;N) = \boldsymbol{Re}\sum_{\gamma}\sum_{s=1}\frac{1}{s}\left(\frac{e^{-\frac{(N+1)sl_{\mathbb{C}}(\gamma)}{2}}}{1-e^{-sl_{\mathbb{C}}(\gamma)}}\right)^2$$

Large N leading part : Bekenstein-Hawking entropy $S_{\text{BH}} = \frac{(g-1)L^2\pi}{2G_4} = \frac{(g-1)vol(M_3)}{3\pi}N^3$

Log N term : match with SUGRA zero-mode analysis $3(g-1)(1+\boldsymbol{b_1(M_3)})\log L = (g-1)\log N$

2-form ghost **3-form C_3**

[S. Bhattacharyya, A. Grassi, M. Marino, A. Sen;'14]
[J. Liu,1, L. Pando Zayas,V.Rathee,W. Zhao;'17]

Other sub-leadings :

$$\begin{cases} -\frac{N+1}{6\pi}\text{vol}(M_3)(g-1) \\ a(M_3)(g-1),\ \text{b}(M_3;N)\ (g-1) \end{cases}$$

How to understand from M-theory on $AdS_4 \times M_3 \times \widetilde{S}^4$?

From **M2-branes** wrapping $\boldsymbol{\gamma}$ in M_3?

Summary and future directions

We study microstate counting $d^{\text{SUSY}}_{micro}(T_N[M_3], \text{g})$ for 4d magnetically charged BHs made of wrapped M5-branes

Using a 3d-3d relation, the counting to a mathematical problem

$$d^{\text{SUSY}}_{micro}(T_N[M_3], \text{g}) = N^{g-1} \sum_{\alpha} \exp(-2S^{\alpha}(1)[M_3 \,; N]\,)^{g-1}$$

Then, using known mathematical results

$$d^{\text{SUSY}}_{micro}(T_N[M_3], \text{g}) = 2\text{Cos}\big[\theta_N[\boldsymbol{M_3}]\big]\exp\left((g-1)\left(\frac{2N^3 - N - 1}{6\pi}\text{vol}(M_3) + \underline{\log N} + a(M_3) + \text{b}(M_3; N)\right)\right) + (\dots)$$

(Relative phase) *(Bekenstein-Hawking)* (Logarithmic correction $3(g-1)\log L$)

Future work : 1) **6D derivation** of the 3d-3d relation for twisted index

2) Perturbative corrections (a, b) from **quantum gravity**? (b from M2?)

3) Curious **integral properties of ray-singer torsions** on hyperbolic 3-manifolds

[Work in progress with Seonhwa kim, Seokbeom Yoon]

Thank you
for your attention !!