Soft Hair on Generic Horizons
and
Black Hole Microstates

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Based on my recent papers with

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And upcoming work with

D. Grumiller, A. Perez, R. Troncoso, C. Zwikel

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Outline

- Some words on black holes, theoretical and observational
- Status of black hole microstate problem
- Residual symmetries and soft charges
- Soft Hair on generic horizons
- Near horizon symmetry algebras and membrane paradigm
- Horizon fluff as black hole microstates
- Summary and Outlook

Black holes.....

- Gravity Waves has opened a window to the blackness or darkness.

- Questions around black holes (BH) touches upon the deepest issues in our understanding of notions like spacetime, Quantum Theory and Gravity.

- Observationally, BHs appear in a wide range of mass and spin in many physically relevant systems.

- Theoretically, BHs constitute a big class of known solutions to (Einstein's) General Relativity (GR).

- BH solutions come in various families with very different properties.

- Regardless of the details, classically

  any BH has horizon which separates spacetime into two causally disconnected regions, inside and outside.

- Semiclassically BHs have a thermodynamical description:

  they Hawking-radiate and have entropy and evolve as governed by laws of thermodynamics.

- Unitarity of this evolution at quantum level necessitates existence of BH microstates.

Equivalence Principle and Diffeomorphisms

- Einstein GR is based on Equivalence Principle which stipulates that all observers should give (exactly) the same description of local events in regions of spacetime to which they have causal access.

- Each observer is specified by a coordinate system and vice versa.

- Equivalence Principle at theory level is made manifest through general covariance, invariance of the action under diffeomorphisms.

- Physical observables in the Einstein GR are all defined through local diffeomorphism invariant quantities.

- In particular, any two metric tensors related by diffeomorphisms are physically equivalent:

$$x^\mu \to x^\mu + \xi^\mu(x), \quad g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}, \ \delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

- The above is shared between all theories with local gauge symmetries: Action and physical observables should be gauge invariant.

- Gauge symmetry is in fact a redundancy of description which should be removed by gauge fixing, but yet, there may be nontrivial gauge transformations for a prescribed falloff and boundary conditions and boundary terms.

- Among thermodynamical quantities entropy is very special:

it is the only extensive, dimensionless and observer independent quantity.

- Note: “corrections” to entropy may be ensemble dependent.

- While may not agree on mass (energy), angular momentum, temperature, all observers must measure the same value for entropy,

- Features and expectations of BH entropy
  - It should be accounted by the BH microstates
  - Its ubiquity is a result of ubiquitous property of BHs, the horizon.
  - For the cases with Killing horizon, BH entropy is a conserved (Noether) charge given by the Wald formula. It reduces to the Bekenstein-Hawking area law for the case of Einstein gravity.

- According to Wald's derivation, black hole entropy depends only on the BH solution (metric) and the gravitational part of the action.

- Intensive thermodynamical quantities like temperature and horizon angular velocity only depend on the metric and ,

- Extensive quantities like, mass and angular momentum depend on metric and the gravitational part of the theory.

- From the above one can deduce

A) *Entropy of a classical, large BH is a gravitational effect and BH microstates should be sought for ONLY in the gravitational sector.*

B) *Recalling the uniqueness theorems, not all the physically observable gravitational effects can be removed in a local accelerating frame, hence*

C) the simple and strong statement of Equivalence Principle should be amended.

“Softly” moving away the equivalence principle

- Diffeo’s are ‘’local redundancies”. There are, however, nontrivial diffeo’s to which one can associate well-defined surface charges not measured by local observables/observations.

- To extract the non-trivial diffeo’s and the associated surface charges we may use covariant phase space method (CPSM):

  i) All field configurations (histories) may form a Phase Space,

  ii) with the symplectic structure systematically constructed from the action of the theory:

- Consider a field configuration Φ and perturbations around it $\delta\Phi$.

- On-shell field configurations $\bar{\Phi}$ satisfy field equations and on-shell perturbations $\delta\Phi$ satisfy linearized field equations.

- Set of Φ and $\delta\Phi$ may be viewed as a phase space and one-forms in the corresponding cotangent space.

- On-shell cotangent space includes two important directions:

  – $\delta\Phi$ generated by gauge and/or diffeo's transformations on Φ;

  – parametric variations, generated by moving in the parameter space of the solutions Φ, e.g. the difference between two Sch'd solutions with masses m and $m + \delta m$.

Symplectic structure

- Symplectic current $\boldsymbol{\omega}$ is a *finite*, *closed*, *nondegenerate* two-form over tangent space and a $d-1$ form in space time:

$$\boldsymbol{\omega} = \boldsymbol{\omega}[\delta_1\Phi, \delta_2\Phi; \Phi]$$

- Symplectic structure $\boldsymbol{\Omega}_\Sigma$ is defined through integration of $\boldsymbol{\omega}$ over a Cauchy surface Σ:

$$\boldsymbol{\Omega}_\Sigma\ [\delta_1\Phi, \delta_2\Phi; \Phi] = \int_\Sigma \boldsymbol{\omega}[\delta_1\Phi, \delta_2\Phi; \Phi]$$

- We build $\boldsymbol{\omega}$ within the *covariant phase space method*, constructed in [Lee-Wald '1990, Wald '1993] and refined in [Barnich-Brandt '2002, Barnich-Compère '2008].

Construction of the symplectic current

- Presymplectic potential $\boldsymbol{\theta}[\delta\Phi;\Phi]$: $\boldsymbol{\omega} = \delta\boldsymbol{\theta}$, or

$$\boldsymbol{\omega}[\delta_1\Phi,\delta_2\Phi;\Phi] = \delta_1\boldsymbol{\theta}[\delta_2\Phi;\Phi] - \delta_2\boldsymbol{\theta}[\delta_1\Phi;\Phi]$$

- The presympelctic structure $\boldsymbol{\theta}$ is a spacetime $d-1$ form and a one-form over the phase space.

- The Lee-Wald contribution to $\boldsymbol{\theta}$:

$$\delta\boldsymbol{L}\big|_{on-shell} = d\boldsymbol{\theta}_{(LW)}.$$

- Consistency of symplectic structure may require addition of *boundary terms* $\mathbf{Y}$:

$$\boldsymbol{\theta} = \boldsymbol{\theta}_{(LW)} + d\mathbf{Y}.$$

$\boldsymbol{Y}$ is a $d-2$ form on spacetime and one-form on phase space.

- Consistency of symplectic structure means its

  – Conservation:

  $d\boldsymbol{\omega}[\delta_1\Phi, \delta_2\Phi; \Phi] \approx 0$ for all on-shell fields and perturbations.

  – Non-degeneracy: $\boldsymbol{\Omega}_\Sigma$ has no degenerate directions, is conserved and is independent of Σ.

The conserved charges

- Fundamental Theorem of Covariant Phase Space Method

$$\boldsymbol{\omega}[\delta\Phi, \delta_\chi\Phi; \Phi] \approx \boldsymbol{d}\boldsymbol{K}_\chi[\delta\Phi; \Phi]$$

- $\delta_\chi\Phi$ is a specific transformation generated by a symmetry χ,

- $\boldsymbol{K}$ is a spacetime $d-2$ form, while a one-form on the tangent space of the phase space.

- Given $\boldsymbol{K}$ one can define charge variations:

$$\delta Q_\chi = \oint_{\partial\Sigma} \boldsymbol{K}_\chi[\delta\Phi; \Phi]$$

- Charge Q_χ is integrable if

$$\delta_1\delta_2 Q_\chi - \delta_2\delta_1 Q_\chi = 0$$

Integrability [Lee-Wald '1991]:

$$\oint_{\partial\Sigma} \chi\cdot\boldsymbol{\omega}[\delta_1\Phi, \delta_2\Phi; \Phi] = 0, \quad \forall\chi, \delta\Phi$$

There usually exists $\boldsymbol{Y}$ terms which guarantee the above.

- Using integrability one can define surface charges Q_χ:

$$Q_\chi[\Phi] = \int_\gamma \oint_{\partial\Sigma} \boldsymbol{K}_\chi[\delta\Phi; \Phi] + N_\chi[\Phi]$$

where N is the zero point charge.

- If δQ_χ is zero everywhere on the phase space, χ is called pure gauge transformation. These are the "real gauge d.o.f".

- The charges are given by surface integrals over the boundary of the Cauchy surface $\partial\Sigma$.

- The charge Q_χ is non-zero if Cauchy surface is non-compact.

- Examples of $\partial\Sigma$:
  - Flat d dimensional Minkowski space: is the $d-2$ dimensional sphere at infinity, i^0.
  - Sch'ld black hole: $\partial\Sigma = \mathcal{H} \cup i^0$, where $\mathcal{H}$ is the bifurcate horizon.
  - AdS-Sch'ld BH: $\partial\Sigma = \mathcal{H} \cup S^{d-2}_{\text{b'dry}}$.

    (Note: AdS is not globally hyperbolic and hence one should take special care here.....)

- Algebra of charges:

$$\{Q_\chi, Q_\xi\} = Q_{[\chi,\xi]} + \text{possible central terms}$$

- Notes:
  - Charges are functions over the phase space,
  - the bracket is Poisson bracket among these functions, and
  - $[\chi, \xi]$ is the Lie bracket of generators.

- The charges Q_ξ may be used to label states/configurations in the phase space, and hence how to account for them.

Focusing on BHs, e.g. generic Kerr black hole,

- How can we describe BH thermodynamics in terms of these conserved charges?

- what are non-trivial diffeos and what is their algebra?

- How do we specify our phase space?

- How can this resolve the BH microstate problem?

General picture and results for BH thermodynamics

- $\partial\Sigma$ has two separated parts $\mathcal{H}$ and i^0.

- We can hence have two distinct set of charges, the Near Horizon (NH) charges and the asymptotic (AS) charges.

- Common part of the two accounts for mass and angular momenta associated with exact symmetries of the BH background solution.

- Smoothness of the geometry implies that NH and AS observers should measure the same exact charges.

- We can understand thermodynamics of stationary BHs only in terms of these exact charges, assoicated with Killing/exact symmetries.

- Exact charges are symplectic symmetries and may be computed on any codimension to compact surface. [Hajian-MMShJ, 2015].

- First law of thermodynamics for stationary BHs with Killing horizon is generically reduced to the equation relating Killing vectors, .e.g for Kerr BH

$$\zeta = \frac{1}{\kappa}(\xi_t + \Omega \xi_\phi)$$

- $1/\kappa$ factor is necessitated by integrability of the charge associated with ζ, the entropy.

■ General picture/idea and results for BH microstates

- States charged under non-trivial residual diffeos are soft, they commute with the Hamiltonian.

- We hence have NH soft hair and AS soft states/config's.

- NH soft hair account for BH microstates while AS soft states are irrelevant to the BH entropy.

- NH soft modes weakly interact with each other.

- NH and AS soft states also weakly interact at quantum level. (Recall the potential barrier separating the NH and AS regions.)

■ Summary of results[D. Grumiller, A. Perez, MMShJ, R. Troncoso, C. Zwikel, coming soon]

- For Sch'ld or Kerr BH, AS algebra is BMS_4 and we find that NH algebra consists of NH APD's and "Horizon superrotations" plus "supertranslations".

- Besides our NH boundary (falloff) conditions, depending on what we keep fixed on the horizon, we get a different algebra.

- This is like moving between different ensembles in stat. mech.

- Our NH analysis is true for generic (non-degenerate) black hole or cosmological horizons.

Some details of charge analysis

- Any metric with a Killing horizon, in the NH region takes the form

$$ds^2_{NH} = -\kappa^2\rho^2 dt^2 + d\rho^2 + \Omega_{ab}(x)dx^a dx^b + \cdots$$

where $a, b = 1, 2, \cdots, d-2$.

- We assume the bifurcate horizon $\mathcal{H}$ ($\kappa \neq 0$) with metric Ω_{ab}, is compact and non-degenerate $\det\Omega \neq 0$ with a finite volume.

- NH falloff behavior

$$g_{tt} = -\kappa^2\rho^2 + \mathcal{O}(\rho^3),\ \ g_{t\rho} = \mathcal{O}(\rho^2),\ \ g_{ta} = f_{ta}\rho^2 + \mathcal{O}(\rho^3),$$

$$g_{\rho\rho} = 1 + \mathcal{O}(\rho),\ \ g_{\rho a} = f_{\rho a}\ \rho + \mathcal{O}(\rho^2),\ \ g_{ab} = \Omega_{ab} + \mathcal{O}(\rho),$$

- The residual diffeo's which respect these NH falloff behavior are

$$\xi^t = \frac{\eta(t;x)}{\kappa} + \mathcal{O}(\rho),\ \ \xi^\rho = \mathcal{O}(\rho^2),\ \ \xi^a = \eta^a(t;x) + \mathcal{O}(\rho^2),$$

  where $\partial_t + \eta^a \partial_a \kappa = \delta\kappa$.

- η, η^a may be viewed as diffeo's on a codimension one surface.

- Here we choose to work in Einstein gravity. But our results may be readily generalized to any generally covariant higher derivative theory.

- In our NH metric κ can be a function of t, x^a.

- Standard computation of charges leads to

$$\delta Q[\eta,\eta^a] = \int_{\mathcal{H}} \eta\delta\mathcal{P} + \eta^a \delta\mathcal{J}_a$$

where

$$\mathcal{P} = \frac{\sqrt{\Omega}}{8\pi G}, \qquad \mathcal{J}_a = \frac{\sqrt{\Omega}}{16\pi\kappa\ G}\left(\partial_t f_{\rho a} - 2 f_{ta}\right)$$

- $\delta Q[\eta,\eta^a]$ are integrable if η,η^a are field independent, but we have other choices too.

- Charge variations

$$\delta\mathcal{P} = \eta^a\partial_a\ \mathcal{P} + \mathcal{P}\ \partial_a\eta^a,$$

$$\delta\mathcal{J}_a = \mathcal{P}\partial_a\eta + \mathcal{J}_a\ \partial_b\eta^b + \eta^b\ \partial_b\mathcal{J}_a + \mathcal{J}_b\ \partial_a\eta^b.$$

- The charge algebra

$$\{\mathcal{P}(x), \mathcal{P}(y)\} = 0 \qquad \text{as} \qquad \delta_\eta \mathcal{P} = 0$$

$$\{\mathcal{J}_a(x), \mathcal{J}_b(y)\} = \left(\mathcal{J}_a(y)\frac{\partial}{\partial x^b} - \mathcal{J}_b(x)\frac{\partial}{\partial y^a}\right)\delta^{d-2}(x-y)$$

$$\{\mathcal{J}_a(x), \mathcal{P}(y)\} = \left(\mathcal{P}(y)\frac{\partial}{\partial x^a} - \mathcal{P}(x)\frac{\partial}{\partial y^a}\right)\delta^{d-2}(x-y)$$

- The charges $\mathcal{P}(x), \mathcal{J}_a(x)$ are densities over the horizon $\mathcal{H}$.

- $\mathcal{J}_a(x)$ generate general diffeomorphisms on the horizon.

- $\mathcal{P}(x)$ generate supertranslations on the horizon.

More on boundary conditions

- The above choice of η, η^a are not the only choices leading to integrable charges.

- We may choose

$$\tilde{\eta} = F(\mathcal{P}, \mathcal{J}_b)\eta + F_a(\mathcal{P}, \mathcal{J}_b)\eta^a, \qquad \tilde{\eta}^a = G^a(\mathcal{P}, \mathcal{J}_b)\eta + G^a{}_b(\mathcal{P}, \mathcal{J}_b)\eta^b,$$

with appropriate redefinition of charges

$$\tilde{\mathcal{P}} = \tilde{\mathcal{P}}(\mathcal{P}), \qquad \tilde{\mathcal{J}}_a = \tilde{\mathcal{J}}_a(\mathcal{J}_a, \mathcal{P})$$

- Below we show two such examples.

NH "generalized BMS algebra"

Choose

$$\tilde{\eta} \equiv \eta_{(r)} = \mathcal{P}^{-r}\eta, \qquad \tilde{\mathcal{P}} \equiv \mathcal{P}_{(r)} = \frac{\mathcal{P}^{r+1}}{r+1},$$

with $\tilde{\eta}^a = \eta^b$, $\quad \tilde{\mathcal{J}}_a = \mathcal{J}_a$. Then

$$Q[\eta_{(r)}, \eta^a] = \int_{\mathcal{H}} \left[\eta_{(r)}\mathcal{P}_{(r)} + \eta^a \mathcal{J}_a\right]$$

yielding the algebra

$$\{\mathcal{P}_{(r)}(x), \mathcal{P}_{(r)}(y)\} = 0$$

$$\{\mathcal{J}_a(x), \mathcal{J}_b(y)\} = \left(\mathcal{J}_a(y)\frac{\partial}{\partial x^b} - \mathcal{J}_b(x)\frac{\partial}{\partial y^a}\right)\delta^{d-2}(x-y)$$

$$\{\mathcal{J}_a(x), \mathcal{P}_{(r)}(y)\} = \left(r\mathcal{P}_{(r)}(y)\frac{\partial}{\partial x^a} - \mathcal{P}_{(r)}(x)\frac{\partial}{\partial y^a}\right)\delta^{d-2}(x-y)$$

- The above is true in any dimension.

- There is no central term in the above NH algebras.

- If Ω_{ab} is topologically an S^{d-2}, $\mathcal{J}_a$ part of the algebra includes $d-2$ dimensional Euclidean conformal algebra $so(d-1,1)$, i.e. d dimensional Lorentz algebra.

- For $r=0$ and $d=3,4$ this recovers the DGGP algebra [Donnay, Giribet, Gonzalez, Pino, 2015, 2016].

- For $r=1, d=3$ we recover BMS_3.

- For $d=3$, generic r, we get $W(0;-r)$ algebra [Parsa, Safari, MMShJ, 2018].

- For $d = 4$, if restrict

$$\Omega_{ab} = \Phi^2 \; \Omega^{S^2}_{ab}$$

- for $r = 1/2$ we get BMS_4.

- For generic r we get W(-r/2,-r/2;-r/2,-r/2) [Safari, MMShJ, 2019].

- For $d > 4$ restricting horizon metric Ω_{ab} to confrmally sphere ones, for $r = \frac{1}{d-2}$ we get an algebra which may be called BMS_d.

- For generic r we get a higher spin version of BMS, "supertranslations $\mathcal{P}$ have generic spin $s = r(d-2)$.

NH Heisenberg-like algebra, another special case:

- Define

$$\eta^a_{\text{H}} = \sqrt{\Omega}\ \eta^a, \qquad \mathcal{J}^{\text{H}}_a = \frac{\mathcal{J}_a}{\sqrt{\Omega}}$$

- $\mathcal{J}^{\text{H}}_a$ is a one-form (not a density).

- Then

$$Q_{\text{H}}[\eta_{\text{H}}, \eta^a_{\text{H}}] = \int_{\mathcal{H}} \eta_{\text{H}}\mathcal{P} + \eta^a_{\text{H}}\mathcal{J}^{\text{H}}_a$$

with

$$\delta\mathcal{P} = \frac{1}{8\pi G}\partial_a\eta^a_{\text{H}}$$

$$\delta\mathcal{J}^{\text{H}}_a = \frac{1}{8\pi G}\Big[\partial_a\eta_{\text{H}} - \frac{\eta^b_{\text{H}}}{\mathcal{P}}\Big(\partial_a\mathcal{J}^{\text{H}}_b - \partial_b\mathcal{J}^{\text{H}}_a\Big)\Big], \qquad \eta_{\text{H}} = \eta + \eta^a\mathcal{J}^{\text{H}}_a.$$

- Q_{H} become integrable if we assume η_{H}, η^a are field independent.

$$\{\mathcal{P}(x), \mathcal{P}(y)\} = 0$$

$$\{\mathcal{J}_a^{\mathsf{H}}(x), \mathcal{J}_b^{\mathsf{H}}(y)\} = \frac{1}{8\pi G\ \mathcal{P}(x)} F_{ba}(x) \delta^{d-2}(x-y)$$

where $F_{ab}(x) \equiv \partial_a \mathcal{J}_b^{\mathsf{H}}(x) - \partial_b \mathcal{J}_a^{\mathsf{H}}(x)$

$$\{\mathcal{J}_a^{\mathsf{H}}(x), \mathcal{P}(y)\} = \frac{1}{8\pi G}\ \frac{\partial}{\partial x^a} \delta^{d-2}(x-y)$$

The above implies

$$\{F_{ab}(x), \mathcal{P}(y)\} = 0.$$

- If $F_{ab}(x) = 0$, e.g. as in the Sch'ld case, then

$$\mathcal{J}^{\mathsf{H}}_a(x) \equiv \partial_a \mathcal{Q}(x).$$

- Hence $\{\mathcal{Q}(x), \mathcal{Q}(y)\} = 0$ and

$$\{\mathcal{Q}(x), \mathcal{P}(y)\} = \frac{1}{8\pi G}\ \delta^{d-2}(x-y)$$

- Upon "quantization," replacing $\{,\}$ with $-i[,]$, the NH algebra reduces to "a Heisenberg type algebra", with $\hbar$ equal to $1/(8\pi G)$.

- In general, one can decompose the one-form $\mathcal{J}^{\mathsf{H}}_a$ into exact and coexact parts. The exact part $\mathcal{Q}$ is the "conjugate" to $\mathcal{P}$.

- NH Hamiltonian

$$H_{\mathsf{NH}} = Q_{\mathsf{H}}[\eta_{\mathsf{H}} = \kappa \equiv \eta_0, \eta^a_H = 0] = \int_{\mathcal{H}} \kappa \mathcal{P} \equiv \kappa \mathcal{P}_0$$

- $\mathcal{P}_0$ commutes with all the other charges as $\delta_{\eta_0}\mathcal{P} = 0$, $\delta_{\eta_0}\mathcal{J}^{\mathsf{H}}_a = 0$.

- Therefore, all states charged under $\mathcal{P}, \mathcal{J}^{\mathsf{H}}_a$ are soft.

- Note that the above happens only for the "Heisenberg-type" boundary conditions, generically the near horizon Hamiltonian need not commute with the other charges.

- "Heisenberg-type" boundary conditions are hence more natural.

- The BH (Bekenstein-Hawking) entropy and *the NH first law*

$$S_{\mathrm{BH}} = 2\pi \mathcal{P}_0 = \frac{1}{4G}\int_{\mathcal{H}} \sqrt{\Omega} = \frac{Area}{4G},$$

$$dH_{\mathrm{NH}} = T_H \; dS_{\mathrm{BH}}, \qquad T_H = \frac{\kappa}{2\pi}$$

- *Entropy must be a charge commuting with all the charges labelling microstates.*

- The above is true for any horizon in any dimension.

- Had we used a more general theory of gravity, we would recover Wald's entropy.

▶ 4d Kerr and NUT BHs, example

- In this case $\mathcal{J}_a^{\mathsf{H}}$ does not have an exact part and

$$\mathcal{P} = \frac{2Mr_+}{8\pi G}\,\sin\theta, \qquad \mathcal{J}_a^{\mathsf{H}} = \epsilon_a{}^b\partial_b\psi$$

where

$$\psi = \frac{1}{8\pi G}\left[2\arctan U + \frac{r_+ - r_-}{2M}U\right], \qquad U = \sqrt{r_-/r_+}\,\cos\theta.$$

- For the Kerr case, $\int_{\mathcal{H}} F = 0$.

- One can show that the NUT charge is proportional to $\int_{\mathcal{H}} F$.

Discussion, Concluding Remarks and Outlook

⊛ The need for revisiting Einstein General Invariance:

- Not all diffeomorphic/gauge equivalent field configurations are physically equivalent.

- Only a measure-zero subset of gauge transformations, defined on codimension two surfaces can be non-trivial.

- This codim. two surface may or may not be the boundaries of Cauchy surfaces $\partial\Sigma$; it may be bifurcation surface of a horizon.

- One may distinguish classes of gauge equivalent/diffeomorphic configurations by the non-local surface charges labelling them.

- In view of these charges one should extend the physical Hilbert space of the theory:

$$\mathcal{H} = \mathcal{H}_{local\ gauge\ inv.} \otimes \mathcal{H}_{soft}$$

- Their number is infinite (coming from continuous local transformations).

- They form an algebra which may admit central extensions.

- For the case of black holes,

$$\partial\Sigma = \mathcal{H} \cup i_0, \qquad \mathcal{H}_{soft} = \mathcal{H}_{horizon} \cup \mathcal{H}_{\infty}.$$

⊛ Question: Are these charges and their algebra gauge invariant? Do they depend on the choice of gauge fixing and "boundary conditions"?

- Presence of surface charges do not change the n-p't functions of local gauge invariant operators, or the S-matrix.

⊛ Question: Do these charges have any physical relevance?

- Quantum Gauge Field and Gravity Theories
  - memory effect;
  - "soft theorems" and IR dynamics of gauge theories;
  - identifying and counting of BH microstates;
  - resolving the BH unitarity problem?!

- Working within Hawking-Perry-Strominger “soft hair” paradigm, we have over shooting problem: there are too many soft states,

We need to cut the soft hair off

- In series of our works on 3d BHs, we made horizon fluff proposal in which we proposed a way for the cutoff.[Afshar, Grumiller, MMShJ, 2016; MMShJ, Yavartanoo, 2016 & Afshar, Grumiller, MMShJ, Yavartanoo, 2017].

- A paper of today [W. Merbis, D. Grumiller] have elaborated further on the cutoff procedure.

- We also extended horizon fluff proposal to 4d extremal Kerr BH [Hajian, MMShJ, Yavartanoo, 2017].

- Key question in the horizon fluff proposal is to single out a set of NH soft hairs which “add up” giving the mass and angular momentum.

- Thermodynamical features of large classical BHs are universal.

- It is hence expected that the BH microstate identification should have a universal answer.

- With this motivation, we made such a universal analysis for NH soft charges, algebras and states.

- There are various consistent choices for the boundary conditions, for a given NH falloff, yielding different NH Symmetry Algebras.

- Our NH SA, includes generic diffeos on $d-2$ dim. bifurcation horizon surface, plus a supertranslation part.

- Our NH SA includes BMS_d, *without a central term,* as a subalgebra.

- We also have a generic Heisenberg-type algebra, with $1/(4G)$ playing the role of Planck's constant.

- One can compute the on-shell boundary action:

$$I_B = \int dt d^{d-2}x \, \left(\mathcal{N}_H \, \mathcal{P} + \mathcal{N}_H^a \, \mathcal{J}_a^H\right)$$

where $\mathcal{N}_H, \mathcal{N}_H^a$ are the usual lapse and shift functions.

- The above may be viewed as the "action" for a physical membrane at the horizon, as membrane paradigm suggests.

NH SA and the membrane paradigm[D. Grumiller, MMShJ, 2018]

- The idea is that a universal question like BH microstates should have a universal answer through membrane paradigm.

- That is, membrane action as "hydrodynamical" description of the BH thermodynamics.

- Upon its "semiclassical" Bohr-type quantization we should be able to identify BH microstates.

- Consider a $d-2$ dimensional brane wrapping the stretched horizon.

- Membrane action is the volume it swips over the spacetime.

- The action is invariant under $d-1$ dimensional diffeos and the only physical parameter in this action is membrane tension T.

- Fixing the static gauge on the membrane and choosing the natural light-cone time coordinate

$$\tau = t$$

- The membrane e Hamiltonian becomes

$$H_{membrane} = \frac{T}{2\pi} \int_{\mathcal{H}} \sqrt{\det \Omega}$$

- Comparing this with our H_{NH}, we learn

$$T = \frac{\kappa}{4G}$$

Our proposal for BH microstates:

BH microstates are certain states among the near horizon soft hair
and
are indistinguishable (degenerate) from the asymptotic symmetry viewpoint.

This Heisenberg algebra arises as a result of Rindler wedge, ubiquitously found in any nonextreme NH geometry.

Membrane paradigm may be providing the way to identify BH microstates.

Thank You For Your Attention