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FOR FUNDAMENTAL PHYSICS

CFTs at Large Charge

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based on arXiv:1505.01537, 1610.04495, 1707.00710, 1809.06371, 1902.09542, 1905.00026, and work in progress with:

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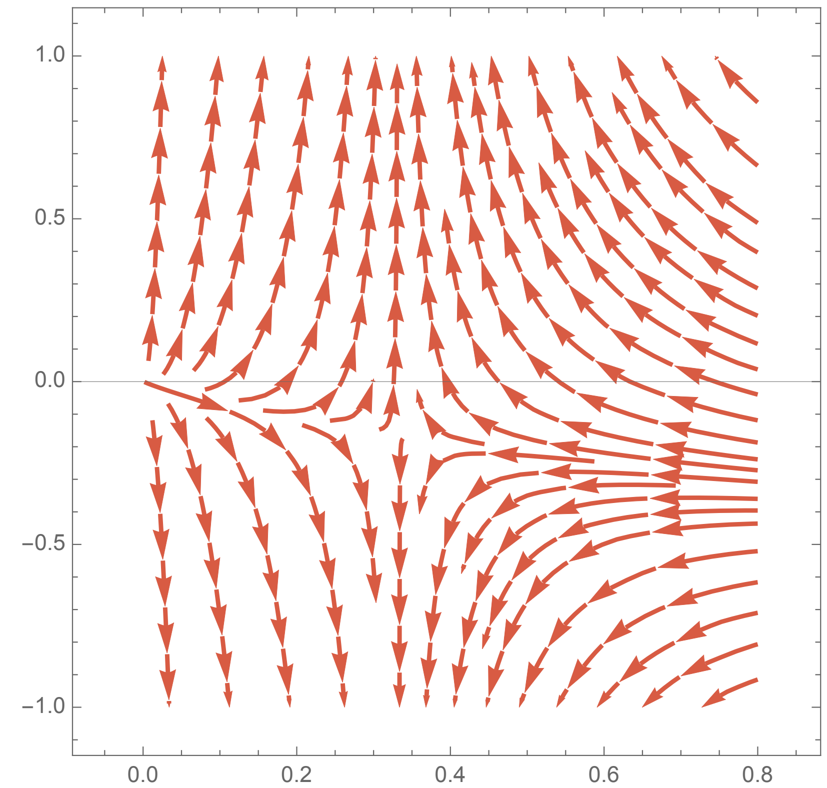
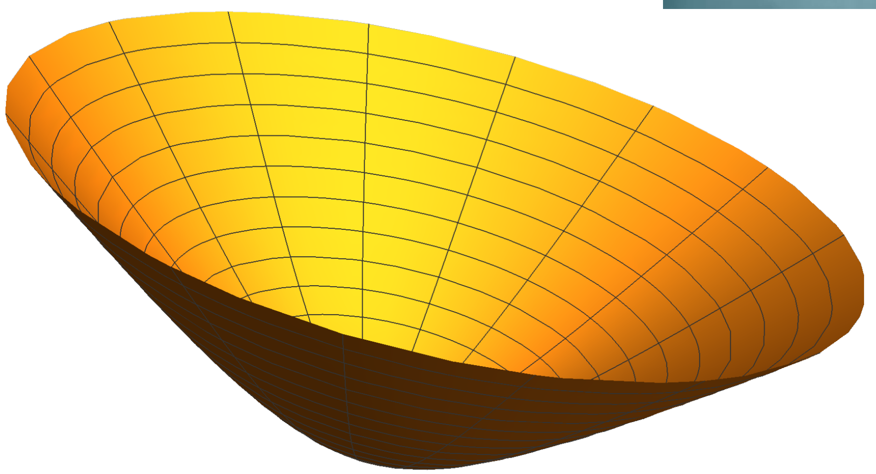
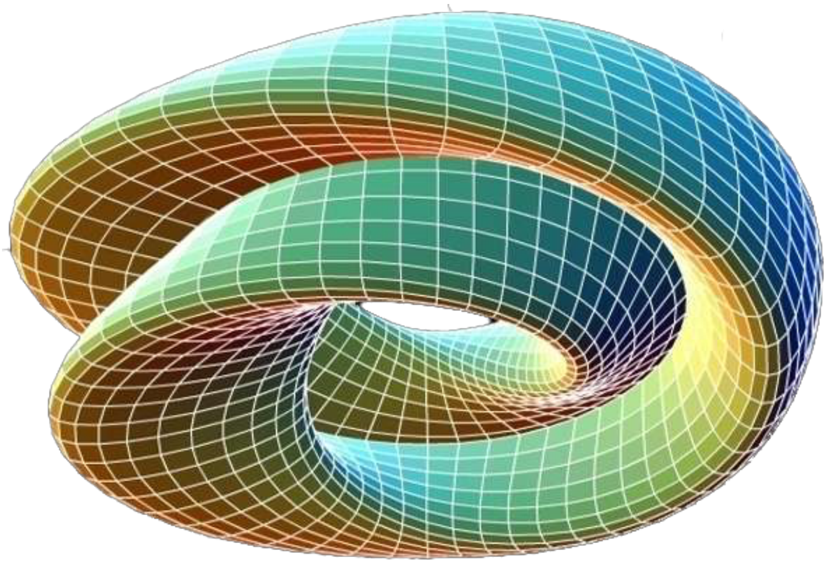
Introduction

Conformal field theories (CFTs) play an important role in theoretical physics:

- fixed points in RG flows
- critical phenomena
- quantum gravity
- string theory







Introduction

BUT: most CFTs do not have small parameters in which to do a perturbative expansion: couplings are O(1).

Difficult to access.
Possibilities: analytic (2d), conformal bootstrap (d>2), lattice calculations, non-perturbative methods…

Make use of symmetries, look at special limits/ subsectors where things simplify.

Here: study theories with a global symmetry group. Hilbert space of the theory can be decomposed into sectors of fixed charge Q under the action of the global symmetry group.
Study subsectors with large charge Q.

Large charge Q becomes controlling parameter in a perturbative expansion!

Introduction

The large-charge approach consists of 2 steps:

1. identify the possible fixed-charge symmetry breaking patterns for a given order parameter

2. write an effective action for the low-energy DOF and compute physical quantities

Step 1: start from the global symmetries of the system and how they act on the order parameter.

For example, in the superfluid transition of 4He, it is known that the system has an O(2) symmetry. Assume that, just like in the UV, the order parameter is a complex scalar that transforms the same way under O(2).

Introduction

Write down Wilsonian effective action. In general: infinitely many terms - not so useful.
Make self-consistent truncation at large charge:

- Set a cutoff Λ obeying

space dimension

typical scale of the system

$$\frac{1}{L} \ll \Lambda \ll \frac{1}{\ell_Q} = \frac{Q^{1/d}}{L}$$

- write a linear sigma model action for the order parameter. Work at criticality: impose scale invariance of the action, assuming that the fields have vanishing anomalous dimension (at leading order in 1/Q)
- determine the fixed-charge ground state
- compute the quantum fluctuations to verify that they are parametrically small when Q >> 1.

Introduction

In a sector of fixed charge, the classical solution around which the quantum fluctuations are computed will generically break both spacetime (Lorentz) and global symmetries: Goldstone bosons

Step 2: write down EFT encoded by Goldstones. Similar techniques to chiral perturbation theory. Important difference: the symmetry breaking comes from fixing the charge (NOT dynamical).

Use EFT to calculate the CFT data (anomalous dimensions, 3-pt functions).

Wilsonian action has only a handful of terms that are not suppressed by the large charge. Useful!

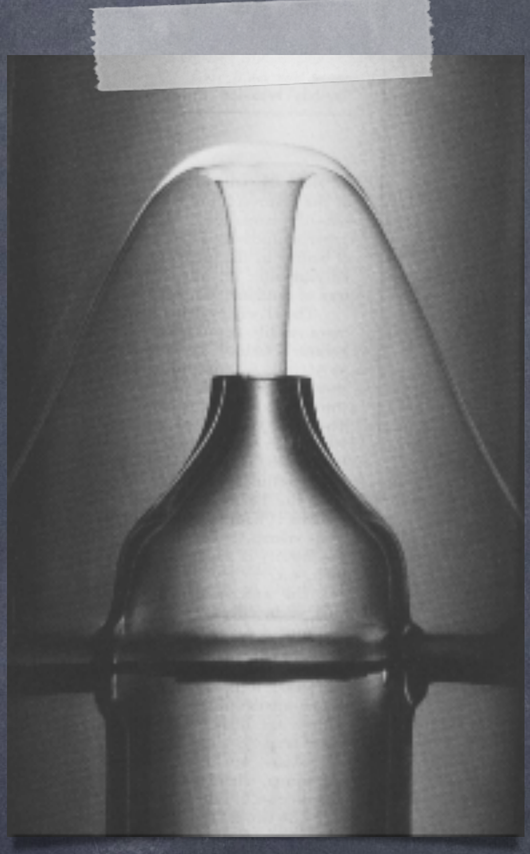
Introduction

Some questions:

- Does it work?
- For what kinds of theories does it work?
- In how many space-time dimensions?
- For what kinds of global symmetries does it work?
- What happens if we fix several charges independently?
- What can we learn via this approach?

Overview

- Introduction
- The O(2) model
  - semi-classical treatment
  - quantum treatment
  - results and lattice comparison
- Beyond O(2)
  - O(2n) vector model
  - an asymptotically safe CFT
  - non-relativistic CFTs
- Summary/Outlook



The O(2) model

The O(2) model

Consider simple model: O(2) model in (2+1) dimensions

$$\mathcal{L}_{\mathrm{UV}} = \partial_\mu \phi^* \, \partial^\mu \phi - g^2 (\phi^* \phi)^2$$

Flows to Wilson-Fisher fixed point in IR.

Assume that also the IR DOF are encoded by cplx scalar

Global U(1) symmetry: $\varphi_{IR} = a \, e^{ib\chi} \qquad \chi \to \chi + \mathrm{const.}$

Look at scales: put system in box (2-sphere) of scale R

Second scale given by U(1) charge Q: $\rho^{1/2} \sim Q^{1/2}/R$

Study the CFT at the fixed point in a sector with

$$\frac{1}{R} \ll \Lambda \ll \frac{Q^{1/2}}{R} \ll g^2$$

UV scale

cut-off of effective theory

Write Wilsonian action.

The O(2) model

Assume large vev for a: $\Lambda \ll a^2 \ll g^2$

scalar curvature

$$\mathcal{L}_{\mathrm{IR}} = \tfrac{1}{2}\,\partial_\mu a\,\partial^\mu a + \tfrac{1}{2} b^2 a^2\,\partial_\mu \chi\,\partial^\mu \chi - \frac{R}{16} a^2 - \frac{\lambda}{6} a^6 + \text{higher derivative terms}$$

dimensionless constants

Lagrangian is approximately scale-invariant.

φ has approximately mass dimension 1/2 and the action has a potential term $\propto |\varphi|^6$

Do semi-classical analysis: solve classical e.o.m. at fixed Noether charge.

$$\rho = \frac{\delta \mathcal{L}_{\mathrm{IR}}}{\delta \dot{\chi}} = b^2 a^2 \dot{\chi} \qquad\qquad Q \sim 4\pi R^2 b\sqrt{\lambda}a^4$$

Classical solution at lowest energy and fixed global charge becomes the vacuum of the quantum theory.

The O(2) model

Classical solution:

non-const. vev

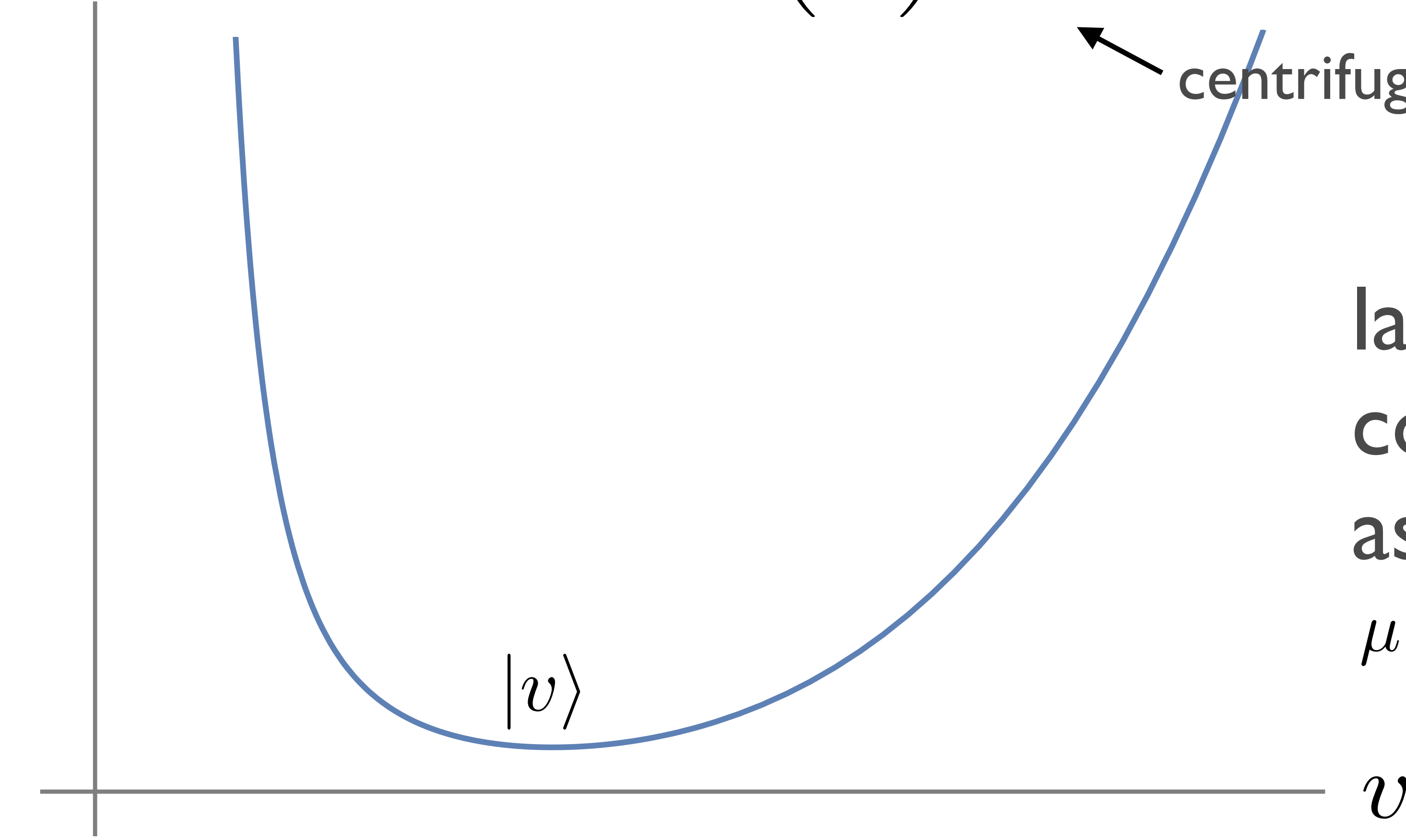
$$\langle a \rangle = v, \qquad \langle \dot{\chi} \rangle = \mu = \frac{Q}{V \cdot v^2}, \qquad \langle \chi \rangle = \mu\, t$$

Fixed-charge ground state is homogeneous in space.

Determine radial vev v by minimizing the classical potential:

$$V_{class} = \left(\frac{Q}{V}\right)^2 \frac{1}{2v^2} + \frac{R}{16} v^2 + \frac{\lambda}{6} v^6$$

centrifugal term



$v \sim Q^{1/4}$

large condensate is compatible with our assumption $a \gg 1$

$\mu \sim \rho^{1/2}$

The O(2) model

Ground state at fixed charge breaks symmetries:

$$SO(1,4)_{\text{spacetime}} \times O(2)_{\text{global}} \xrightarrow{\text{expl.}} SO(3)_{\text{space}} \times D \times O(2)_{\text{global}} \xrightarrow{\text{spont.}} SO(3)_{\text{space}} \times D'$$

$D' = D - \mu O(2)$

Quantum story: study the low-energy spectrum

Parametrize fluctuations on top of the classical vacuum

$$a = v + \hat{a} \qquad \chi = \mu\, t + \frac{\hat{\chi}}{v}$$

Goldstone

massive mode, not relevant
for low-energy spectrum $m \sim \mathcal{O}(\sqrt{Q})$

Go to NLSM: Integrate out a (saddle point for LO).

Dynamics is described by a single Goldstone field χ:

$$\mathcal{L}_{LO} = k_{3/2}(\partial_\mu \chi\, \partial^\mu \chi)^{3/2}$$

can get this purely by
dimensional analysis

The O(2) model

Use dimensional analysis and scale invariance to determine (tree-level) operators in effective action beyond LO (scalar operators of scaling dimension 3, including curvatures of the background metric)

Use ρ-scaling to determine which terms appear:

$$\partial\chi \sim \rho^{1/2}, \quad \partial \dots \partial\chi \sim \rho^{-1/4}$$

$\mathcal{O}(\rho^{3/2})$: $\mathcal{O}_{3/2} = |\,\partial\chi|^3$ LO Lagrangian

conf. inv. combination, negative ρ-scaling

$\mathcal{O}(\rho^{1/2})$: $\mathcal{O}_{1/2} = R|\,\partial\chi| + 2\dfrac{(\partial|\,\partial\chi|)^2}{|\,\partial\chi|}$

scale-inv. but NOT conformally inv.

For homogeneous solutions, there are no other terms contributing to the effective Lagrangian at non-negative ρ-scaling for d>1.

The O(2) model

Result:

$$\mathcal{L} = k_{3/2}(\partial_\mu \chi \partial^\mu \chi)^{3/2} + k_{1/2} R (\partial_\mu \chi \partial^\mu \chi)^{1/2} + \mathcal{O}(Q^{-1/2})$$

dimensionless parameters

suppressed by inverse powers of Q

To be understood as an expansion around the classical ground state $\mu t + \hat{\chi}$

Expand action to second order in fields:

$$\mathcal{L} = k_{3/2}\mu^3 + k_{1/2} R \mu + (\partial_t \hat{\chi})^2 - \tfrac{1}{2}(\nabla_{S^2} \hat{\chi})^2 + \ldots$$

Compute zeros of inverse propagator and get dispersion relation:

$$\omega_{\vec{p}} = \frac{|\vec{p}|}{\sqrt{2}}$$

dictated by conf. invariance $1/\sqrt{d}$

Spontaneous symmetry breaking

⇒ χ is relativistic Goldstone (type I)

⇒ superfluid phase of O(2) model

The O(2) model

Are also the quantum effects controlled?

All effects except Casimir energy are suppressed (negative ρ-scaling)

Effective theory at large Q:

vacuum + Goldstone + 1/Q-suppressed corrections

Energy of classical ground state at fixed charge:

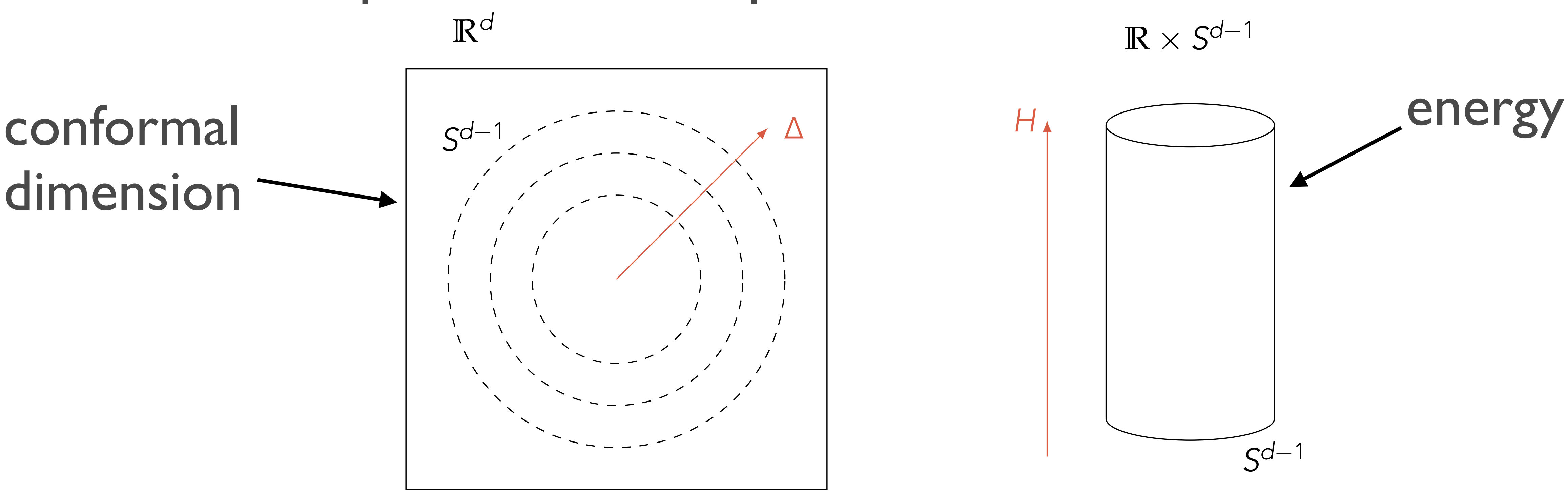
2 dimensionless parameters (b, λ)

$$E_\Sigma(Q) = \frac{c_{3/2}}{\sqrt{V}} Q^{3/2} + \frac{c_{1/2}}{2} R\sqrt{V} Q^{1/2} + \mathcal{O}(Q^{-1/2})$$

dependence on manifold

The O(2) model

Use state-operator correspondence of CFT:



Conformal dimension of lowest operator of charge Q:

one-loop vacuum energy of Goldstone

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

S. Hellerman, D. Orlando, S. R., M. Watanabe, arXiv:1505.01537 [hep-th]

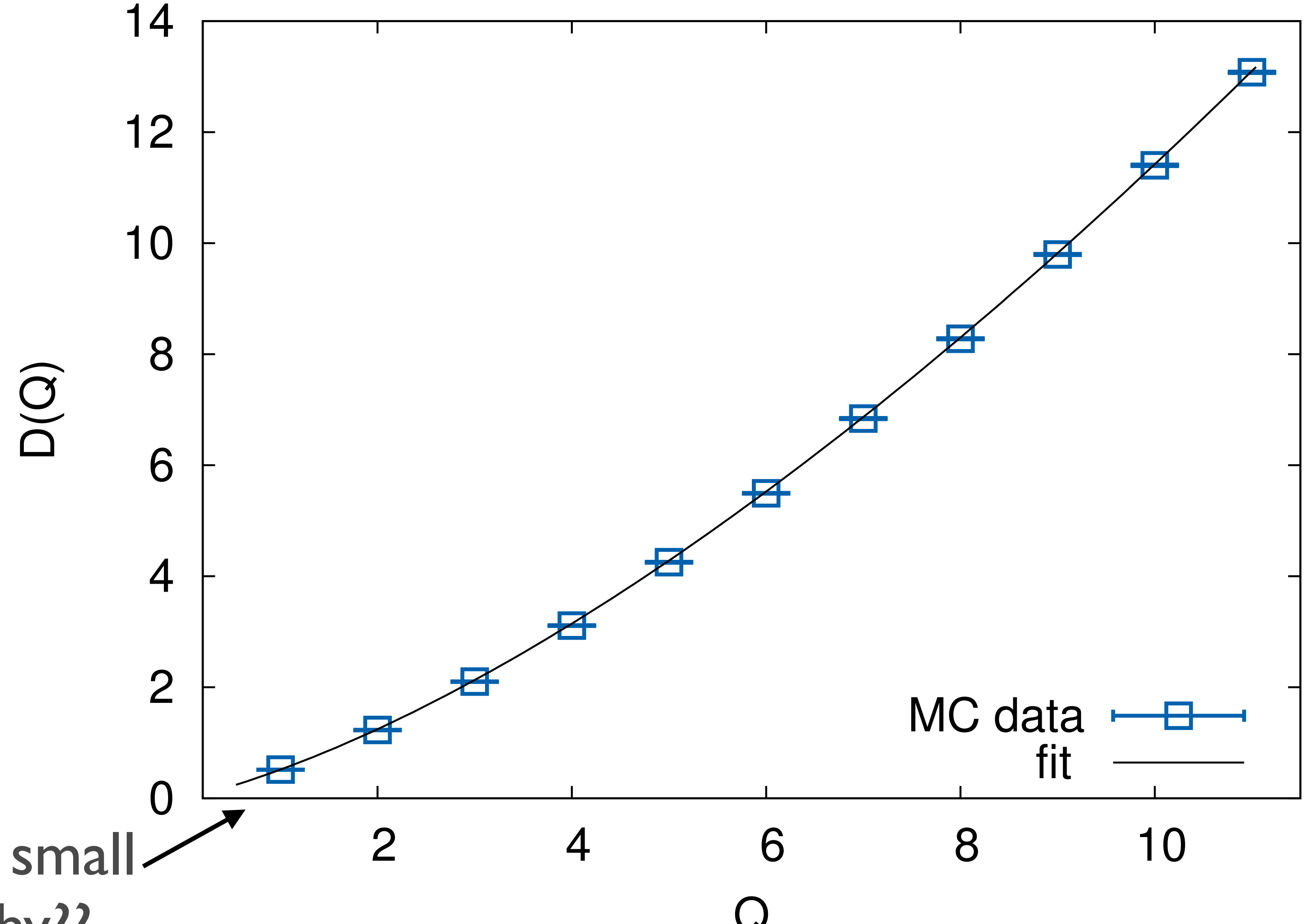
$$E_{\text{VAC}} = \frac{1}{2\sqrt{2}r} \int \frac{\mathrm{d}\omega}{2\pi} \sum_{l=0}^{\infty} (2l+1) \log(\omega^2 + l(l+1)) = \frac{1}{2\sqrt{2}r} \zeta(-1/2|S^2) = -\frac{0.0937\ldots}{r}$$

The O(2) model

Our prediction:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}}Q^{3/2} + 2\sqrt{\pi}\,c_{1/2}Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

Independent confirmation from the lattice:



works for small charge. Why??

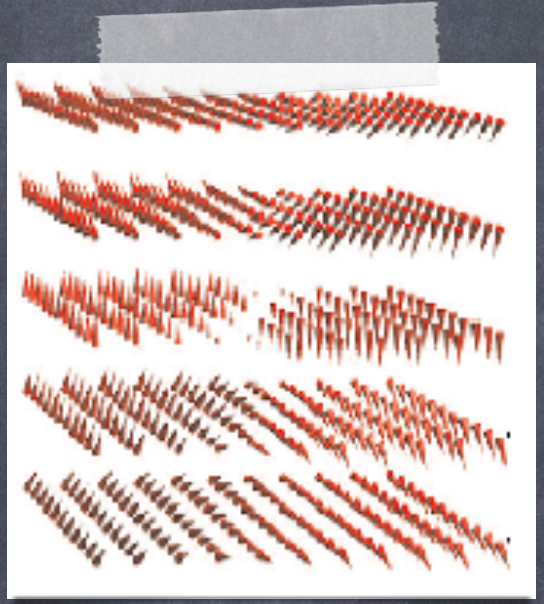
D. Banerjee, Sh. Chandrasekharan, D. Orlando [hep-th/1707.00711]

Excellent agreement!!

$c_{3/2} = 1.195(10)$

$c_{1/2} = 0.075(10)$

Large-charge expansion works extremely well for O(2). Where else?



Beyond O(2):
3d O(2n) vector model

Beyond O(2)

Where else can apply the large-charge expansion?

Try out other known CFTs/assume they exist.

- Obvious generalization in 3d: O(2n) vector model
  non-Abelian global symmetry group: new effects

  SU(N) matrix model in 3d.

  Not many examples of (non-susy) CFTs known in 4d.
- Asymptotically safe CFT (UV fixed point)

  Superconformal CFTs in 3d and 4d. Cases with moduli space work differently!

- Non-relativistic CFTs (Schrödinger symmetry) in 3d, 4d

The O(2n) vector model

Generalize to O(2n).

$$\mathcal{L} = \tfrac{1}{2}\,\partial_\mu \phi^a\,\partial^\mu \phi^a - \tfrac{1}{2}\sum_{i=1}^{2n}\left(\frac{1}{8}R\phi_a^2 + \frac{\lambda}{12}\,\phi_a^6\right), \qquad a = 1,\dots,2n \qquad \mathbb{R}_t \times \mathbb{R}^2$$

$U(n) \subset O(2n)$

$$\varphi_1 = \frac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right), \qquad \varphi_2 = \frac{1}{\sqrt{2}}\left(\phi_3 + i\phi_4\right), \qquad \dots,$$

Fix $k \leq n$ U(1) charges:

$$\int \mathrm{d}^{d-1}x\, i\left(\dot\varphi_i\varphi_i^* - \dot\varphi_i^*\varphi_i\right) = \overline{Q}_i = \text{vol.} \times \bar\rho_i$$

Solution for homogeneous ground state:

$$\begin{cases} \varphi_i = \frac{1}{\sqrt{2}}A_i\, e^{i\mu t}, & i = 1,\dots,k\,, \\ \varphi_{k+j} = 0, & j = 1,\dots,n-k\,, \end{cases}$$

same for all fields!

$$A_i^2 = \frac{Q_i}{4\pi\mu}, \qquad \mu = \frac{1}{4}\sqrt{R + \sqrt{R^2 + \frac{2}{\pi^2}\lambda\left(\sum_i Q_i\right)^2}}$$

The O(2n) vector model

Fixing k charges explicitly breaks O(2n) to O(2n-2k) x U(k).

We can always rotate $\langle\vec{\varphi}\rangle = \frac{1}{\sqrt{2}}(A_1, \dots, A_k, 0, \dots)$ by a U(k) transformation into $\left(0, \dots, 0, \sqrt{\frac{A_1^2+\cdots+A_k^2}{2}}, 0, \dots\right)$

Vacuum breaks symmetry spontaneously to O(2n-2k) x U(k-1).

We also see that all homogeneous states of minimal energy with fixed total charge $(Q_1 + Q_2 + \cdots + Q_k)$ are related by an U(k) transformation and have the same energies (and conformal dimensions).

What happens if instead, we choose a configuration with k different chemical potentials that cannot be rotated into the state $\Big(\underbrace{0, \dots, 0}_{k-1}, \frac{v}{\sqrt{2}}, \underbrace{0, \dots, 0}_{n-k}\Big)$?

Ground state must be inhomogeneous!

The O(2n) vector model

For quantum description, write effective theory for fluctuations around the ground state.

Expand Lagrangian around the ground state

$$(\underbrace{0\,,\,\ldots\,,0}_{k-1},\,\tfrac{v}{\sqrt{2}}\,,\underbrace{0\,,\,\ldots\,,0}_{n-k})$$

U(1) sector: $\varphi_k = \frac{1}{\sqrt{2}}\,e^{i\mu t + i\hat{\phi}_{2k}/v}\left(v + \hat{\phi}_{2k-1}\right)$

$$\begin{cases} \hat{\phi}_{2k-1} \to \hat{\phi}_{2k-1} \\ \hat{\phi}_{2k} \to \hat{\phi}_{2k} + \theta\,, \end{cases}$$

U(k-1) sector: $\varphi_i = e^{i\mu t}\hat{\varphi}_i$

$$\hat{\varphi}_i \mapsto \tilde{U}_i^{\ j}\hat{\varphi}_j$$

Developing to second order in fields:

$$\mathcal{L}^{(2)} = \sum_{i=1}^{k}(\partial_t - i\mu)\varphi_i^*(\partial_t + i\mu)\varphi_i + \sum_{i=k+1}^{n}\dot{\varphi}_i^*\dot{\varphi}_i - \sum_{i=1}^{n}\nabla\varphi_i^*\nabla\varphi_i$$
$$- \sum_{i=1}^{n}\mu^2\varphi_i^*\varphi_i - 2\mu^2\phi_{2k-1}^2$$

Find inverse propagators and dispersion relations.

The O(2n) vector model

We expect dim[U(k)/U(k-1)] = 2k-1 Goldstone d.o.f.

Massless modes:

$$\omega_{nr}^2 = \frac{p^4}{4\mu^2} - \frac{p^6}{8\mu^4} + \mathcal{O}(\mu^{-6}) \qquad k-1 \text{ times}$$

$$\omega_r^2 = \frac{1}{2}p^2 + \frac{p^4}{32\mu^2} + \mathcal{O}(\mu^{-4}) \qquad \text{one time}$$

"conformal" Goldstone

There are

- 1 relativistic Goldstone $\omega \propto p$
- k-1 non-relativistic Goldstones (count double) $\omega \propto p^2$

Nielsen and Chadha; Murayama and Watanabe

$$1 + 2 \times (k-1) = 2k - 1 = \dim(G/H)$$

Non-relativistic Goldstones have no zero-point energy and do not contribute to the conformal dimensions. Ground-state energy again determined by a single relativistic Goldstone.

The O(2n) vector model

Same formula for anomalous dimensions as for O(2):

n-dependent

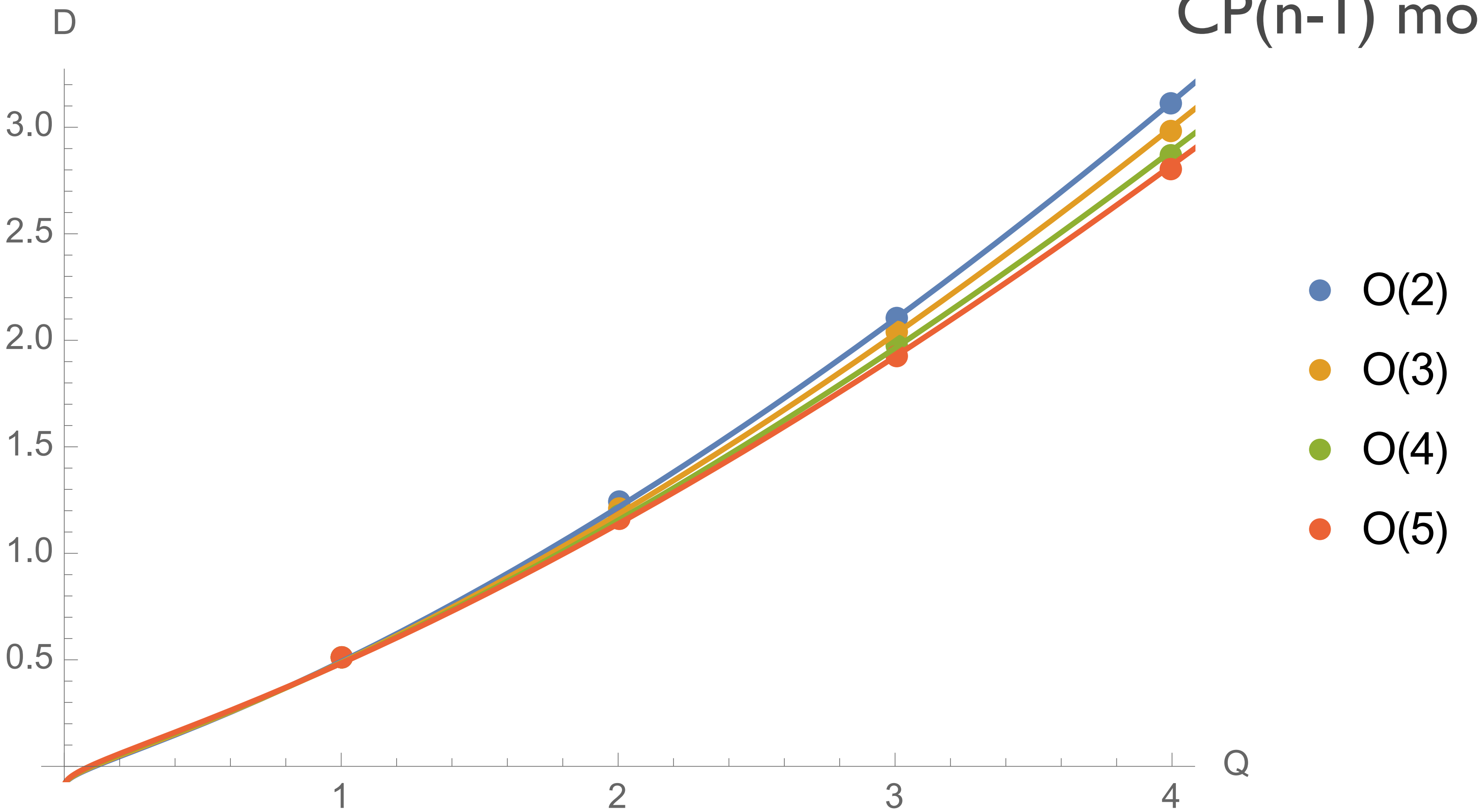
universal for O(2n)

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

L. Alvarez-Gaume, O. Loukas, D. Orlando and S. R., arXiv:1610.04495 [hep-th]

verified at large n for CP(n-1) model de la Fuente

Comparison with old lattice data:

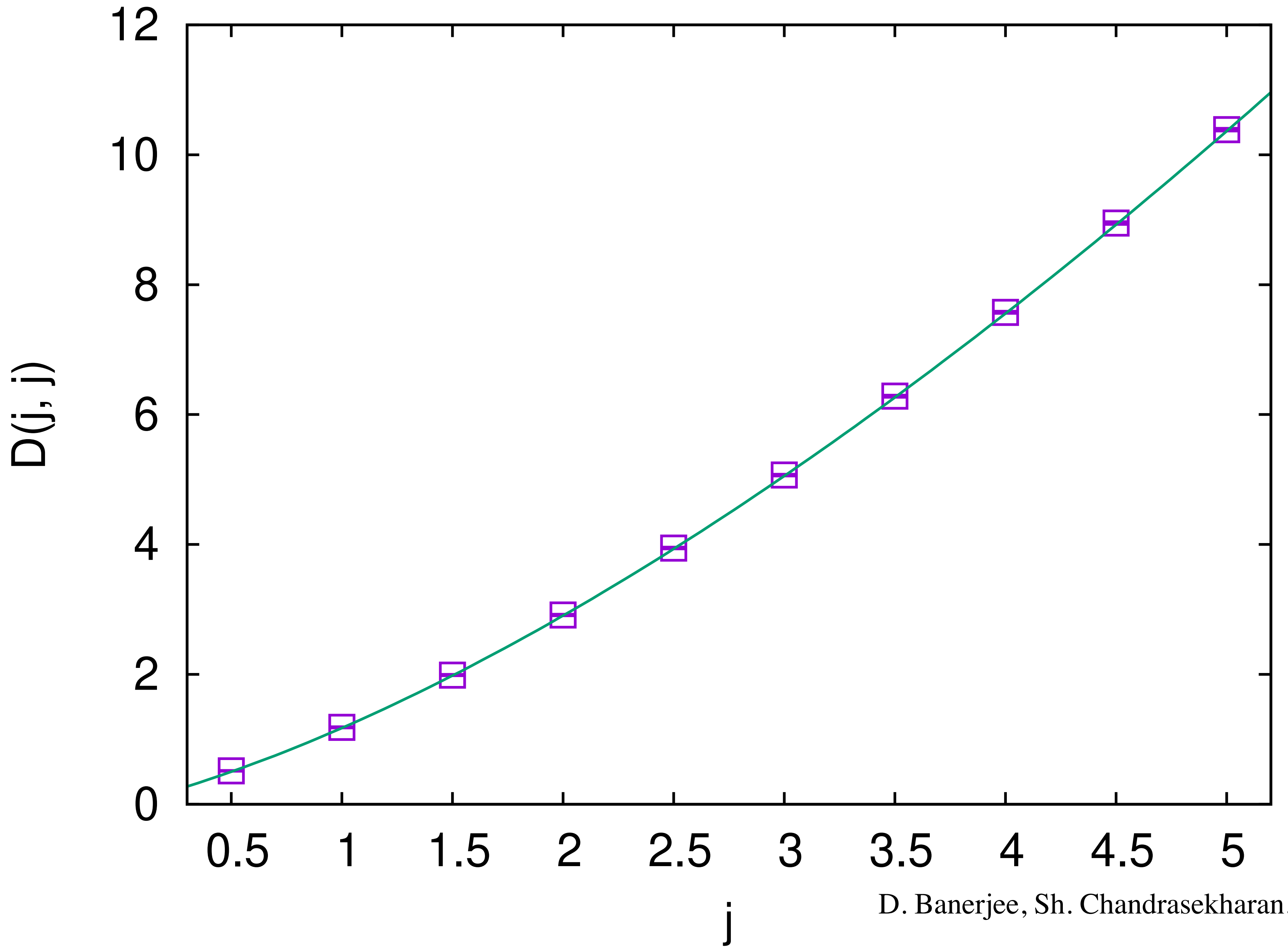


Hasenbusch, Vicari

$c_{3/2}$ decreases, $c_{1/2}$ increases with increasing n

The O(2n) vector model

New lattice data for O(4) model:



$$c_{3/2} = 1.068(4)$$
$$c_{1/2} = 0.083(3)$$

D. Banerjee, Sh. Chandrasekharan, D. Orlando, S.R. 1902.09542

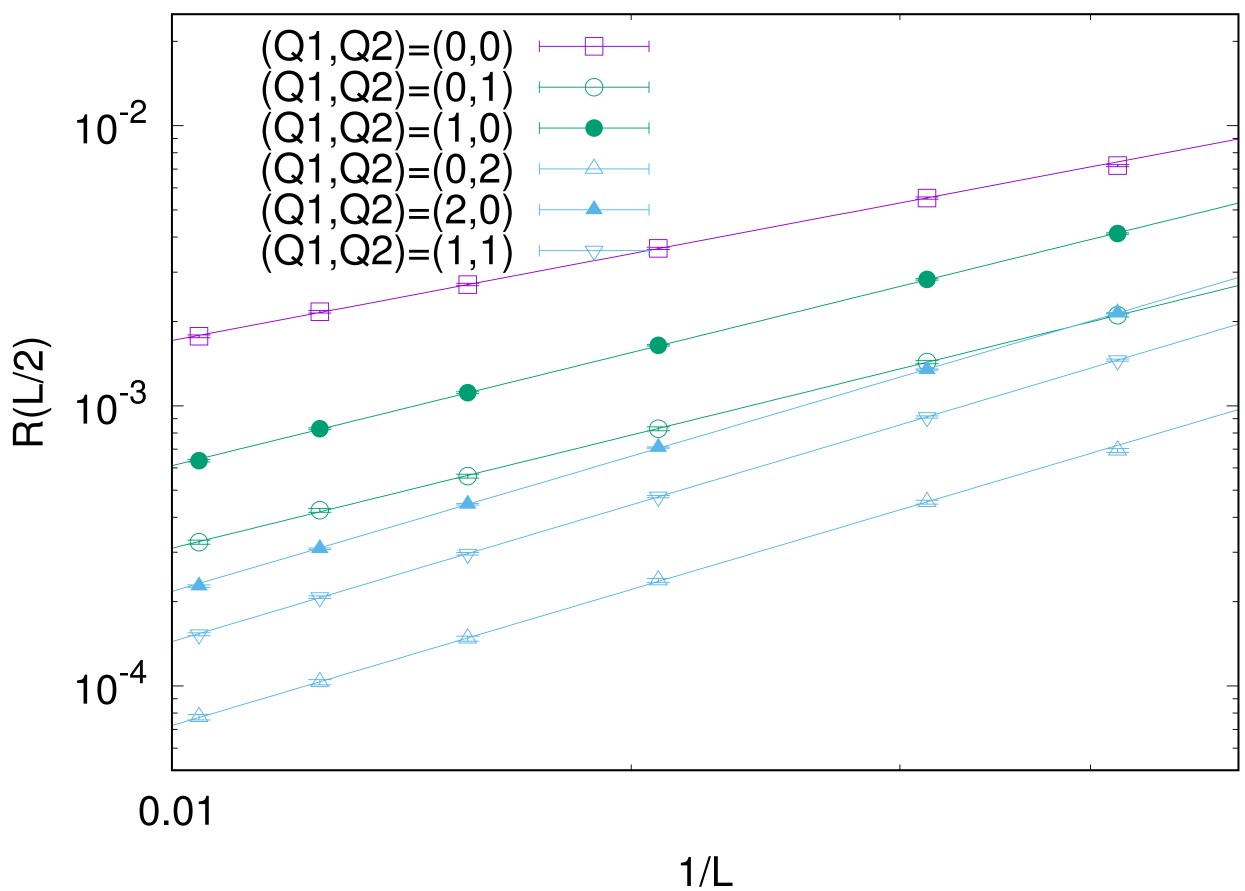
Again excellent agreement with large-Q prediction!

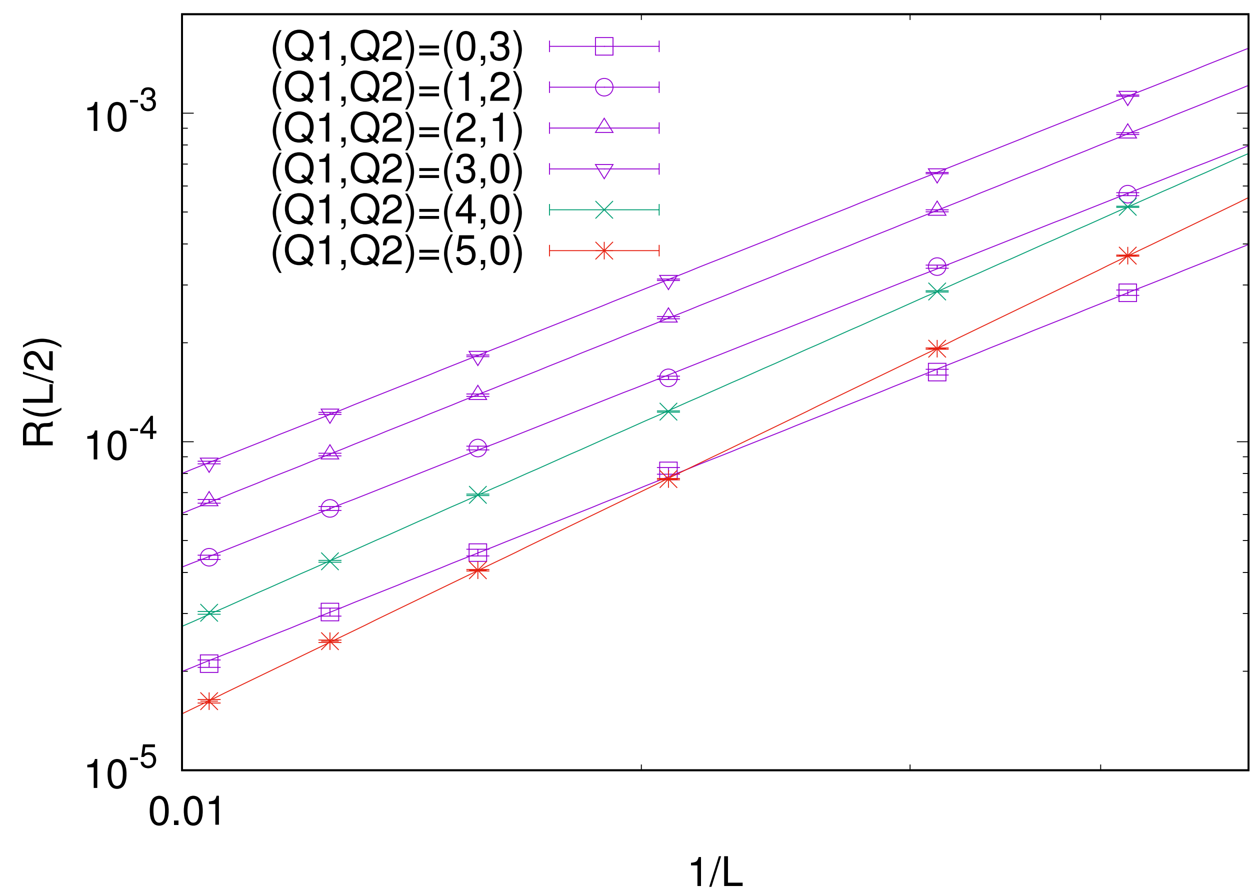
The O(2n) vector model

Only total charge matters for homogeneous case:

Correlation function:

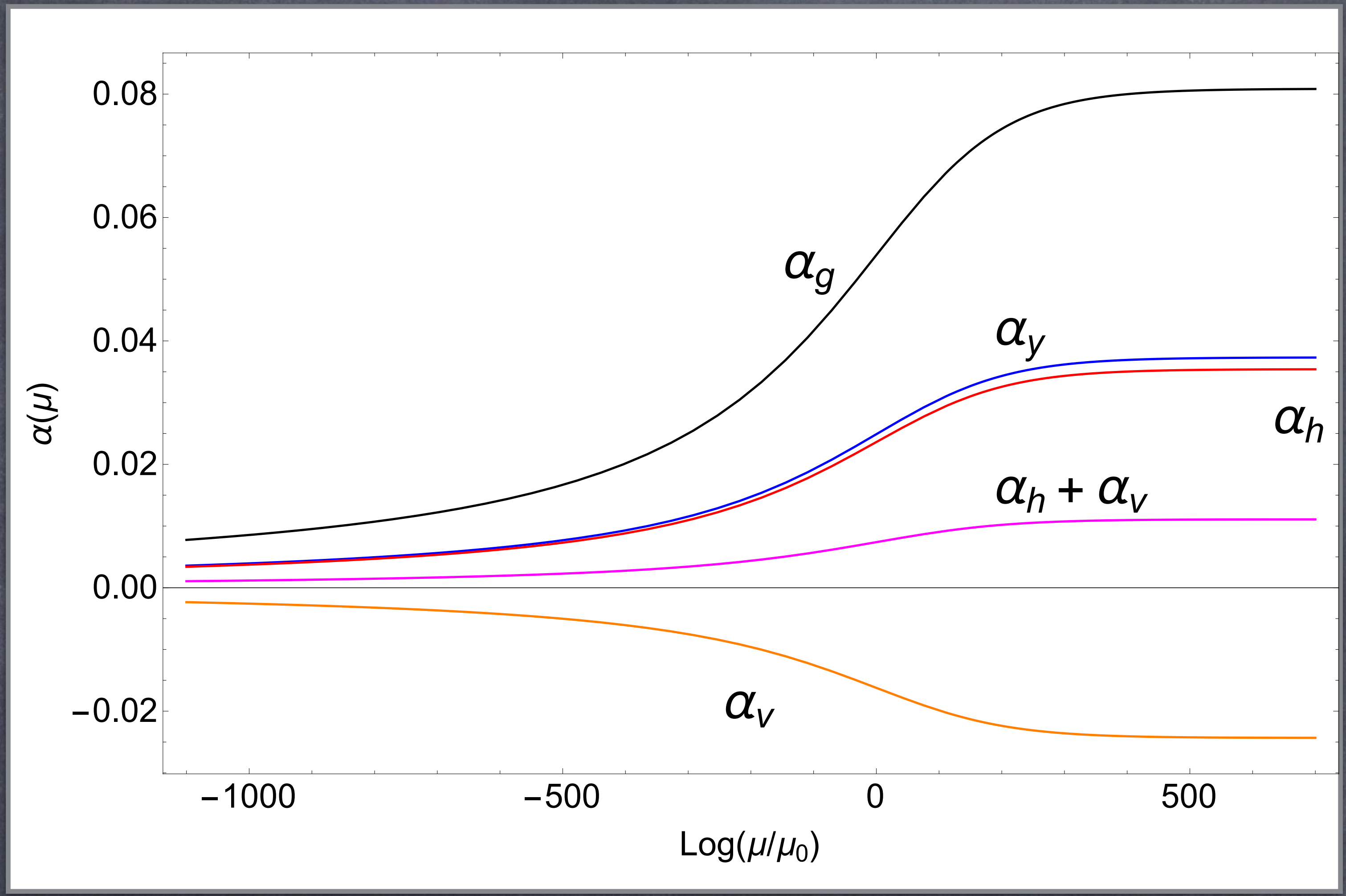
$$C_Q(r) \sim \frac{a(Q)}{|\vec{r}|^{2D(Q)}} \qquad R(L/2) = \frac{C_Q(r = L/2)}{C_{Q-1}(r = L/2)} \qquad R(L) \sim 1/L^{2(D(Q)-D(Q-1))}$$





D. Banerjee, Sh. Chandrasekharan, D. Orlando, S.R. unpublished

Parallel lines in log/log plot: conformal dimensions are the same!



An example in 4d: asymptotically safe CFT

An asymptotically safe CFT

Look for CFTs in 4D! Start with a QCD-inspired theory with quarks, gluons and scalars:

N_F flavors of fermions

gauge group $SU(N_C)$

$N_F \times N_F$ matrix of cplx scalars

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \mathrm{Tr}\big(\bar{Q}i\not{D}Q\big) + y\,\mathrm{Tr}\big(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\big)$$
$$+ \mathrm{Tr}\big(\partial_\mu H^\dagger \partial^\mu H\big) - u\,\mathrm{Tr}\big(H^\dagger H\big)^2 - v(\mathrm{Tr}\,H^\dagger H)^2 - \frac{R}{6}\mathrm{Tr}\big(H^\dagger H\big)$$

$Q_{L/R} = \frac{1}{2}(1 \pm \gamma_5)Q$

Rescaled couplings

$$\alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \qquad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}, \qquad \alpha_h = \frac{u N_F}{(4\pi)^2}, \qquad \alpha_v = \frac{v N_F^2}{(4\pi)^2}$$

Control parameter $\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$

In the limit $N_F \to \infty,\ N_C \to \infty$ with N_F/N_C fixed: asymptotically safe.

Litim, Sannino

Perturbatively controlled UV fixed point with

$$\alpha_g^* = \frac{26}{57}\epsilon + \ldots \qquad \alpha_y^* = \frac{4}{19}\epsilon + \ldots \qquad \alpha_h^* = \frac{\sqrt{23}-1}{19}\epsilon + \ldots \qquad \alpha_{v1}^* = -0.1373\,\epsilon + \ldots$$

An asymptotically safe CFT

Study this theory at large charge.

$$\mathcal{L} = -\frac{1}{2}\operatorname{Tr}(F^{\mu\nu}F_{\mu\nu}) + \operatorname{Tr}\big(\bar{Q}i\not{D}Q\big) + y\operatorname{Tr}\big(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\big)$$
$$+\operatorname{Tr}\big(\partial_\mu H^\dagger \partial^\mu H\big) - u\operatorname{Tr}\big(H^\dagger H\big)^2 - v(\operatorname{Tr} H^\dagger H)^2 - \frac{R}{6}\operatorname{Tr}\big(H^\dagger H\big)$$

Global symmetry: $SU(N_F)_L \times SU(N_F)_R \times U(1)_B$

New elements compared to vector model:

- H is a matrix field, large non-Abelian global symmetry
- fermions and gluons are present
- 4D, different scalings
- UV fixed point, perturbatively controlled, trustable LSM

Large-charge expansion: focus on scalar sector

An asymptotically safe CFT

Noether currents:

$$J_L = i\,\mathrm{d}H\,H^\dagger, \qquad J_R = -iH^\dagger\,\mathrm{d}H$$

Corresponding charges:

$$\mathcal{Q}_L = \int \mathrm{d}^3x\,J_L^0, \qquad \mathcal{Q}_R = \int \mathrm{d}^3x\,J_R^0$$

$$\mathrm{spec}(\mathcal{Q}_L) = \{J_1^L, J_2^L, \ldots, J_{N_F}^L\} \qquad \mathrm{spec}(\mathcal{Q}_R) = \{J_1^R, J_2^R, \ldots, J_{N_F}^R\}$$

Ansatz for homogeneous ground state:

Cartan subalgebra

self-adjoint

$$H_0(t) = e^{iM_Lt}Be^{-iM_Rt}$$

Impose charge conservation:

$$\dot{\mathcal{Q}}_L = -iVe^{iM_Lt}\big(-M_L\big[M_L, BB^\dagger\big] + \big[M_L, BM_RB^\dagger\big]\big)e^{-iM_Lt} = 0,$$

$$\dot{\mathcal{Q}}_R = iVe^{iM_Rt}\big(-M_R\big[M_R, B^\dagger B\big] + \big[M_R, B^\dagger M_LB\big]\big)e^{-iM_Rt} = 0$$

M commutes or anti-comm with B

diagonal

$$\Rightarrow H_0 = e^{2iMt}B$$

An asymptotically safe CFT

We find: $\mathcal{Q}_L = -2VMB^2, \qquad \mathcal{Q}_R = 2VB^2M = -\mathcal{Q}_L$

Simple choice for charges:

$$\mathrm{M} = \mu\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & -\mathbb{1} \end{array}\right), \qquad \mathrm{B} = \mathrm{b}\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & \mathbb{1} \end{array}\right) \qquad \mathcal{Q}_\mathrm{L} = \mathrm{J}\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & -\mathbb{1} \end{array}\right)$$

in su(N), traceless

$$J = 2Vb^2\mu$$

EOM on ansatz $H_0 = e^{2iMt}B$:

$$2\mu^2 = (u + vN_F)b^2 - \frac{R}{12}$$

Assume J large, expand in series:

$$\mu = \left(\frac{2\pi^2}{V}\right)^{1/3} \mathcal{J}^{1/3} + \frac{R}{72}\left(\frac{V}{2\pi^2}\right)^{1/3} \mathcal{J}^{-1/3} + \mathcal{O}\left(\mathcal{J}^{-5/3}\right)$$

Natural expansion parameter:

$$\mathcal{J} = J\frac{(u + vN_F)}{8\pi^2} = 2J\frac{\alpha_h + \alpha_v}{N_F} = 2J_{\mathrm{tot}}\frac{\alpha_h + \alpha_v}{N_F^2} \gg 1$$

$$J_{\mathrm{tot}} = JN_F$$

Consistent for $\mathrm{J}_{\mathrm{tot}} \gg \frac{\mathrm{N}_\mathrm{F}^2}{\epsilon}$

huge

tiny

An asymptotically safe CFT

Ground-state energy:

not universal

$$E = \frac{3}{2}\frac{N_F^2}{\alpha_h + \alpha_v}\left(\frac{2\pi}{V}\right)^{1/3}\left[\mathcal{J}^{4/3} + \frac{R}{36}\left(\frac{V}{2\pi^2}\right)^{2/3}\mathcal{J}^{2/3} - \frac{1}{144}\left(\frac{R}{6}\right)^2\left(\frac{V}{2\pi^2}\right)^{4/3}\mathcal{J}^0 + \mathcal{O}\left(\mathcal{J}^{-2/3}\right)\right]$$

Specialize to 3-sphere: $E = \frac{3}{2r_0}\frac{N_F^2}{\alpha_h + \alpha_v}\left[\mathcal{J}^{4/3} + \frac{1}{6}\mathcal{J}^{2/3} - \frac{1}{144}\mathcal{J}^0 + \mathcal{O}\left(\mathcal{J}^{-2/3}\right)\right]$

Classical result. What about Goldstone contributions, what about fermions, gluons?

At large charge, the fermions receive large masses and decouple:

kinetic term

Yukawa term

$$m_\psi = (\mu^2 + y^2 b^2)^{1/2} = \left(\frac{2\pi^2}{V}\right)^{1/3}\left(1 + 2\frac{N_F}{N_c}\frac{\alpha_y}{\alpha_h + \alpha_v}\right)^{1/2}\mathcal{J}^{1/3} + \mathcal{O}\left(\mathcal{J}^{-1/3}\right)$$

Below the fermion mass scale, also gluons decouple.

Gap:

$$\Lambda_{YM} = m_\psi \exp\left[-\frac{3}{22\alpha_g(m_\psi)}\right]$$

$\approx \mathcal{O}(\epsilon)$

Low-energy physics described by Goldstones only!

An asymptotically safe CFT

Symmetry-breaking pattern: $H_0 - e^{2iMt}B$

$$SU(N_F) \times SU(N_F) \times U(1) \xrightarrow{\text{exp.}} SU(N_F/2) \times SU(N_F/2) \times U(1)^2 \times SU(N_F)$$
$$\xrightarrow{\text{spont.}} SU(N_F/2) \times SU(N_F/2) \times U(1)^2$$

Expect $\dim(SU(N_F)) = N_F^2 - 1$ Goldstone DoF

Do quadratic expansion of the Lagrangian around the ground state, find dispersion relations.

$$\omega = \frac{p^2}{4\mu} + \ldots$$ $(N_F/2)^2$ type II Goldstone modes

$$\omega = \frac{p}{\sqrt{3}} + \ldots$$ conformal Goldstone (type I)

$$\omega = \sqrt{\frac{\alpha_h}{3\alpha_h + 2\alpha_v}} p + \ldots$$ $N_F^2/2 - 2$ type I Goldstones

causality constraint: $0 < \alpha_h/(3\alpha_h + 2\alpha_v) < 1$

Constraint satisfied at fixed point.

An asymptotically safe CFT

Goldstones are organized in reps of the unbroken symmetry group:

adjoint

bifundamental

$\mathrm{SU}(\mathrm{N_F}/2) \times \mathrm{SU}(\mathrm{N_F}/2)$ representation	$(\mathbf{1},\mathbf{1})$	$\left(\begin{smallmatrix}\square\square\\ \square\\ \vdots\\ \square\end{smallmatrix}, \mathbf{1}\right)$	$\left(\mathbf{1}, \begin{smallmatrix}\square\square\\ \square\\ \vdots\\ \square\end{smallmatrix}\right)$	$(\square, \square)$
type	I	I	I	II
DOF	1	$\mathrm{N}^2/4 - 1$	$\mathrm{N}^2/4 - 1$	$2 \times \mathrm{N}^2/4$
velocity	$1/\sqrt{3}$	$\sqrt{\frac{\alpha_h}{3\alpha_h+2\alpha_v}}$	$\sqrt{\frac{\alpha_h}{3\alpha_h+2\alpha_v}}$	n/a

Vacuum energy of the type I Goldstones:

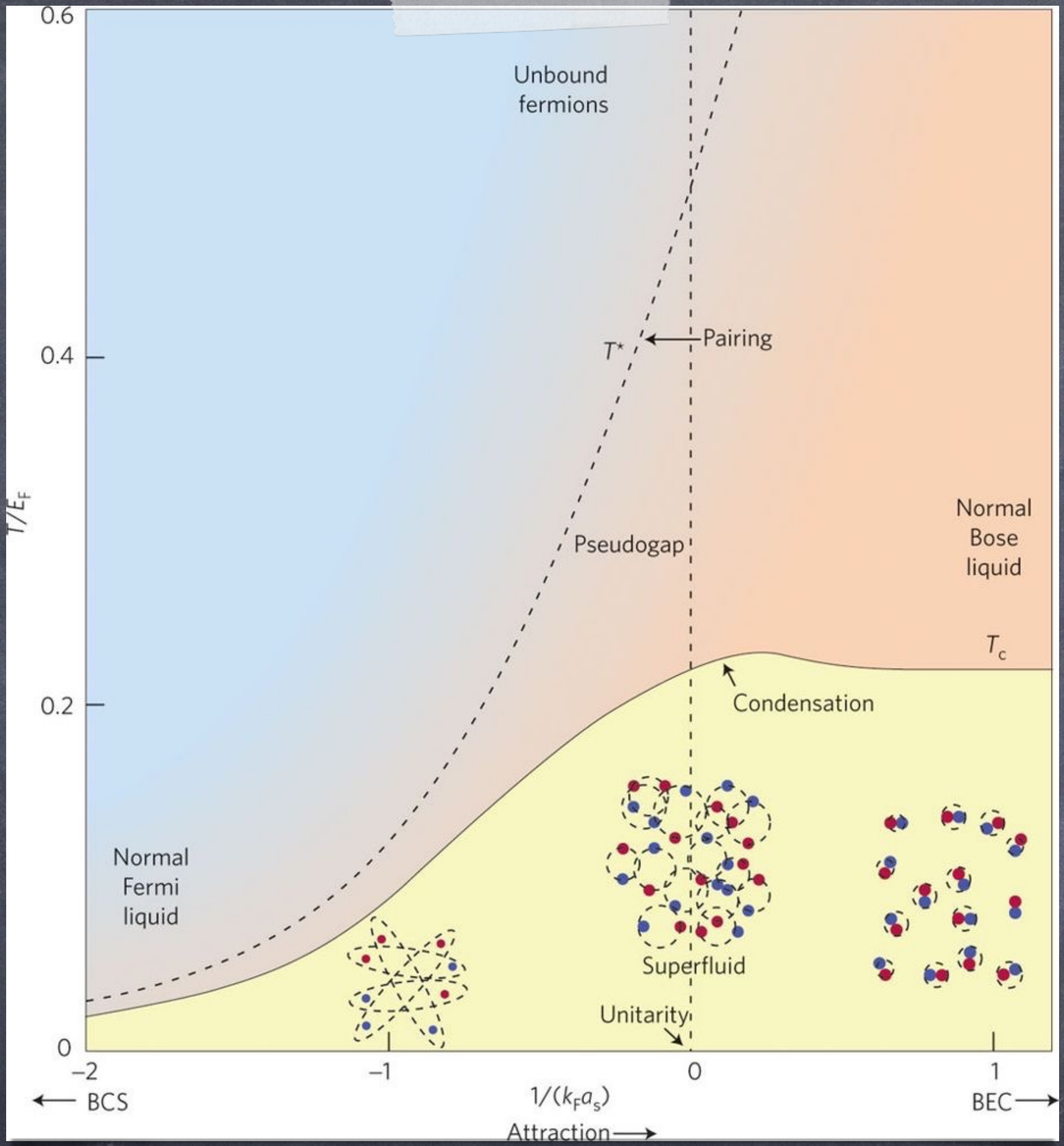
$$\zeta(-1/2|S^3) = -\frac{0.414\ldots}{r_0}$$

$$E_0 = \frac{1}{2}\left(2 \times \left(\frac{N_F^2}{4} - 1\right)\sqrt{\frac{\alpha_h}{3\alpha_h + 2\alpha_v}} + \frac{1}{\sqrt{3}}\right)\zeta(-1/2|M_3).$$

Conformal dimension (via state-operator corr.):

$$\Delta(J) = r_0 E(S^3) = \frac{3}{2}\frac{N_F^2}{\alpha_h + \alpha_v}\left[\mathcal{J}^{4/3} + \frac{1}{6}\mathcal{J}^{2/3} - \frac{1}{144}\mathcal{J}^0 + \mathcal{O}\left(\mathcal{J}^{-2/3}\right)\right]$$
$$-\left(\left(\frac{N_F^2}{2} - 2\right)\sqrt{\frac{\alpha_h}{3\alpha_h + 2\alpha_v}} + \frac{1}{\sqrt{3}}\right) \times 0.212\ldots$$

D.Orlando, S.R., F. Sannino, arXiv:1905.00026

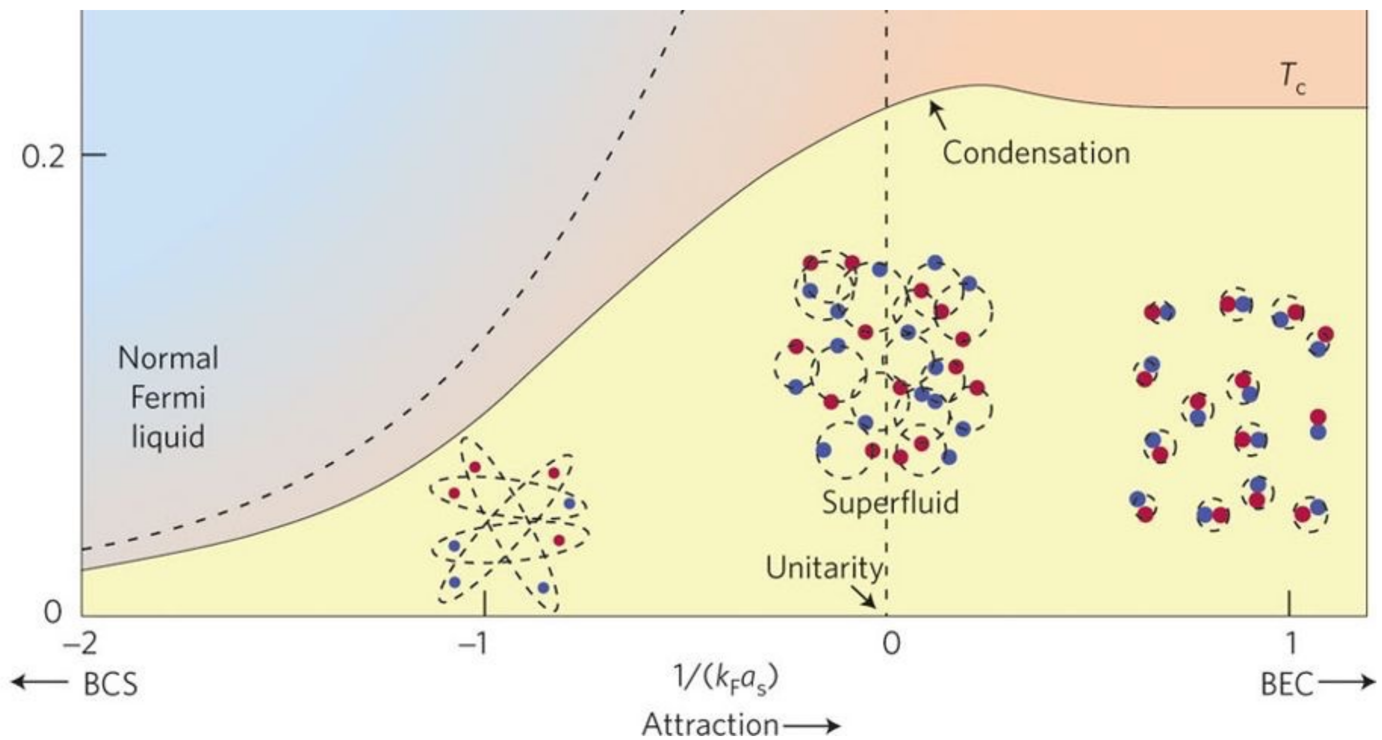


Non-relativistic CFTs

Nonrelativistic CFTs

Motivation: unitary Fermi gas (3+1)D

Can be realized in the lab via cold atoms in a trap.
Tuning via Feshbach resonances: unitary point, correlation length = ∞, interaction length = 0



At unitary point: described by a non-relativistic superfluid
Effective action (small momentum expansion)

Son & Wingate

Nonrelativistic CFTs

Non-relativistic systems are not invariant under the full conformal group.

Schrödinger algebra: contains the Galilean algebra with central extension plus

real parameters

scale transform. $(t, x_i) \to (t', x_i') = (e^{2\tau} t, e^{\tau} x_i)$

special conf. trans $(t, x_i) \to (t', x_i') = \left(\frac{t}{1+\lambda t}, \frac{x_i}{1+\lambda t} \right)$

The Schrödinger Lagrangian (in d space-dim) is invariant under Schrödinger symmetry:

$$\mathcal{L}(\psi) = \frac{i}{2}(\psi^* \partial_t \psi - \psi \partial_t \psi^*) - \frac{\hbar}{2m} \partial_i \psi^* \partial_i \psi - \frac{k}{m} \hbar^{\frac{d-2}{d}} (\psi^* \psi)^{\frac{d+2}{d}}$$

scale

Nonrelativistic CFTs

System has again a global U(1) symmetry.

Follow the same recipe as for O(2): $\psi = a\, e^{i\theta}$

Homogeneous ground state:

$$\theta = \mu t + \chi \qquad\qquad \mu = k \frac{d+2}{d} \frac{\hbar}{m} \rho^{2/d}$$

The leading piece of the effective action for θ can be found by dimensional analysis:

$$\mathcal{L}^{(0)} = c_0\ \hbar^{(2-d)/2} m^{d/2} U^{(d+2)/2}$$

$$U = \partial_t \theta - \frac{\hbar}{2m}\, \partial_i \theta\, \partial_i \theta$$

The first quantum correction to this (semi-classical) result is the Casimir energy, it goes as $Q^{1/d}$

Nonrelativistic CFTs

Check for higher-derivative terms at tree level in the effective action (here w/o curvature terms = flat space).

Use Schrödinger symmetry to constrain the terms that can appear in the action (d=3):

Generic operator allowed by dimensional analysis and compatible with scale and SC transformations:

$$\mathcal{O}_\beta \propto \hbar^{\beta-1/2} m^{3/2-\beta}\, \partial_i^{2\beta} U^{5/2-\beta}$$

Invoke μ-scaling to exclude highly suppressed terms:

$$U \sim \mu, \qquad \partial_i \theta \sim \mu^{-1/4}, \qquad \partial_i U \sim \mu^{-1/4}$$

Term with highest μ-scaling:

$$\mathcal{O}_\beta^{\max} \propto \hbar^{\beta-1/2} m^{3/2-\beta} U^{5/2-\beta}\, \partial_i^n \theta\, \partial_i^m \theta \sim \mu^{2-\beta}, \quad n+m = 2\beta$$

For positive μ-scaling, β<3.

Nonrelativistic CFTs

Check terms explicitly.

Result for d=2 and 3:

$\beta = 0$

$\beta = 1$

$\beta = 2$

$$\begin{aligned}\mathcal{L}(\theta) = {} & c_0 \hbar^{1-d/2} m^{d/2} U^{(d+2)/2} \\ & + c_1 \hbar^{2-d/2} m^{-1+d/2} U^{(d-4)/2}\, \partial_i U\, \partial_i U \\ & + c_2 \hbar^{3-d/2} m^{-2+d/2} U^{(d-2)/2} (\partial_i \partial_i \theta)^2 + \mathcal{O}(\mu^{-2})\end{aligned}$$

Check loop corrections to the effective action.

Both quantum corrections and tree-level higher derivative terms are suppressed by inverse powers of μ for d>1.

Nonrelativistic CFTs

Speed of sound (leading order): $c_s^2 = \frac{2}{d}\frac{\hbar\mu}{m}$ ← different from relativistic case!

NLO-correction to the dispersion relation:

$$\omega = c_s p \left(1 - d_0^2 \frac{\hbar}{m}\left(2c_1 + dc_2\right)\frac{p^2}{\mu} + \mathcal{O}(\mu^{-2})\right)$$

again linear in p!

from NLO tree-level terms

Quantum corrections enter at higher order.

Energy of ground state (on the torus):

different from relativistic case!

$$E_{T^d} = \frac{\hbar^2}{m}\left[V b_1^2 \rho^{(d+2)/d} + \underbrace{\frac{b_1}{V^{1/d} d}\sqrt{\frac{d+2}{2}}\rho^{1/d}\zeta_{T^d}(-2) + \frac{b_2}{V^{2/d}}}_{\text{Casimir energy}}\right] + \mathcal{O}\left(\frac{1}{\rho^{2/d}}\right)$$

class. ground state energy

All other classical and quantum corrections are suppressed by inverse powers of ρ.

Nonrelativistic CFTs

Large-Q expansion also works for non-relativistic CFTs.

Reproduce results of Son, Wingate (different approach)

Further directions:

- include curvature
- work in harmonic potential to use non-relativistic state-operator correspondence Kravec, Pal
- make connection to experimental results.

Summary

Summary

We studied various CFTs in sectors of large global charge

Concrete examples where a (strongly-coupled) CFT simplifies in a special sector.

- O(2N) model in 3d: in the limit of large U(1) charge Q, we computed the conformal dimensions in a controlled perturbative expansion:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

  - Excellent agreement with lattice results for O(2), O(4)
- Can be applied beyond vector model: SU(N) matrix models, SCFT

Summary

- Asymptotically safe CFT in 4d (scalars, fermions and gauge fields). Controllable UV fixed point.
  - fermions and gluons decouple
  - large-charge expansion for scalar sector
  - interesting Goldstone spectrum
- Non-relativistic CFTs with global U(1).
  - Large-charge expansion exists, quantum corrections and higher-derivative terms are suppressed
  - results in 3+1D match eff. theory for unitary Fermi gas
  - qualitatively different behavior to relativistic case

Summary

Some questions:

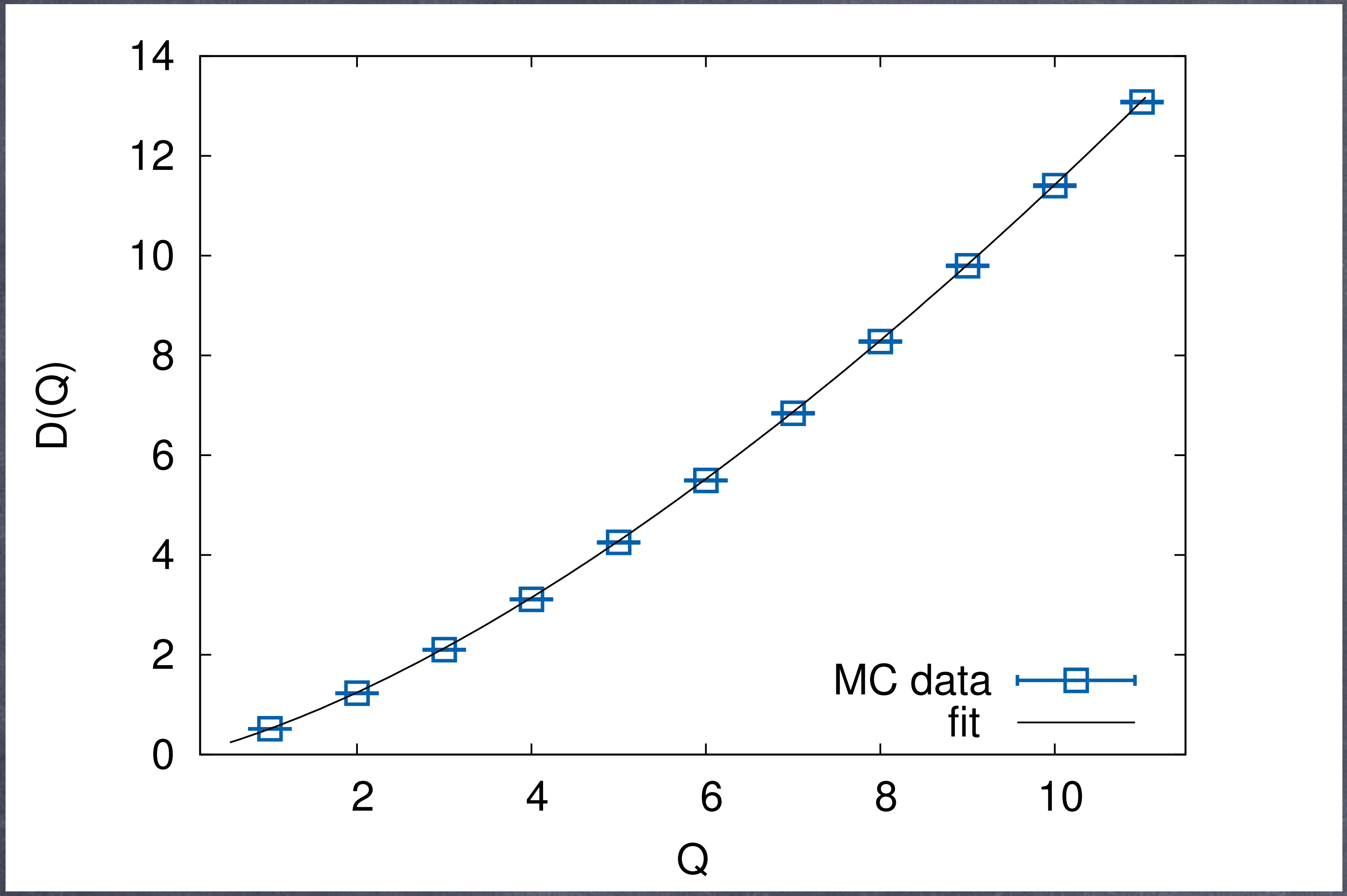
- Does it work?
  - For all the examples, we tried, yes! Confirmation from lattice data (O(2) and O(4))
- For what kinds of theories does it work?
  - (S)CFTs and non-relativistic CFTs
- In how many space-time dimensions?
  - $d>1$ space dimensions
- For what kinds of global symmetries does it work?
  - we checked U(1), O(2n) vector models, SU(N) matrix models

Summary

- What happens if we fix several charges?
  - k charges with same chemical potential: homogeneous solution with type I and type II Goldstones.
  - different chemical potentials: inhomogeneous solutions
- What can we learn via this approach?
  - calculate CFT data at large charge!

Outlook

- Further study of supersymmetric models at large R-charge (higher-dim. moduli spaces) Hellerman, Maeda, Orlando, Reffert, Watanabe
- Connection to holography (gravity duals) Loukas, Orlando, Reffert, Sarkar
- Connection to large-spin results Rattazzi et al.
- Understanding dualities semi-classically at large charge
- Use/check large-charge results in conformal bootstrap Jafferis and Zhiboedov
- Comparison with large-N expansion
- Further lattice simulations: inhomogeneous sector, general O(N)
- strongly coupled CFTs in 4d at IR fixed point
- Fishnet CFTs (non-unitary)
- Study fermionic theories. Can large-charge approach be used for QCD (e.g. large baryon number)?



Thank you for your attention!