Vertical Vafa-Witten Invariants

Vafa-Witten invariants have recently been defined in algebraic geometry by Tanaka and Thomas, and virtually enumerate certain sheaves on a local surface. We study contributions of sheaves in the counting problem, that can be described by flags of torsion free sheaves of rank 1 on the underlying surface (the vertical contributions). We discuss a universality result, which allows us to explicitly compute some of these contributions. The results can be used to verify precise formulas, conjectured by Göttsche and Kool, for the vertical contributions to rank 2 and 3 Vafa-Witten invariants.