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Lecture 16:
Low-energy nuclear reactions – Part 1

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Nuclear reactions & Nuclear Scattering

Nuclear reaction:

The process in which two "particles" collide to produce one or more "particles" that are different from the those that began the process (parent "particles"). A nuclear reaction must cause a transformation of at least one particle to another.

"Nuclear" ? "Particles" ?

Nuclear Scattering:

The process in which a "particle" interacts with another "particle" and they then separate without changing their "nature" of any nuclide.

"Nuclear" ? "Particles" ? "Nature" ?

Nuclear "reactions" – in general

We use nuclear reactions to study nuclear properties

(structure and dynamics)

- Coulomb excitation
- Transfer and knockout reactions
- Reactions in the resonance region to study resonances and spins-parities
- Compound nucleus reactions
- Heavy ion reactions – fusion evaporation reactions to study structure properties of neutron-deficient nuclei
- Fission and deep inelastic scattering to study nuclear structure or neutron-rich nuclei
- Photonuclear reactions and Nuclear Resonance Fluorescence to study the electromagnetic response of the nucleus (Giant Dipole Resonance, Pygmy excitations, etc.)
- Inelastic scattering to low-lying states to extract spins

Nuclear “reactions” – in general

We study nuclear reactions to understand how ions interact with nuclear matter and how nuclear species are produced

- Surface and bulk analysis of materials
- Radiation transport in materials
- Production of radioisotopes for medical applications
- Nuclear reactor inventories – production of neutrons, fission products, delayed neutrons
- Fusion plasma erosion of structural material
- Production of nuclei in the universe: nucleosynthesis (astrophysical reaction rates) etc.

Nuclear reactions & Q-values

reactants — exit channel 1 — exit channel 2

$$a + A \rightarrow (b + B) + (d + D) + \ldots$$

target nucleus → **A(a,b)B** ← residual nucleus *(undetected part)*

projectile *(incident beam)*

ejectile *(detected particle)*

Q-value = masses (before) – masses (after)

$= M_a + M_A - M_B - M_b$ (in energy units)*

Q-value > **0** : exothermic (exoergic)

Q-value < **0** : endothermic (endoergic)

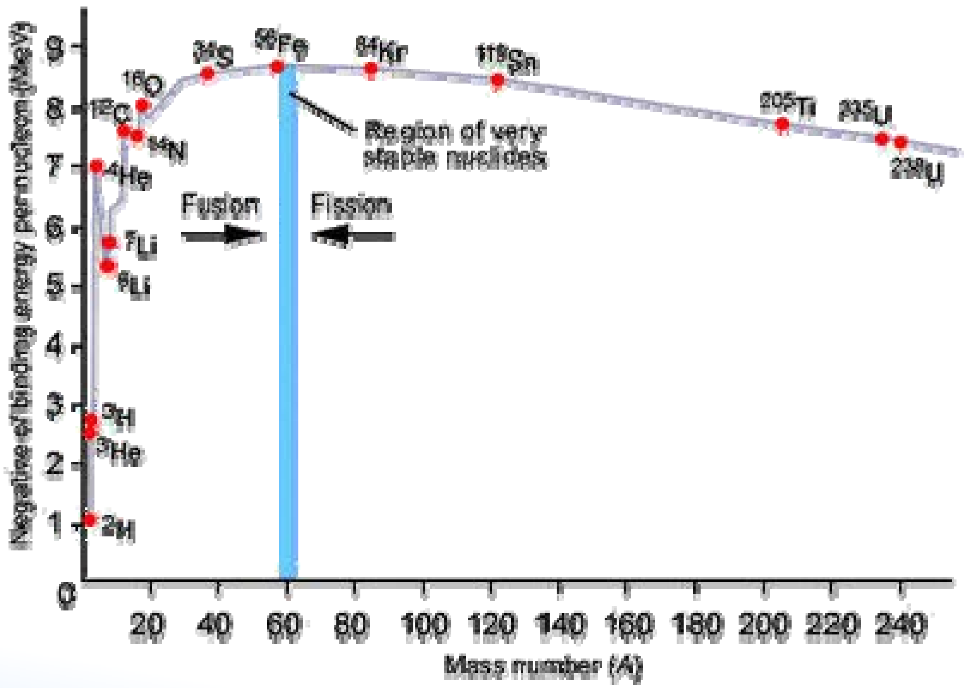
Q-value = **0** : elastic scattering

Binding energy – Nuclear & Atomic mass – Mass Excess

The sum of masses of nucleons is more than **the total nuclear mass**

$$\boldsymbol{m_{nuc}} = Zm_p + Nm_n - \Delta m = Zm_p + Nm_n - (\boldsymbol{\Delta E}/c^2)$$

Nuclear Binding Energy = $\boldsymbol{B(Z,N)} = (Zm_p + Nm_n - m_{nuc})c^2$



the atomic mass

$$\boldsymbol{m_{atom}}(A,Z) = \boldsymbol{m_{nuc}}(A,Z) + Zm_e - B_e(Z)$$

Nuclear reactions conserve the total charge, i.e. in nuclear reactions:

$$\boldsymbol{m_{atom}}(A,Z) \approx \boldsymbol{m_{nuc}}(A,Z)$$

atomic mass excess

$$M.E. \equiv (m_{atom}(A,Z) - Am_u)c^2$$

Reaction A(a,b)B: $Q - value = ME_a + ME_A - ME_B - ME_b$

p + ^{17}O → ^{14}N + α : Q – value = 1191.83 keV

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The AME2016 atomic mass evaluation *

(II). Tables, graphs and references

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N	Z	A	Elt.	Orig.	Mass excess (keV)		Binding energy per nucleon (keV)		Beta-decay energy (keV)			Atomic mass μu	
1	0	1	n		8071.3171	0.0005	0.0	0.0	β^-	782.346	0.001	1 008664.9158	0.0005
0	1		H		7288.97061	0.00009	0.0	0.0		*		1 007825.03224	0.00009
1	1	2	H		13135.72176	0.00011	1112.283	*a*		*		2 014101.77811	0.00012
2	1	3	H		14949.80993	0.00022	2827.265	*a*	β^-	18.592	*a*	3 016049.28199	0.00023
1	2		He		14931.21793	0.00021	2572.680	*a*		*		3 016029.32265	0.00022
0	3		Li	-pp	28670#	2000#	-2270#	670#	β^+	13740#	2000#	3 030780#	2150#
3	1	4	H	-n	24620	100	1720	25	β^-	22200	100	4 026430	110
2	2		He		2424.91561	0.00006	7073.915	*a*		*		4 002603.25413	0.00006
1	3		Li	-p	25320	210	1150	50	β^+	22900	210	4 027190	230
10	4	14	Be	x	39950	130	4994	9	β^-	16290	130	14 042890	140
9	5		B		23664	21	6101.6	1.5	β^-	20644	21	14 025404	23
8	6		C		3019.893	0.004	7520.319	*a*	β^-	156.476	0.004	14 003241.988	0.004
7	7		N		2863.41672	0.00019	7475.614	*a*		*		14 003074.00446	0.00021
6	8		O		8007.781	0.025	7052.278	0.002	β^+	5144.364	0.025	14 008596.706	0.027
5	9		F	-p	31960	40	5285.2	2.9	β^+	23960	40	14 034320	40
12	5	17	B	x	43720	200	5270	12	β^-	22680	200	17 046930	220
11	6		C	2p−n	21032	17	6558.0	1.0	β^-	13162	23	17 022579	19
10	7		N	+p	7870	15	7286.2	0.9	β^-	8679	15	17 008449	16
9	8		O		-808.7635	0.0007	7750.728	*a*		*		16 999131.7566	0.0007
8	9		F		1951.70	0.25	7542.328	0.015	β^+	2760.47	0.25	17 002095.24	0.27
7	10		Ne		16500.4	0.4	6640.499	0.021	β^+	14548.7	0.4	17 017714.0	0.4
6	11		Na	x	35170	1000	5500	60	β^+	18670	1000	17 037760	1080

Systems of reference in nuclear collisions: A(a,b)B

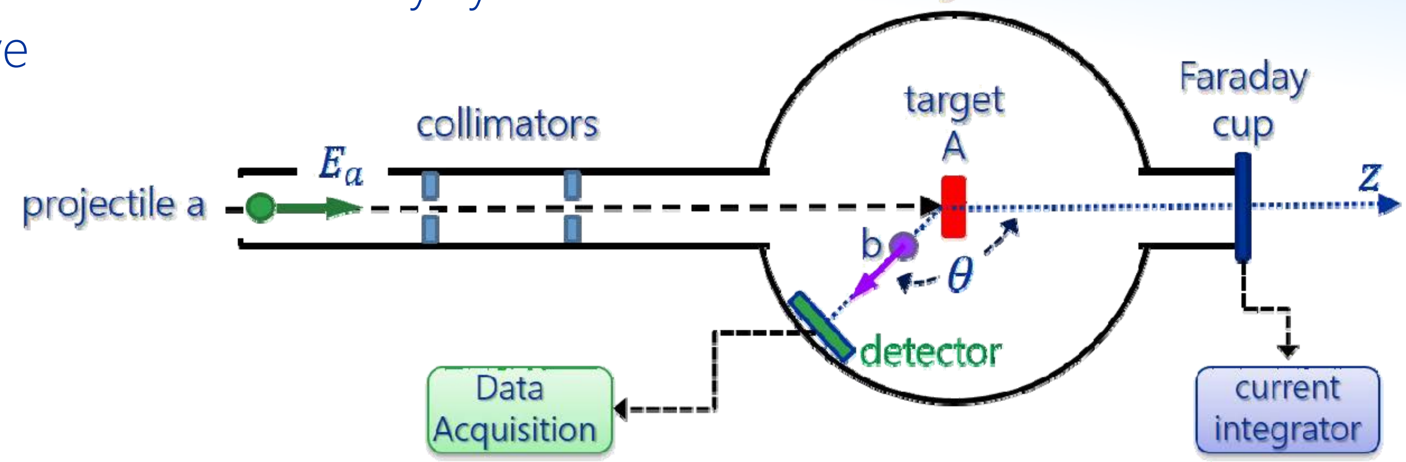
Target A in rest
Projectile a in move

$m_A > m_a$

B heavy product
b light product

$m_B > m_b$

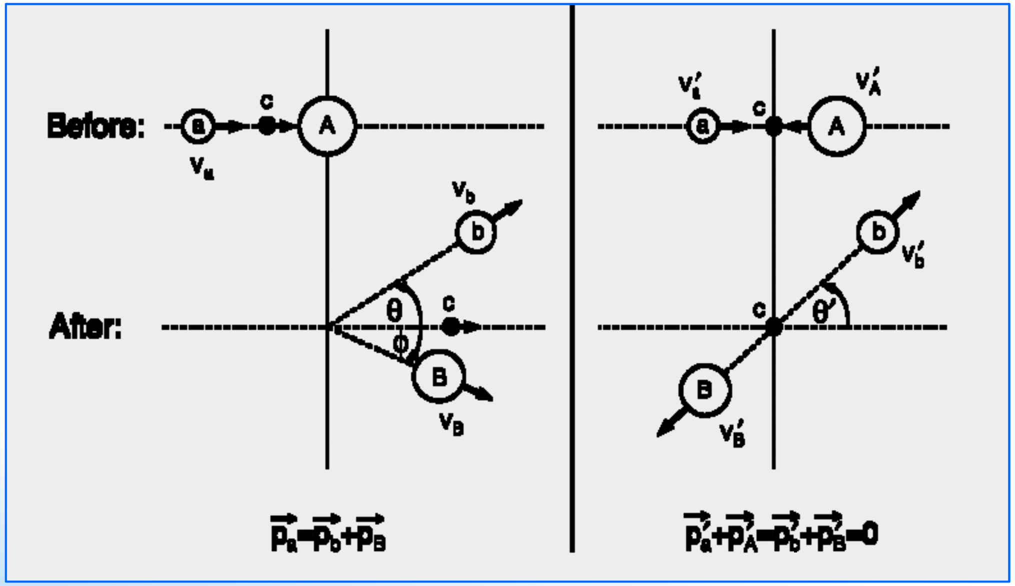
Laboratory system



Laboratory system

Projectile energy

E_a



Center-of-mass system

"Projectile" energy

$$E_{cm} = \frac{m_A}{m_A + m_a} E_a$$

Reaction thresholds & reaction kinematics (1/2)

Energy and linear momentum conservation allows to calculate the energy of the ejectiles

$$\sqrt{E_b} = \frac{\sqrt{m_a m_b E_a}\cos\theta}{m_b + m_B} \pm \frac{\sqrt{\{m_a m_b E_a cos^2\theta + (m_b+m_B)[m_B Q + (m_B - m_a)E_a]\}}}{m_b + m_B} \quad \{..\}\leftarrow C$$

Eq. 1

Non-relativistic kinematic formula for a two-body nuclear reaction

Similarly for E_B by permuting symbols b and B and replacing θ with φ

Q-value **< 0 :** endothermic

Threshold energy E_{th}

$$E_{th} = -Q\frac{m_b + m_B}{m_b + m_B - m_a}$$

if $E_\alpha < E_{th}$ → no reaction

- at E_{th} part. b emerge at $\theta=0^0$
- with increasing E_α, part. b are emitted in a cone that becomes wider until its angle is maximized: $2\theta_{max}=180^0$

Eq. 1: Two possible solutions for E_b, acceptable only if:

$$m_B Q + (m_B - m_A)E_a < 0 \Rightarrow \boxed{E_a < \frac{-m_B Q}{m_B - m_a} := E_{max}}$$

When $E_{th} < E_\alpha < E_{max}$ two groups of part. b are observed and the emission angle θ is between $0^0 < \theta < \theta_{max} < 90^0$

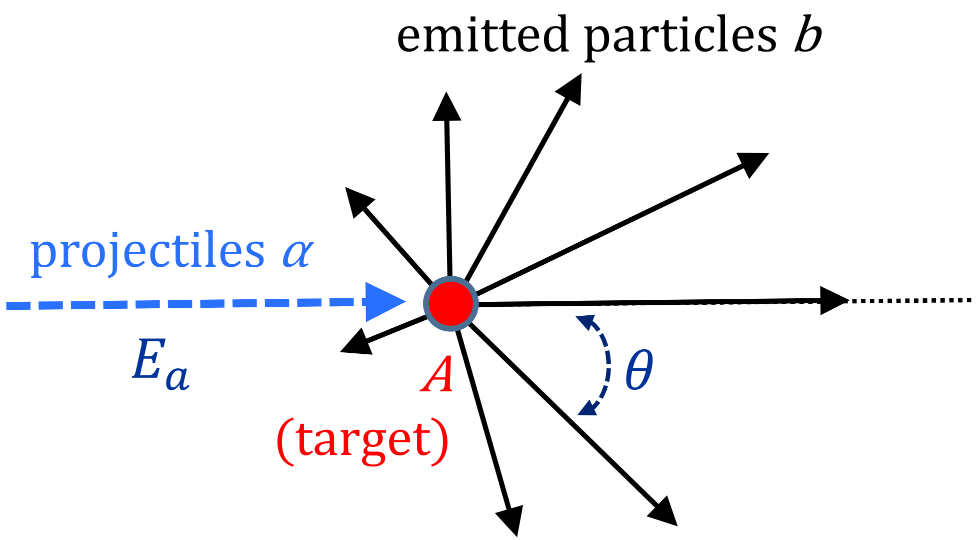
C →{..}=0

$$cos^2\theta_{max} = \frac{-(m_b + m_B)[m_B Q + (m_B - m_A)E_a]}{m_a m_b E_a}$$

Reaction thresholds & reaction kinematics (2/2)

Q>0 **exothermic:** E_b is single-valued function of θ; decreasing with increasing θ, if $m_\alpha < m_B$

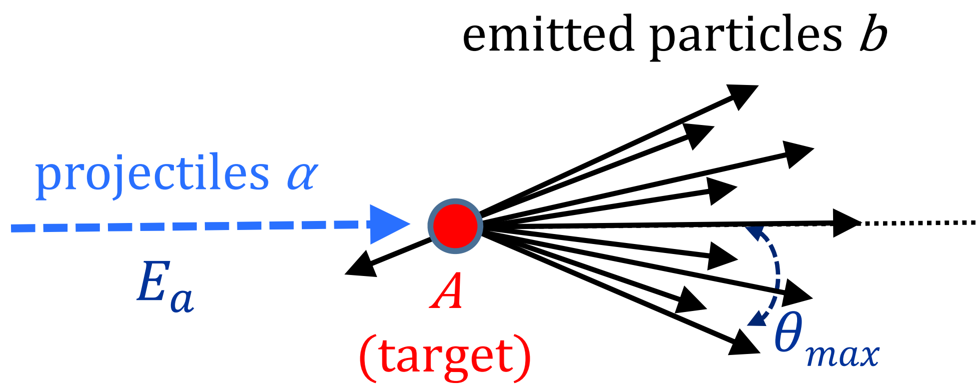
$Q > 0\ (m_a < m_B)$ ***or*** $Q > 0\ (E_a < E_{max})$



- Particles b are emitted in all directions;
- E_b increases at forward angles and reaches maximum at $\theta = 0^o$;
- E_b decreases at backward angles and reaches minimum at $\theta = 180^o$

$Q > 0\ (m_a > m_B)$ ***or*** $Q < 0\ (m_a < m_B)$

$E_{th} < E_a < E_{max}$



- Particles b are emitted at forward angles only $(\theta \leq \theta_{max})$
- At each emission angle θ, two particle energy groups are detected.

Exercise 1: the $^{7}Li(p,n)^{7}Be$ reaction

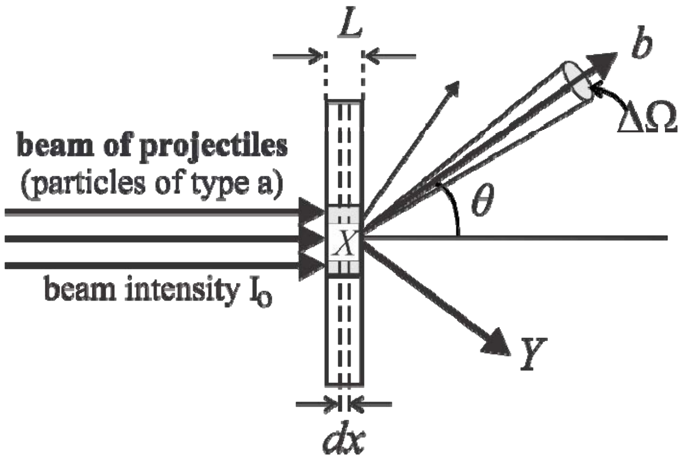
- Which is the Q-value of the $^{7}Li(p,n)^{7}Be$ reaction?
- Has the reaction a threshold? If yes what is the threshold energy?
- Which is the E_{max} of the reaction, if applicable?
- If the proton-beam energy is 1.9 MeV, which direction(s) will the emitted neutron(s) take?
- What energies can the emitted neutron have?
- Which energy has the proton beam to have so that the $^{7}Li(p,2n)$ reaction occurs?

Exercise 2: elastic scattering of α-particles

- Calculate the final energy of a 2-MeV α-particle scattered at 90^{o} by ^{40}Ca.
- In case ^{40}Ca is replaced by ^{197}Au, the energy of the scattered α-particle will decrease?
- Which will be the final energy of the α-particle if subsequently scattered at 60^{o} ?

Reaction cross section

$$a + X \longrightarrow Y + b$$



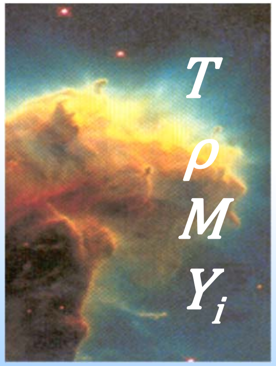
$$\sigma = \frac{\textit{reactions per time}}{(\textit{projectiles per area per time}) \times (\textit{target nuclei})}$$

$$\sigma = \frac{A\,N_b}{N_a\,N_A\,\rho\,L} = \frac{A\,N_b}{N_A\,N_a\,\xi} \qquad 1\ b = 10^{-24}\ cm^2$$

A := target's atomic weight (g/mol)
N_A := Avogadro's number (at/mol)
ξ := target radial density (g/cm^2)
N_a := number of projectiles a
N_b := number of ejectiles b

measure for probability that a certain reaction takes place at a given "projectile" energy E

Reaction rate $\langle \sigma v \rangle$ ....the link to nuclear astrophysics



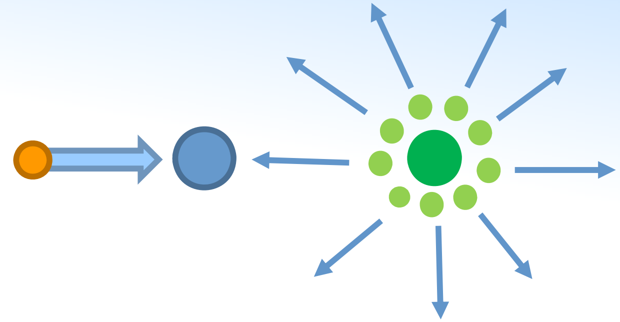
$$< \sigma v > = \left(\frac{8}{\pi \mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{kT}\right) dE$$

reactions / sec / cm^3

Types of measured cross sections

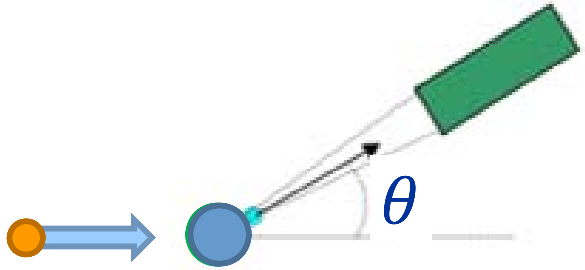
Total cross section:

σ_T



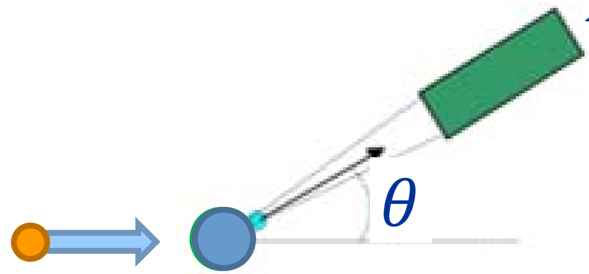
Differential cross section: $\left(\frac{d\sigma}{d\Omega}\right)$

"angular distributions"

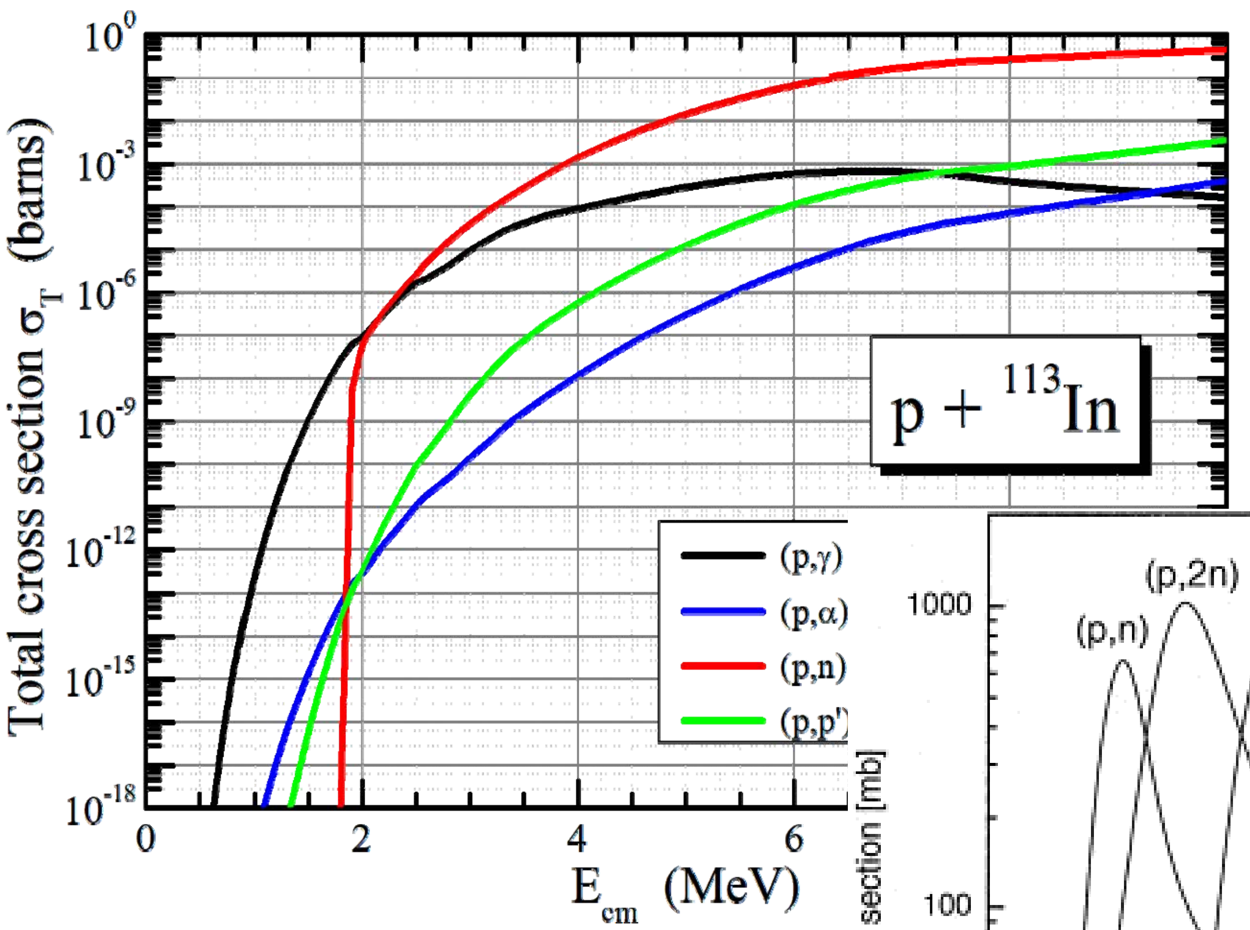


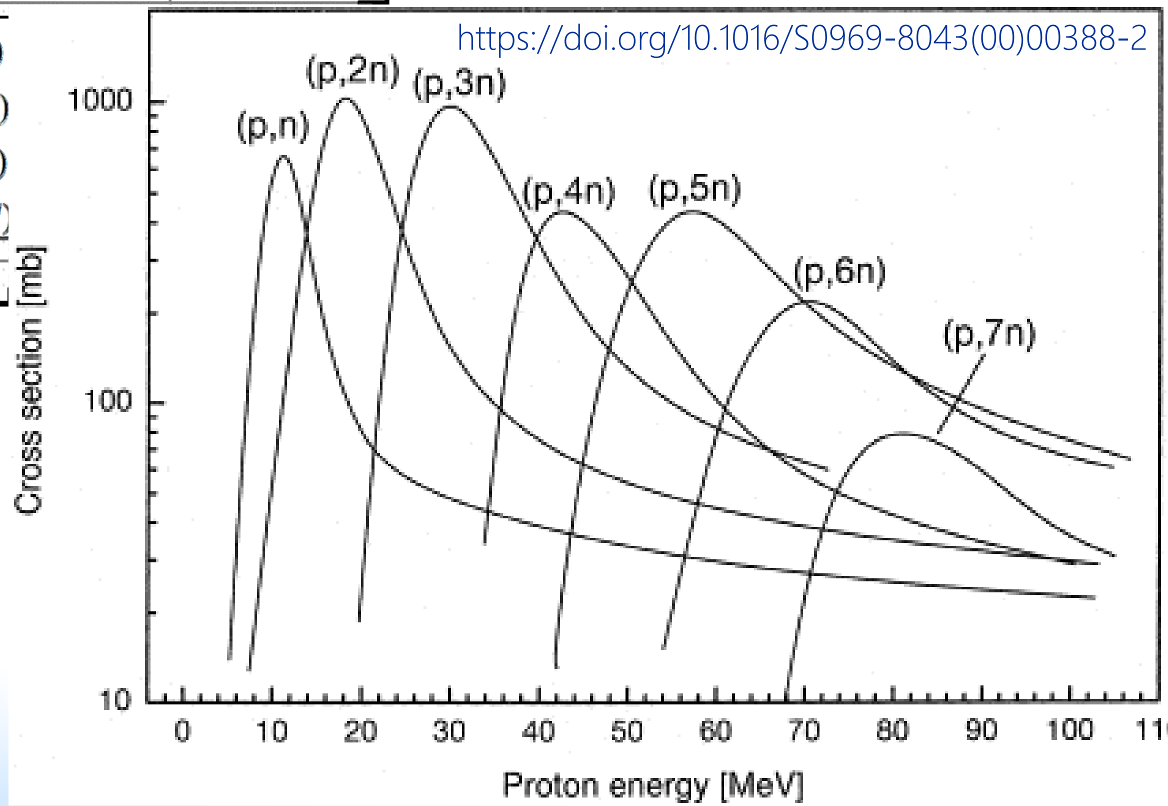
Double Differential cross section:

$$\left(\frac{d^2\sigma}{dE\,d\Omega}\right)$$



Reaction thresholds – Q values – Cross sections





Classification of nuclear reactions *)

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Reactions

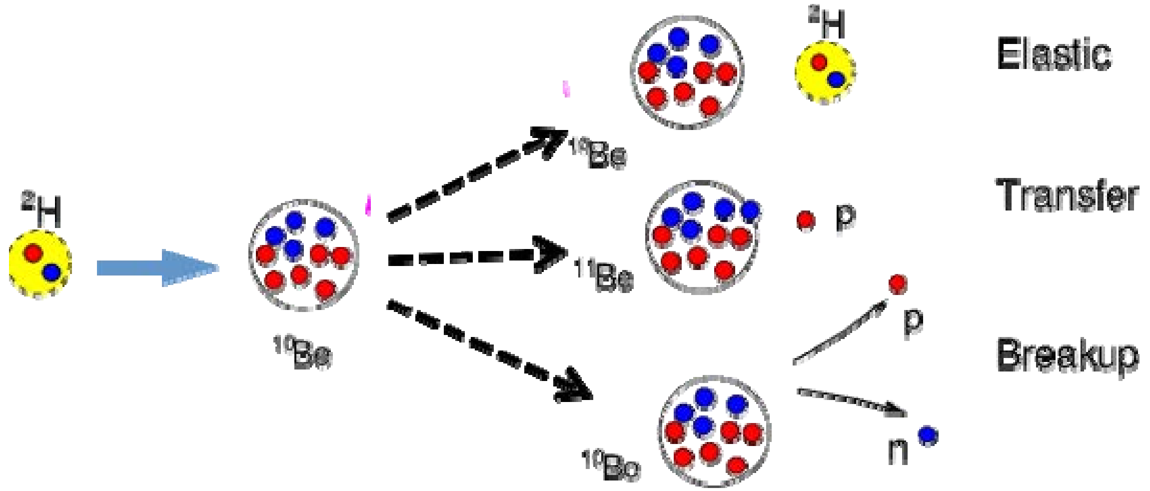
• Elastic Scattering	$\boldsymbol{a + A \rightarrow A + a}$ $(Q = 0)$	Always possible; can be due to simple Coulomb repulsion or by more complicated nuclear interactions. When Coulomb forces dominant, then Coulomb or Rutherford scattering. Plays key role in surface layer analysis.
• Inelastic Scattering	$\boldsymbol{a + A \rightarrow A^* + a' + Q}$ $E_{th} \cong -Q\frac{m_a + m_A}{m_A}$ $(Q < 0)$	Both A and α can be in excited state; if so then excited state of A* decays via γ-ray emission (for analysis purposes, γ-rays are preferred to be detected, instead of α'
• Rearrangement Collisions	$\boldsymbol{a + A \rightarrow B_\iota + b + Q_i}$	$d + {}^{14}N \rightarrow {}^{12}C + \alpha + 13.575 \text{ MeV}$ or $d + {}^{14}N \rightarrow {}^{12}C^* + \alpha + 9.142 \text{ MeV}$ ${}^{12}C^* at\ E_X = 4.433 \text{ MeV}$
• Many body reactions	$\boldsymbol{a + A \rightarrow B + b_1 + b_2 + \cdots + Q}$	$\alpha + {}^{197}Au \rightarrow {}^{198}Tl + n + n + n - 25.4\ MeV$ or ${}^{197}Au(\alpha, 3n)^{198}Tl$
• Photonuclear reactions	$\boldsymbol{\gamma + A \rightarrow B + b + Q}$ $(Q < 0)$	$\gamma + {}^{12}C \rightarrow {}^{11}C + n - 18.72\ MeV$ Carbon trace detection of ^{12}C; highly sensitive analytical method
• Radiative Capture	$\boldsymbol{a + A \rightarrow C^* \rightarrow C + \gamma}$ $(Q > 0)$	${}^{27}Al(p,\gamma)^{28}Si$ Energy calibration of electrostatic accelerators (Resonances)

*) before 1990

Reaction mechanisms -1 (Direct reactions)

The incoming projectile interacts with one or few nucleons of the target nucleus and transfers energy or picks up or loses nucleons to the nucleus in a single quick event

$$\mathbf{(10^{-20} \leq \Delta t \leq 10^{-22})}$$



- Elastic scattering
- Inelastic scattering
- Transfer reactions (stripping, pickup)
- Knock-out reactions
- Break-up reactions
- Direct Capture reactions

- Inelastic scattering: individual collisions between the projectile and a single target nucleon; the incident particle emerges with reduced energy
- Pickup reactions: projectile collects additional nucleons from the target

$$d + {}^{16}_{8}O \rightarrow {}^{15}_{8}O + {}^{3}_{1}H \qquad (d,\ {}^{3}H)$$

$${}^{3}_{2}He + {}^{41}_{20}Ca \rightarrow {}^{40}_{20}Ca + \alpha \quad ({}^{3}He, a)$$

- Stripping reactions: projectile leaves one more nucleons behind in the target

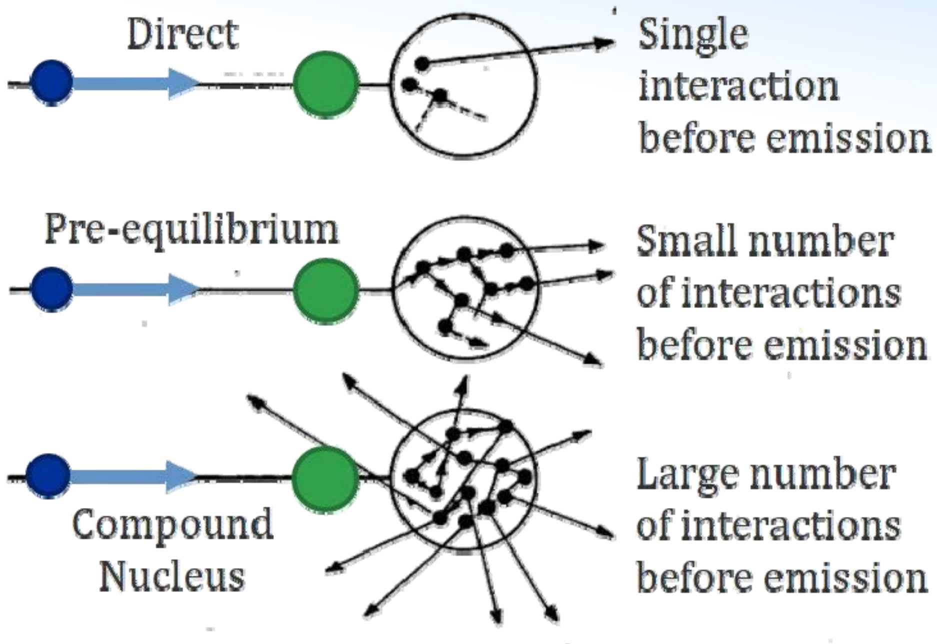
$$d + {}^{90}_{40}Zr \rightarrow {}^{91}_{40}Zr + p \qquad (d, p)$$

$${}^{3}_{2}He + {}^{23}_{11}Na \rightarrow {}^{24}_{12}Mg + d \quad ({}^{3}He, d)$$

Reaction mechanisms -2 (Statistical / Compound reactions)

$$(10^{-16} \leq \Delta t \leq 10^{-20})$$

When a properly low energetic projectile (< 50 MeV) enters the range of nuclear forces it can be scattered or begin a series of collisions with the nucleons. The products of these collisions, including the incident particle, will continue in their course, leading to new collisions and new changes of energy. During this process one or more particles can be emitted and they form with the residual nucleus the products of a reaction known as **pre-equilibrium.**



At low energies, the largest probability is the continuation of the process so that the initial energy is distributed among all nucleons, with no emitted particle. The final nucleus with A + 1 nucleons has an excitation energy equal to the kinetic energy of the projectile plus the binding energy the projectile nucleus has in the new, highly unstable system, the **compound nucleus.**

Pre-equilibrium

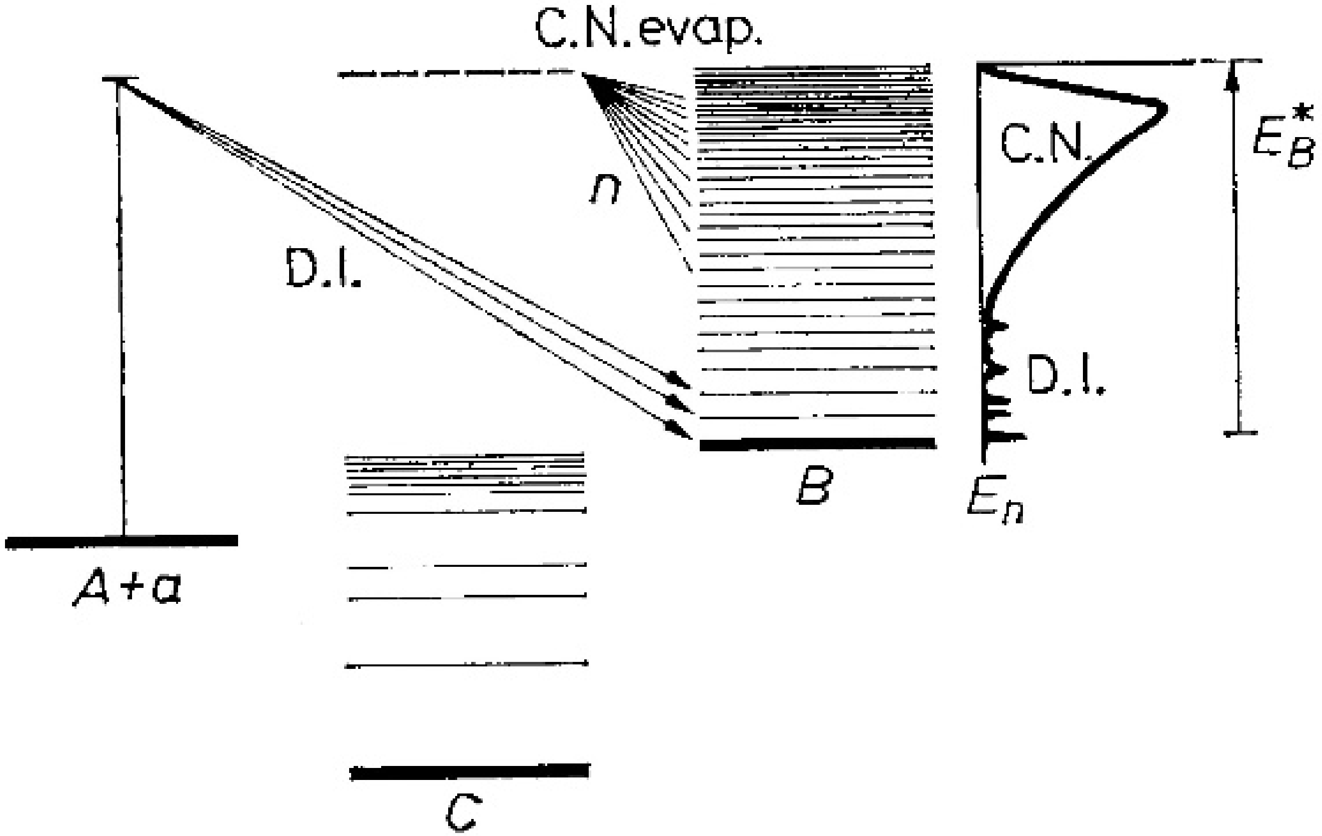
- Inelastic scattering
- Transfer reactions (stripping, pickup)
- Knock-out
- HI reactions

Above ≈10 MeV Incident energy

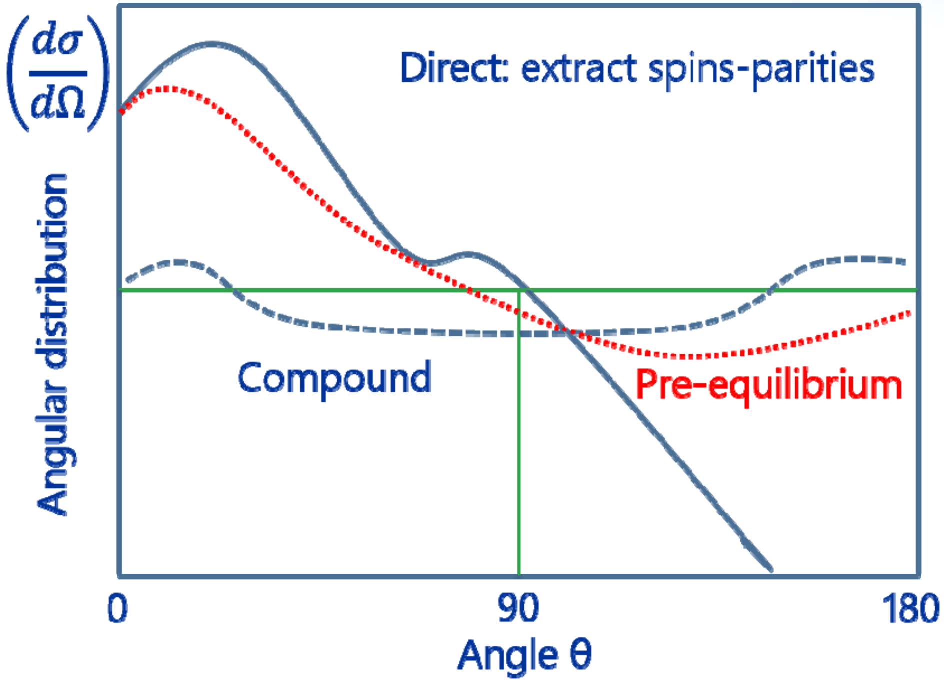
Compound Nucleus

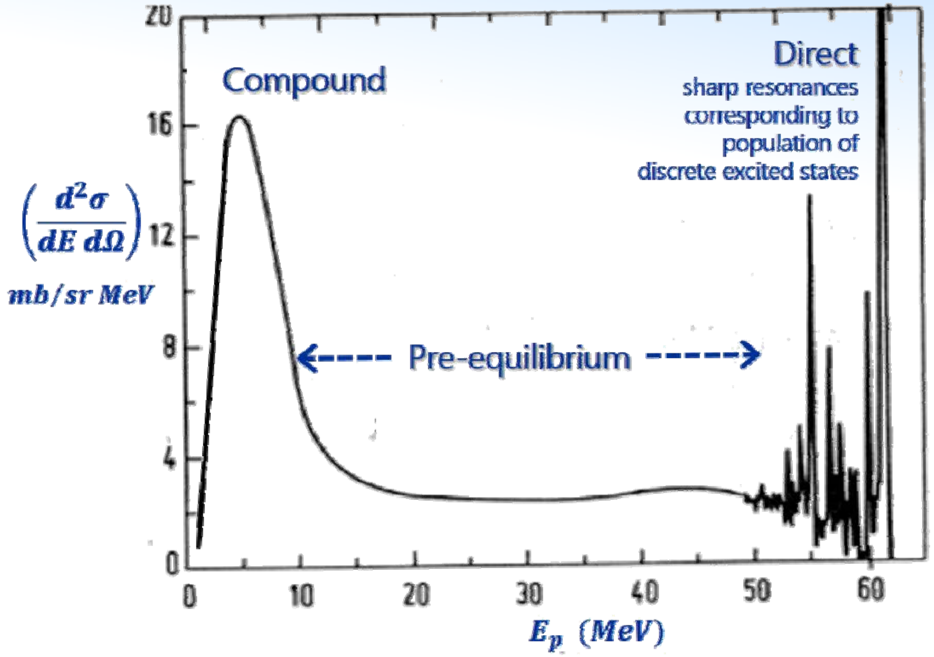
- Resonance scattering
- Evaporation (incl. radiative capture)
- Fission

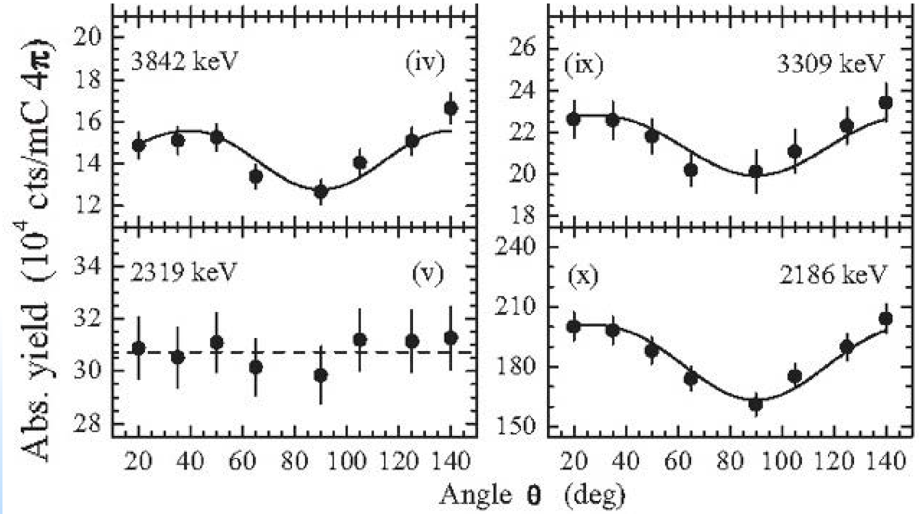
Reaction mechanisms -3 (particle spectra)

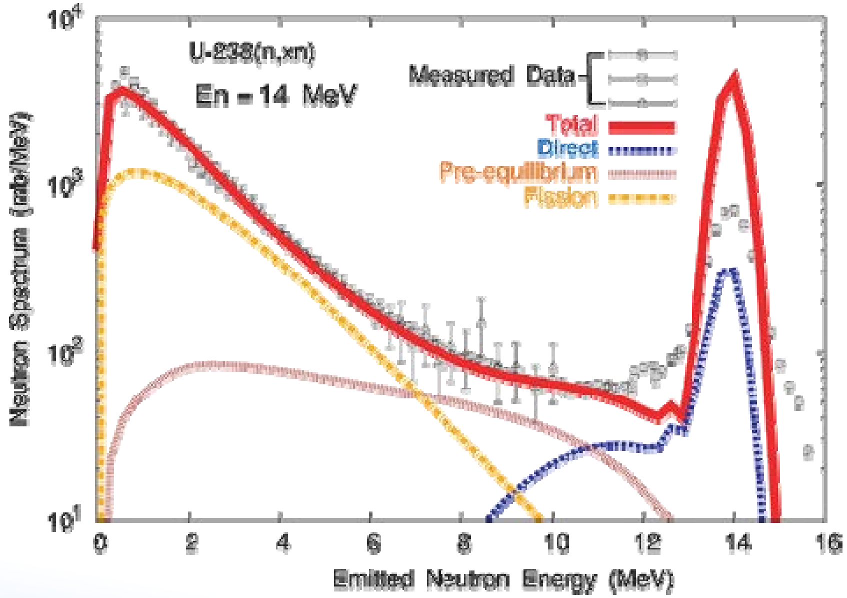


Reaction mechanisms and differential cross sections

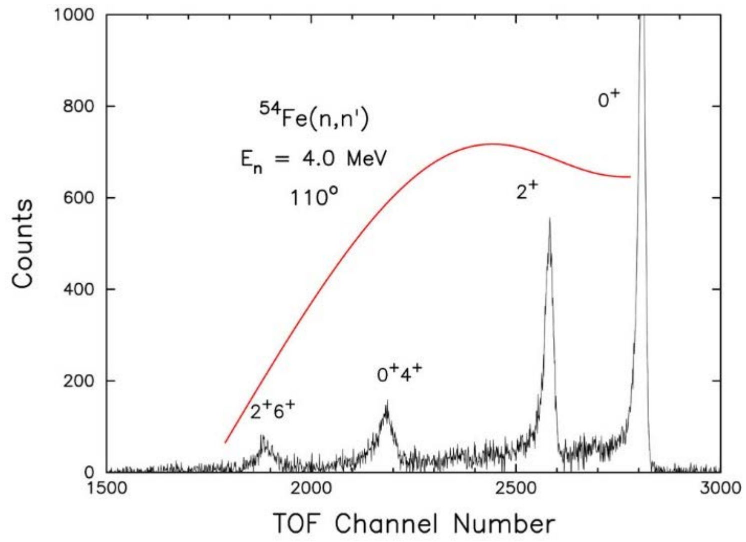


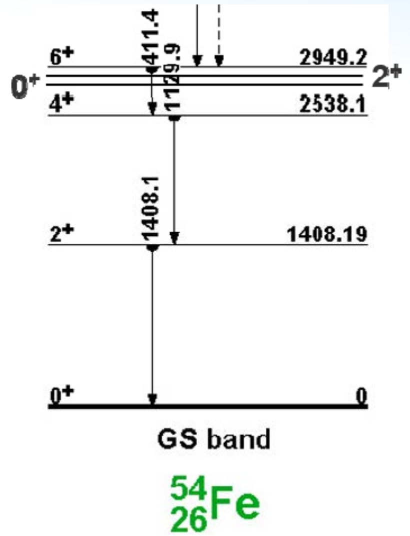


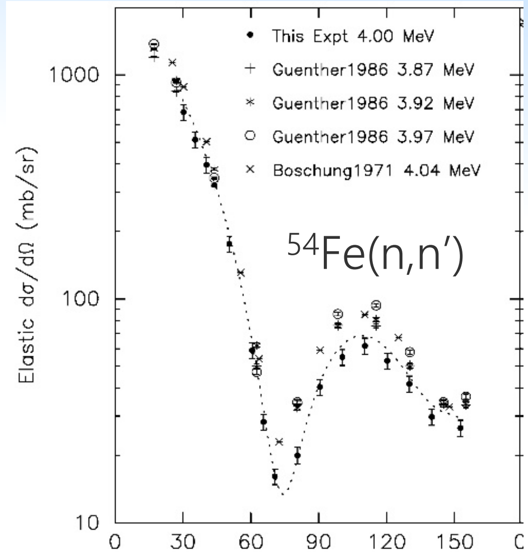


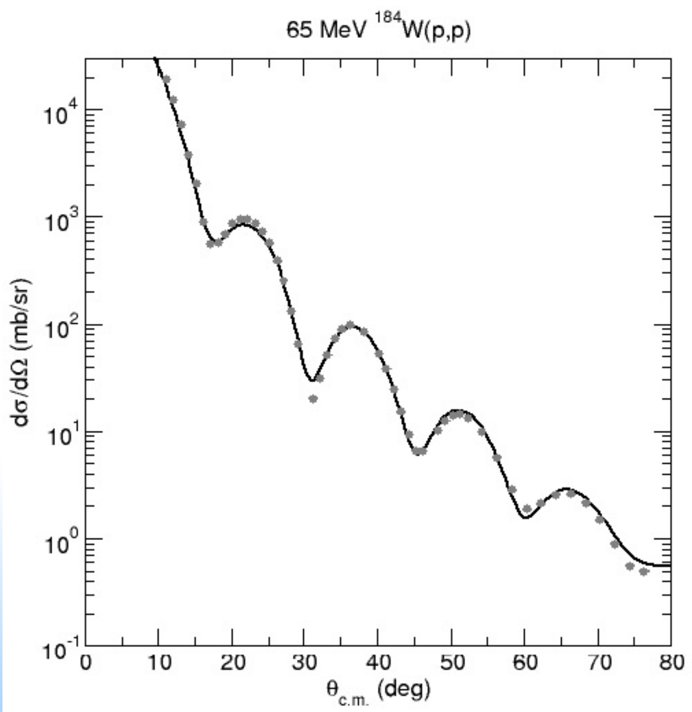


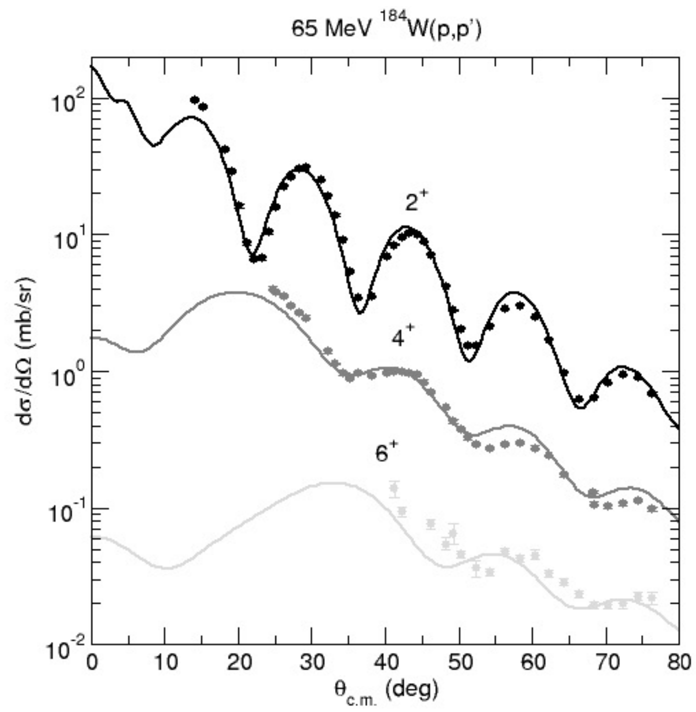
Elastic & Inelastic reactions and nuclear structure

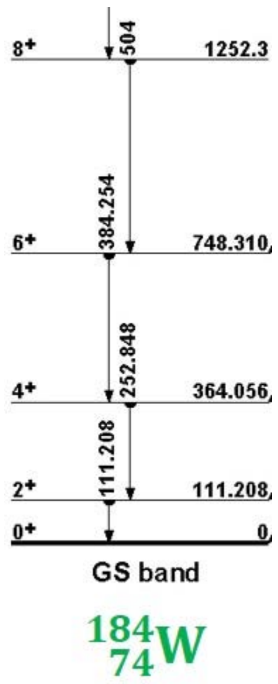












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Thank you!
Ευχαριστώ !

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