F-theory landscape with SCFT sectors

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Machine Learning Landscape, ICTP

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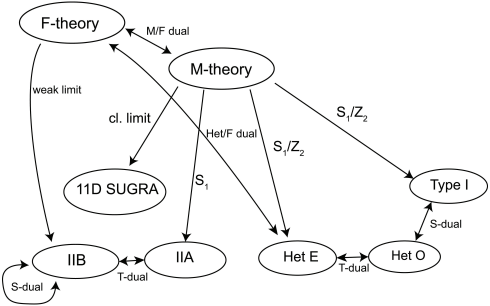
String landscape(s)

- Superstring theory has many versions



- 4D string compactification models with $\mathcal{N} \leq 1$ SUSY:

(1) 10D IIA/IIB superstring on CY3 orientifold

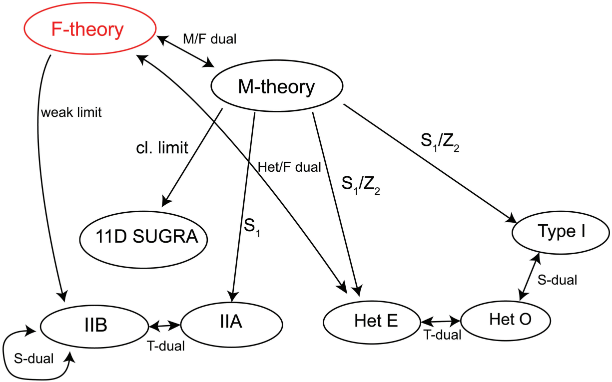
(2) 10D heterotic string on CY3

(3) M-theory on G_2 manifold

(4) F-theory on elliptic CY4

(5) F-theory on Spin(7) manifold; non-SUSY heterotic

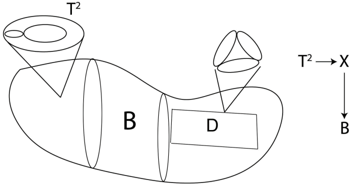
- Which is the correct/best ensemble of geometric models? Unknown
- In this talk, focusing on the F-theory geometric ensemble on elliptic CY4



- F-theory ensemble has the largest finite numbers: $\gg 10^{3,000}$ base geometries, $10^{224,000}$ self-dual flux vacua on a single geometry.

• F-theory is a geometric description of strongly coupled IIB superstring theory in presence of 7-branes.

• Compactification on an elliptic Calabi-Yau $(d+1)$-fold X with complex d-fold base $B \rightarrow$ Supergravity theory on $\mathbb{R}^{9-2d,1}$.



• The elliptic fibration X over B is described by a Weierstrass form:

$$y^2 = x^3 + fx + g \tag{1}$$

• $f \in \mathcal{O}(-4K_B)$ and $g \in \mathcal{O}(-6K_B)$ are holomorphic functions of base coordinates on B.

• 7-branes locates at the discriminant locus: $\Delta = 4f^3 + 27g^2 = 0$, where the elliptic fiber is singular.

• Non-Abelian geometric gauge group on the stack of 7-branes

ord(f)	ord(g)	ord(Δ)		Gauge group
0	0	2	I_2	SU(2)
0	0	$n \geq 3$	I_n	$\mathrm{Sp}\lfloor \frac{n}{2} \rfloor$ or $\mathrm{SU}(n)$
1	≥ 2	3	III	SU(2)
≥ 2	2	4	IV	SU(2) or SU(3)
≥ 2	≥ 3	6	I_0^*	G_2 or SO(7) or SO(8)
2	3	$6+n$	I_n^*	$\mathrm{SO}(8+2n)$ or $\mathrm{SO}(7+2n)$
≥ 3	4	8	IV^*	F_4 or E_6
3	≥ 5	9	III^*	E_7
≥ 4	5	10	II^*	E_8
≥ 4	≥ 6	≥ 12	not allowed	

- Example: $y^2 = x^3 + u^4 x + u^5$
- E_8 gauge group on the hypersurface $D : u = 0$
- Localized charged matter fields on codimension-two locus $C \subset D$: open string modes between two stacks of 7-branes.

• Cod-2 (4,6) criteria: when f vanishes to order ≥ 4 and g vanishes to order ≥ 6 on codimension-two loci $C : u = v = 0$ over B, there exists strongly coupled matter on C. For example:

$$y^2 = x^3 + u^4 x + u^5 v + (higher\ order\ terms) \tag{2}$$

• In 6D F-theory, localized 6D "conformal matter" at the point $u = v = 0$. (del Zotto, Heckman, Morrison, Rudelius, Tomasiello, Vafa, ...)

• After decoupling gravity sector and the E_8 gauge group $\rightarrow$ A non-trivial 6D (1,0) SCFT with E_8 global symmetry: E-string theory

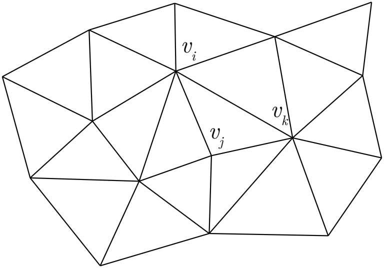
• In 4D F-theory, C is a genus-g complex curve. Localized strongly coupled matter: 6D E-string theory compactified on C, 4D conformal matter. (Apruzzi, Heckman, Kim, Morrison, Razamat, Tizzano, Vafa, Zafrir)

1. Classify the base threefolds B: over each B, there exists a generic fibration X_{gen} over B, s. t. the geometric gauge groups are smallest among all the possible fibrations.
   - Non-Higgsable gauge groups: SU(2), SU(3), G_2, SO(7), SO(8), F_4, E_6, E_7, E_8, U(1)(*)
2. For each B, classify different fibrations X over B; could have other gauge groups such as SU(5)
3. For each B and X, classify other discrete quantities such as the G_4 flux; responsible for breaking GUT gauge group, chiral generations
4. For each B, X and G_4, classify and study the scalar potential and vacua structure

Classification of base geometries

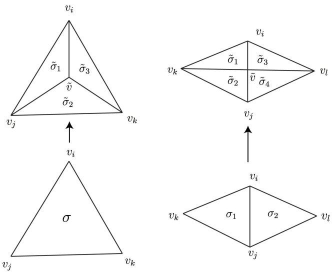
- The subset of toric threefold has been probed and partially classified. (Taylor, YNW 15, 17'; Halverson, Long, Sung 17')
- A toric threefold is given by a set of discrete data:

1. A set of 1D rays σ_1: $v_i = (x_i, y_i, z_i) \in \mathbb{Z}^3$, $(i = 1, \ldots, n)$
2. A set of 2D cones σ_2: (v_i, v_j)
3. A set of 3D cones σ_3: (v_i, v_j, v_k)

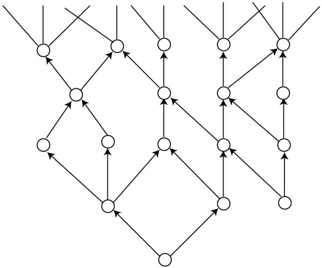


1D ray $\leftrightarrow$ complex surfaces carrying gauge groups; 2D cone $\leftrightarrow$ complex curve; 3D cone $\leftrightarrow$ point.

- Topological transitions of toric threefolds: blow up/blow down.



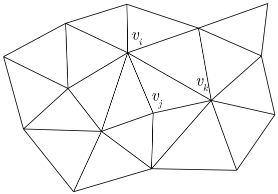
- The web of toric base threefolds



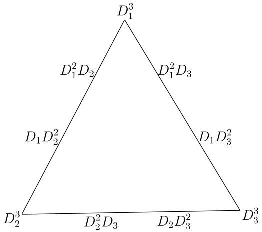
- Requirement on B: generic fibration X_{gen} over B does not have cod-1 (4,6), cod-2 (8,12) or cod-3 (12,18)
- Almost all the bases have non-Higgsable gauge groups, the geometric gauge groups are typically a product of SU(2), G_2, F_4 and E_8
- Almost all the bases non-Higgsable strongly coupled matter sectors from cod-2 (4,6) lcous; out of $\sim 10^{3,000}$ bases, only $\sim 10^{250}$ bases do not have them (Taylor, YNW 17')
- It is typically hard to enhance the gauge symmetry by choosing a different fibration X over B. SU(5) construction is unfavored.
- In general, the existence of weakly coupling limit is unlikely (Halverson, Long, Sung 17')

- What about non-toric threefold bases?
- In fact, it's unknown how to compute the non-Higgsable gauge group G from local geometric data near a complex surface D.
- Using machine learning to construct the map (YNW, Z. Zhang 18')
- Input data: an input vector of local triple intersection numbers between D and its neighbors
- Output data: the non-Higgsable gauge group G, only 10 possible choices
- Supervised machine learning with multiple classes

Triple Intersection numbers

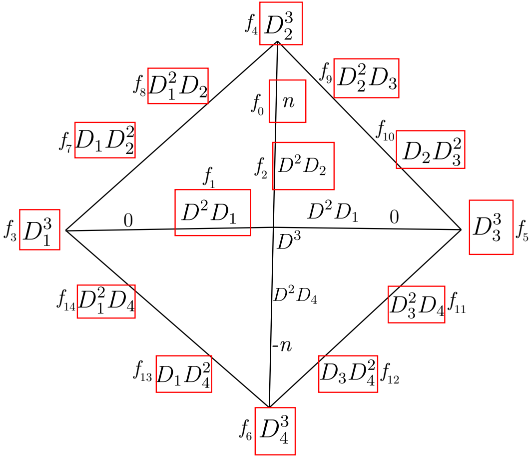


The triple intersection numbers among D_1, D_2, D_3 are labeled as



$$D_1 \cdot D_2 \cdot D_3 = 1 \tag{3}$$

- For divisors $D = \mathbb{F}_n$ (Hirzebruch surface) with 4 neighbors:



- Take the Hirzebruch surfaces inside a particular class of "end point bases" in a Monte Carlo program (Taylor, YNW 17')

Gauge groups	number of samples	Fraction in the whole data set
$\varnothing$	2053638	0.700
SU(2)	520783	0.178
G_2	286592	0.098
F_4	66934	0.023
E_8	4374	0.001
SU(3)	24	8e-06
SO(8)	8	3e-06

- Extremely unbalanced: up/down resampling to modify the number of samples in each category to $\sim 200,000$.

Classification method	Class-weighted Accuracy	RunTime (s)
Decision Tree	0.995	22
Feedforward Neural Network	0.968	11485
Logistic Regression	0.776	324
Random Forest	0.996	53
Support Vector Machine	0.974	24

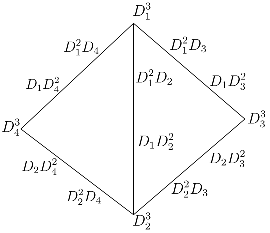
- Neural network: 2 hidden layers $(10, 10)$, epochs=5, unoptimized.
- Decision tree is the best in terms of speed, accuracy and interpretability

- A huge decision tree with 66441 nodes and 33221 leaves, maximal depth 49.
- We can generate a large set of rules in form of inequalities:

d	$\|S(l)\|$	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_{13}	other f_i	G
2	6831	-	≤ -9	-	-	-	-	-	-	-	E_8
7	4698	-	-6	-	-	-	≤ 0	-	-	$f_7 \geq 0$	F_4
8	52528	-	-5	-	-	-	≤ 3	≤ 5	≤ -1	$f_7 \geq 0,\ f_{12} \leq -1$	F_4
10	4478	-	-5	$-7 \sim -4$	≥ 2	-	≥ 4	≤ 3	≤ -1	-	F_4
9	4285	0	-5	-	-	-	≤ 2	≥ 6	≤ -1	$f_7 \geq 0$	F_4
9	5680	-	-4	≥ -12	-	$1 \sim 3$	≥ 5	-	-	-	G_2
10	4249	-	-4	≥ -12	≥ 5	$1 \sim 4$	≤ 4	-	-	$f_7 \geq 0$	G_2
12	4248	≥ 7	-4	≤ -13	-	3	-	-	-	$f_7 \geq 0,\ f_{10} \geq 0$	G_2
8	160214	-	-3	-	-	≥ 6	≥ -5	-	-	$f_{12} \leq -1$	G_2
9	42972	-	-3	-	-	≤ 2	≥ -5	-	-	$f_{11} = 0,\ f_{12} \leq -2$	G_2
9	42871	-	-3	-	-	$3 \sim 5$	≥ -5	≥ 7	-	$f_{12} \leq -1$	G_2
16	17985	-	-3	-2	≥ 6	$0 \sim 2$	≥ 12	$-4 \sim 2$	-	$f_{12} \geq 0,\ f_{14} \leq -1$	SU(2)
12	11200	-	-3	≤ -3	-	≥ 7	≥ -5	-	-	$f_{12} \geq 0,\ f_{14} \geq -2$	G_2
14	8483	-	-3	≥ -12	-	≤ 1	$-5 \sim 11$	-	≤ -2	$f_{12} \geq -1,\ f_{14} = 0$	G_2
13	3751	≥ 1	-3	≥ -5	-	$3 \sim 5$	≥ -5	≤ 6	-	$f_7 \geq 0,\ f_{10} \geq 0,\ f_{12} \leq -1$	G_2
16	3536	-	-3	≤ -13	≤ 6	$-7 \sim 2$	$-5 \sim 3$	≥ 10	-	$f_{11} \geq 0,\ f_{12} \geq -1$	SU(2)
16	3355	≥ 2	-3	$-12 \sim -7$	≥ -11	2	$-5 \sim 11$	-	≤ -1	$f_7 \geq 0,\ f_{12} \geq -1$	G_2
20	3230	-	-3	≥ -4	≥ 3	2	$3 \sim 11$	-	≤ -1	$f_7 \geq 0,\ f_{10} \geq 0,\ f_{11} \geq 0,\ f_{12} \geq -1,\ f_{14} \geq 0$	SU(2)

- For divisors with different $h^{1,1}(D)$, the out-of-sample accuracy is 85%$\sim$ 99%

- We can apply the similar method to learn whether a complete intersection curve $C = D_1 \bigcap D_2$ contains strongly coupled matter



- Out-of-sample accuracy $\sim 95\%$
- Example of machine derived rule: if $D_1^2 D_2 \leq -2$, $D_1 D_2^2 \leq -2$, then (f, g) vanishes to at least order $(4, 6)$ on $C = D_1 \bigcap D_2$; can be checked analytically

• In F-theory geometric ensemble, the existence of SU(5) GUT gauge group and SU(3)×SU(2)×U(1) is rare

• The dominating majority of geometries have geometric gauge group:

$$G = SU(2)^a \times G_2^b \times F_4^c \times E_8^d . \qquad (4)$$

• The only way to realize standard model is embedding it into a single E_8 on a complex surface S, and then break $E_8 \to H = SU(3) \times SU(2) \times U(1) \times U(1)^4$ by flux.

• However, one cannot get any chiral standard model generations from the bulk fields (Tatar, Watari 06')

• The only localized matter field of E_8 in F-theory is strongly coupled matter!

$$y^2 = x^3 + u^4 x + u^5 v \qquad (5)$$

• E-string theory compactified on the curve $C : u = v = 0$, coupled with E_8 gauge group and gravity

- E-string theory is a strongly coupled 6D (1,0) SCFT with no Lagrangian description (Chang 18')
- What we know: the fields in E-string theory are rep. of E_8;
- Question: what are the E_8 rep. and the quantum numbers such as the Lorentz rep., R-charge under SU(2)...
- Strategy: study the M-theory dual picture, where the singular elliptic CY3 X

$$y^2 = x^3 + u^4 x + u^5 v \tag{6}$$

is resolved to a smooth one $\hat{X}$. The E_8 gauge field theory with conformal matter goes to the Coulomb branch in 5D.

- M2-brane wrapping 2-cycles C on $\hat{X}$ $\rightarrow$ Particle in 5D M-theory picture with mass $m \propto$Area(C)
- Shrink the size of fiber direction to zero $\rightarrow$ 6D F-theory picture
- If Area(C) shrinks, this M2-brane wrapping mode corresponds to massless particle in 6D

• Pick a global Weierstrass model X: generic fibration over the Hirzebruch surface $\mathbb{F}_{11}$

• E_8 gauge group on the (-11)-curve: $u = 0$ in $\mathbb{F}_{11}$

$$y^2 = x^3 + u^4 x + u^5 v \tag{7}$$

• Resolution of this singular Weierstrass model was studied (Lawrie, Schäfer-Nameki 13')

• Another equivalent way: construct a 4D toric ambient space T in form of a $\mathbb{P}^{2,3,1}$ bundle over $\mathbb{F}_{11}$; The singular CY3 X is an anticanonical hypersurface of T

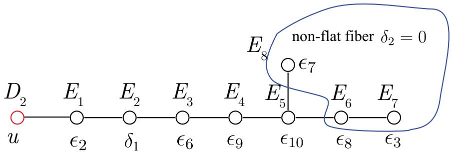
• Resolution of the $X \to \hat{X} \leftrightarrow$ blowing up $T \to T_{res}$

• T_{res} is a reflexive polytope, one can compute Hodge numbers of $\hat{X}$: $h^{1,1} = 12$, $h^{2,1} = 462$; triple intersection number of $\hat{X}$

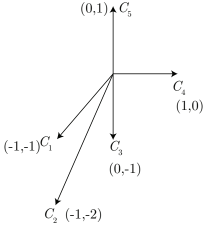
Spectrum of E-string theory

The blow up	The 4D toric ray corresponding to exceptional divisor
$(x, y, u; \zeta_1)$	$(0, -1, -1, -2)$
$(x, y, \zeta_1; \zeta_2)$	$(0, -1, 0, -1)$
$(y, \zeta_1; \delta_1)$	$(0, -1, -1, -1)$
$(y, \zeta_2; \delta_2)$	$(0, -1, 0, 0)$
$(\zeta_2, \delta_1; \epsilon_1)$	$(0, -2, -1, -2)$
$(\zeta_1, \delta_1; \epsilon_2)$	$(0, -2, -2, -3)$
$(\zeta_2, \delta_2; \epsilon_3)$	$(0, -2, 0, -1)$
$(\delta_1, \delta_2; \epsilon_4)$	$(0, -2, -1, -1)$
$(\delta_2, \epsilon_1; \epsilon_5)$	$(0, -3, -1, -2)$
$(\epsilon_1, \epsilon_4; \epsilon_6)$	$(0, -4, -2, -3)$
$(\delta_2, \epsilon_4; \epsilon_7)$	$(0, -3, -1, -1)$
$(\delta_2, \epsilon_5; \epsilon_8)$	$(0, -4, -1, -2)$
$(\epsilon_4, \epsilon_5; \epsilon_9)$	$(0, -5, -2, -3)$
$(\epsilon_5, \epsilon_7; \epsilon_{10})$	$(0, -6, -2, -3)$

- The exceptional divisor $\delta_2 = 0$ is a non-flat fiber S_{nf}!



- Non-flat fiber S_{nf} has the topology of generalized del Pezzo surface gdP_2



- C_1, C_2, C_3 are intersection curve between S_{nf} and exceptional divisors E_8, E_6, E_7; generates the Mori cone of S_{nf}
- M2 brane wrapping any curve class $C \subset S_{nf} \to$ massless particle in 6D F-theory picture
- E_8 representation $\leftrightarrow$ intersection number between C and exceptional divisors
- Representation under little group SO(4) $(N, 1/2) \oplus 2(N, 0) \leftrightarrow$ dimension N of the moduli space of C (Witten 96')

Spectrum of E-string theory

- Some lowest representations of E_8:
  1. 248 rep. : a 6D hypermultiplet and the vector multiplet of E_8 gauge field (Klemm, Mayr, Vafa 96')
  2. 3875 rep. : a non-adjoint 6D vector multiplet
  3. 147250 rep. : a 6D Rarita-Schwinger multiplet
  4. 6696000 rep. : $(3/2, 1/2) \oplus 2(3/2, 0)$
- We generate an infinite massless higher spin tower!
- Interpretation in SCFT is not clear yet

• 4D spectrum after compactified on a complex curve Σ with genus $g = 0$ and the presence of gauge flux

$$SO(1,5)\times SU(2)_R \to SO(1,3)\times U(1)_J\times U(1)_R = SU(2)_L\times SU(2)_R\times U(1)_J\times U(1)_R. \tag{8}$$

• To preserve 4D N=1 SUSY, the 6D theory needs to be topologically twisted, $U(1)_J \times U(1)_R$ is broken to the diagonal subgroup $U(1)_{J_{top}}$,

$$J_{top} = J - \frac{1}{2}R. \tag{9}$$

• The gauge flux characterized by line bundle $L_\Sigma \in \mathcal{O}(\Sigma)$

(1) 6D hypermultiplet

$$(\mathbf{4}', \mathbf{1}) \to (\mathbf{2}, \mathbf{1}, -\frac{1}{2}, 0) + (\mathbf{1}, \mathbf{2}, \frac{1}{2}, 0), \tag{10}$$

$$(\mathbf{1}, \mathbf{2}) \to (\mathbf{1}, \mathbf{1}, 0, 1) + (\mathbf{1}, \mathbf{1}, 0, -1) \tag{11}$$

After twisting, the fermionic field components are

$$(\mathbf{2}, \mathbf{1})_{-1/2} + (\mathbf{1}, \mathbf{2})_{1/2} \tag{12}$$

Net chiral generations: $h^0(K_\Sigma^{1/2} \otimes L_\Sigma, \Sigma) - h^0(K_\Sigma^{1/2} \otimes L_\Sigma^*, \Sigma) = \chi(K_\Sigma^{1/2} \otimes L_\Sigma, \Sigma)$

(Beasley, Heckman, Vafa 08')

(2) 6D vector multiplet

• Bosonic part:

$$(\mathbf{6},\mathbf{1}) \to (\mathbf{2},\mathbf{2},0,0) + (\mathbf{1},\mathbf{1},1,0) + (\mathbf{1},\mathbf{1},-1,0). \tag{13}$$

• Fermionic part: two 6D Weyl spinors χ^1, χ^2 in the $(\mathbf{4},\mathbf{2})$ representation of $SO(5,1) \times SU(2)_R$, satisfying the symplectic Majorana conditions
After twisting, the fermionic components under $SU(2)_L \times SU(2)_R \times U(1)_{J_{top}}$ are

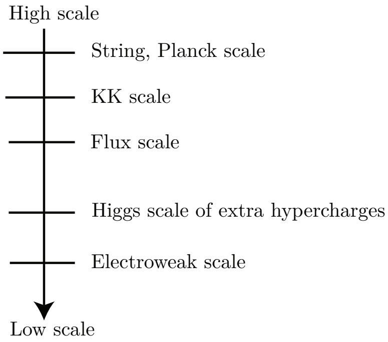
$$(\mathbf{2},\mathbf{1})_0 + (\mathbf{1},\mathbf{2})_{-1} + (\mathbf{2},\mathbf{1})_1 + (\mathbf{1},\mathbf{2})_0 \tag{14}$$

• Net chiral generations from the 6D vector multiplet:

$$\frac{1}{2}(h^0(L_\Sigma,\Sigma) - h^0(L_\Sigma^*,\Sigma) - h^0(K_\Sigma \otimes L_\Sigma,\Sigma) + h^0(K_\Sigma \otimes L_\Sigma^*,\Sigma)). \tag{15}$$

• Not an Euler characteristic! Topological invariant for line bundle L_Σ on $g=0$ curve Σ

- Consider a local model with a stack of E_8 7-branes on a complex surface S
- Strongly coupled matter on a curve $\Sigma = \mathbb{P}^1 \subset S$, which is 6D E-string theory reduced on $\mathbb{P}^1$
- $E_8 \to G_{sm} \times U(1)^4 = SU(3) \times SU(2) \times U(1) \times U(1)^4$ by gauge flux
- Bulk fields on $S \to$ non-chiral charged matter fields, Higgs sector
- Localized strongly coupled matter on $\Sigma \to$ Chiral fermionic matter



- What fields to include in the infinite higher spin tower?
- Not actually infinite d.o.f.; in usual QCD, there exists infinite higher spin composite particles
- Criterion of "fundamental particles": lowest spin, chiral anomaly free

(1) Fields from 6D hypermultiplet in rep. 248: cannot give rise to chiral SM generation

(2) Fields from 6D vector multiplet in rep. 3875: branching rule under $E_8 \to SU(5)_1 \times SU(5)_2$

$$\begin{aligned} \mathbf{3875} \to & (\mathbf{15},\bar{\mathbf{5}}) + (\overline{\mathbf{40}},\mathbf{5}) + (\overline{\mathbf{45}},\overline{\mathbf{10}}) + (\mathbf{75},\mathbf{1}) + (\mathbf{24},\mathbf{24}) + (\mathbf{24},\mathbf{1}) \\ & + (\mathbf{45},\mathbf{10}) + (\mathbf{5},\mathbf{15}) + (\mathbf{5},\mathbf{40}) + (\mathbf{5},\mathbf{10}) + (\mathbf{40},\bar{\mathbf{5}}) + (\mathbf{10},\overline{\mathbf{45}}) \\ & + (\mathbf{10},\bar{\mathbf{5}}) + (\overline{\mathbf{15}},\mathbf{5}) + (\overline{\mathbf{10}},\mathbf{45}) + (\overline{\mathbf{10}},\mathbf{5}) + (\bar{\mathbf{5}},\overline{\mathbf{40}}) + (\bar{\mathbf{5}},\overline{\mathbf{15}}) \\ & + (\bar{\mathbf{5}},\overline{\mathbf{10}}) + (\mathbf{1},\mathbf{75}) + (\mathbf{1},\mathbf{24}) + (\mathbf{1},\mathbf{1}). \end{aligned} \tag{16}$$

- Requirement: 3 chiral generations of $\mathbf{5}$, $\overline{\mathbf{10}}$, no exotics in representation **15**, **40**, **45**.
- One can find a number of gauge bundle choices! Chiral anomaly free.

• On the majority of the geometries in F-theory geometric landscape, it seems that the only way to embed standard model is use an E_8 GUT and strongly coupled matter fields.

• Under our assumptions, we can find flux vacua with 3 chiral SM generations. Reinforcement learning to systematically find these vacua?

• Need to study the dynamics of E-string theory and its compactification to 4D in more detail. Difficult because it's a non-Lagrangian UV theory.

• Haven't solved more detailed problems such as triplet-doublet splitting, break of SUSY, cosmology, . . .

• From string theory perspective, provided a new way of studying strongly coupled SCFT from M/F-duality. Study the different Coulomb branches/flops of the E-string theory?

• Thanks!