Quantized pseudo-fixed-point subalgebras and the reflection equation

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Let g be a finite-dimensional simple Lie algebra over the complex numbers and let k be the fixed-point subalgebra of an involutive automorphism of g. In the 1990s, M. Noumi and collaborators and independently G. Letzter studied a coideal subalgebra $U_q(k)$ of the quantized enveloping algebra $U_q(g)$ (as q goes to 1, we recover the enveloping algebras U(k) and U(g)). More recently M. Balagović and S. Kolb showed that $U_q(k)$ is quasitriangular with respect to the category of finite-dimensional representations of $U_q(g)$. In particular, associated to $U_q(k)$ there is a solution of the universal reflection equation: the quartic or type B braid relation. This story can be told in a more general setting. Roughly speaking, a pseudo-fixed-point subalgebra of q is a subalgebra k which intersects the root spaces of g in the same way as the fixed-point subalgebra of an involution of g. For such k there exists a quasitriangular coideal subalgebra $U_q(k)$ of $U_q(g)$. Conjecturally, all quasitriangular coideal subalgebras of $U_q(g)$ arise this way. Time permitting, we will also outline how this works in the setting of quantized Kac-Moody Lie algebras, in particular those of affine type whose representations are relevant in quantum integrability. Joint work with V. Regelskis (arXiv:1807.02388 and in progress).