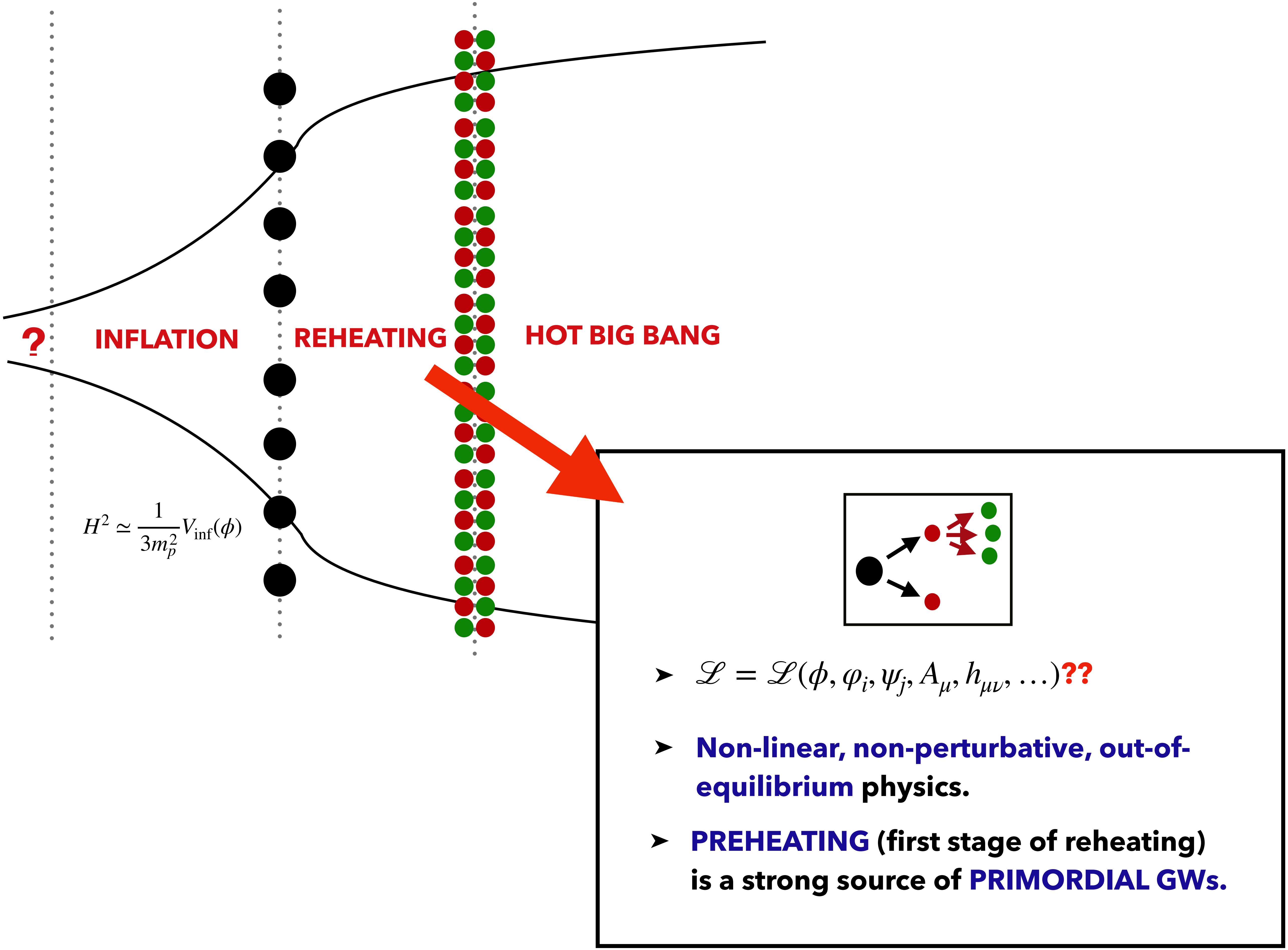
Gravitational waves from preheating:
parameter dependence

Francisco Torrentí
University of Basel

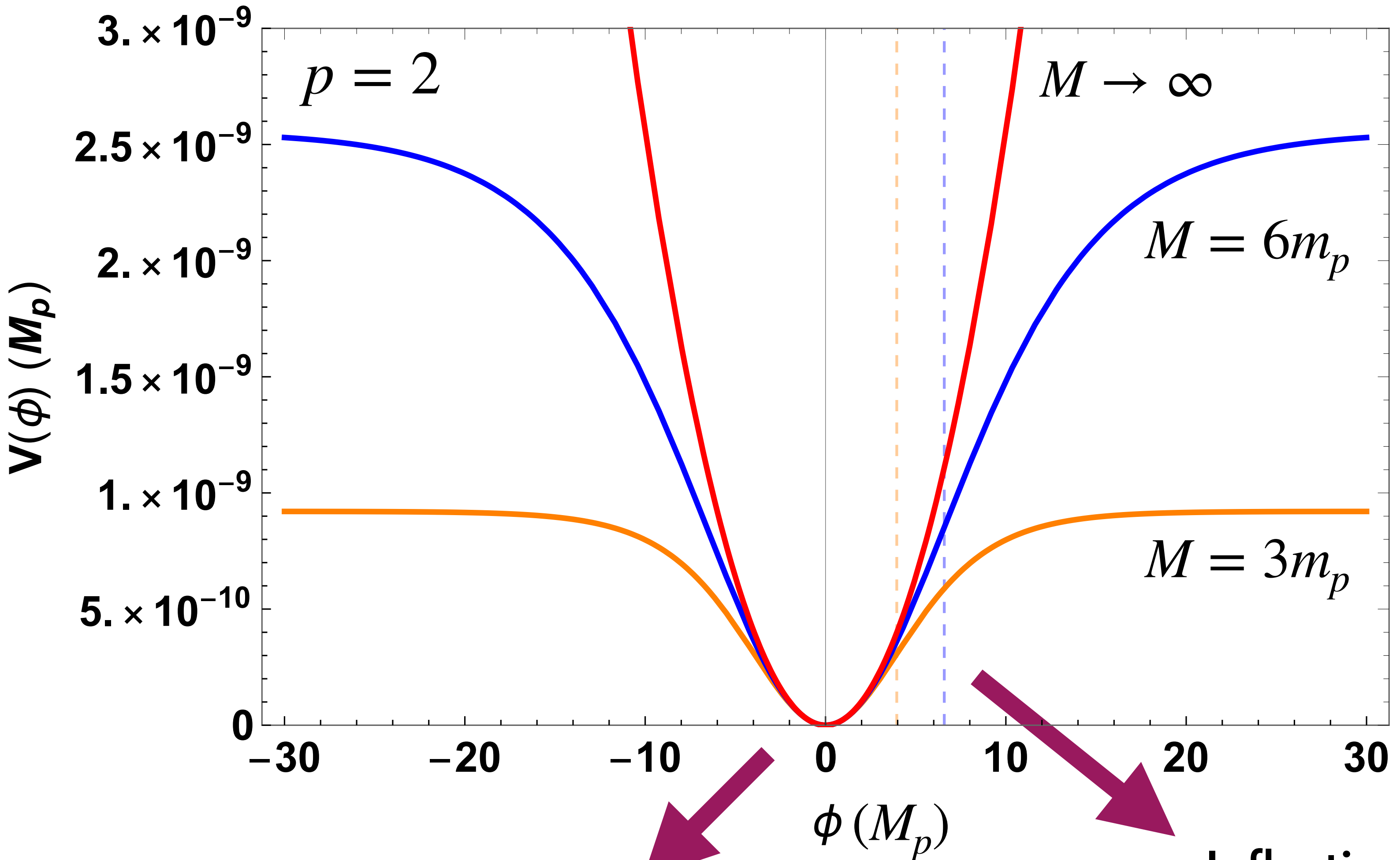
Challenges and opportunities for high-frequency GW detection
ICTP, 15th October 2019

Universität Basel


?
INFLATION
REHEATING
HOT BIG BANG
$H^2 \simeq \frac{1}{3m_p^2} V_{\rm inf}(\phi)$
➤ $\mathscr{L} = \mathscr{L}(\phi, \varphi_i, \psi_j, A_\mu, h_{\mu\nu}, \dots)$??
➤ Non-linear, non-perturbative, out-of-equilibrium physics.
➤ PREHEATING (first stage of reheating) is a strong source of PRIMORDIAL GWs.

Inflationary potential

$$V(\phi) = \Lambda^4 \tanh^p\left(\frac{|\phi|}{M}\right) \rightarrow V(\phi) = \Lambda^4\left(\frac{|\phi|}{M}\right)^p + \ldots$$



CMB constraints:

$M \lesssim 10 M_p$

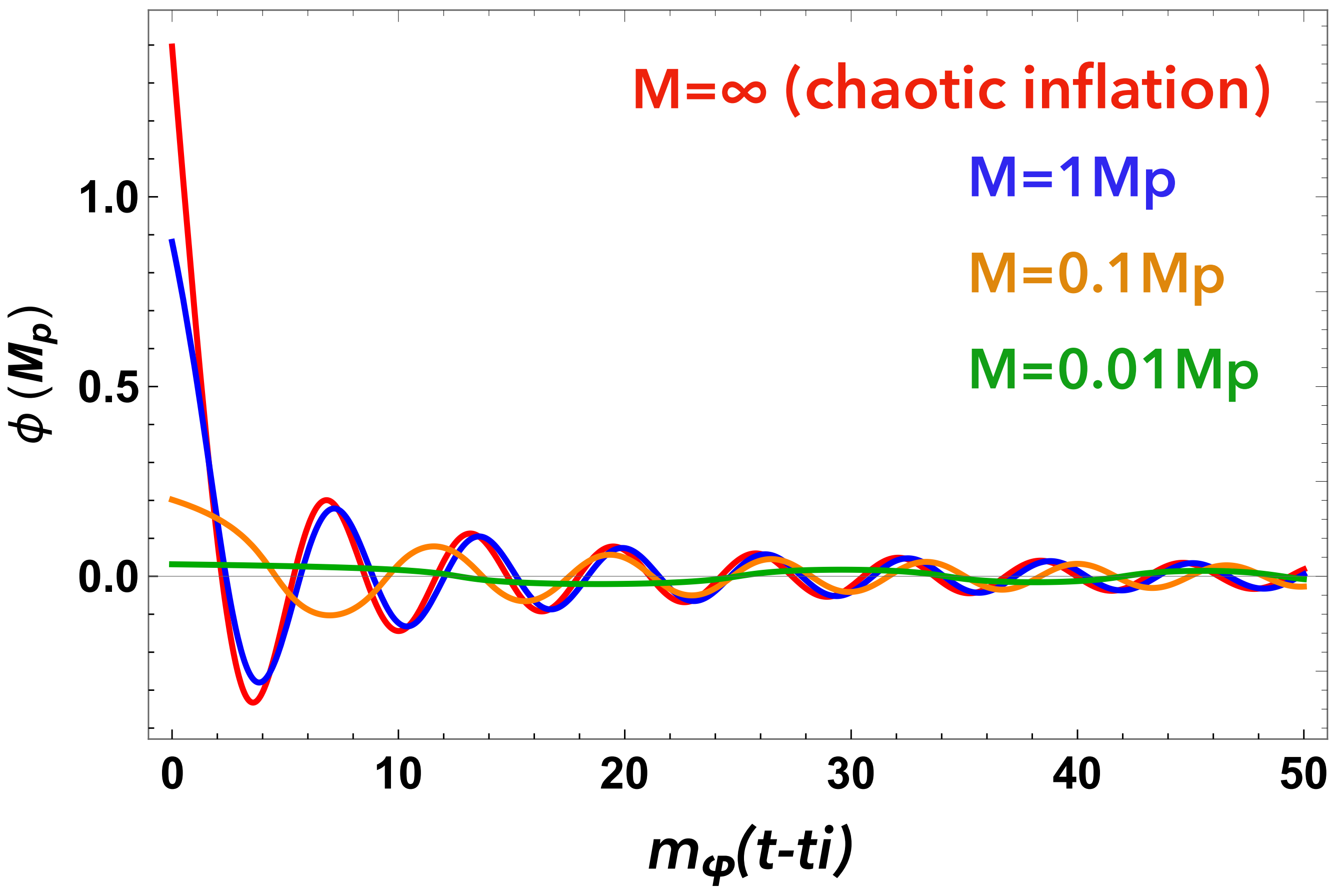
$\Lambda = \Lambda(M)$

$$m_\phi^2 \equiv \frac{2\Lambda^4}{M^2} \simeq \text{const}$$

Inflection point

$$V''(\phi) = 0$$

Post-inflationary oscillations



TWO REGIMES:

$M \gtrsim M_p \rightarrow V''(|\phi_i|) > 0$

$M \lesssim M_p \rightarrow V''(|\phi_i|) < 0$

for $M \gtrsim M_p$, we can approximate the potential with a power-law form

$\phi_i \equiv \phi(t_i)$ inflaton amplitude at the end of inflation

Frequency of oscillations:
($M \gtrsim M_p$)

$$\Omega^2_{\rm osc} \approx \omega_*^2 \left(t/t_i\right)^{2-4/p} \quad , \quad \omega_*^2 \equiv \frac{p}{\Lambda^4} M^p \phi_i^{p-2}$$

Parametric resonance after inflation

PARAMETRIC RESONANCE after inflation:
power-law potential + quadratic interaction term

$$V(\phi) = \frac{\Lambda^4}{M^p}\phi^p + \frac{1}{2}g^2\phi^2\chi^2$$

ϕ inflaton
χ daughter field

- **Redefinitions for**
  - **Spacetime:** $\tau \to z \equiv \omega_* \tau \qquad \vec{x} \to \vec{y} \equiv \omega_* \vec{x}$
  - **Fields:** $X^{(c)} \equiv a(t)\frac{X}{\phi_i} \qquad \varphi = a(t)\frac{\phi}{\phi_i}$

Eom for daughter field modes

$$\frac{d^2}{dz^2}X^{(c)}_{\mathbf{k}} + \left(\kappa^2 + q\varphi^2(t)\right) X^{(c)}_{\mathbf{k}} \simeq 0$$

$\kappa \equiv k/\omega_*$

Dynamics depend on **resonance parameter**:

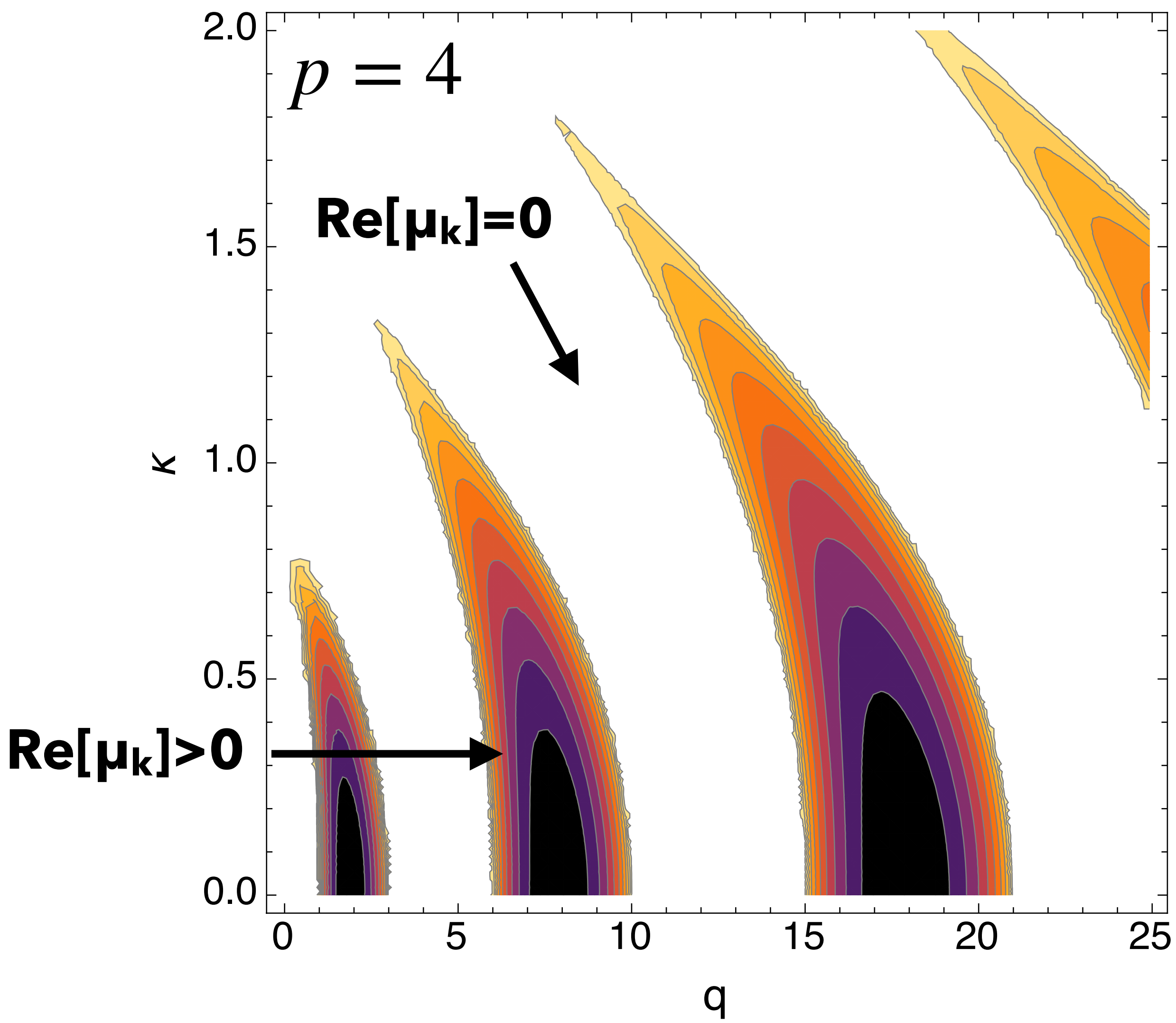
$$q \equiv \frac{g^2\phi_i^2}{2^{(4-p)}\omega_*^2}$$

Parametric resonance after inflation

$$\frac{d^2}{dz^2}X_{\mathbf{k}}^{(c)} + \left(\kappa^2+q\varphi^2(t)\right) X_{\mathbf{k}}^{(c)} \simeq 0 \quad \longrightarrow \quad |X_{\mathbf{k}}^{(c)}|^2 \sim e^{2\mu_\kappa(q,a)t}$$

Floquet index

Kofman et al (1994, 1997)



- **Two scenarios:**

$q \gtrsim 1$ **BROAD RESONANCE:** wide resonance bands, stronger particle production

$q \lesssim 1$ **NARROW RESONANCE:** small resonance band, weaker particle production

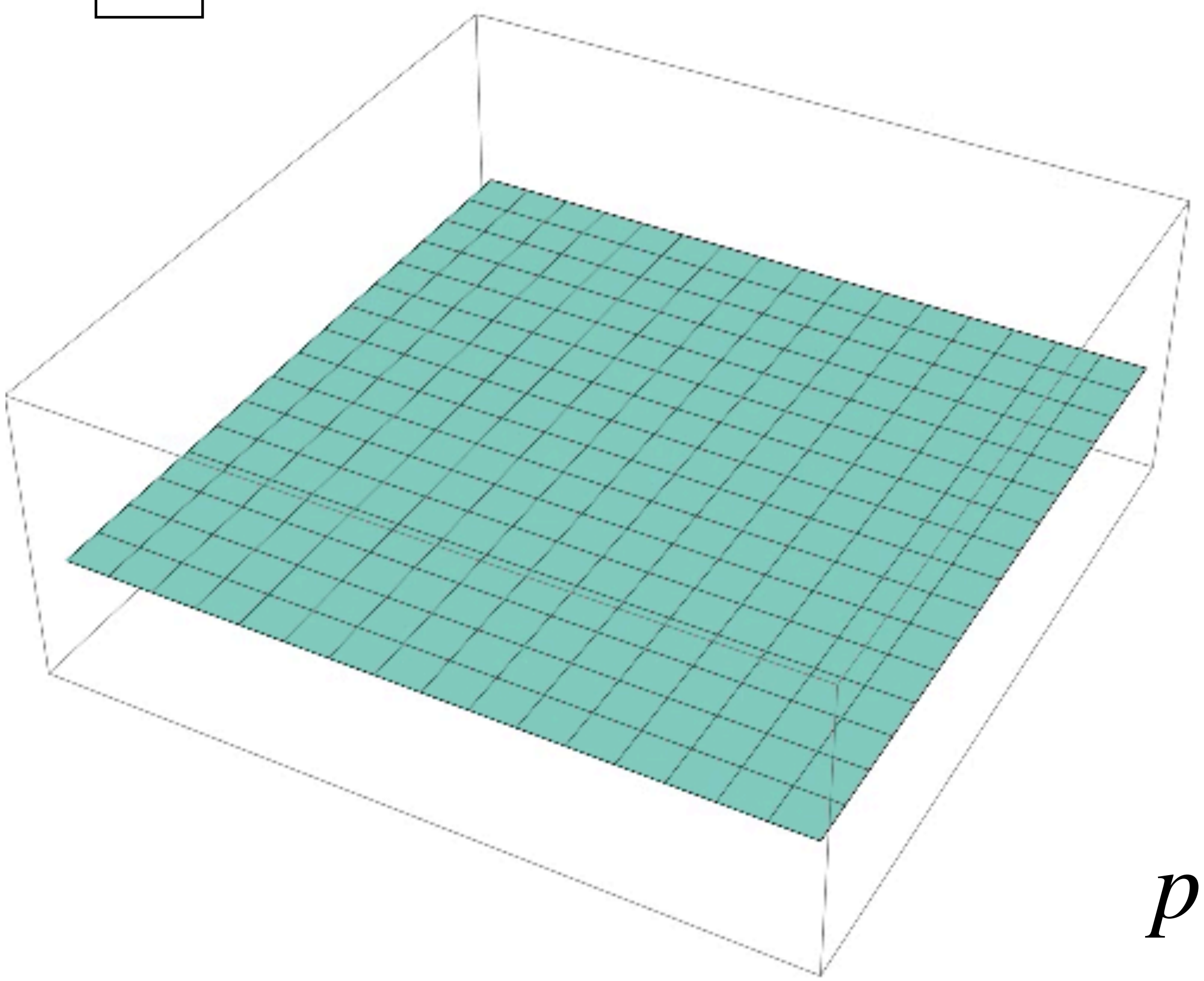
- **With expansion:** $q_{\rm eff} \equiv q a^{-6\frac{4-p}{2+p}}$

For p<4: **BROAD** $\longrightarrow$ **NARROW**

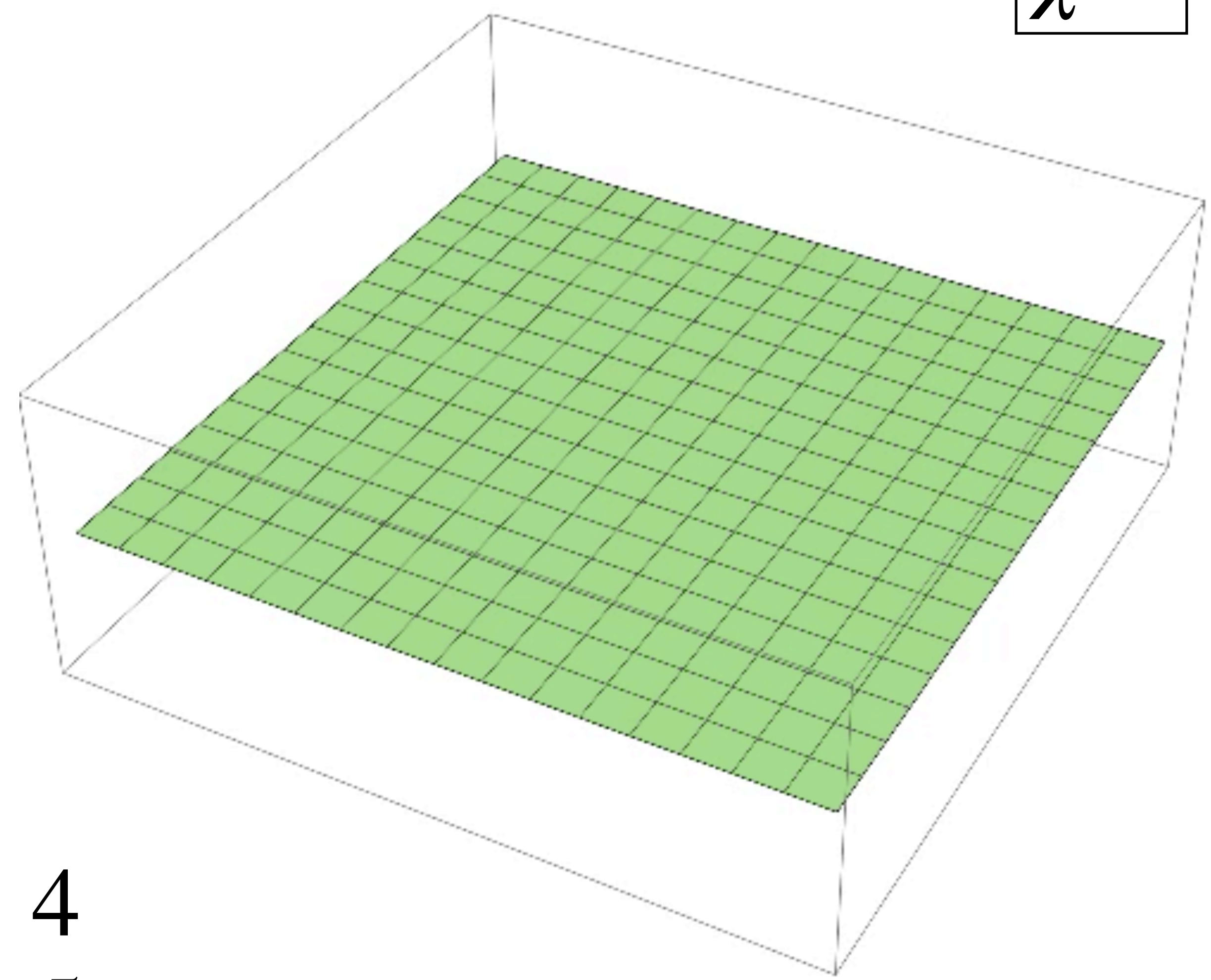
Parametric resonance after inflation

Preheating generates huge gradients in field distributions

φ



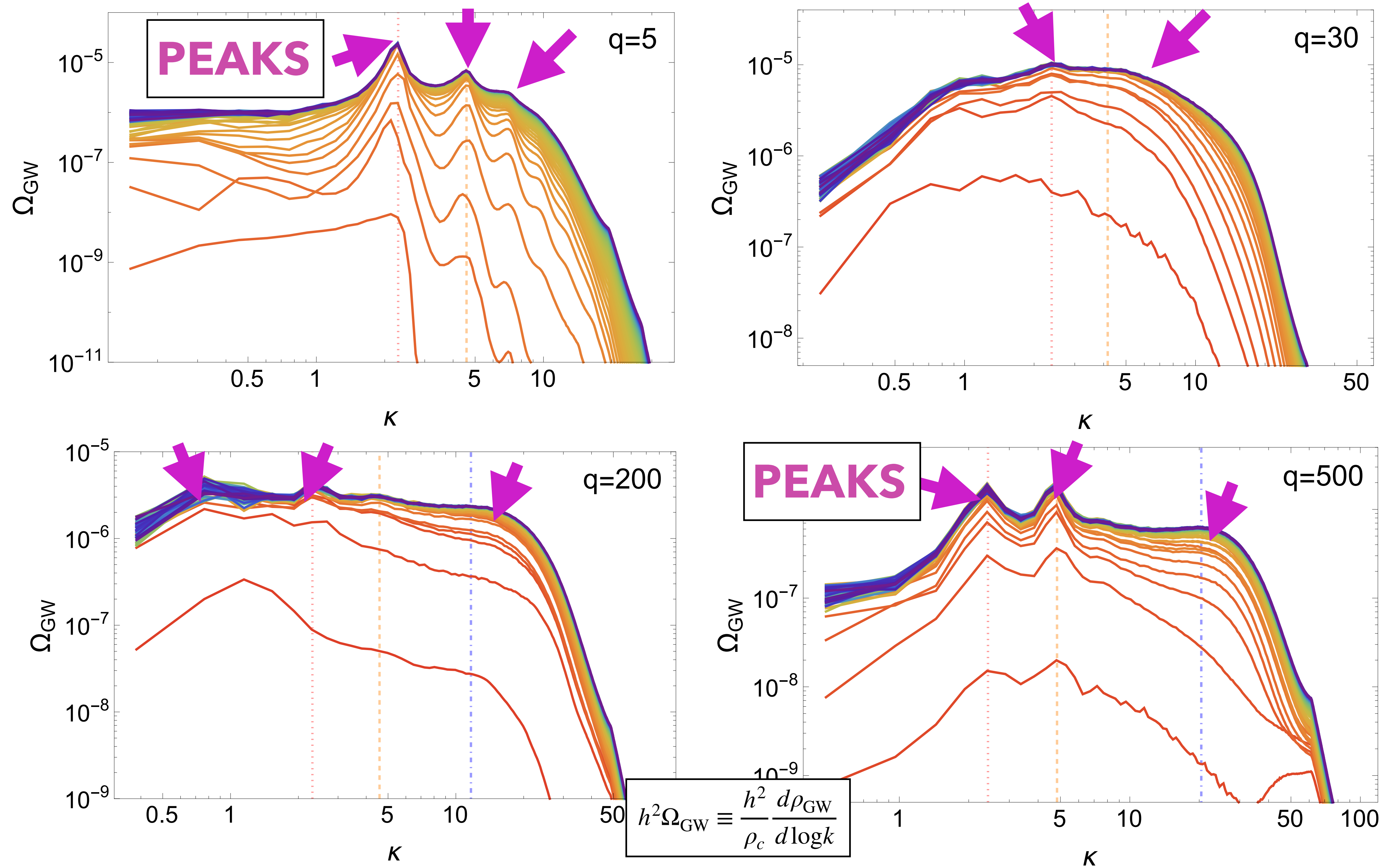
$\chi^{(c)}$



$$p = 4$$
$$q = 5$$

GWs from preheating

$$\ddot{h}_{ij} + 2\mathcal{H}\dot{h}_{ij} - \nabla^2 h_{ij} = \frac{2}{m_p^2}\Pi_{ij}^{TT}\,, \qquad \Pi_{ij}^{TT} = \left\{\partial_i\phi\,\partial_j\phi + \partial_i\chi\partial_j\chi\right\}^{TT}$$



$$h^2\Omega_{GW} \equiv \frac{h^2}{\rho_c}\frac{d\rho_{GW}}{d\log k}$$

GWs from preheating

- **Prediction for peaks in GW spectra from parametric resonance, linear regime: [Figueroa & F.T., JCAP 2017]**

$$f_p \simeq 8 \cdot 10^9 \left(\frac{\omega_*}{\rho_{\rm i}^{1/4}} \right) \epsilon_f^{\frac{1}{4}} q^{\frac{1}{4}+\eta} \, \mathrm{Hz}$$

$$h^2 \Omega_{\rm GW}^{(\rm f)}(f_p) \sim \mathcal{O}(10^{-9}) \times \epsilon_f \frac{C}{8\pi^4} \frac{\omega_*^6}{\rho_i m_p^2} q^{-\frac{1}{2}+\delta}$$

ρ_i : energy density at end of inflation

ϵ_f : expansion rate between end of GW production and radiation-domination

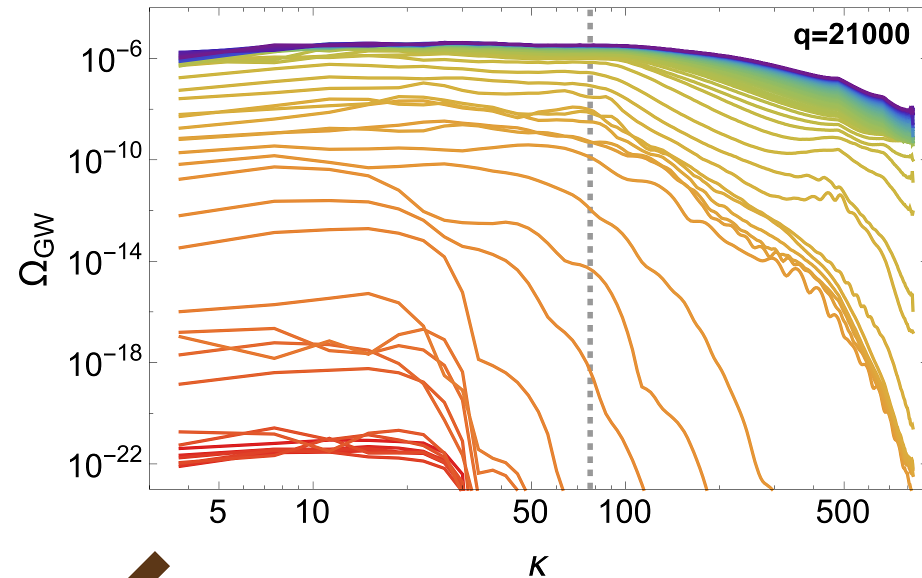
$$\epsilon_f \equiv \left(\frac{a_f}{a_{\rm RD}} \right)^{1-3w} \begin{cases} < 1 & \text{if} \quad w < 1/3 \quad \textbf{(MD)} \\ = 1 & \text{if} \quad w = 1/3 \quad \textbf{(RD)} \\ > 1 & \text{if} \quad w > 1/3 \end{cases}$$

- **Parameters C, δ, η: fixed with lattice simulations:** $(\eta, \delta \ll 1\,?)$

Frequency increases with q. Amplitude decreases with q.

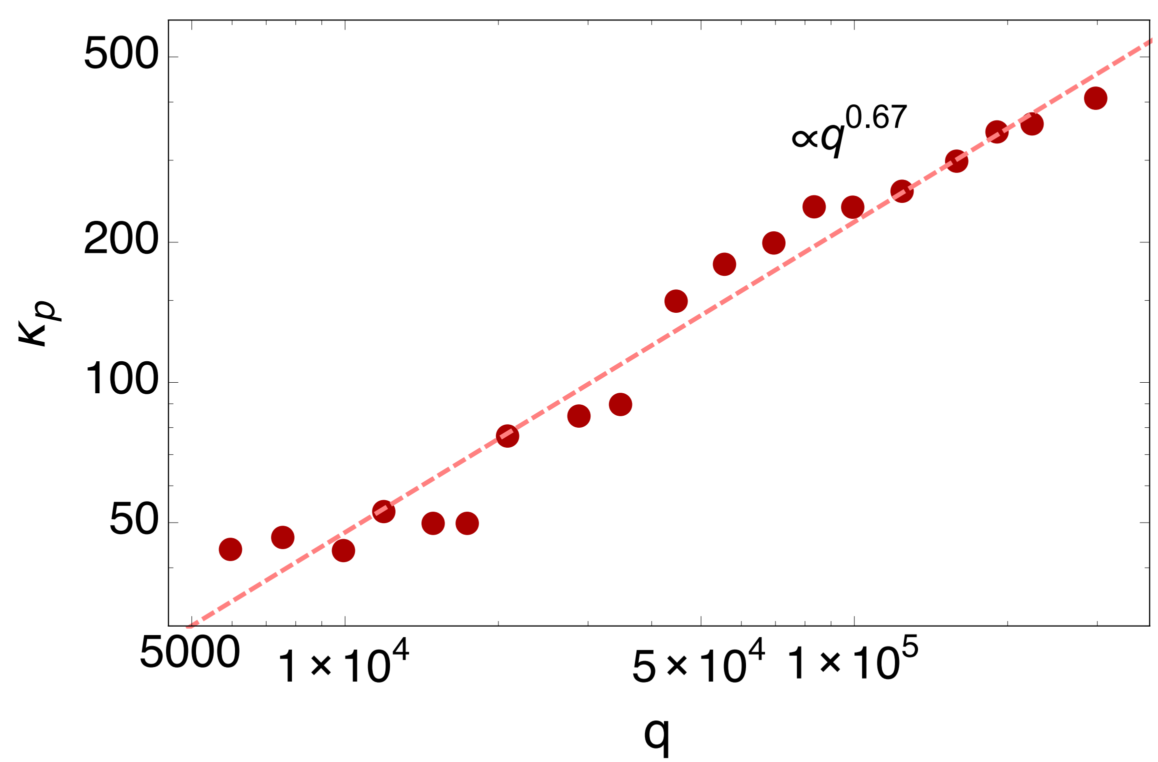
GWs from preheating - p=2

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

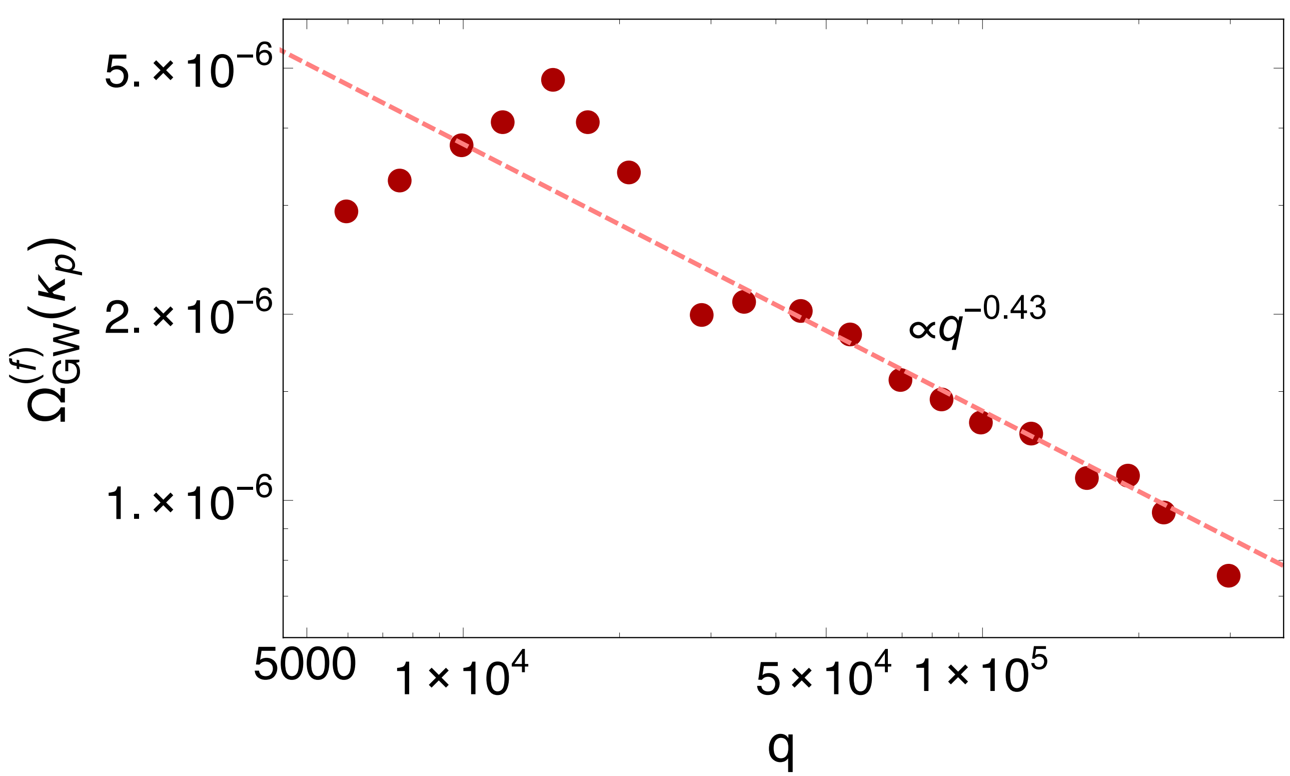


$$q = \frac{g^2\phi_i^2}{4m^2}$$

Peaks frequency



Peaks amplitude



GWs from preheating today

$$\epsilon_f \equiv \left(\frac{a_f}{a_{\rm RD}}\right)^{1-3w}$$

GW predictions today:

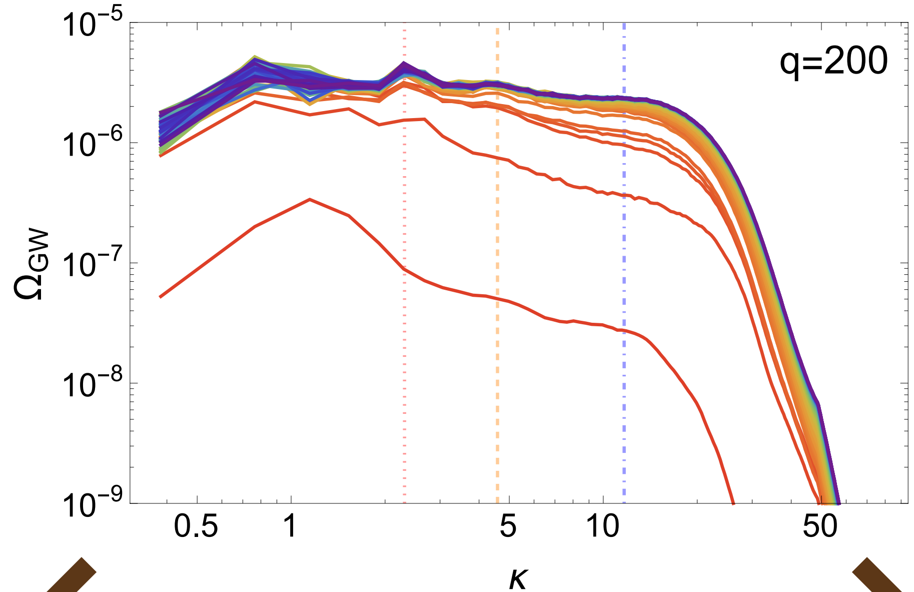
$$p = 2$$

$$f_p = \epsilon_{\rm f}^{1/4}\left(\frac{q}{10^4}\right)^{0.67} \times 2.0 \cdot 10^8 \text{ Hz}$$

$$h^2\Omega_{\rm GW}(f_p) = \epsilon_{\rm f}\left(\frac{q}{10^4}\right)^{-0.43} \times 1.5 \cdot 10^{-11}$$

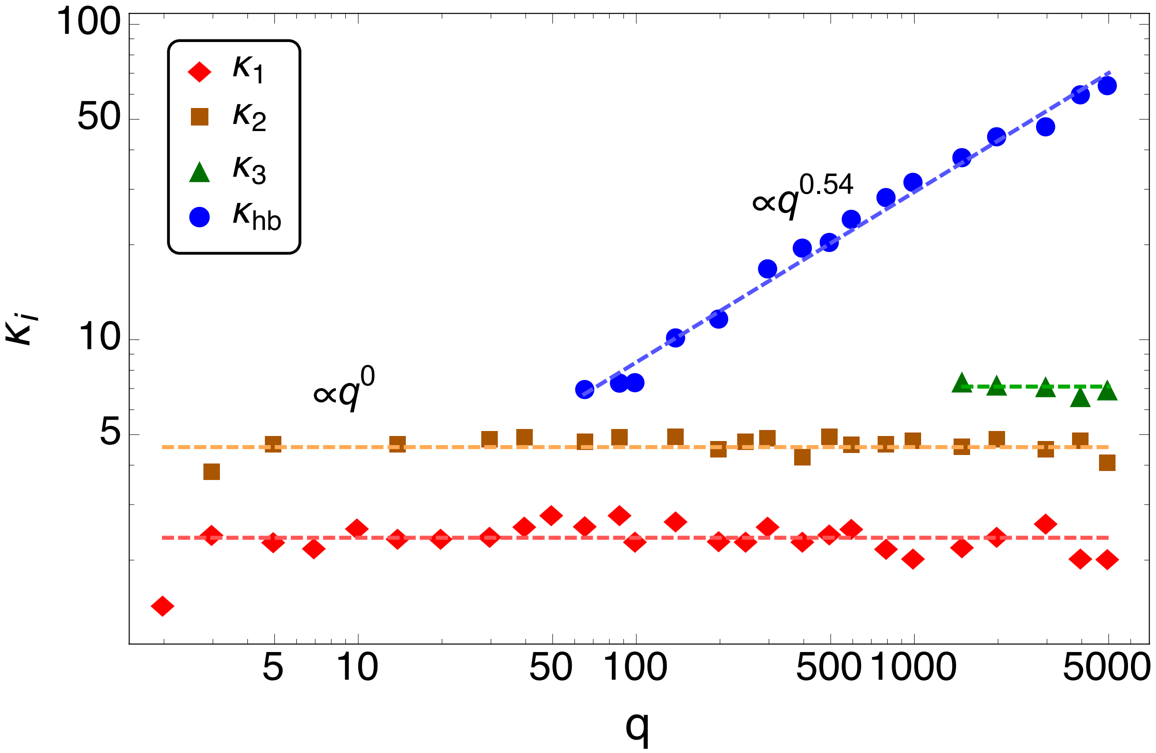
GWs from preheating - p=4

$$V(\phi) = \frac{1}{4}\lambda\phi^4$$

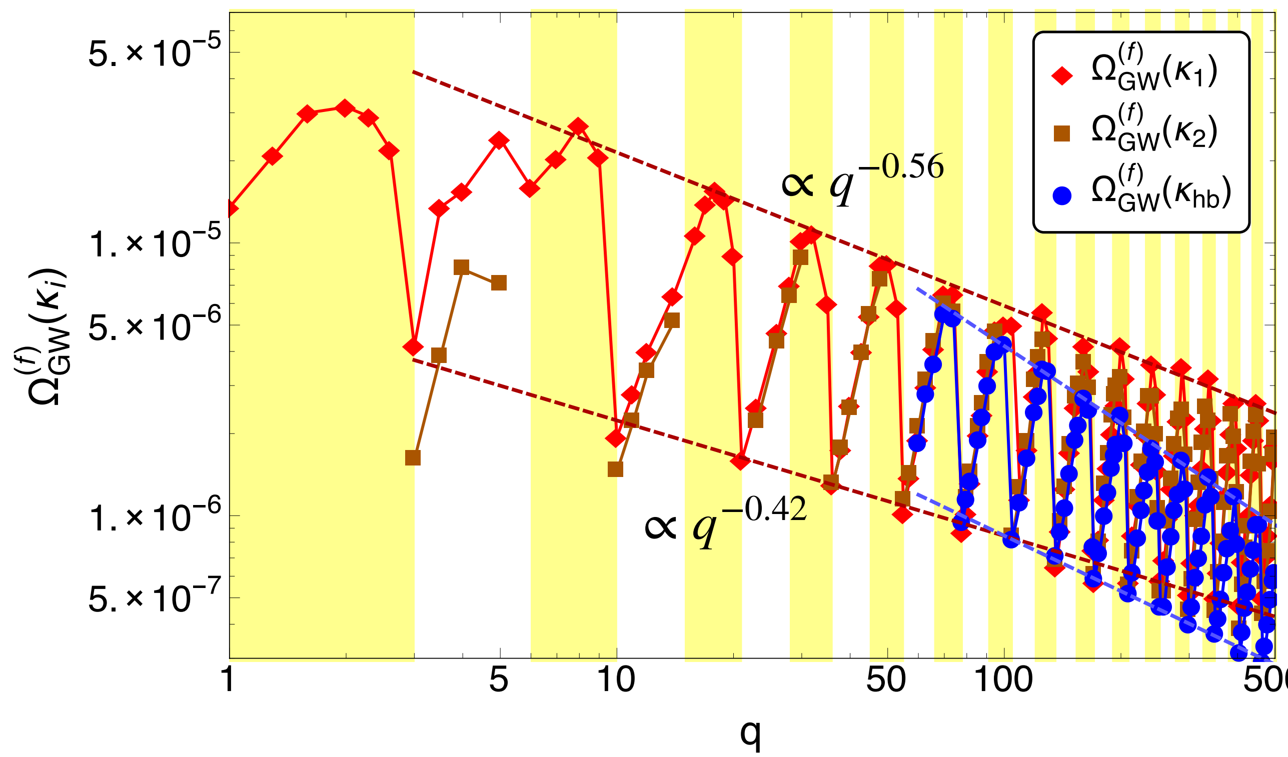


$$q = \frac{g^2}{\lambda}$$

Peaks frequency



Peaks amplitude



GWs from preheating today

$$\epsilon_f \equiv \left(\frac{a_f}{a_{\rm RD}}\right)^{1-3w}$$

GW predictions today:

$p = 2$

$$f_p = \epsilon_{\rm f}^{1/4} \left(\frac{q}{10^4}\right)^{0.67} \times 2.0 \cdot 10^8 \text{ Hz}$$

$$h^2\Omega_{\rm GW}(f_p) = \epsilon_{\rm f} \left(\frac{q}{10^4}\right)^{-0.43} \times 1.5 \cdot 10^{-11}$$

$p = 4$

$$f_1 \approx 1.5 \cdot 10^7 \text{ Hz}$$

$$f_2 \approx 2.8 \cdot 10^7 \text{ Hz}$$

$$f_{\rm hb} \approx \left(\frac{q}{100}\right)^{0.54} \times 5.3 \cdot 10^7 \text{ Hz}$$

$$3.4 \cdot 10^{-12} \left(\frac{q}{100}\right)^{-0.42} \lesssim h^2\Omega_{\rm GW}(f_{1,2}) \lesssim 2.4 \cdot 10^{-11} \left(\frac{q}{100}\right)^{-0.56}$$

$$3.4 \cdot 10^{-12} \left(\frac{q}{100}\right)^{-0.68} \lesssim h^2\Omega_{\rm GW}(f_{\rm hb}) \lesssim 1.6 \cdot 10^{-11} \left(\frac{q}{100}\right)^{-0.94}$$

$$f \sim 10^7 - 10^8 \text{Hz}$$

$$h^2\Omega_{\rm GW} \sim 10^{-11} - 10^{-12}$$

GWs from excitation of other species

- GW from parametric excitation of **other species:**
  - **BOSONS** $\mathscr{L} \in g^2\chi^2\phi^2$ $\Omega_{\rm GW} \propto q^{-1/2}$
  - **FERMIONS** $\mathscr{L} \in g\bar{\psi}\psi\phi$ $\Omega_{\rm GW} \propto q^{3/2}$ Figueroa (2014)
  - **GAUGE BOSONS** $\mathscr{L} \in (D_\mu\phi)^\dagger(D_\mu\phi)$ $\Omega_{\rm GW} \propto q^{3/2}$ Figueroa, Garcia-Bellido, F.T. (2015)

… but still $f_p \propto q^{1/2}$

Conclusions

- **Preheating generates huge field gradients –> strong source of PRIMORDIAL GRAVITATIONAL WAVES.**

- **Typical frequencies and amplitudes:**

$$f \sim 10^7 - 10^8 \text{Hz}$$
$$h^2\Omega_{\rm GW} \sim 10^{-11} - 10^{-12}$$

- **GW spectra depend on details of inflationary potential and interactions. Scaling of peaks:**

$$h^2\Omega_{\rm GW} \propto q^{-1/2} \qquad f \propto q^{1/2}$$

- **GW spectroscopy?**

THANK YOU!