

Modern technologies for quantum photonics II

Quantum photonics in multi-dimensional systems



Benni Brecht

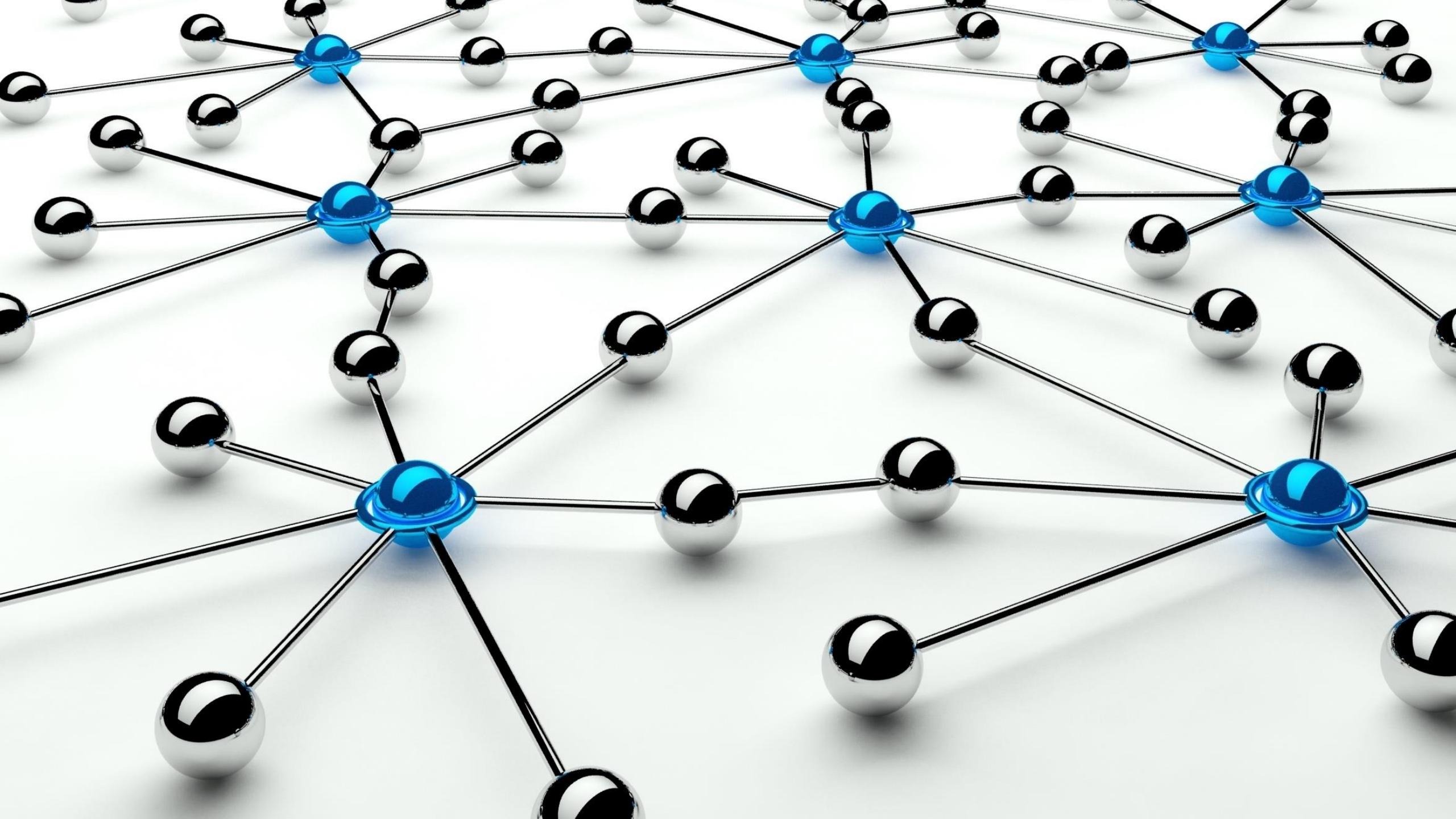
Christine Silberhorn

Integrated Quantum Optics

Paderborn University

ICTP Winter College on Optics:
Quantum Photonics and Information

13/02/2020



SCALABILITY

COST

SIMPLICITY

RESOURCES

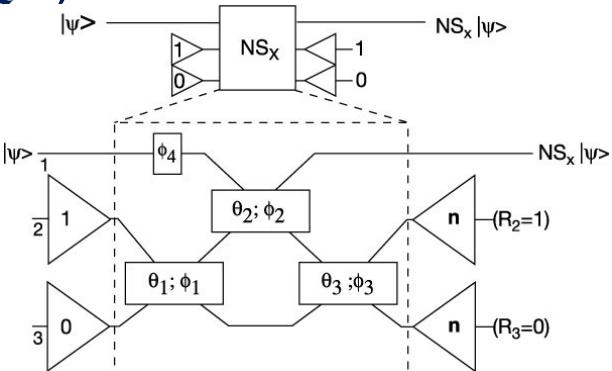
COMPATIBILITY

ALPHABET

RECONFIGURABILITY

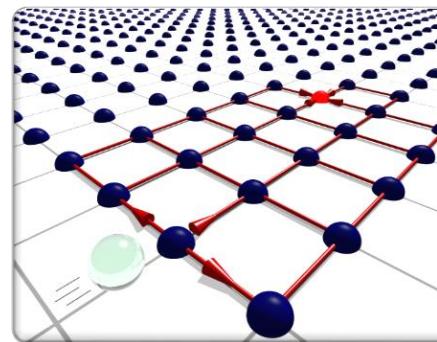
Goal: to build multi-dimensional systems

► Optical quantum computation (LOQC)



Knill, Laflamme, Milburn, Nature (2001)

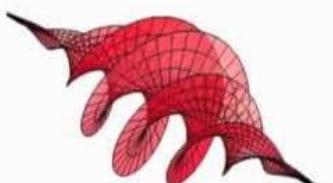
► Quantum walks and quantum simulation



Schreiber et al., Science (2012)

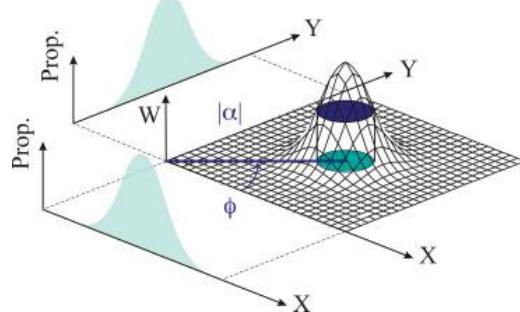
► High-dimensional information coding for quantum communication

Orbital angular momentum states

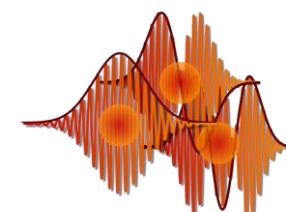


Padgett, Barnett, Boyd et al., Science (2010)

Continuous variable / Photon number states



Temporal modes of pulsed quantum states



Brecht et al., Phys. Rev. X 5, 041017 (2015)

few-D coin,

many positions,
few photons

HD alphabet,

one channel,
few photons



Outline

Optical
modes

Parametric
down-conversion

Quantum
pulse gate

Applications



Optical modes

Definition:

Modes are eigenfunctions of the wave equations, including boundary conditions, e.g. resonators or waveguides.

Properties:

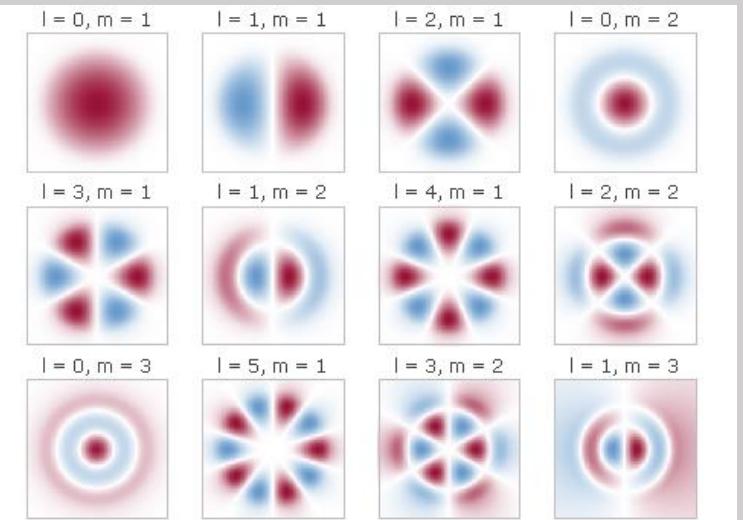
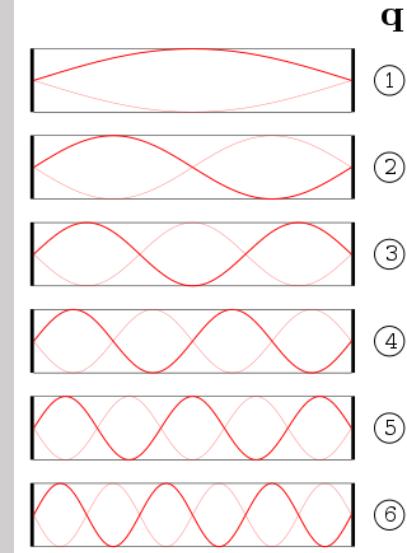
- ▶ they are orthogonal; dissimilar modes do not interfere
- ▶ within the same mode light is coherent and does interfere

Temporal modes: in direction of propagation (time, frequency)

Spatial modes: transvers to direction of propagation (x , and \vec{k})

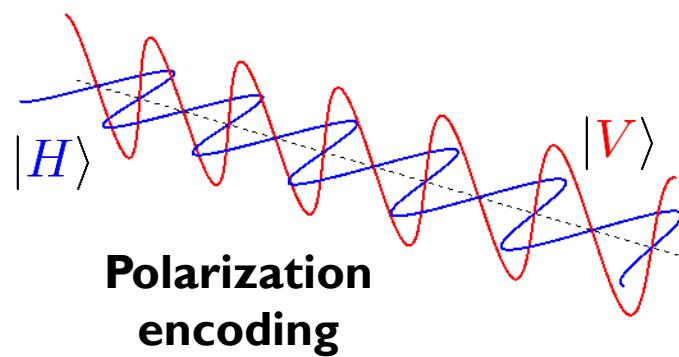
Example: cavity modes

longitudinal modes



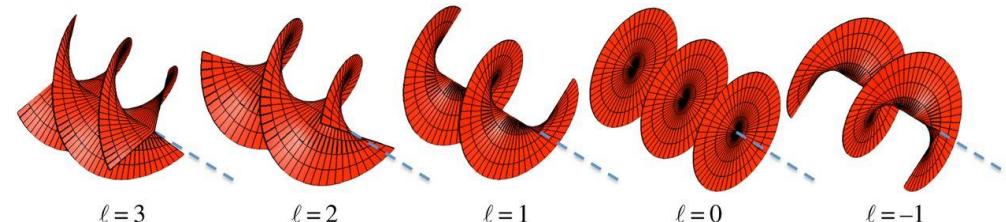
transverse modes

High-dimensional information encoding



$|0\rangle$ or $|1\rangle$ or $|2\rangle$ or $|3\rangle$ or $|4\rangle$ or $|5\rangle$ or $|6\rangle$ or $|7\rangle$ or $|8\rangle$

Spatial encoding

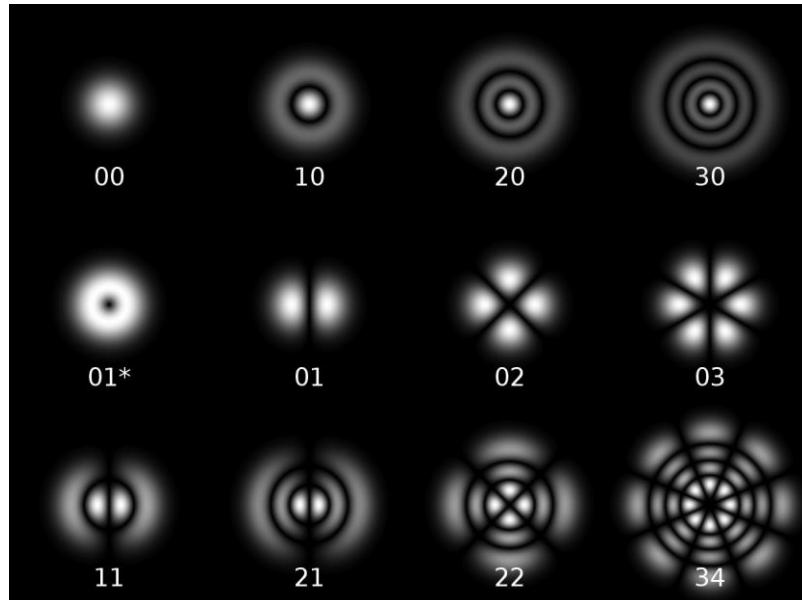


Leach et al., Science 329, 662-665 (2010)
Bent, et al, Phys. Rev. X 5, 041006 (2015)
Naidoo, et al, Nat. Photon. 10, 327 (2016)

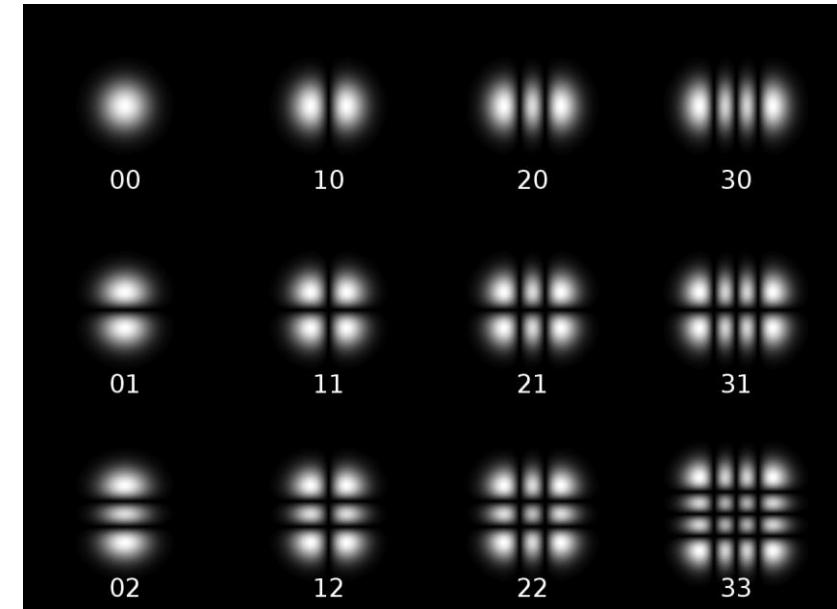
- ▶ More information encoded per photon
- ▶ Enhanced resilience to loss and noise
- ▶ High dimensional entanglement

Transverse spatial modes

► Laguerre-Gauss modes



► Hermite-Gaussian modes



Spatial modes are orthogonal

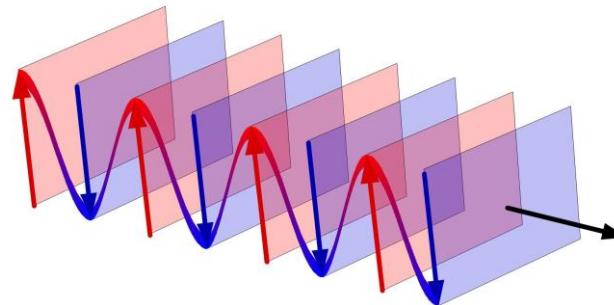
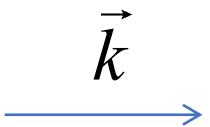
$$\int_{-\infty}^{\infty} u_n^*(x, z) \tilde{u}_m(x, z) dx = \delta_{nm}$$

$$\tilde{E}(x, y, z) = \sum_n \sum_m c_{nm} \tilde{u}_n(x, z) \tilde{u}_m(y, z) e^{-j k z}$$

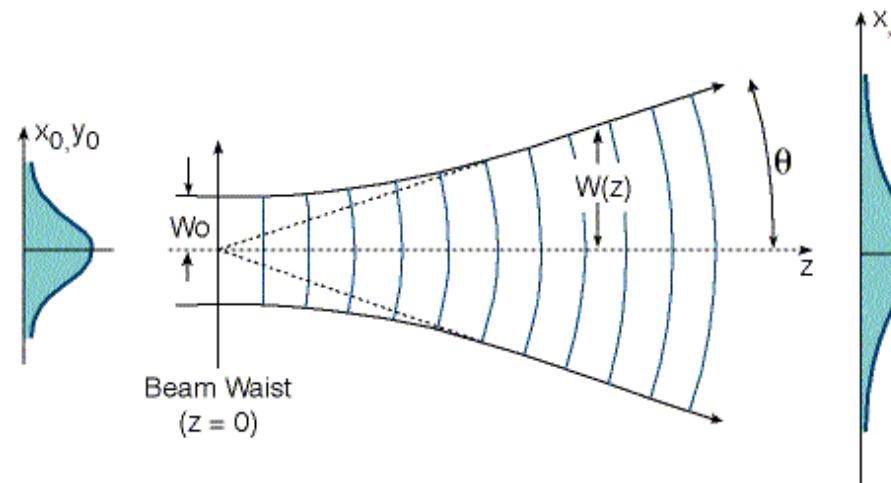
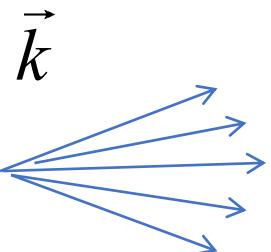


Gaussian beam

► Plane wave: single k-vector

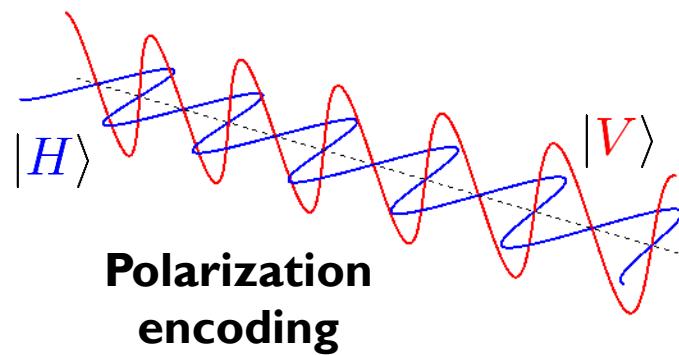


► Gaussian beam: superpositions of wave vectors



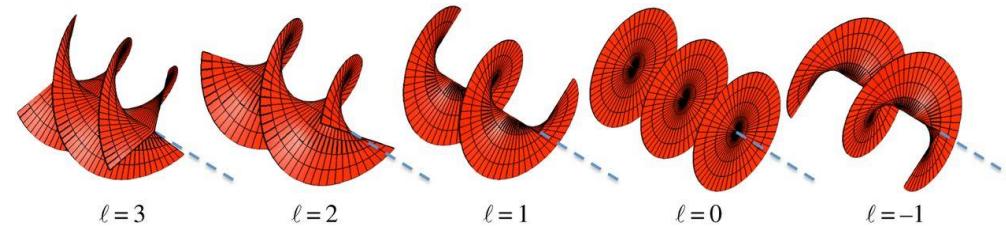
$$E_0 \propto \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \int_{-\infty}^{\infty} \frac{dk_y}{2\pi} \exp \left(ik_x x + ik_y y + ikz - i \frac{k_x^2 + k_y^2}{2k} z - \frac{w_0^2}{4} (k_x^2 + k_y^2) \right).$$

High-dimensional information encoding



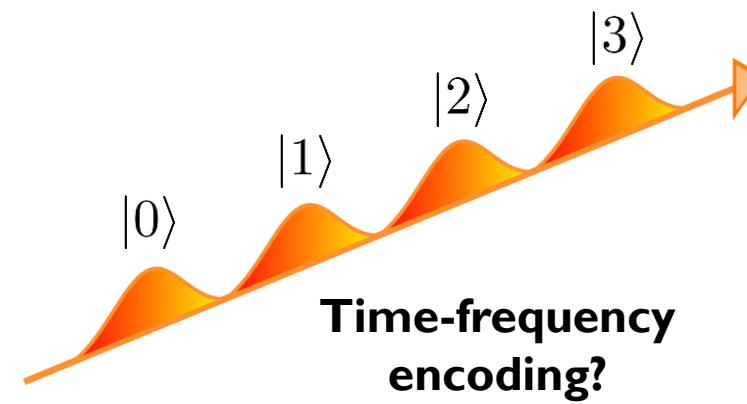
$|0\rangle$ or $|1\rangle$ or $|2\rangle$ or $|3\rangle$ or $|4\rangle$ or $|5\rangle$ or $|6\rangle$ or $|7\rangle$ or $|8\rangle$

Spatial encoding

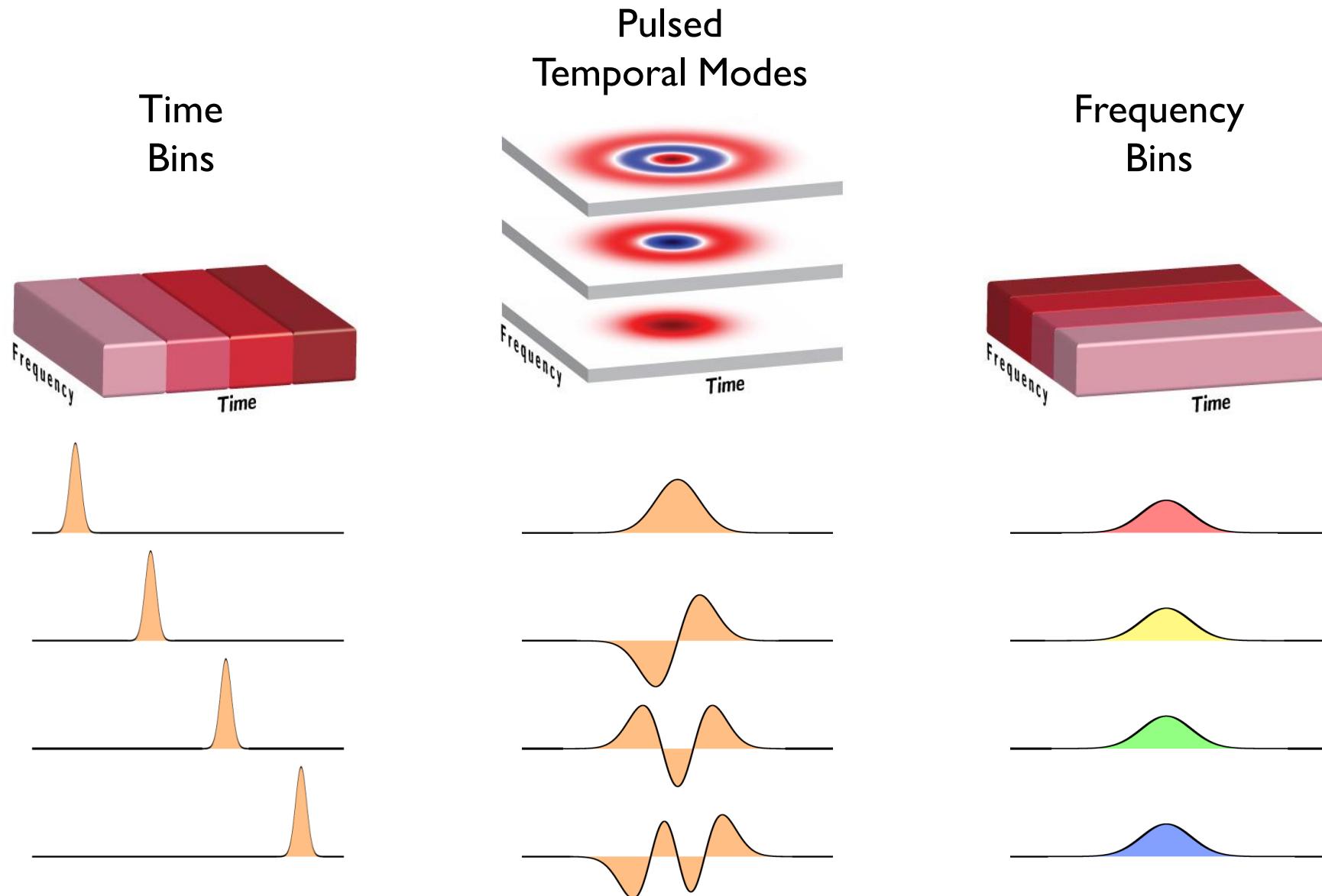


Leach et al., Science 329, 662-665 (2010)
Bent, et al, Phys. Rev. X 5, 041006 (2015)
Naidoo, et al, Nat. Photon. 10, 327 (2016)

- More information encoded per photon
- Enhanced resilience to loss and noise
- High dimensional entanglement

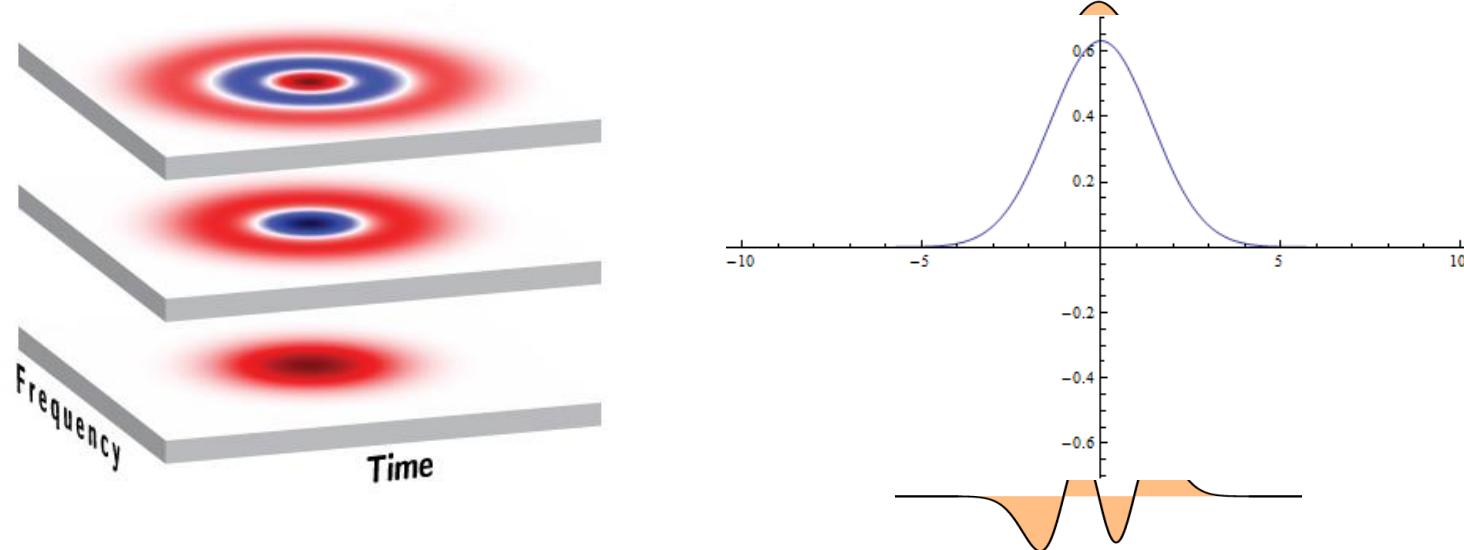


Time-frequency modes

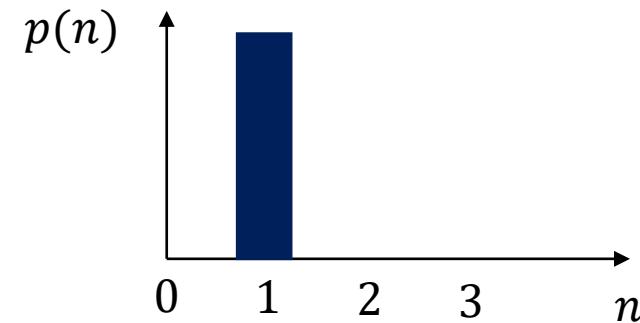
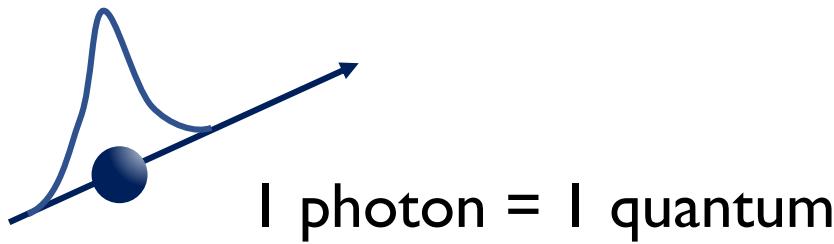


Pulsed temporal modes

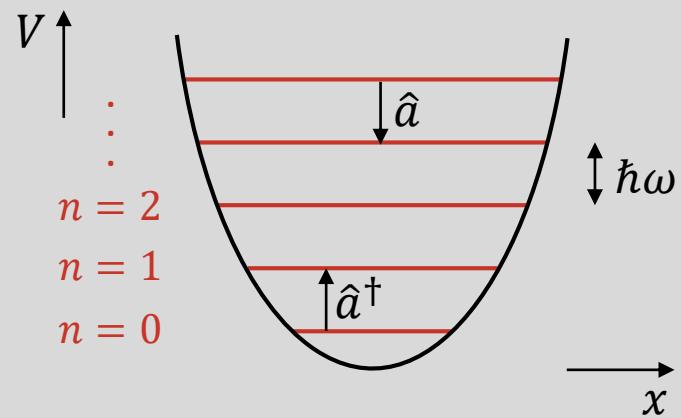
- ▶ Hermite-Gaussian envelopes in frequency and time
- ▶ Overlapping intensities but orthogonal field amplitudes
- ▶ Naturally compatible with waveguides and fibers
- ▶ Pulse and spectral width scale as $\sqrt{2n + 1}$



What is a single photon?



Quantisation of an optical field mode



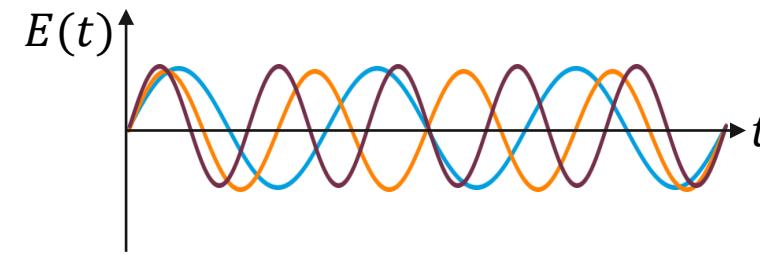
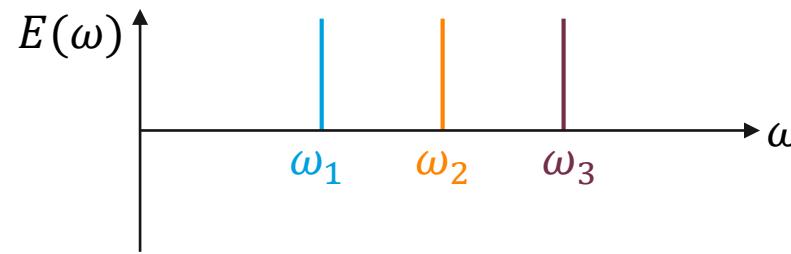
plane wave = harmonic oscillator

$$\hat{E} = \hat{a}e^{-i(\omega t - \vec{k}\vec{r})} + \hat{a}^\dagger e^{i(\omega t - \vec{k}\vec{r})}$$

$$|\psi\rangle = \sum_{n=0}^{\infty} c_n \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}} |0\rangle$$

Pulsed temporal mode states of single photons

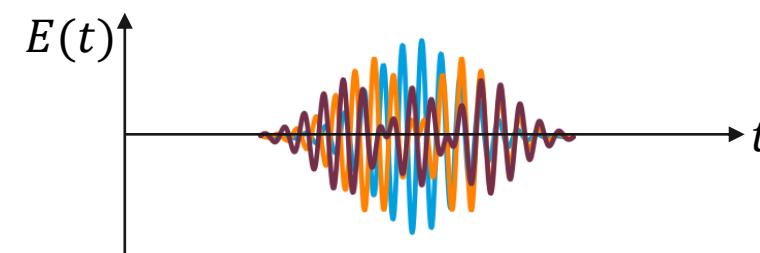
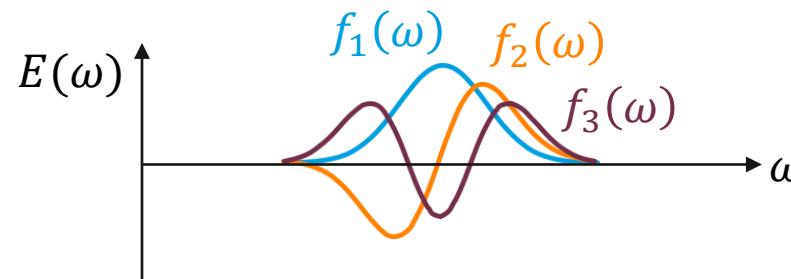
Monochromatic modes



$$[\hat{a}(\omega_i), \hat{a}^\dagger(\omega_j)] = \delta(\omega_i - \omega_j)$$

State: $|\omega_j\rangle = \hat{a}^\dagger(\omega_j)|0\rangle$

Temporal modes



$$\hat{A}_j = \int d\omega f_j(\omega) \hat{a}(\omega)$$
$$[\hat{A}_i, \hat{A}_j^\dagger] = \delta_{ij}$$

State: $|A_j\rangle = \hat{A}^\dagger|0\rangle$

Outline

Optical
modes

Parametric
down-conversion

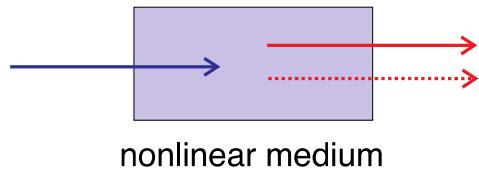
Quantum
pulse gate

Applications



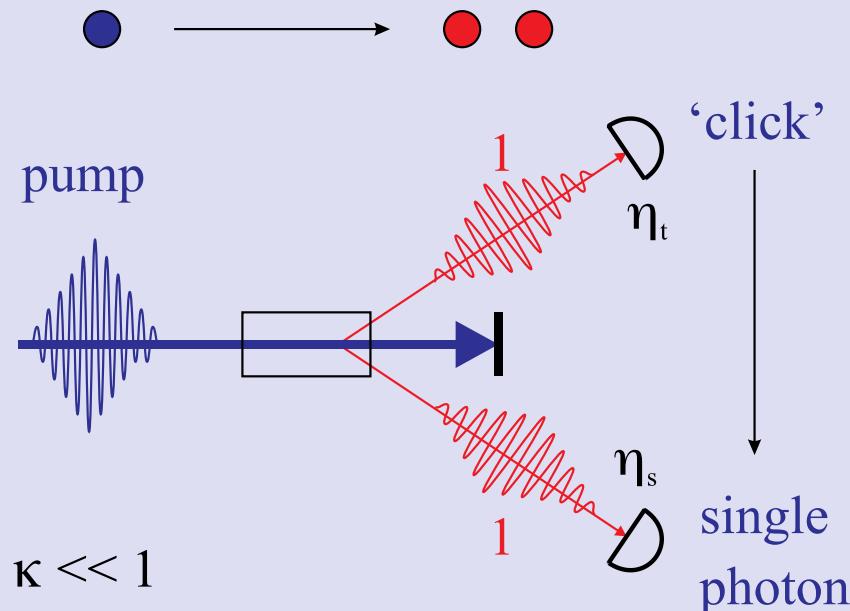
Parametric down-conversion

How can we generate single photons?



$$P = \varepsilon_0 \left[\chi^{(1)} \vec{E} + \chi^{(2)} \vec{E} \vec{E} + \chi^{(3)} \vec{E} \vec{E} \vec{E} + \dots \right]$$

Parametric downconversion



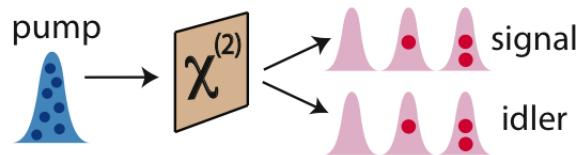
Photons are produced
in pairs

$$R_s = \eta_s \cdot R, \quad R_i = \eta_i \cdot R$$
$$R_{coinc} = \eta_s \eta_i \cdot R$$

$$\eta_s = \frac{R_{coinc}}{R_i}$$

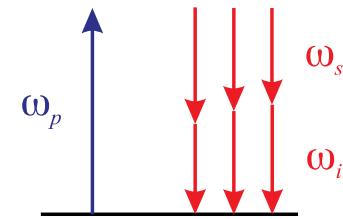


Parametric down-conversion



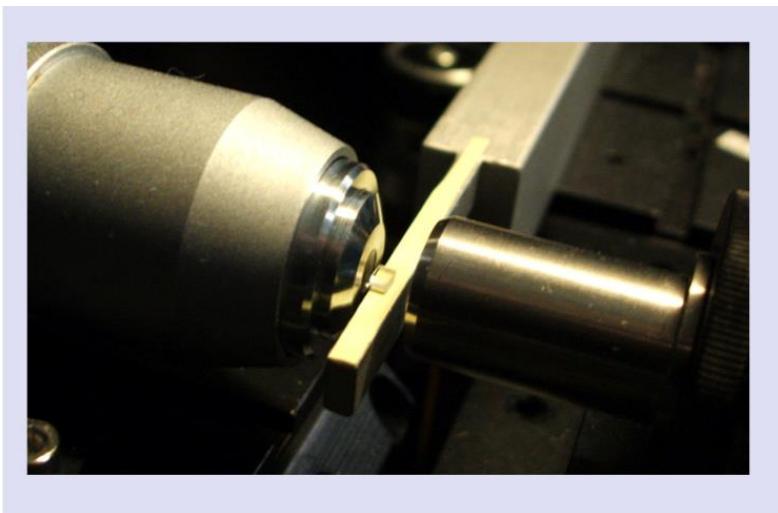
energy conservation

$$W_p = W_s + W_i$$



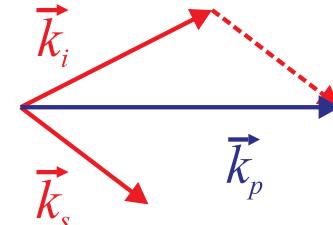
$$\hat{H} = \alpha a^\dagger b^\dagger + h.c.$$
$$|\psi\rangle = \sum \lambda_n |n, n\rangle$$

PDC in waveguides



momentum conservation

$$\vec{k}_p = \vec{k}_s + \vec{k}_i$$



Spatial k-vectors
defined by guide

Parametric down-conversion

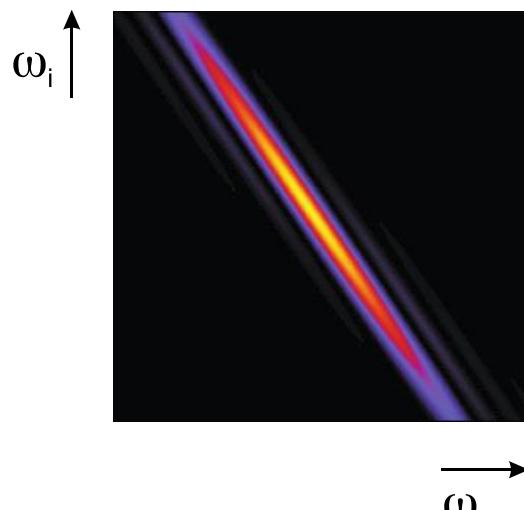
“Ideal” bi-photon state ...

$$|1\rangle |1\rangle = \hat{A}_s^\dagger \hat{B}_i^\dagger |0\rangle = \iint d\omega_s d\omega_i f_A(\omega_s) \cdot g_B(\omega_i) \hat{a}_{\omega_s}^\dagger \hat{a}_{\omega_i}^\dagger |0\rangle$$

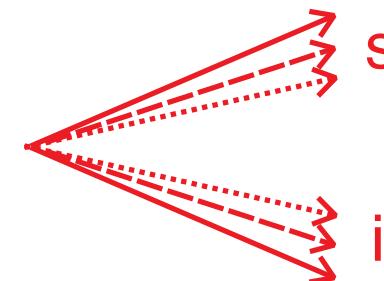
Functions $f_A(\omega_s)$ and $g_B(\omega_i)$ define **one** temporal mode each!

... but PDC is typically **correlated** in:

optical frequencies



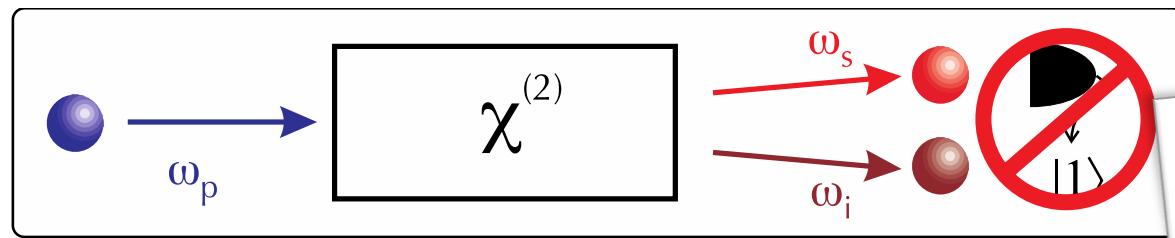
transverse momenta



No correlations
for PDC in wave guides

Heralding single photons

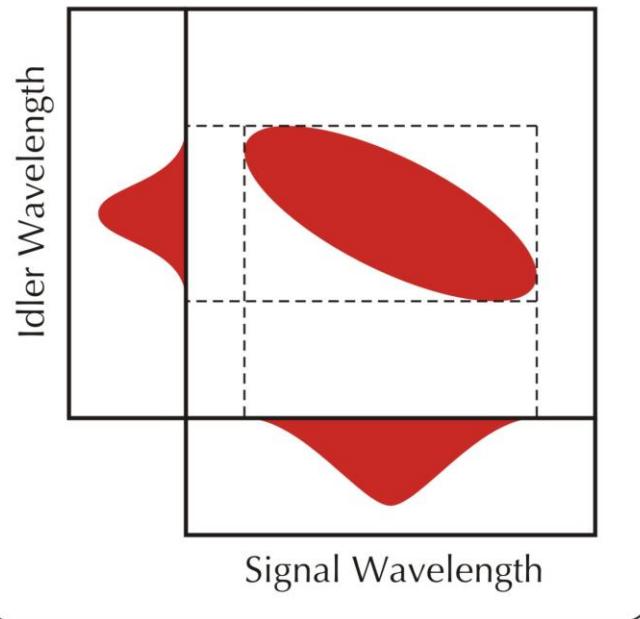
Correlated Bi-photon state



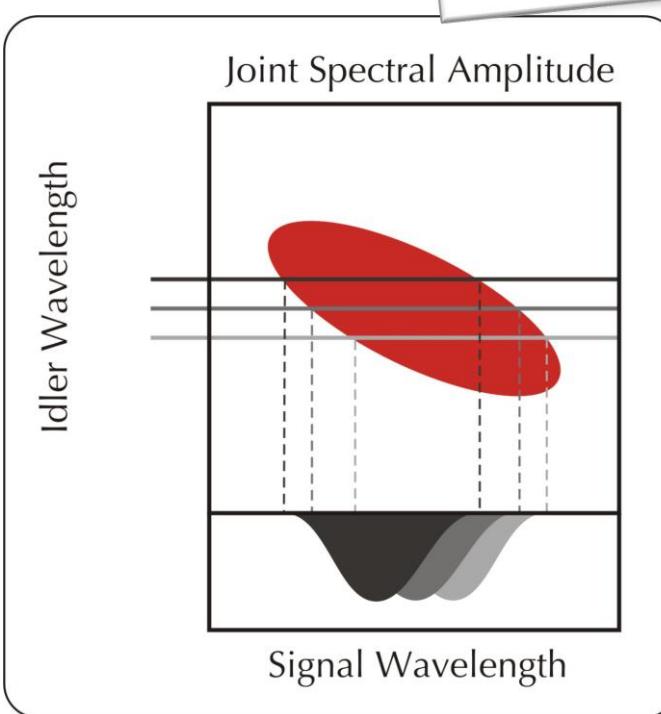
• mixed state:

$$\hat{\rho} = \int d\omega p(\omega) \hat{a}_\omega^\dagger |0\rangle \langle 0| \hat{a}_\omega$$

Joint Spectral Amplitude

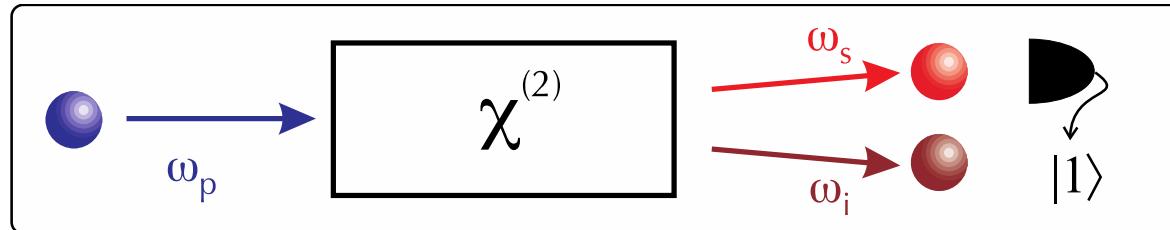


Joint Spectral Amplitude

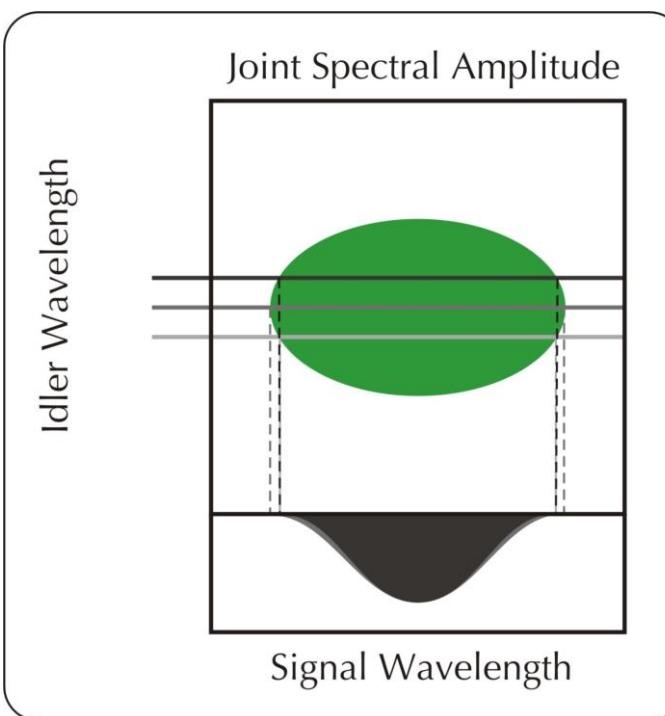
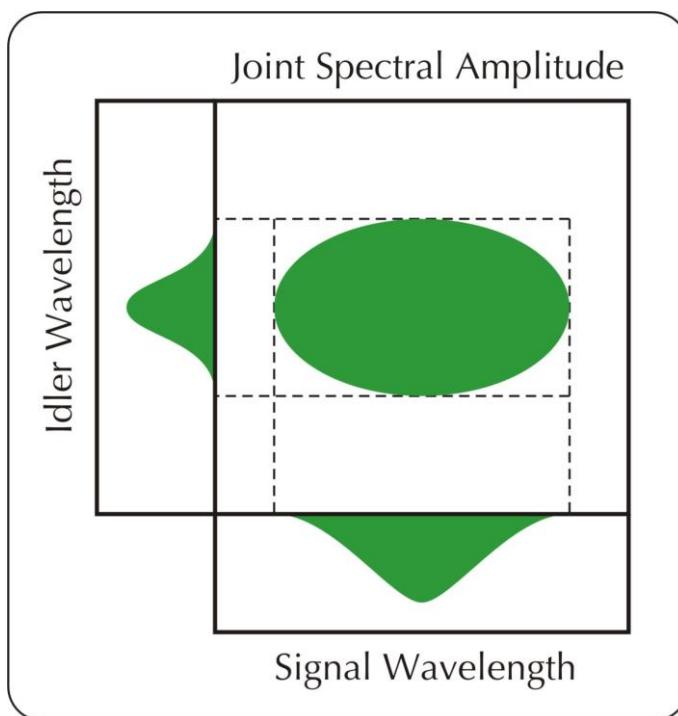
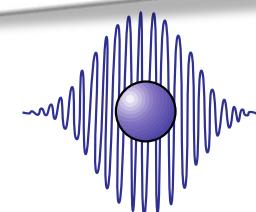


Heralding single photons

Uncorrelated Bi-photon state

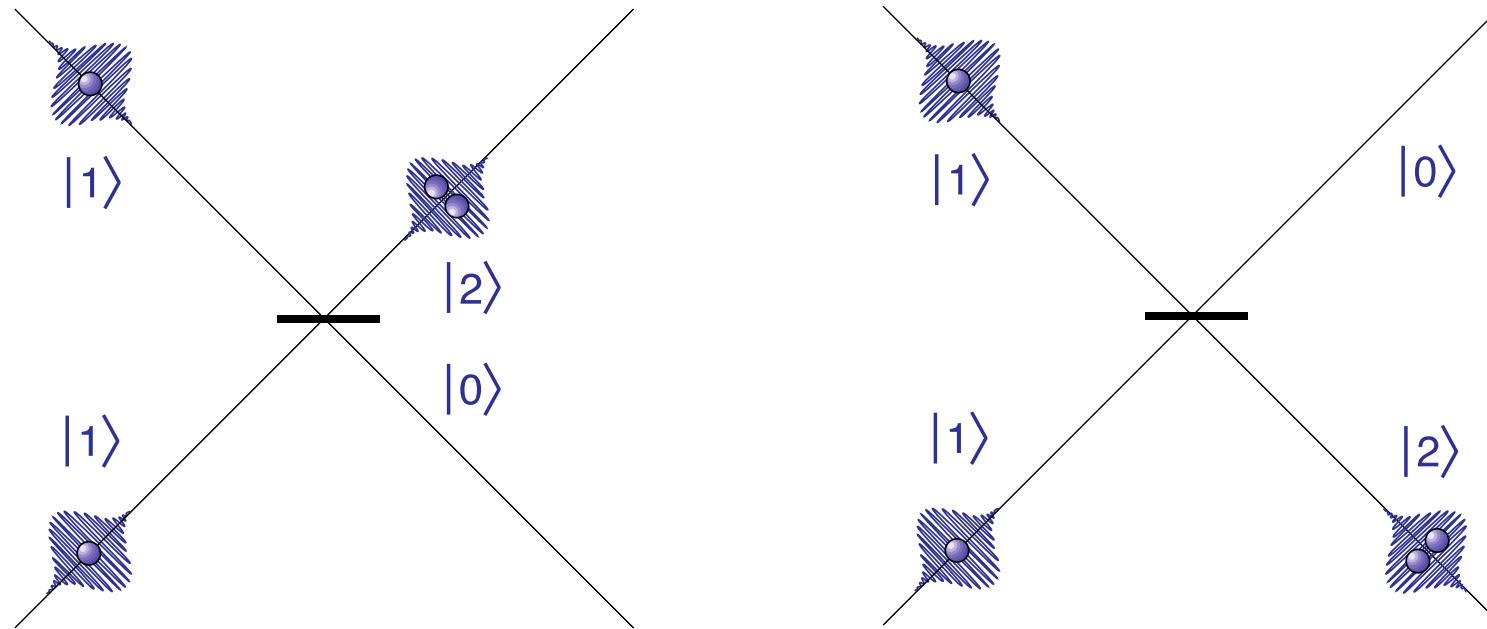
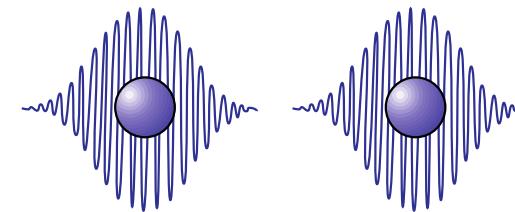


• pure state:
 $|1\rangle = \int d\omega f(\omega) \hat{a}_\omega^\dagger |0\rangle = \hat{A}^\dagger |0\rangle, \quad \hat{\rho} = |1\rangle \langle 1|$



HOM interference

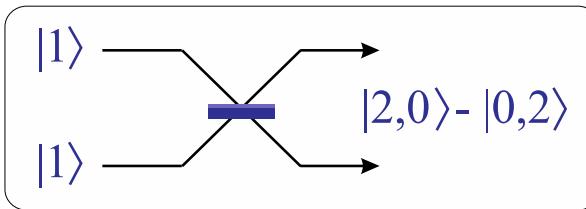
Pure single photons like to team up!



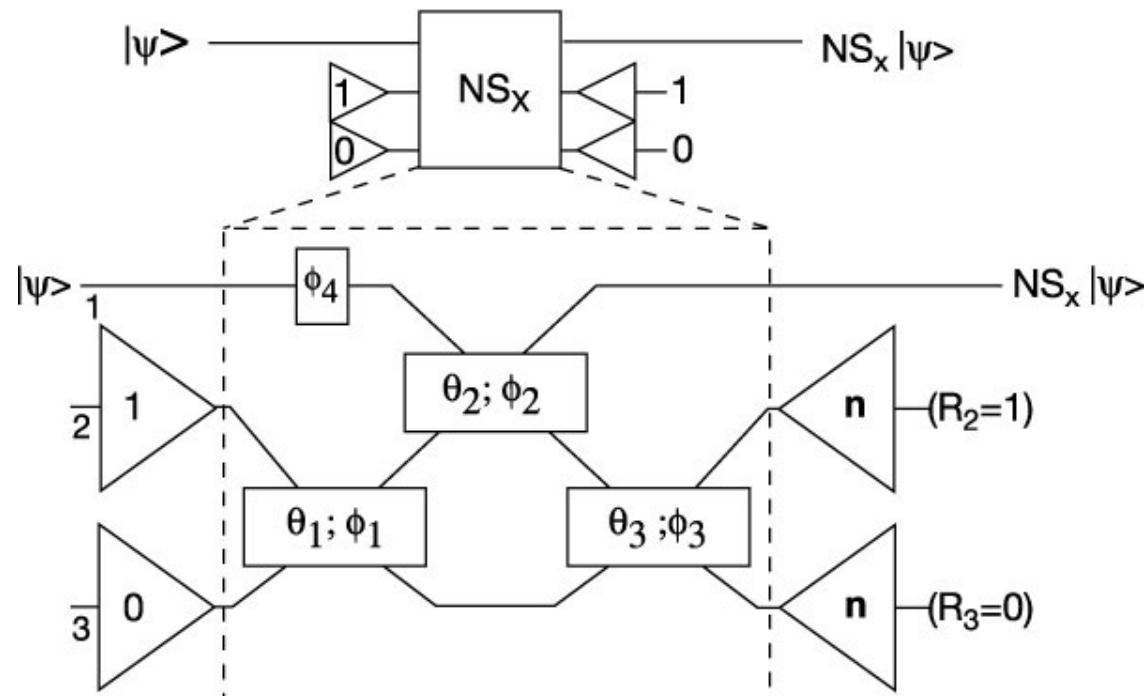
$$|1\rangle |1\rangle \rightarrow |0\rangle |2\rangle - |2\rangle |0\rangle$$

Quantum information processing

What is HOMI good for?



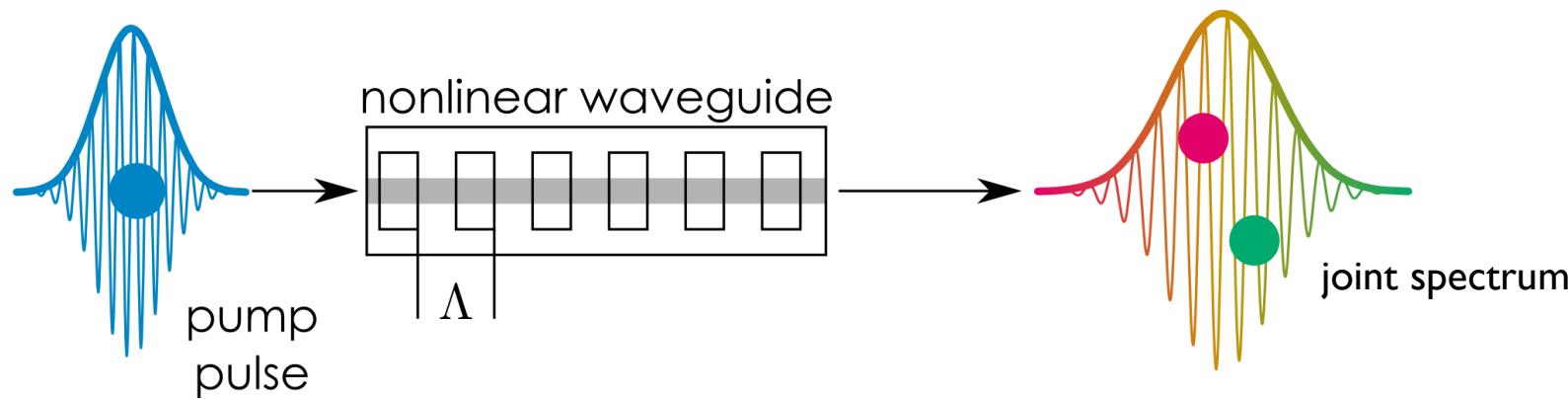
Linear optical quantum computation



Knill, Laflamme, Milburn, Nature **409**, 46, (2001)

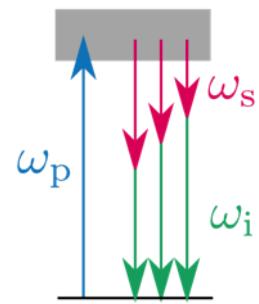


Guided-wave parametric down-conversion



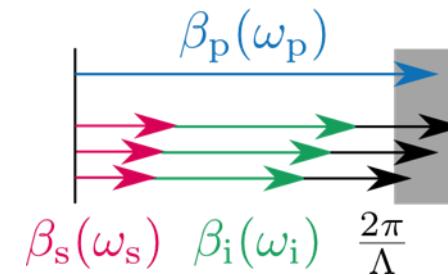
Energy conservation

$$\omega_p = \omega_s + \omega_i$$

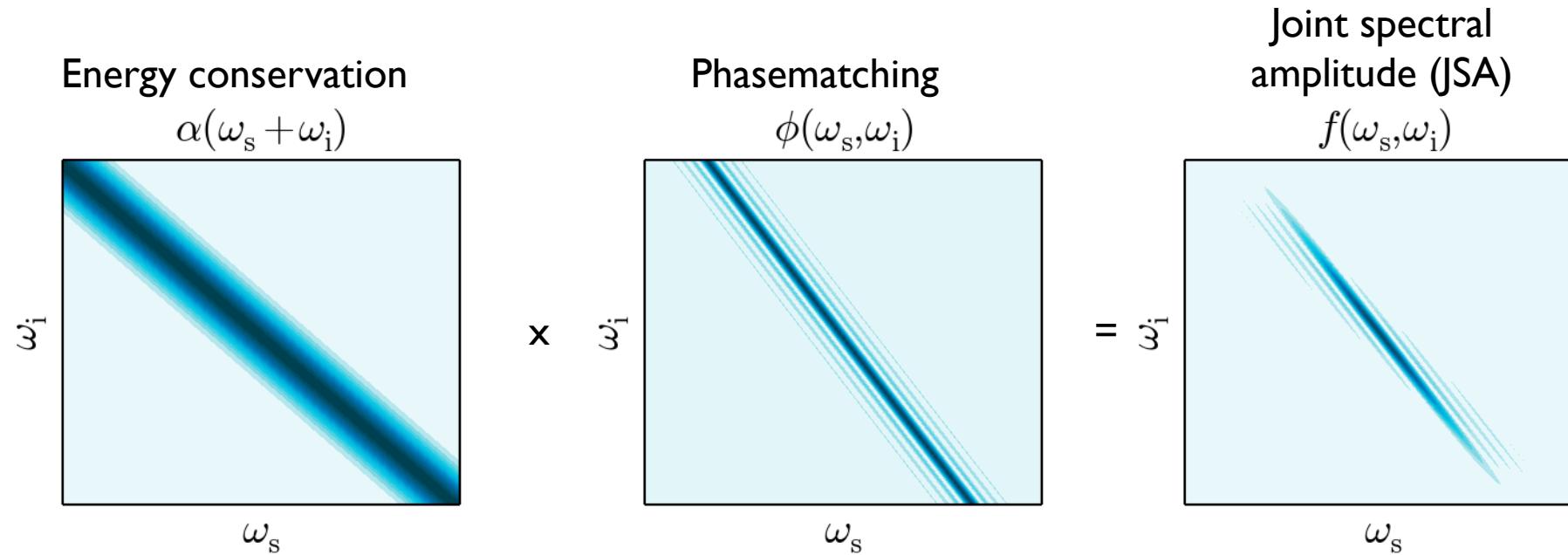


Phasematching

$$\beta_p = \beta_s + \beta_i + \frac{2\pi}{\Lambda}$$



Guided-wave parametric down-conversion



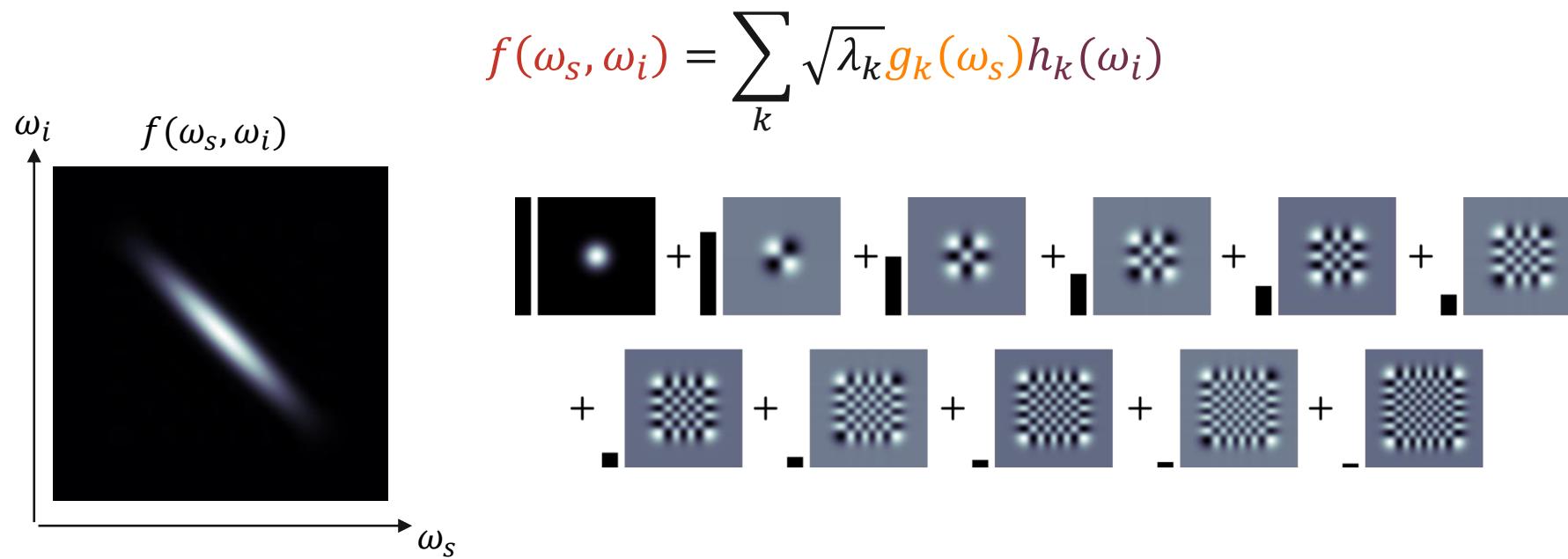
$$|\psi\rangle_{\text{PDC}} = \exp \left[\mathcal{B} \int d\omega_s d\omega_i f(\omega_s, \omega_i) \hat{a}^\dagger(\omega_s) \hat{b}^\dagger(\omega_i) + \text{h.c.} \right] |0\rangle$$

Bi-photon state (general)

$$|\Psi\rangle = |0\rangle|0\rangle + \iint d\omega_s d\omega_i f(\omega_s, \omega_i) \hat{a}_i^\dagger \hat{a}_s^\dagger |0\rangle \neq |0\rangle|0\rangle + \kappa|1\rangle|1\rangle$$

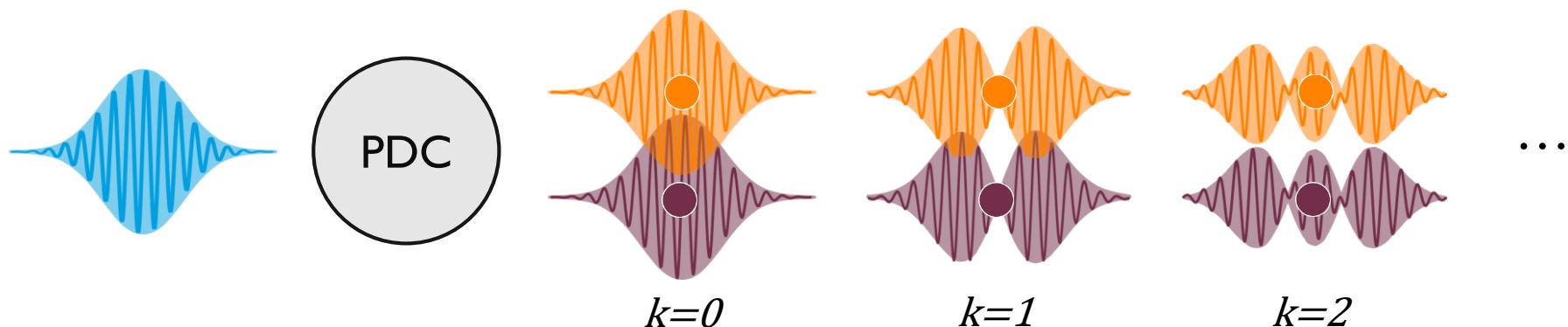


The Schmidt decomposition



$$|\psi\rangle_{\text{PDC}} = \bigotimes_k \exp \left[\theta \sqrt{\lambda_k} \hat{A}_k^\dagger \hat{B}_k^\dagger + h.c. \right] |0\rangle$$

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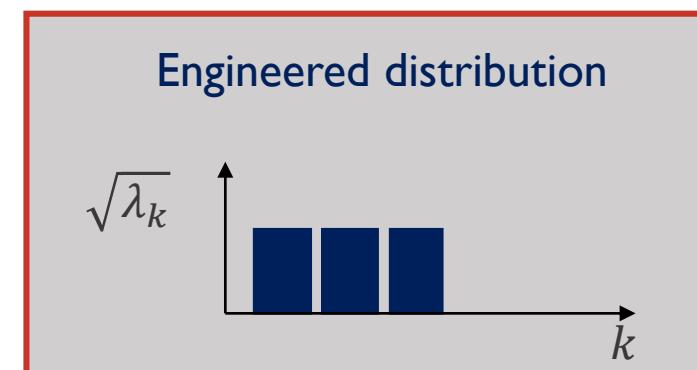
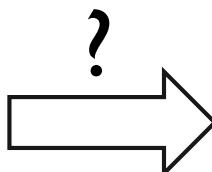
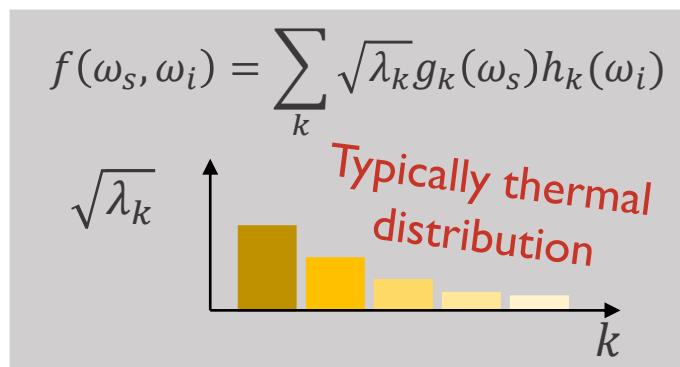
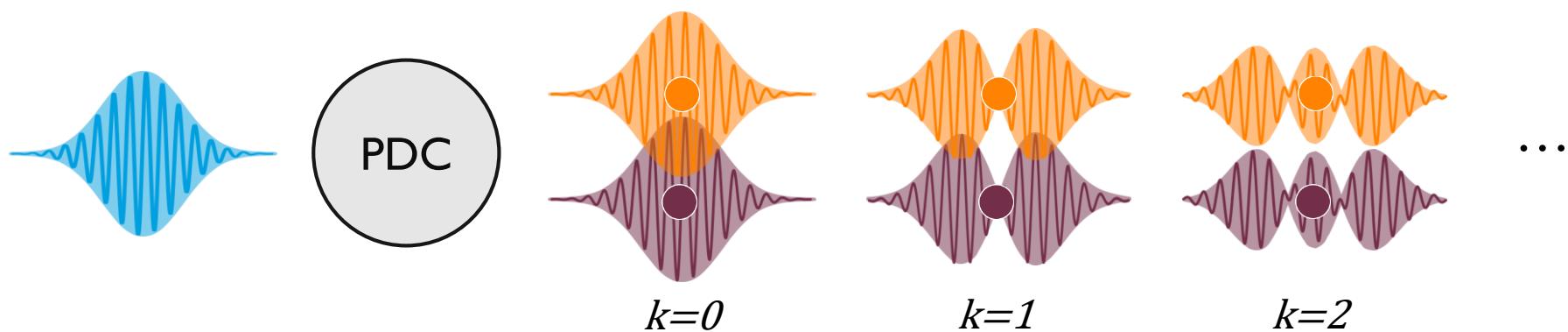
Multimode PDC state

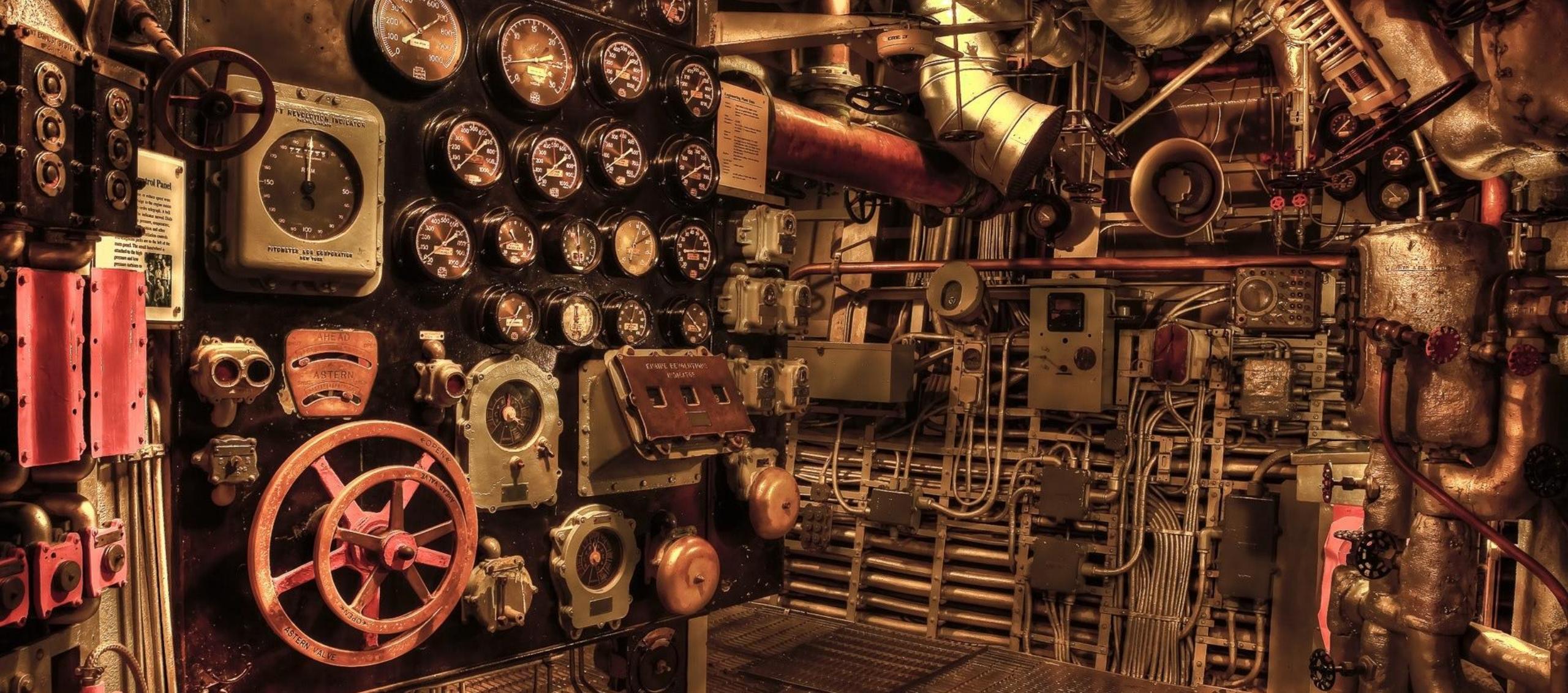
- The **effective number of modes / amount of spectral entanglement** is characterized by the Schmidt number

$$K = 1 / \left(\sum_k \lambda_k^2 \right)$$

- The temporal-mode properties are encoded in the JSA of the PDC

$$|\psi\rangle_{\text{PDC}} = \bigotimes_k \exp \left[\theta \sqrt{\lambda_k} \hat{A}_k^\dagger \hat{B}_k^\dagger + h.c. \right] |0\rangle$$

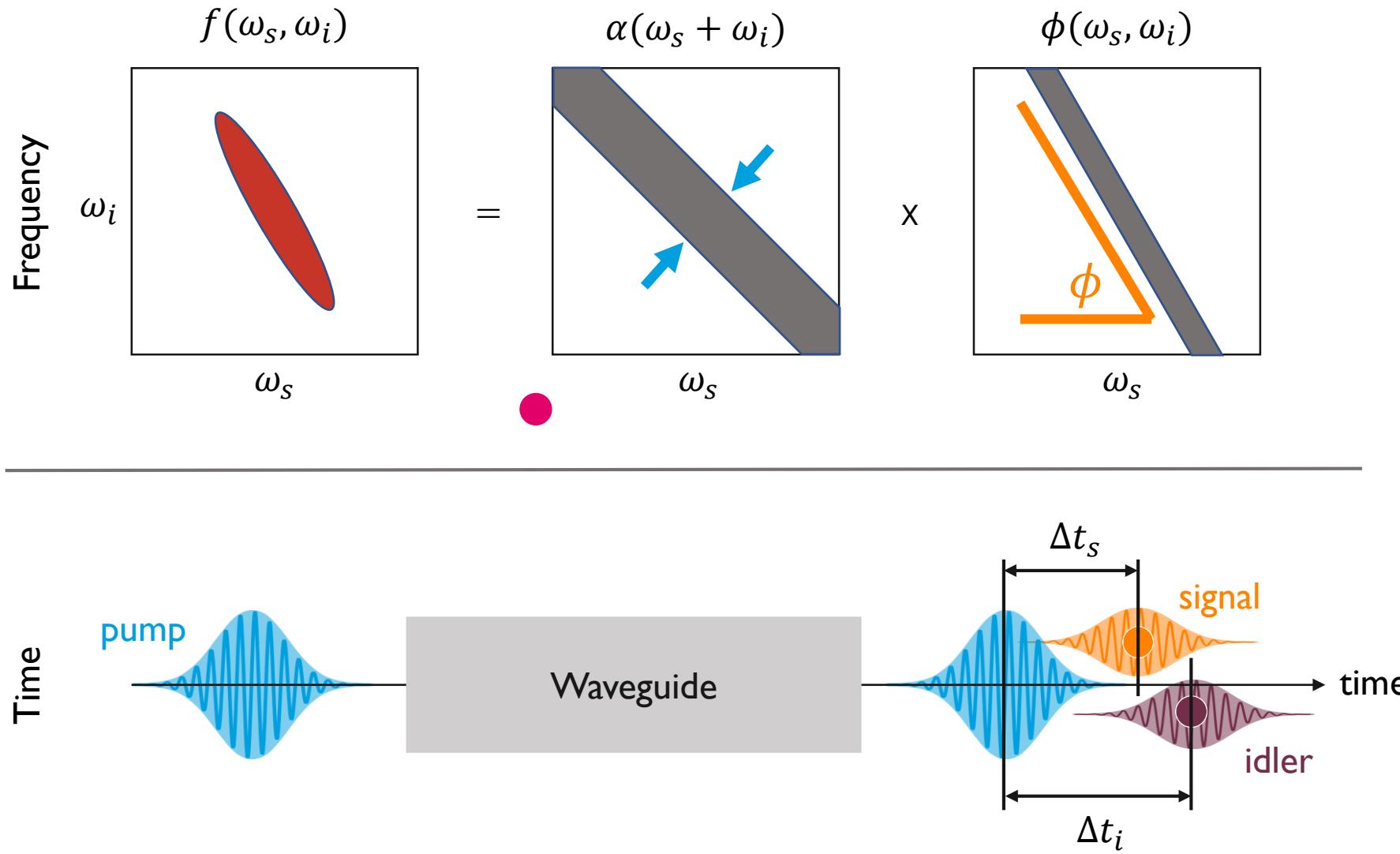




Source engineering – which knobs to turn?

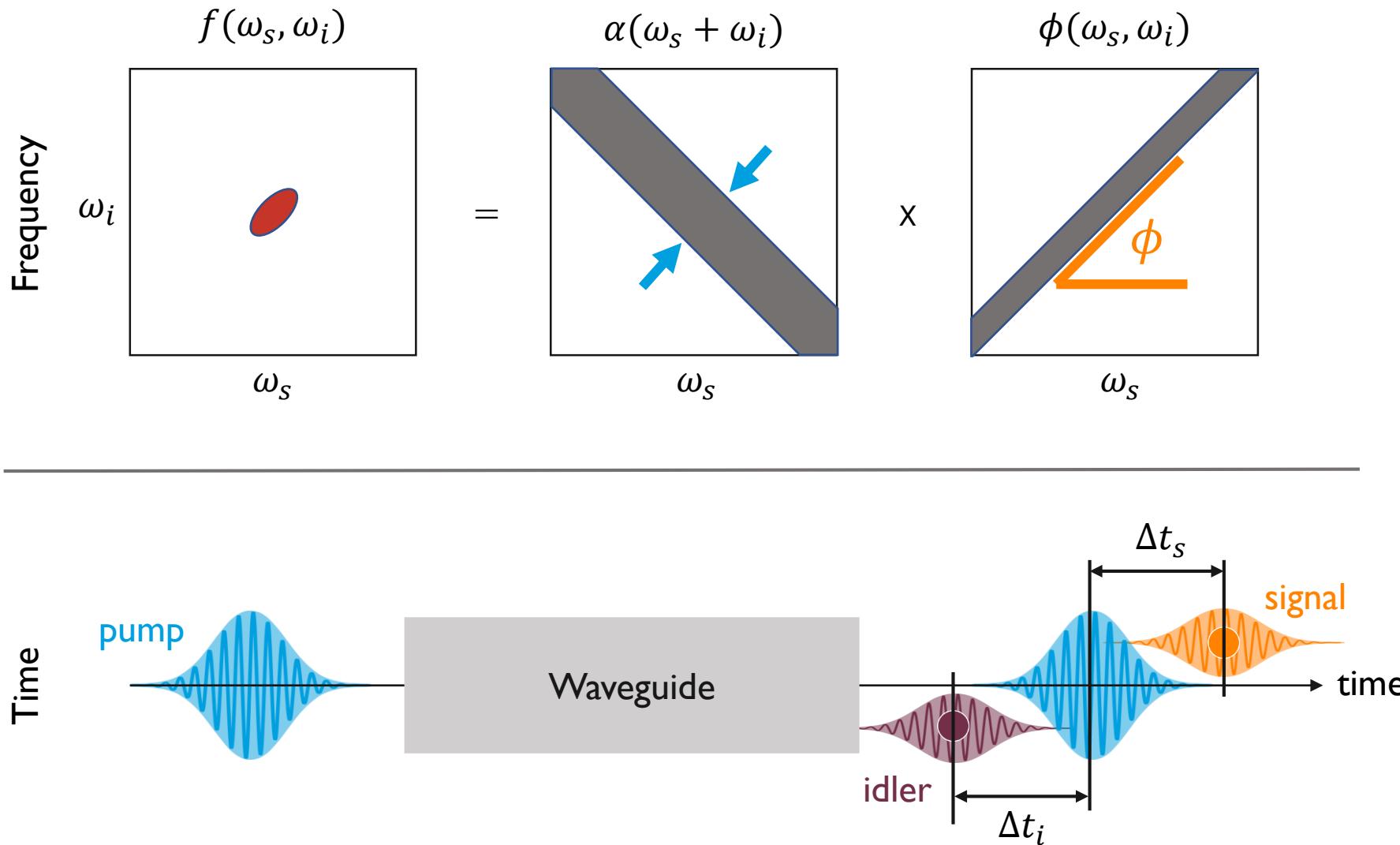


Group velocity matching



$$\tan \phi = -\frac{(\nu_s^{-1} - \nu_p^{-1})}{(\nu_i^{-1} - \nu_p^{-1})}$$

Group velocity matching



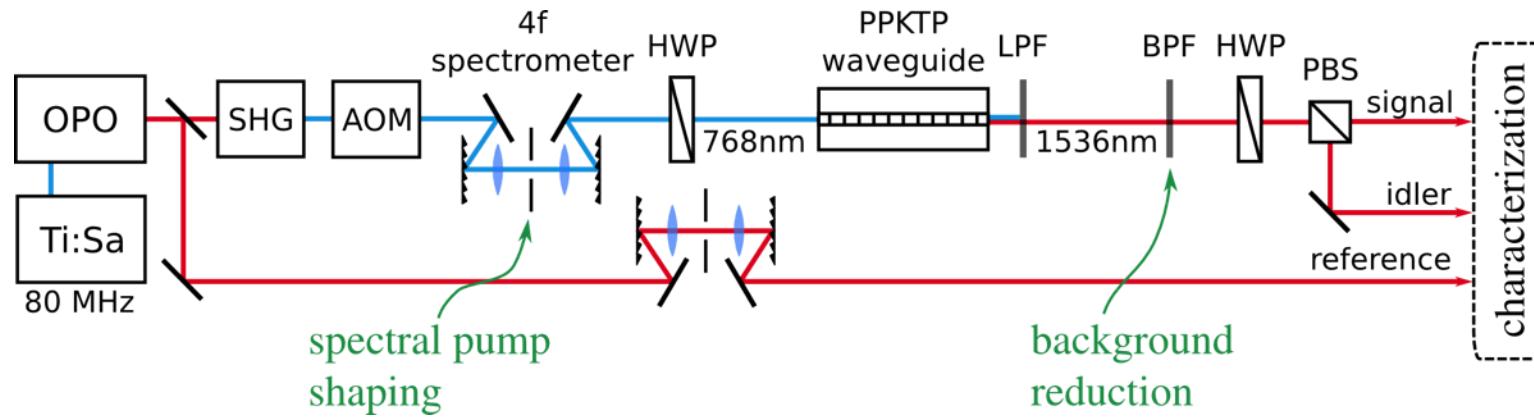
Pump spectrum

- Width
- Shape

Phasematching

- Angle
- (Pattern)

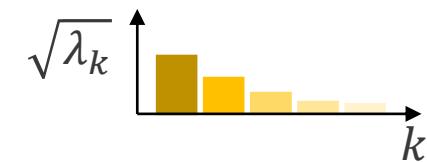
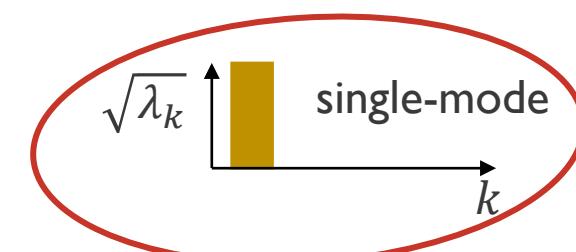
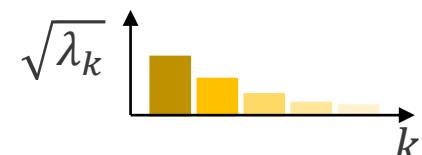
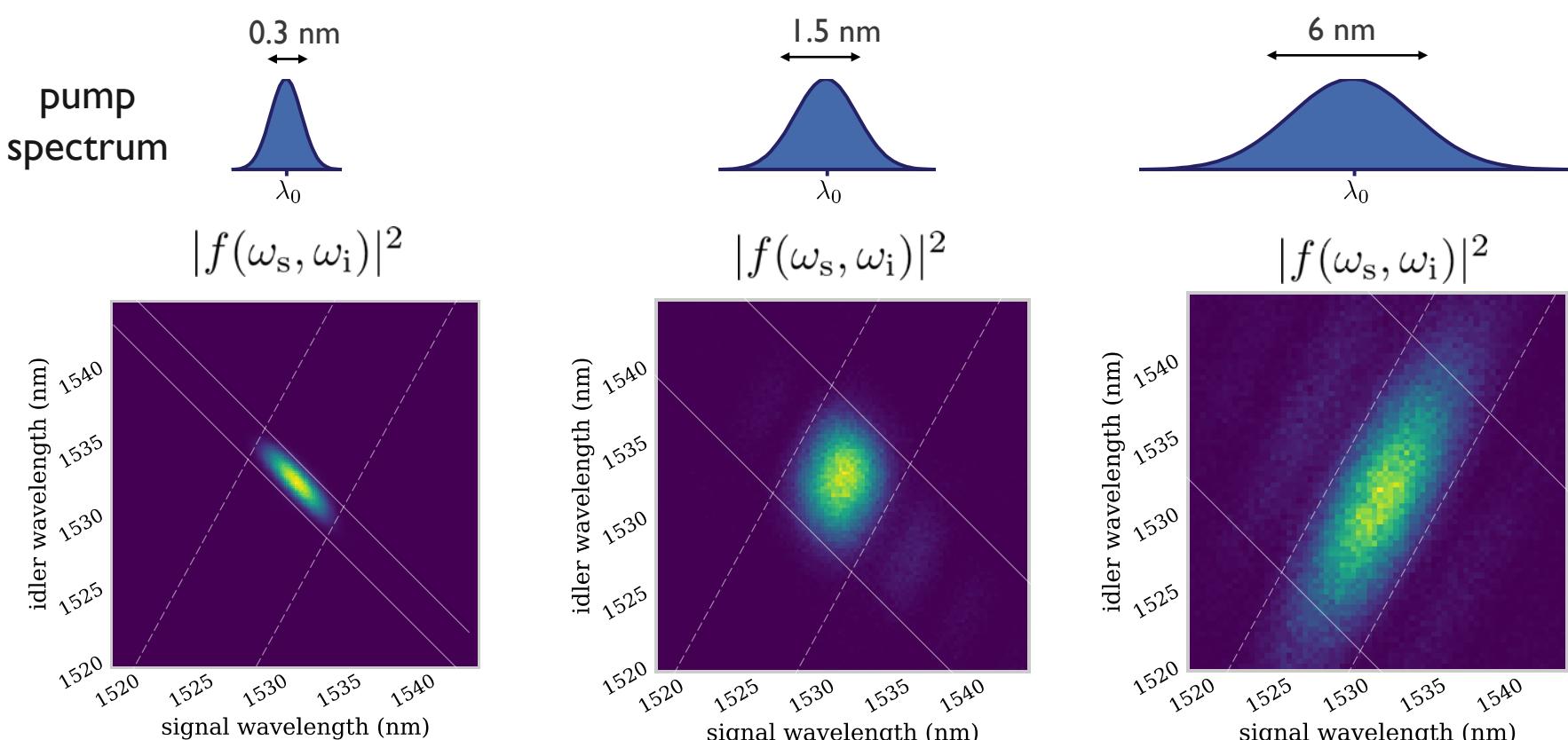
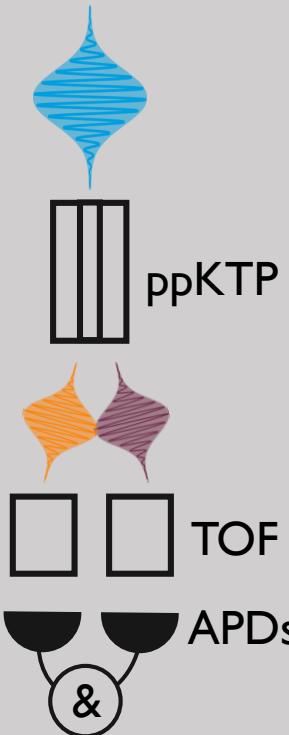
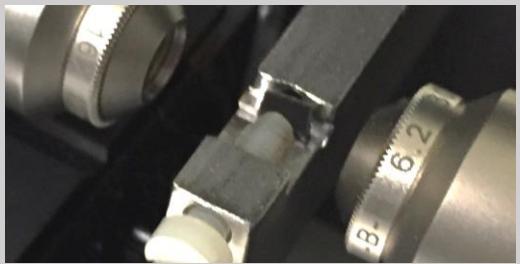
Experiment



- No narrowband spectral filtering
- Coupling into single-mode fibers of **up to 70% and 80%**, corresponding to Klyshko efficiencies of **15.5% and 20.5%**.

ppKTP waveguide

- group velocity matching at telecoms wavelengths
- high brightness
- 8mm length

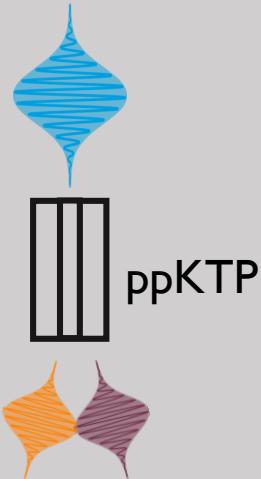
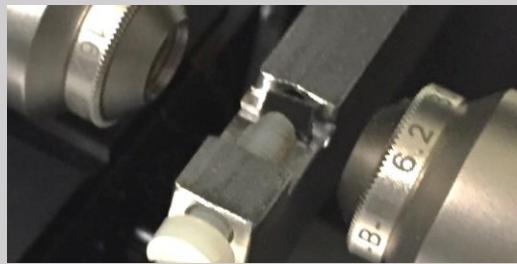


A. Eckstein et al, Phys. Rev. Lett. **106**, 013603 (2011)
 G. Harder et al, Opt. Express **21**, 13975 (2013)

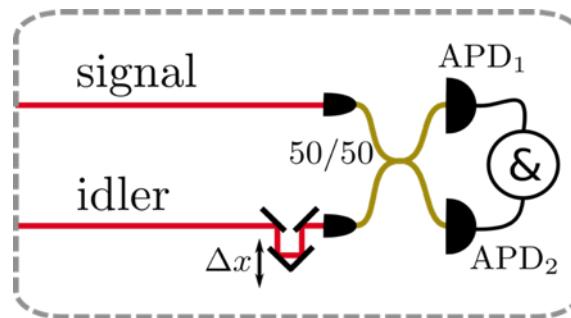


ppKTP waveguide

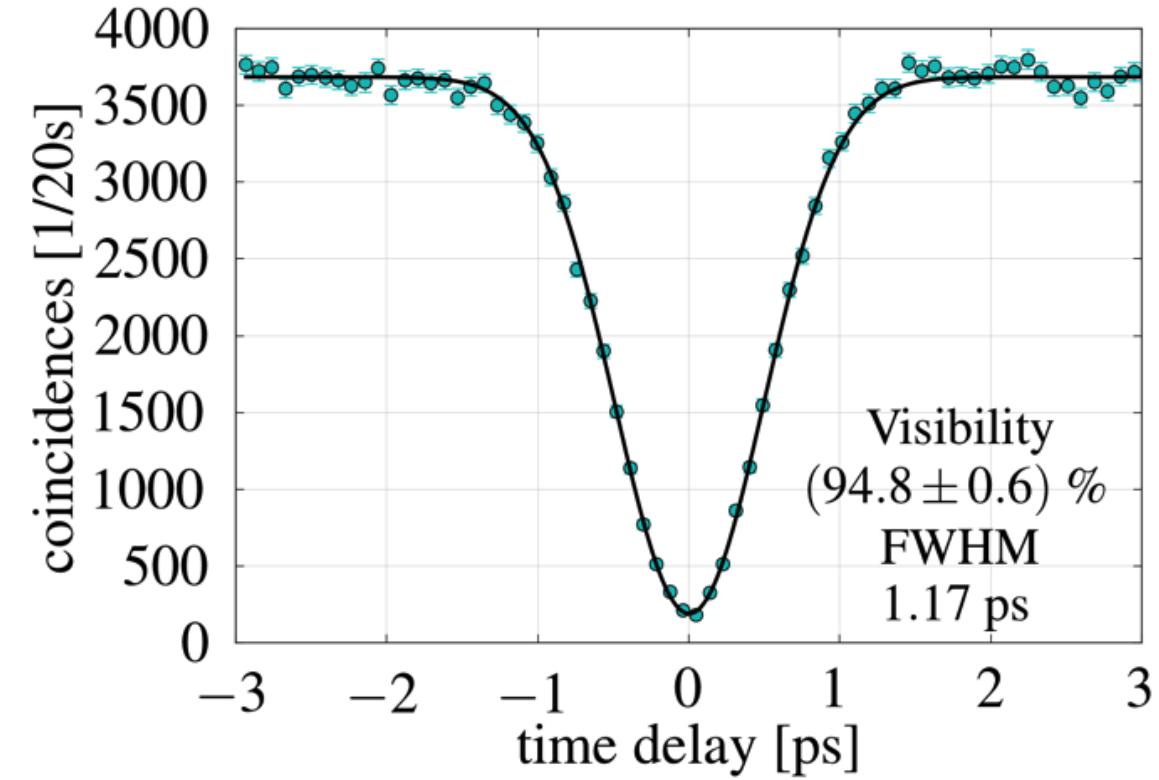
- group velocity matching at telecoms wavelengths
- high brightness
- 8mm length



signal-idler HOMI
→ indistinguishability



For decorrelated joint spectral amplitudes

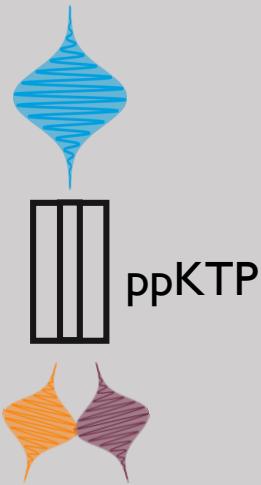
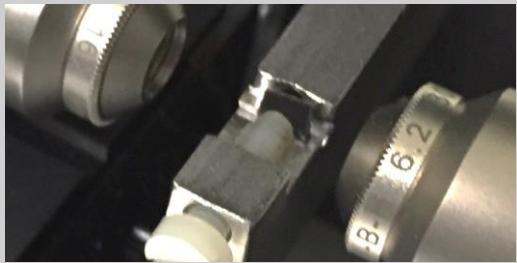


Visibility_{dip} ≈ 95%

⇒ **High indistinguishability**

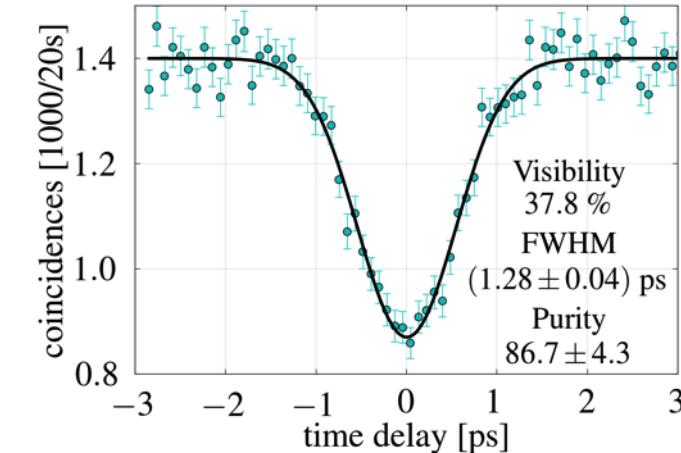
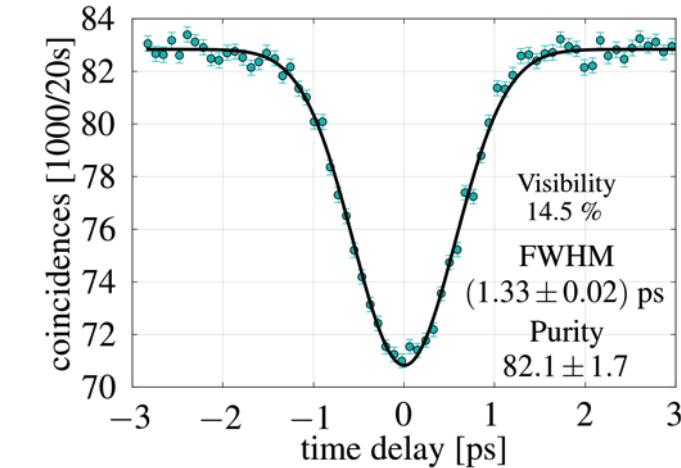
ppKTP waveguide

- group velocity matching at telecoms wavelengths
- high brightness
- 8mm length



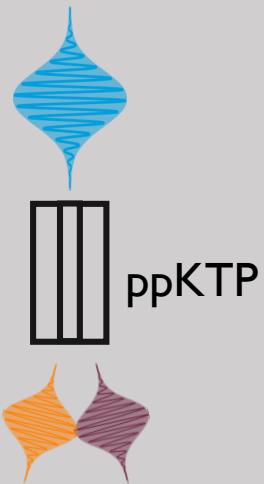
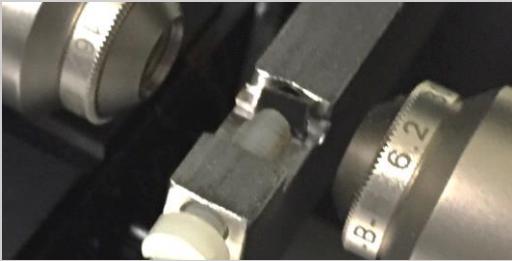
signal-reference HOMI
→ spectral purity

For decorrelated joint spectral amplitude



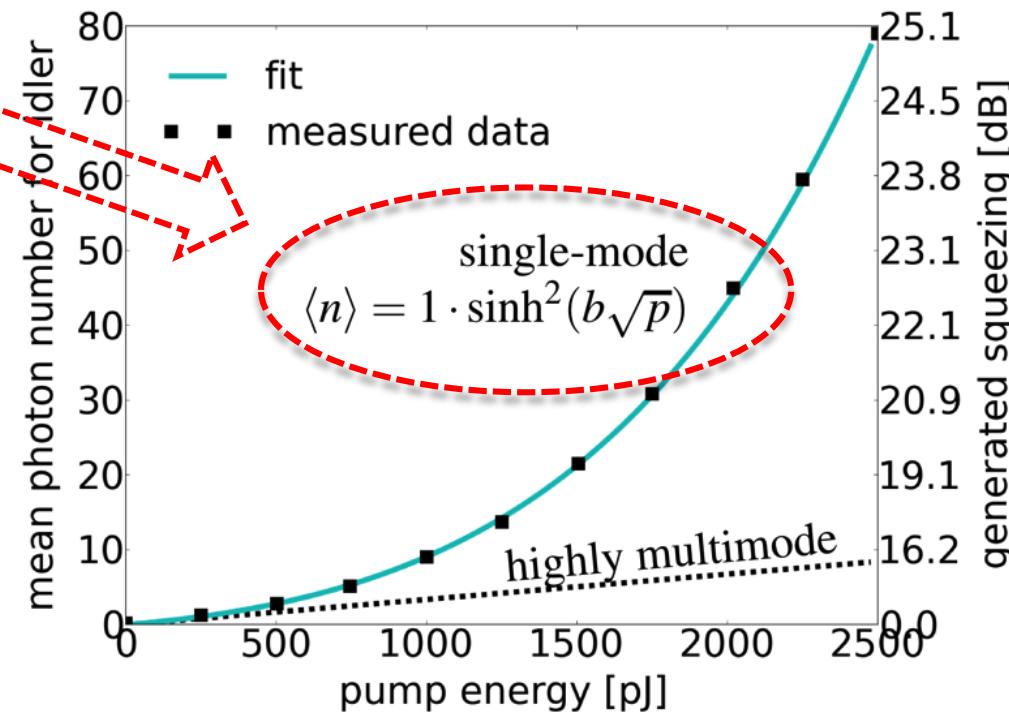
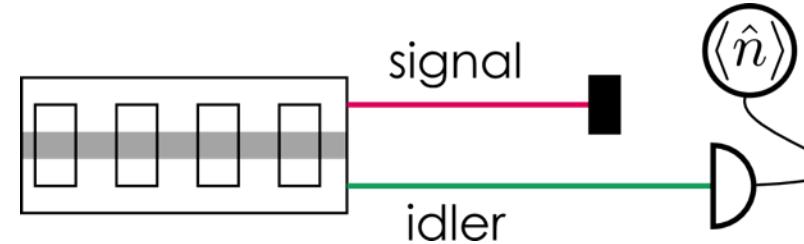
ppKTP waveguide

- group velocity matching at telecoms wavelengths
- high brightness
- 8mm length

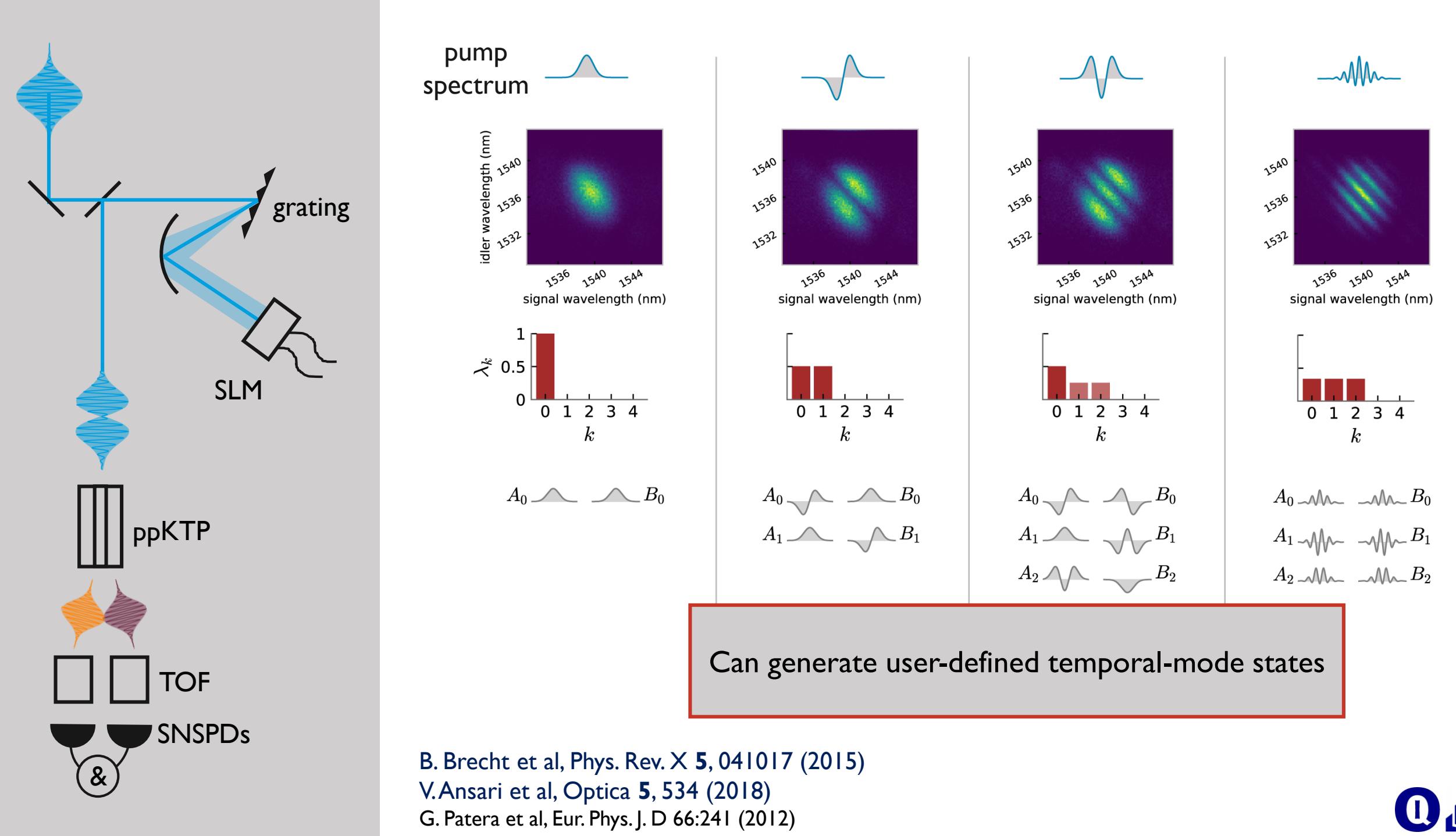


photon counting
→ source brightness

For decorrelated joint spectral amplitude



- High brightness
- Clean single-mode behaviour



Outline

Optical
modes

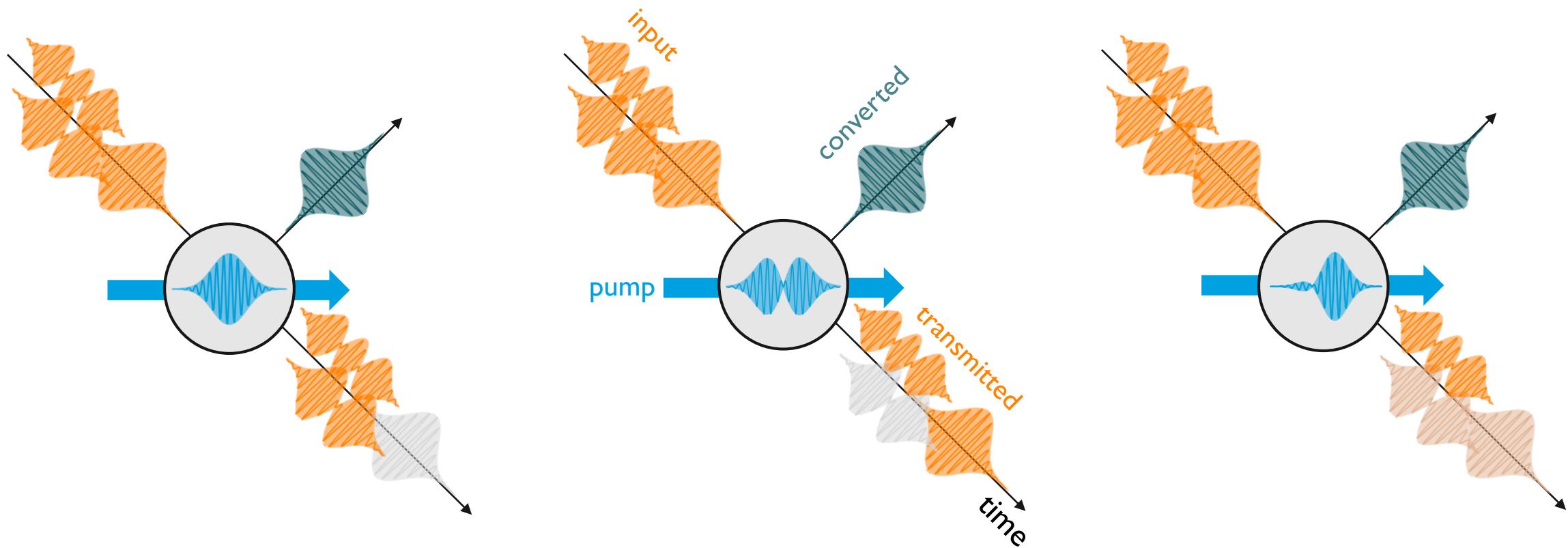
Parametric
down-conversion

Quantum
pulse gate

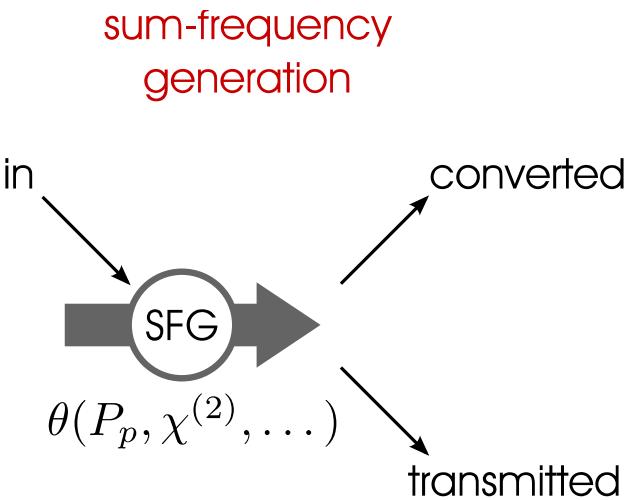
Applications



What would we like to have?

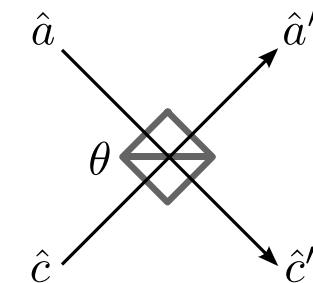


Sum-frequency generation? Maybe???



$$\hat{H}_{\text{SFG}} = \theta \left(\hat{a}_{\text{in}} \hat{c}_{\text{out}}^\dagger + \text{h.c.} \right)$$

beamsplitter



$$\hat{H}_{\text{BS}} = \theta \left(\hat{a} \hat{c}^\dagger + \hat{a}^\dagger \hat{c} \right)$$

Multi-mode theory for SFG

$$|\psi\rangle_{\text{out}} = e^{i\theta \int d\omega_{\text{in}} d\omega_{\text{out}} G(\omega_{\text{in}}, \omega_{\text{out}}) \hat{a}(\omega_{\text{in}}) \hat{c}^\dagger(\omega_{\text{out}}) + \text{h.c.}} |\psi\rangle_{\text{in}}$$

- ▶ Does this operate on one pulsed temporal mode?
- ▶ How can we define temporal modes for SFG?

Revealing the pulsed temporal modes

Starting point

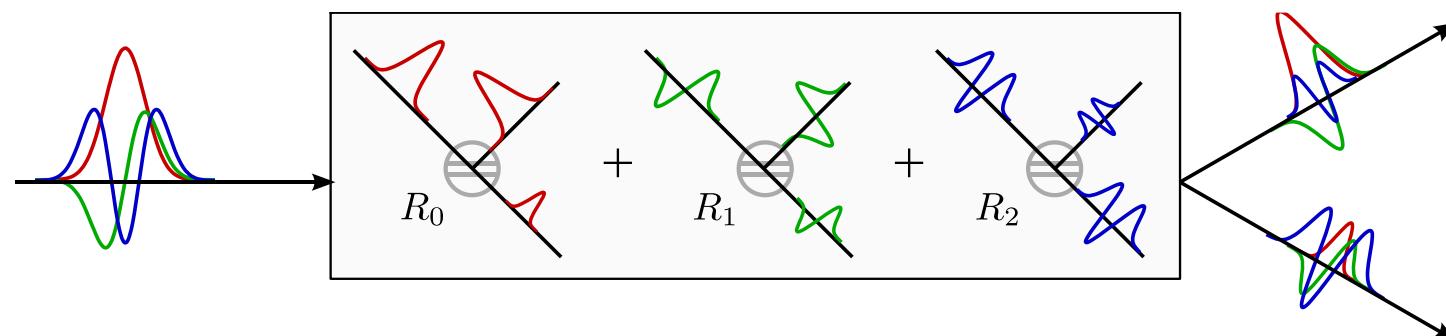
$$|\psi\rangle_{\text{out}} = e^{i\theta \int d\omega_{\text{in}} d\omega_{\text{out}} G(\omega_{\text{in}}, \omega_{\text{out}}) \hat{a}(\omega_{\text{in}}) \hat{c}^\dagger(\omega_{\text{out}}) + \text{h.c.}} |\psi\rangle_{\text{in}}$$

-
- some math
-

analogous to
Schmidt mode decomposition

$$|\psi\rangle_{\text{out}} = e^{i \sum_j \kappa_j \theta (\hat{A}_j \hat{C}_j^\dagger + \hat{A}_j^\dagger \hat{C}_j)} |\psi\rangle_{\text{in}}$$

Array of beamsplitters operating on pulse modes^[6,7]

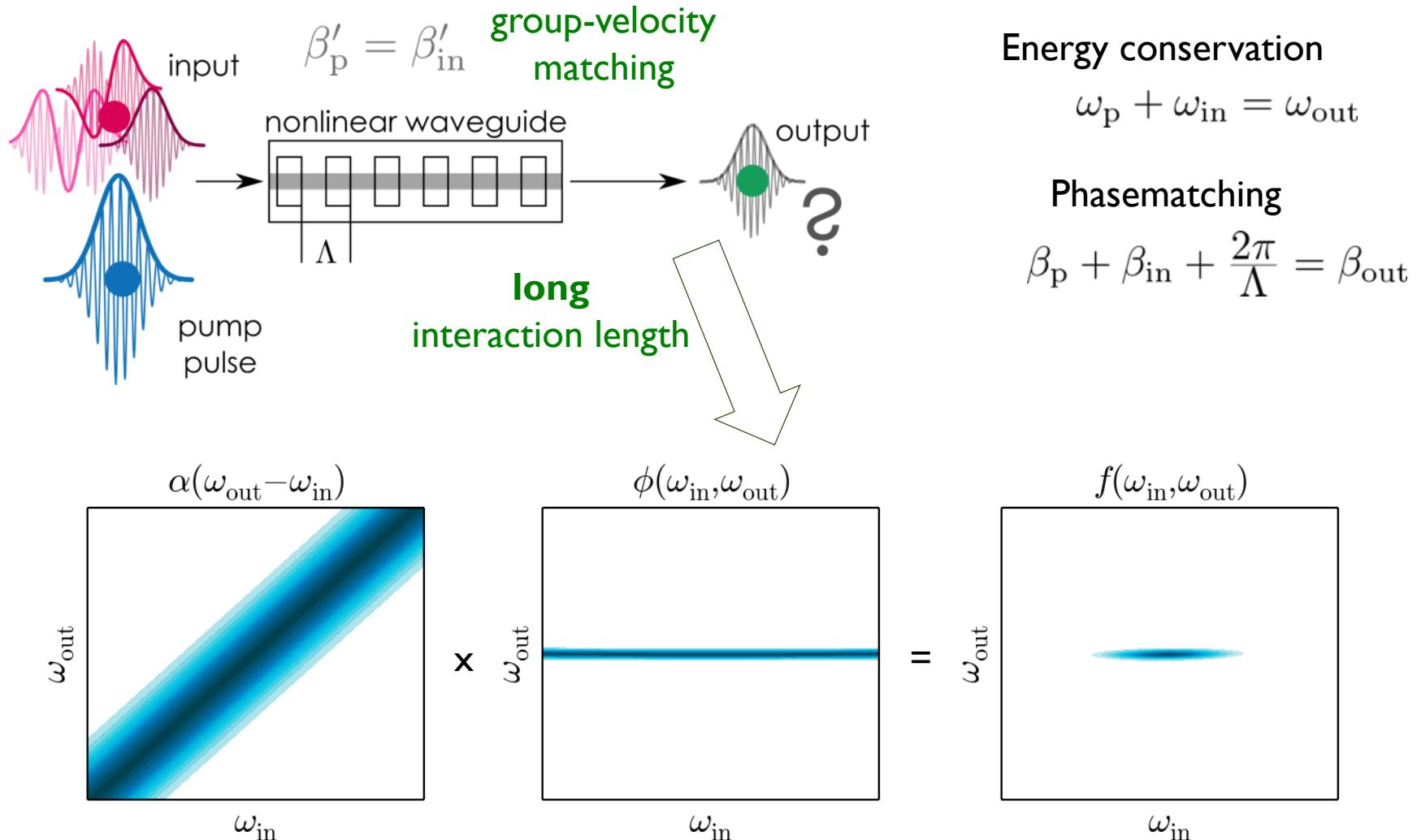


[6] M. G. Raymer et al., Opt. Comm. **283**, 747-752 (2010)

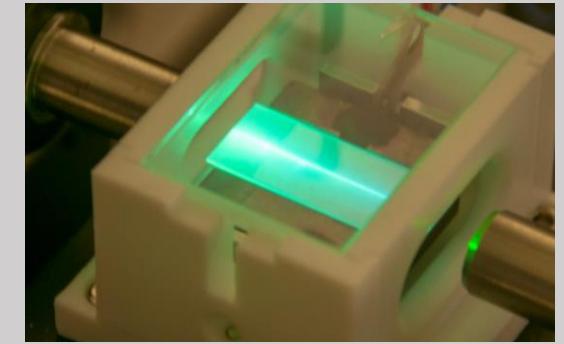
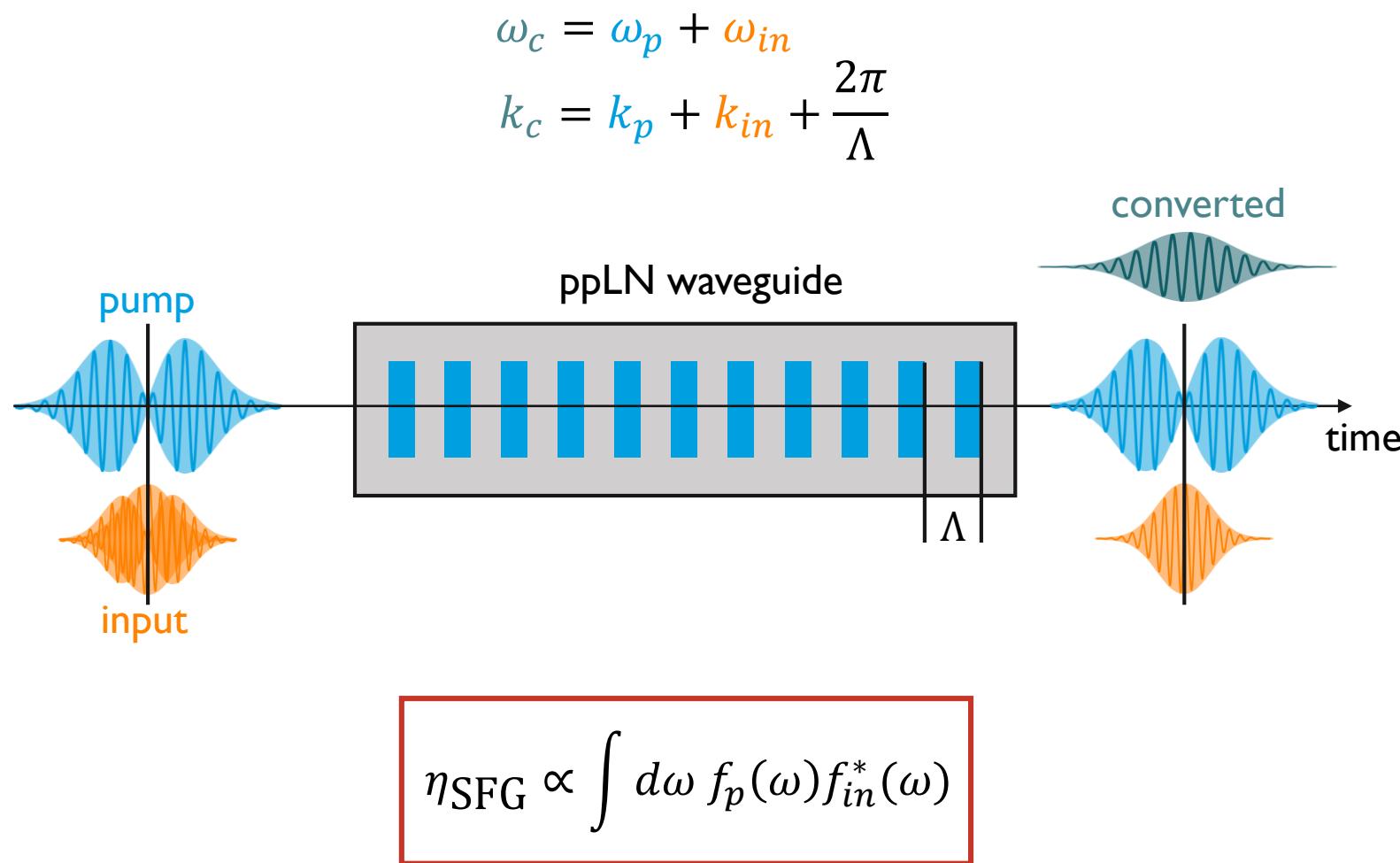
[7] A. Eckstein et al., Opt. Express **19** 13770 (2011),

The quantum pulse gate (QPG)

QPG = single TM SFG

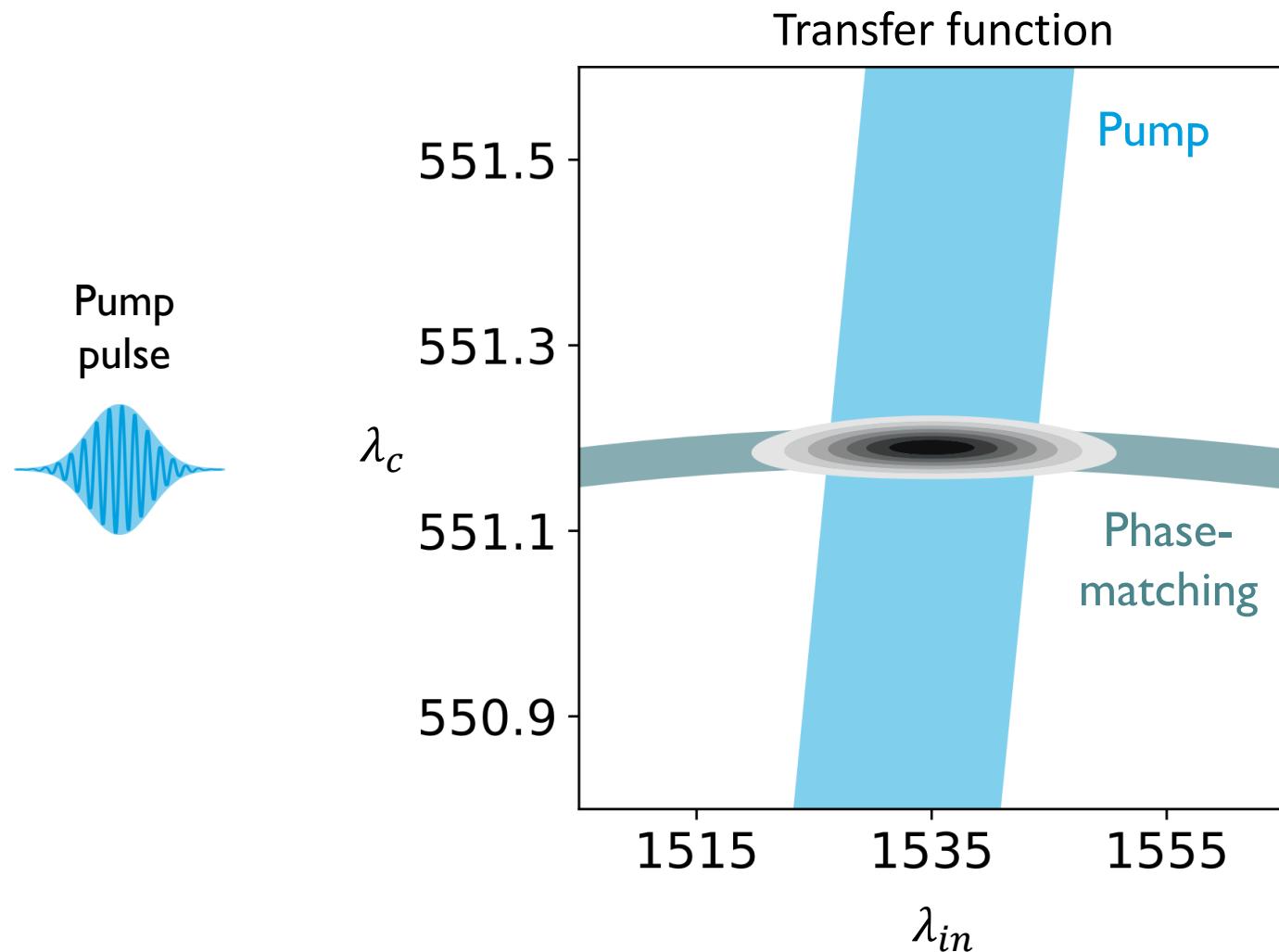


Realisation – group velocity matched sum frequency generation

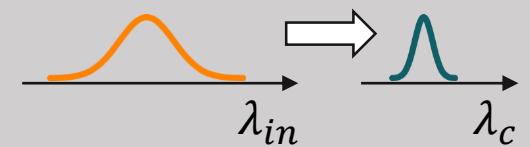


- **homebuilt ppLN waveguide**
- $4.5\mu\text{m}$ poling period
- operated at 190°C
- type II SFG
- $v_g^{(p)} = v_g^{(in)}$
- $\lambda_{in} = 1550\text{nm}$
- $\lambda_p = 860\text{nm}$

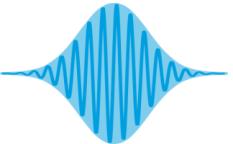
A closer look at the pulse gate operation



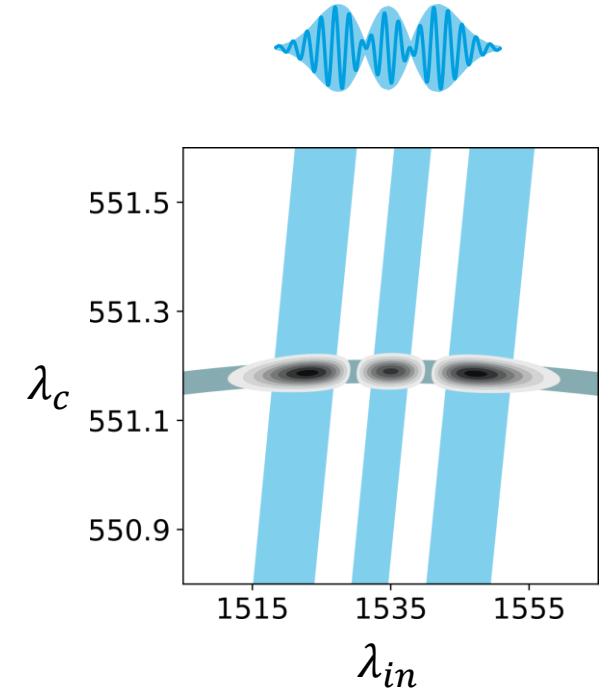
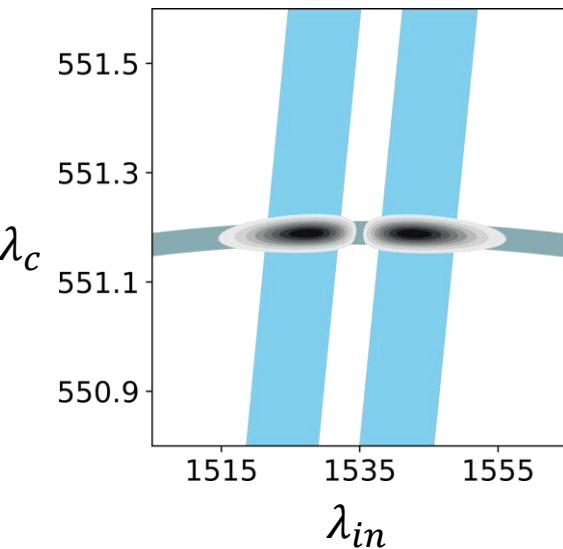
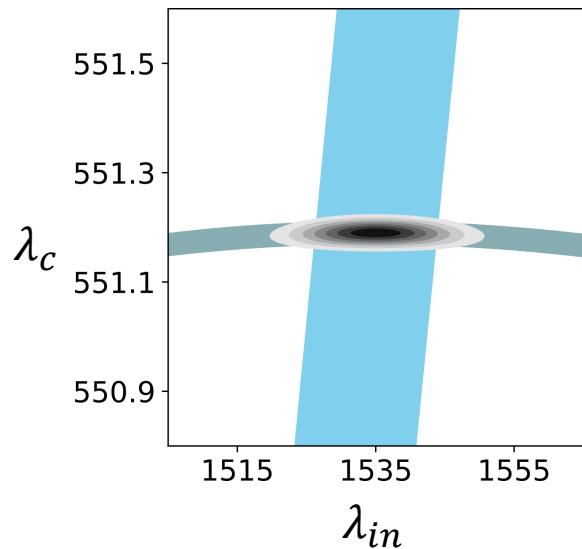
- Transfer function is product of pump and phasematching (c.f. JSA)
- Can use the Schmidt decomposition formalism
- Selected input mode defined by pump
- Converted mode defined by phasematching



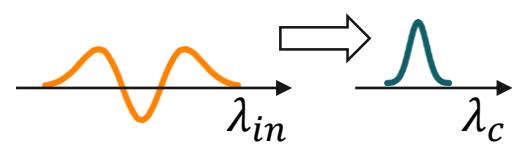
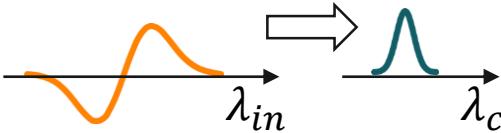
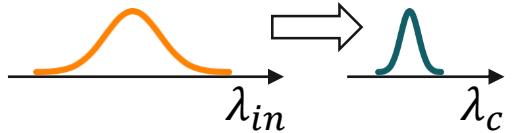
Pump
pulse



Transfer
function



Selected
mode



$$|\psi\rangle_{out} = \exp[\theta \hat{A} \hat{C}^\dagger + \theta^* \hat{A}^\dagger \hat{C}] |\psi\rangle_{in}$$

QPG = special beam
splitter for temporal modes

Outline

Optical
modes

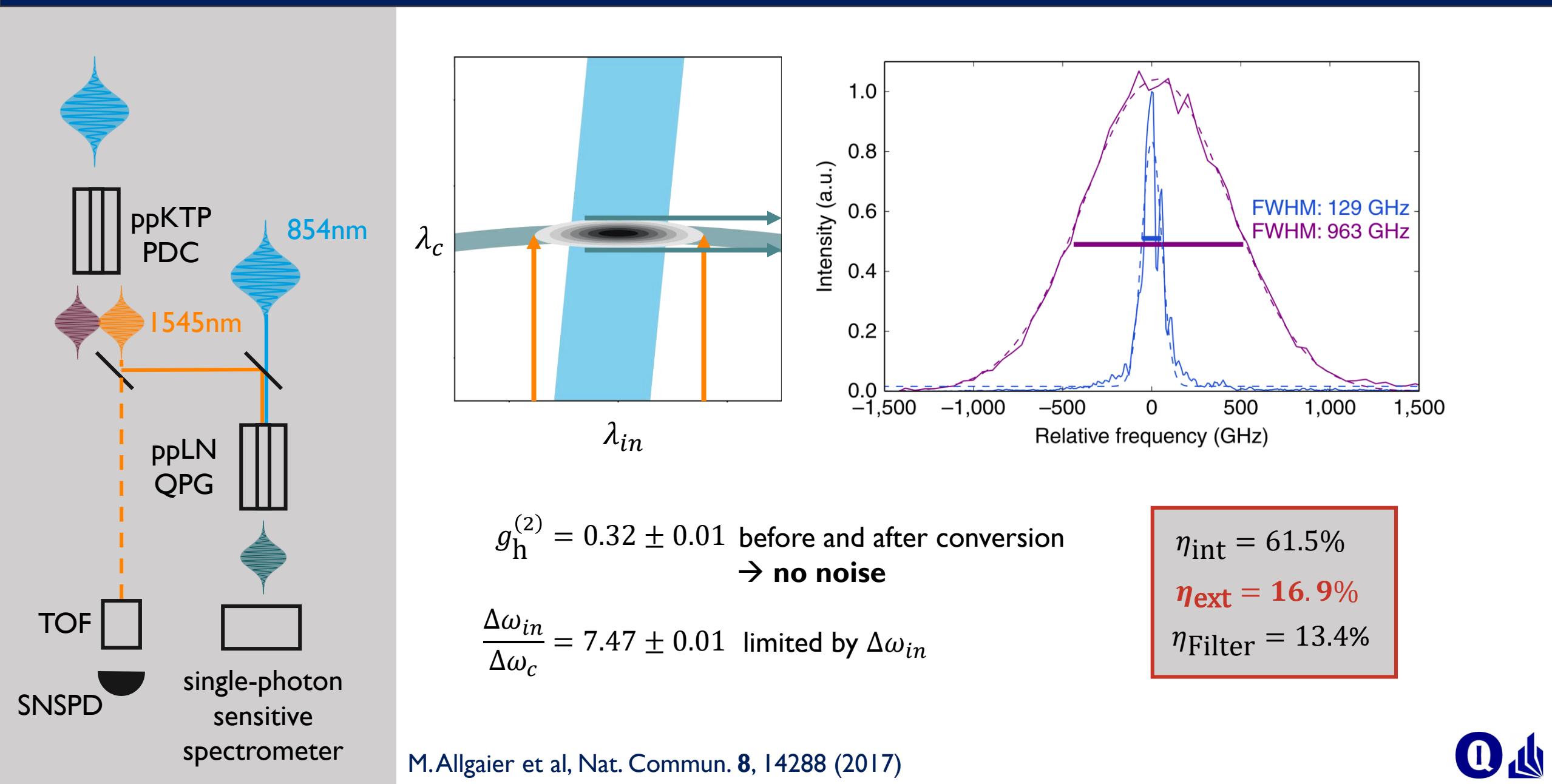
Parametric
down-conversion

Quantum
pulse gate

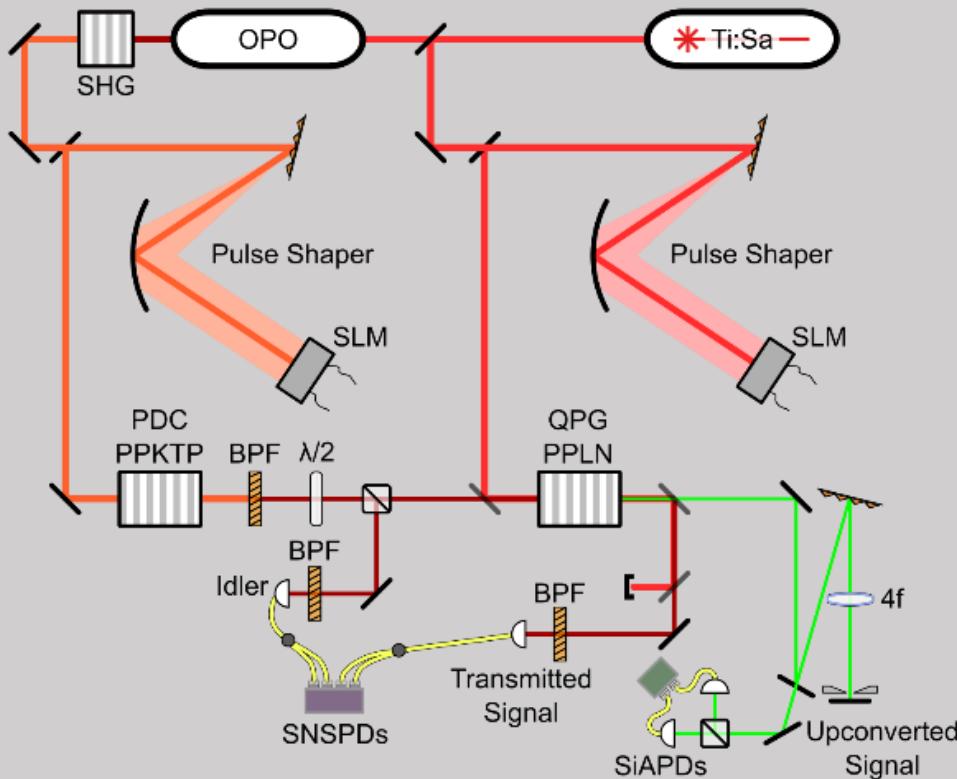
Applications



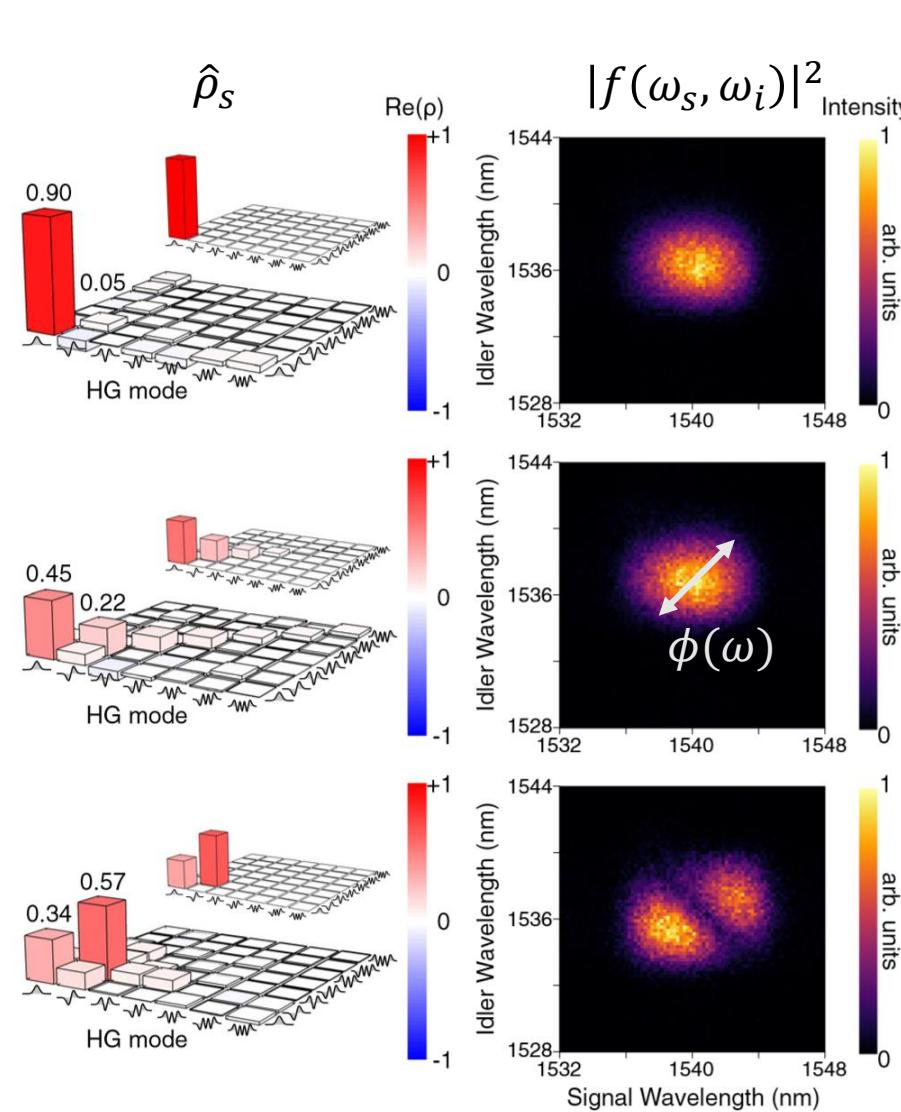
Time-frequency manipulations of single photons



Tomography of PDC photons

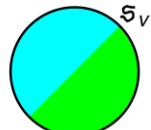


- signal at 1540nm
- pump at 876nm
- bandwidth compression ~ 10
- operated at low efficiency
- $g^{(2)}$ of converted light $> 1.9 \rightarrow$ single mode

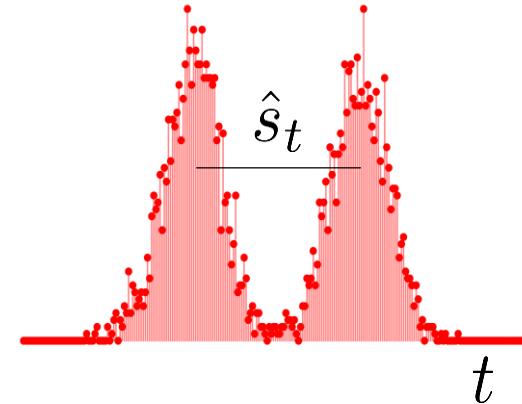
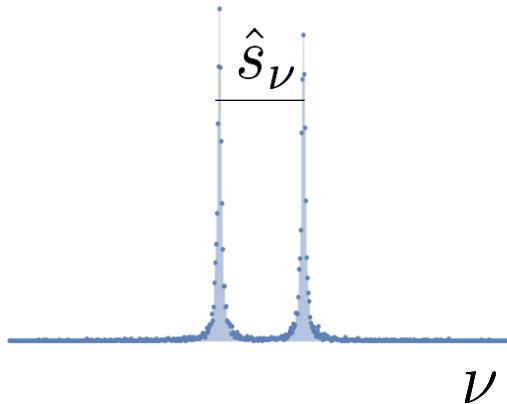
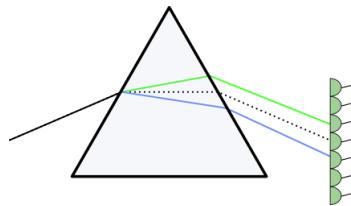
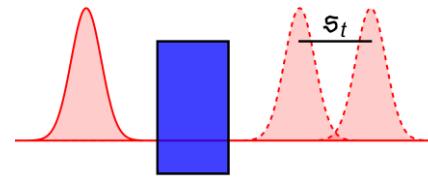


Time-frequency metrology at the quantum limit

Frequency



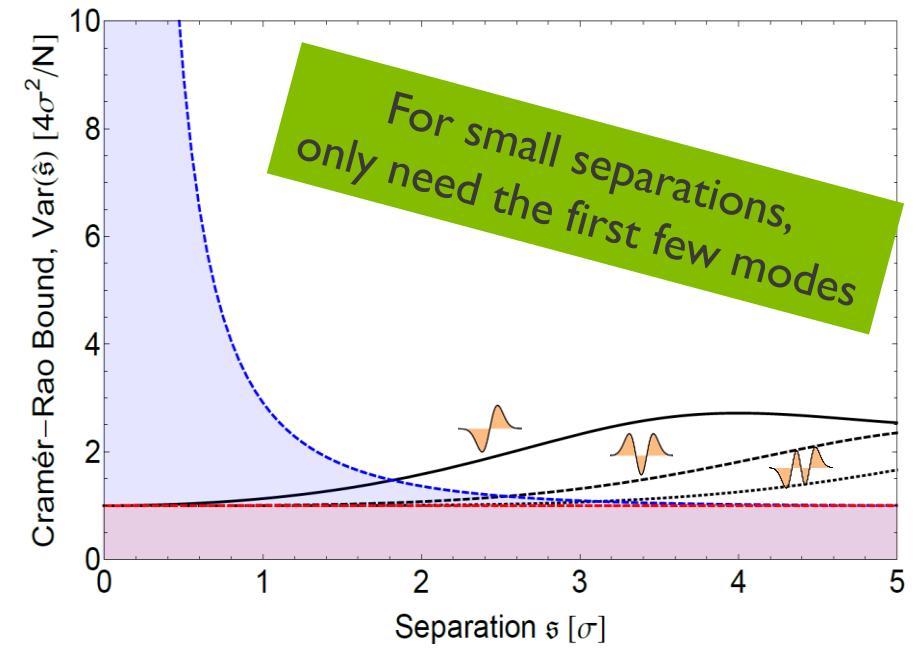
Time



$$\text{Var}(\hat{s}) \geq \frac{1}{N \int dx \frac{1}{I(x,s)} \left(\frac{\partial I(x,s)}{\partial s} \right)^2} \geq \frac{1}{4N\sigma^2}$$

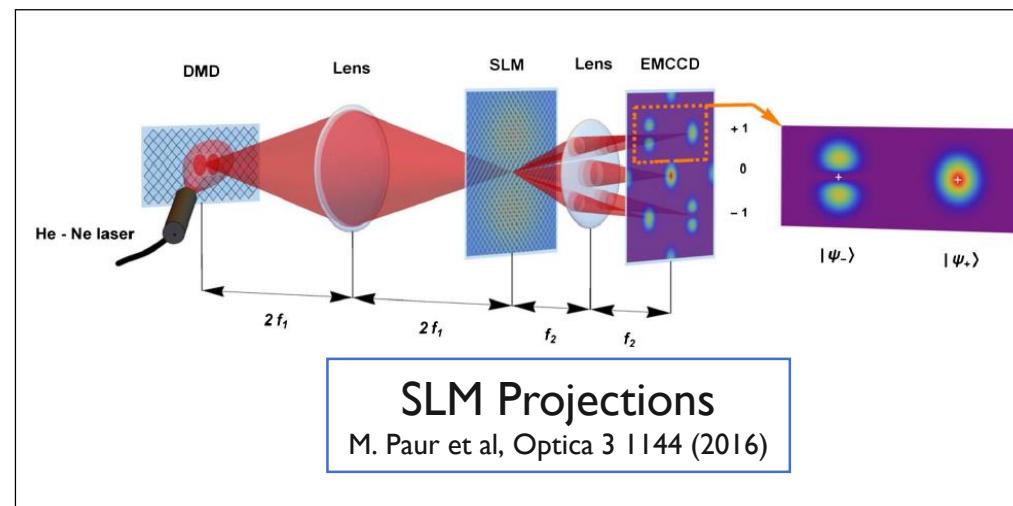
Intensity-Counting
Limit

Quantum
Limit



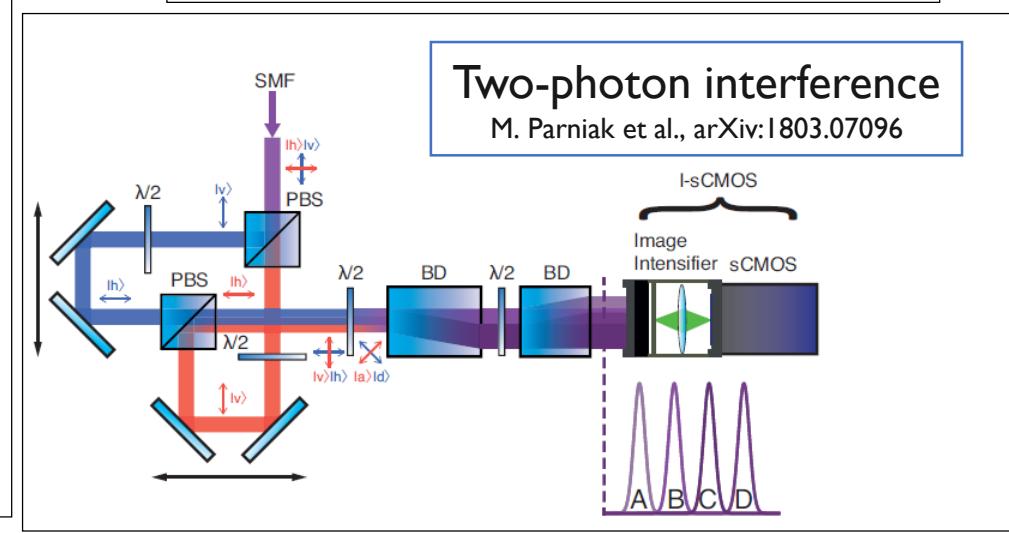
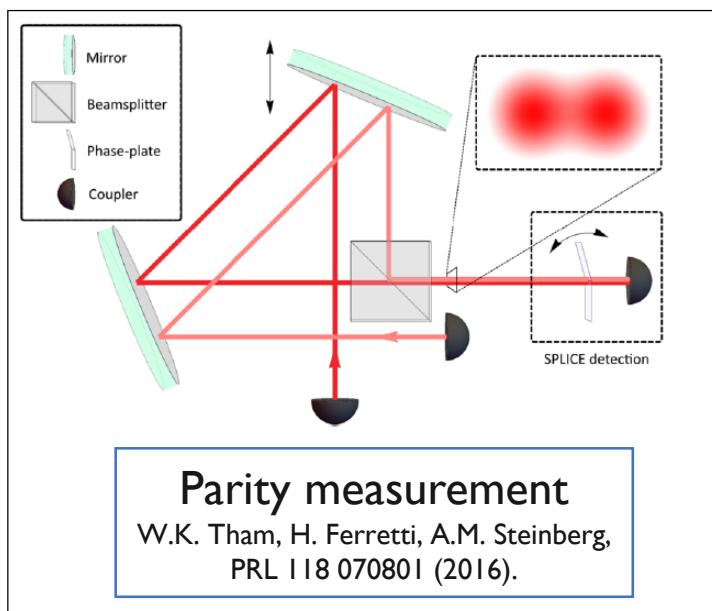
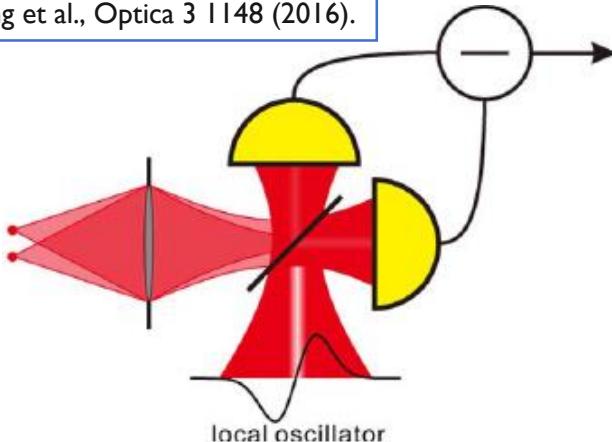
Time-frequency tomography at the quantum limit

Freq



Homodyne Detection

F. Yang et al., Optica 3 1148 (2016).

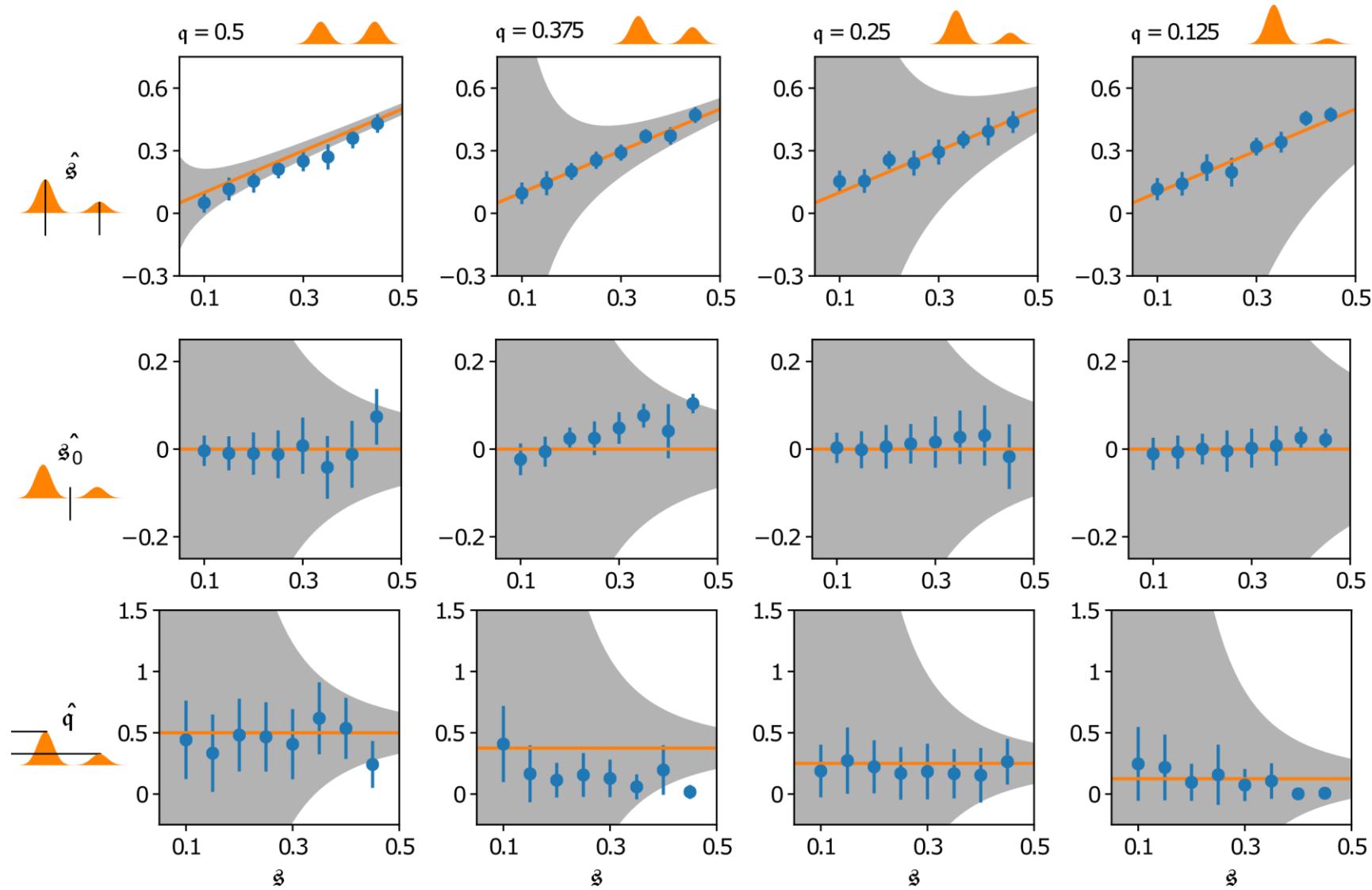
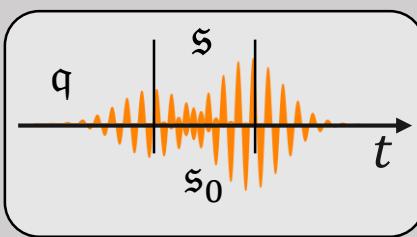


,
modes

5

Multi-parameter estimation

- $\lambda_p = 862\text{nm}$
- $E_p = 150\text{pJ}$
- $\lambda_{in} = 1540\text{nm}$
- $\sigma_{in} = 280\text{GHz}$
- $\tau_{in} = 250\text{fs}$
- $L = 35\text{mm}$
- $T = 193^\circ\text{C}$
- $\sigma_\phi = 17\text{GHz}$



in preparation



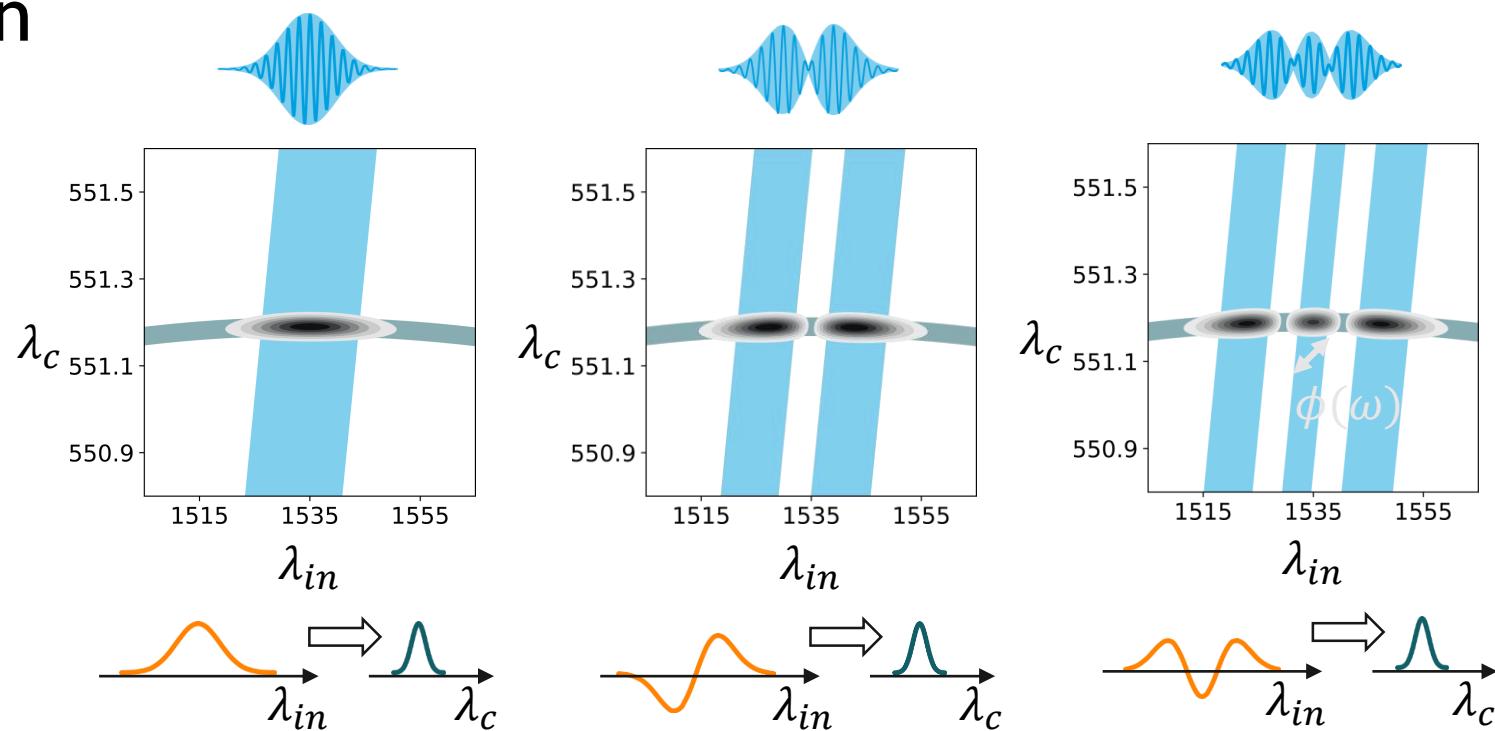
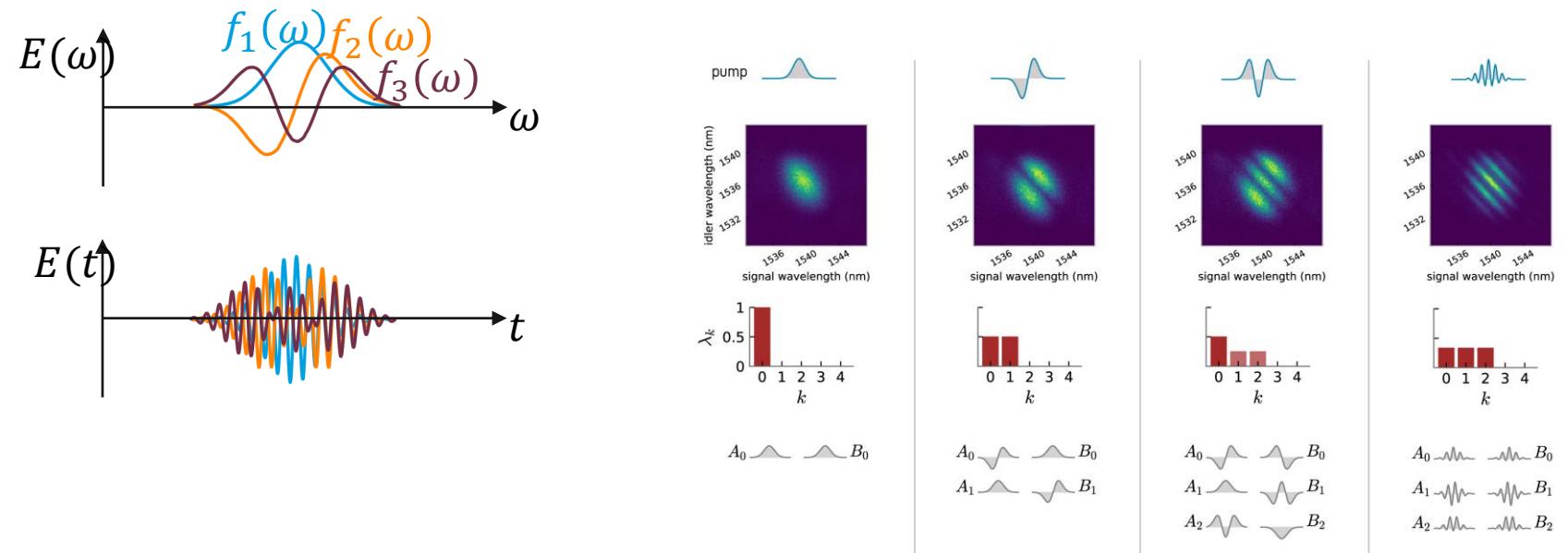
Summary

Optical modes

Parametric down-conversion

Quantum pulse gate

Applications



Thank you for your attention

Quantum Networks

Benjamin Brecht

Sonja Barkhofen

Jan Sperling

Michael Stefszky

Vahid Ansari

Syamsundar De

Thomas Nitsche

Melanie Engelkemeier

Jano Gil Lopez

Johannes Tiedau

Nidhin Prasannan

Rene Pollmann

We have open
positions
(and candy...)



Silberhorn group

