# HIGH-DIMENSIONAL QUANTUM PHOTONICS: *PUSHING THE LIMITS OF QUANTUM TECHNOLOGIES*


MEHUL MALIK

***Institute of Photonic and Quantum Sciences (IPAQS)***
***Heriot-Watt University, Edinburgh, UK***

*Institute for Quantum Optics and Quantum Information (IQOQI) Vienna*
*Vienna, Austria*

# My Academic Journey

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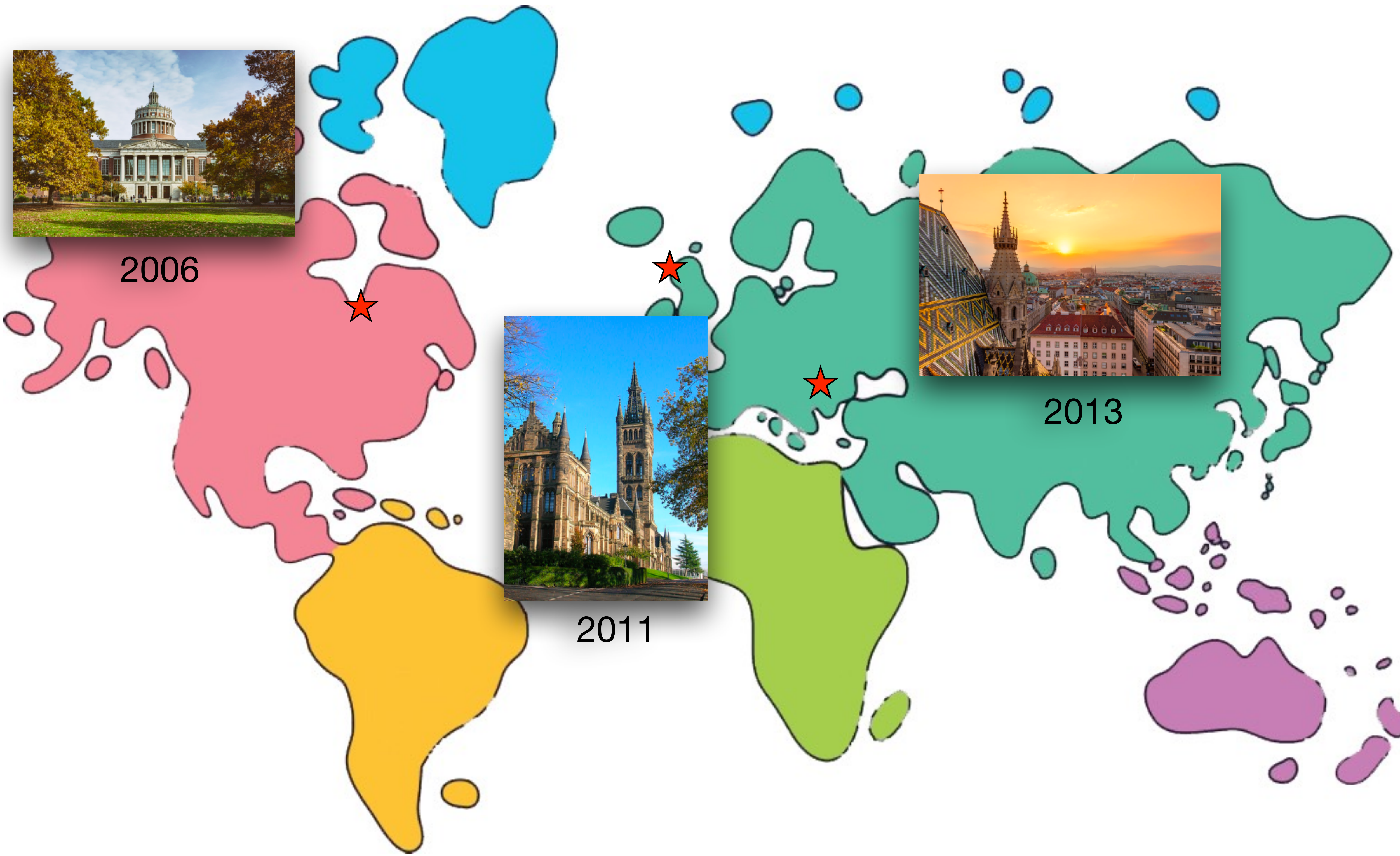

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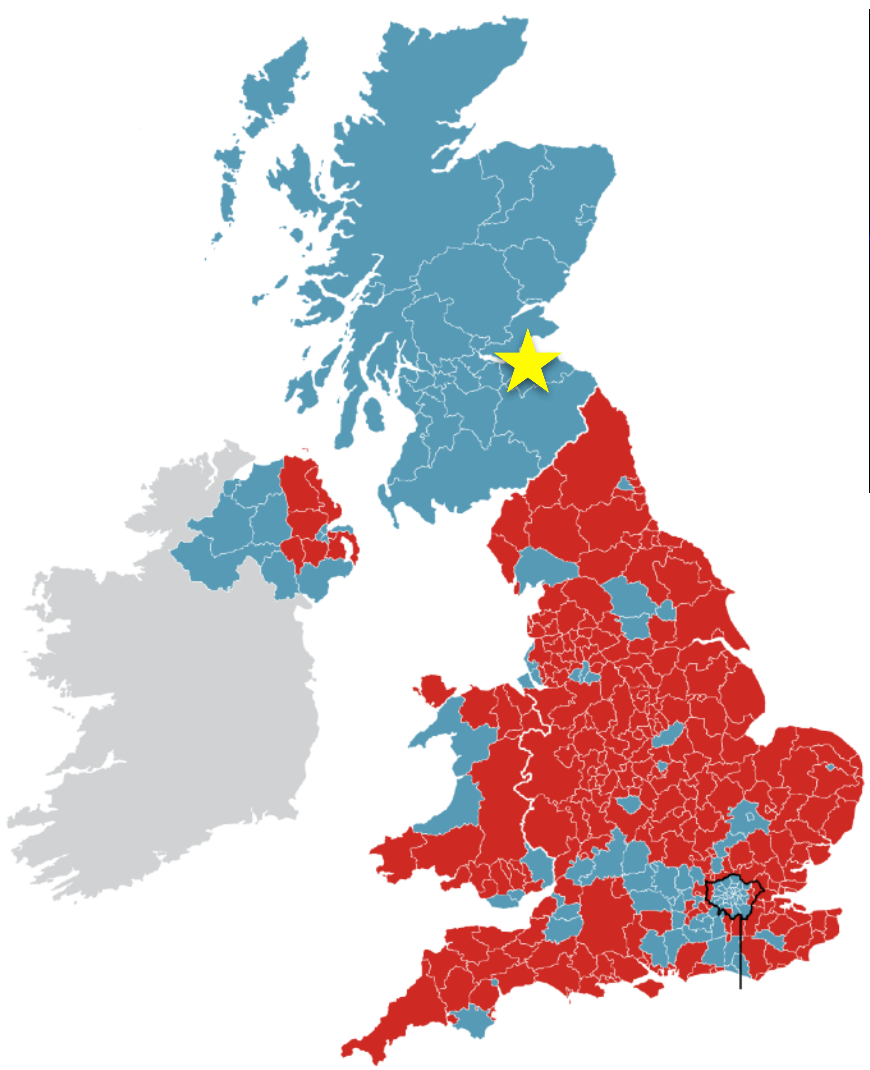
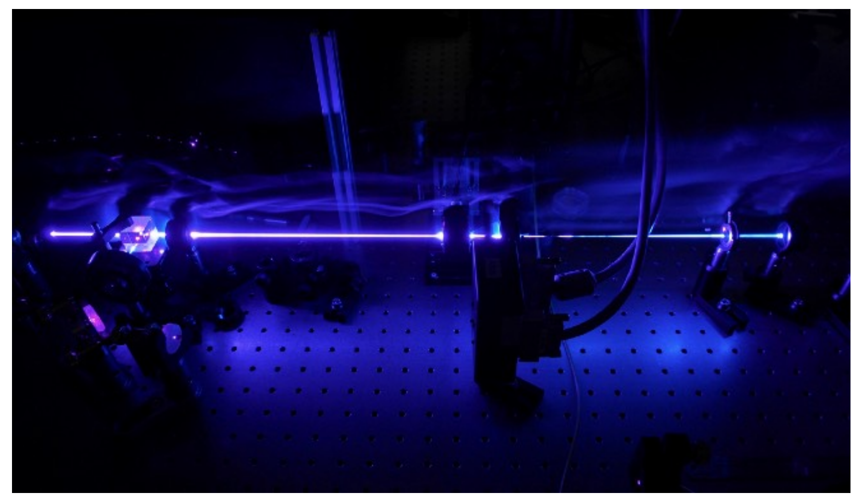
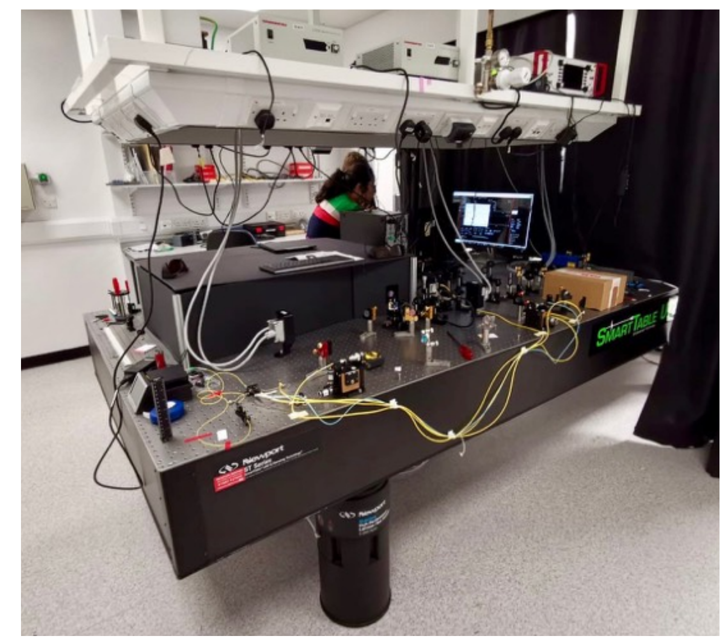
# BBQLAB.ORG

# BEYOND BORING QUBITS!

## Heriot-Watt University, Edinburgh







# LECTURE 1

- Introduction / Qudits
- Superposition / Entanglement
- Measuring HD Entanglement
- Noise Resistance

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## LECTURE 2

- Unscrambling Entanglement through a Complex Medium
- Multi-photon HD entanglement
- Free-space Quantum Communication

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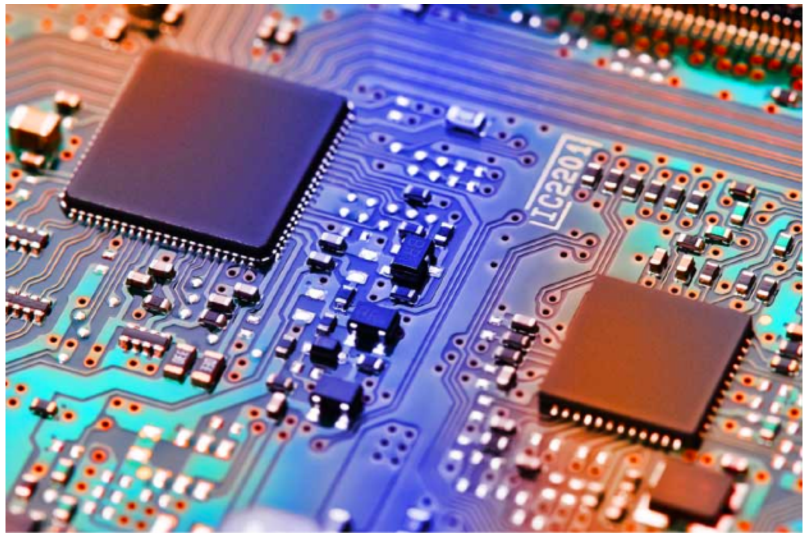


Transistor

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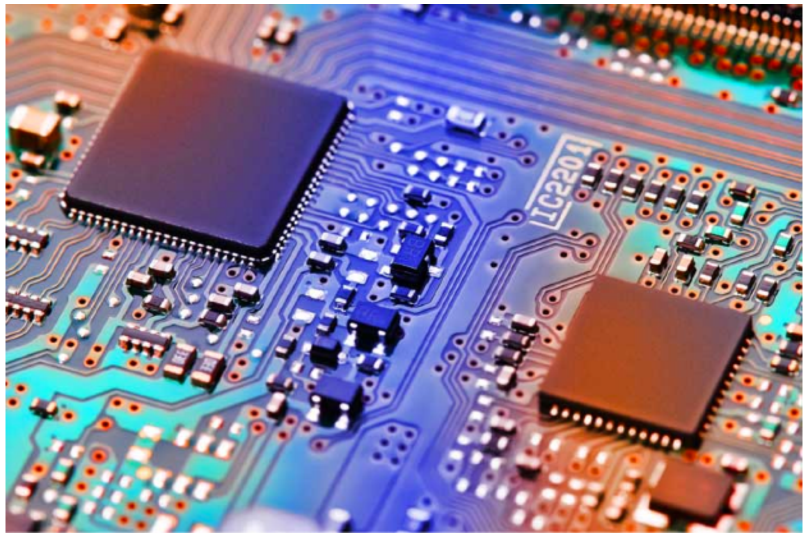


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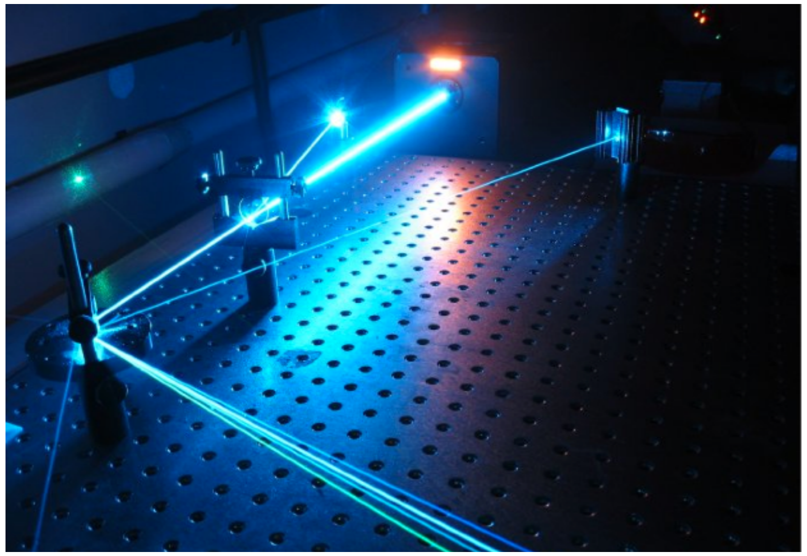

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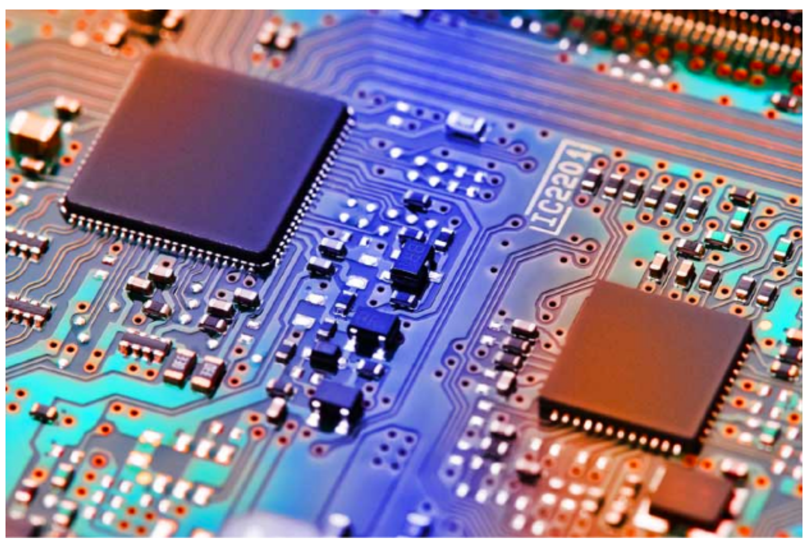


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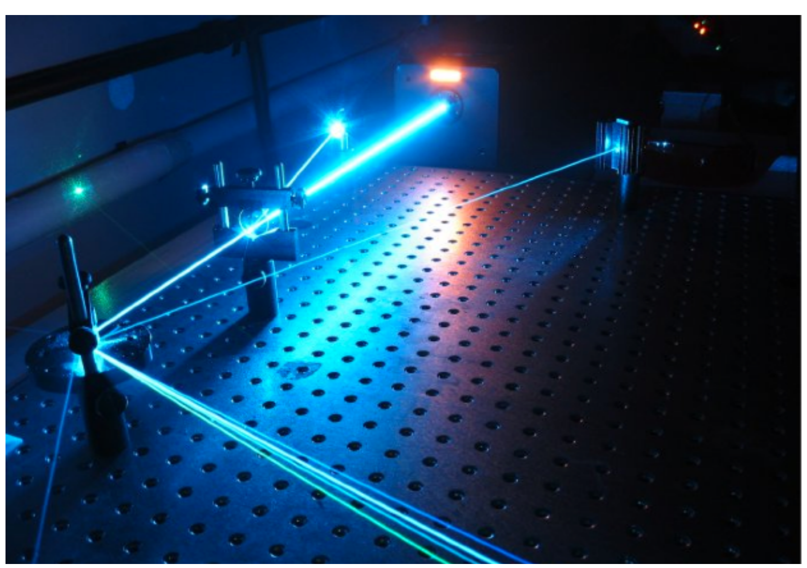
GPS



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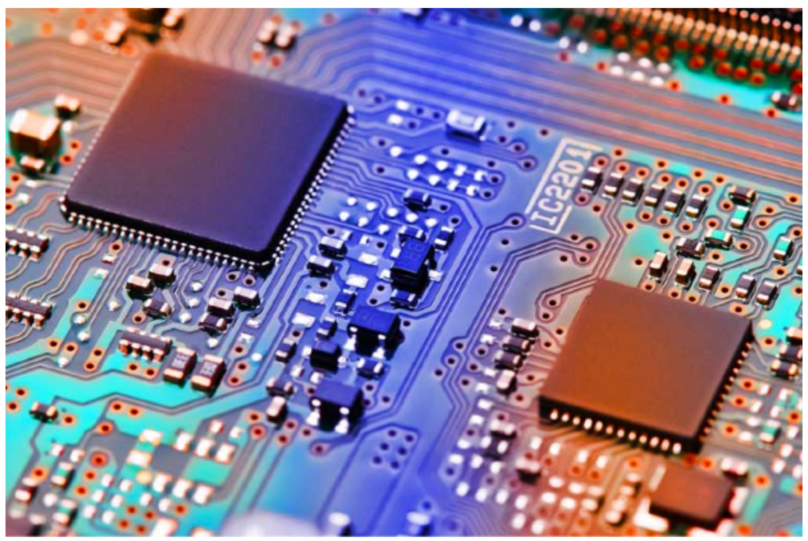

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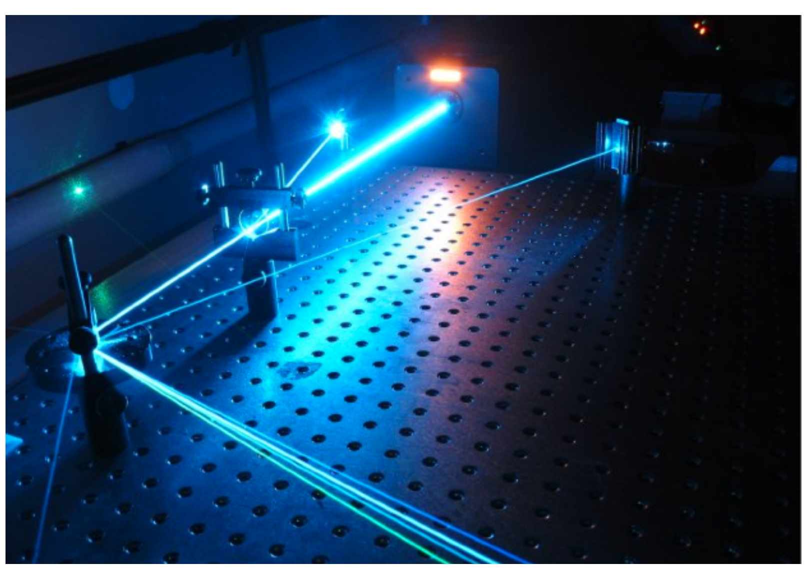

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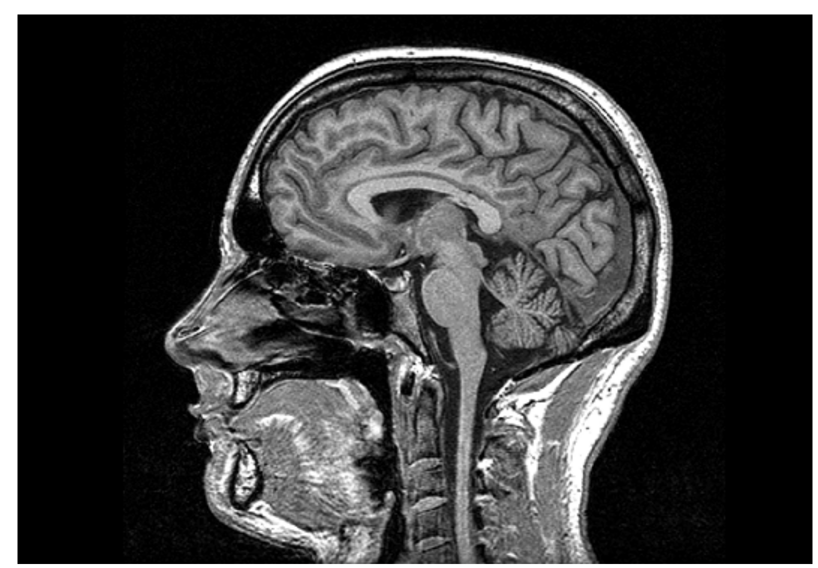

MRI

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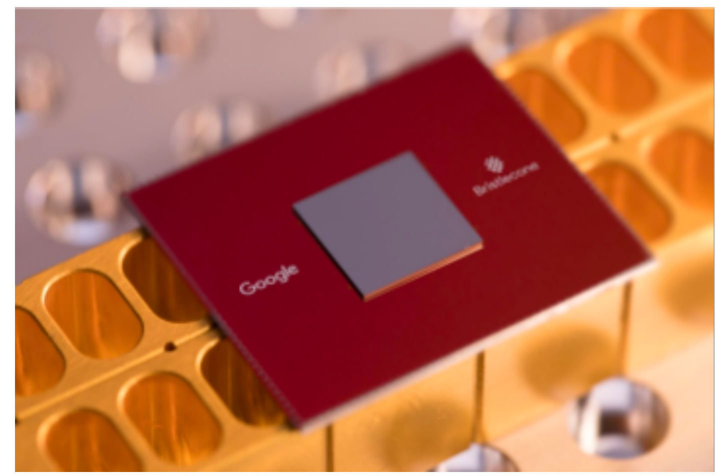
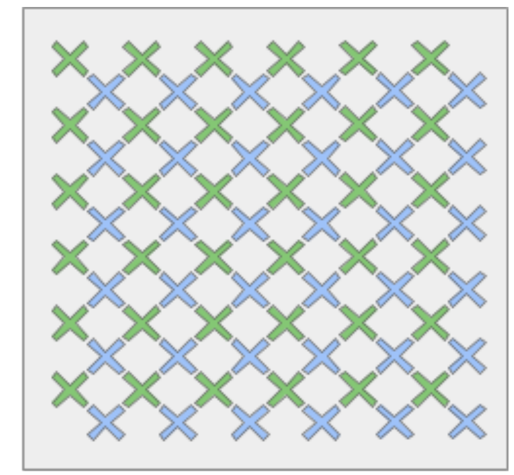
Google AI Blog



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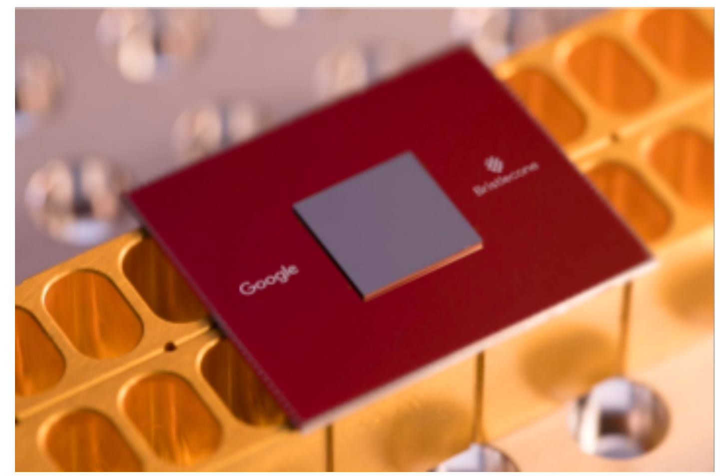
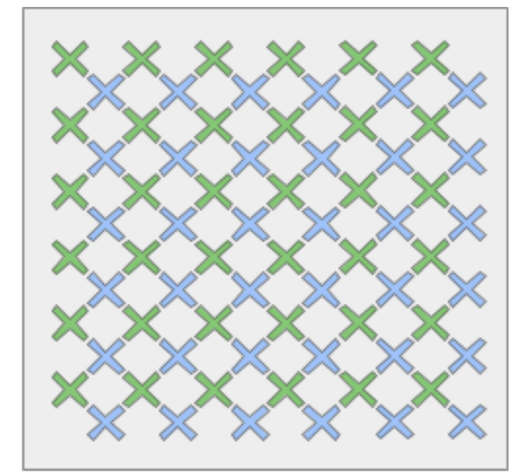
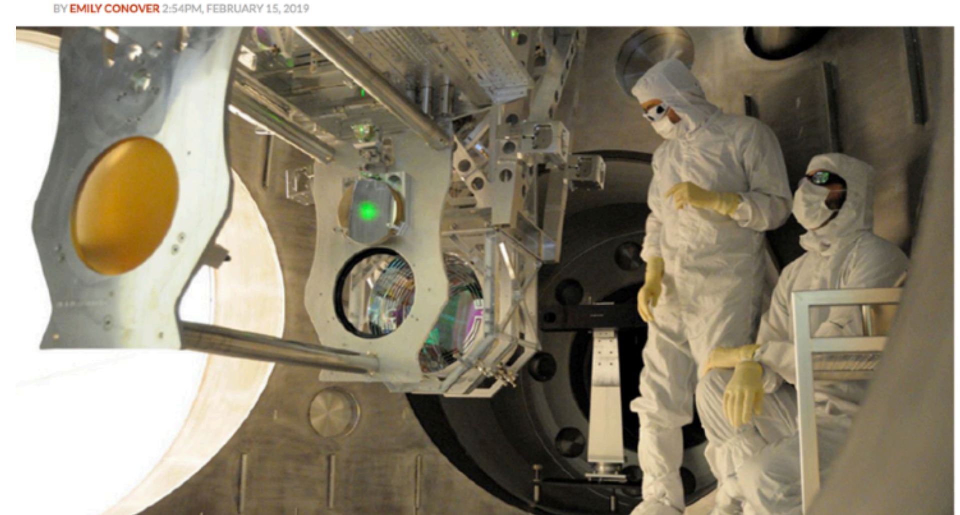


**LIGO will be getting a quantum upgrade**

Quantum 'squeezing' light could lead to daily gravitational wave detections

BY EMILY CONOVER 2:54PM, FEBRUARY 15, 2019



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Science
AAAS

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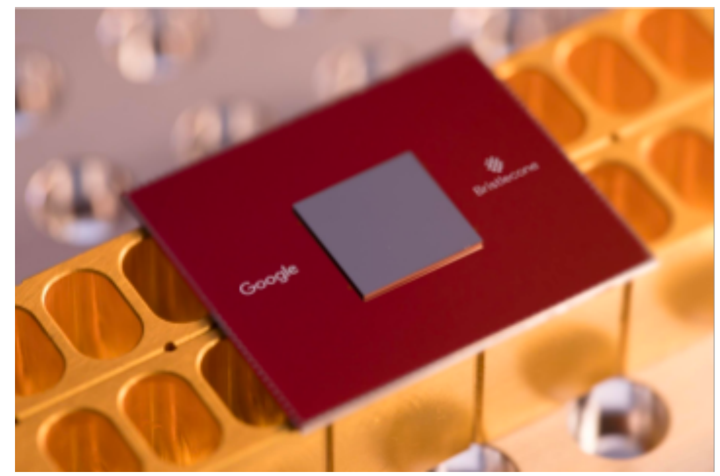

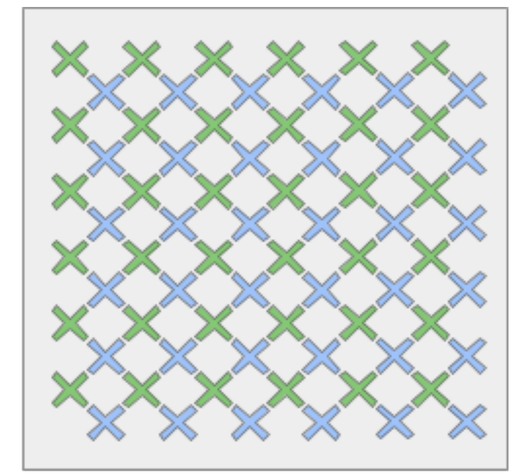

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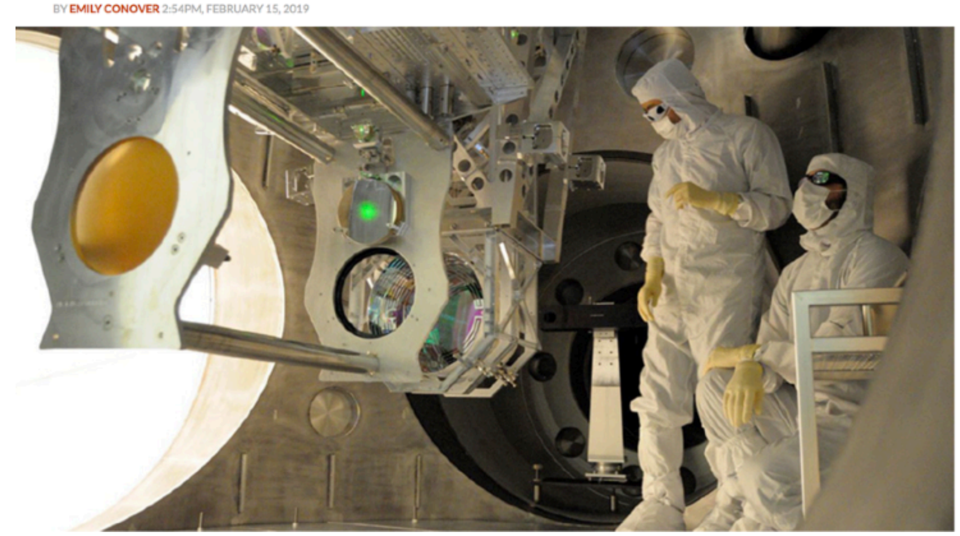

QUANTUM FLAGSHIP

UK NATIONAL QUANTUM TECHNOLOGIES PROGRAMME

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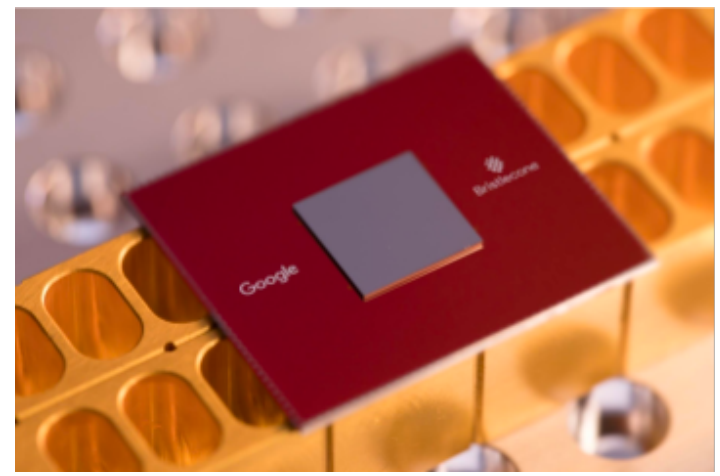


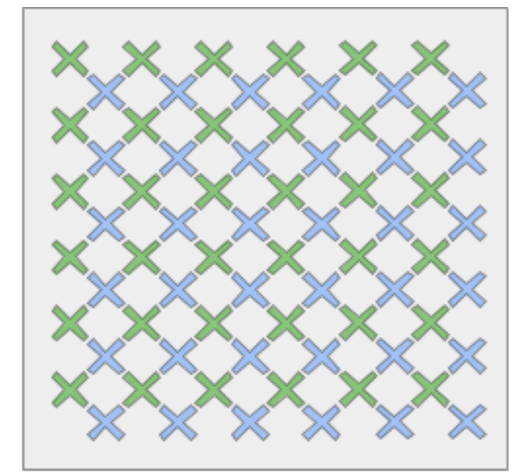

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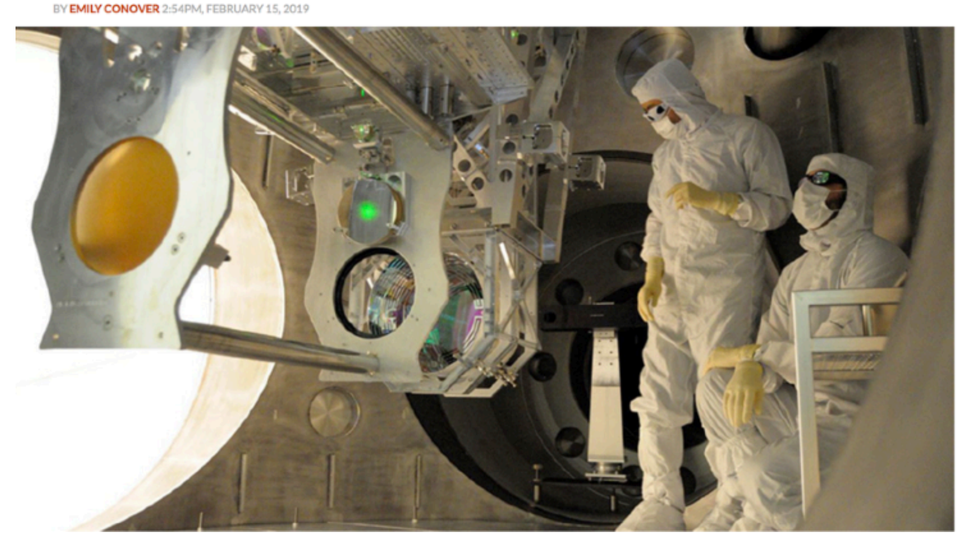

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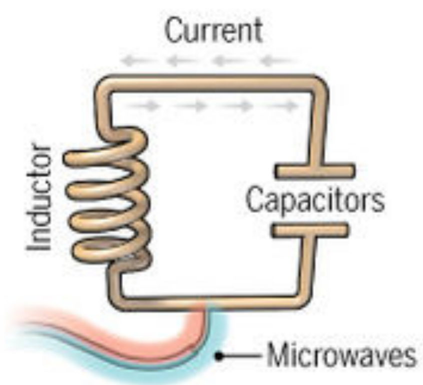


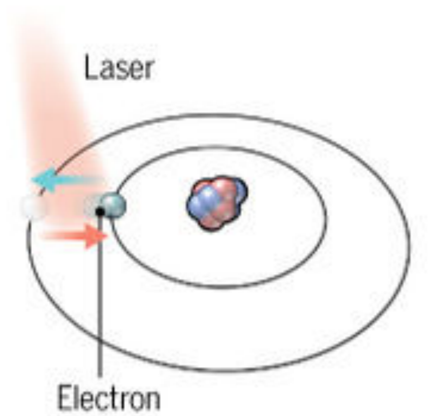



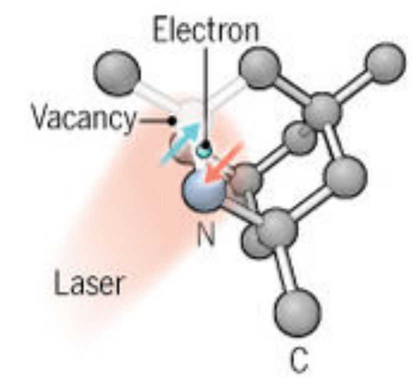

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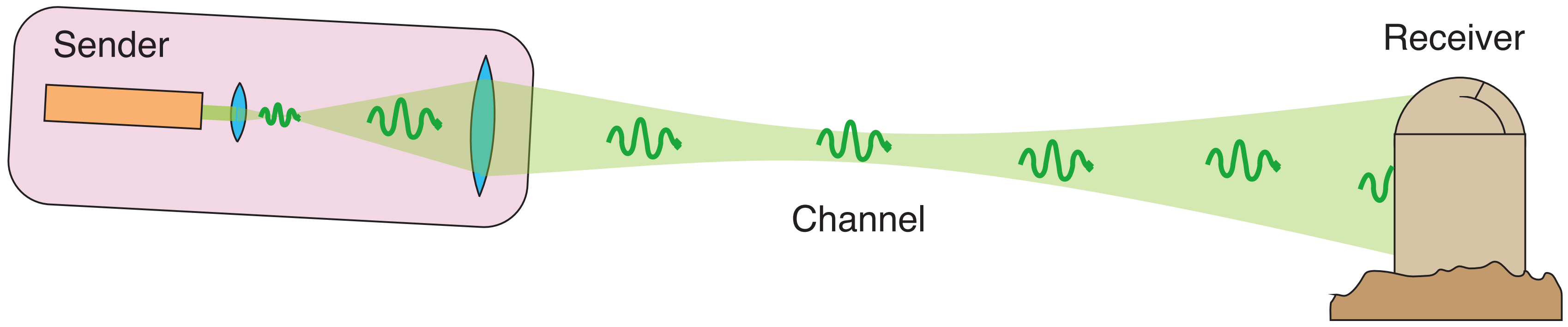

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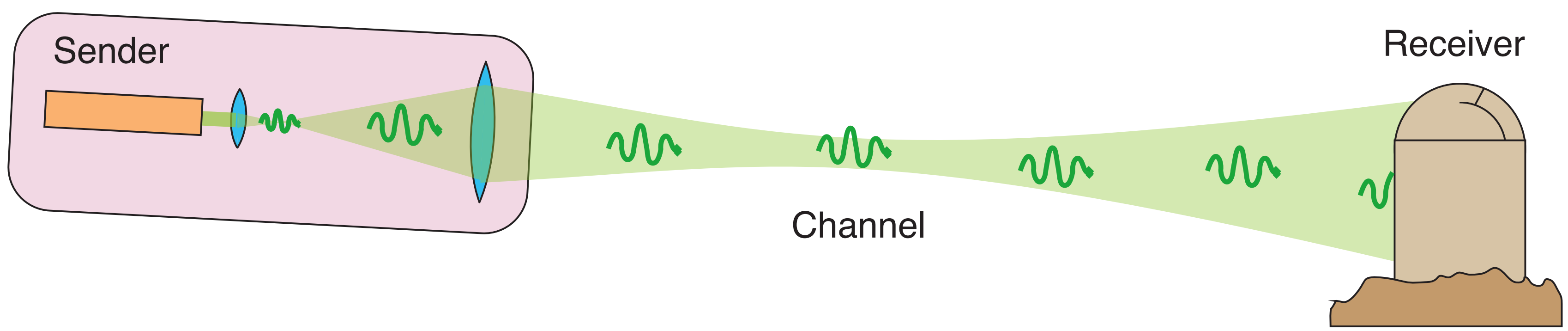


*Fast* and Cheap ££

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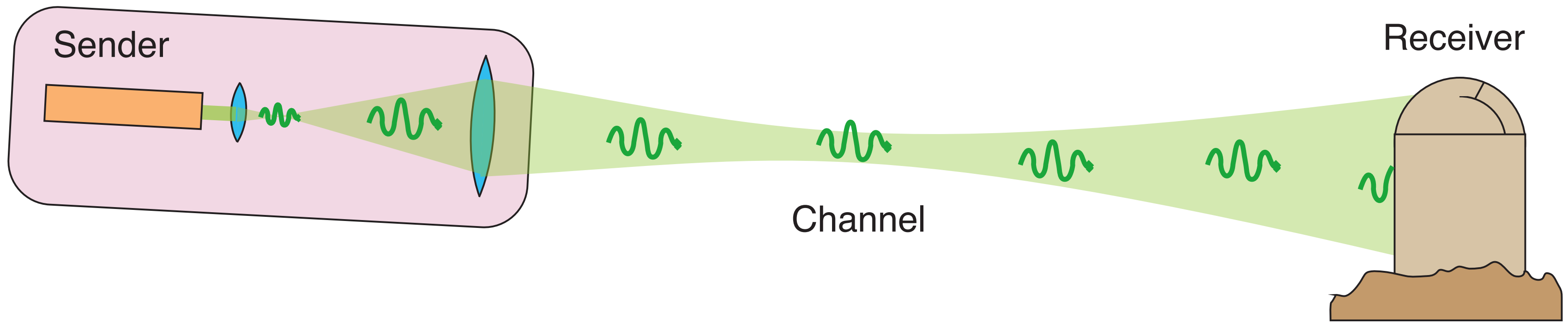


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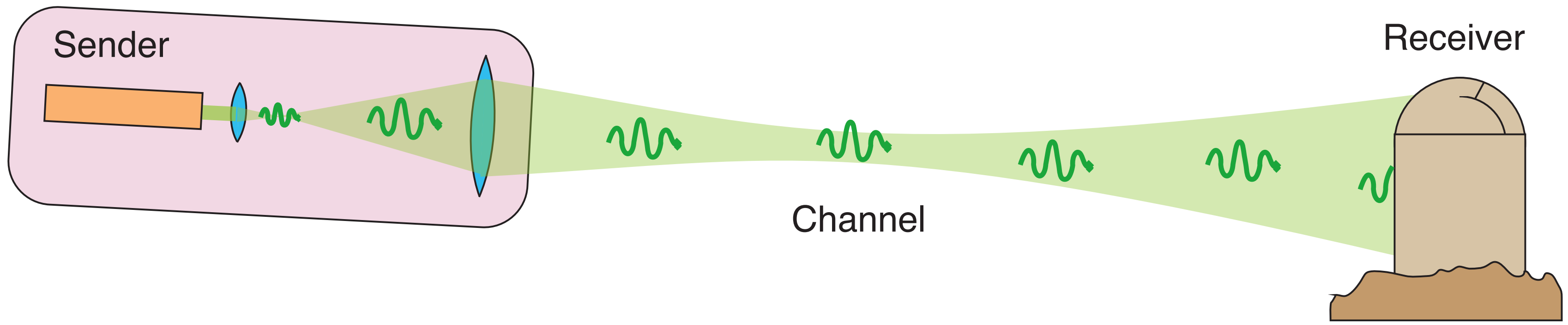


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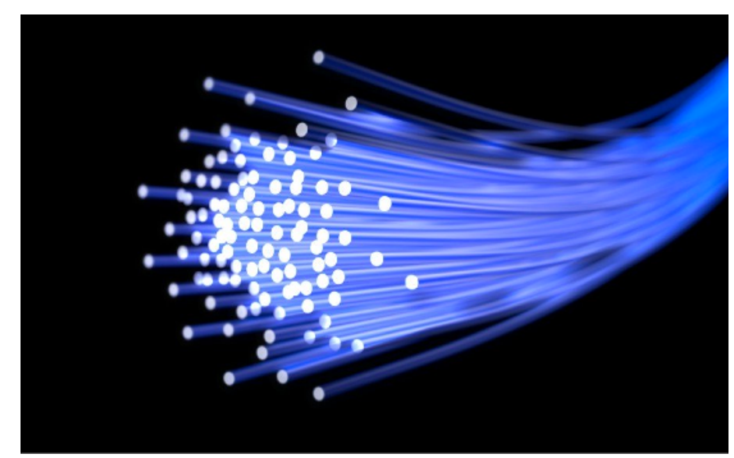
Low-noise

Efficient transport



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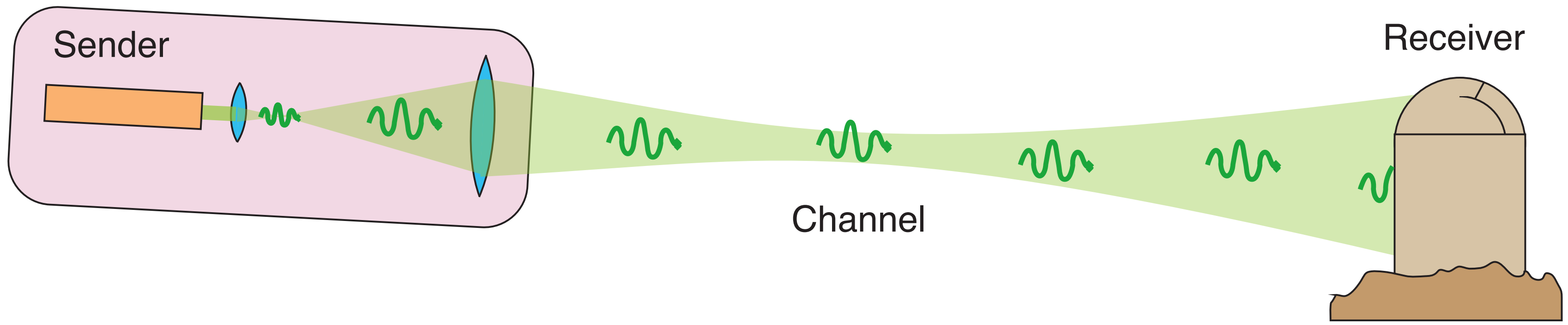


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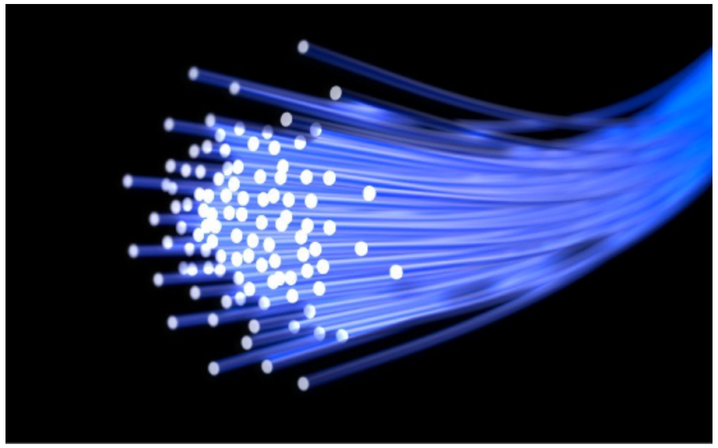

Classical photonics industry and research!

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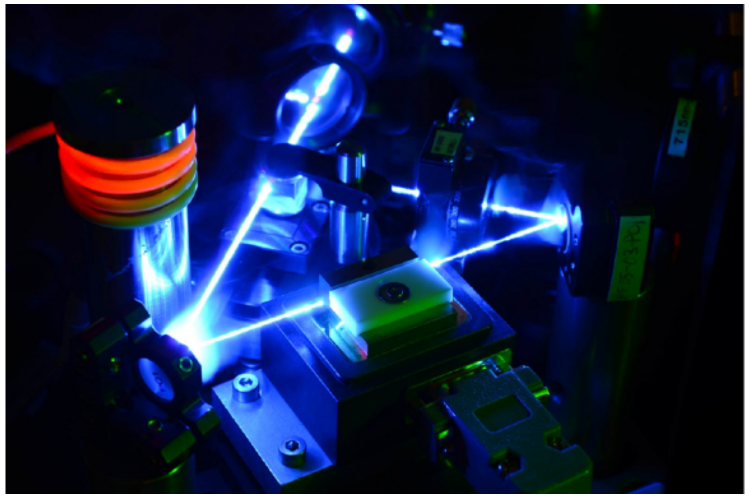

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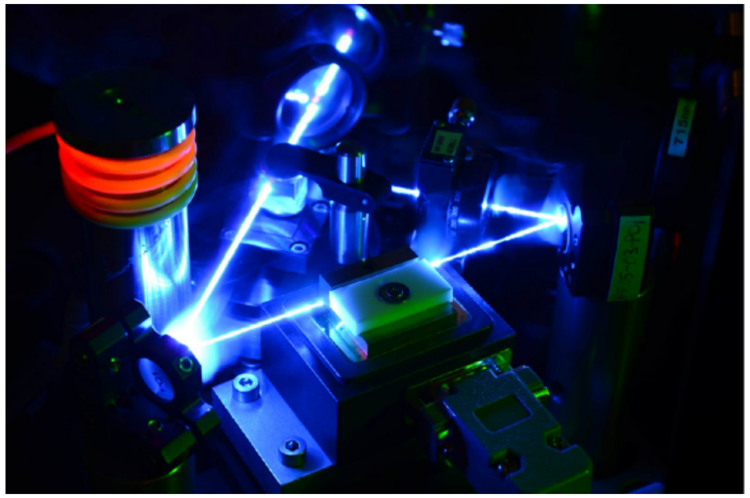

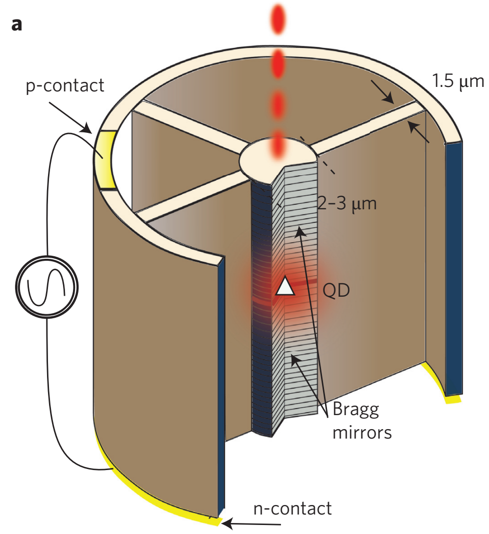

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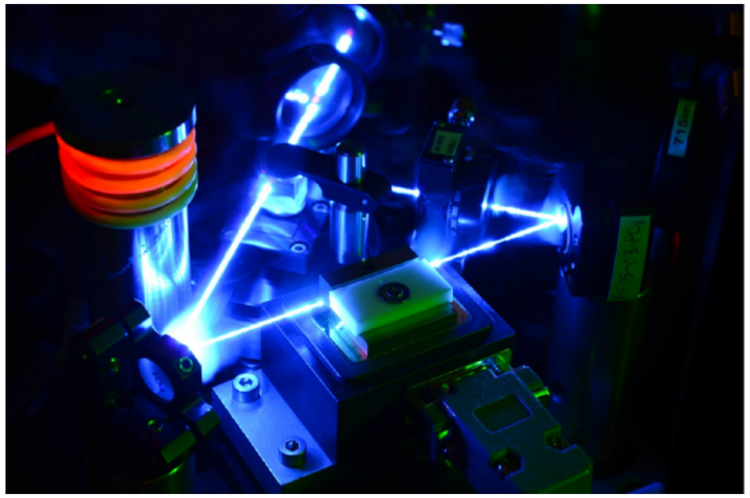

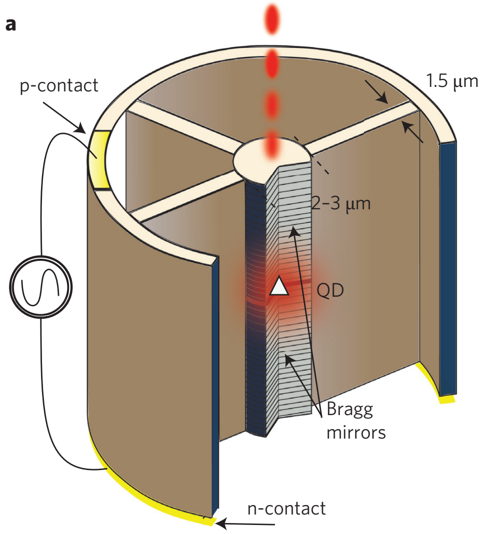


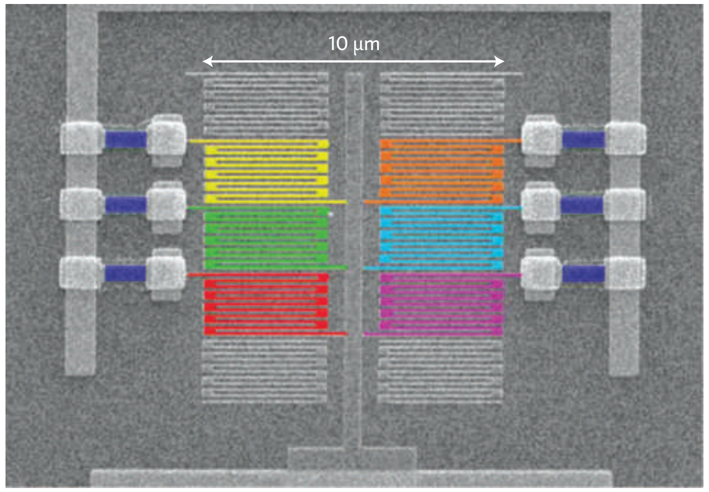

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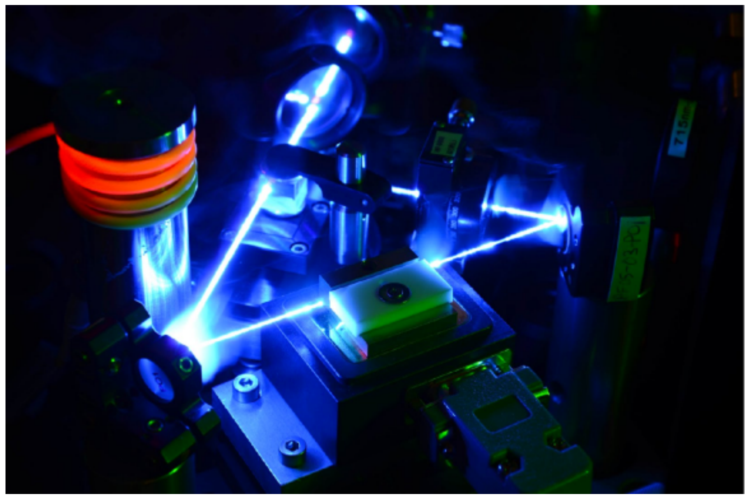

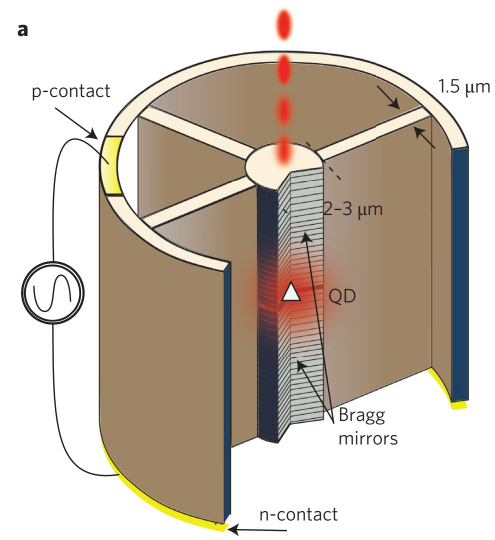


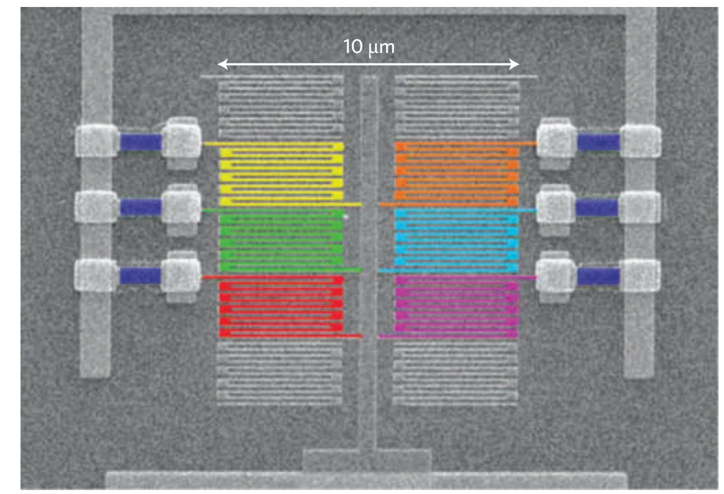


- Quantum optical **interconnects**: <u>links</u> in a global quantum network

Kimble, **Nature 453**, 1023 (2008)

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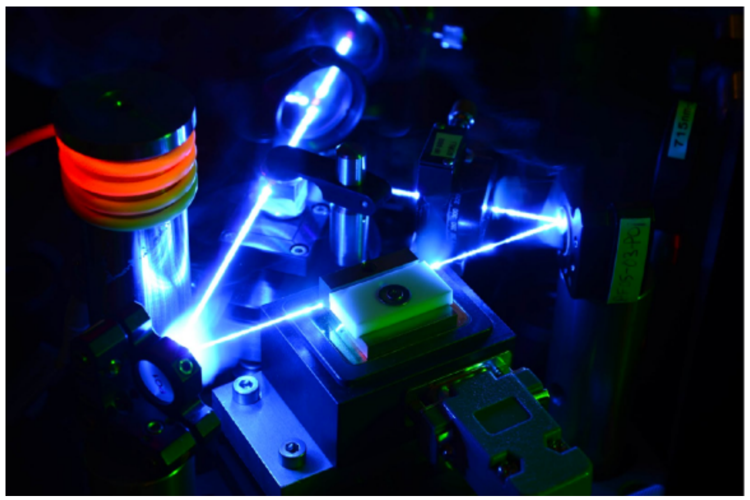

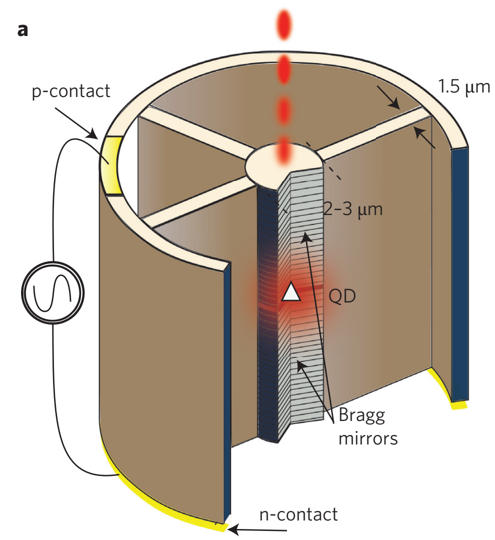



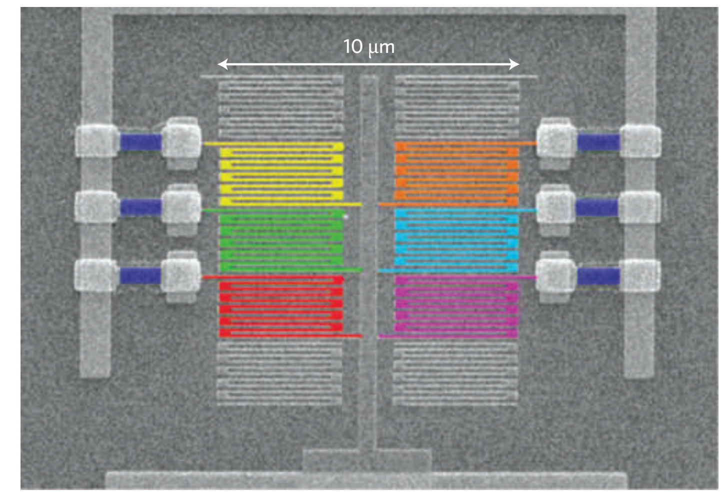



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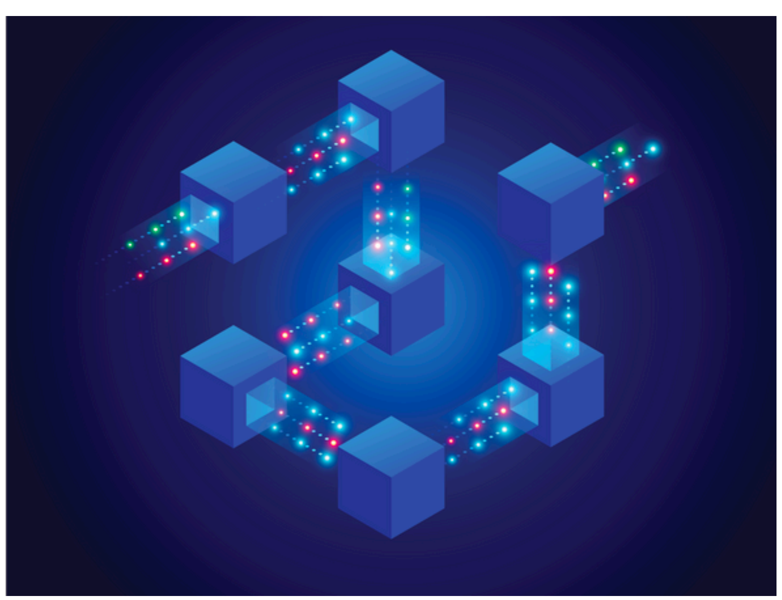

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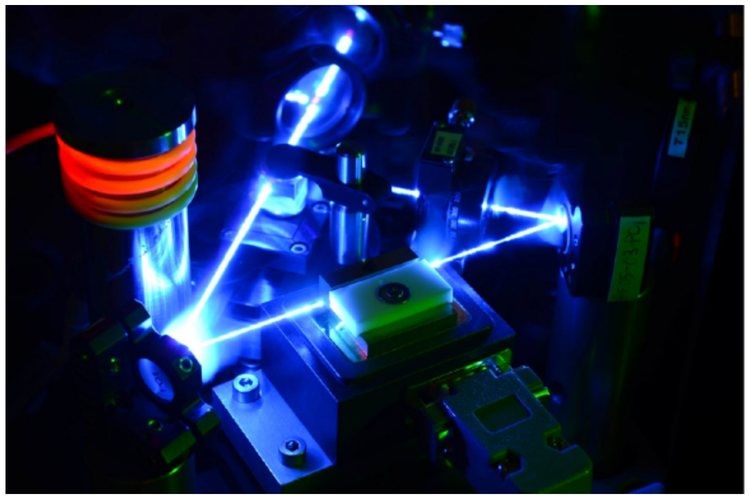
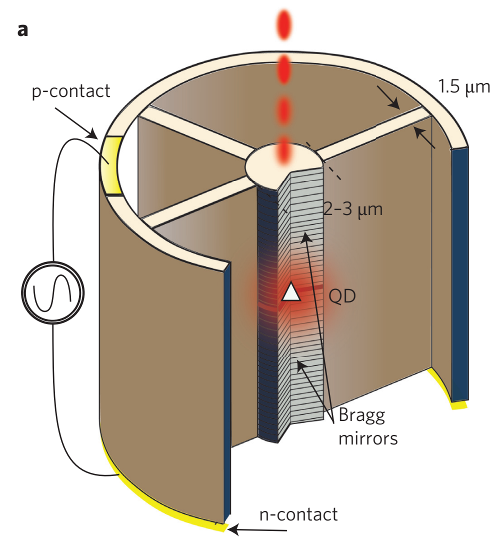

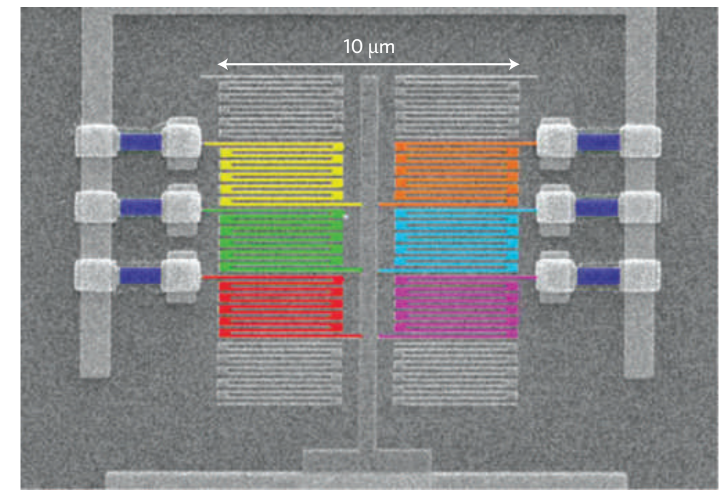


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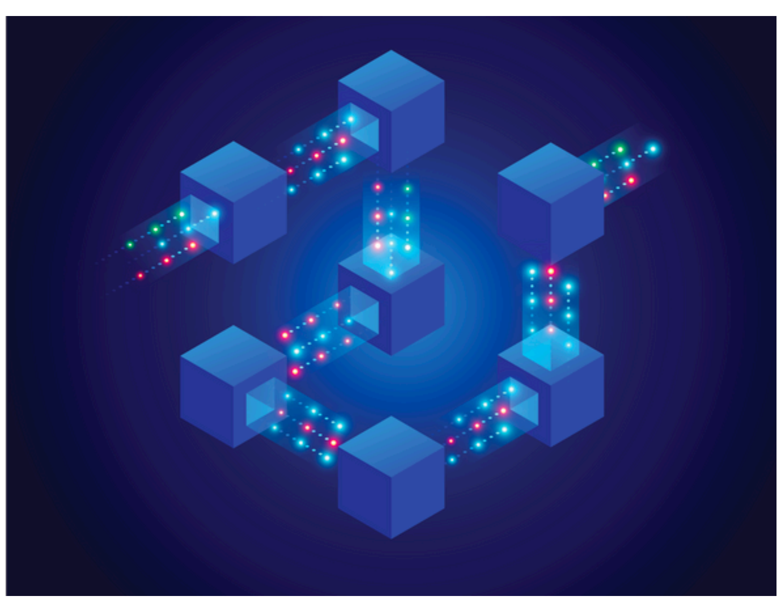
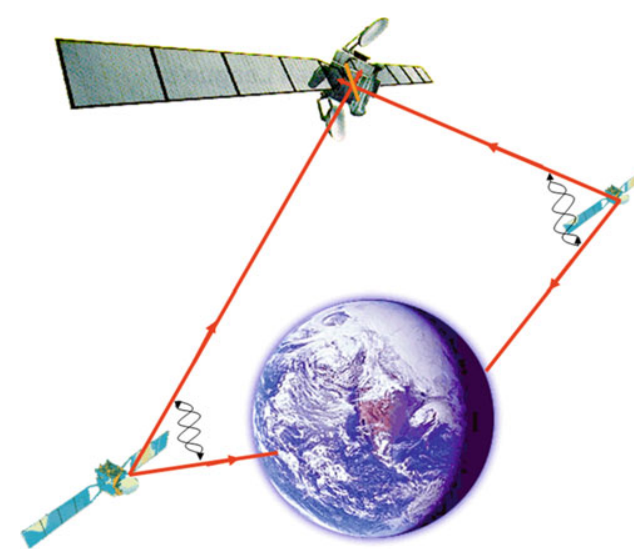

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# From Qubits to Qudits

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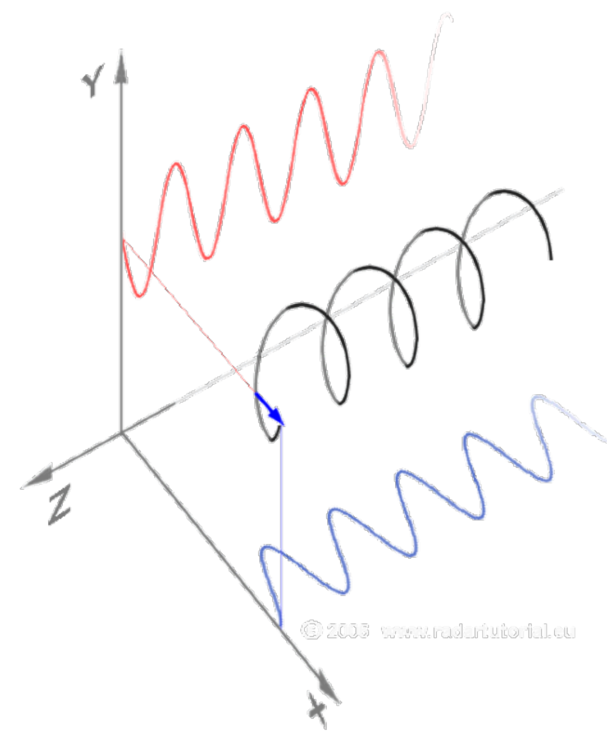
- Polarization: 2-level QM system (*qubit*)
  - Quantum optics workhorse




$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|H\rangle + i|V\rangle]$$

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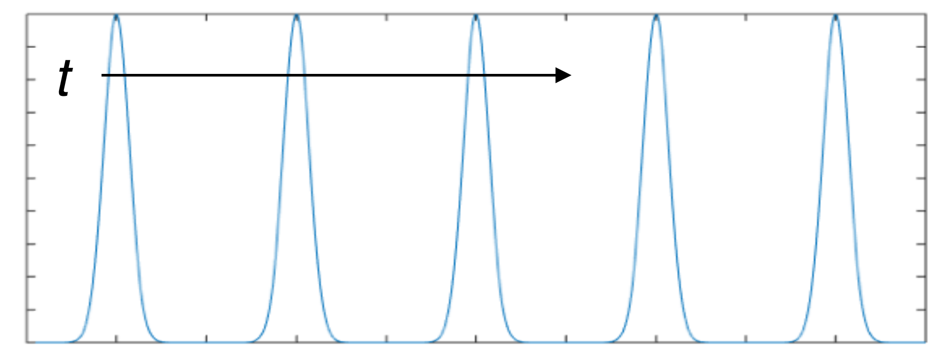


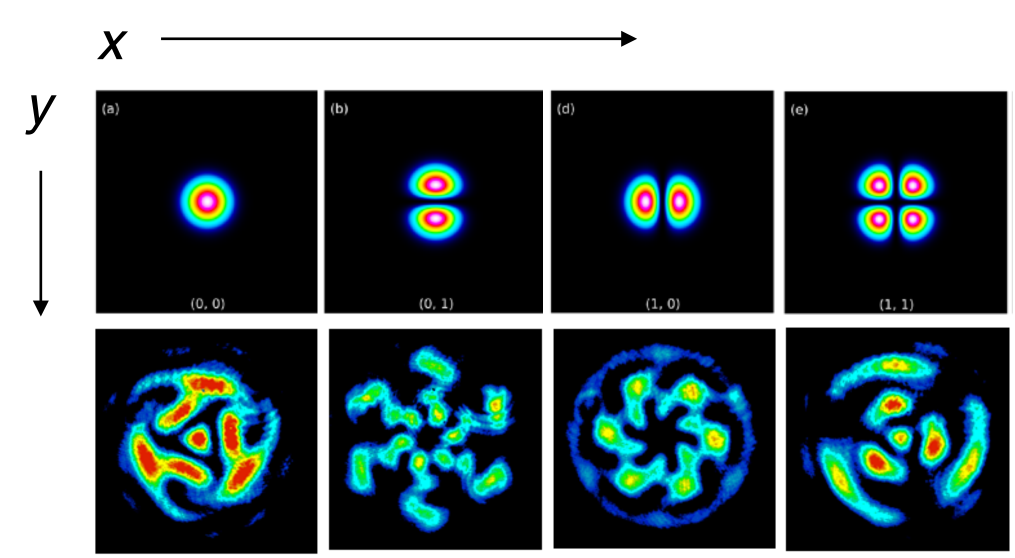

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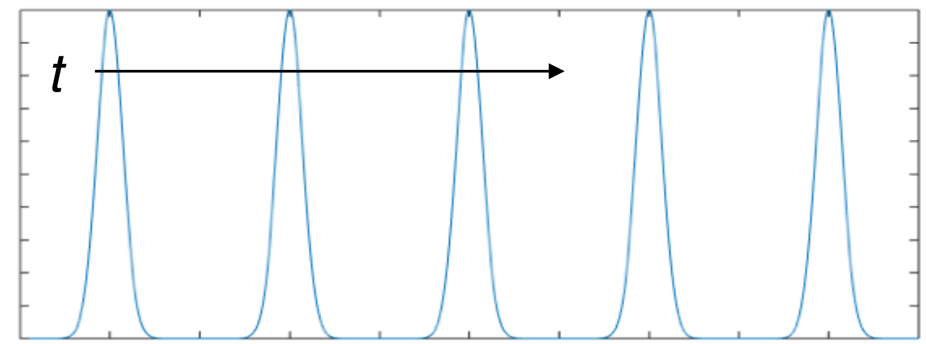
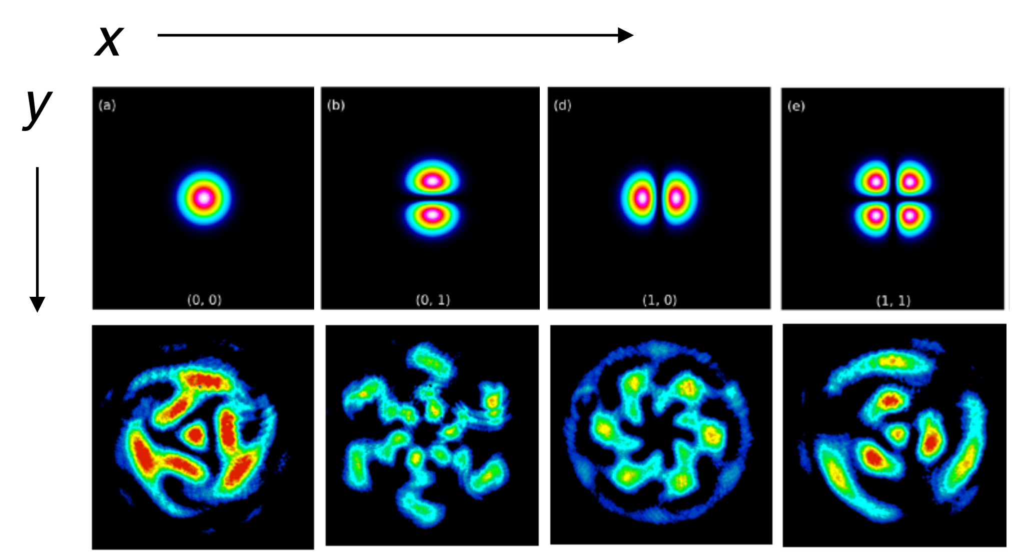



$$|\Psi\rangle = \frac{1}{\sqrt{D}}[|t_0\rangle + |t_1\rangle + |t_2\rangle ... + |t_{D-1}\rangle]$$

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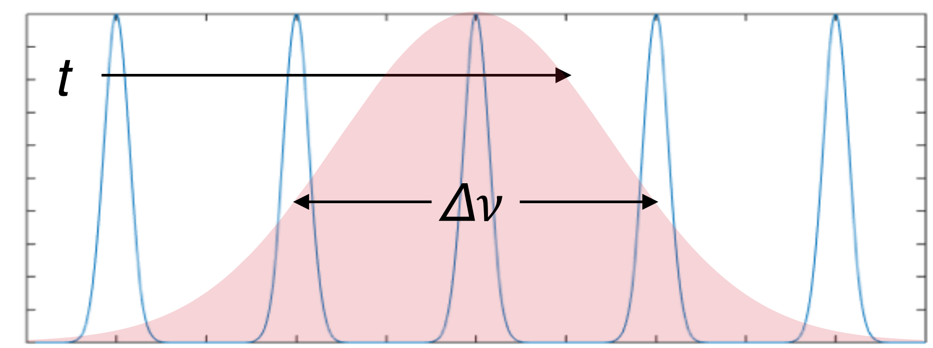
    - *D*-level quantum system! (*qudit*)
- Limits: uncertainty principle + physical system

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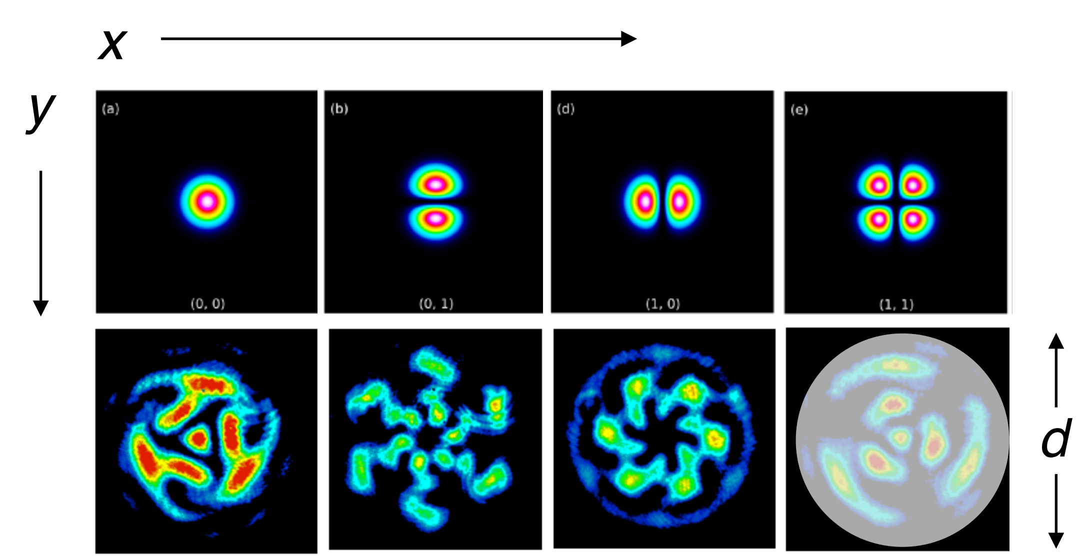


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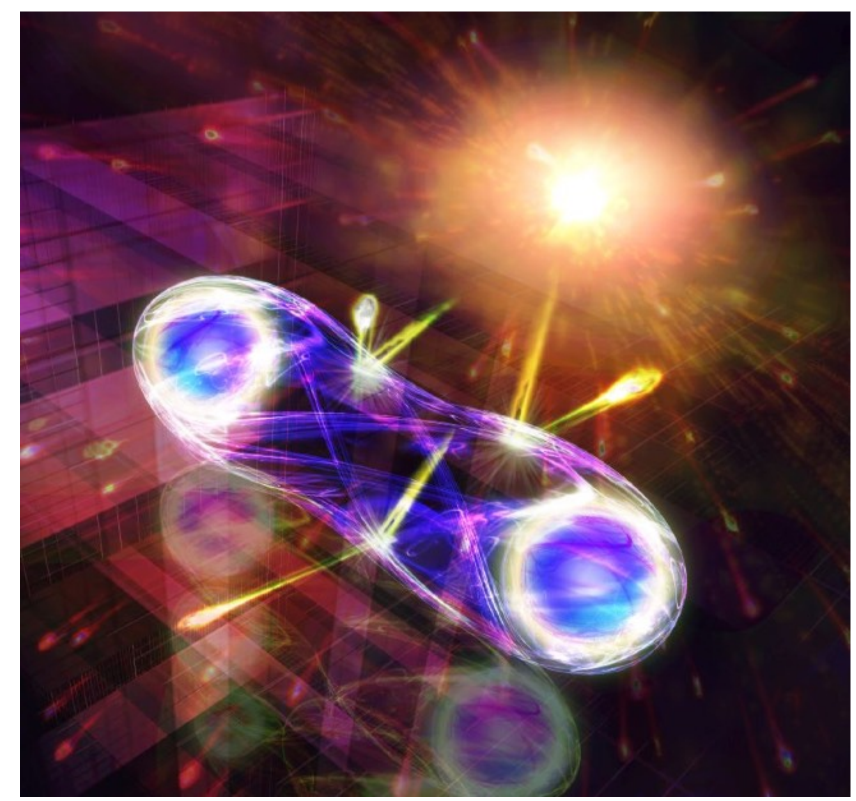

Ecker, **MM** et al, **PRX 9, 041042** (2019)

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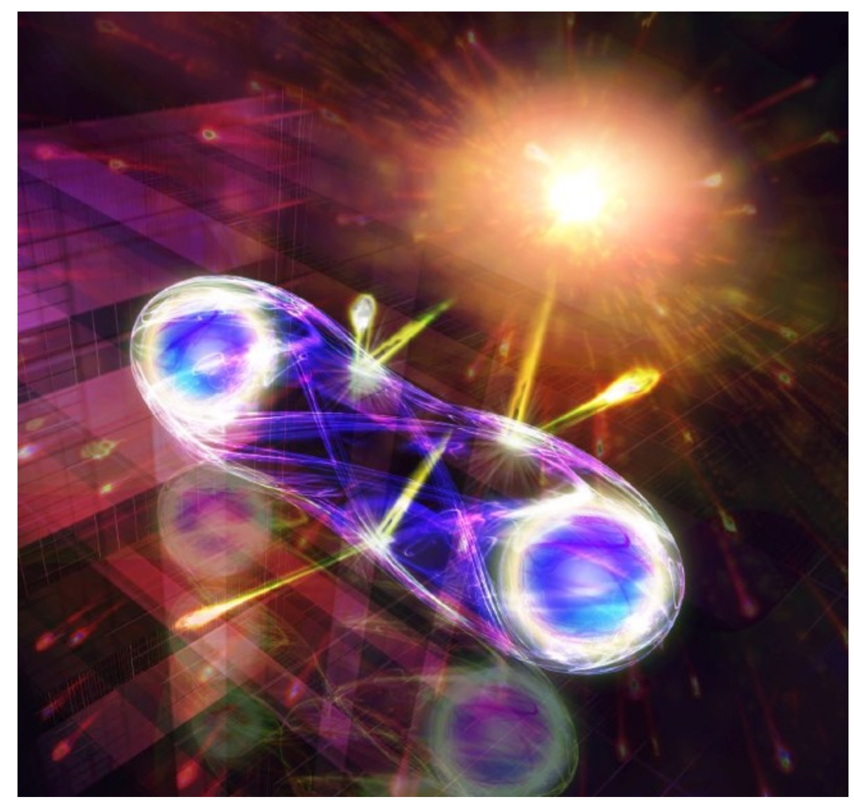

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# Orbital Angular Momentum of Light

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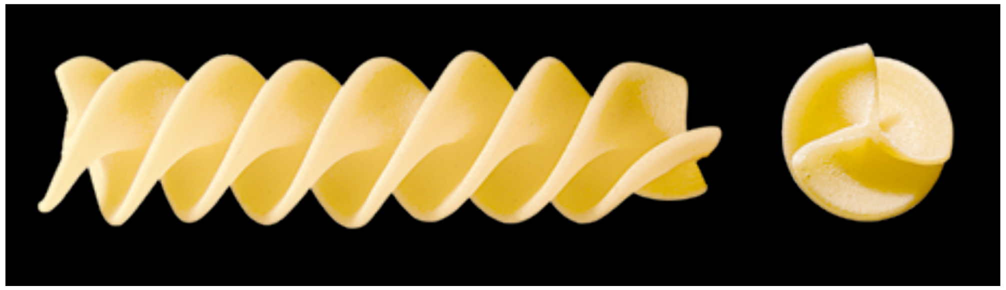

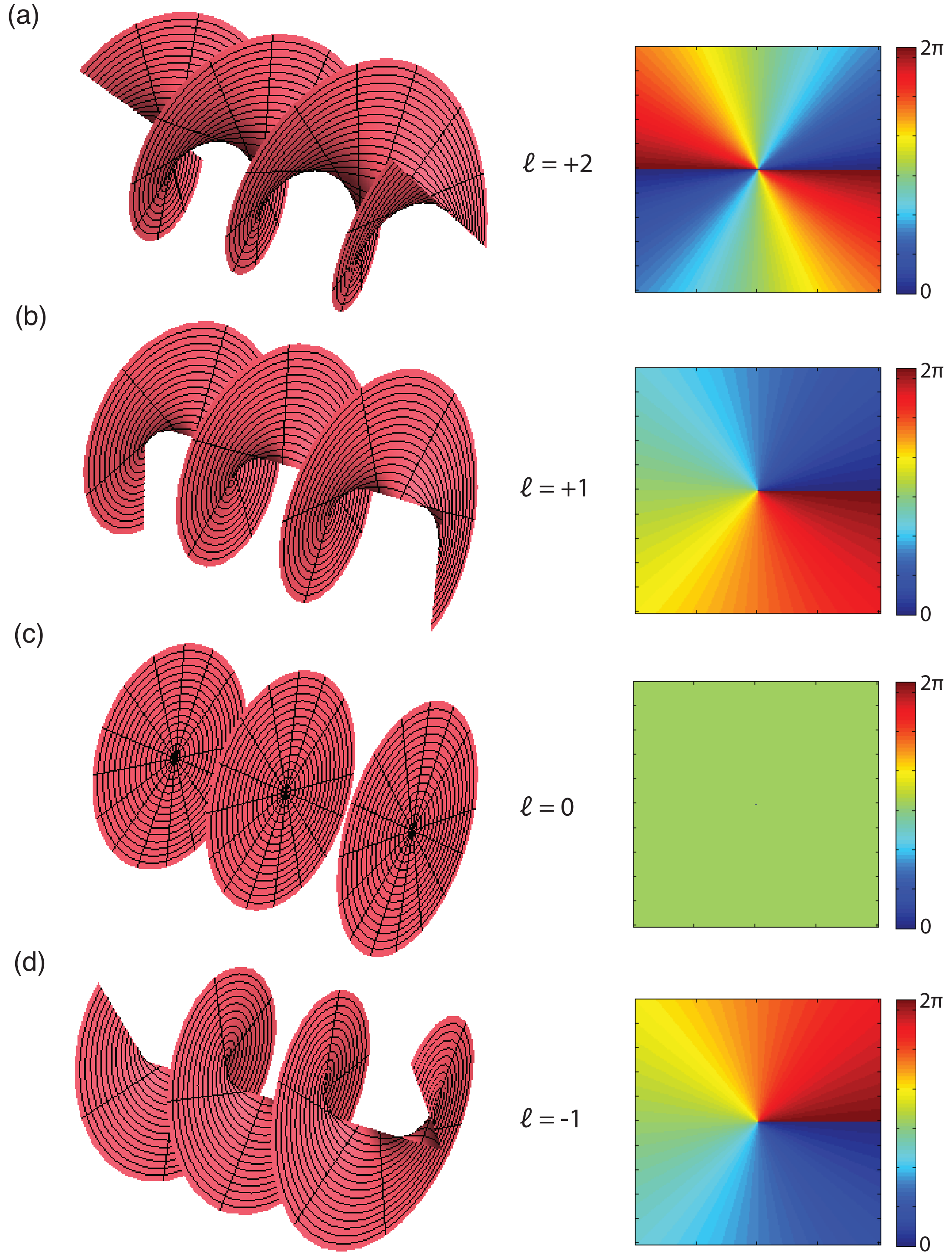


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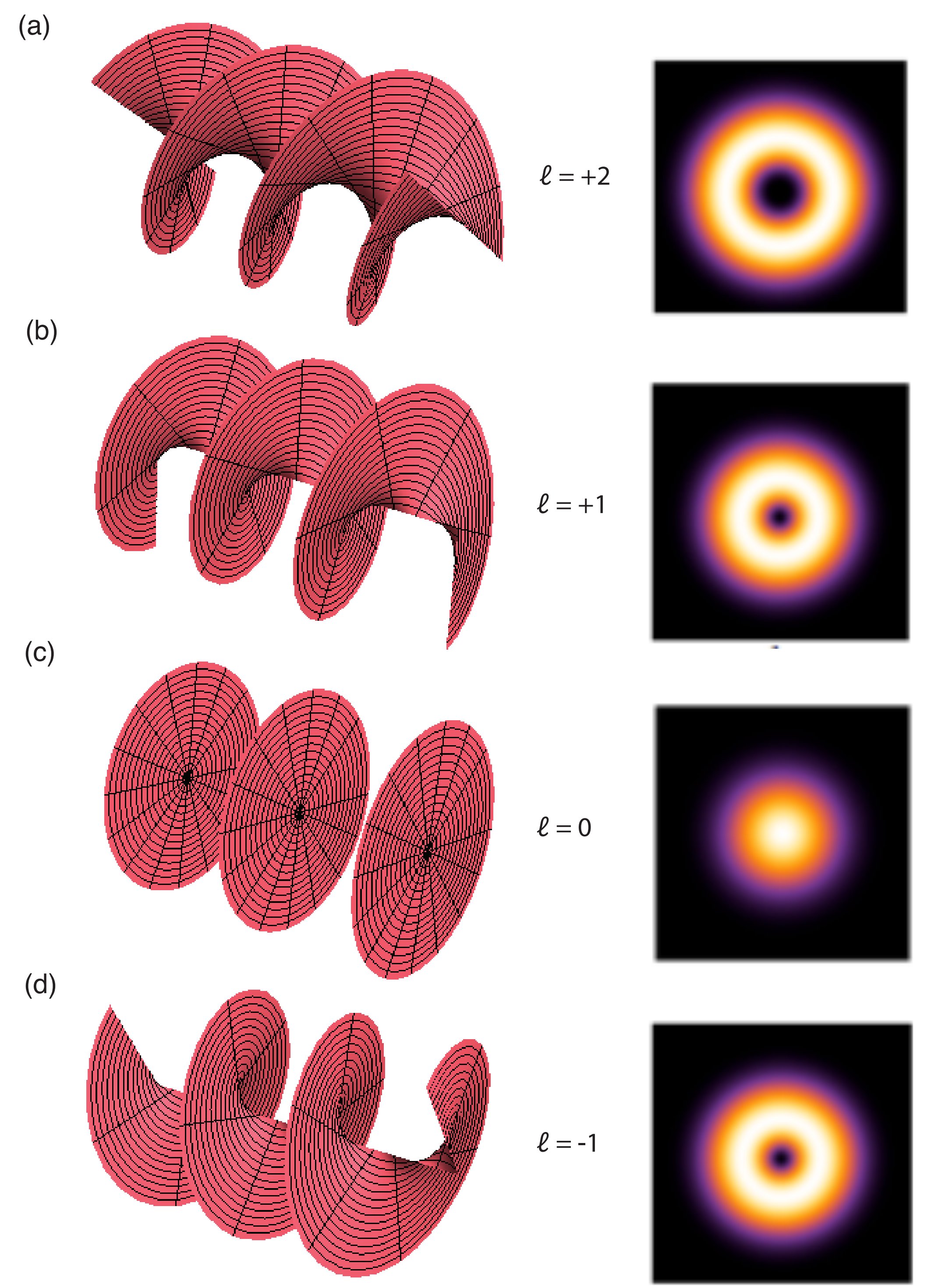


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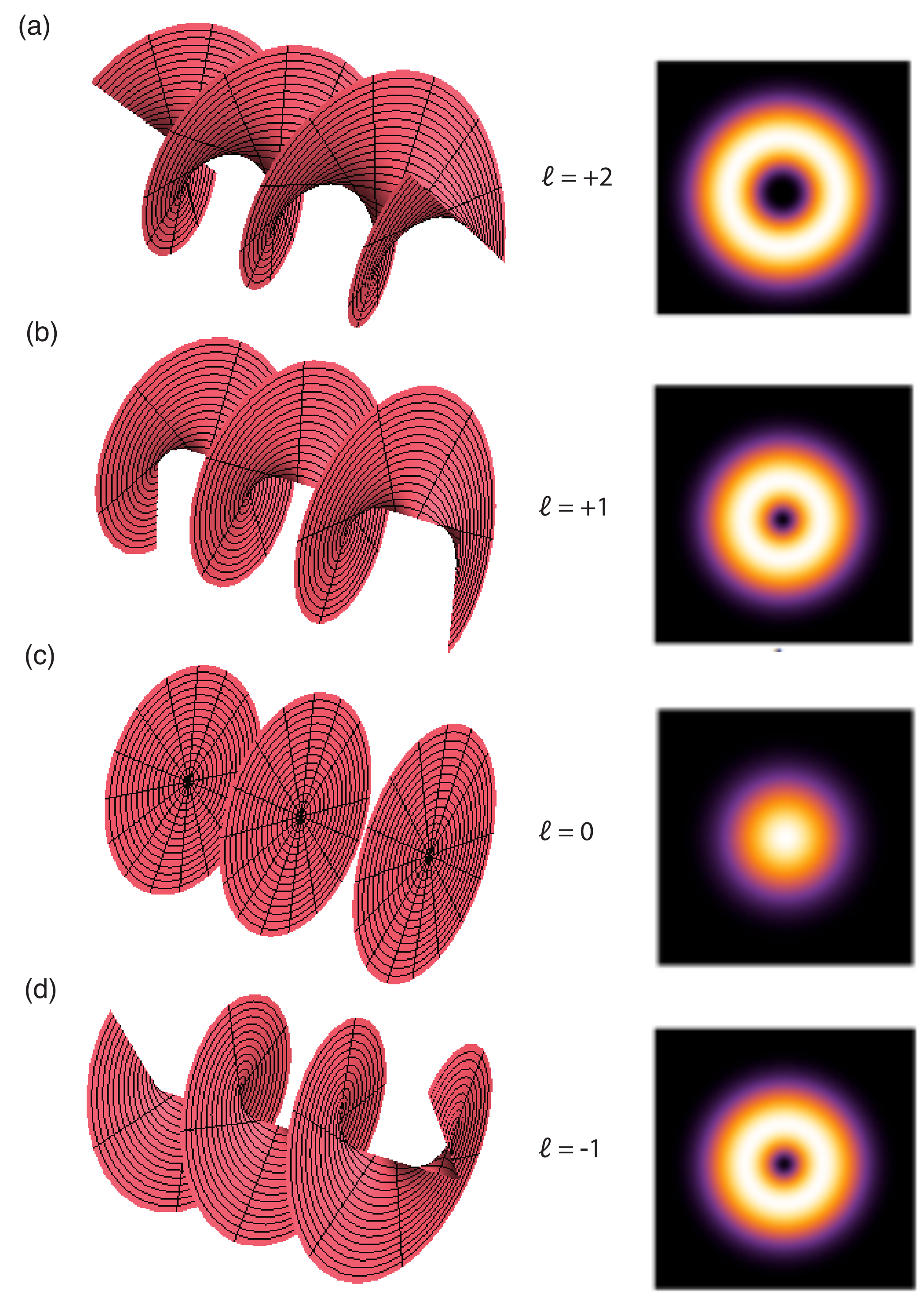


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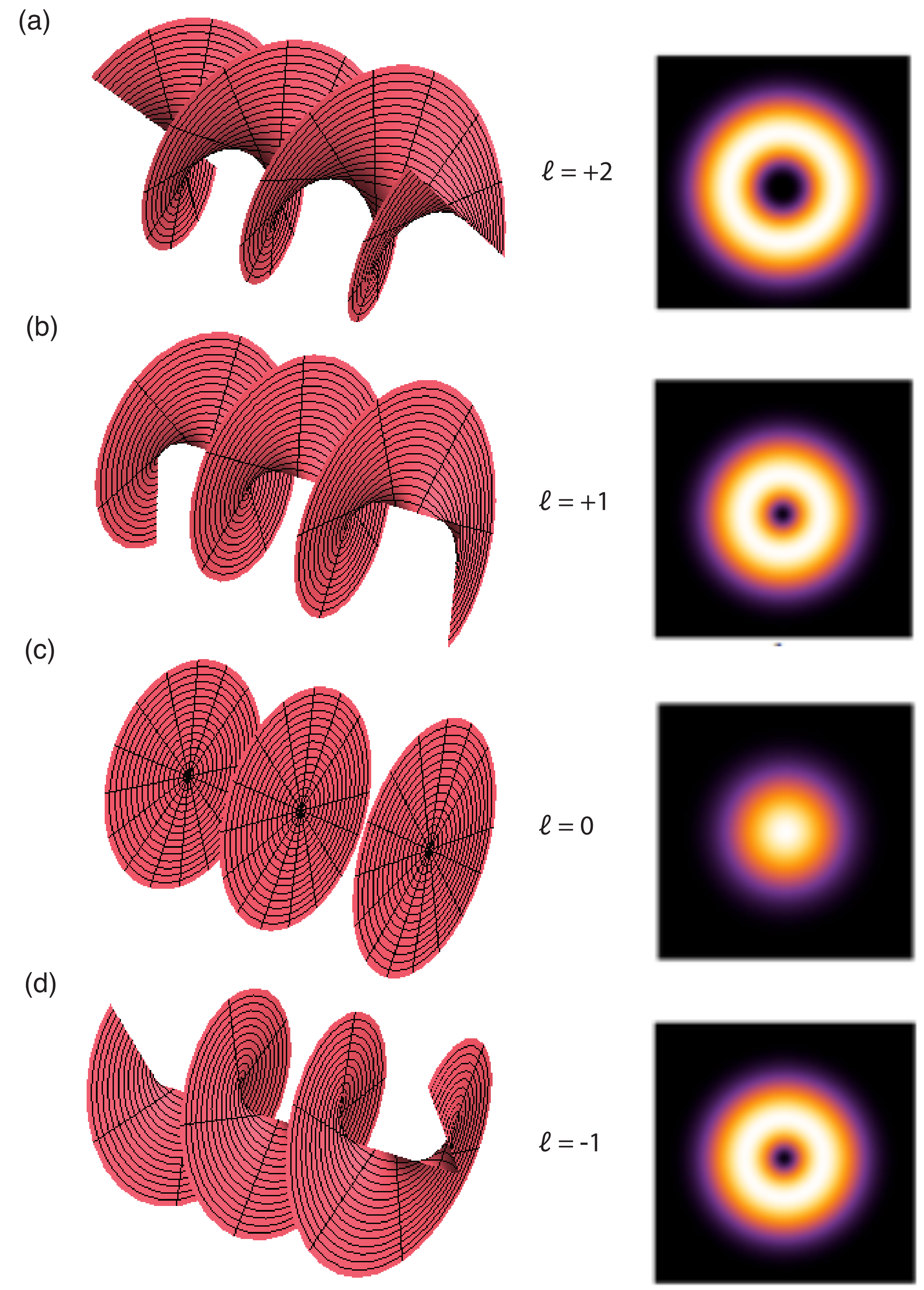


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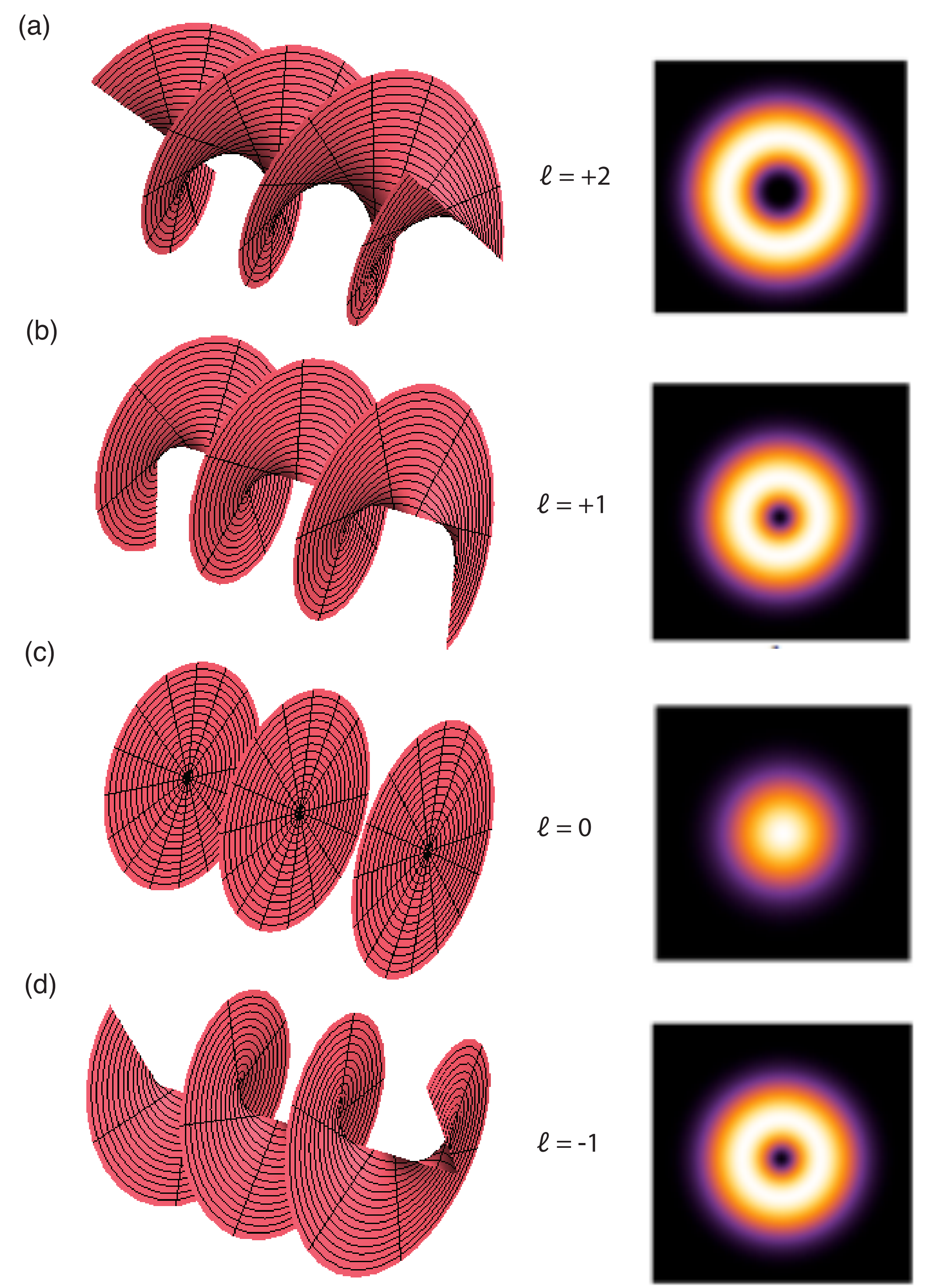


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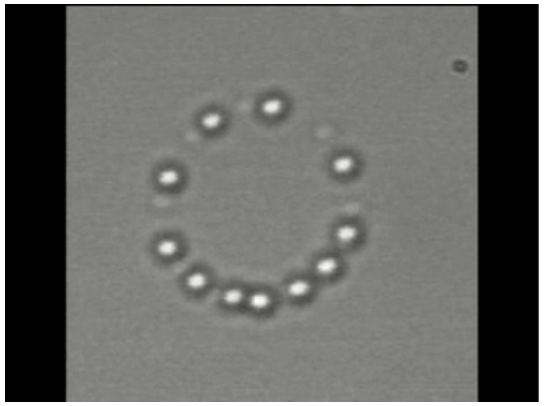

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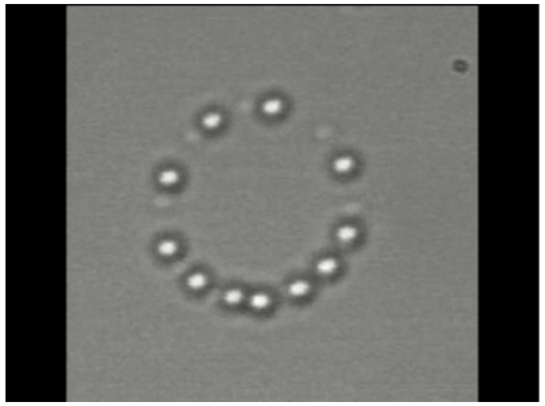
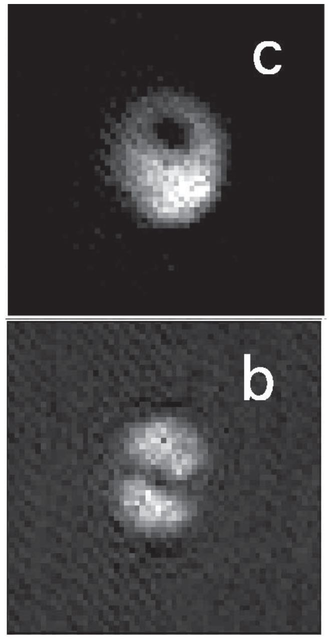


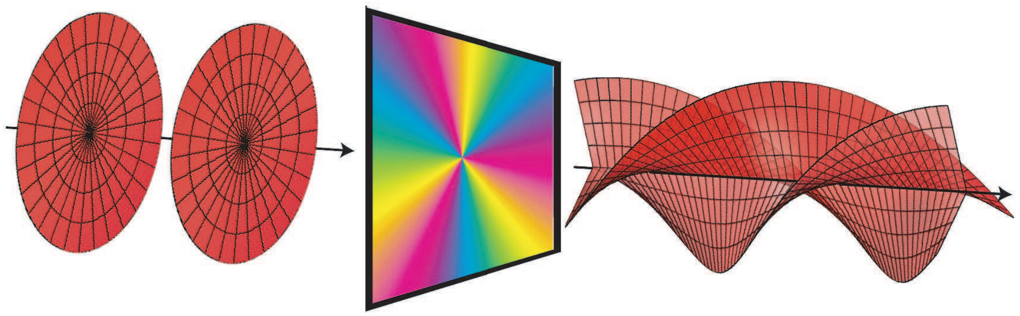



Andersen et al, **PRL 97,** 170406 (2006)

# GENERATING SPATIAL MODES

# Generating Spatial Modes

- Helical phase profile on SLM



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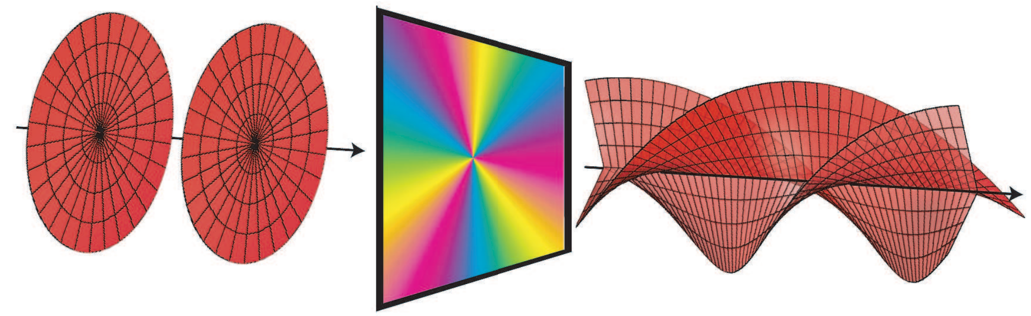
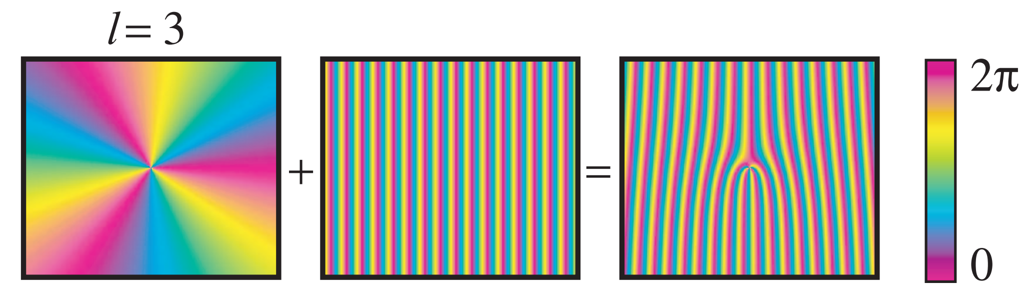
- Added to linear phase ramp (mod 2π)





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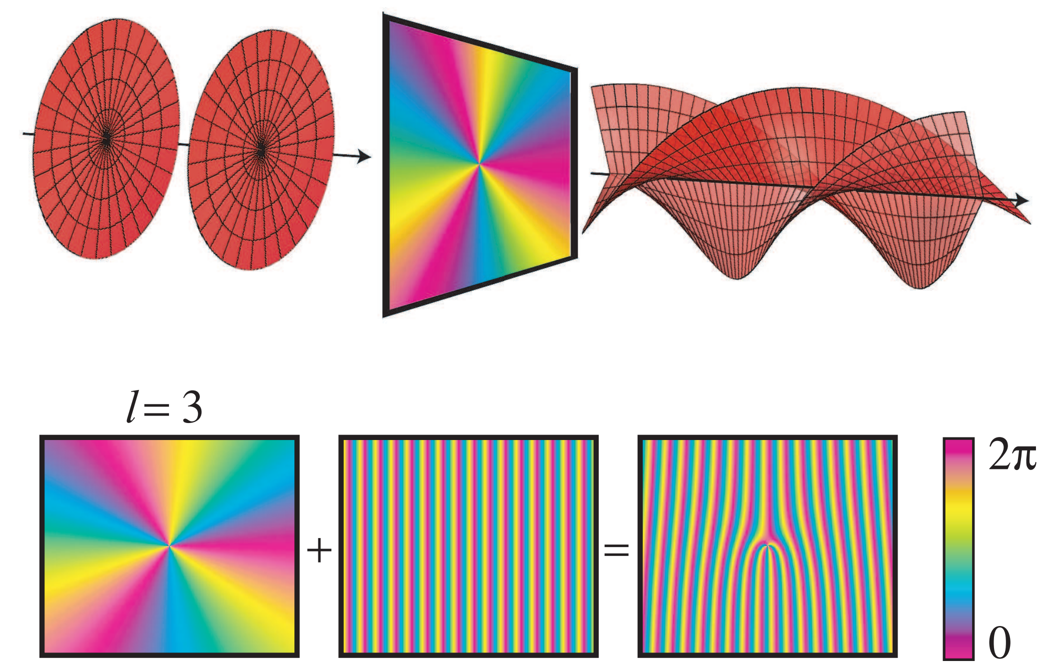


Arrizón et al., **J. Opt. Soc. Am. A 29,** 3500 (2007)
**MM** and R. Boyd, **Riv Nuovo Cimento 37,** 273 (2014)

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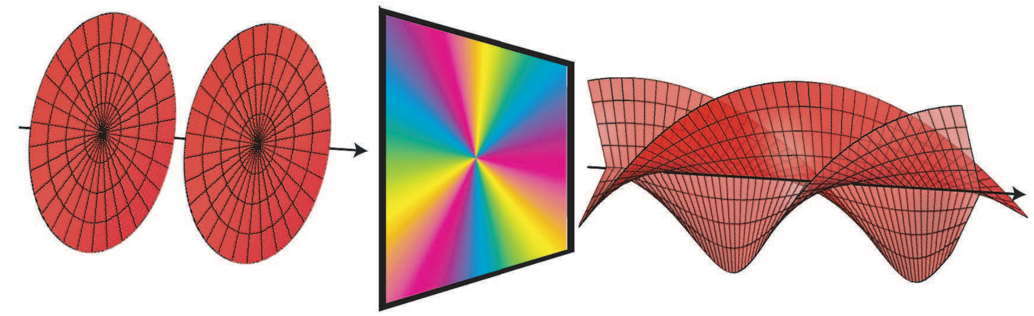

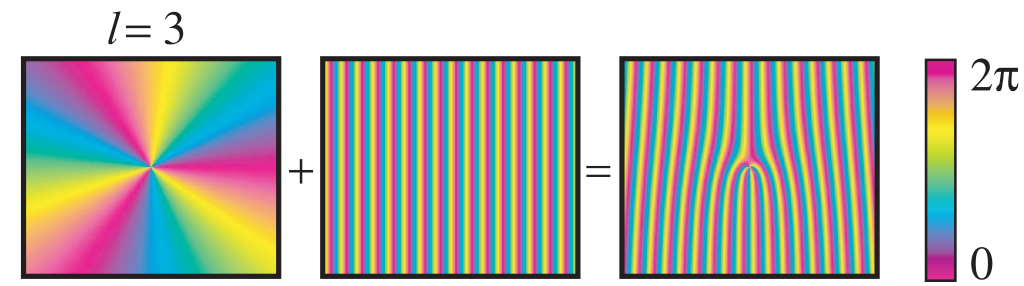


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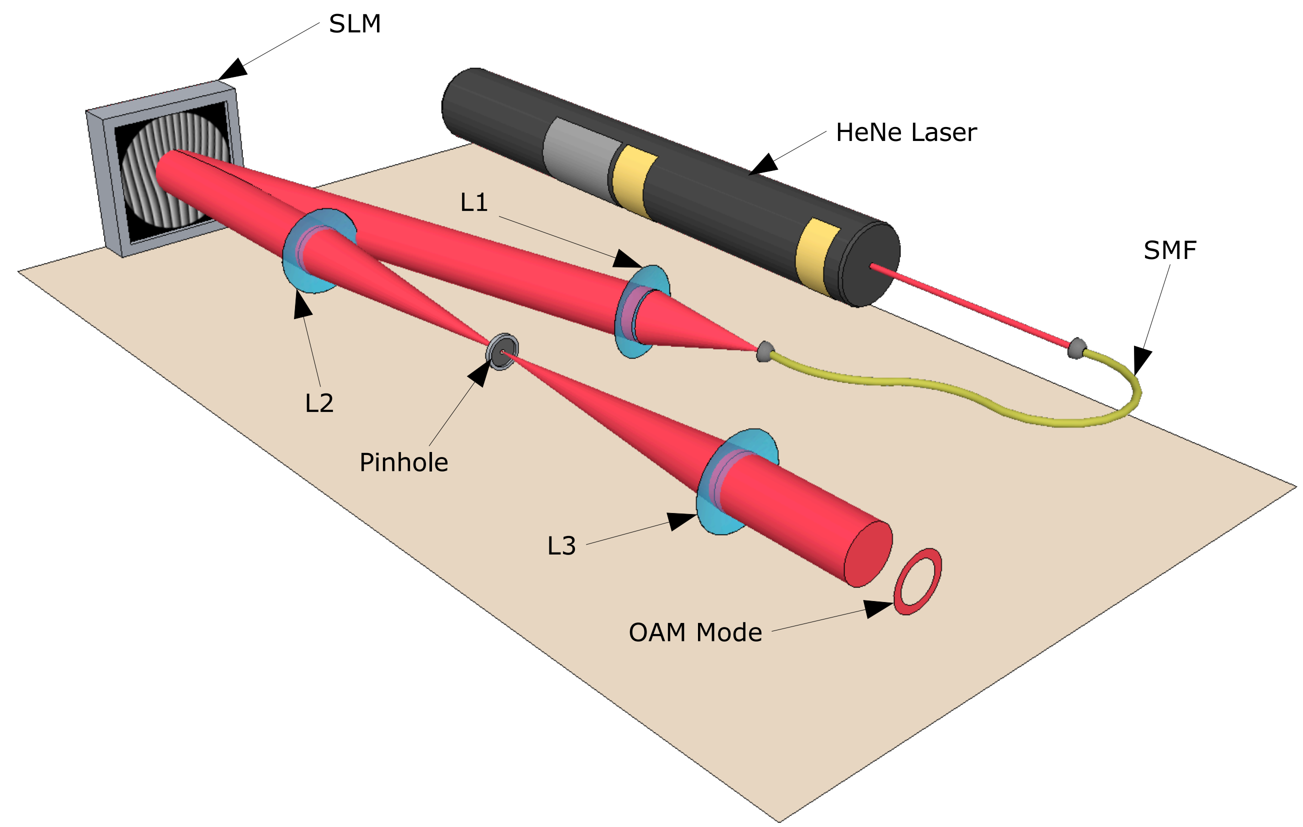


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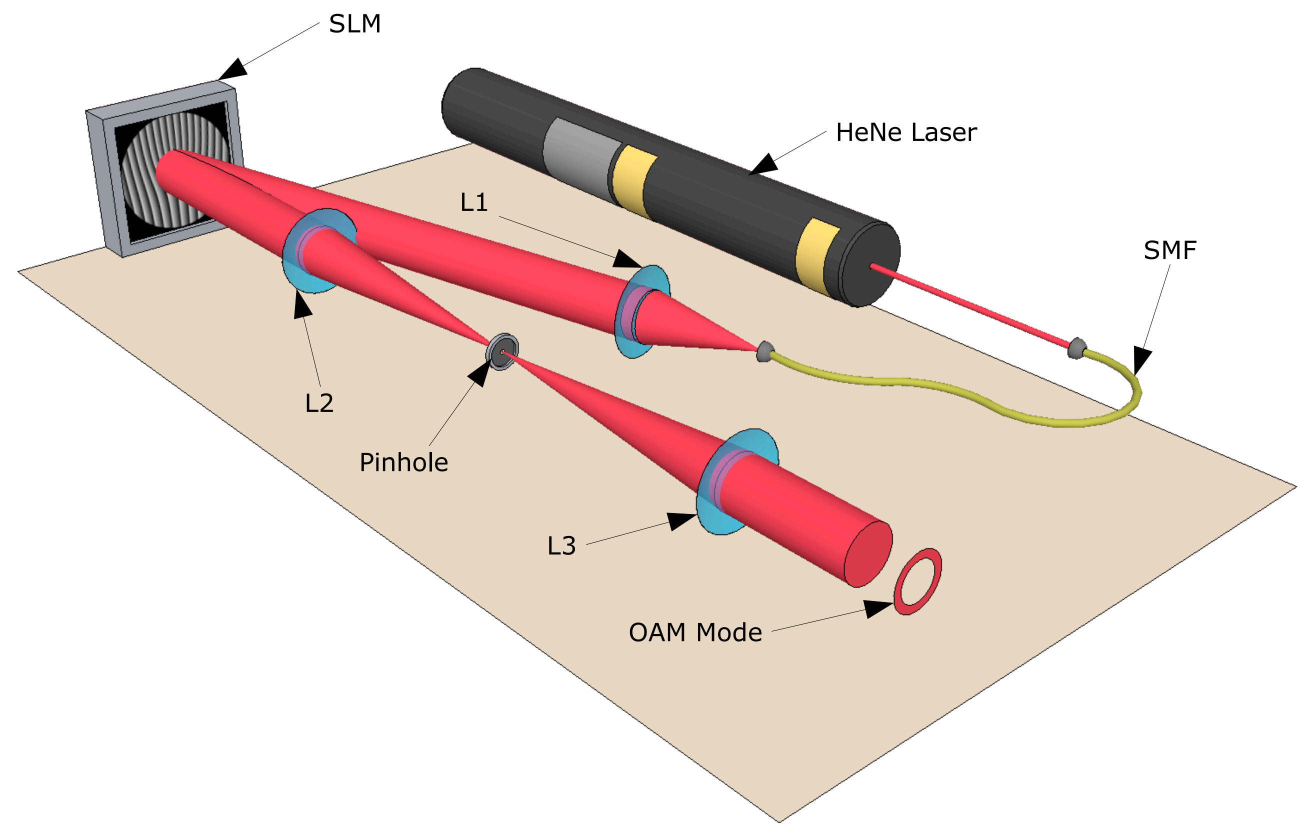
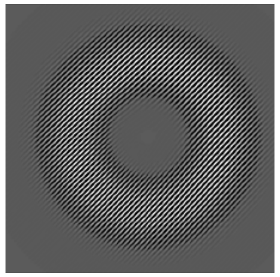
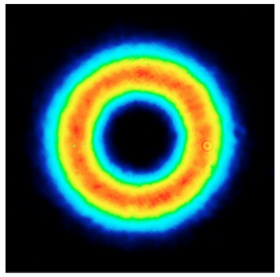


$\ell = -5$



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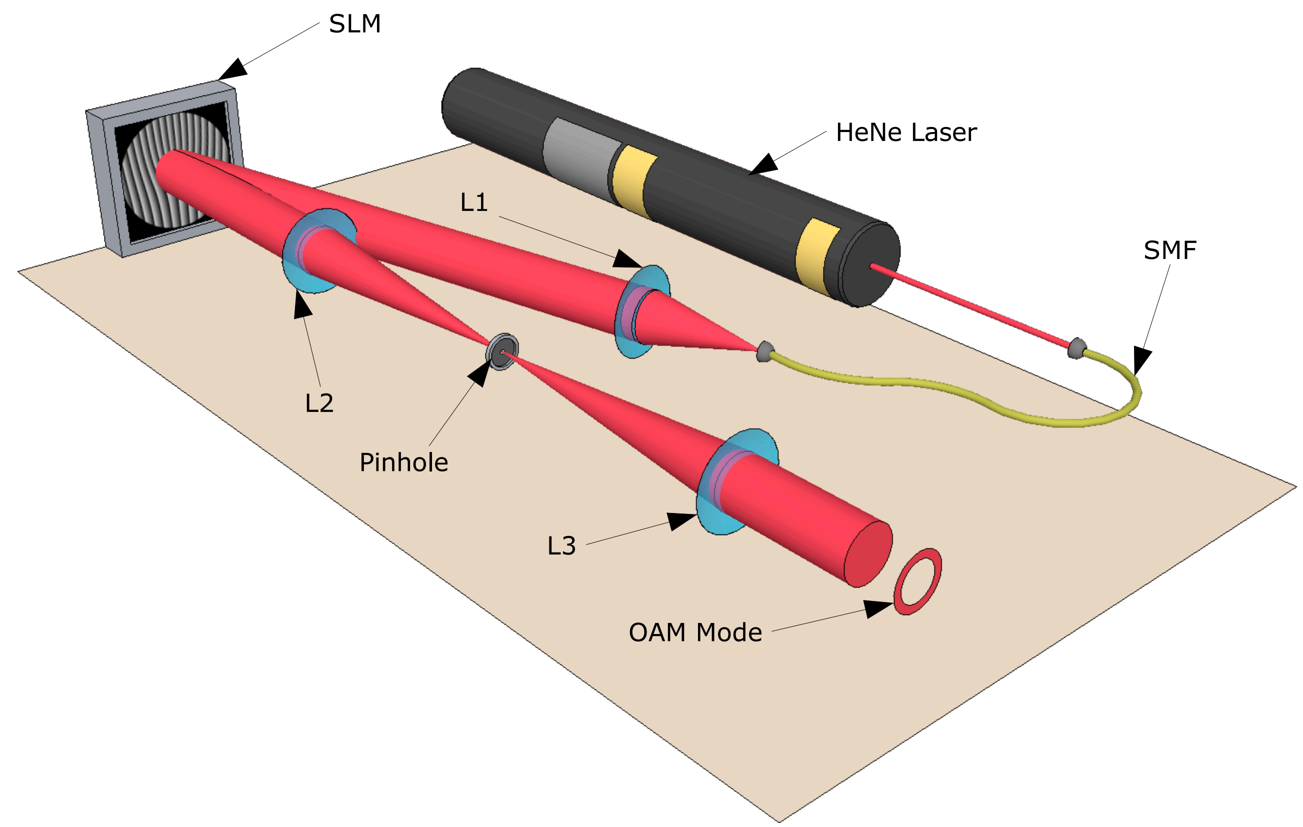


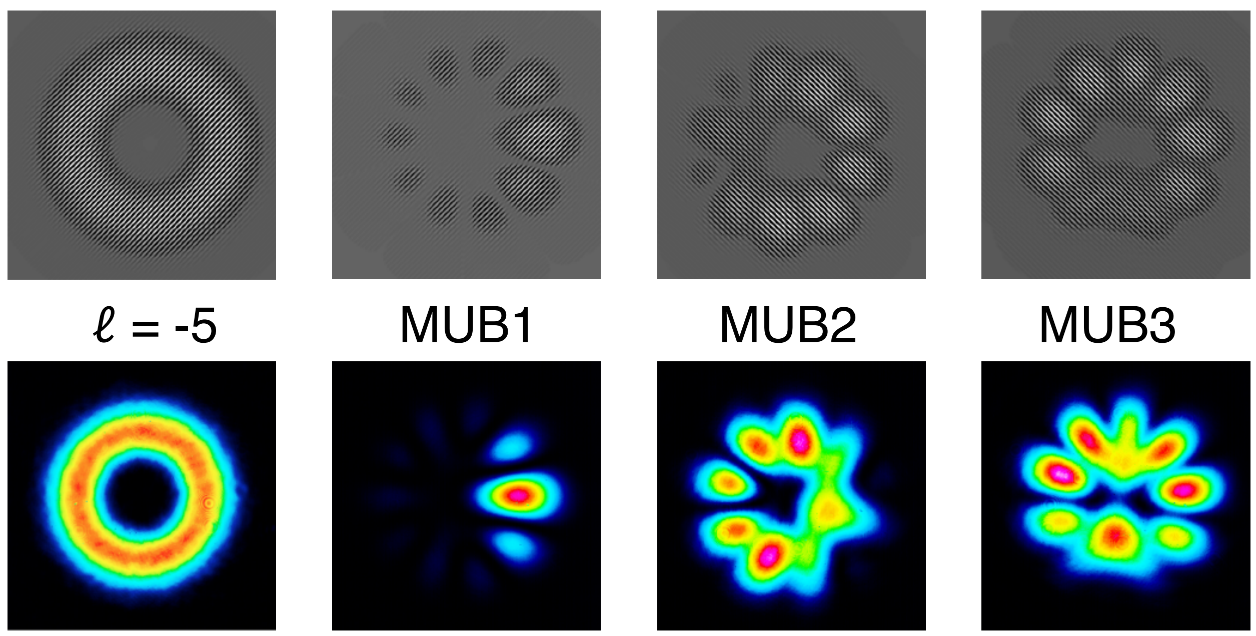


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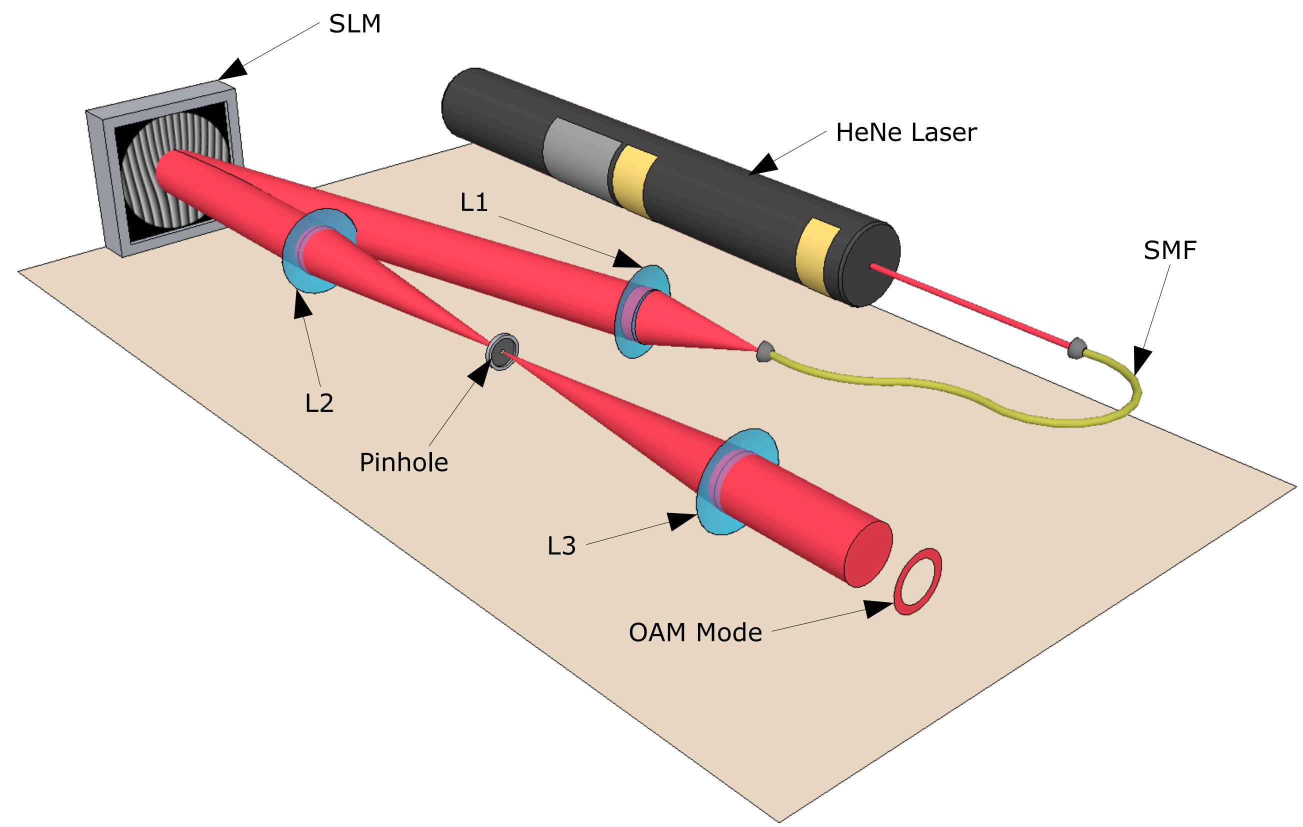
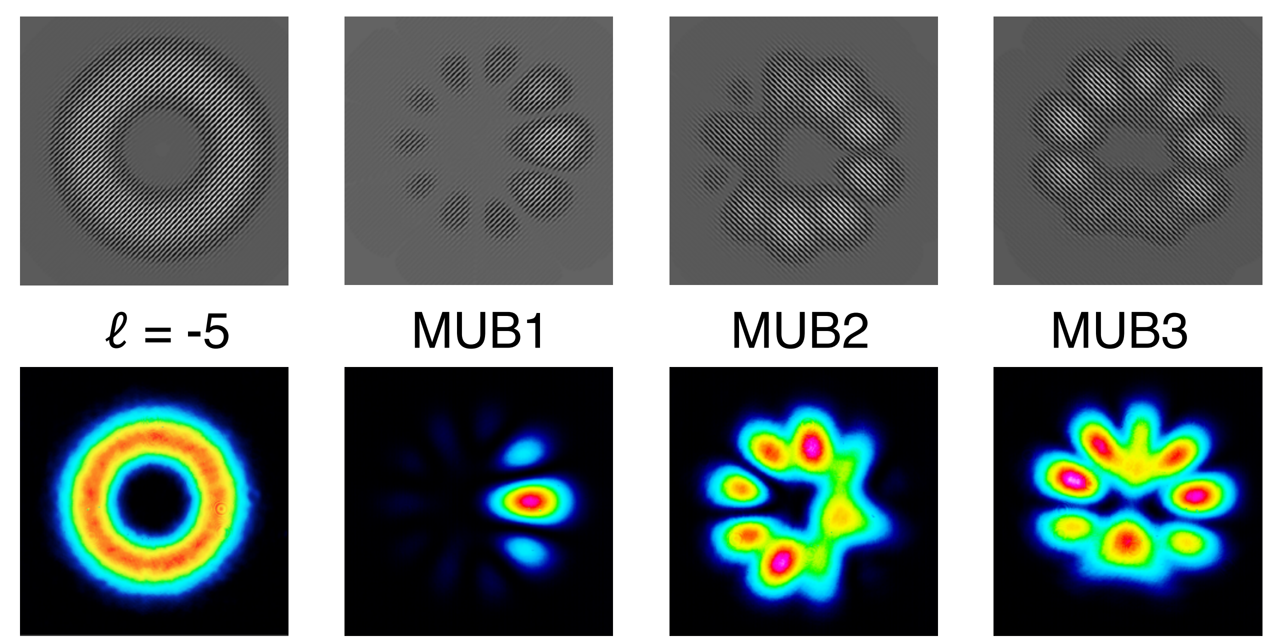
- Reverse process used for detection

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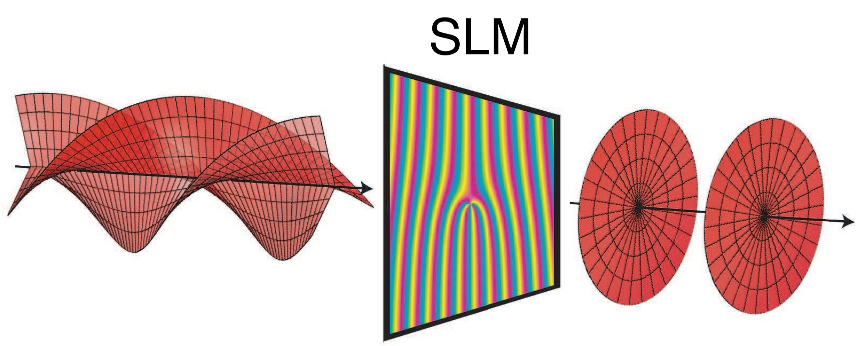
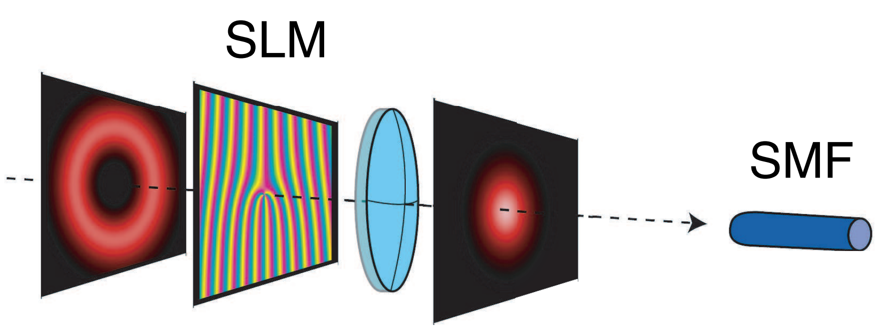
# Measuring Spatial Modes




Bouchard, Herrera, Brandt, Fickler, Huber, and **MM**, Opt. Exp. 26, 31924 (2018)

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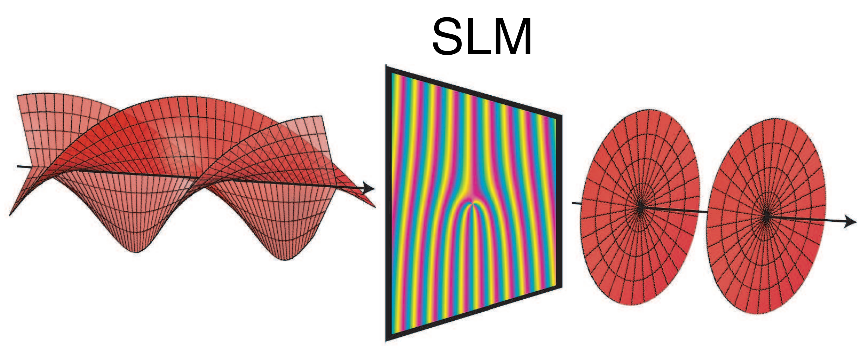


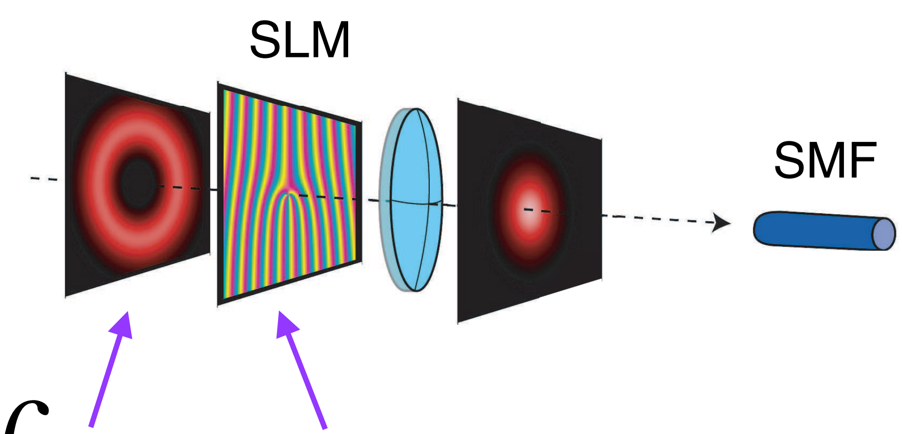


$$\iint A(x,y)\,A^*(x,y)$$

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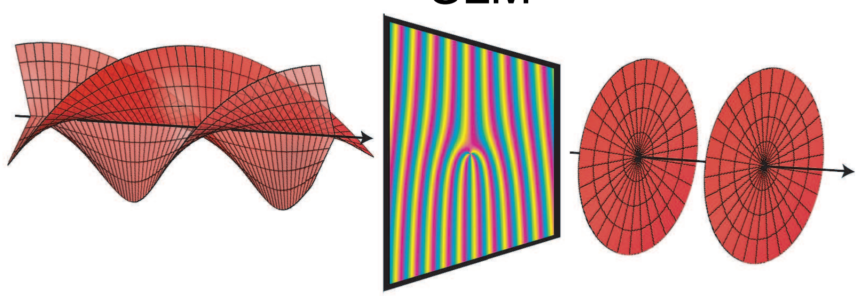


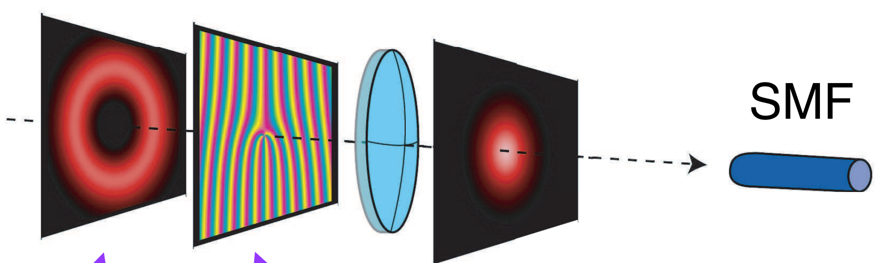


$$\iint A(x,y)\,A^*(x,y)$$

$$A(x,y) \rightarrow \iint |A|^2 \rightarrow \text{non-zero}$$

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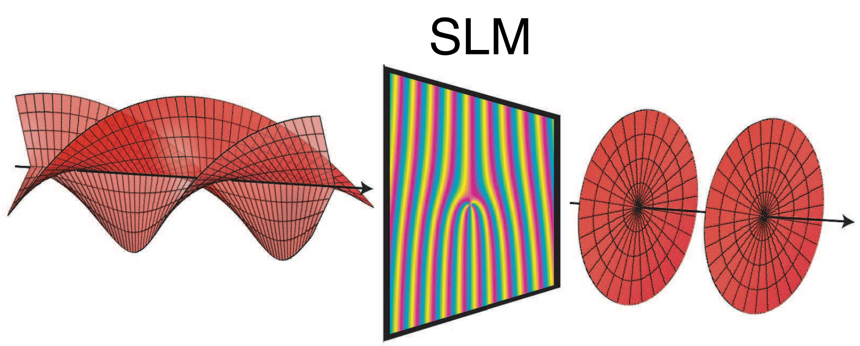


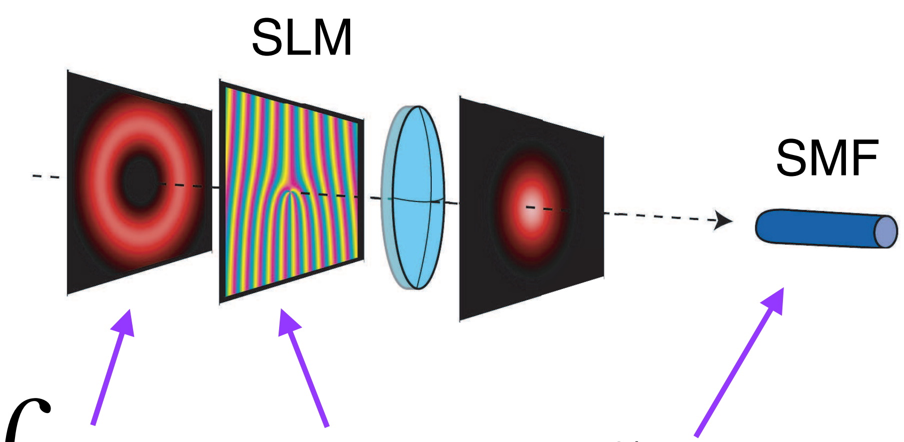


$$\iint A(x,y)\,A^*(x,y)\,\widetilde{LG}_{00}(x,y)$$

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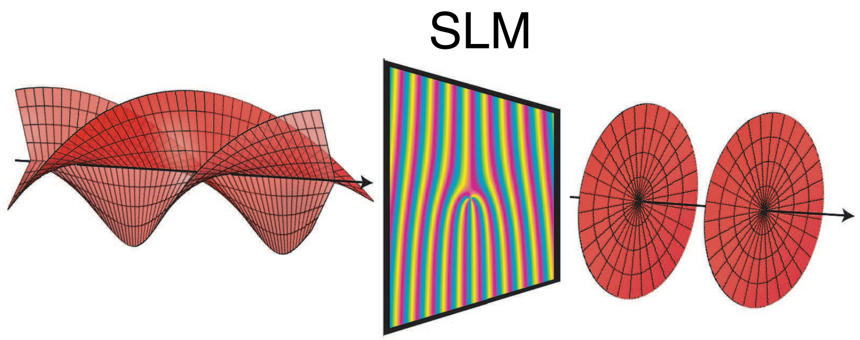


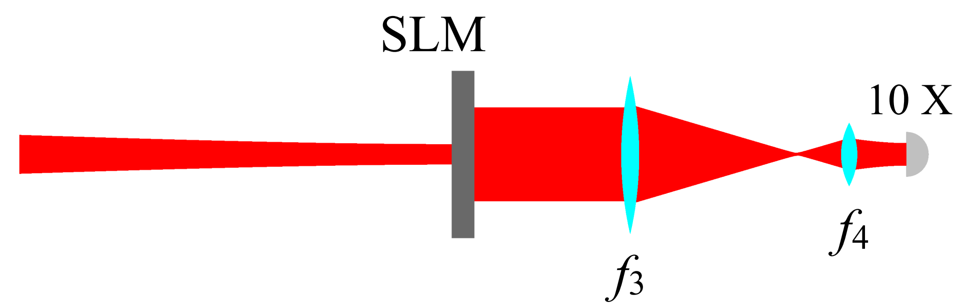


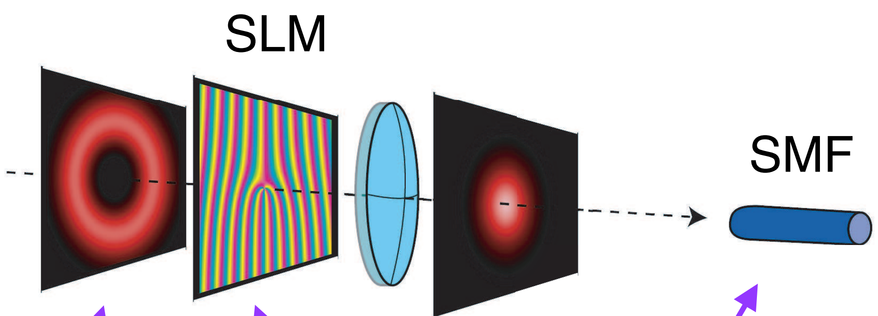


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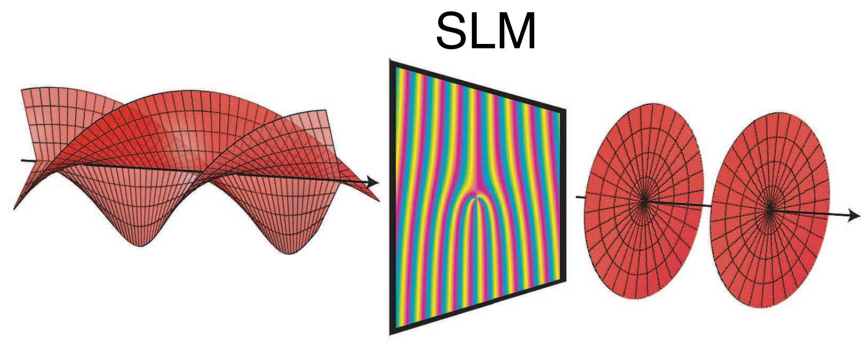


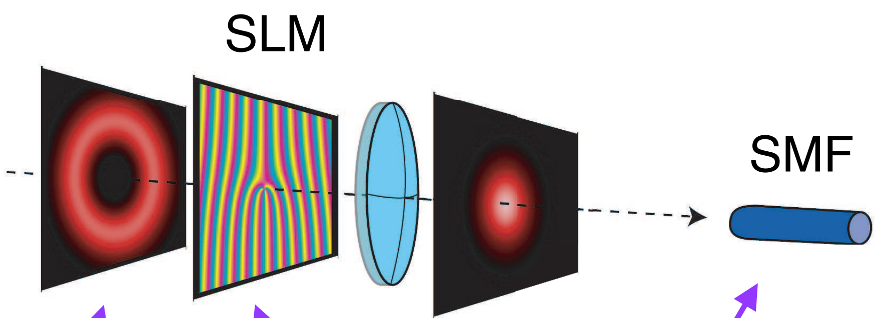


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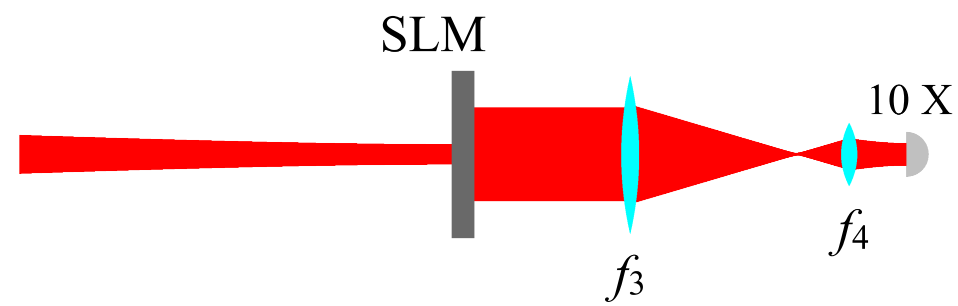


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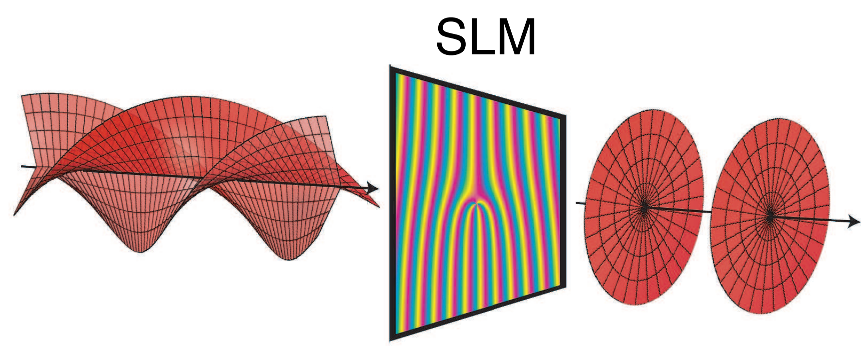


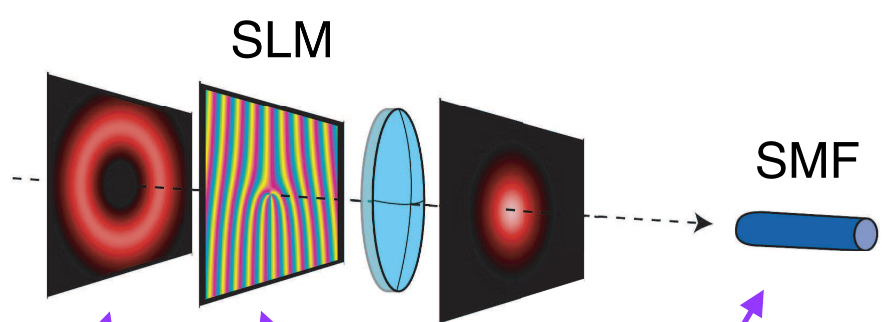


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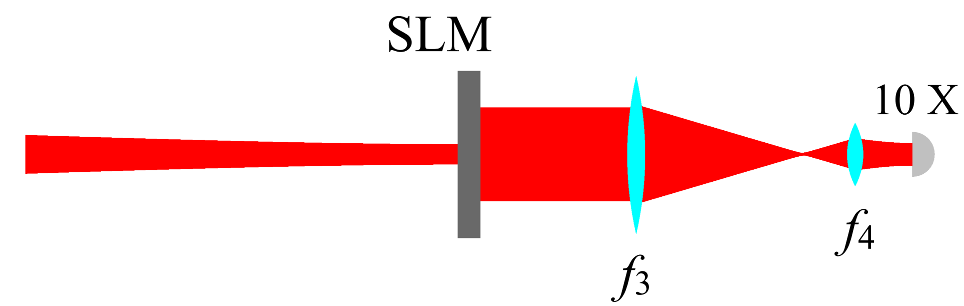


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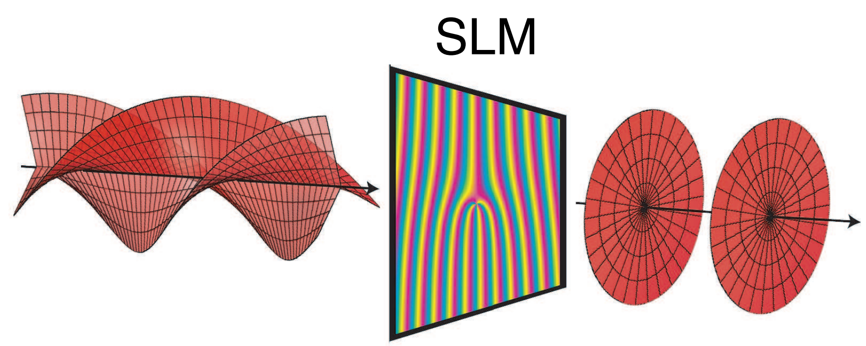
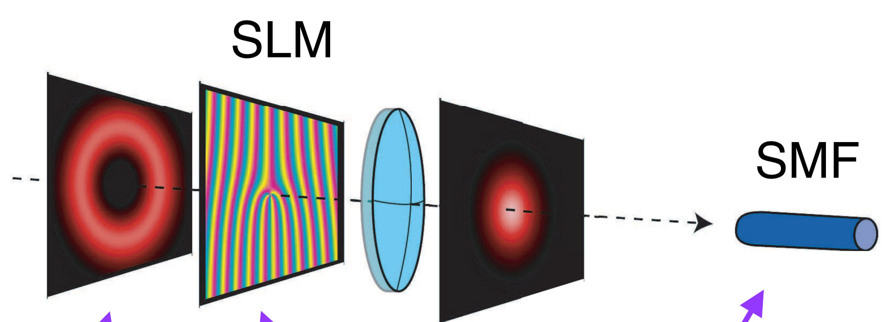
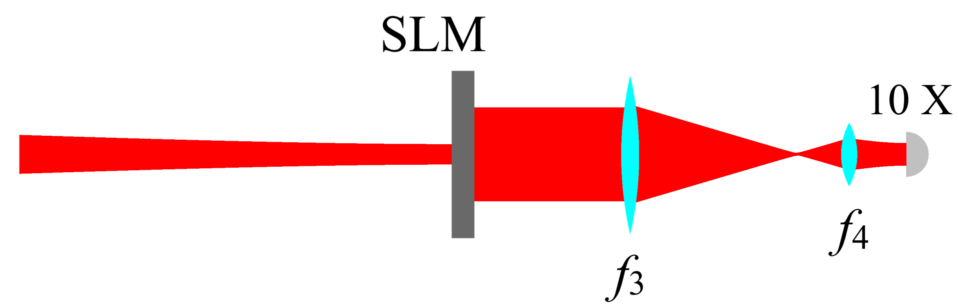
Bouchard, Herrera, Brandt, Fickler, Huber, and **MM**, Opt. Exp. 26, 31924 (2018)

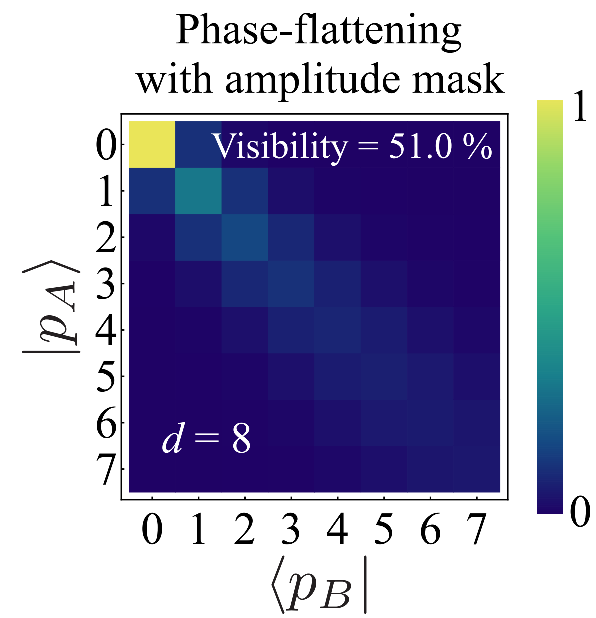
# Measuring Spatial Modes

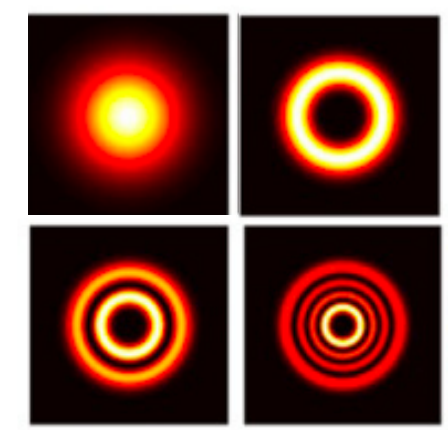



$$\iint A(x,y)\,A^*(x,y)\,\cancelto{k}{L\tilde{G}_{00}}(x,y)$$



$$A(x,y) \rightarrow \iint |A|^2 \rightarrow \text{non-zero}$$

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Bouchard, Herrera, Brandt, Fickler, Huber, and **MM**, Opt. Exp. 26, 31924 (2018)

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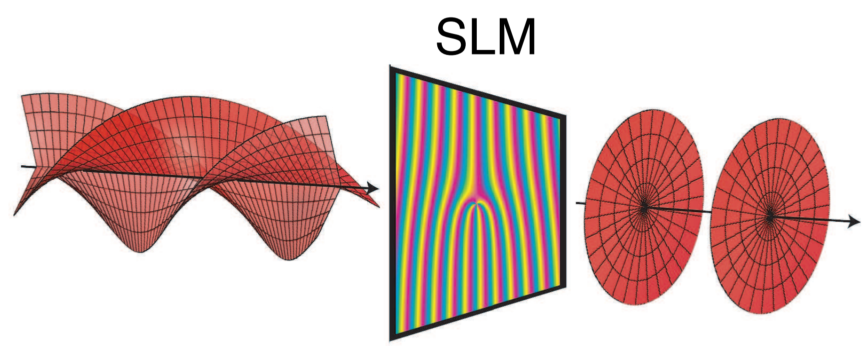


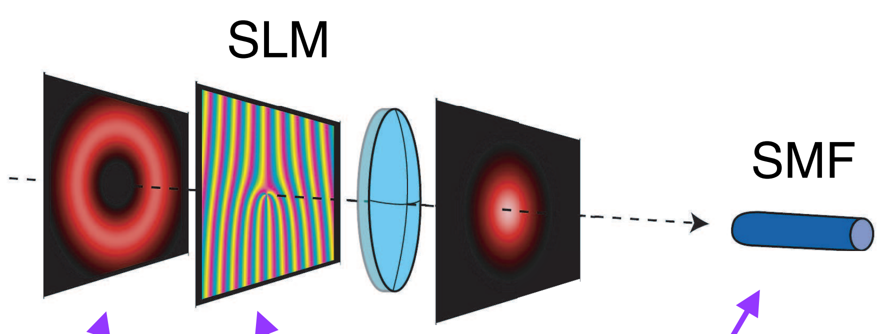


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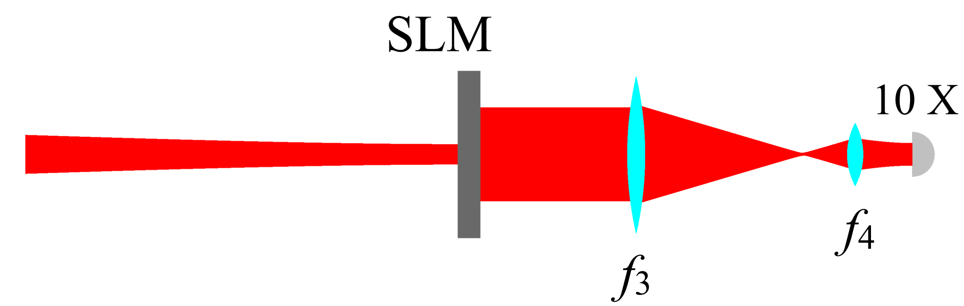


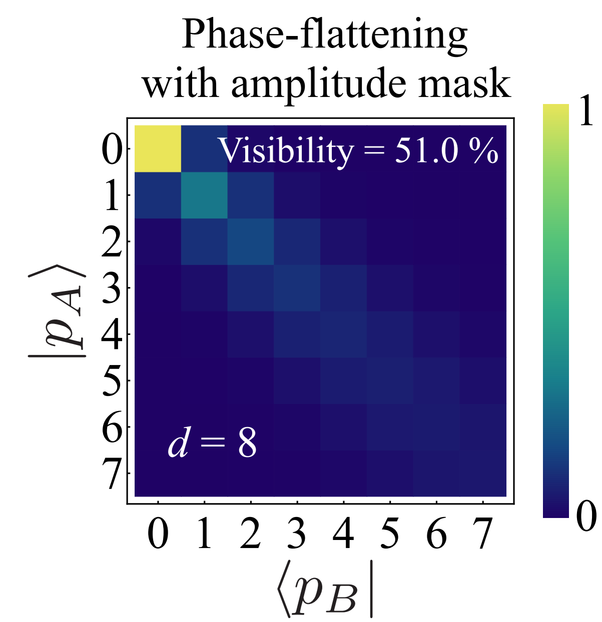


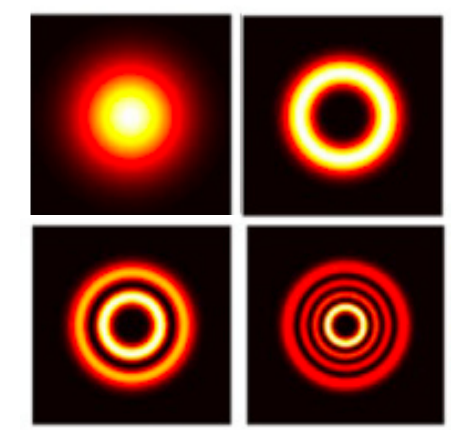

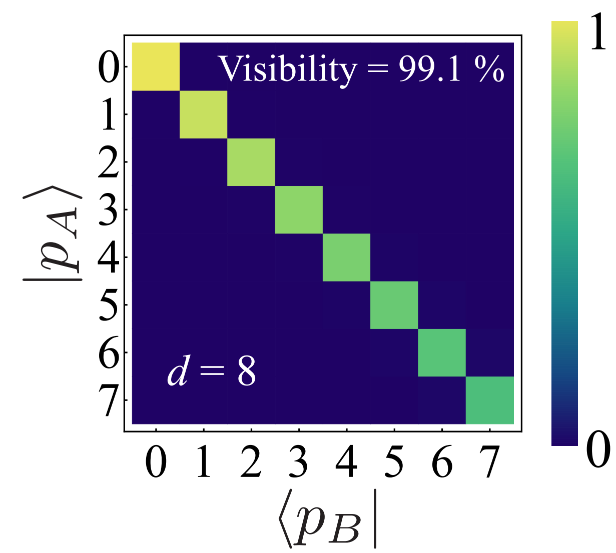


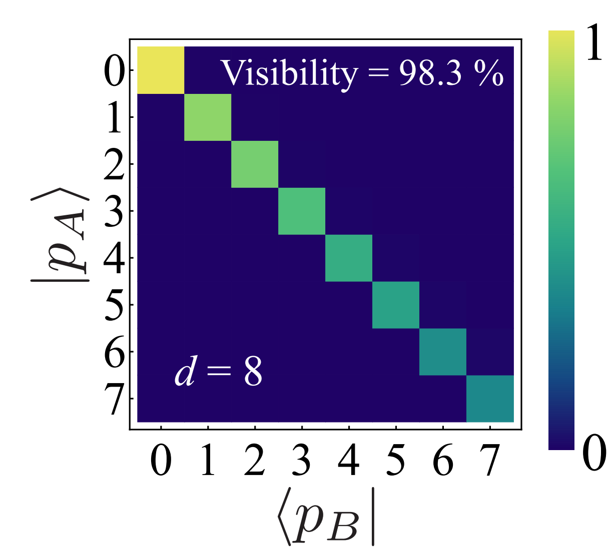


Bouchard, Herrera, Brandt, Fickler, Huber, and **MM**, Opt. Exp. 26, 31924 (2018)

# SUPERPOSITION

# AND

# ENTANGLEMENT

# QUANTUM SUPERPOSITION

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  - Cat put in box
  - Radioactive decay triggers poison
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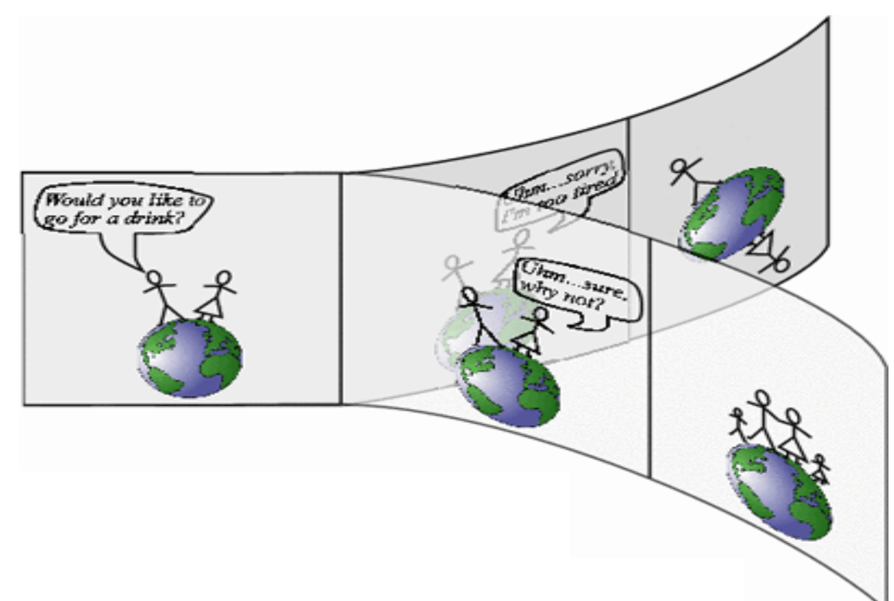
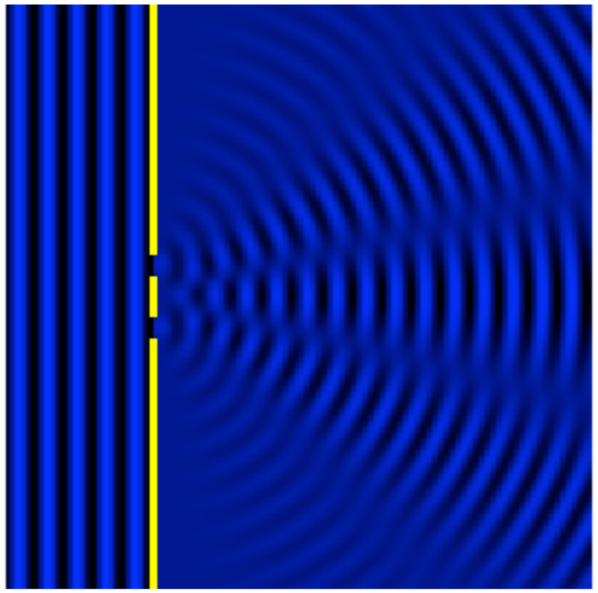
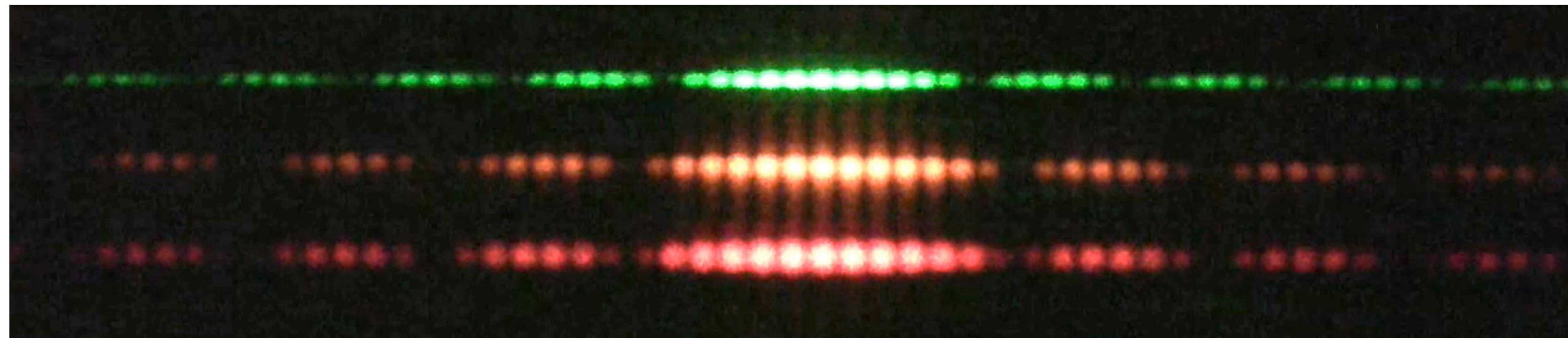


# Quantum Superposition





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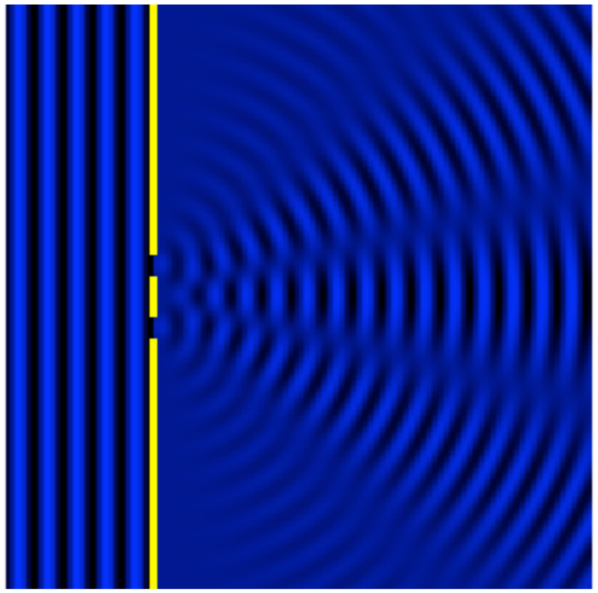

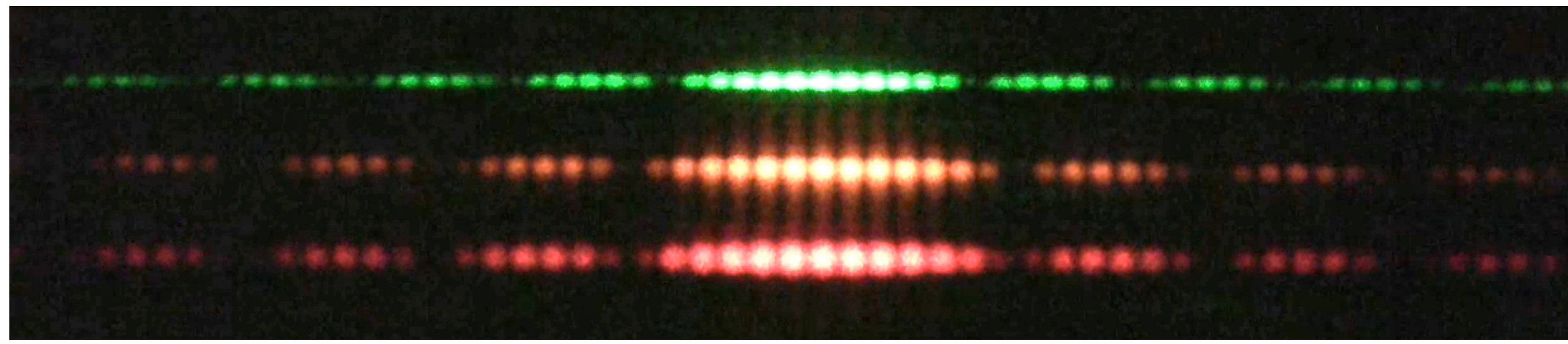

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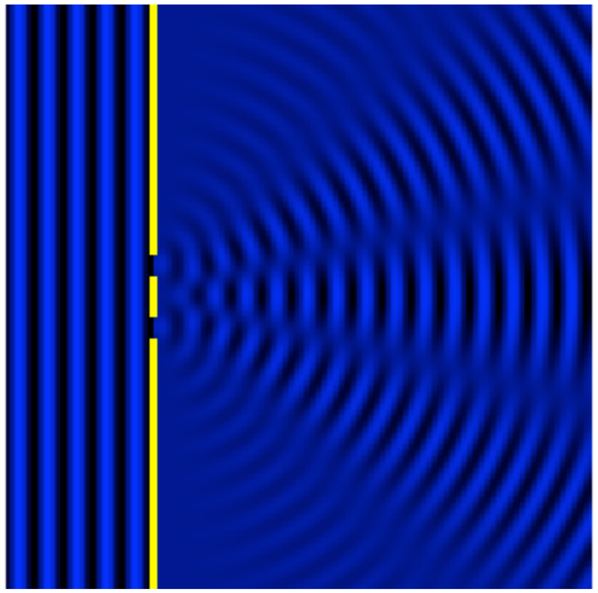

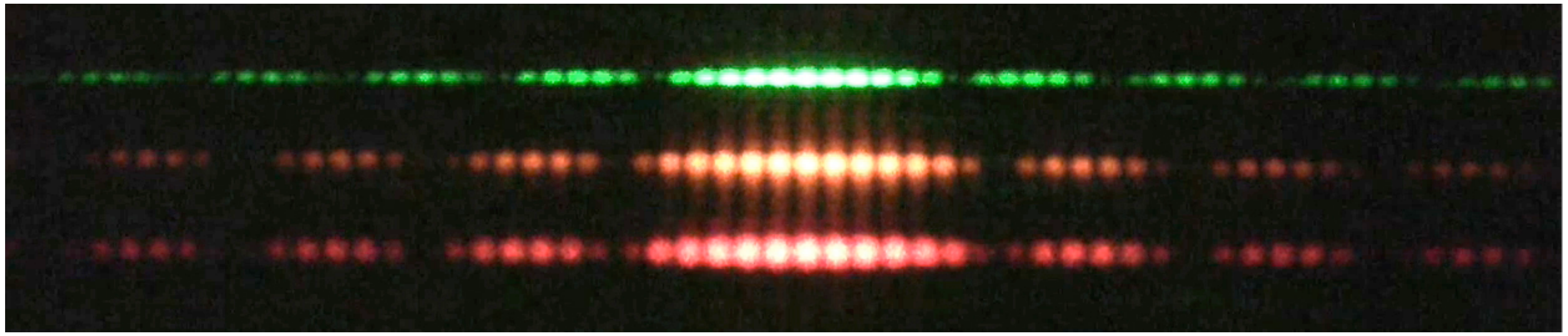

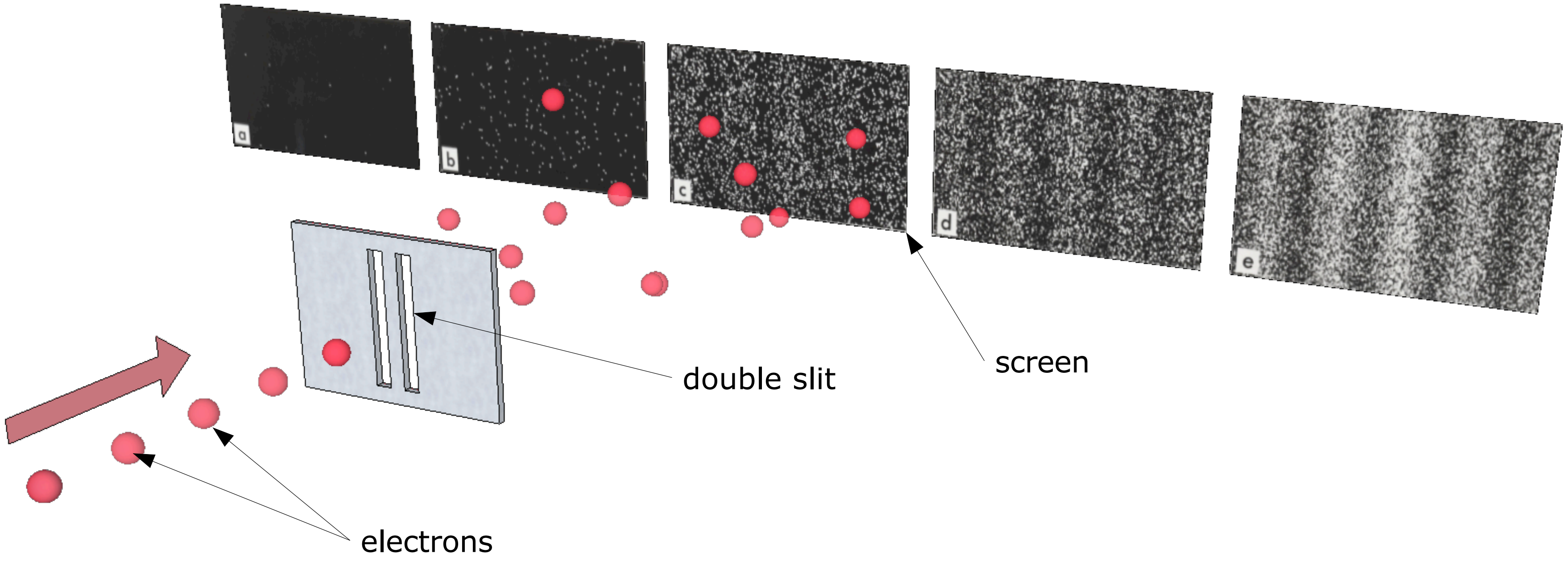

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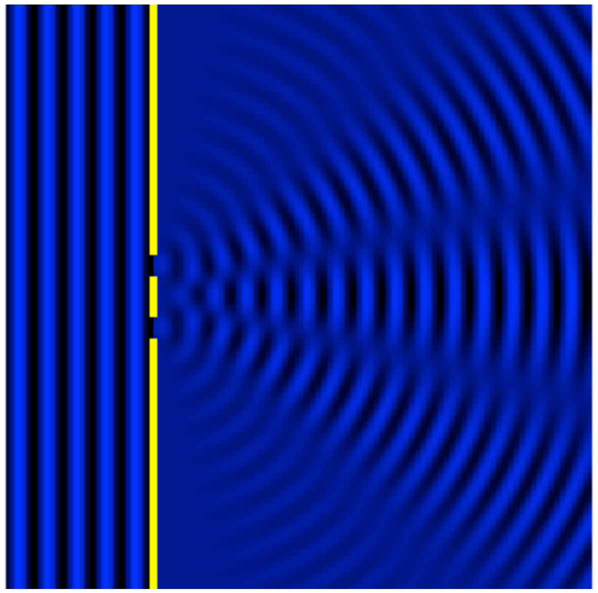

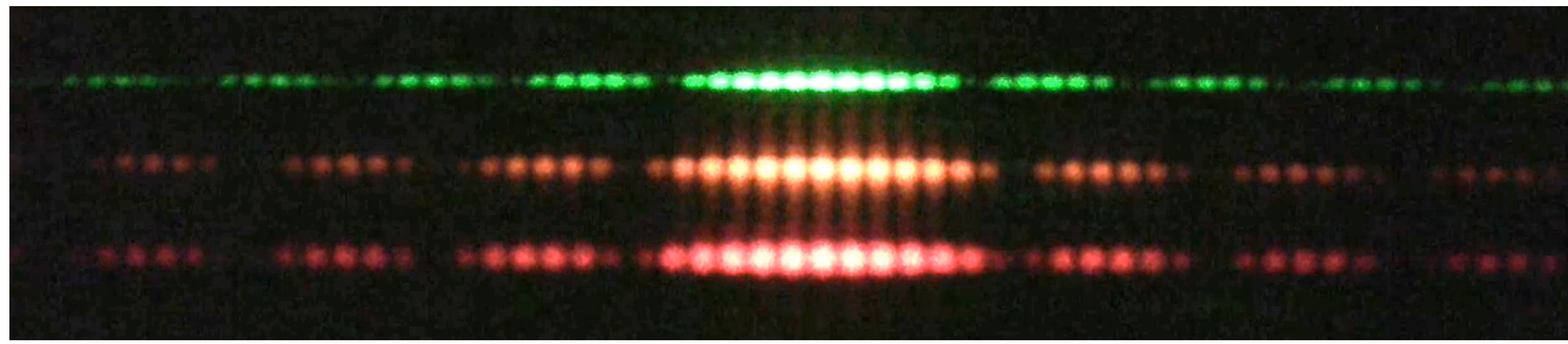

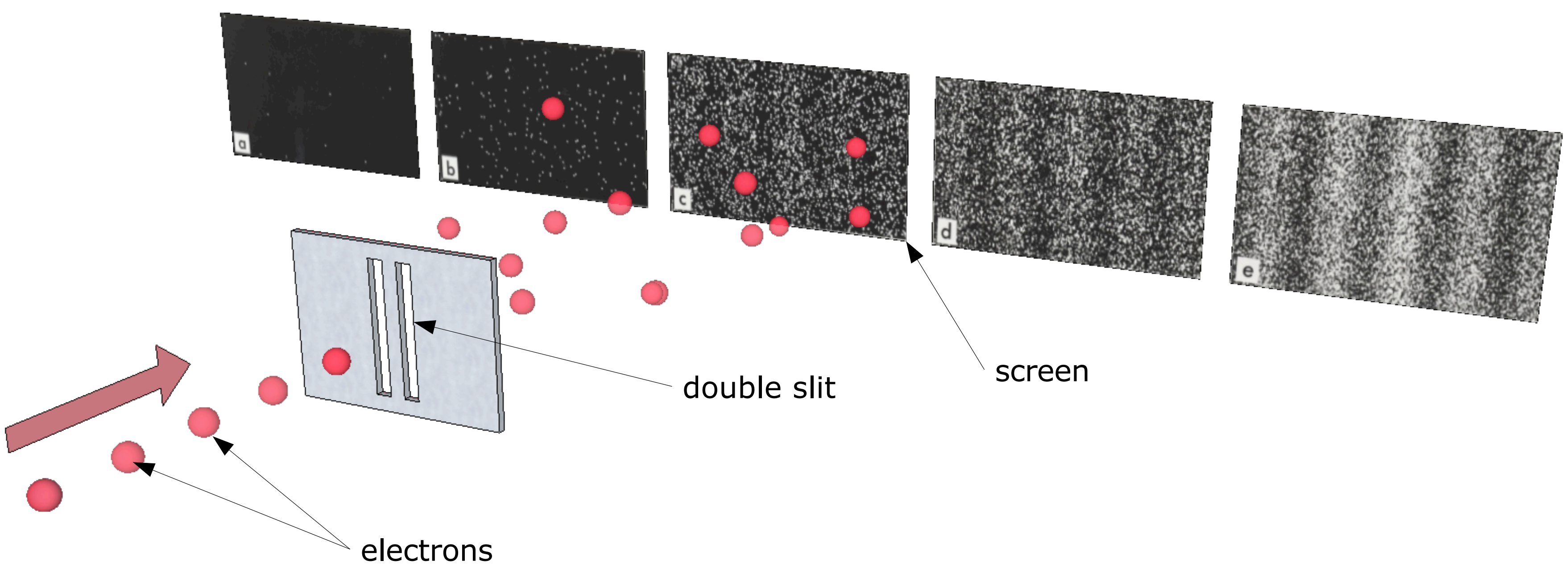


$$|l\rangle + |r\rangle$$

vs

$$|l\rangle\langle l| + |r\rangle\langle r|$$

- *Coherent* superposition of two possible states – “Qubit”

# Density Matrix

$$|\psi\rangle = a\,|H\rangle + b\,|V\rangle$$

Friis, Vitagliano, MM, Huber, Nat. Rev. Phys. 1, 72-87 (2019)

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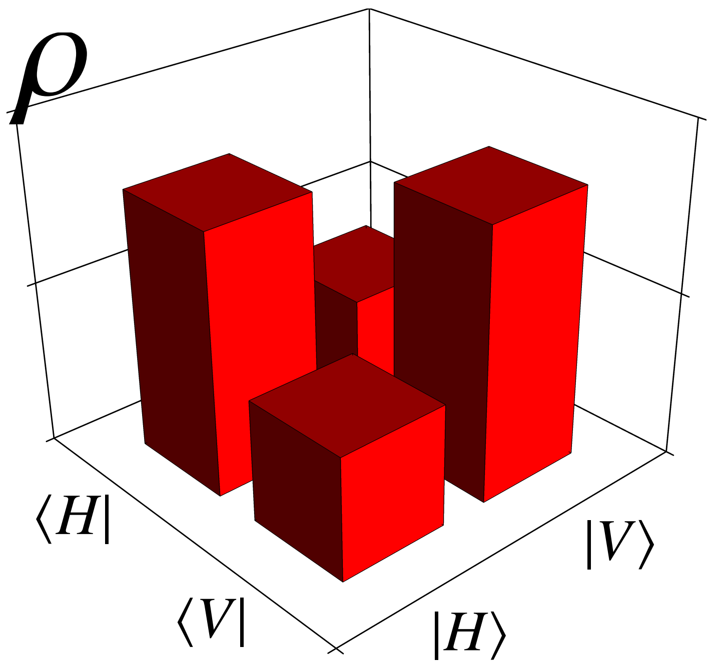


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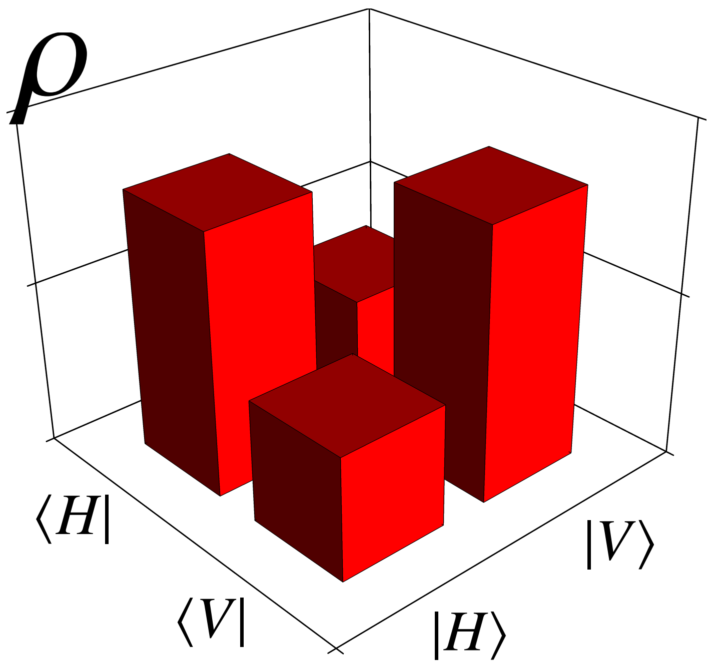


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→ *Diagonals*

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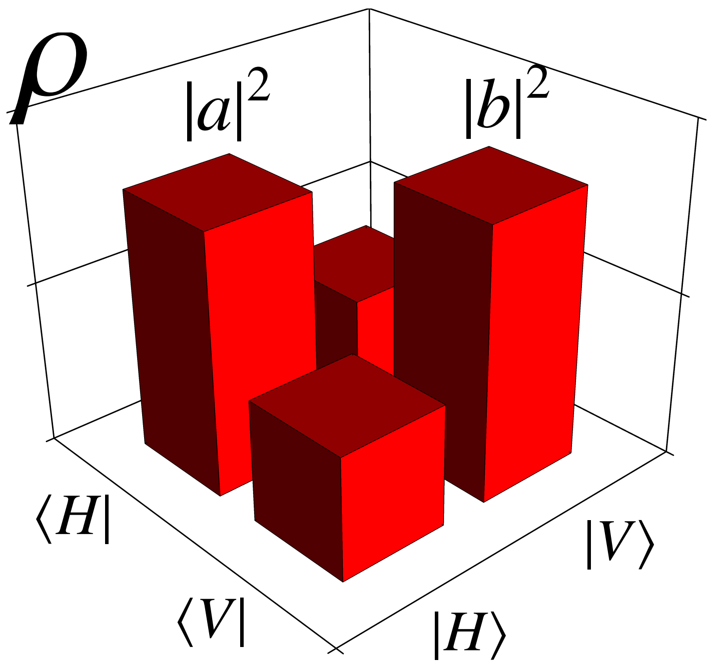


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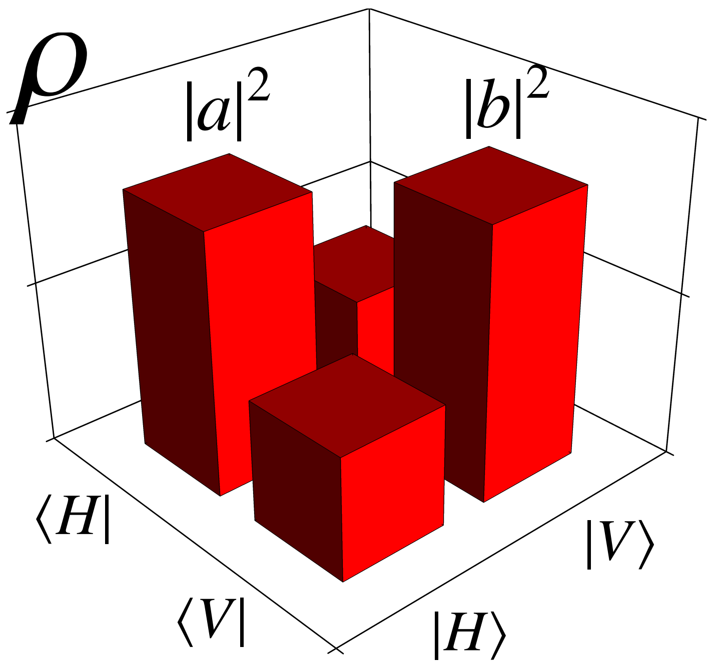


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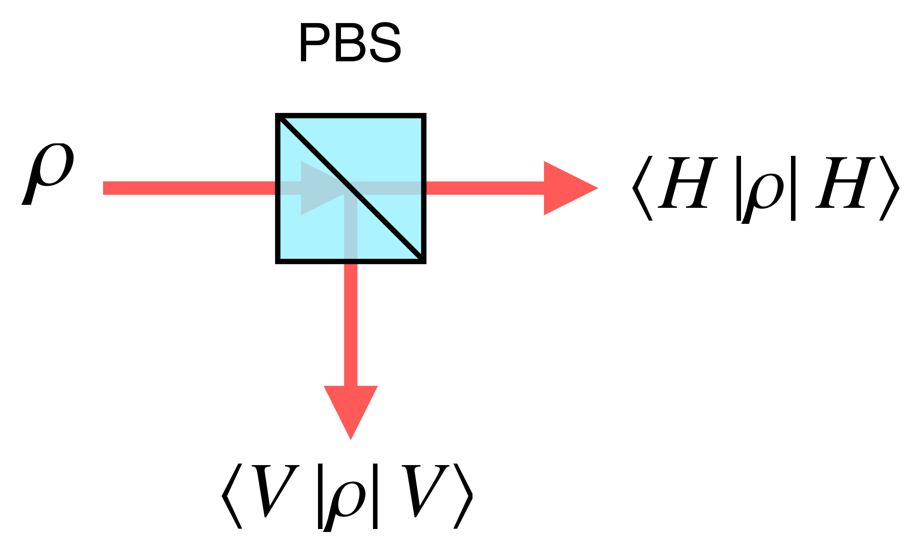


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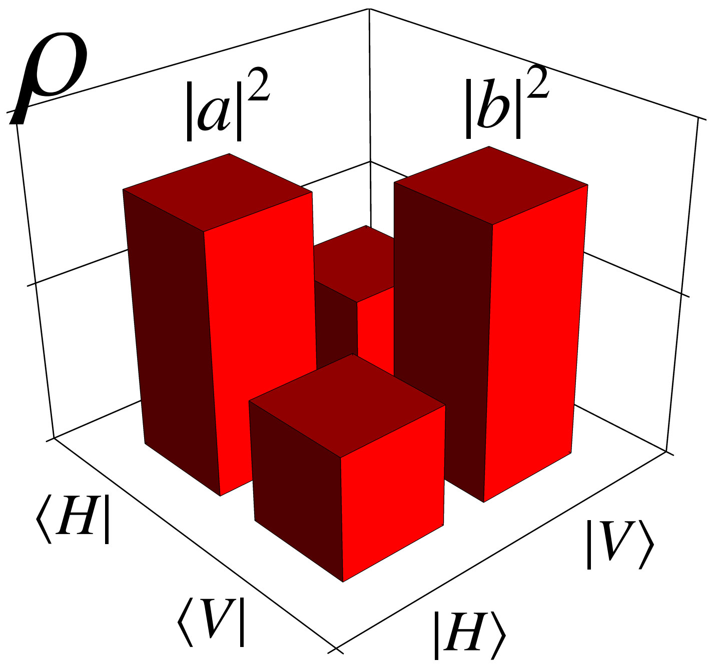


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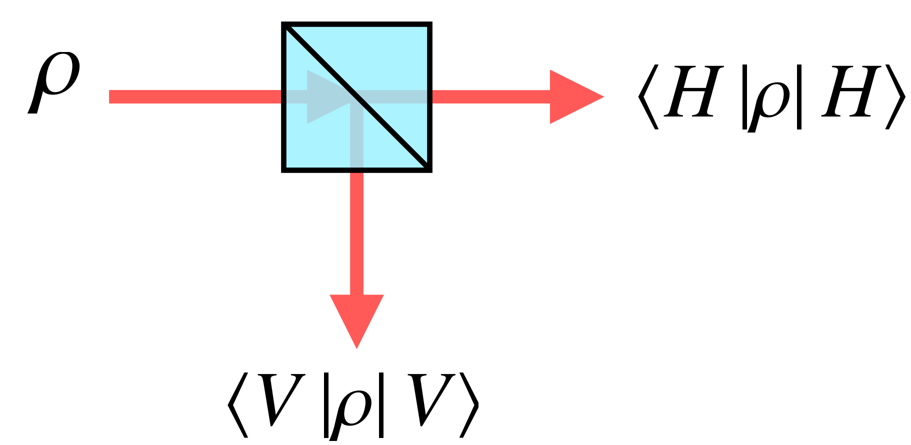


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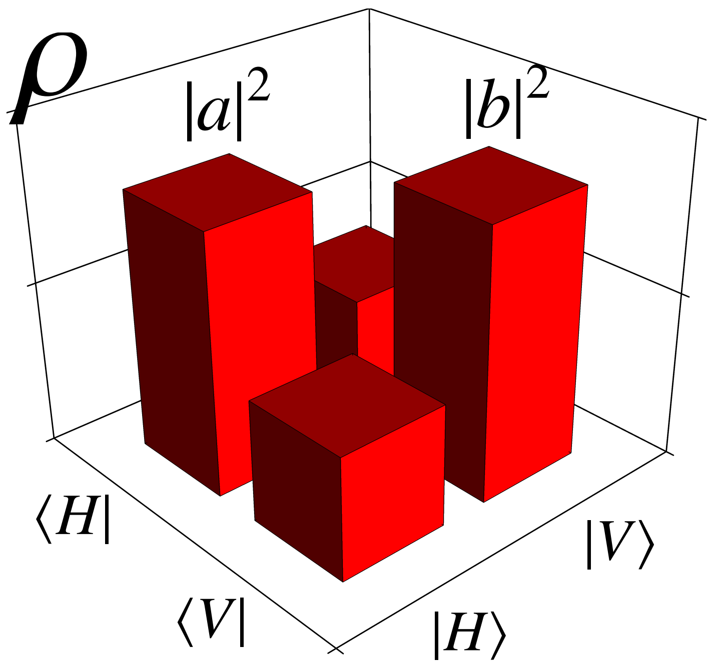


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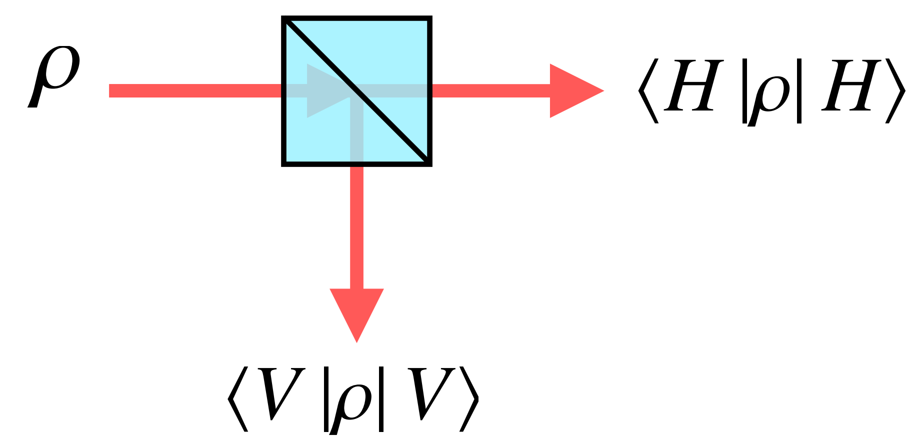


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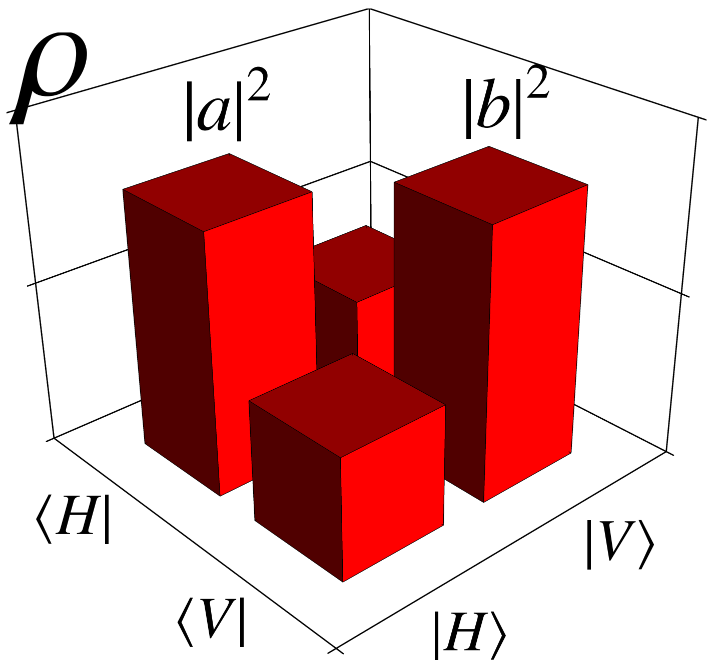


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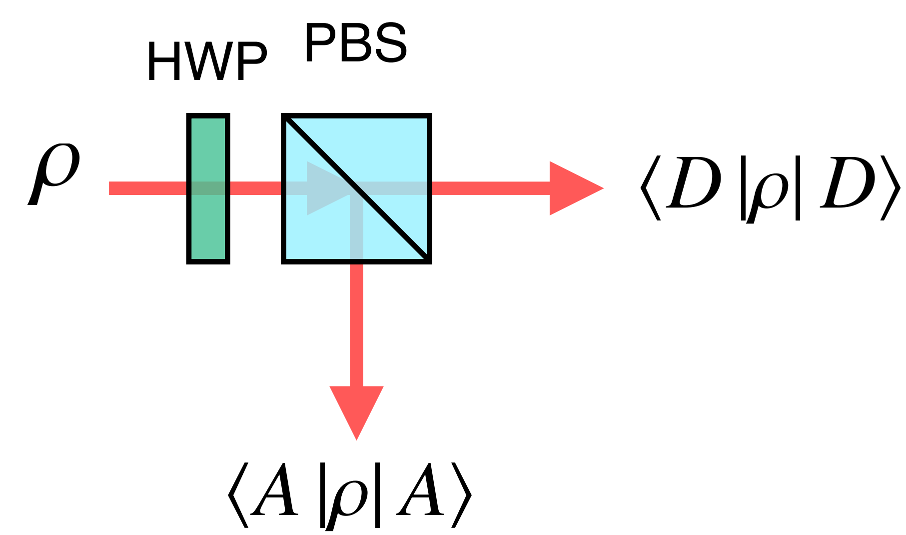


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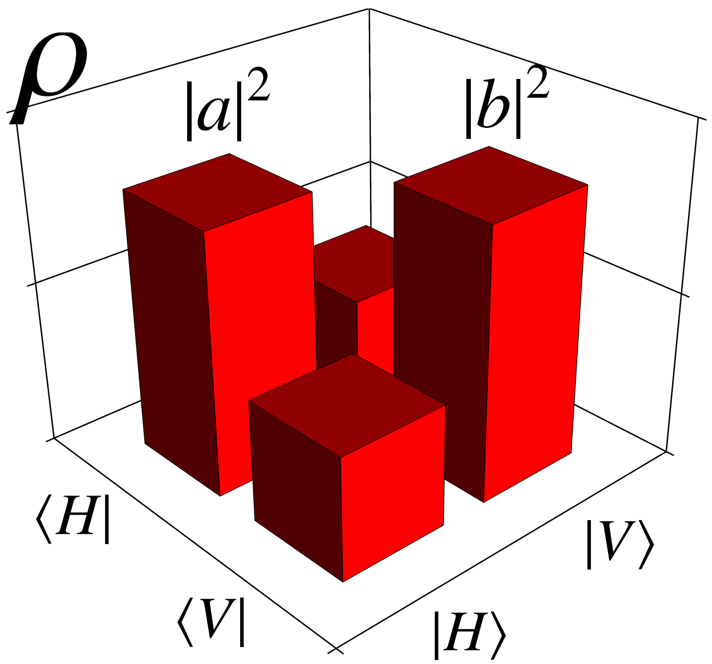


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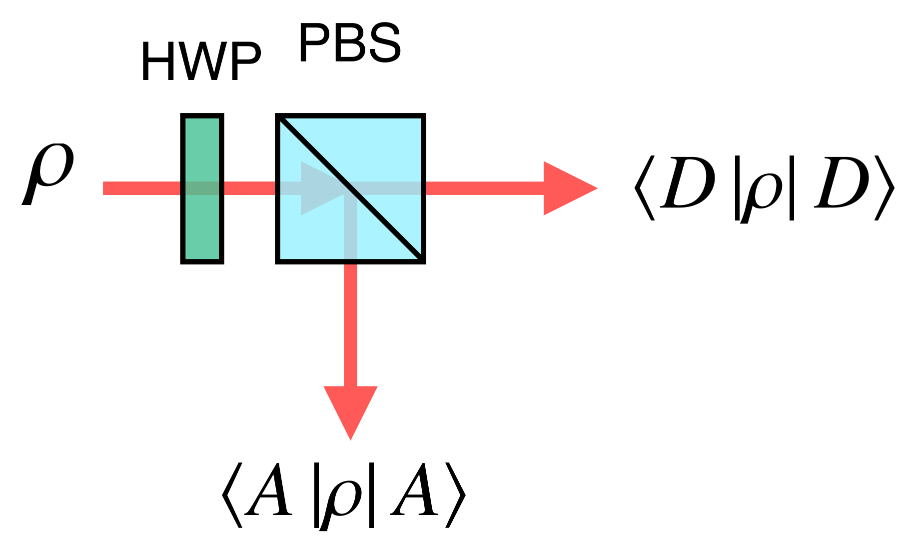


*"Quantum State Tomography"*

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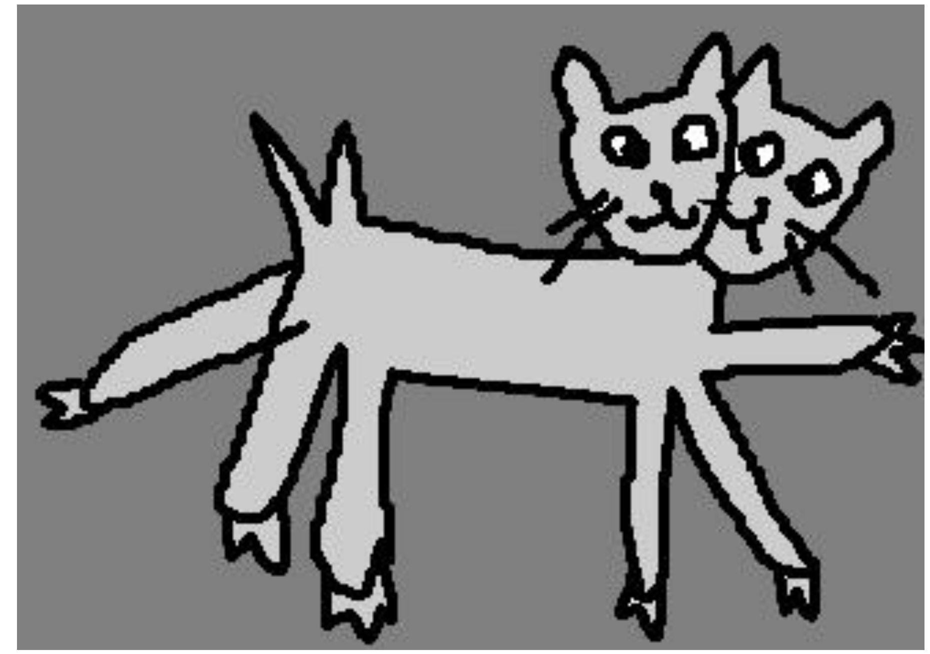
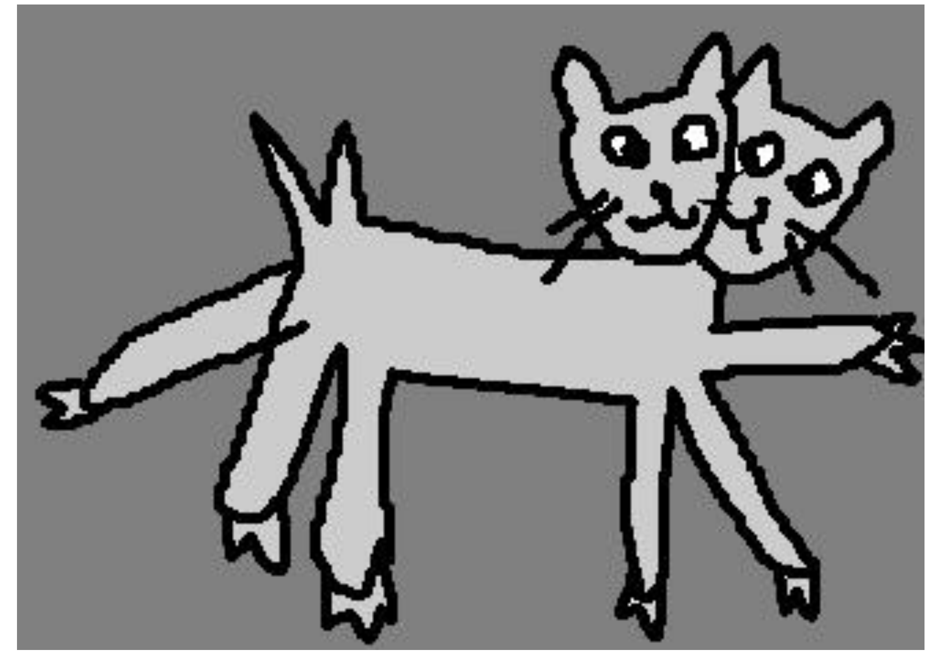
# ENTANGLEMENT



# Entanglement

- Conjoined kittens born in Trieste!



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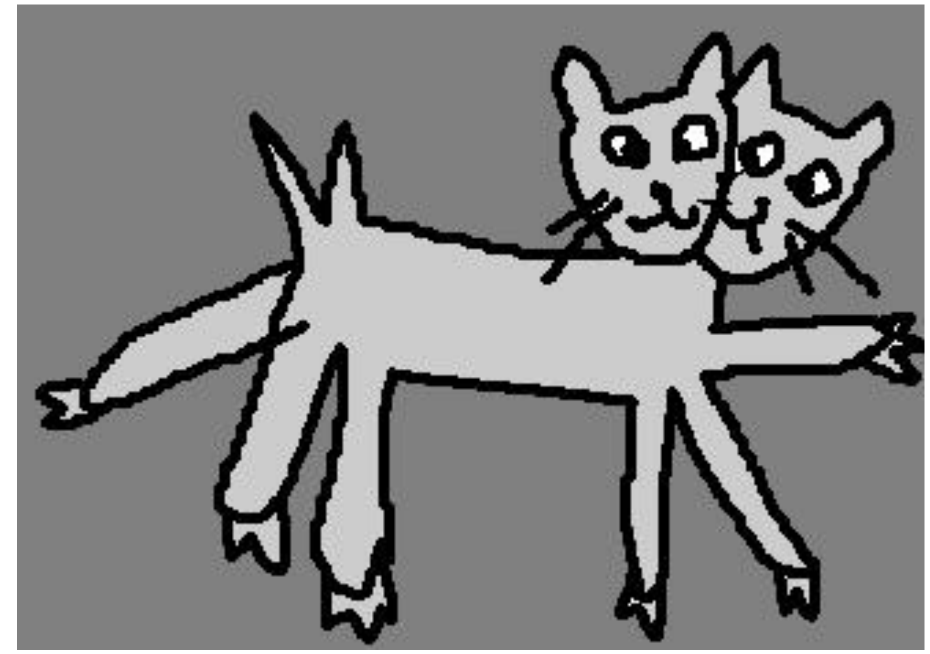

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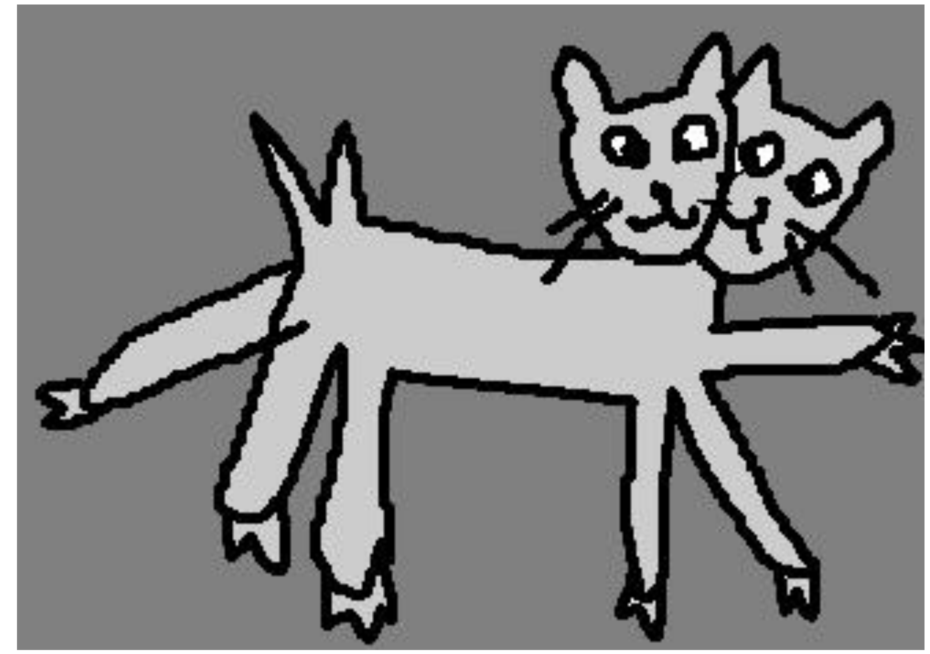

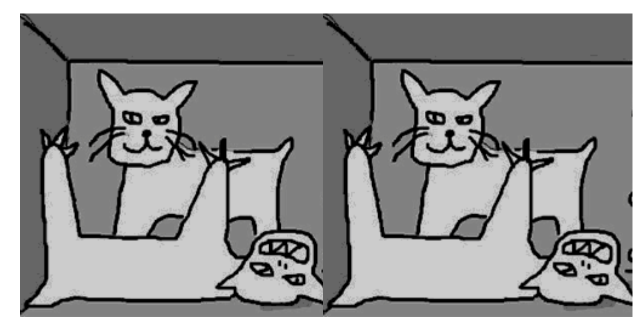

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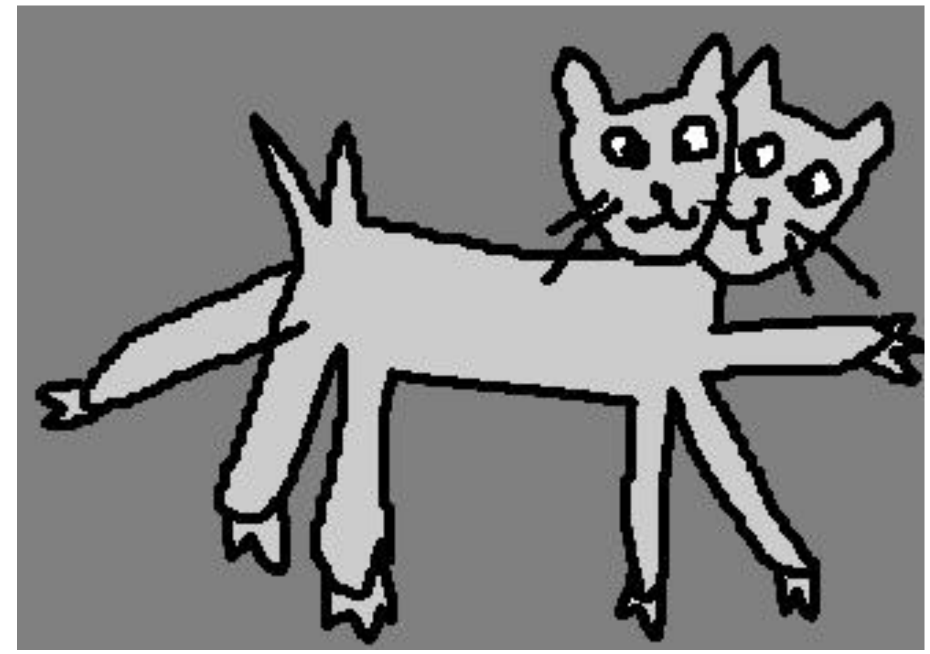

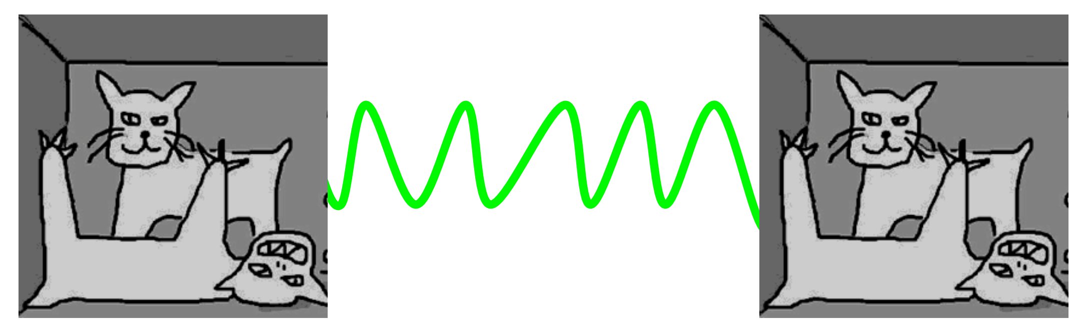

Trieste

Edinburgh

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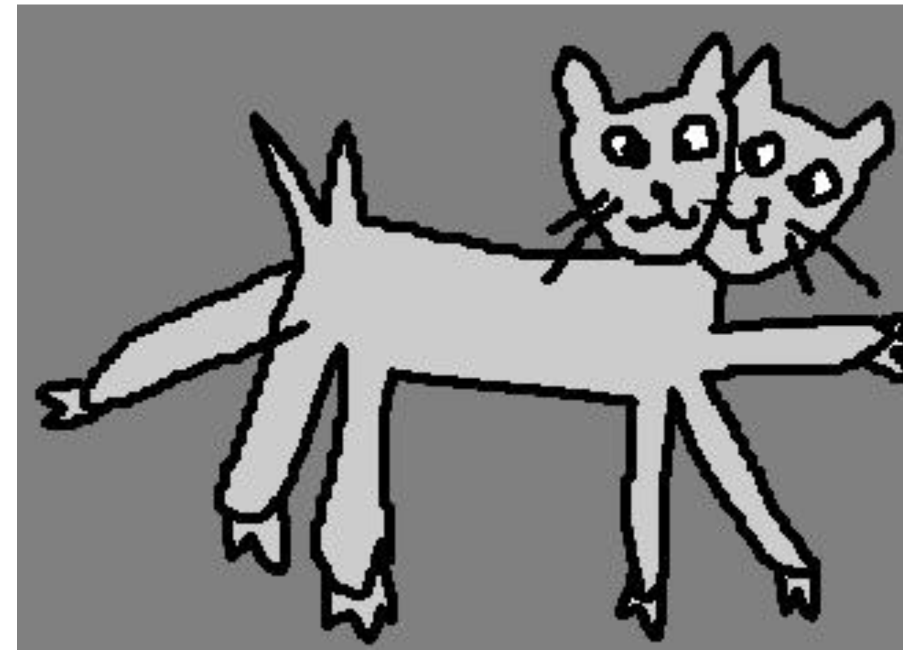

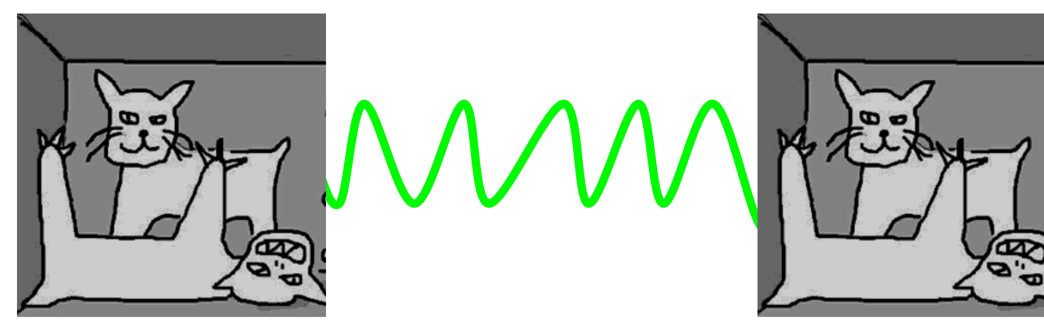

Trieste Edinburgh

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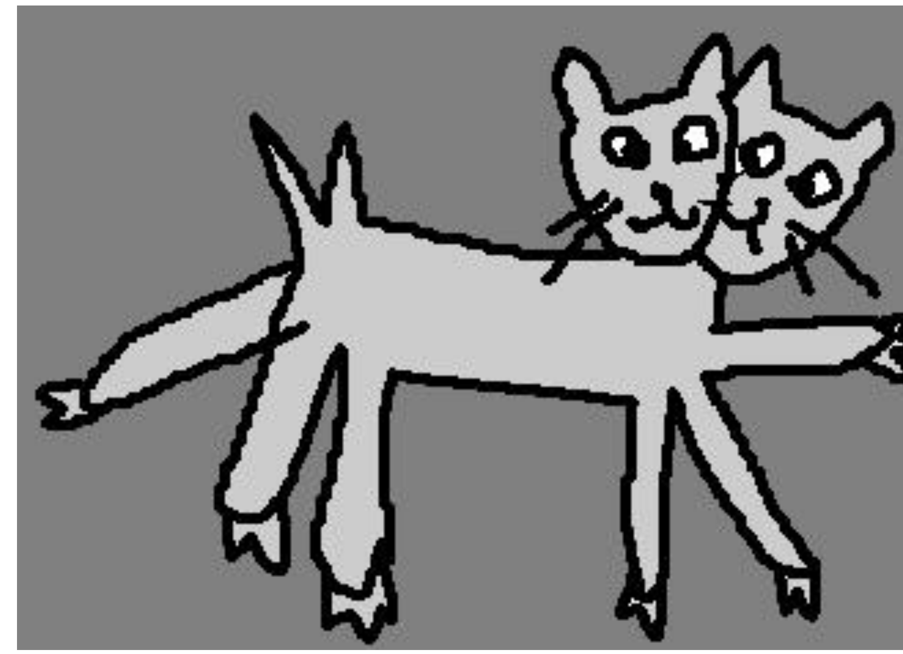

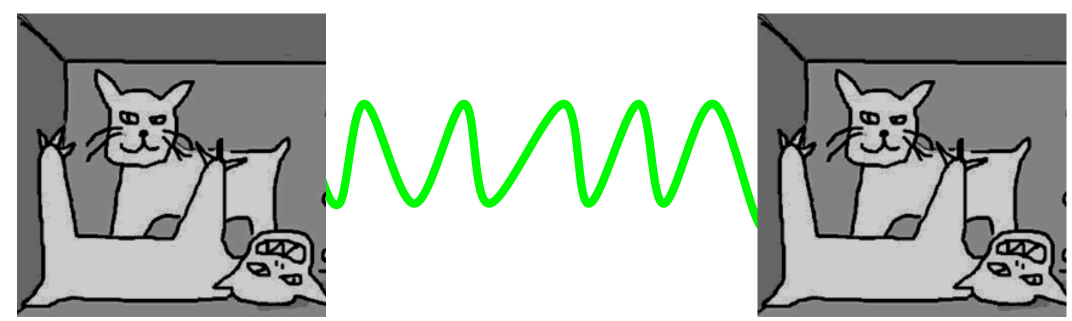

Trieste Edinburgh

$$|\Psi\rangle = |alive\rangle_1 |alive\rangle_2 + |dead\rangle_1 |dead\rangle_2$$

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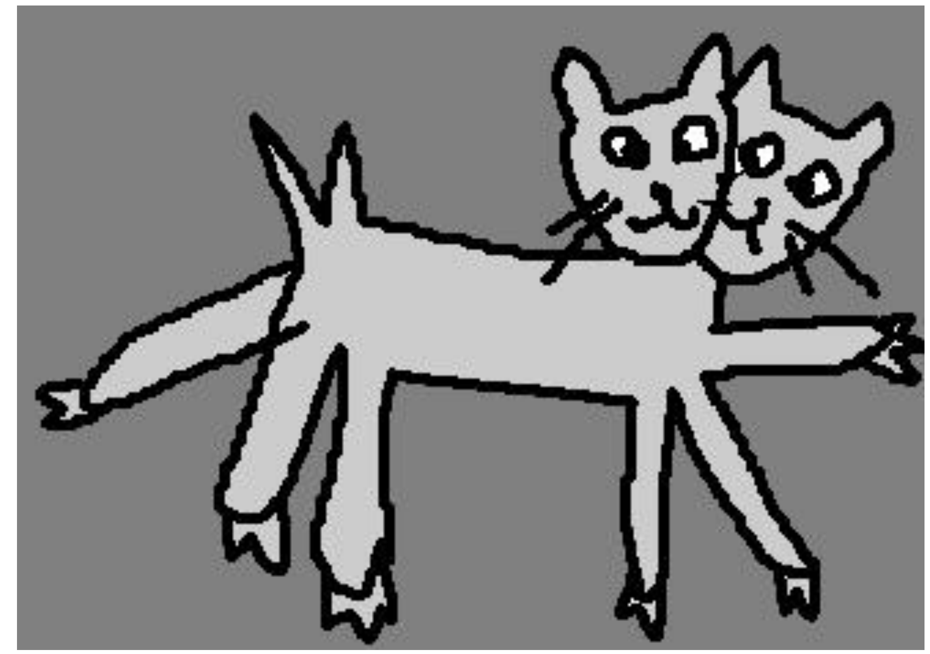
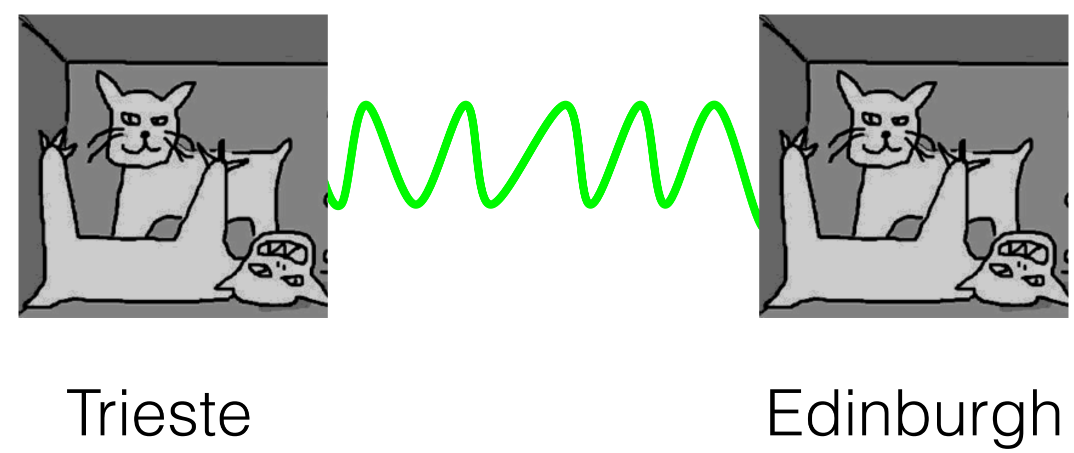
- *Not* a causal connection



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# Entanglement

# Entanglement

- Twist!

# ENTANGLEMENT

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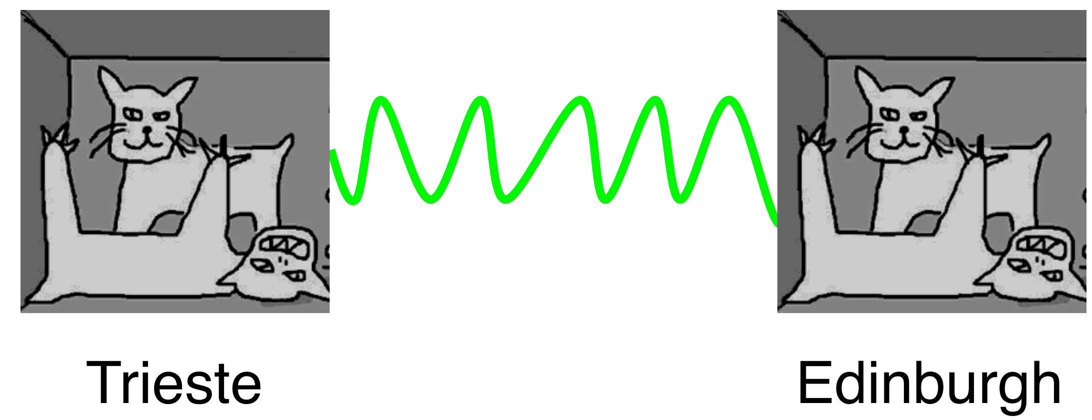
- Opening box in the daytime tells you whether cats are **dead** or *alive*




$$\boxed{|\Psi\rangle = |alive\rangle_1 |alive\rangle_2 + |dead\rangle_1 |dead\rangle_2}$$

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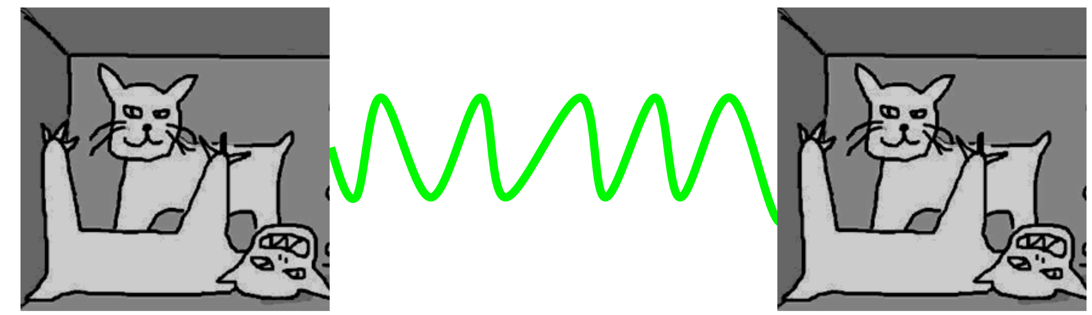
- Opening box at <u>night</u> tells you if the cats are ZOMBIES or VAMPIRES 😱



Trieste Edinburgh

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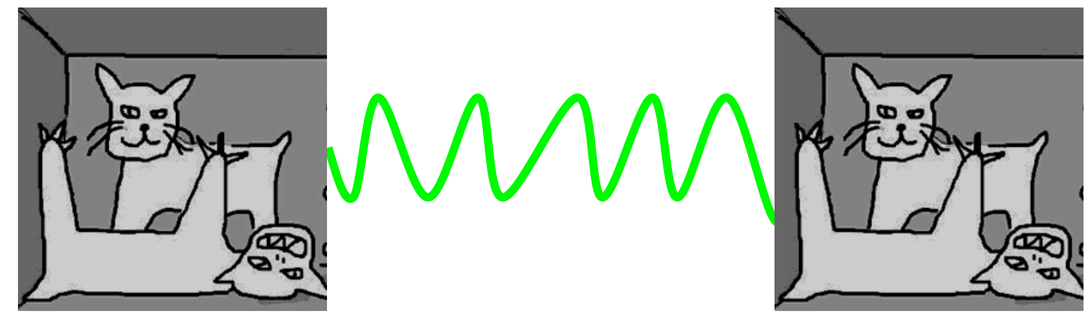

Trieste Edinburgh

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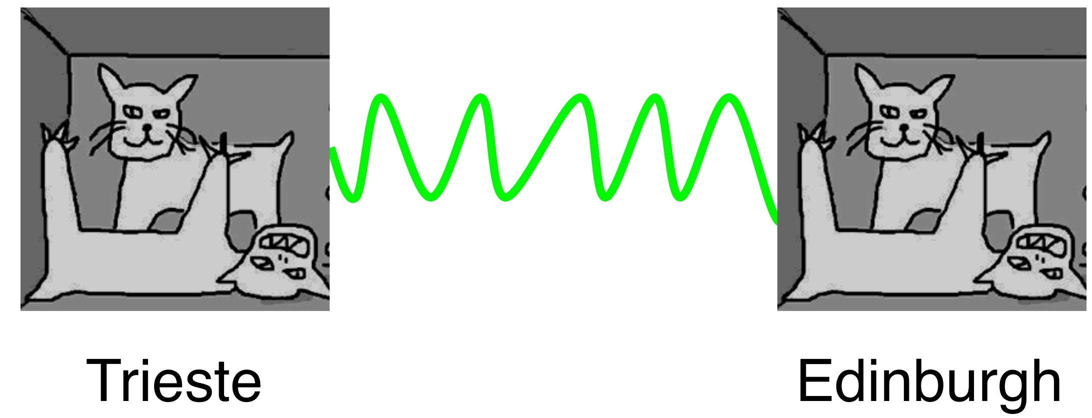
- Magical connection exists no matter when (how) you open the box!

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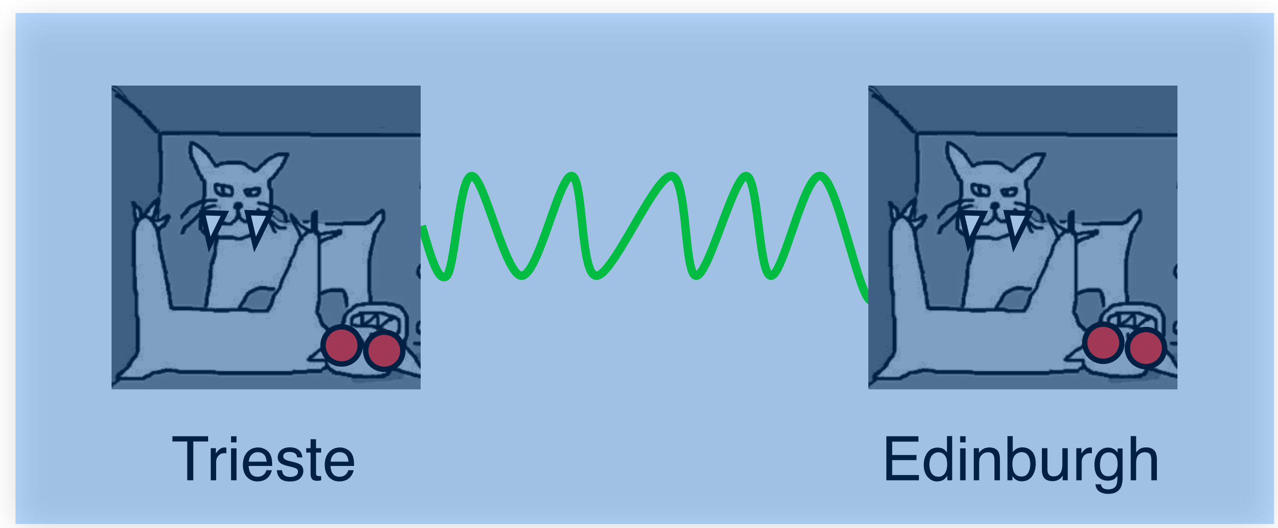


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# ENTANGLED PHOTONS

Friis, Vitagliano, MM, Huber, Nat. Rev. Phys. 1, 72-87 (2019)

# ENTANGLED PHOTONS

- Two photons can be entangled in their <u>polarisation</u>

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2]$$

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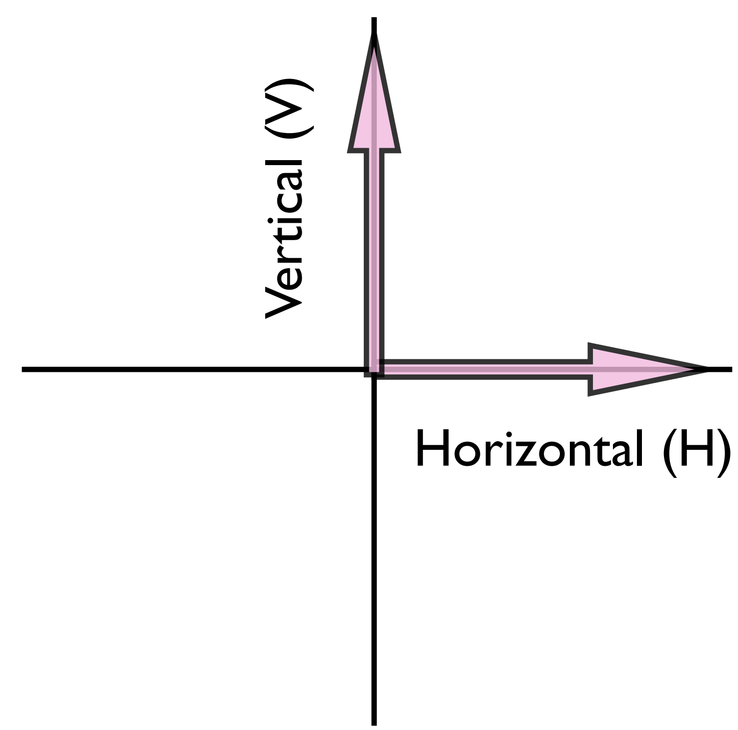


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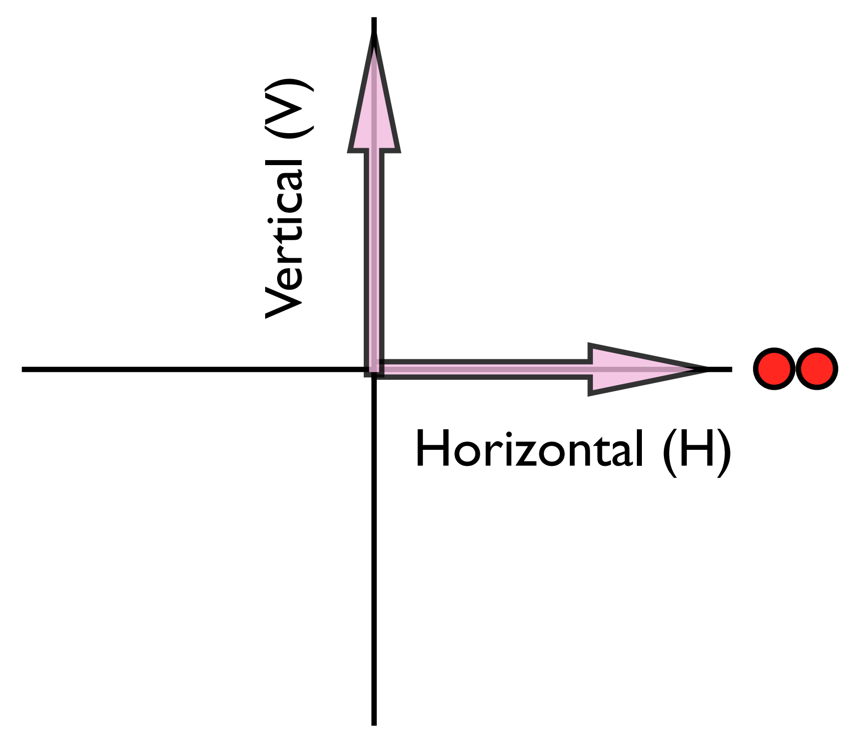


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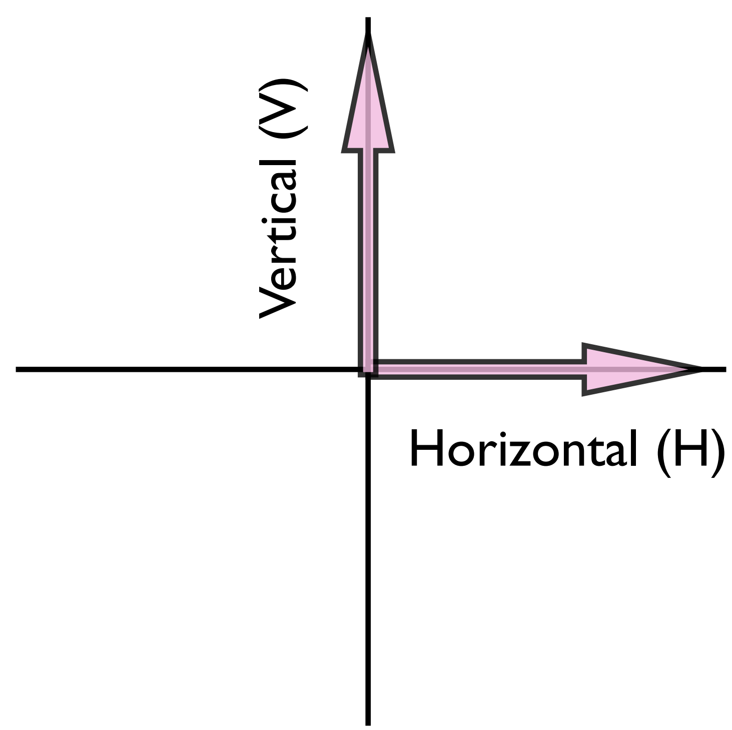


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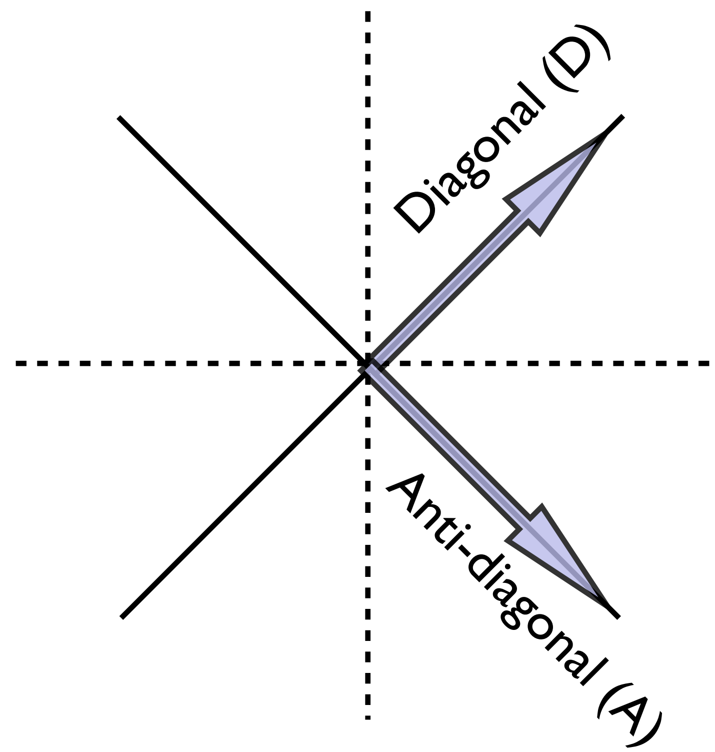


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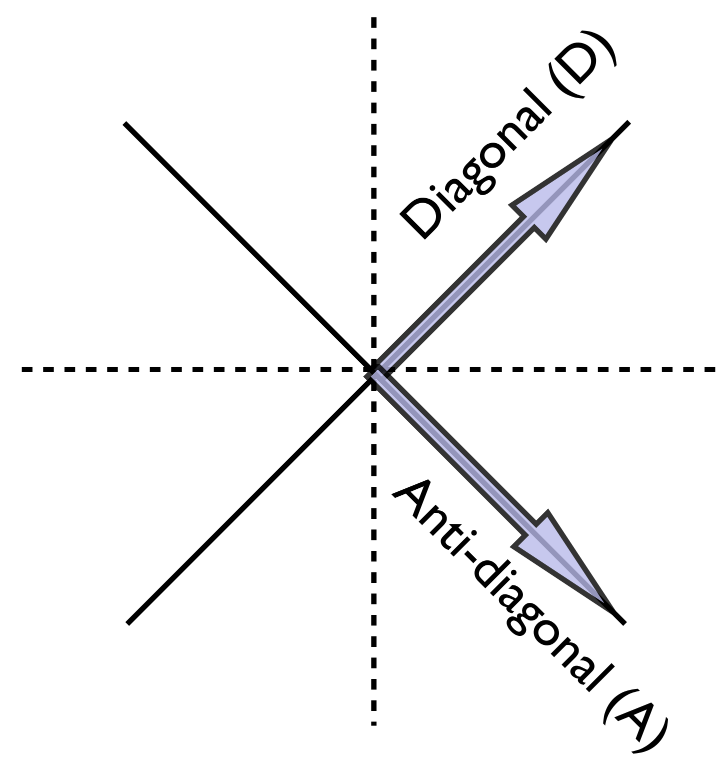


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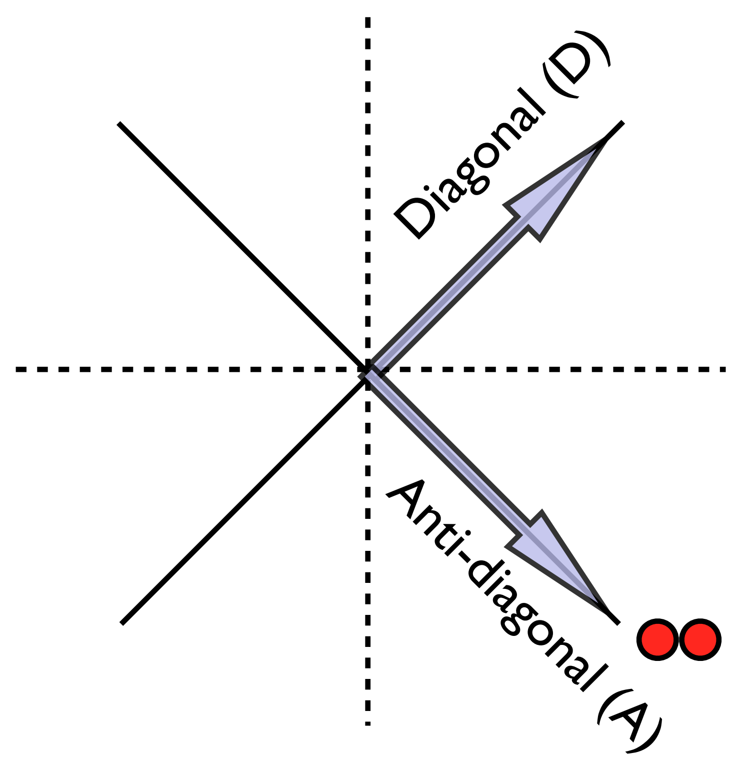


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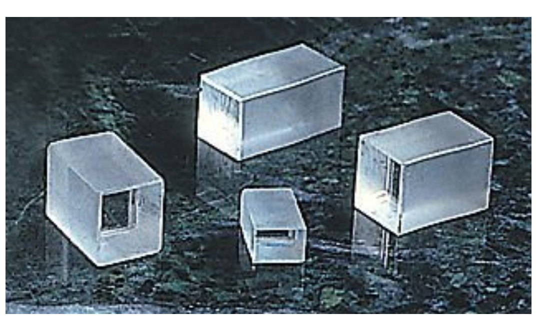

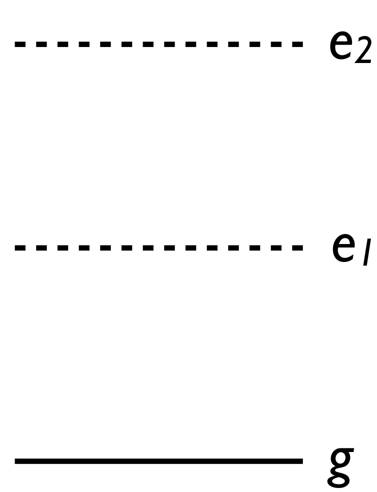


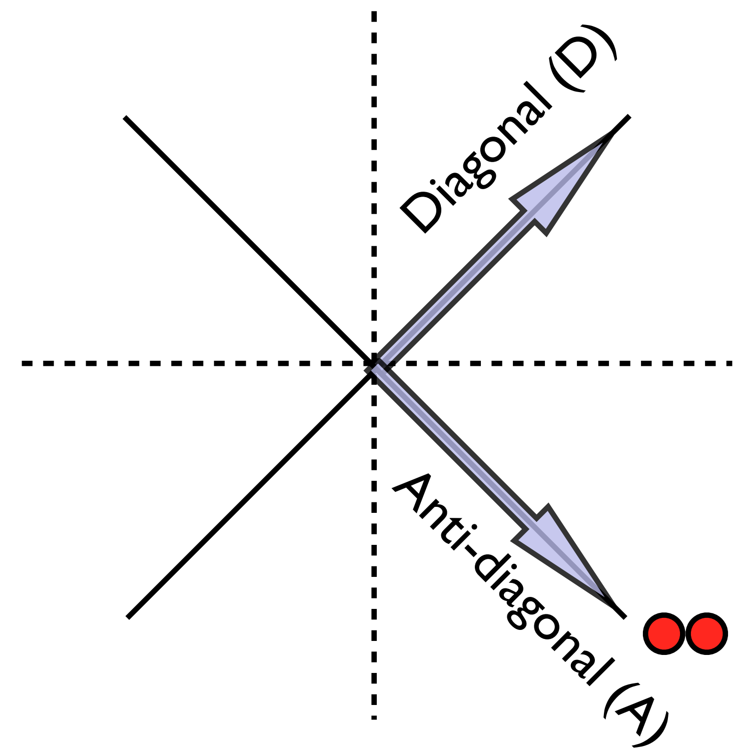


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- Created using type II nonlinear crystals
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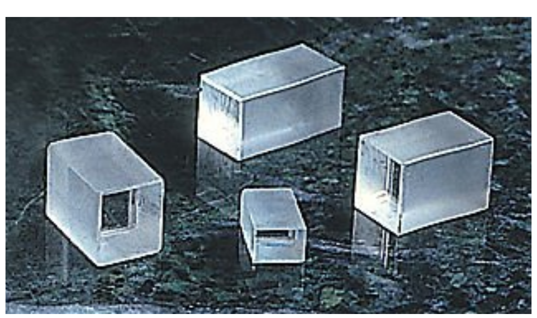

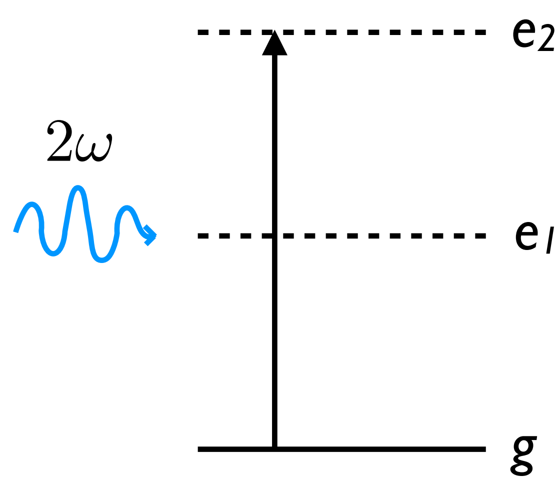


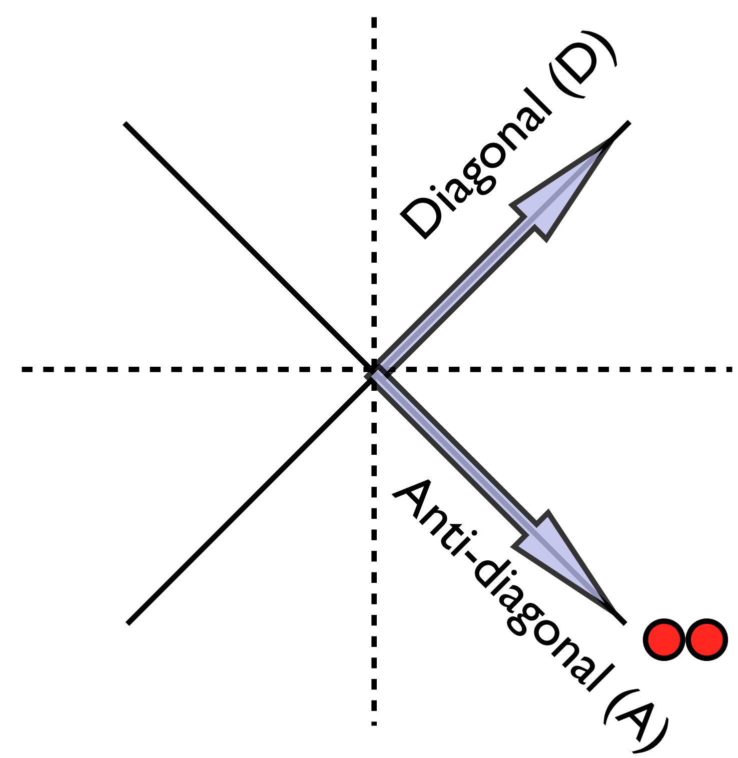


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- Measure in the *HV basis*, polarisations will be **perfectly correlated**
- Measurements in the **DA basis** also show <u>perfect correlations</u>!
- Created using type II nonlinear crystals
- One **blue** photon is absorbed
- Two *<u>red</u>* photons are emitted

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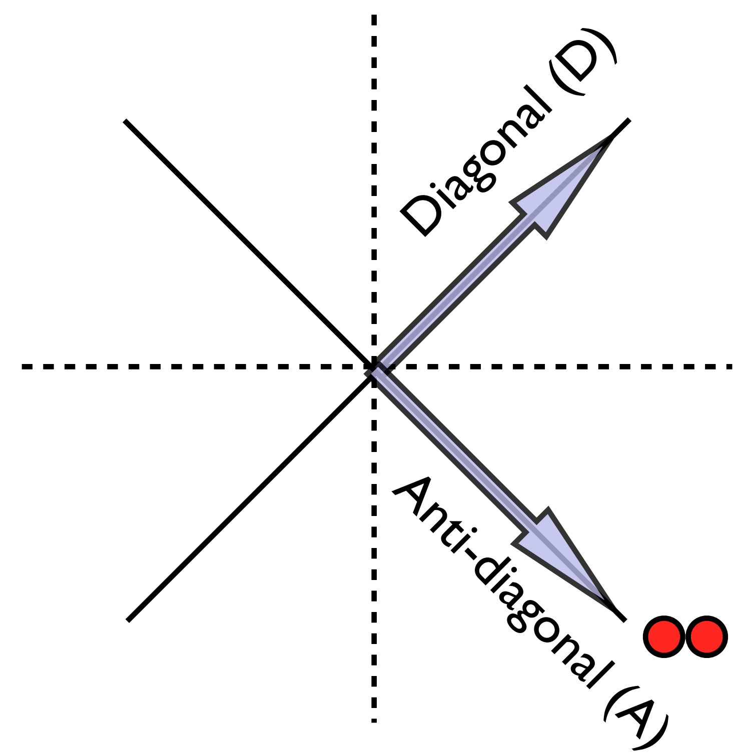


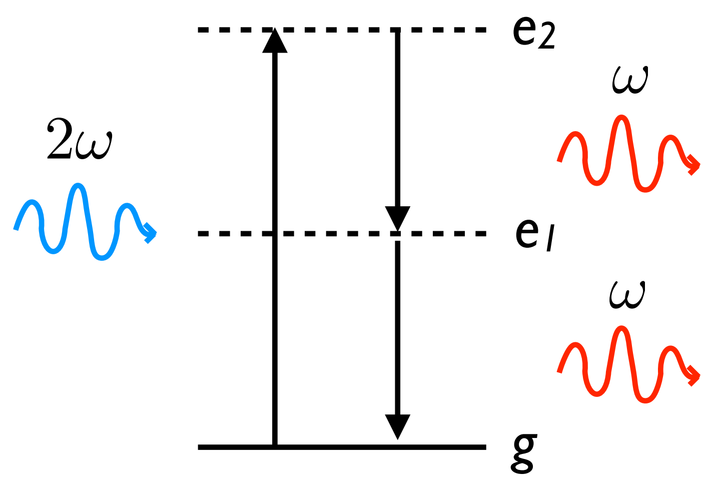


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# ENTANGLEMENT: APPLICATIONS

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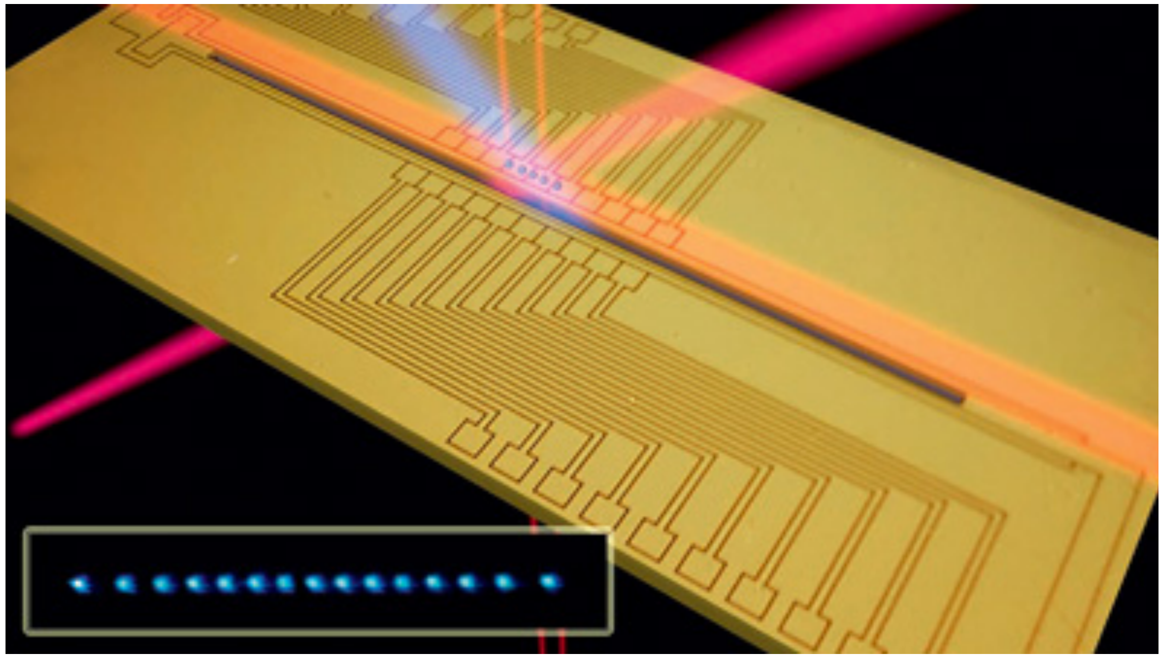

Measurement-based Quantum Computation
Quantum Error Correction

Raussendorf & Briegel, Phys. Rev. Lett. 86, 5188 (2001)
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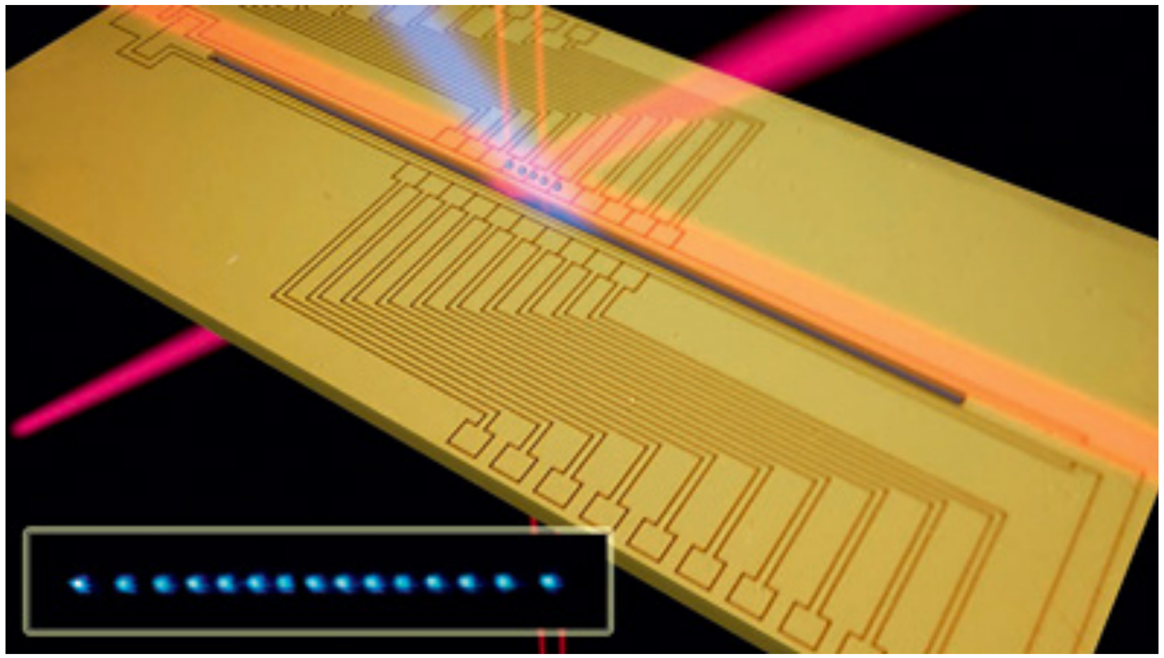

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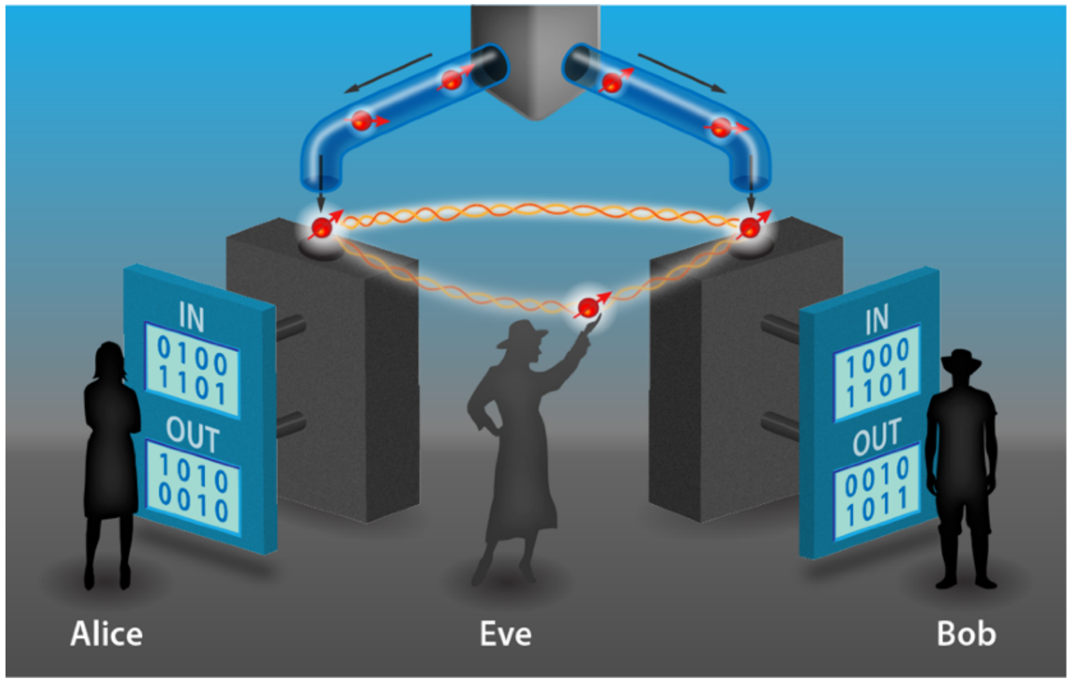


Device-Independent Quantum Cryptography


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Acín, Brunner, Gisin, Massar, Pironio, Scarani, **Phys. Rev. Lett. 98,** 230501 (2007)

# HIGH-DIMENSIONAL ENTANGLEMENT

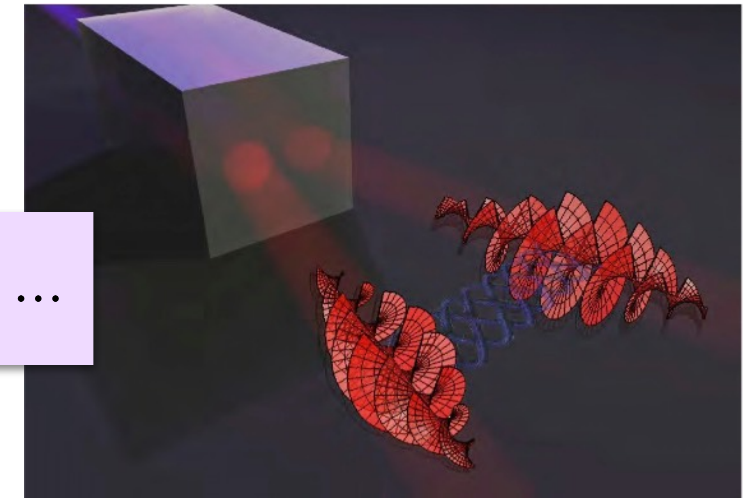
Friis, Vitagliano, MM, Huber, Nat. Rev. Phys. 1, 72-87 (2019)

# High-Dimensional Entanglement

- Orbital angular momentum entanglement

$$|\Psi\rangle = |-1,1\rangle + |0,0\rangle + |1,-1\rangle + |2,-2\rangle + |3,-3\rangle + ...$$

Mair, Vaziri, Weihs, Zeilinger, **Nature 412,** 313 (2001)



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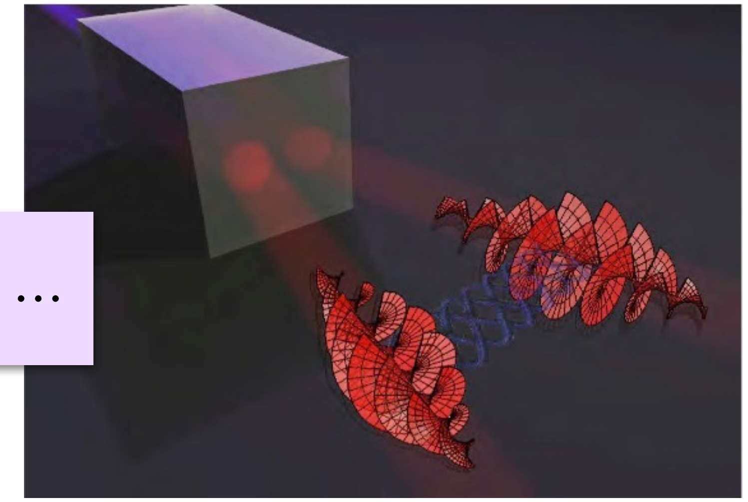

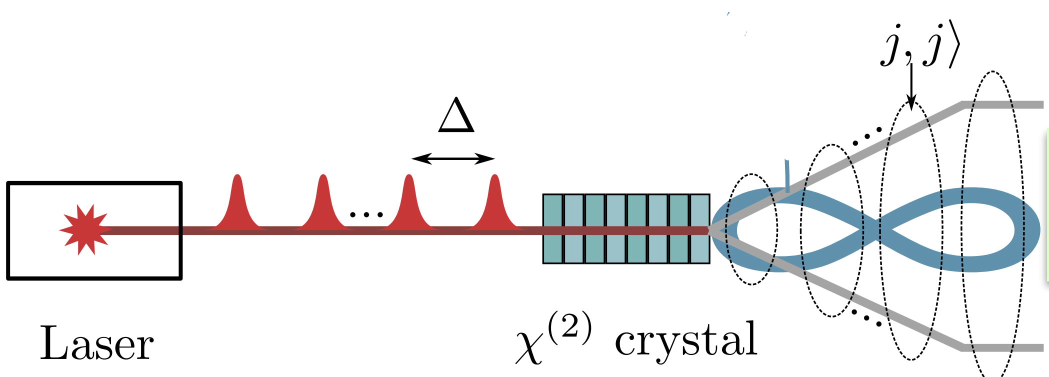


- Time-bin entanglement

$$|\Psi\rangle = |t_0, t_0\rangle + |t_1, t_1\rangle + |t_2, t_2\rangle + |t_3, t_3\rangle + ...$$

Brendel, Gisin, Tittel, Zbinden, **Phys. Rev. Lett. 82**, 2594 (1999)

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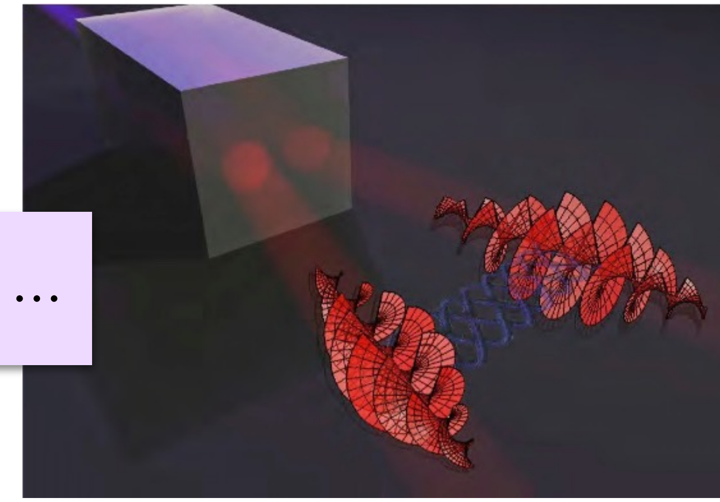

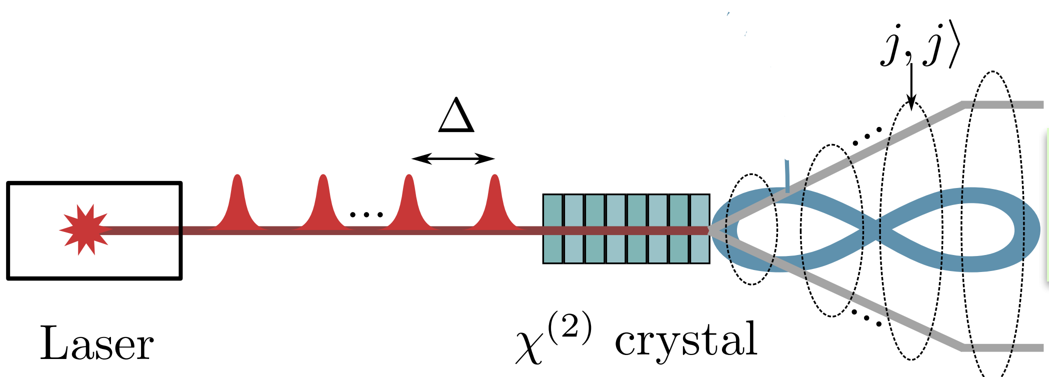


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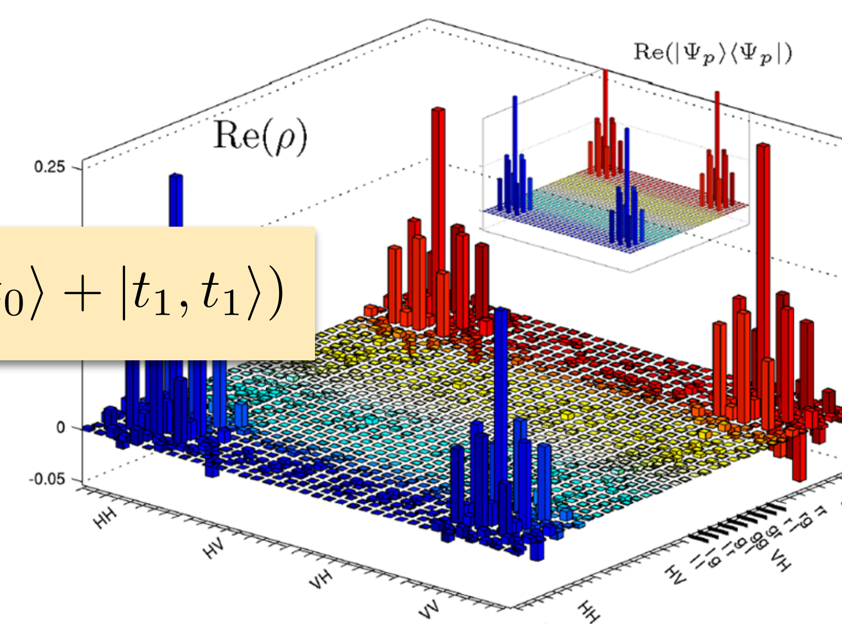
$$|\Psi\rangle = (|H,H\rangle + |V,V\rangle) \otimes (|-1,1\rangle + |0,0\rangle + |1,-1\rangle) \otimes (|t_0,t_0\rangle + |t_1,t_1\rangle)$$

Barreiro, Langford, Peters, Kwiat, **Phys. Rev. Lett. 95,** 260501 (2005)

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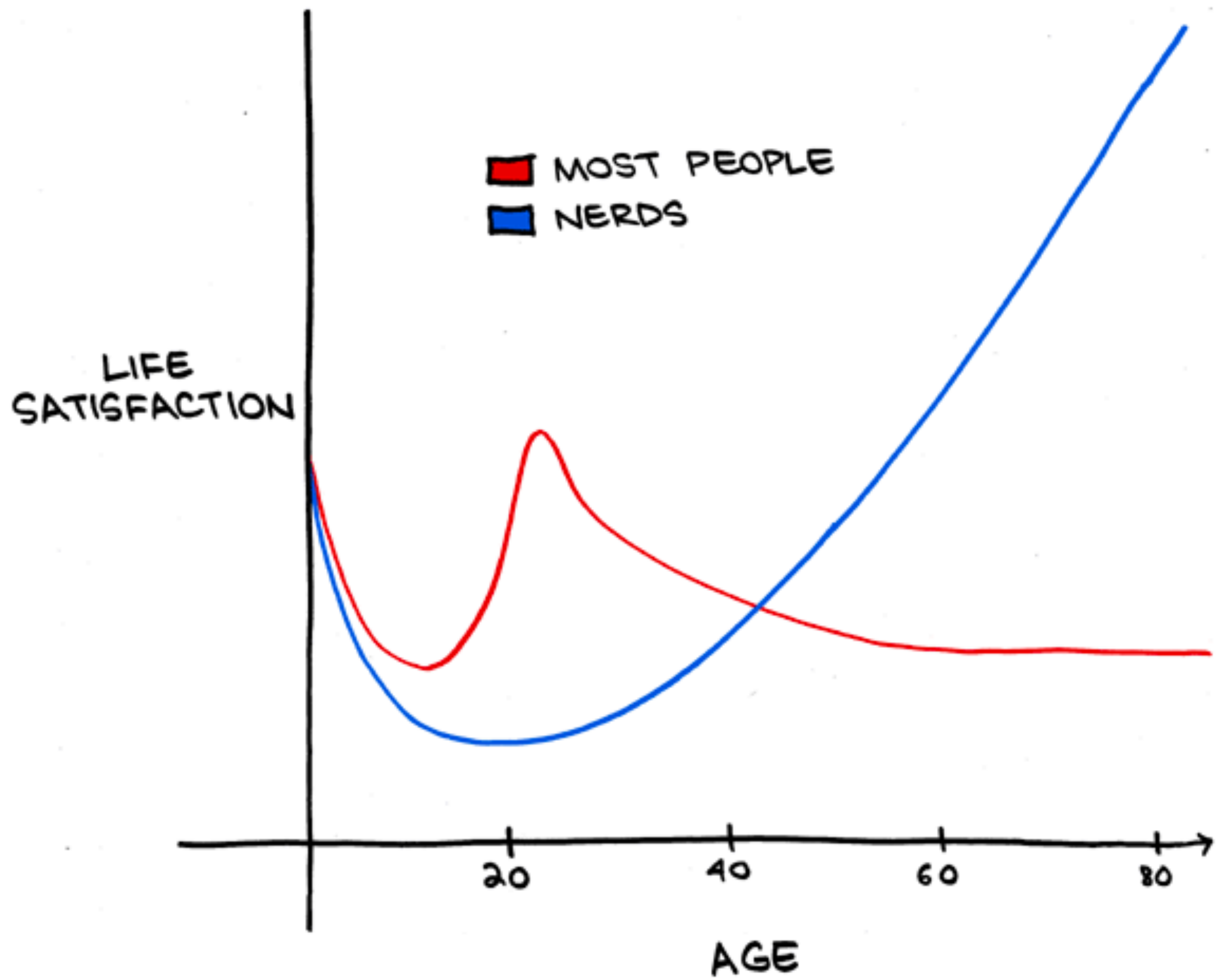

MOST PEOPLE
NERDS
LIFE SATISFACTION
20
40
60
80
AGE

# MEASURING HIGH-DIMENSIONAL ENTANGLEMENT

# Measure of Entanglement Dimensionality

## Schmidt Rank, $k$

Minimum numbers of levels needed to represent a state and its correlations in any local basis

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$$|\Phi\rangle_{AB} = \sum_{m=0}^{k-1} \lambda_m \, |mm\rangle$$

Number of non-zero coefficients

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$$k(|\Phi\rangle_{AB}) = \mathrm{rank}(\mathrm{Tr}_B |\Phi\rangle\langle\Phi|)$$

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Friis, Vitagliano, **MM**, Huber, **Nat. Rev. Phys. 1,** 72-87 (2019)

# ENTANGLEMENT VS CLASSICAL CORRELATIONS

Krenn, MM, Erhard & Zeilinger, Phil Trans R Soc A 375 (2017)

# Entanglement vs Classical Correlations

$$\rho_{1d} = \frac{1}{3}(|00\rangle\langle 00| + |11\rangle\langle 11| + |22\rangle\langle 22|)$$

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$$\rho_{2d} = \frac{1}{3}\left(\left|\psi_{2d}^{01}\right\rangle\left\langle\psi_{2d}^{01}\right| + \left|\psi_{2d}^{02}\right\rangle\left\langle\psi_{2d}^{02}\right| + \left|\psi_{2d}^{03}\right\rangle\left\langle\psi_{2d}^{12}\right|\right)$$

$k = 2$

$$|\psi_{3d}\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$$

$k = 3$

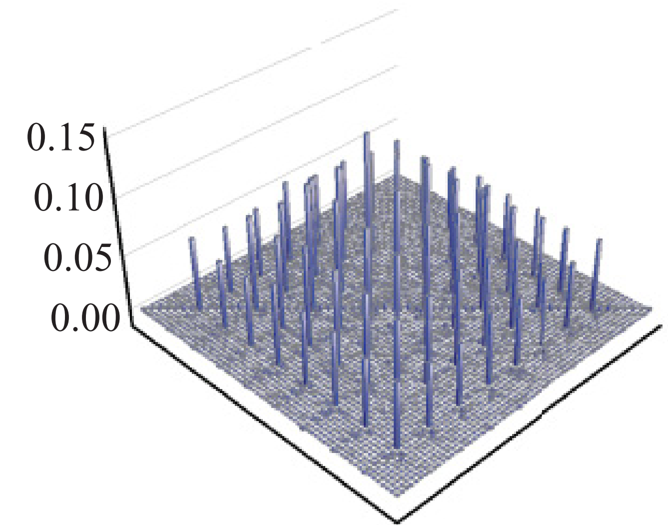
Krenn, **MM**, Erhard & Zeilinger, **Phil Trans R Soc A 375** (2017)

# CERTIFYING ENTANGLEMENT DIMENSIONALITY

PHYSICAL REVIEW A **84**, 062101 (2011)

**Tomography of the quantum state of photons entangled in high dimensions**

Megan Agnew,[1] Jonathan Leach,[1] Melanie McLaren,[2] F. Stef Roux,[2] and Robert W. Boyd[1,3]
[1]*Department of Physics, University of Ottawa, 150 Louis Pasteur, Ottawa, Ontario, K1N 6N5 Canada*
[2]*CSIR National Laser Centre, Pretoria 0001, South Africa*
[3]*Institute of Optics, University of Rochester, Rochester, New York 14627, USA*
(Received 28 September 2011; published 2 December 2011)




★ Full state tomography → k=8 x 8, scales as $d^2(d+1)^2$, time-consuming, unreliable

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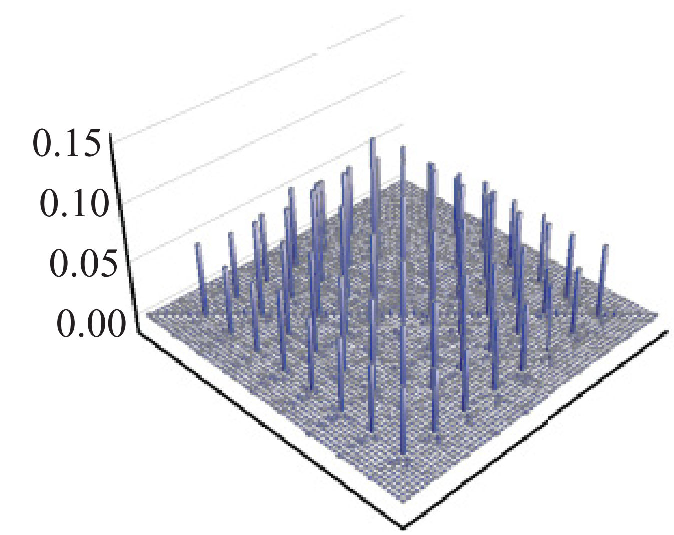


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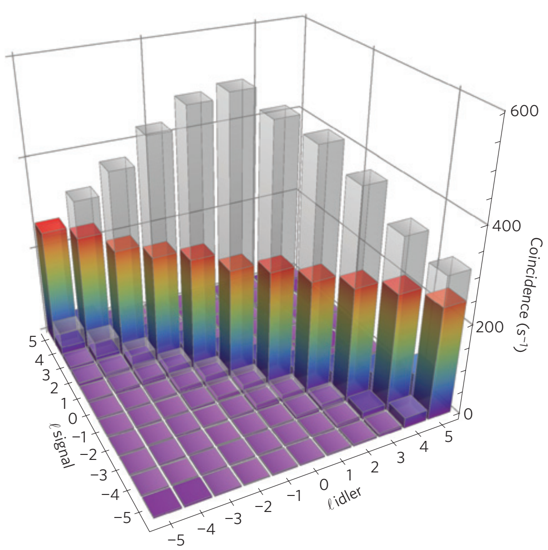



nature physics
LETTERS
PUBLISHED ONLINE: 8 MAY 2011 | DOI: 10.1038/NPHYS1996

**Experimental high-dimensional two-photon entanglement and violations of generalized Bell inequalities**

**Adetunmise C. Dada[1★], Jonathan Leach[2], Gerald S. Buller[1], Miles J. Padgett[2] and Erika Andersson[1]**

- ★ Bell-type (CGLMP) inequality ➞ k=11 x 11, strict criterion, assumptions on state

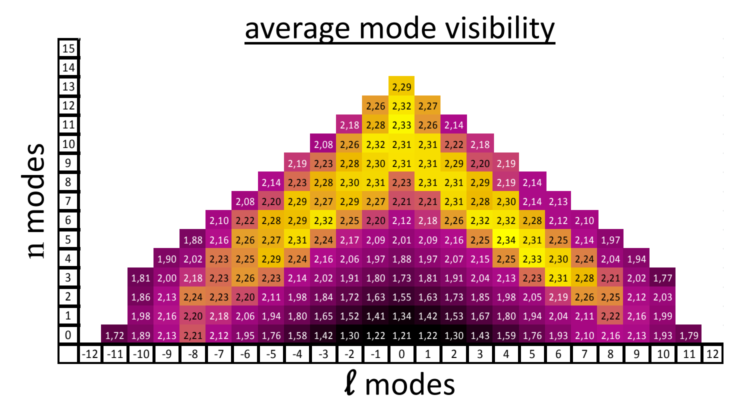
# CERTIFYING ENTANGLEMENT DIMENSIONALITY

## Generation and confirmation of a (100 × 100)-dimensional entangled quantum system

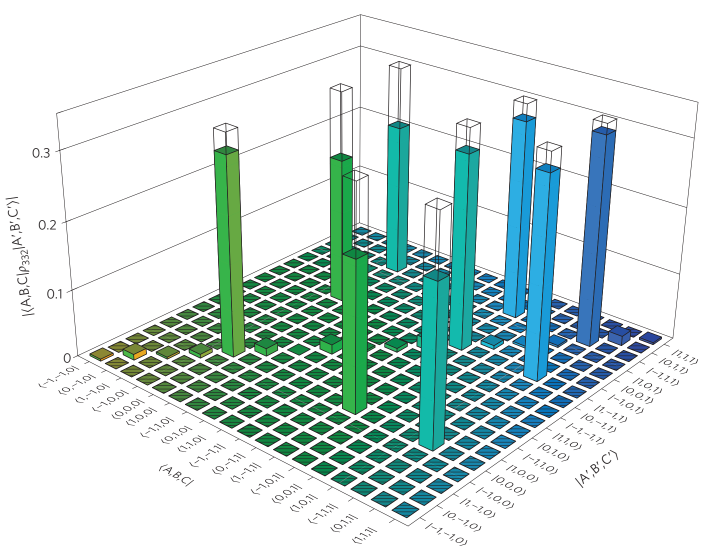
Mario Krenn[a,b,1], Marcus Huber[c,d,e], Robert Fickler[a,b], Radek Lapkiewicz[a,b], Sven Ramelow[a,b,2], and Anton Zeilinger[a,b,1]

[a]Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, A-1090 Vienna, Austria; [b]Vienna Center for Quantum Science and Technology, Faculty of Physics, University of Vienna, A-1090 Vienna, Austria; [c]Department of Mathematics, University of Bristol, Bristol BS8 1TW, United Kingdom; [d]Institut de Ciencies Fotoniques, E-08860 Castelldefels (Barcelona), Spain; and [e]Fisica Teorica: Informacio i Fenomens Quantics, Departament de Fisica, Universitat Autonoma de Barcelona, E-08193 Bellaterra (Barcelona), Spain

Contributed by Anton Zeilinger, February 24, 2014 (sent for review December 15, 2013)




★ Subspace Visibility Witness → k=103 x 103, **d²(d+1)**, assumed conservation of OAM



LETTERS
PUBLISHED ONLINE: 29 FEBRUARY 2016 | DOI: 10.1038/NPHOTON.2016.12
nature photonics

## Multi-photon entanglement in high dimensions

Mehul Malik[1,2★], Manuel Erhard[1,2], Marcus Huber[3,4,5], Mario Krenn[1,2], Robert Fickler[1,2†] and Anton Zeilinger[1,2]

★ 2D Fidelity Witness → k=3 x 3 x 2, scales as **d²(d+1)**

# CERTIFYING ENTANGLEMENT DIMENSIONALITY

PNAS

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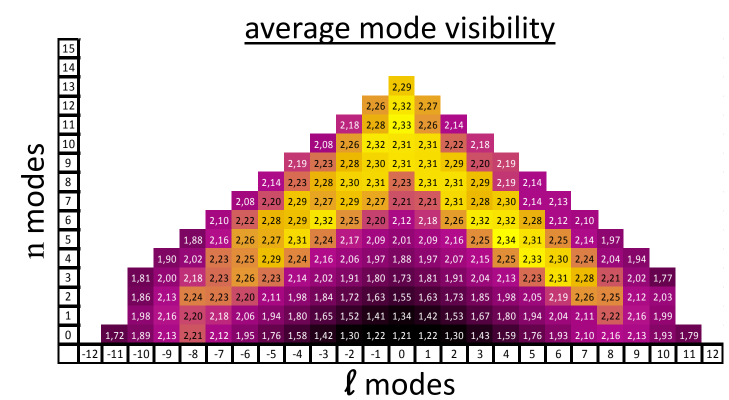


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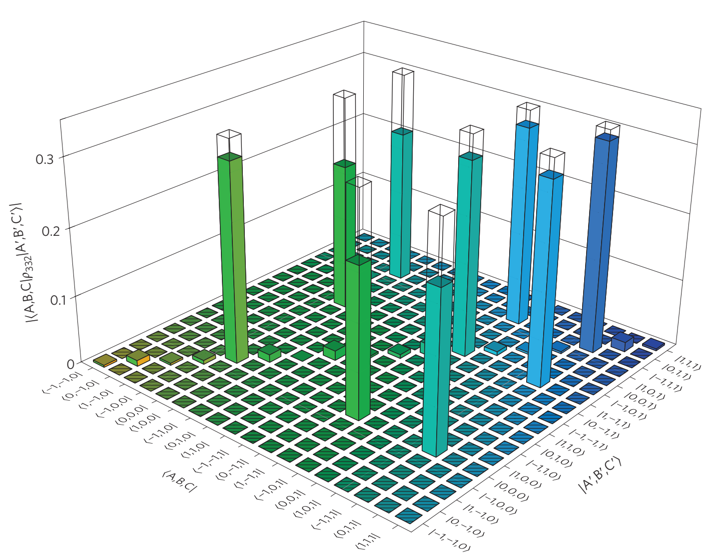

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Assumption-free, efficient entanglement witness? → QKD, Qinfo

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W. K. Wootters and B. D. Fields, Annals of Physics 191, 363–381 (1989).

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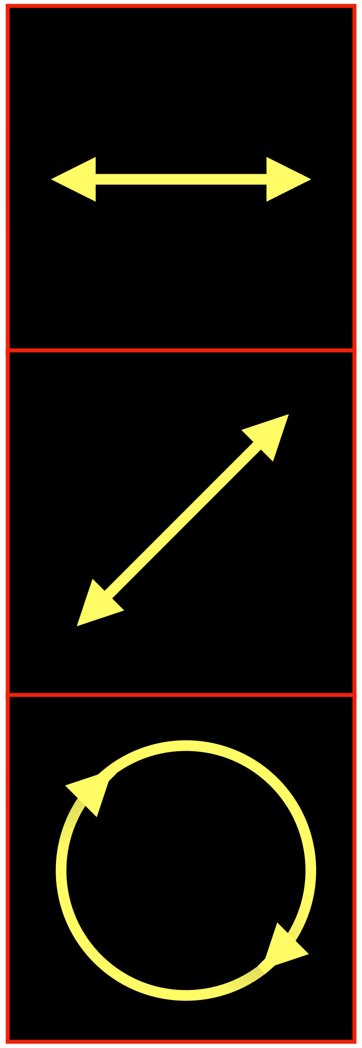

N = 3

$$|D\rangle = \frac{1}{\sqrt{2}}[|H\rangle + |V\rangle] \qquad |A\rangle = \frac{1}{\sqrt{2}}[|H\rangle - |V\rangle]$$

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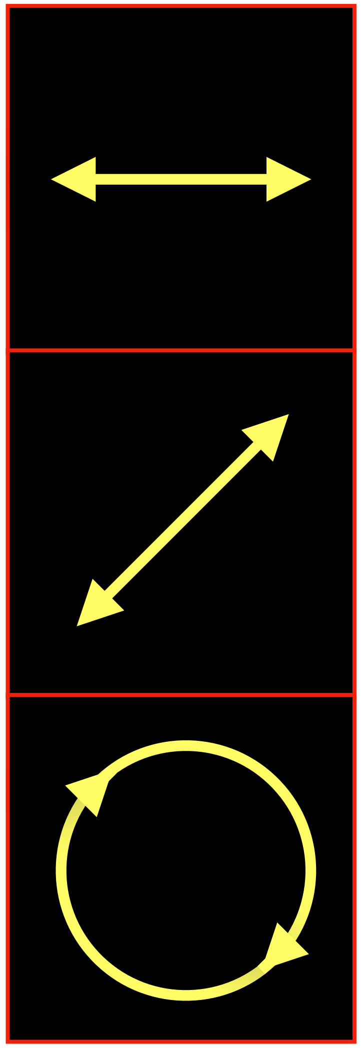

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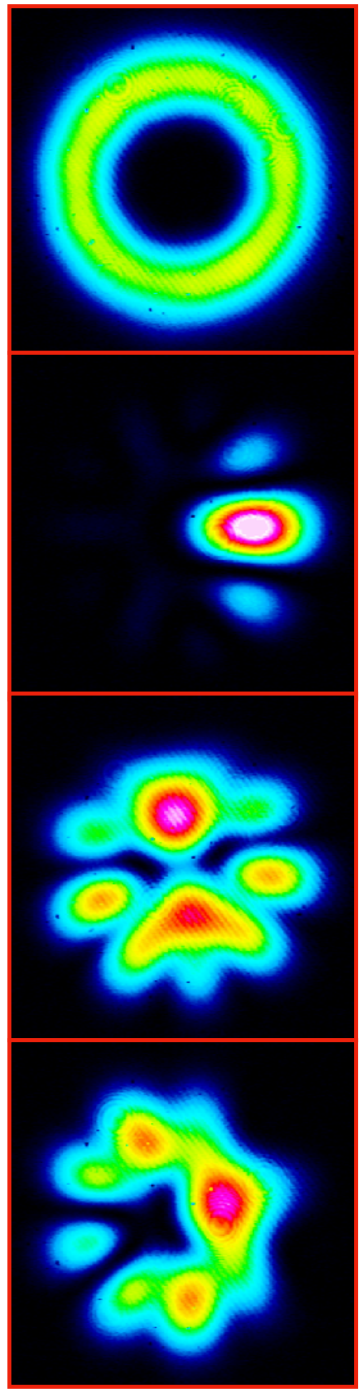

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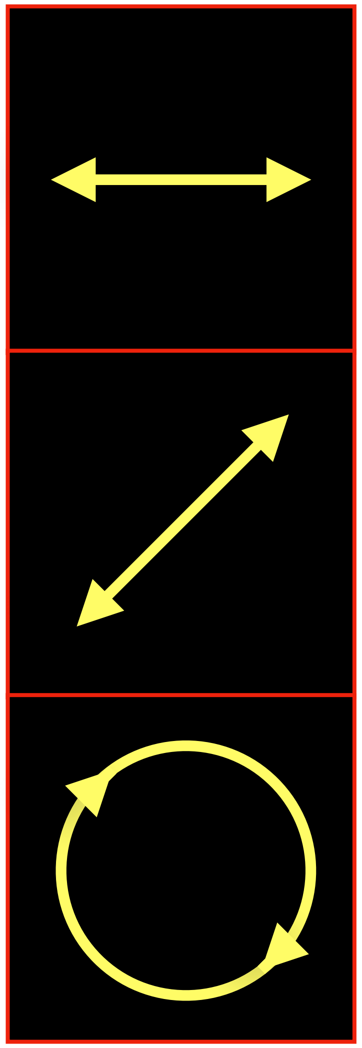

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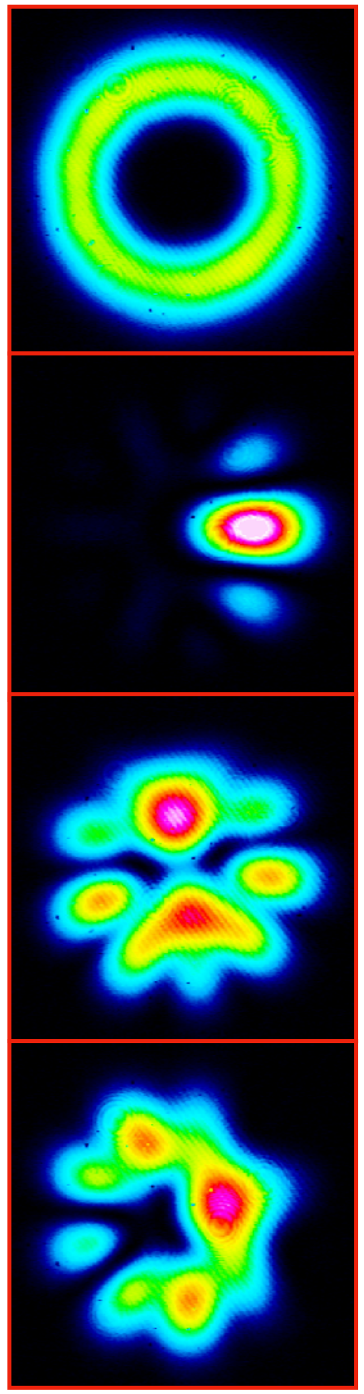

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$$|j\rangle_k = \frac{1}{\sqrt{d}} \sum_{m=0}^{d-1} \omega^{jm+km^2}\, |m\rangle\ , \quad \omega = e^{2\pi i/d}$$

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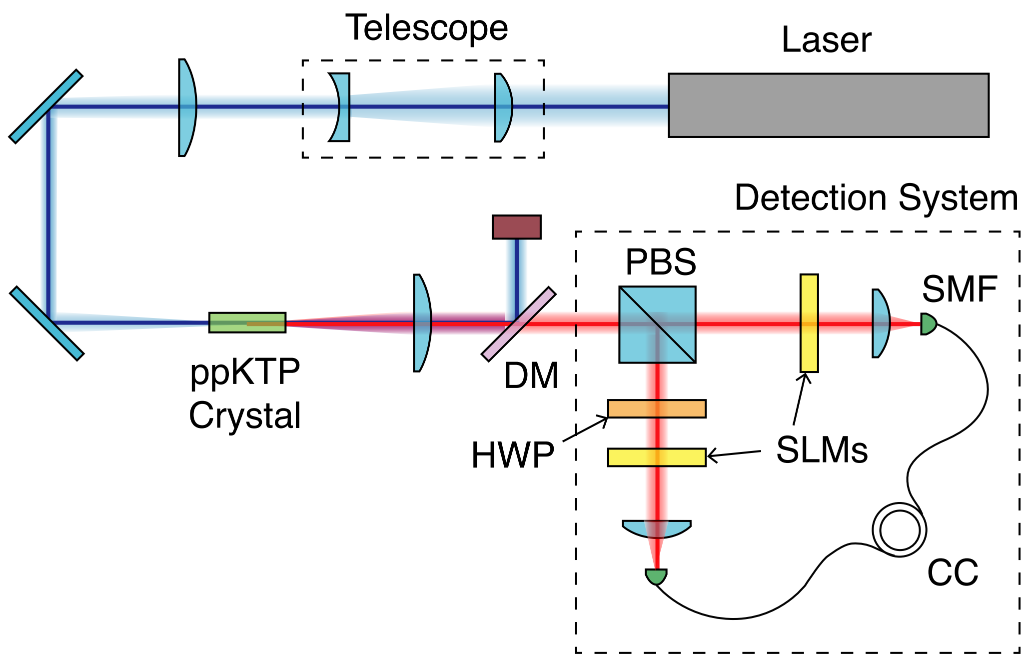
# ADAPTIVE WITNESS: RECIPE

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Bavaresco, Herrera, Klöckl, Pivoluska, Erker, Friis, **MM**, Huber, **Nat. Phys. 14,** 1032 (2018)

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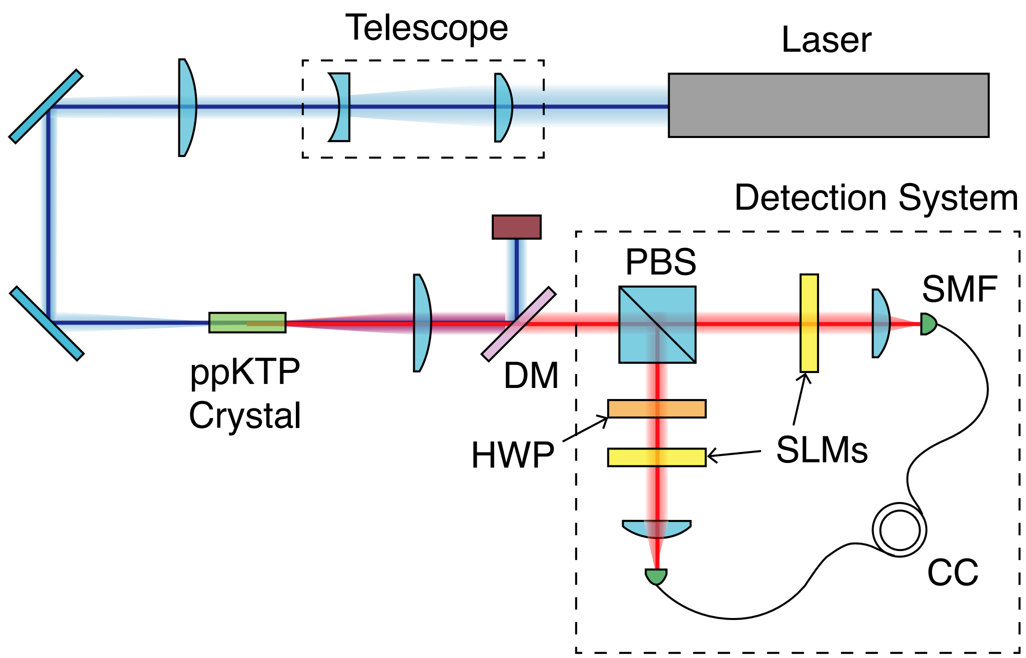
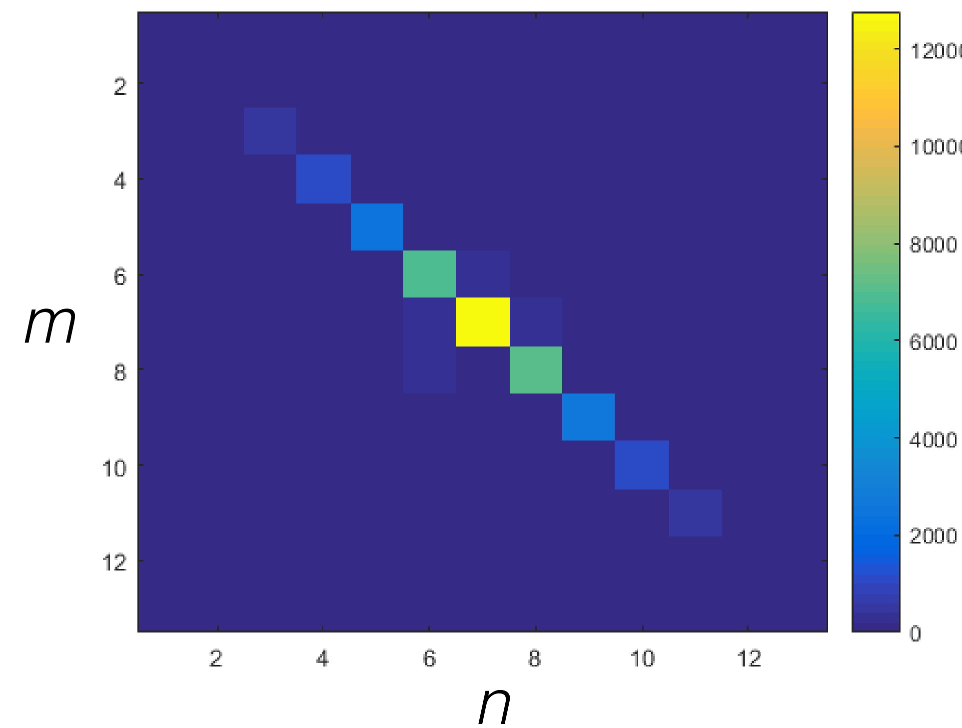
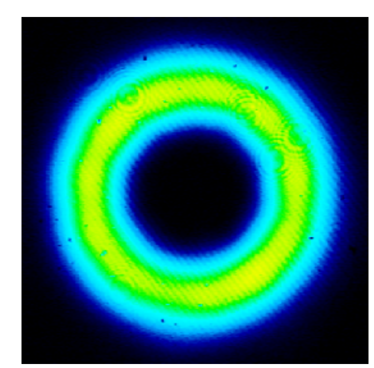



Measure two-photon counts in Schmidt (OAM) basis to obtain diagonal density matrix elements:

**1.**

$$\langle mn|\rho|mn\rangle = \frac{N_{mn}}{\sum_{i,j} N_{ij}}$$



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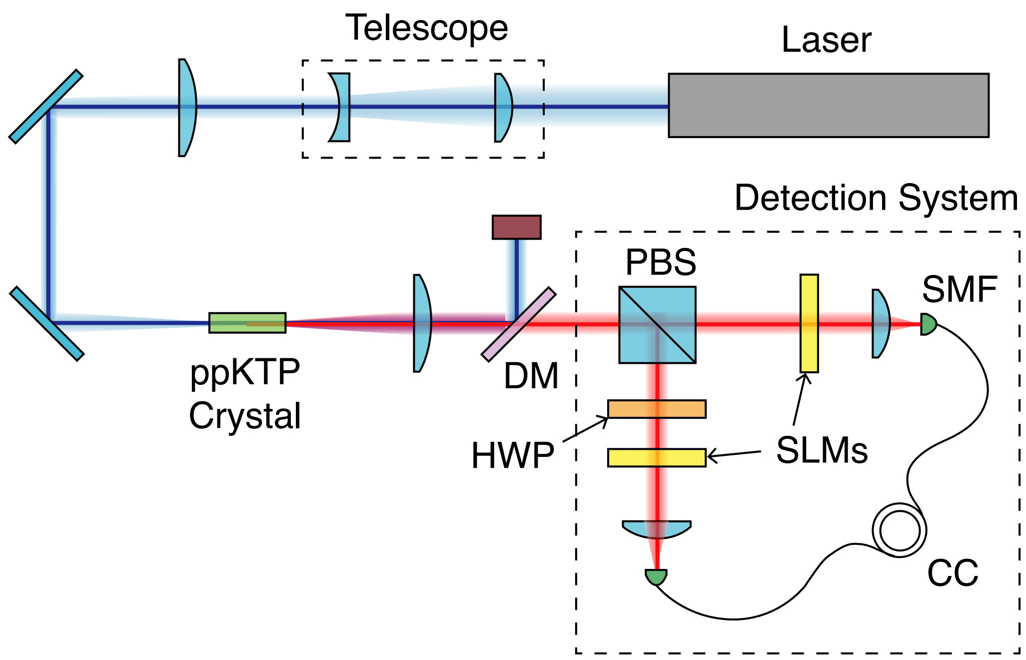


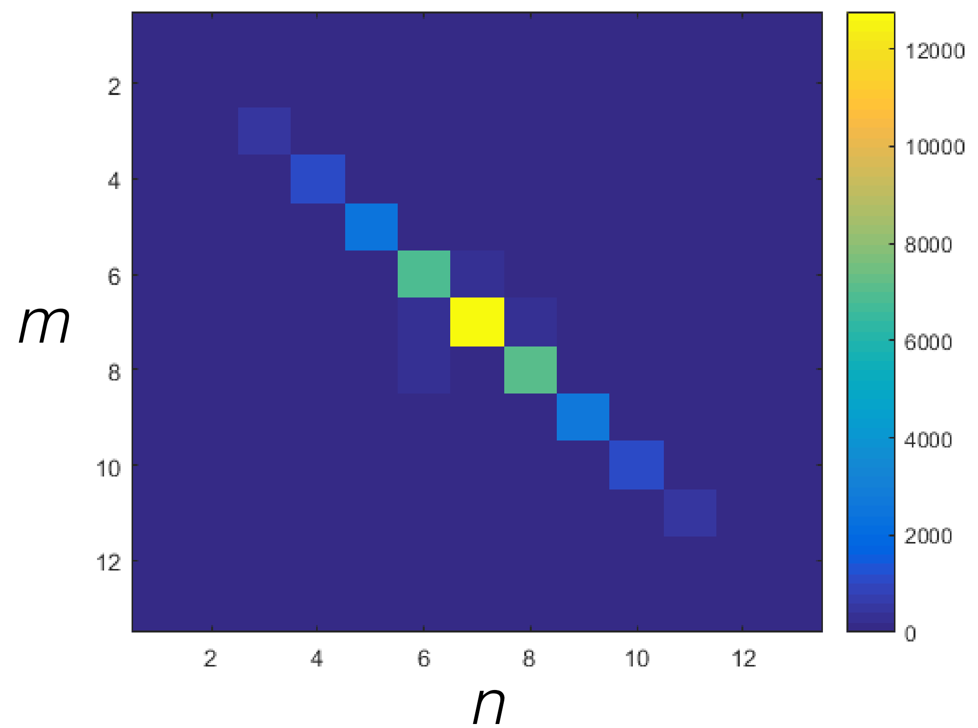


2.

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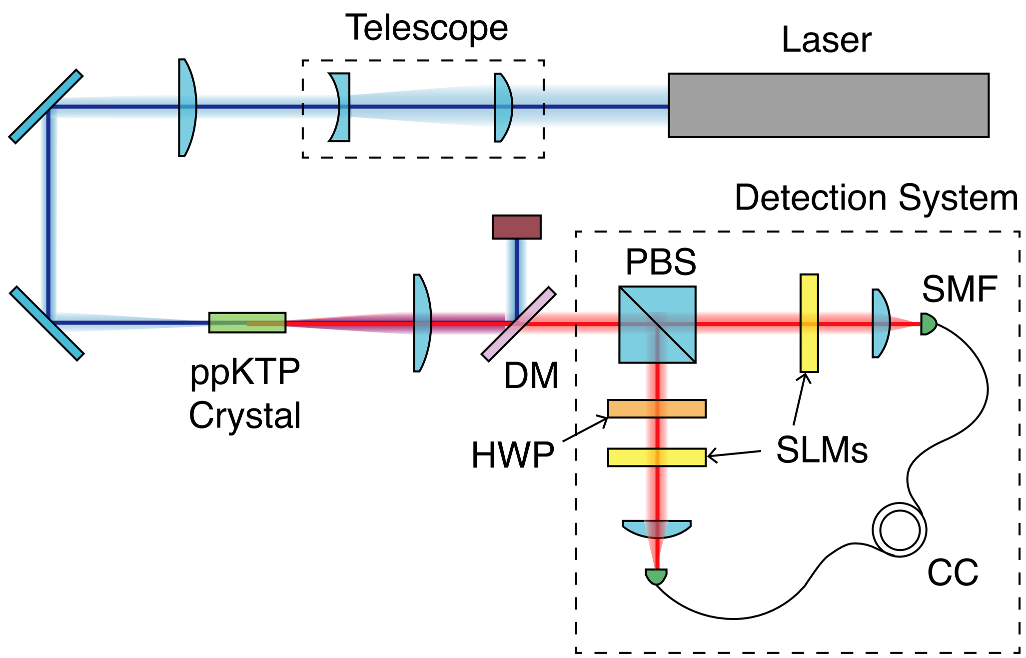
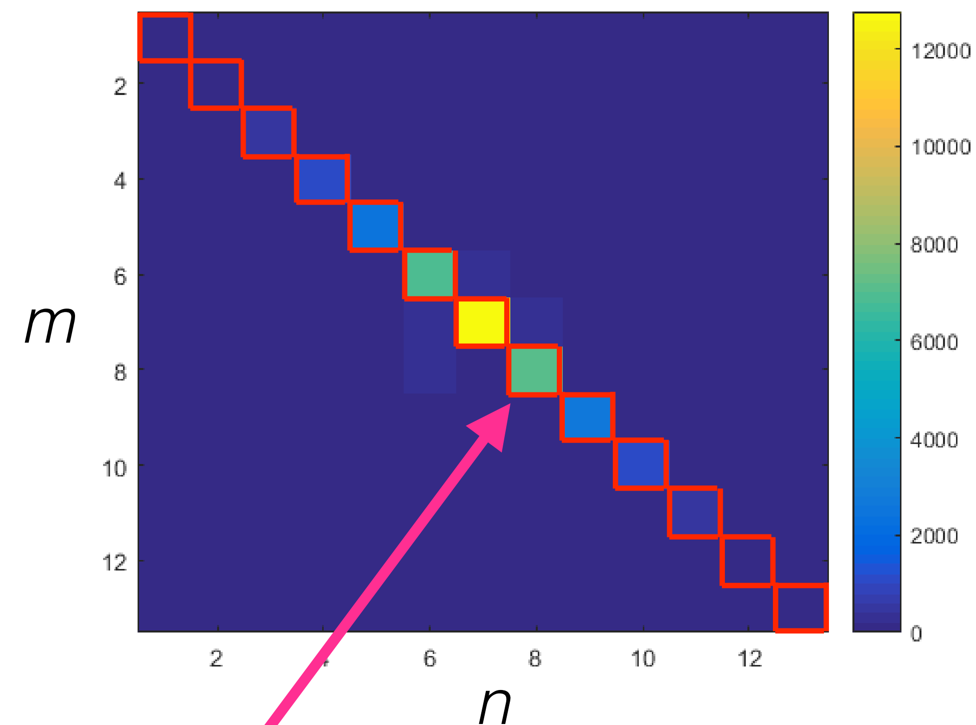
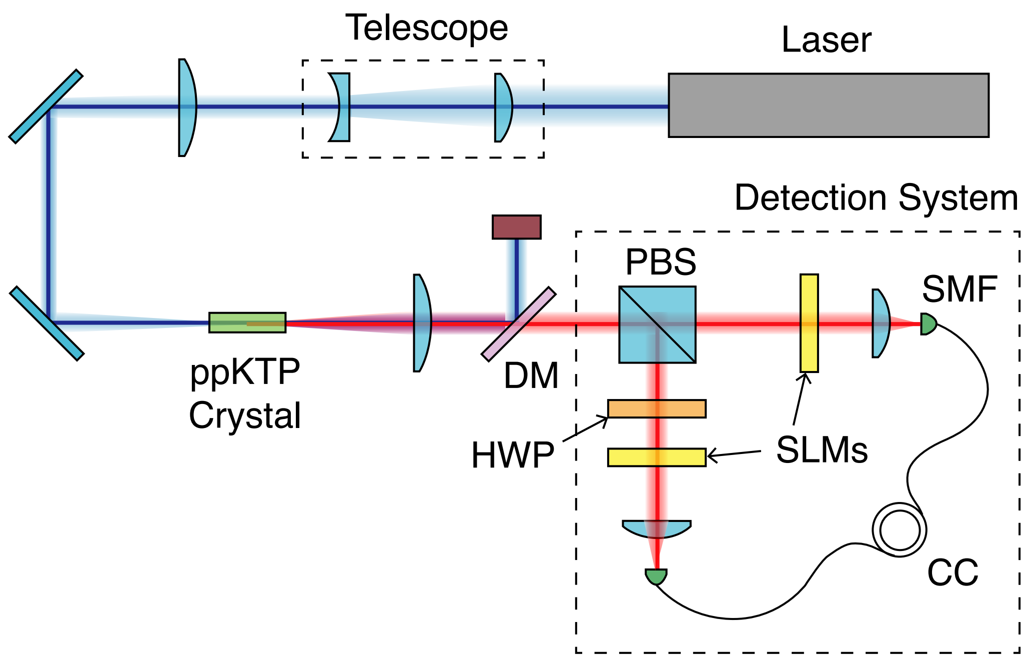


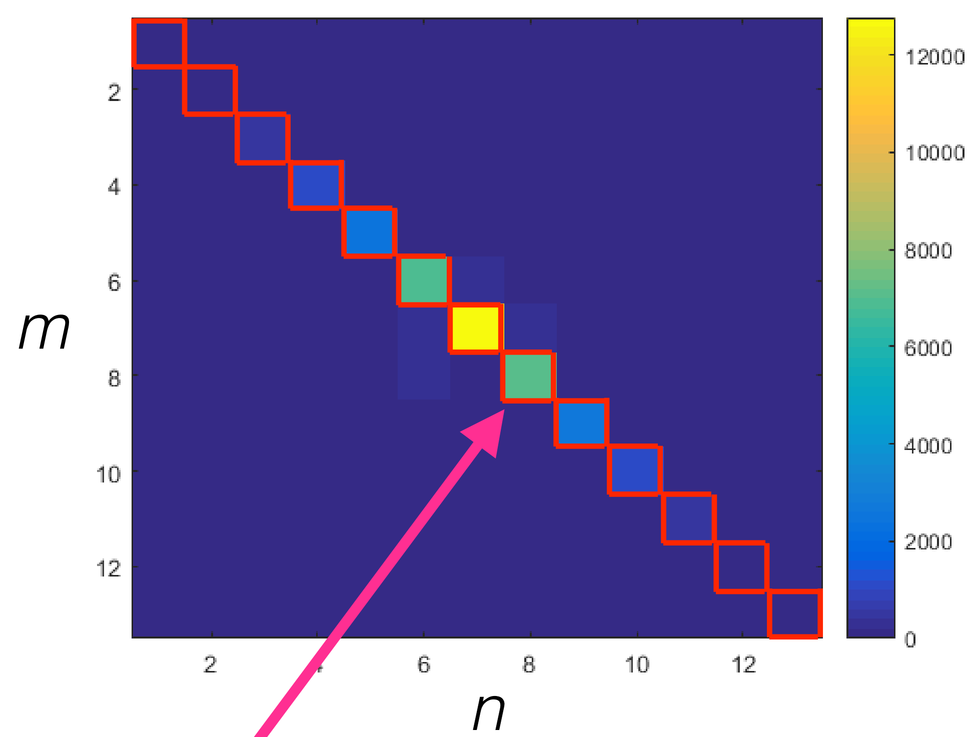




Calculate weights: $\lambda_m = \sqrt{\frac{\langle mm|\rho|mm\rangle}{\sum_n \langle nn|\rho|nn\rangle}}$

2.

# ADAPTIVE WITNESS: RECIPE




2. Calculate weights: $\lambda_m = \sqrt{\dfrac{\langle mm|\rho|mm\rangle}{\sum_n \langle nn|\rho|nn\rangle}}$

Nominate target state: $$\boxed{|\Phi\rangle = \sum_{m=0}^{d-1} \lambda_m |mm\rangle}$$

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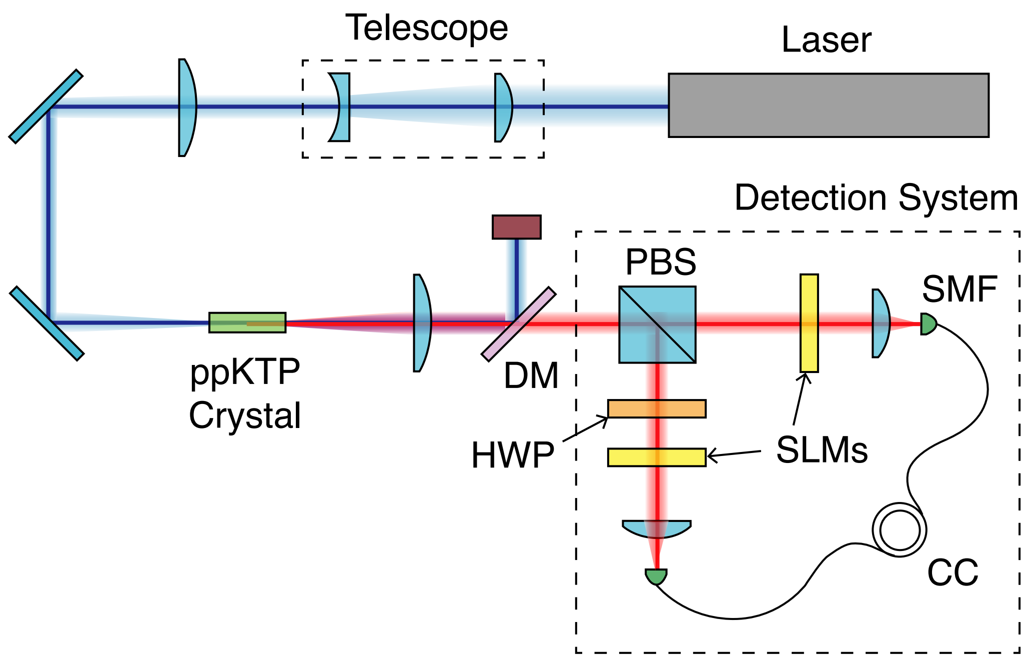


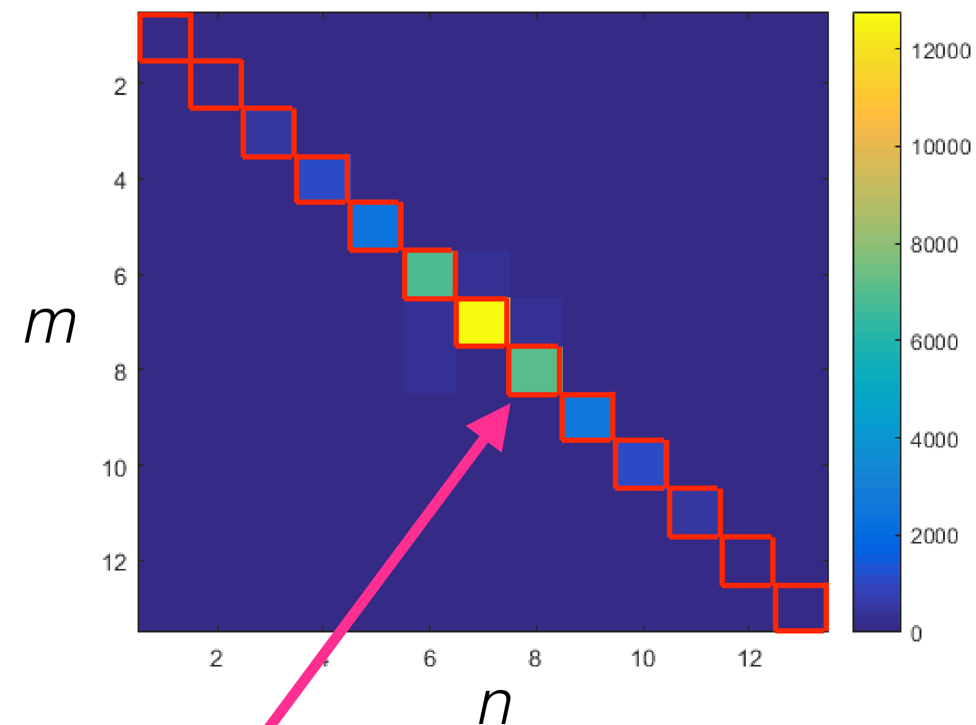


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SCHMIDT NUMBER *d*

# ADAPTIVE WITNESS: RECIPE

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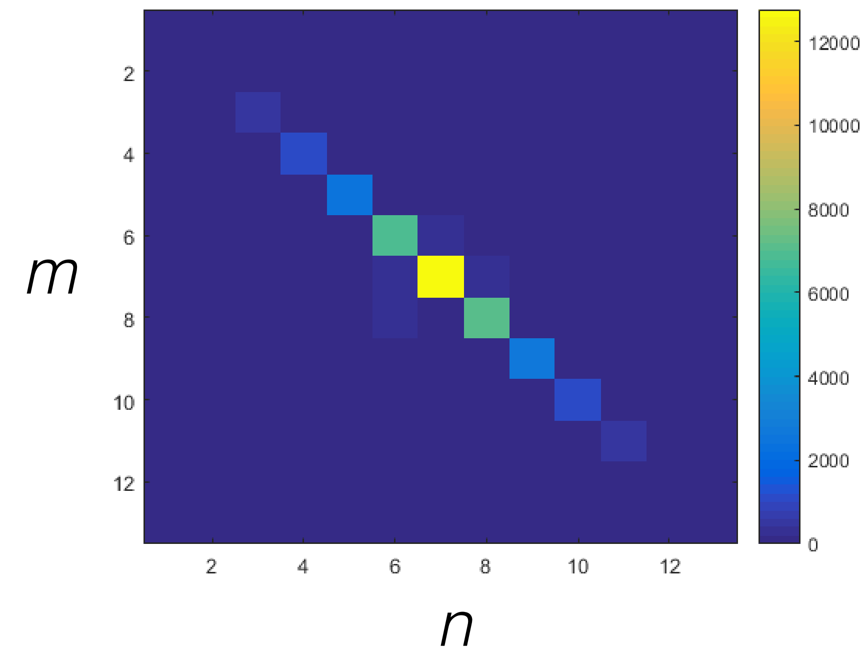


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# ADAPTIVE WITNESS: RECIPE

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For a state of rank ***k* ≤ *d***, fidelity to target state is upper-bounded by:

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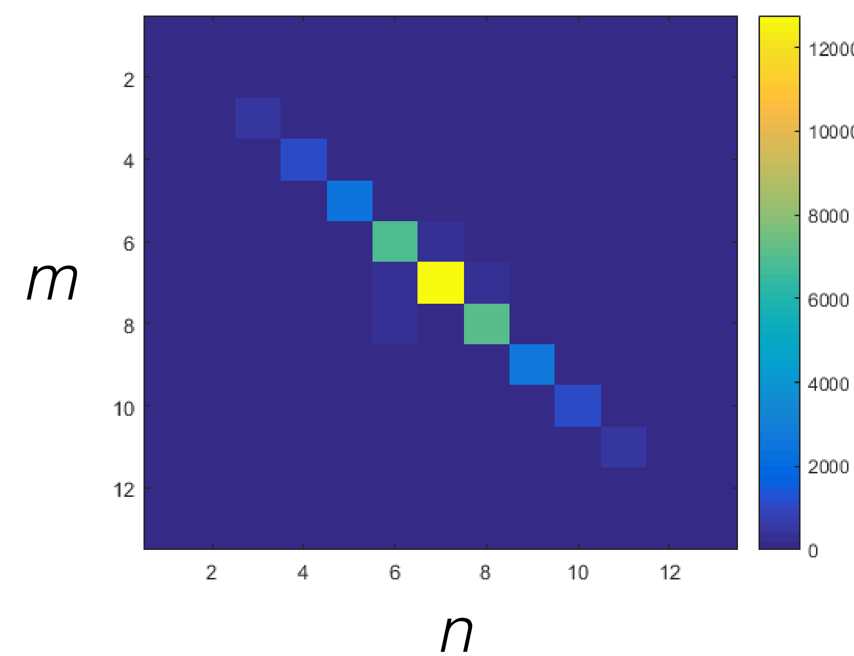
$$B_k(\Phi) := \sum_{m=0}^{k-1} \lambda_m^2$$




# 2.

Fickler et al, **Nat. Commun. 5:4502** (2014)

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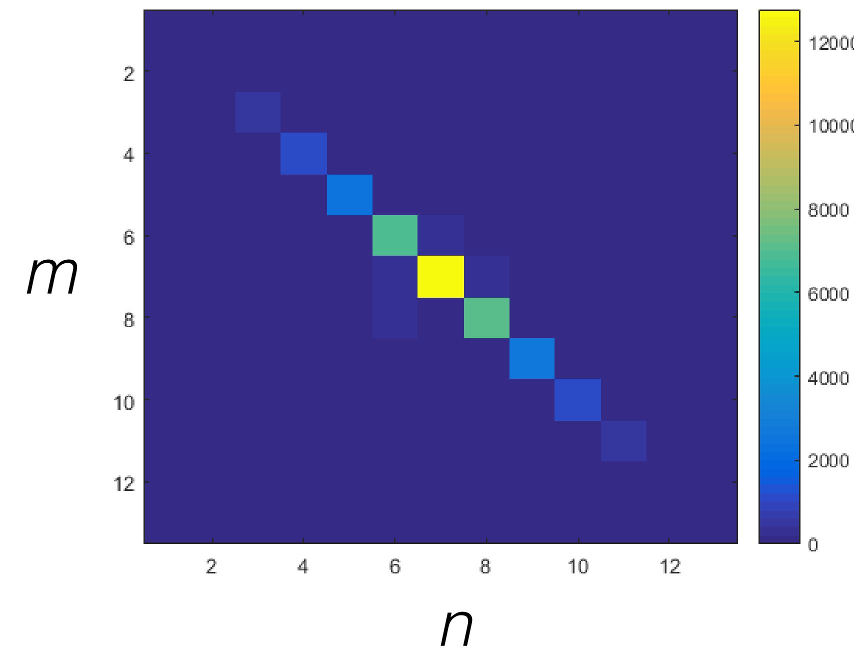


2.

Eg: If state fidelity is greater than

$$F(\rho, \Phi) > B_3(\Phi) := \lambda_0^2 + \lambda_1^2 + \lambda_2^2$$

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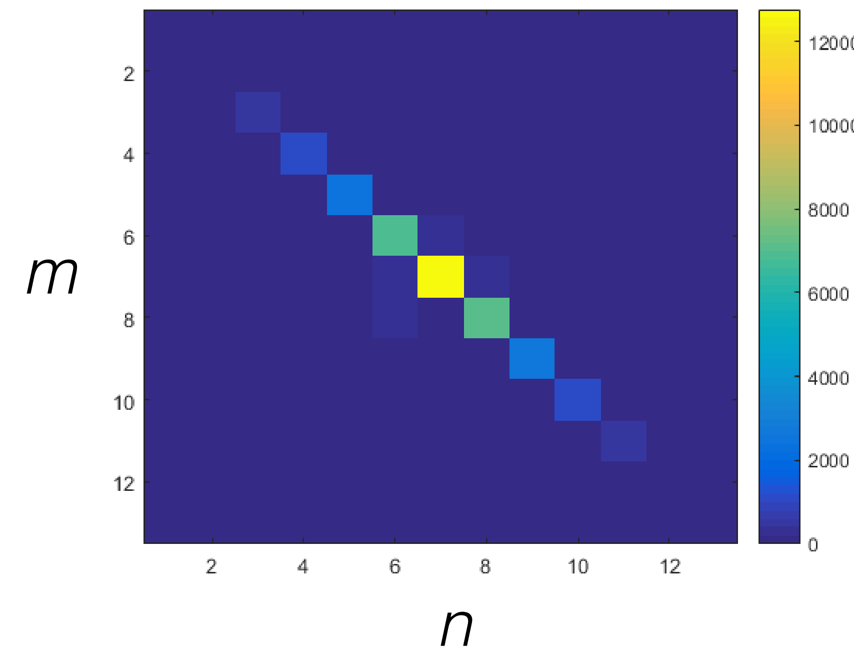


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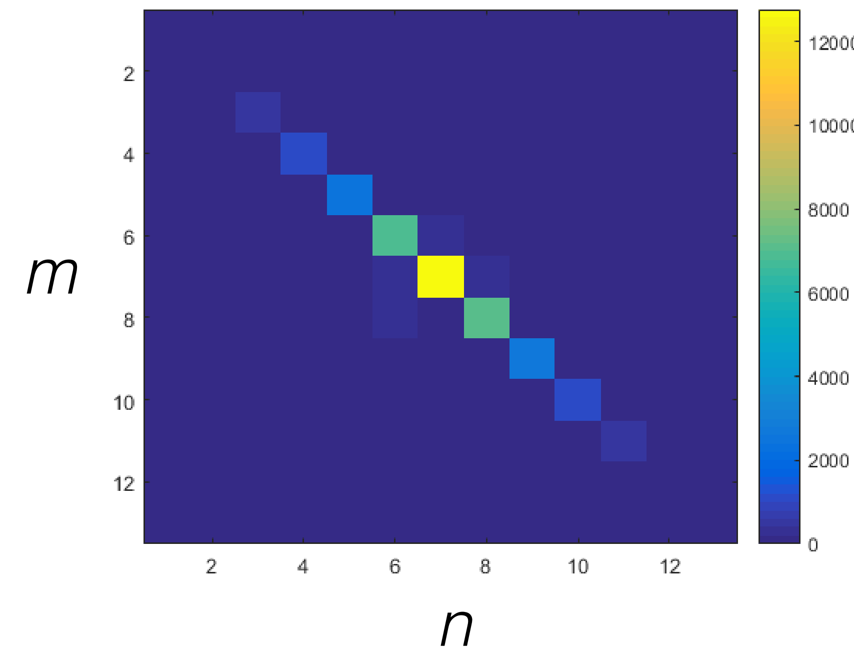


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Entanglement dimensionality = **4**

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Fickler et al, **Nat. Commun. 5:4502** (2014)

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$$= \sum_{m,n=0}^{d-1} \lambda_m \lambda_n \langle mm|\,\rho\,|nn\rangle$$

2.

# Adaptive Witness: Recipe

$$
\begin{aligned}
F(\rho,\Phi) &= \mathrm{Tr}\left(|\Phi\rangle\langle\Phi|\,\rho\right) \\
&= \sum_{m,n=0}^{d-1} \lambda_m \lambda_n \langle mm|\,\rho\,|nn\rangle \\
&= \sum_{m} \lambda_m^2 \langle mm|\,\rho\,|mm\rangle + \sum_{m,n\neq m} \lambda_m \lambda_n \langle mm|\,\rho\,|nn\rangle \\
&= F_1(\rho,\Phi) + F_2(\rho,\Phi)
\end{aligned}
$$

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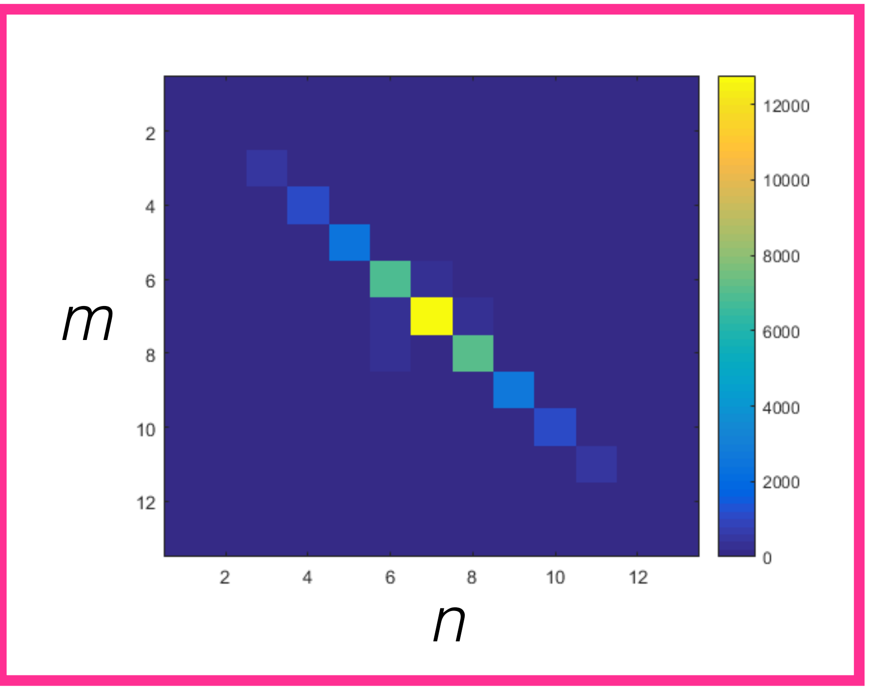
**2.**

**DIAGONAL ELEMENTS**



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**OFF-DIAGONAL ELEMENTS**

**2.**

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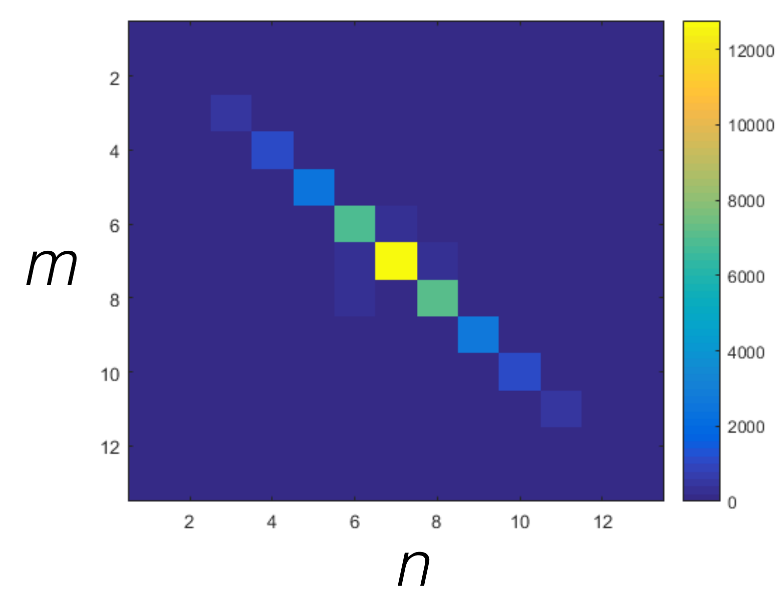

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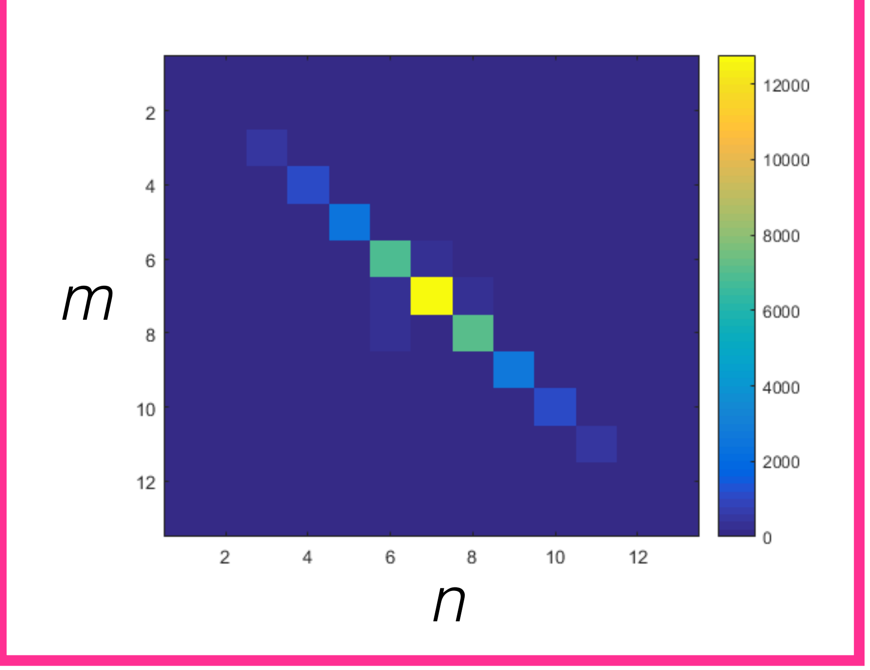
**OFF-DIAGONAL ELEMENTS NEED ANOTHER BASIS!**

# 2.

**DIAGONAL ELEMENTS**



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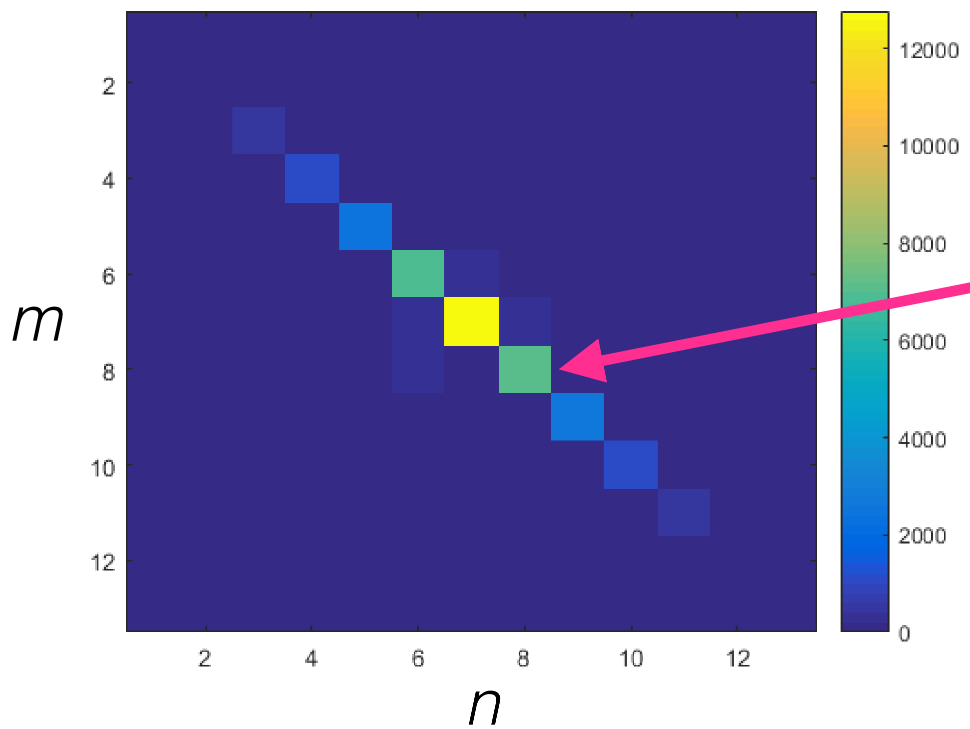
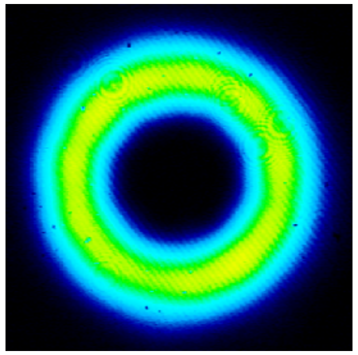



$$\lambda_m = \sqrt{\frac{\langle mm|\,\rho\,|mm\rangle}{\sum_n \langle nn|\,\rho\,|nn\rangle}}$$

3.



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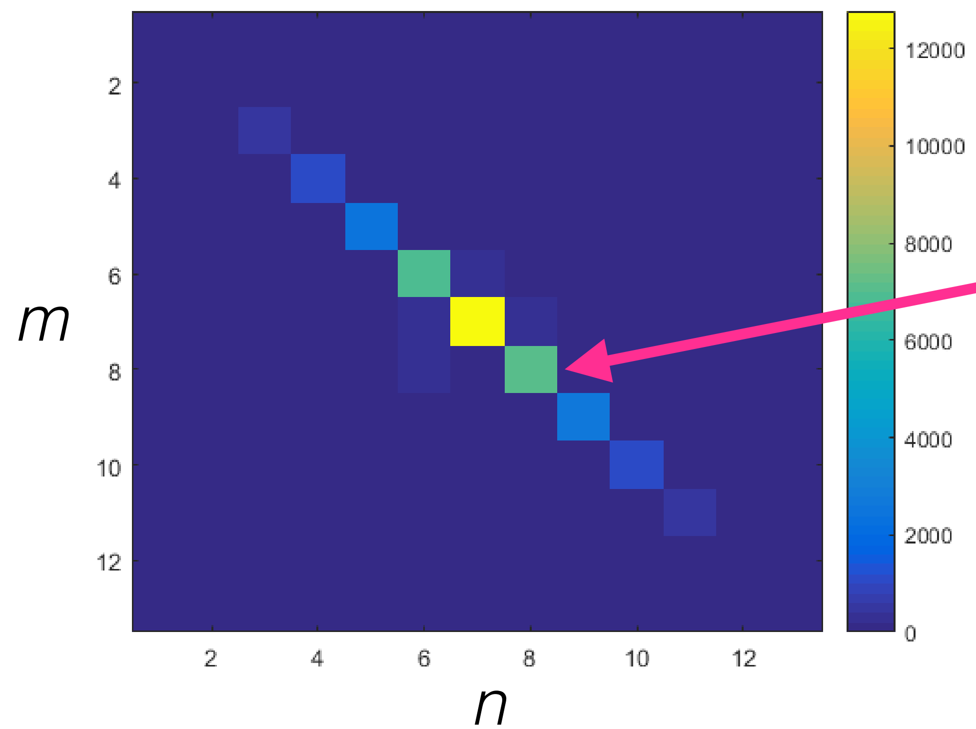
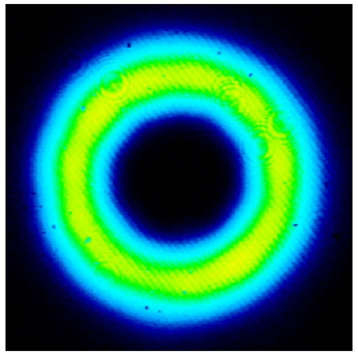



$$\lambda_m = \sqrt{\frac{\langle mm|\,\rho\,|mm\rangle}{\sum_n \langle nn|\,\rho\,|nn\rangle}}$$

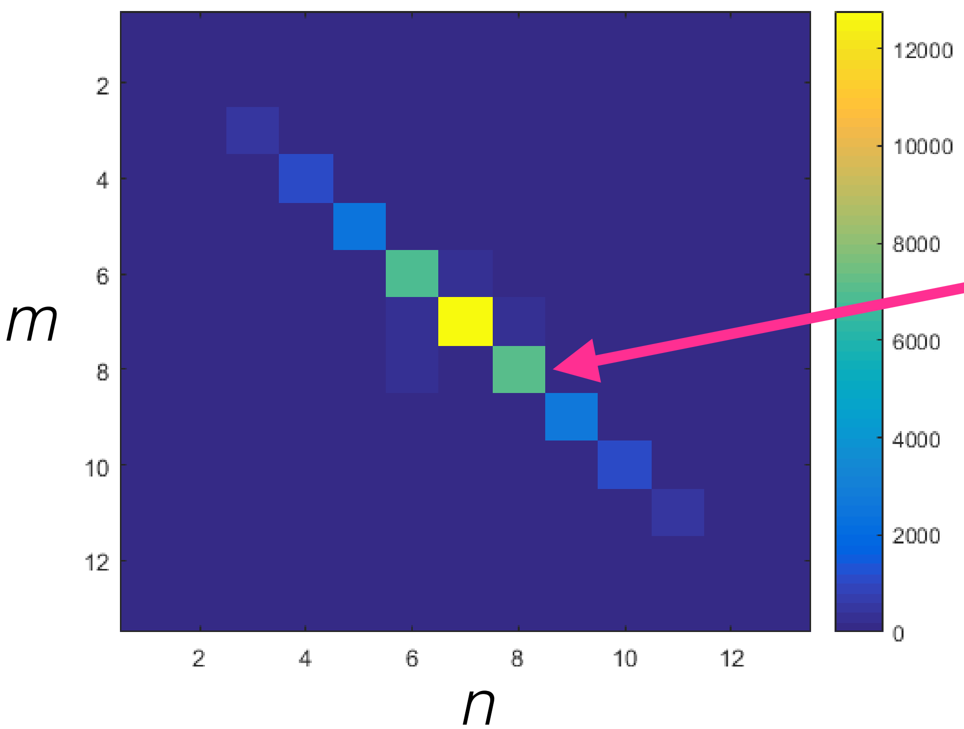
$$\omega = e^{2\pi i/d}$$

Construct tilted basis: $$|\tilde{j}\rangle = \frac{1}{\sqrt{\sum_n \lambda_n}} \sum_{m=0}^{d-1} \omega^{jm} \sqrt{\lambda_m}\,|m\rangle$$

3.



# ADAPTIVE WITNESS: RECIPE




$$\lambda_m = \sqrt{\frac{\langle mm|\,\rho\,|mm\rangle}{\sum_n \langle nn|\,\rho\,|nn\rangle}}$$

$$\omega = e^{2\pi i/d}$$

Construct tilted basis: $$|\tilde{j}\rangle = \frac{1}{\sqrt{\sum_n \lambda_n}} \sum_{m=0}^{d-1} \omega^{jm} \sqrt{\lambda_m}\,|m\rangle$$

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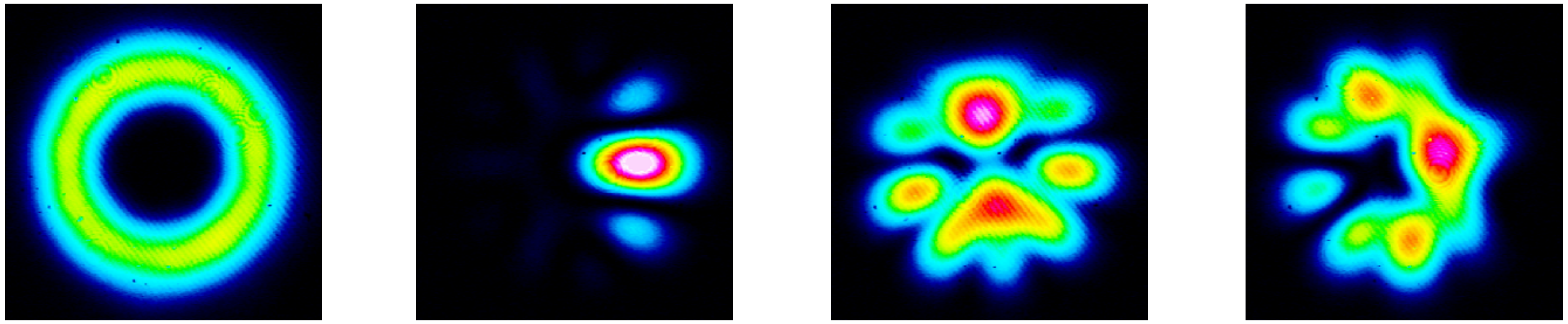
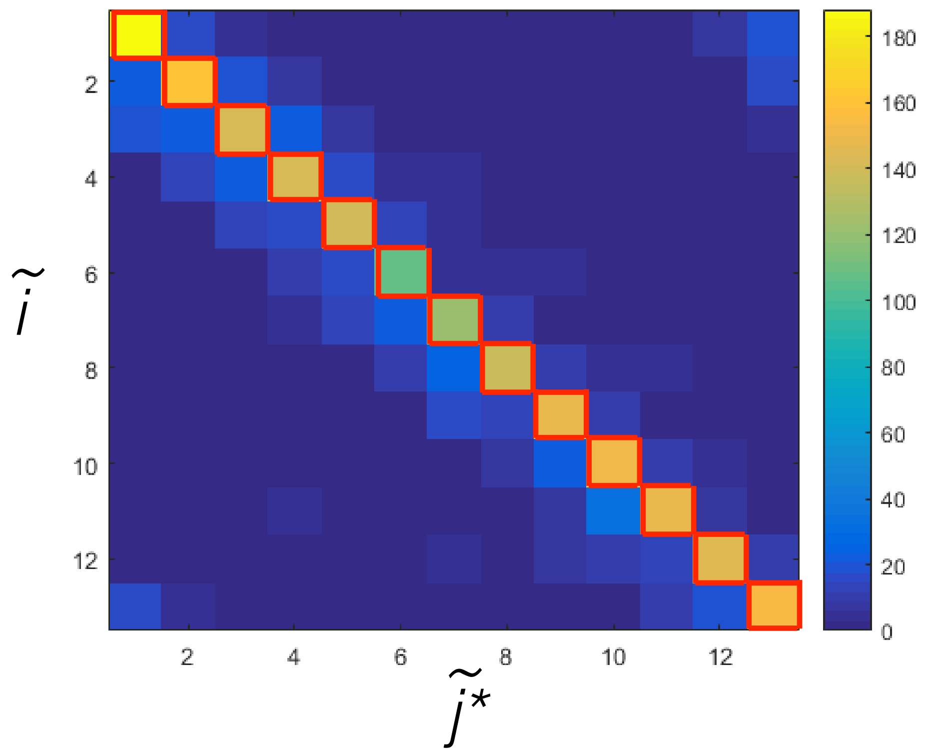
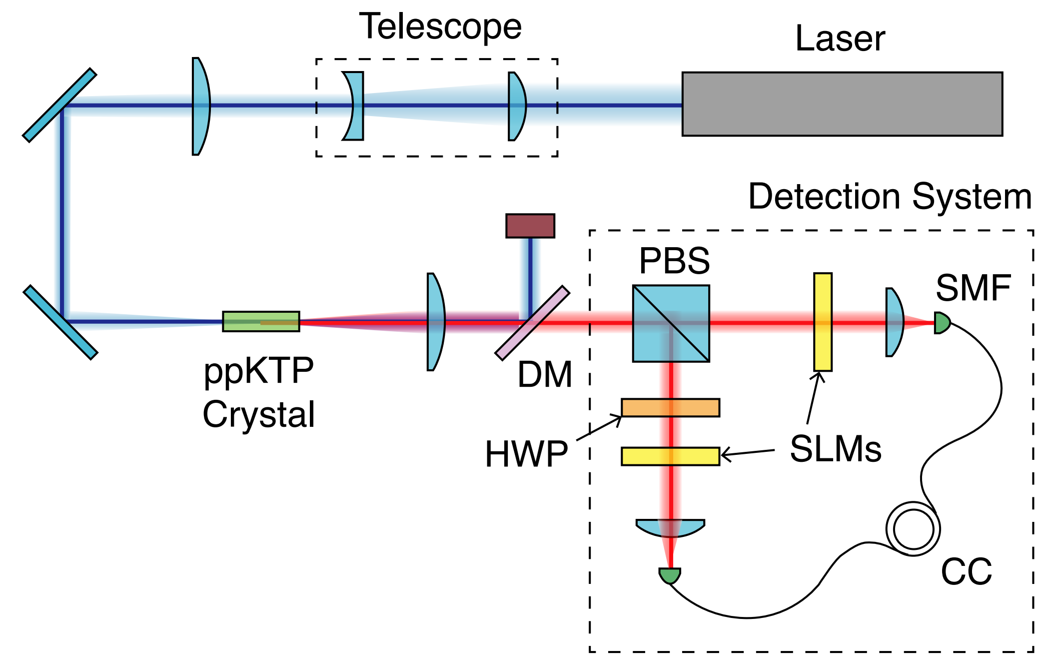


Wootters and Fields, **Ann. of Phys. 191**, 363 (1989)

# ADAPTIVE WITNESS: RECIPE



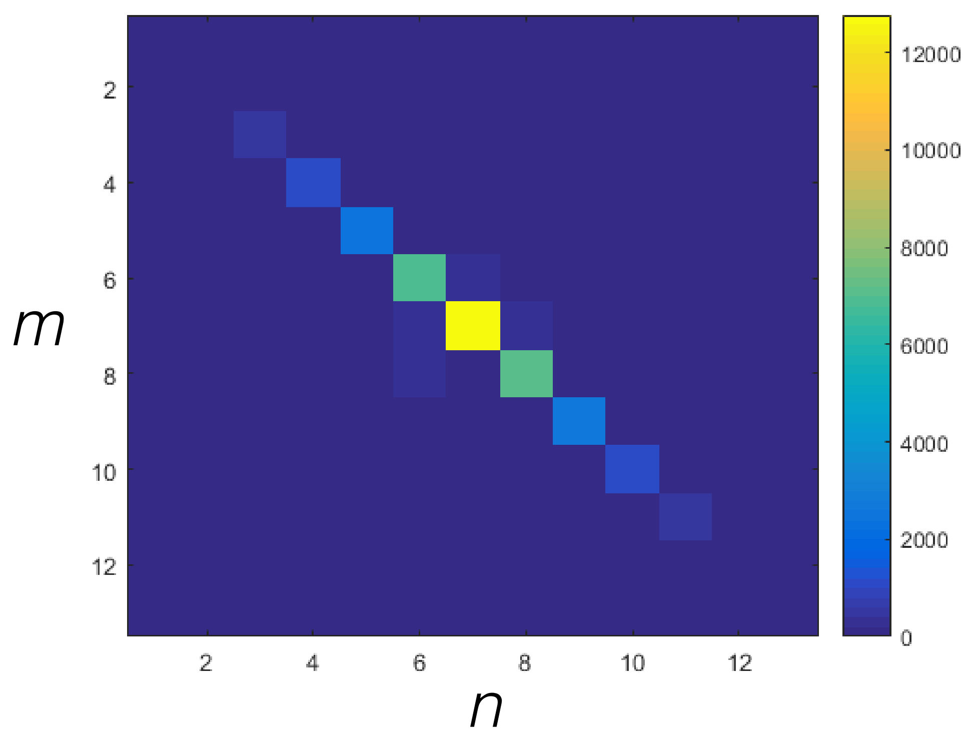



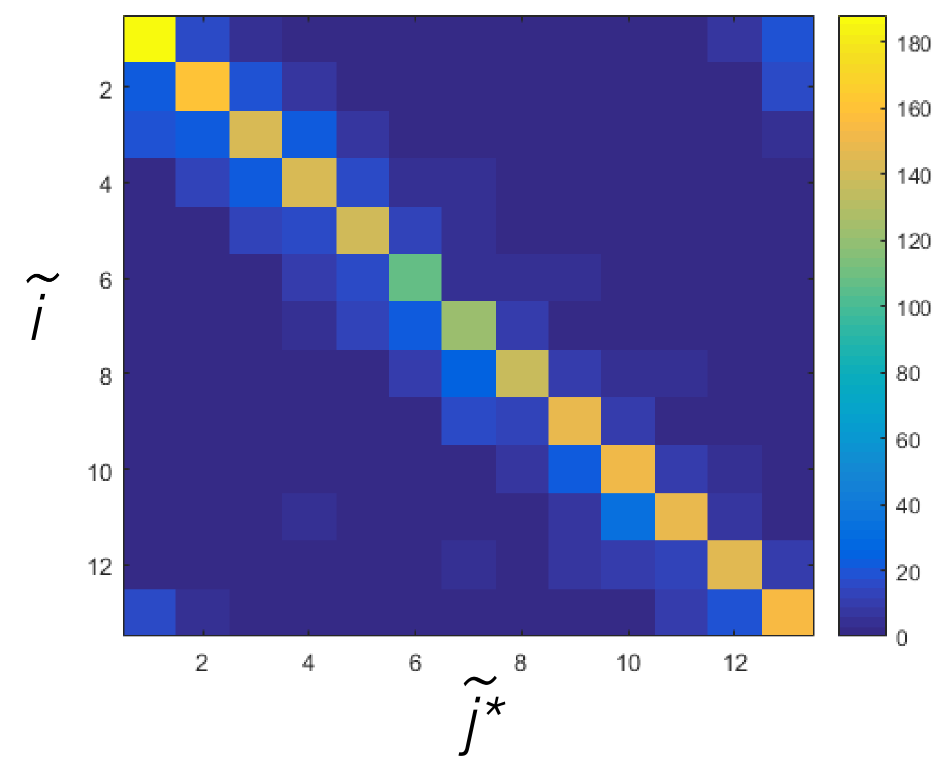

Measure two-photon counts in tilted basis to obtain:

**4.**

$$\boxed{\langle \tilde{j}\tilde{j}^* | \rho | \tilde{j}\tilde{j}^* \rangle = \frac{\tilde{N}_{jj}}{\sum_{i,j} \tilde{N}_{ij}} c_\lambda}$$

# Adaptive Witness: Recipe




Use measurements in two bases to calculate fidelity bound:

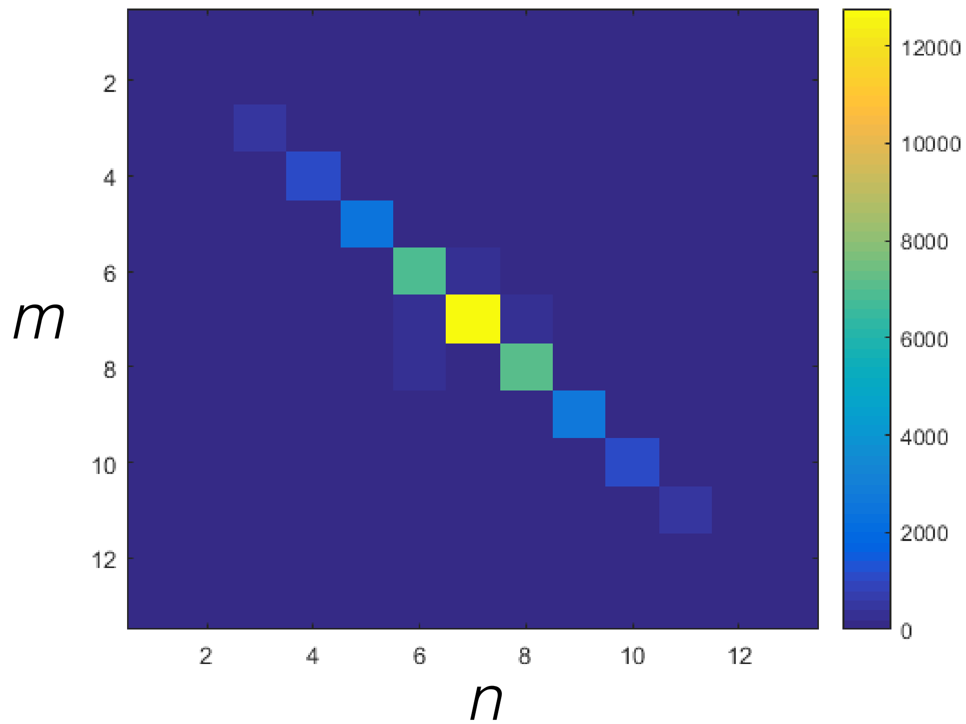
5.

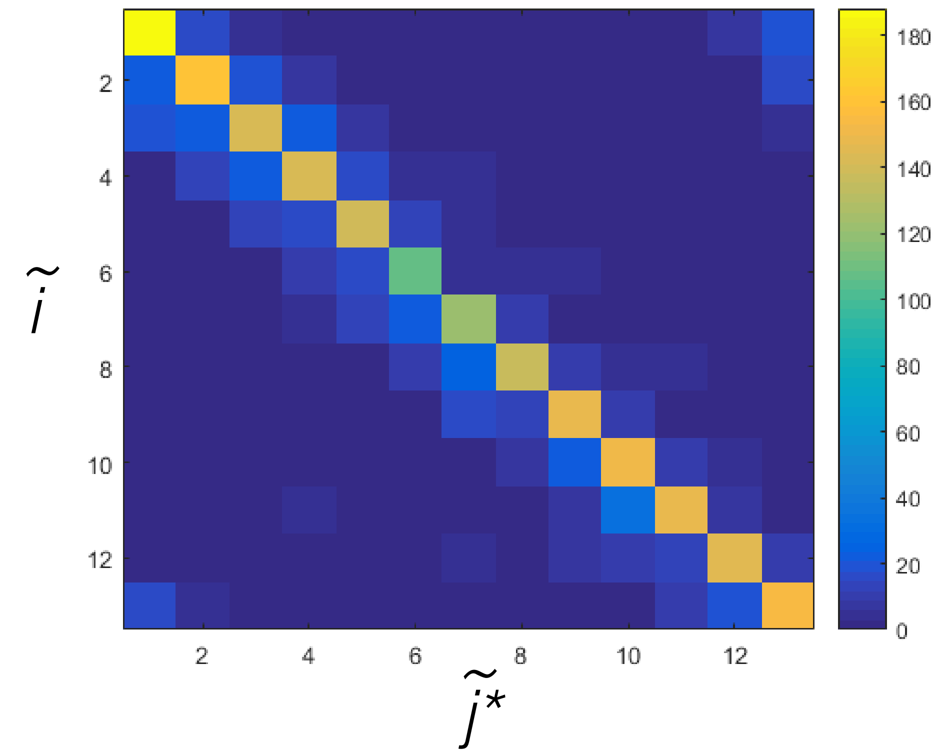
$$F(\rho,\Phi) \geq \sum_m \lambda_m^2 \langle mm|\rho|mm\rangle + \frac{\left(\sum_m \lambda_m\right)^2}{d} \sum_{j=0}^{d-1} \left\langle \tilde{j}\tilde{j}^* \middle| \rho \middle| \tilde{j}\tilde{j}^* \right\rangle$$

$$-\sum_{m,n=0}^{d-1} \lambda_m \lambda_n \langle mn|\rho|mn\rangle - \sum_{\substack{m\neq m', m\neq n \\ n\neq n', n'\neq m'}} \tilde{\gamma}_{mm'nn'} \sqrt{\langle m'n'|\rho|m'n'\rangle \langle mn|\rho|mn\rangle}$$

$$\tilde{\gamma}_{mm'nn'} = \begin{cases} 0 & \text{if } (m - m' - n - n') \bmod d \neq 0 \\ \sqrt{\lambda_m \lambda_{m'} \lambda_n \lambda_{n'}} & \text{otherwise.} \end{cases}$$

# Adaptive Witness: Recipe

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$$-\sum_{m,n=0}^{d-1} \lambda_m\lambda_n \boxed{\langle mn|\rho|mn\rangle} - \sum_{\substack{m\neq m', m\neq n\\ n\neq n', n'\neq m'}} \tilde{\gamma}_{mm'nn'} \boxed{\sqrt{\langle m'n'|\rho|m'n'\rangle\langle mn|\rho|mn\rangle}}$$

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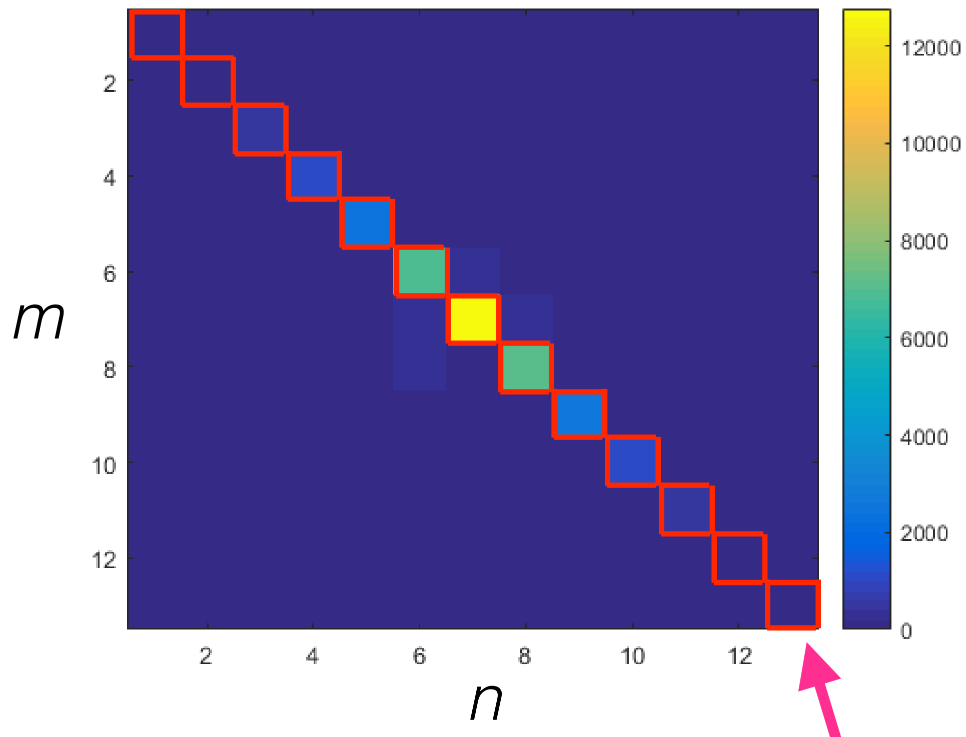


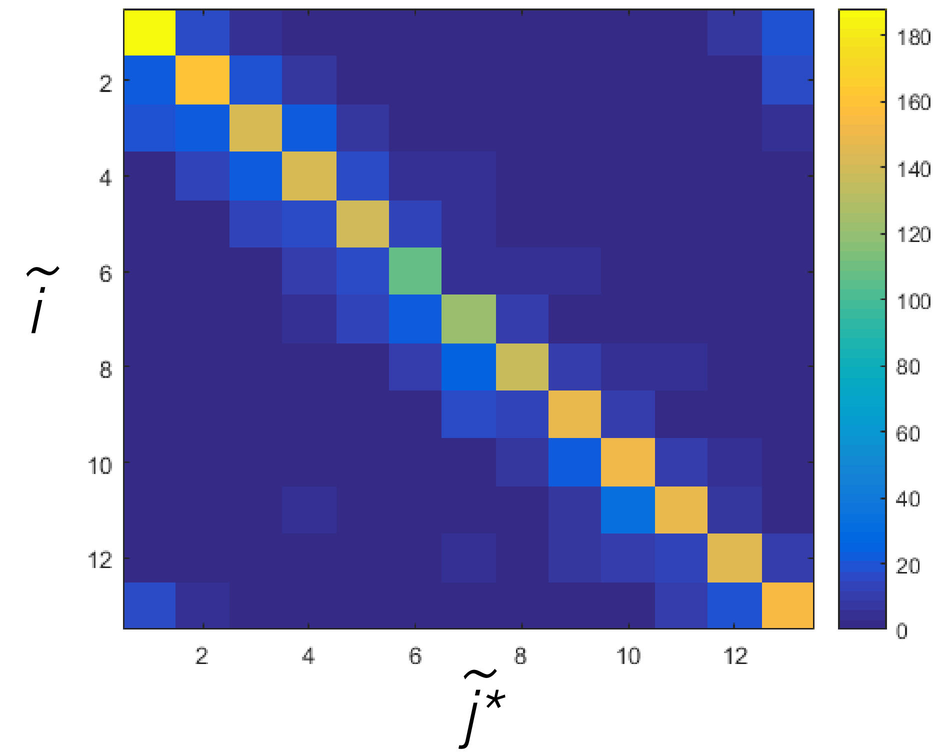


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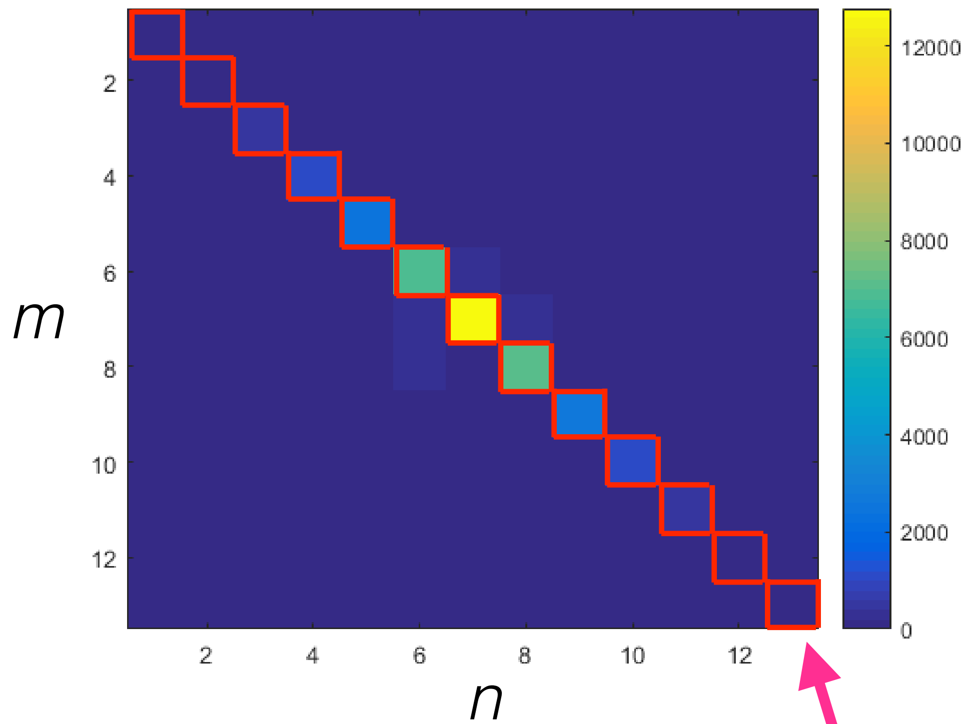
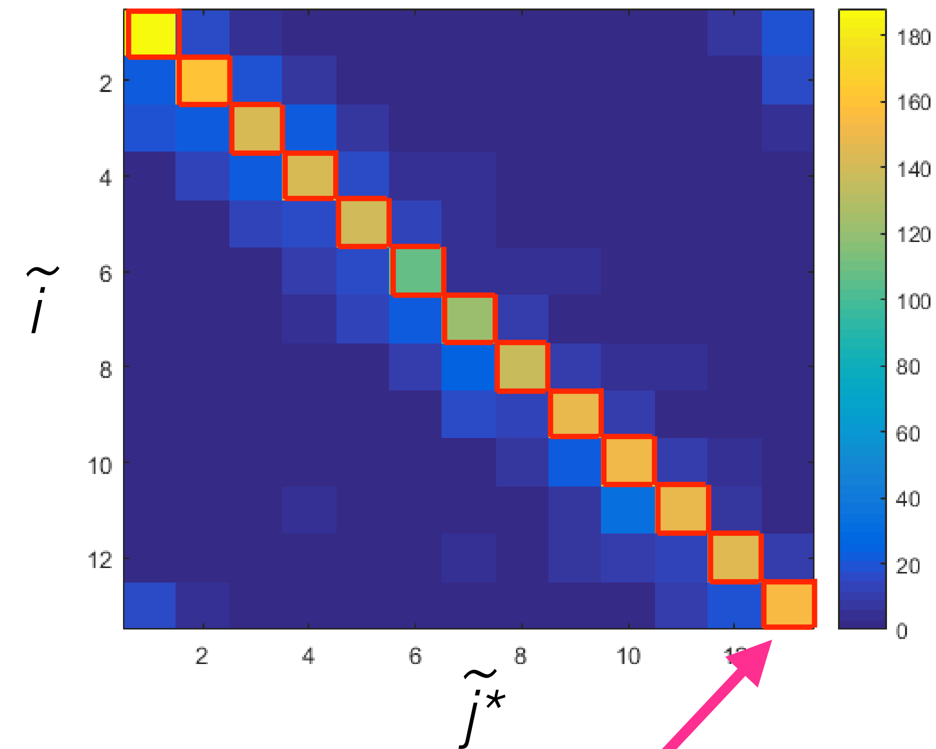
# Adaptive Witness: Recipe




Use measurements in two bases to calculate fidelity bound:

**5.**

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$$- \sum_{m,n=0}^{d-1} \lambda_m \lambda_n \boxed{\langle mn|\rho|mn\rangle} - \sum_{\substack{m\neq m', m\neq n\\ n\neq n', n'\neq m'}} \tilde{\gamma}_{mm'nn'} \boxed{\sqrt{\langle m'n'|\rho|m'n'\rangle \langle mn|\rho|mn\rangle}}$$

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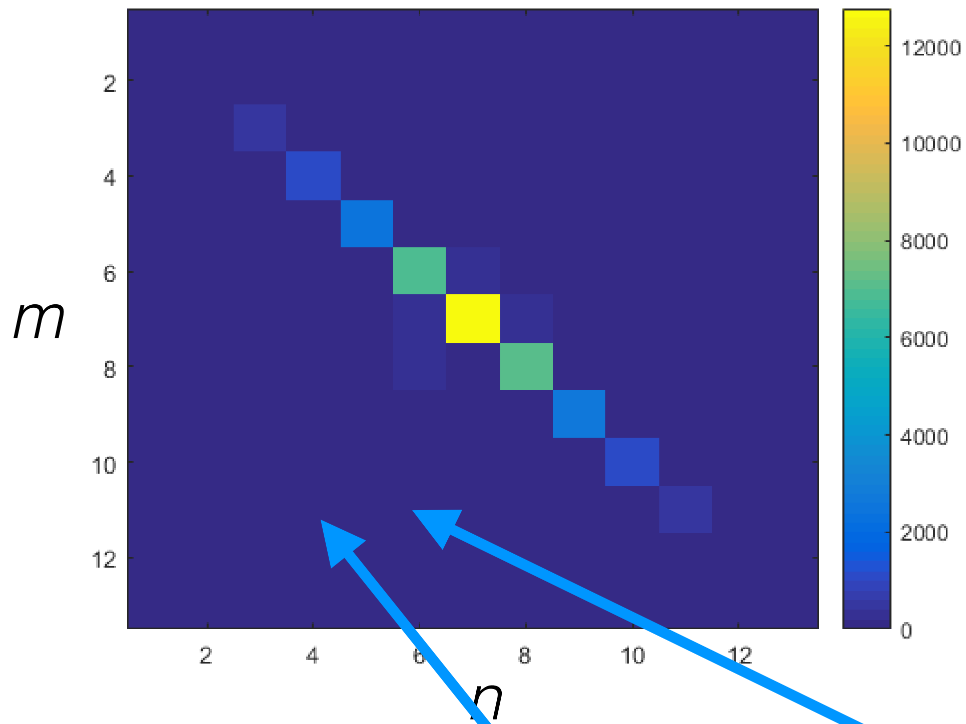


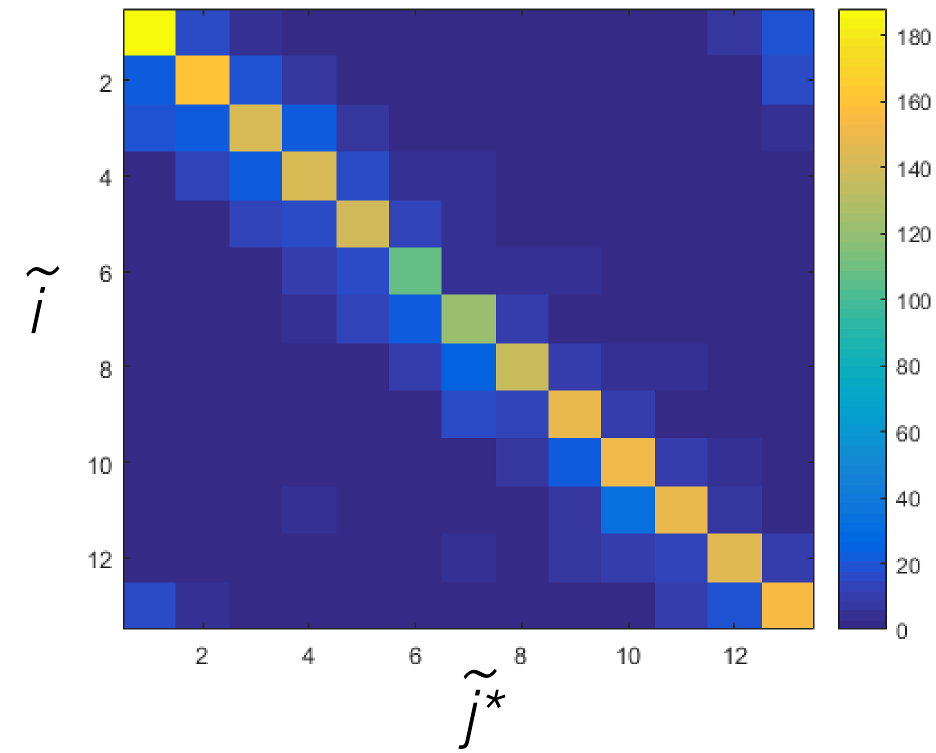


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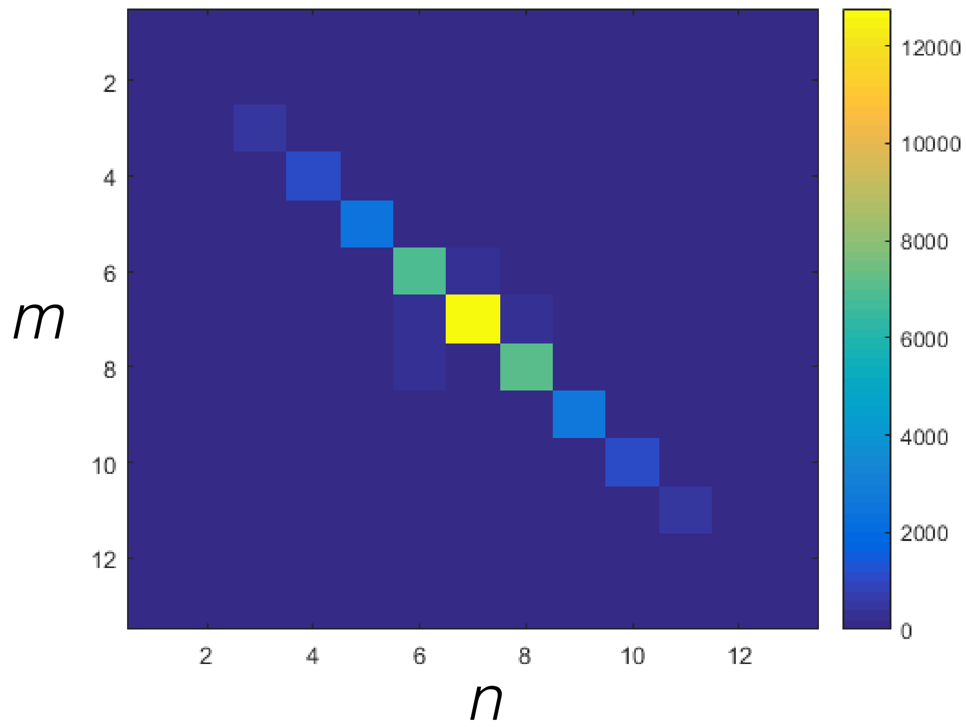
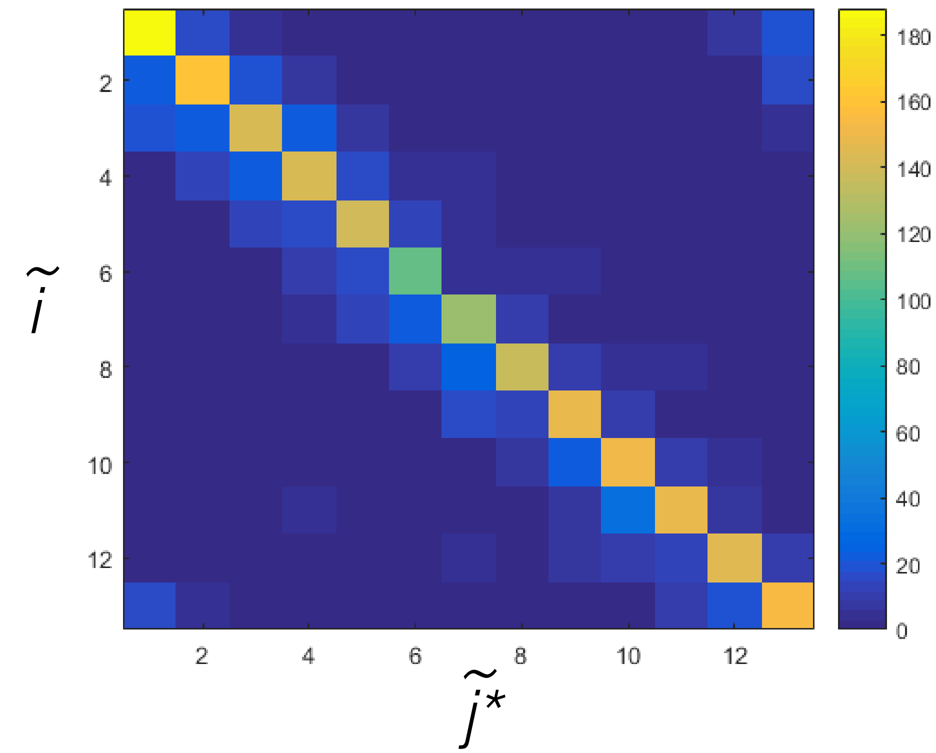


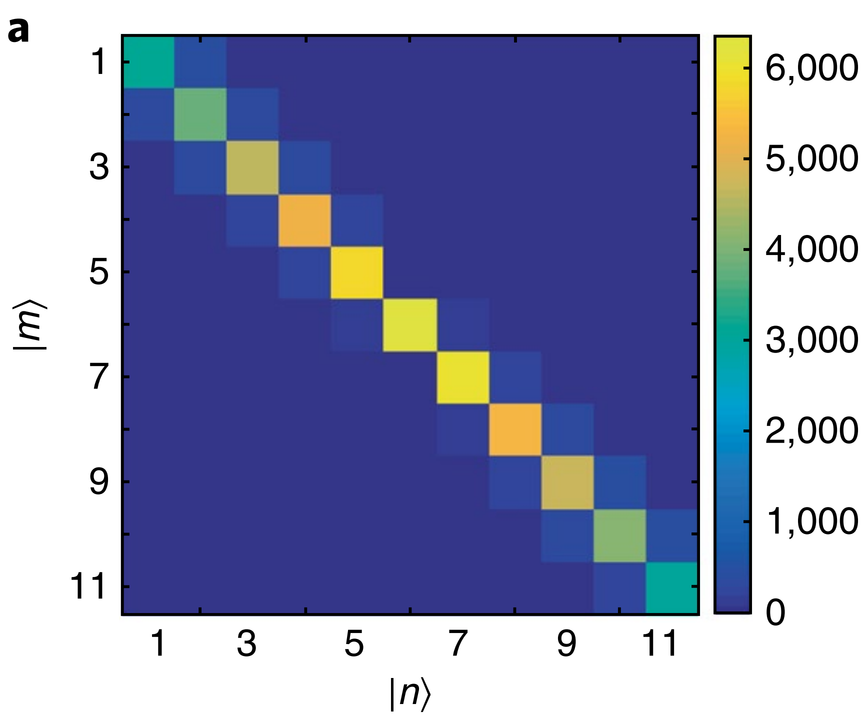



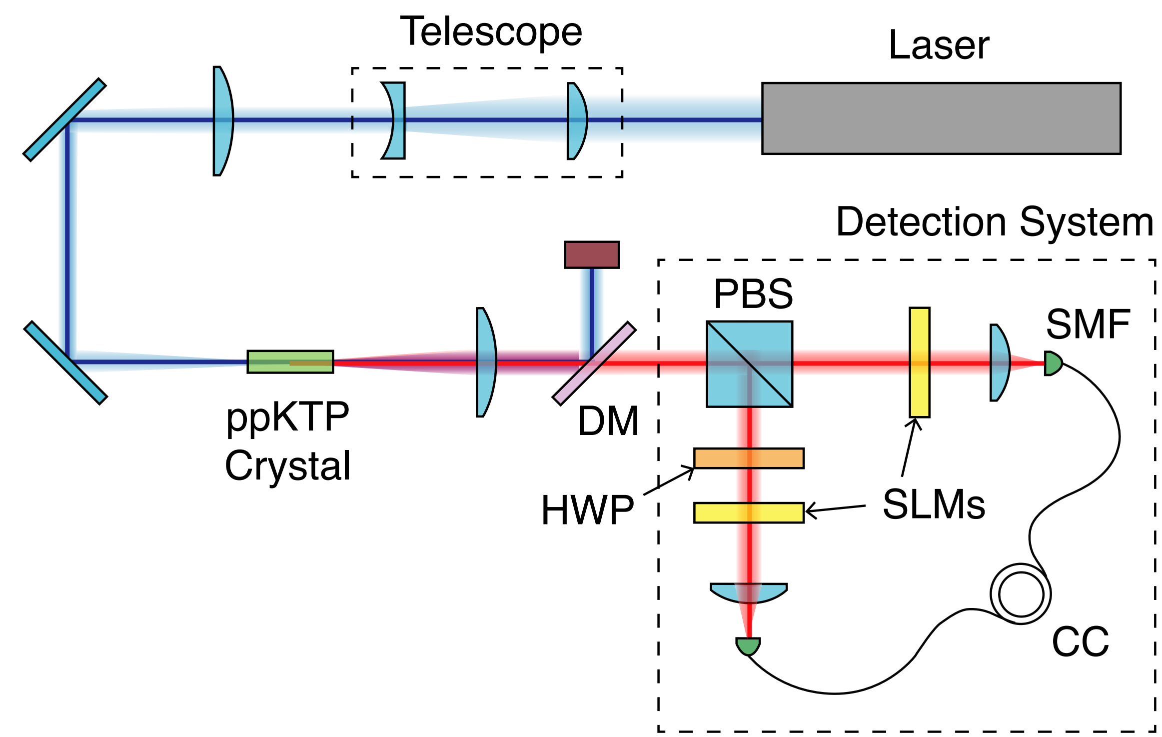

Calculate entanglement dimensionality from target state bound:

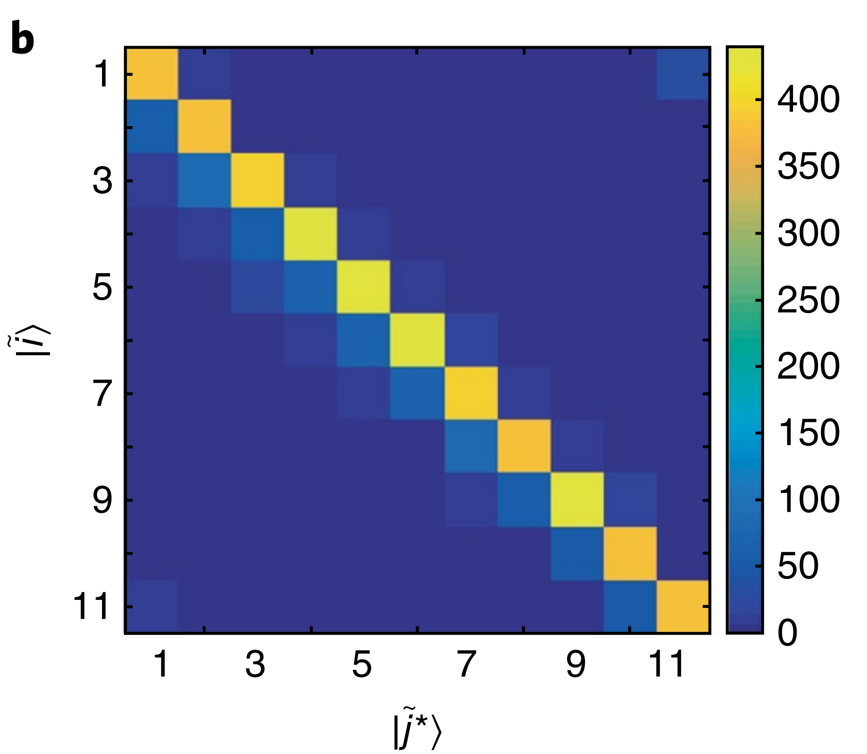
6.

$$F(\rho,\Phi) \leq B_k(\Phi) := \sum_{m=0}^{k-1} \lambda_m^2$$

# Experimental Results



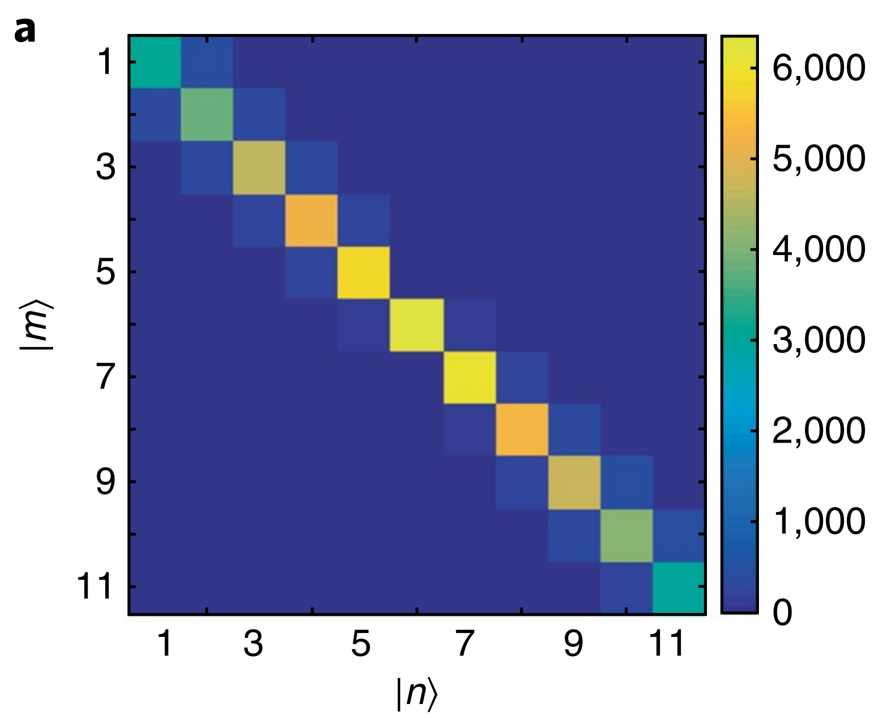

**Experimental results**

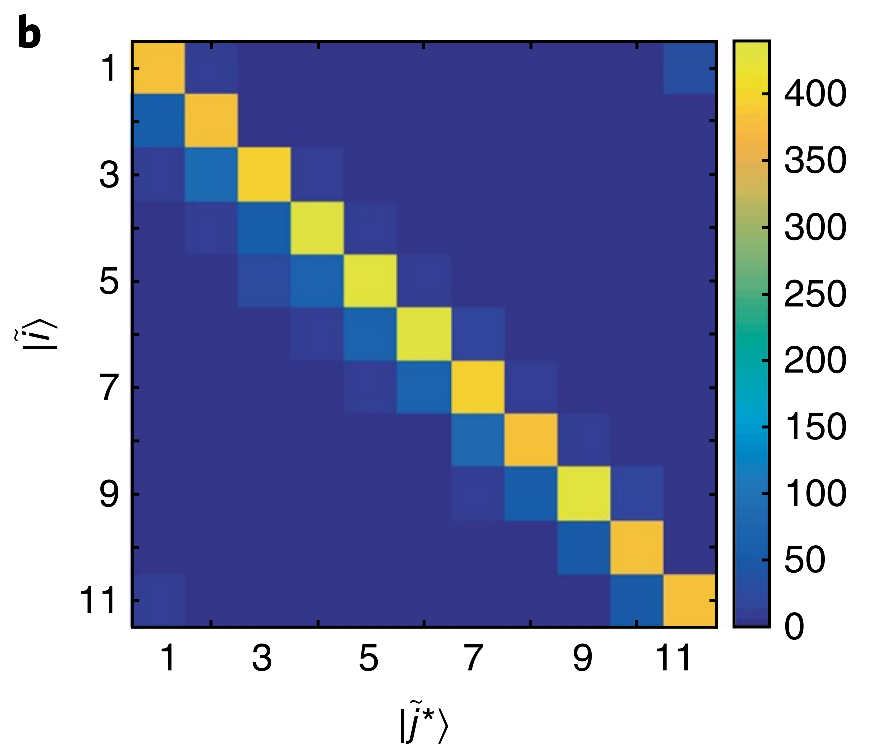
| $d$ | $d_{ent}$ | $\tilde{F}(\rho, \Phi^{+})$ | $\tilde{F}(\rho, \Phi)$ |
|---|---|---|---|
| 3 | 3 | $91.5 \pm 0.4\%$ | $92.5 \pm 0.4\%$ |
| 5 | 5 | $89.9 \pm 0.4\%$ | $90.0 \pm 0.5\%$ |
| 7 | 6 | $84.2 \pm 0.5\%$ | $86.9 \pm 0.6\%$ |
| 11 | 9 | $74.8 \pm 0.4\%$ | $76.2 \pm 0.6\%$ |

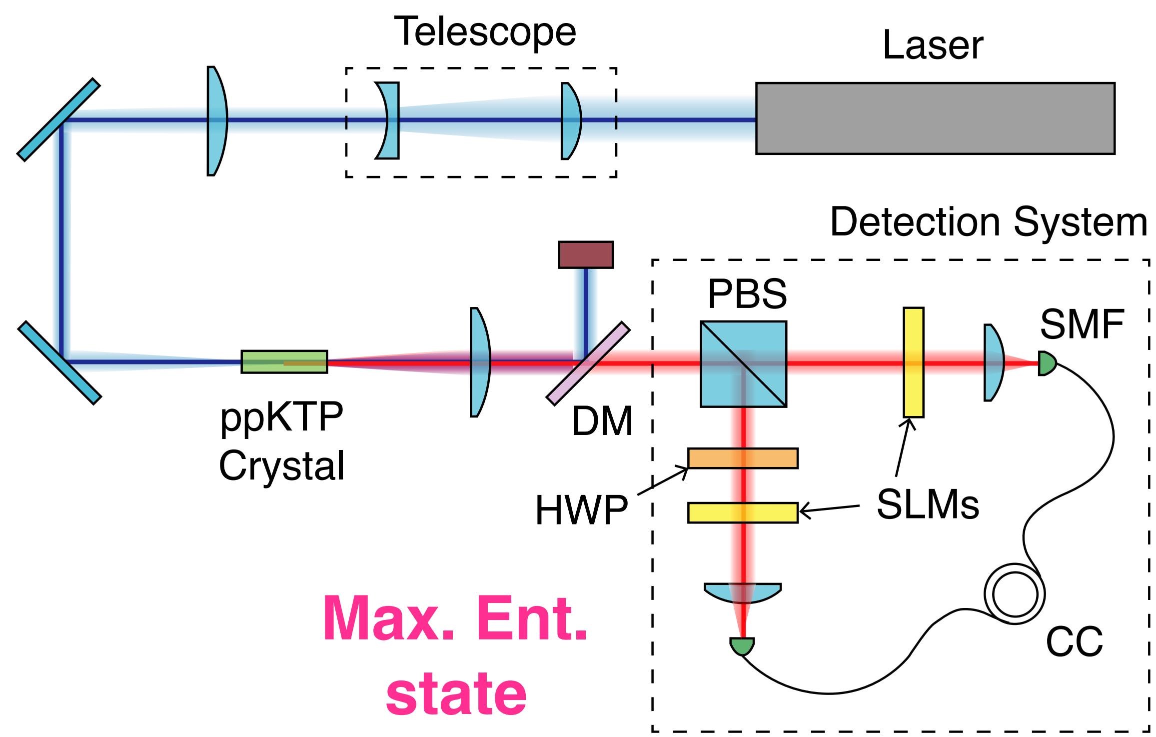
The second column lists the entanglement dimensionality $d_{ent}$ certified using $\tilde{F}_2(\rho, \Phi^{+})$

Bavaresco, Herrera, Klöckl, Pivoluska, Erker, Friis, **MM**, Huber, **Nat. Phys. 14**, 1032 (2018)

# EXPERIMENTAL RESULTS

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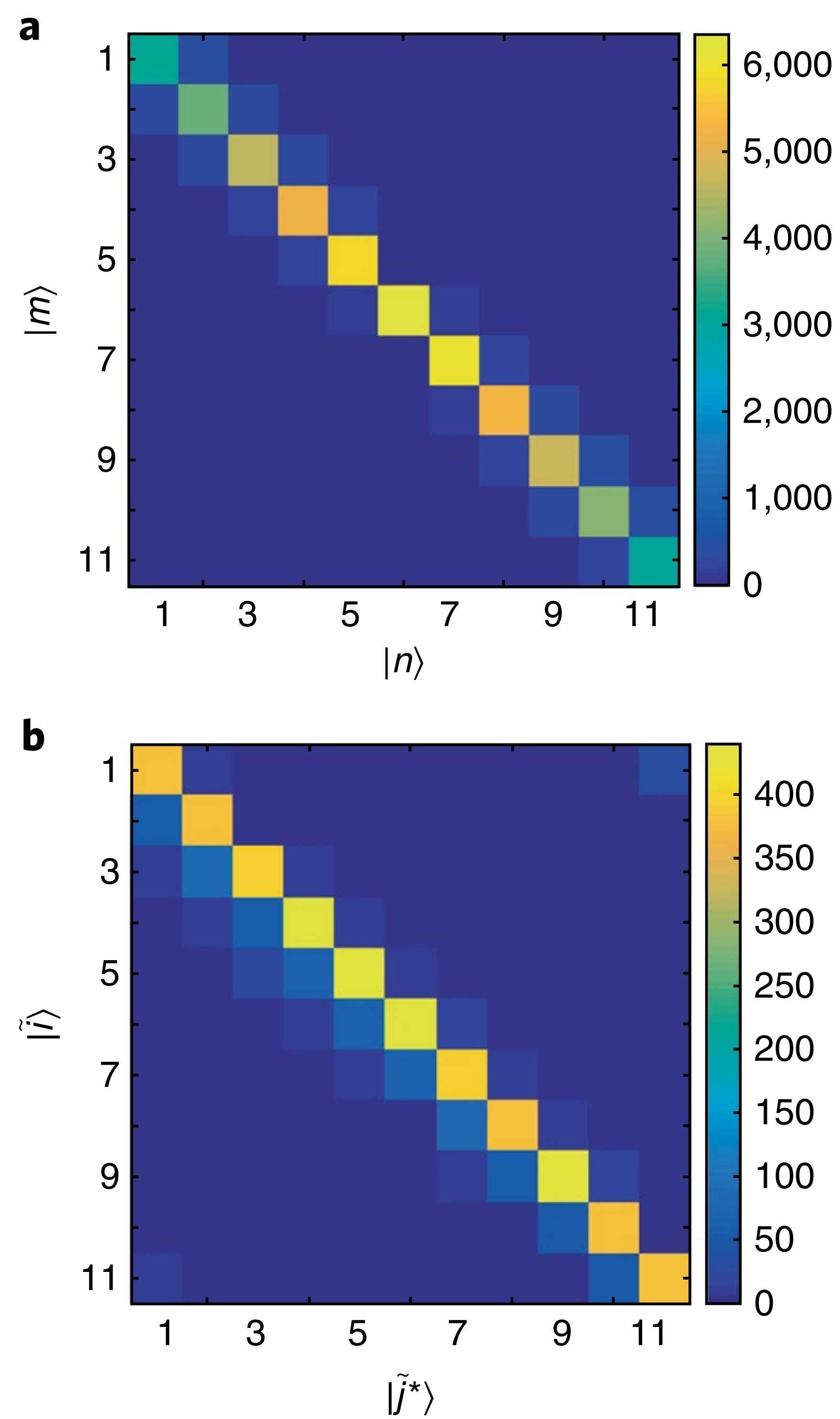


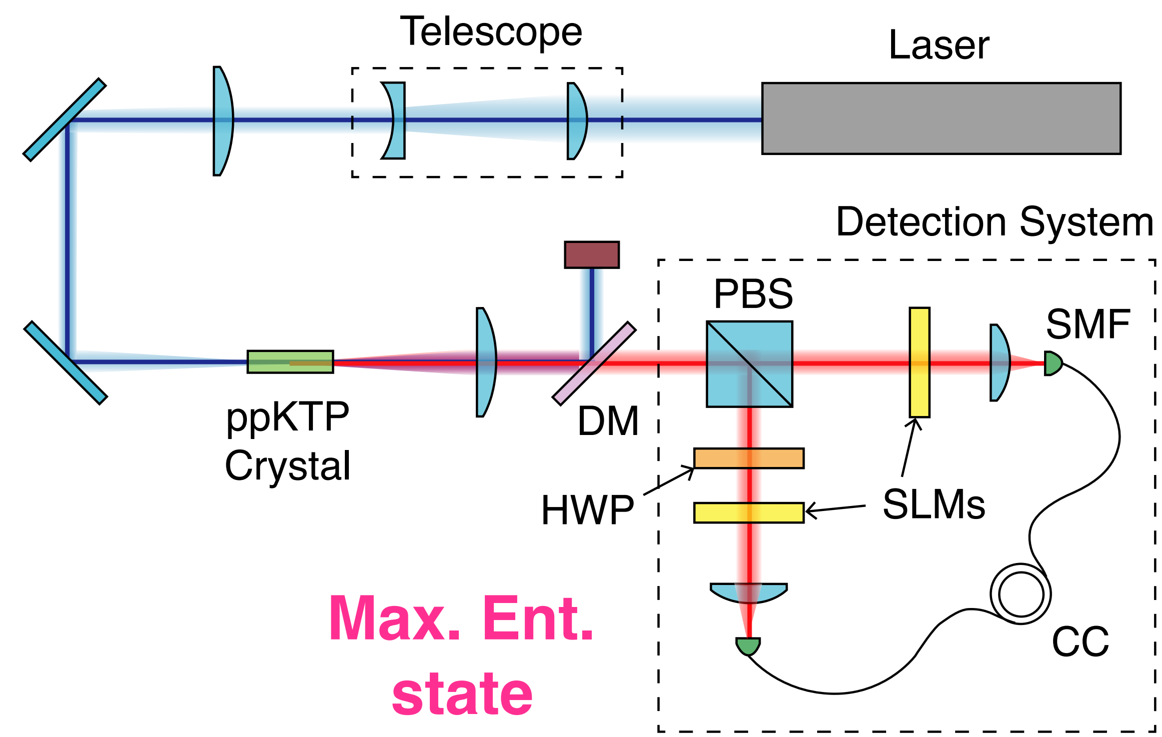


Max. Ent. state

Tilted state

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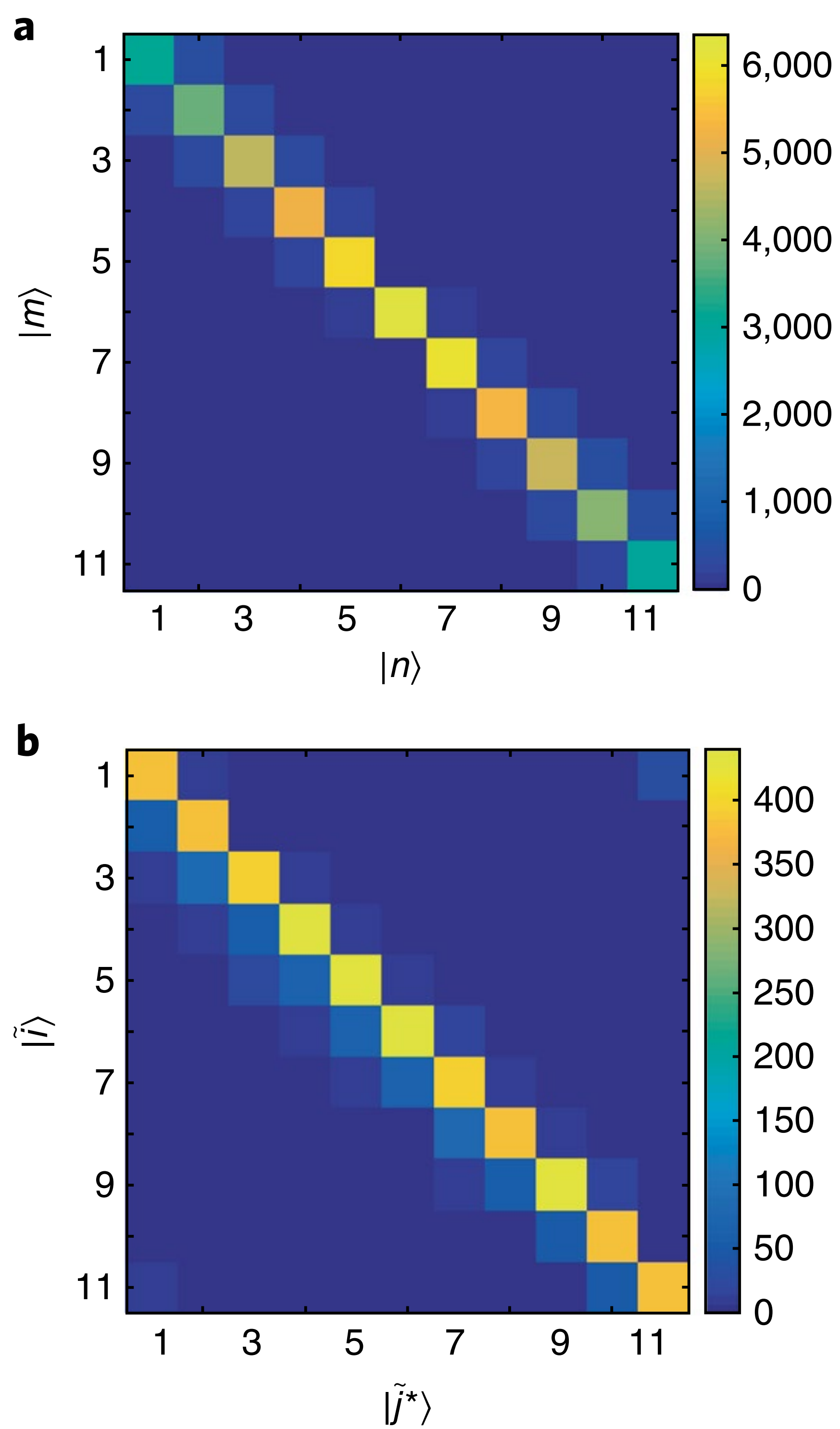


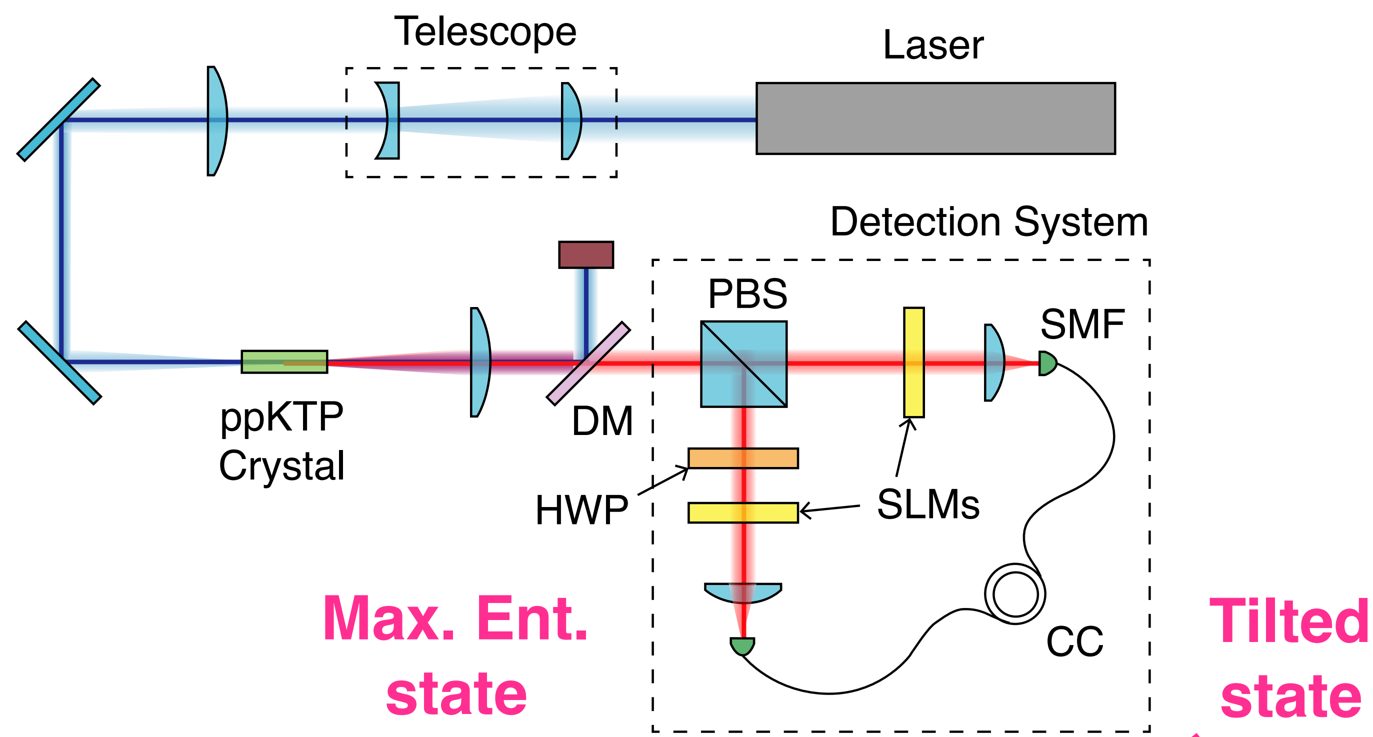


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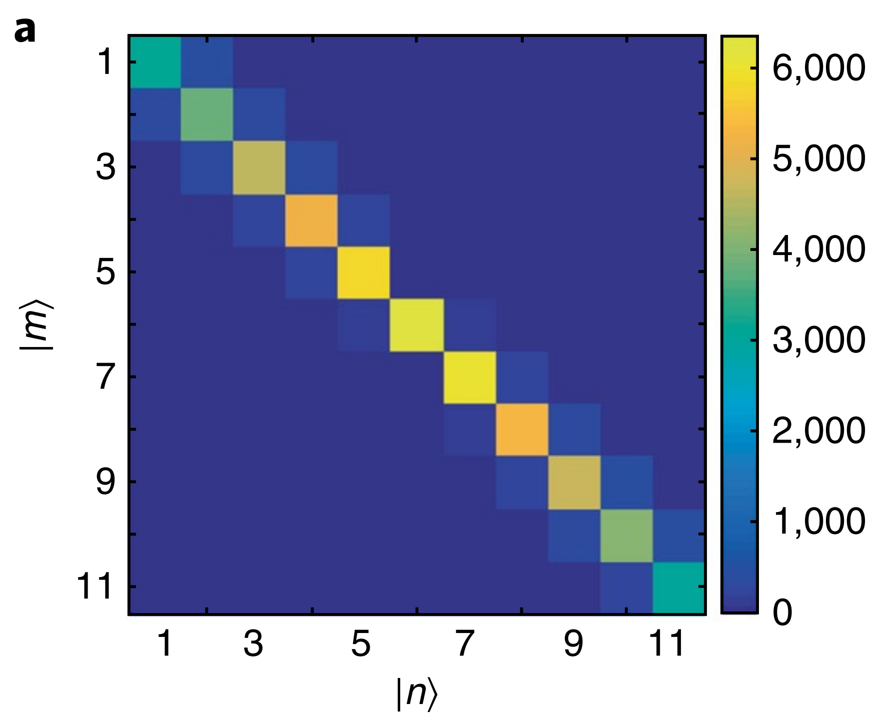


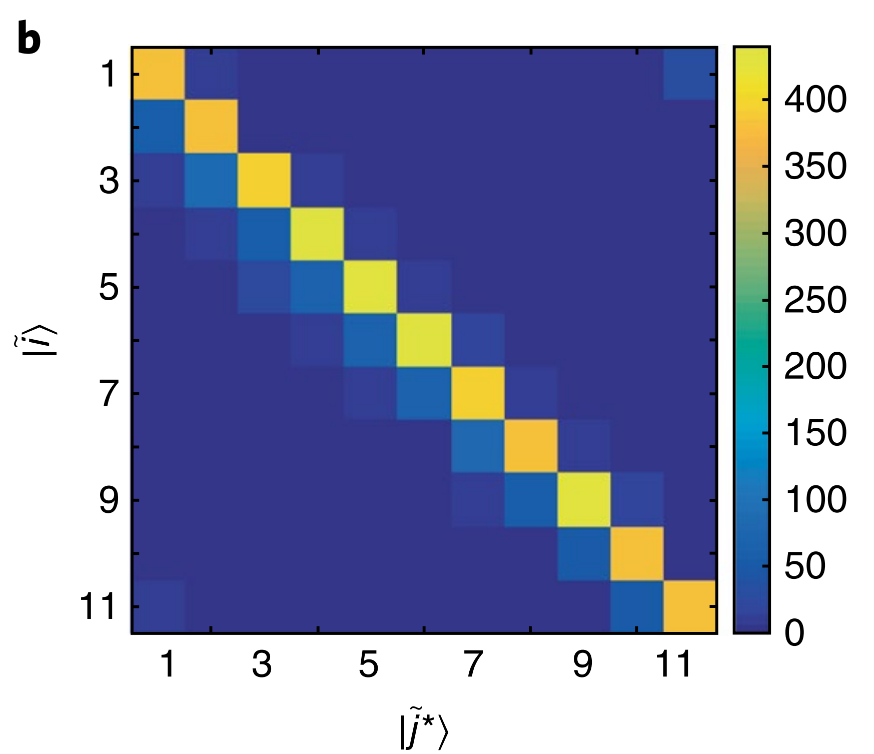


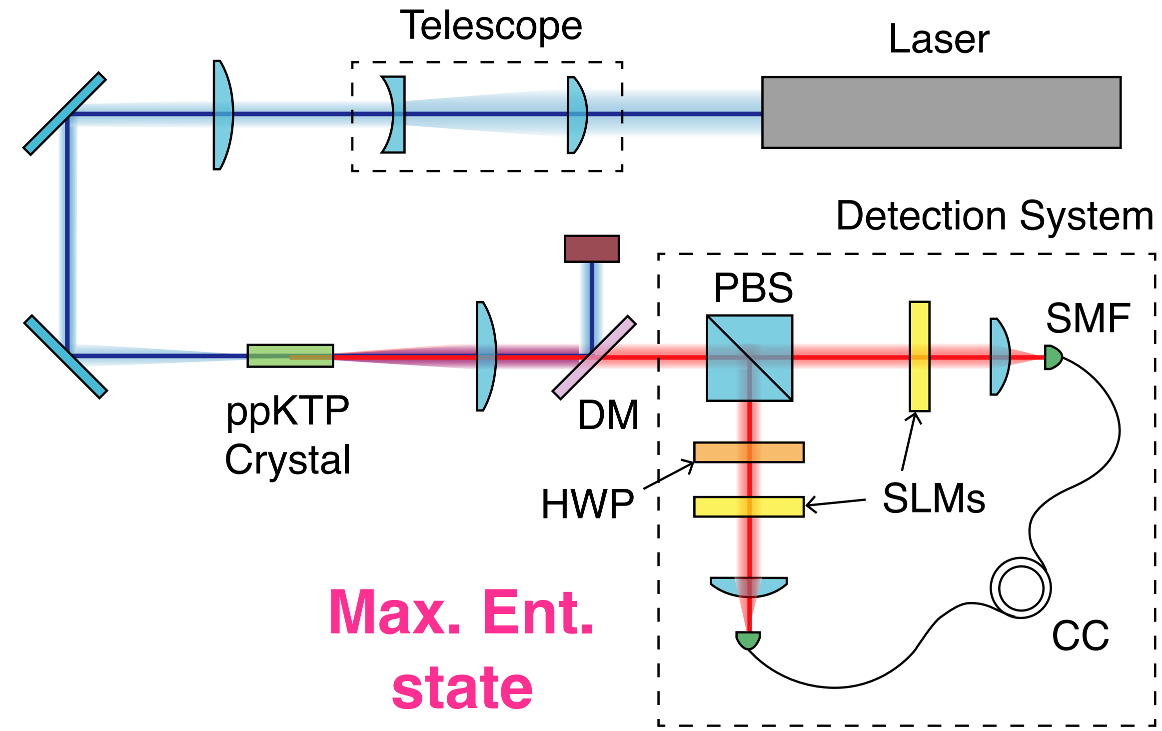


**Max. Ent. state**

**Tilted state**

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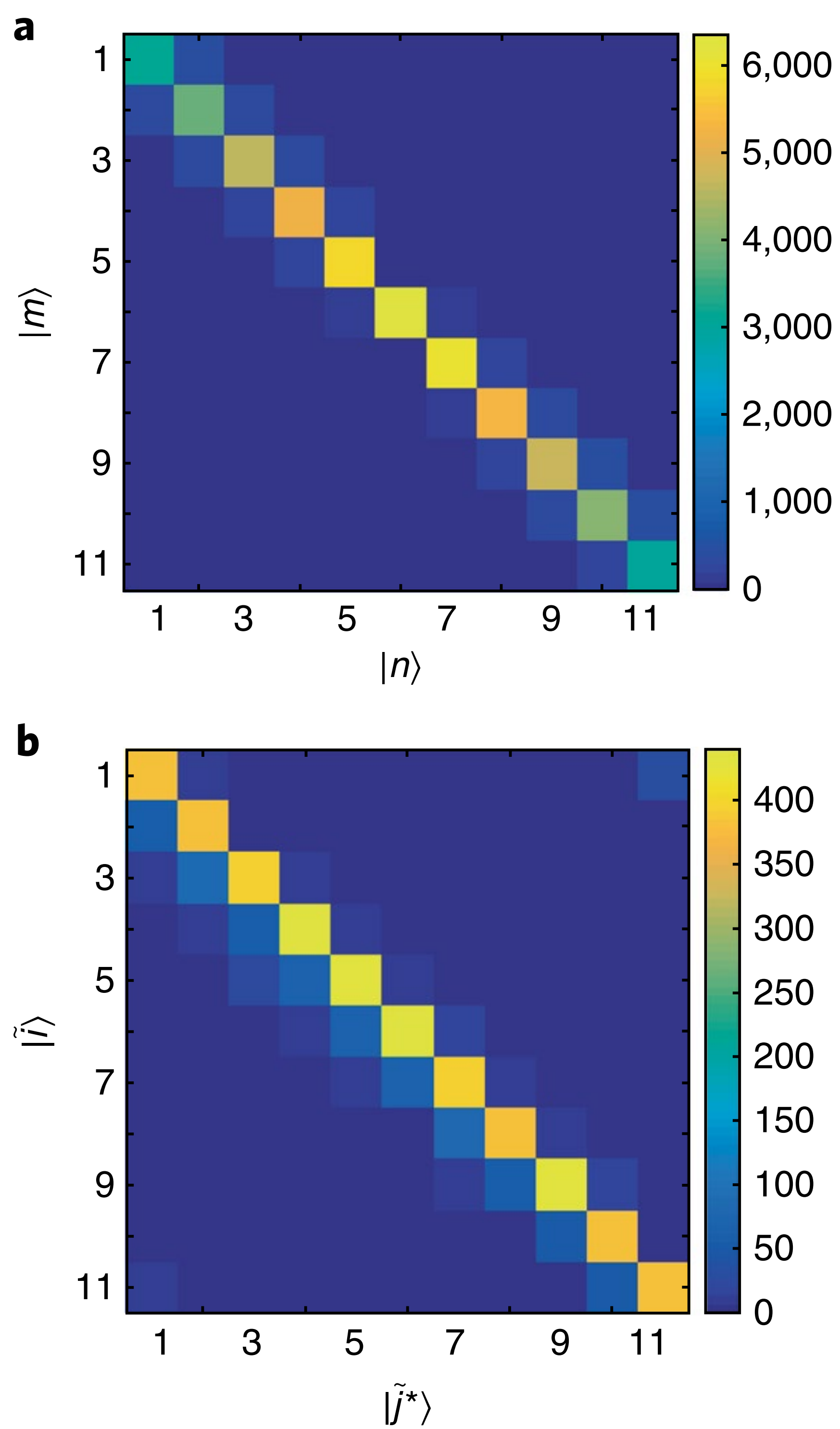


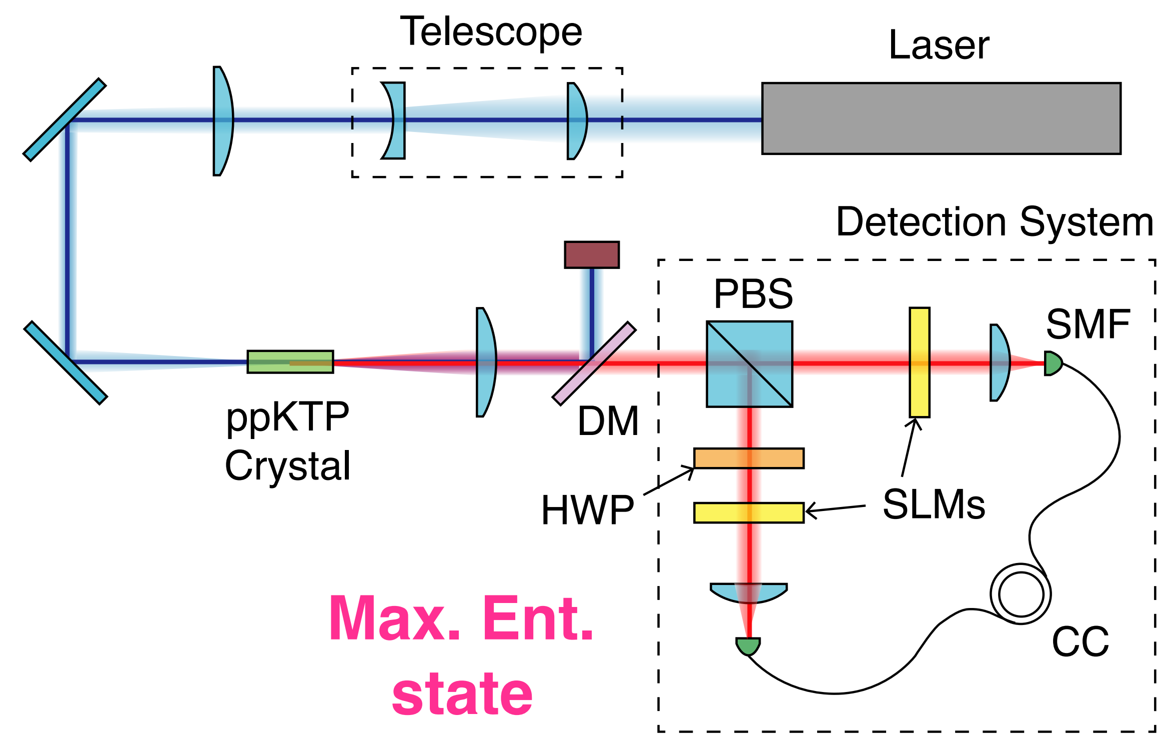


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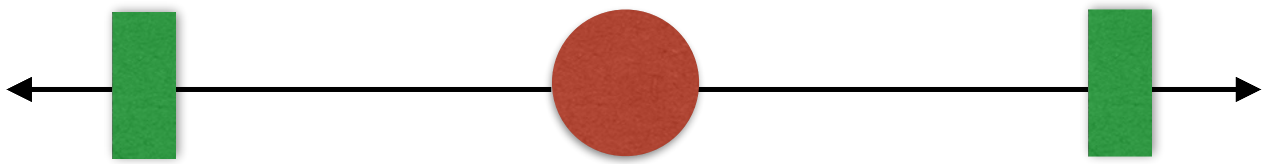
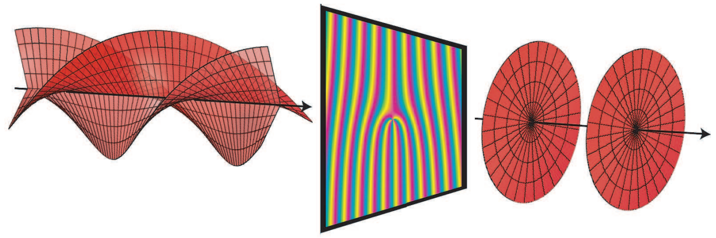
Highest entanglement dimension certified without any assumptions on the state!


Bavaresco, Herrera, Klöckl, Pivoluska, Erker, Friis, **MM**, Huber, **Nat. Phys. 14**, 1032 (2018)

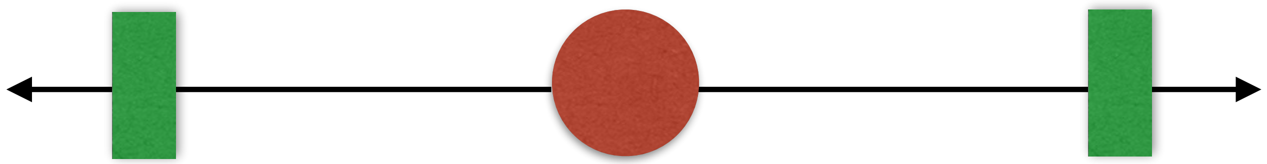
# ADAPTIVE WITNESS: COMPARISON

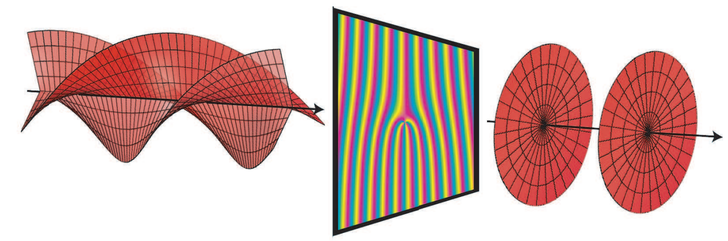




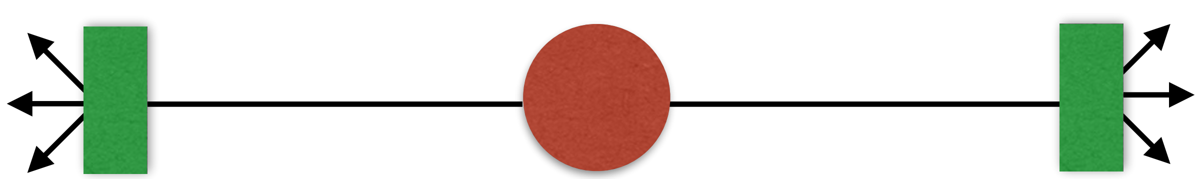
| | 1–outcome | *d* = 11 |
|---|---|---|
| **TOMOGRAPHY** | $d^2(d+1)^2$ | 17,424 |
| **2D WITNESS** | $d^2(d+1)$ | 1452 |
| **ADAPTIVE WITNESS** | $2d^2$ | 242 |

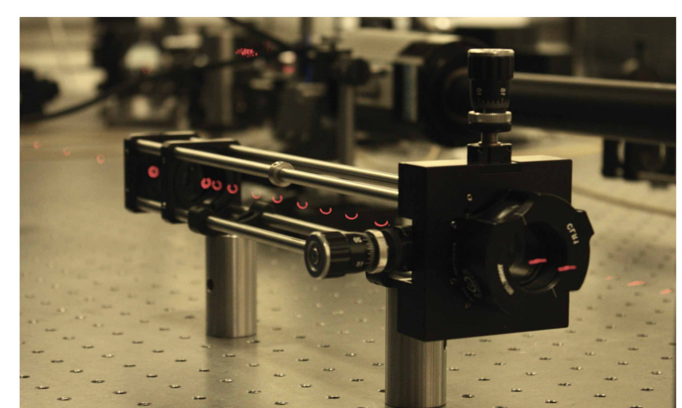
# ADAPTIVE WITNESS: COMPARISON



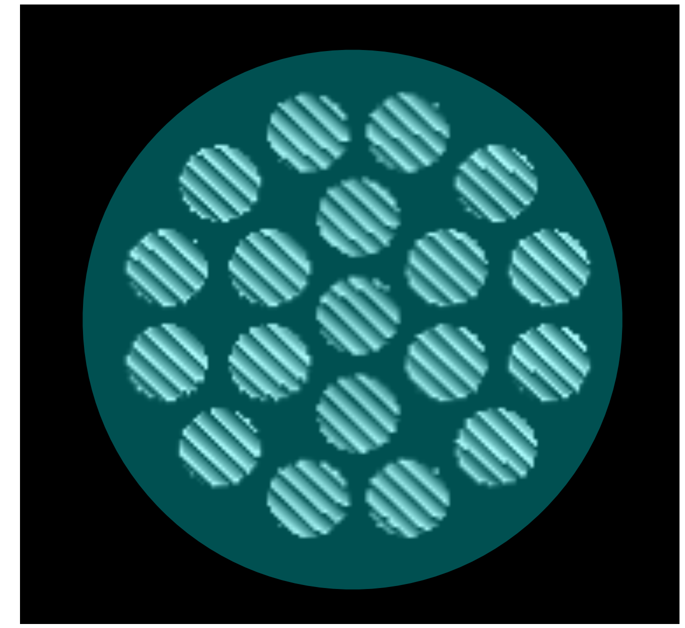


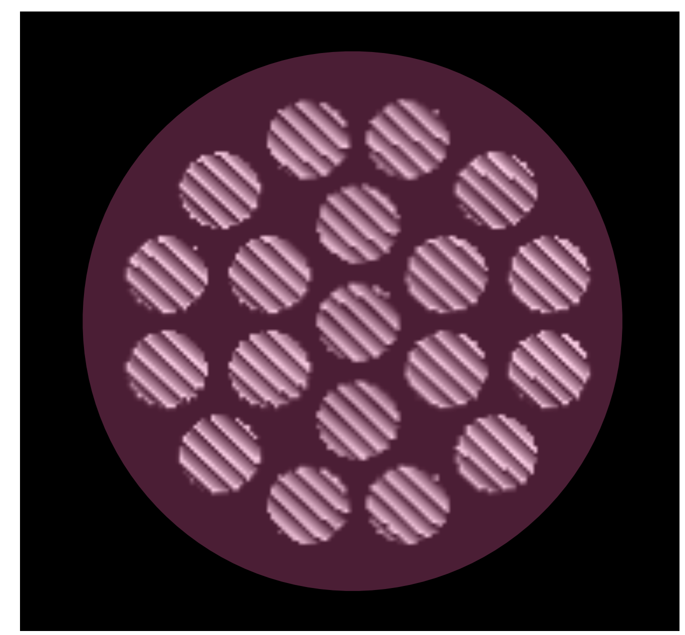
| | 1–outcome | $d = 11$ | $d$–outcome | $d = 11$ |
|---|---|---|---|---|
| TOMOGRAPHY | $d^2(d+1)^2$ | 17,424 | $(d+1)^2$ | 144 |
| 2D WITNESS | $d^2(d+1)$ | 1452 | $d+1$ | 12 |
| ADAPTIVE WITNESS | $2d^2$ | 242 | 2 | 2 |





# HIGH-DIMENSIONAL PIXEL ENTANGLEMENT





Herrera et al, **In Preparation** (2019)

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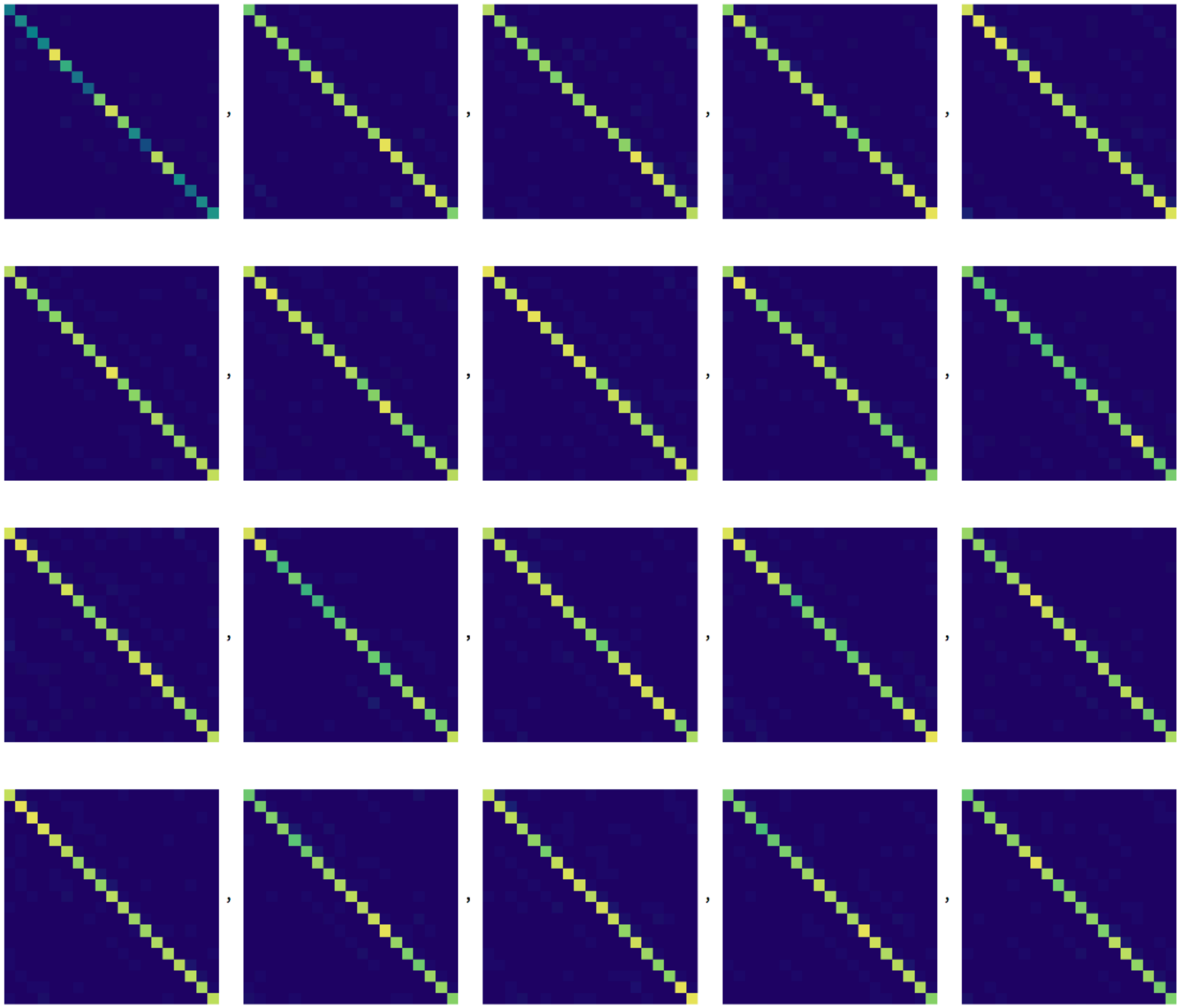
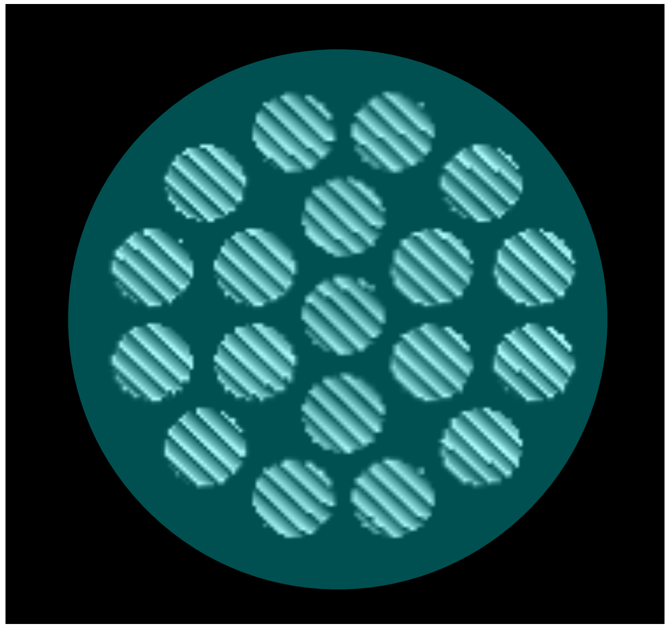
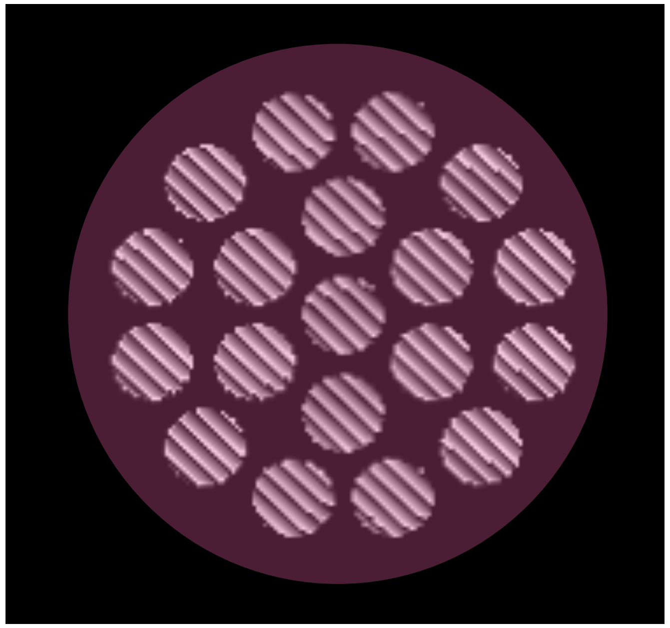
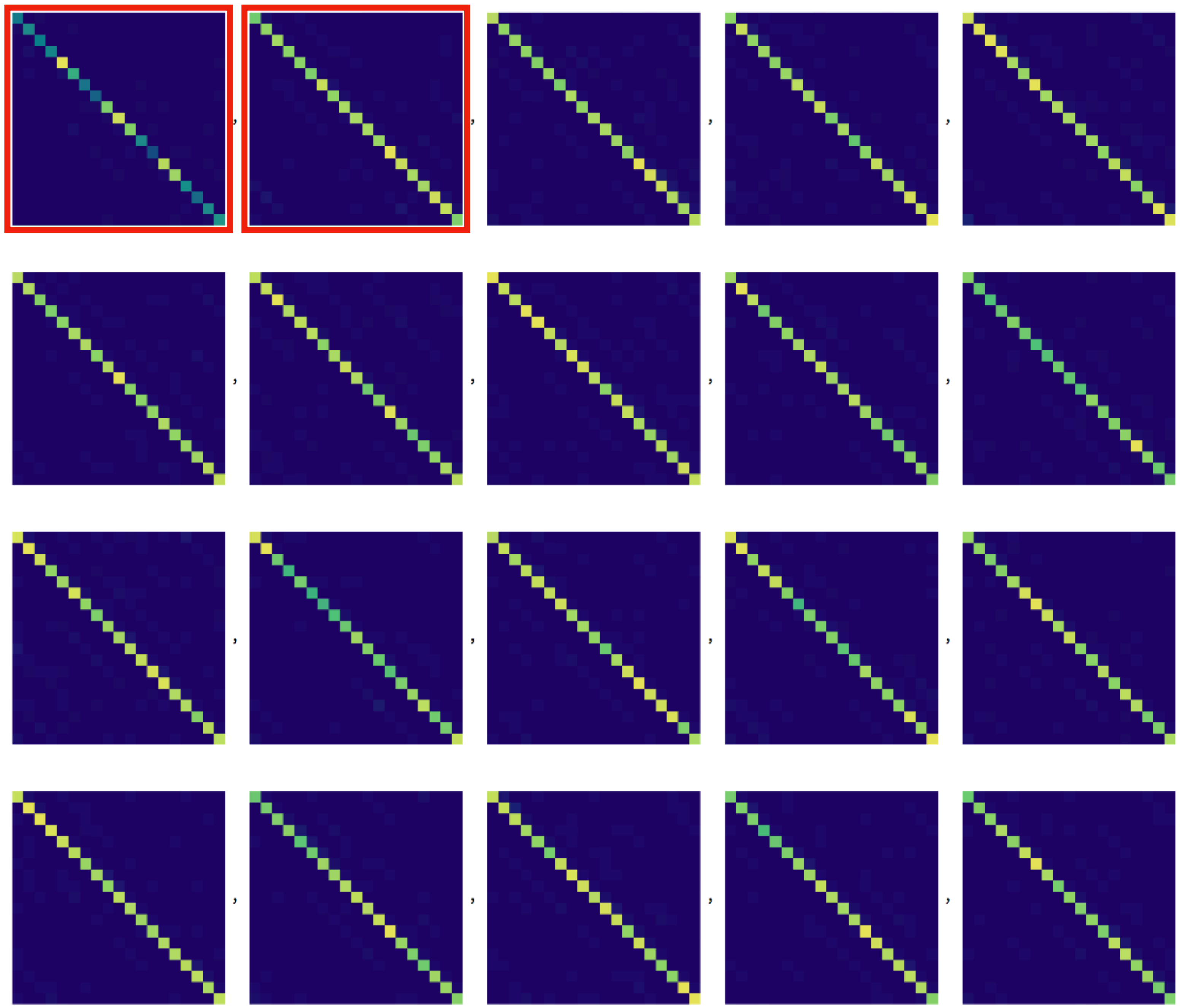


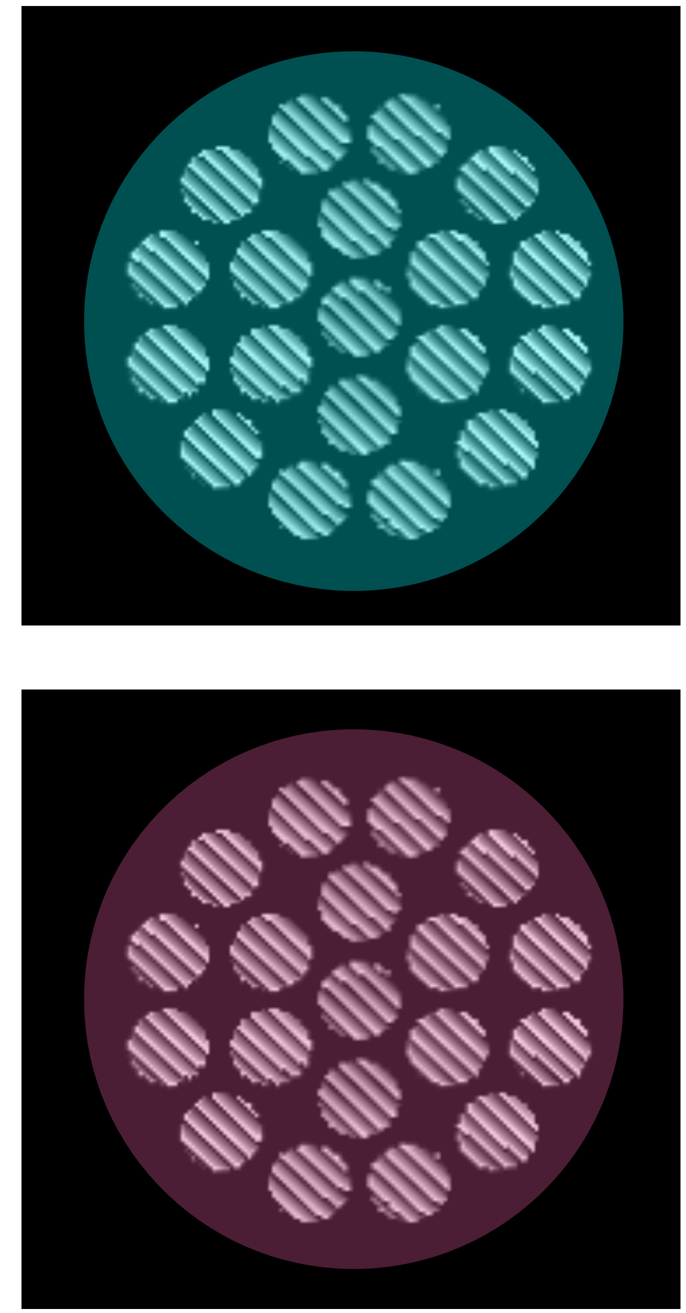




Herrera et al, **In Preparation** (2019)

# High-Dimensional Pixel Entanglement

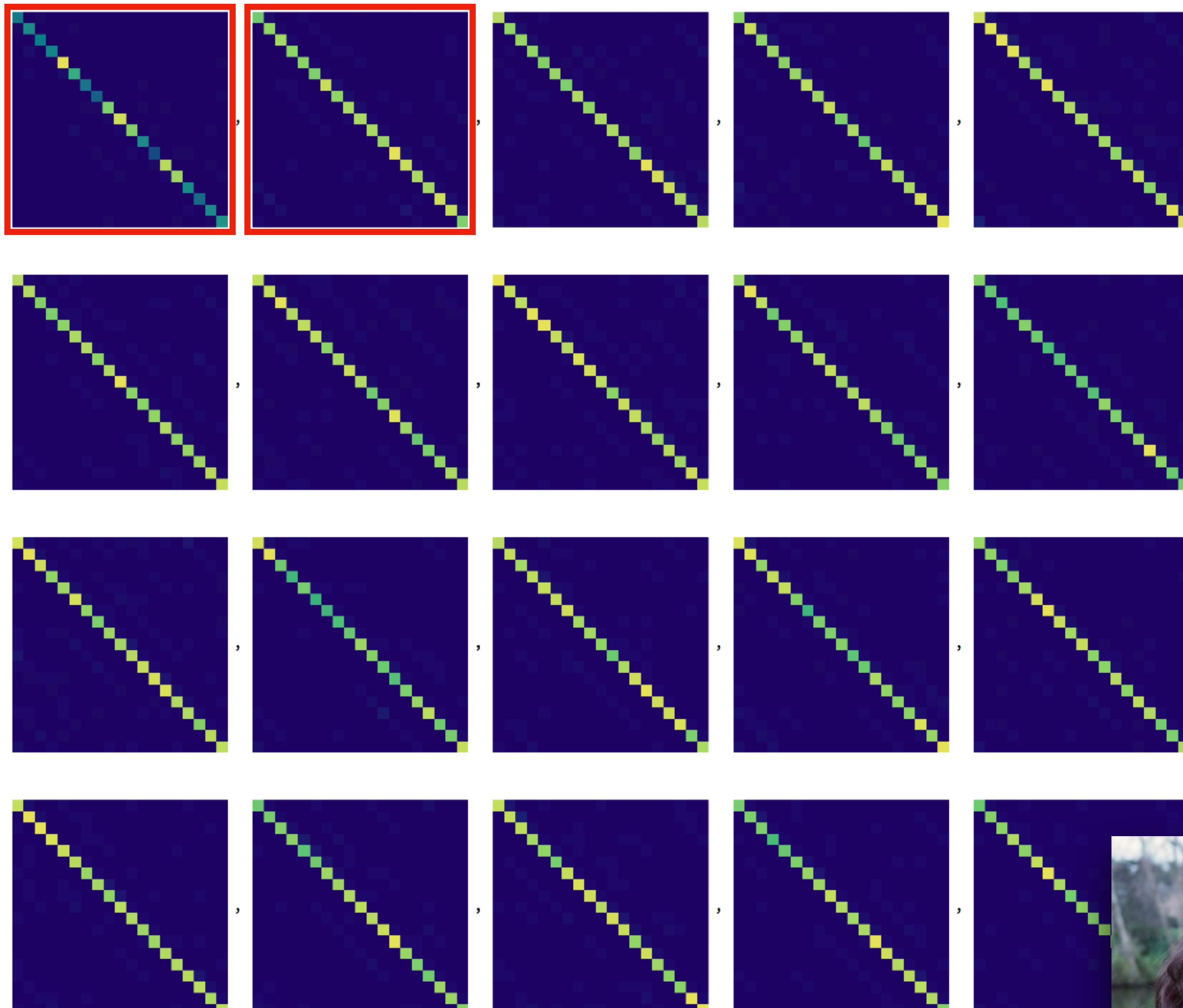


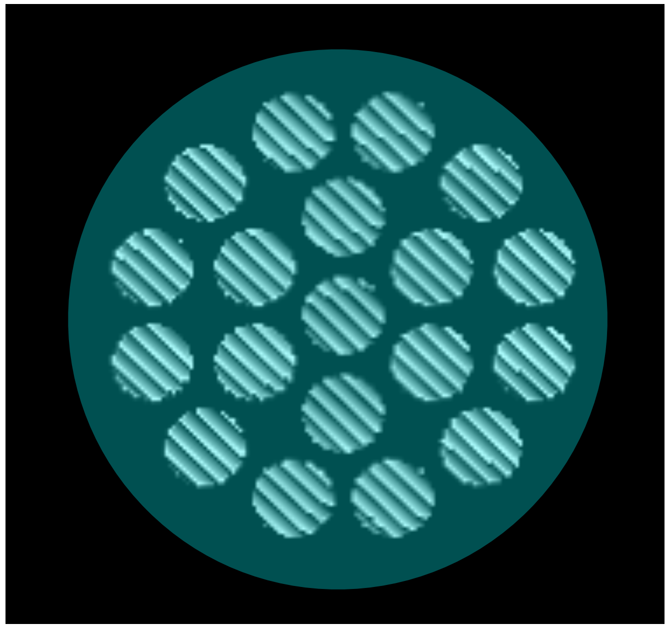


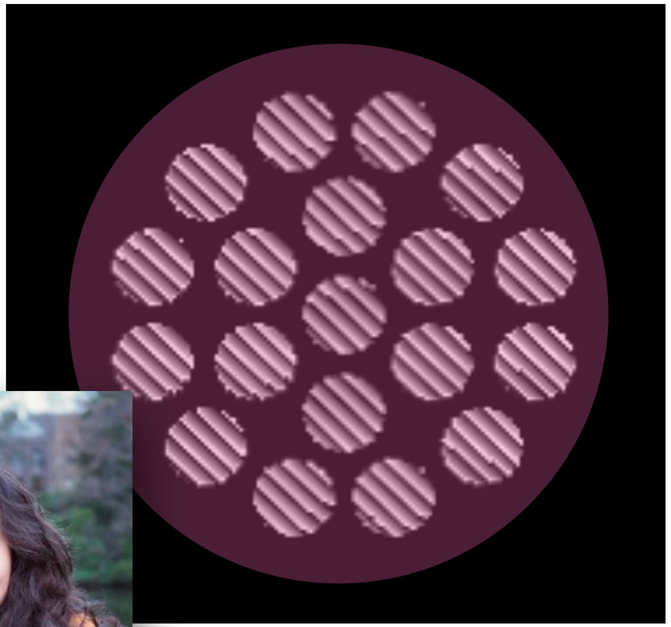
**Fidelity > 94.1%**, <u>18-dim</u> entanglement in a 19-dim space

Herrera et al, **In Preparation** (2019)

# HIGH-DIMENSIONAL PIXEL ENTANGLEMENT







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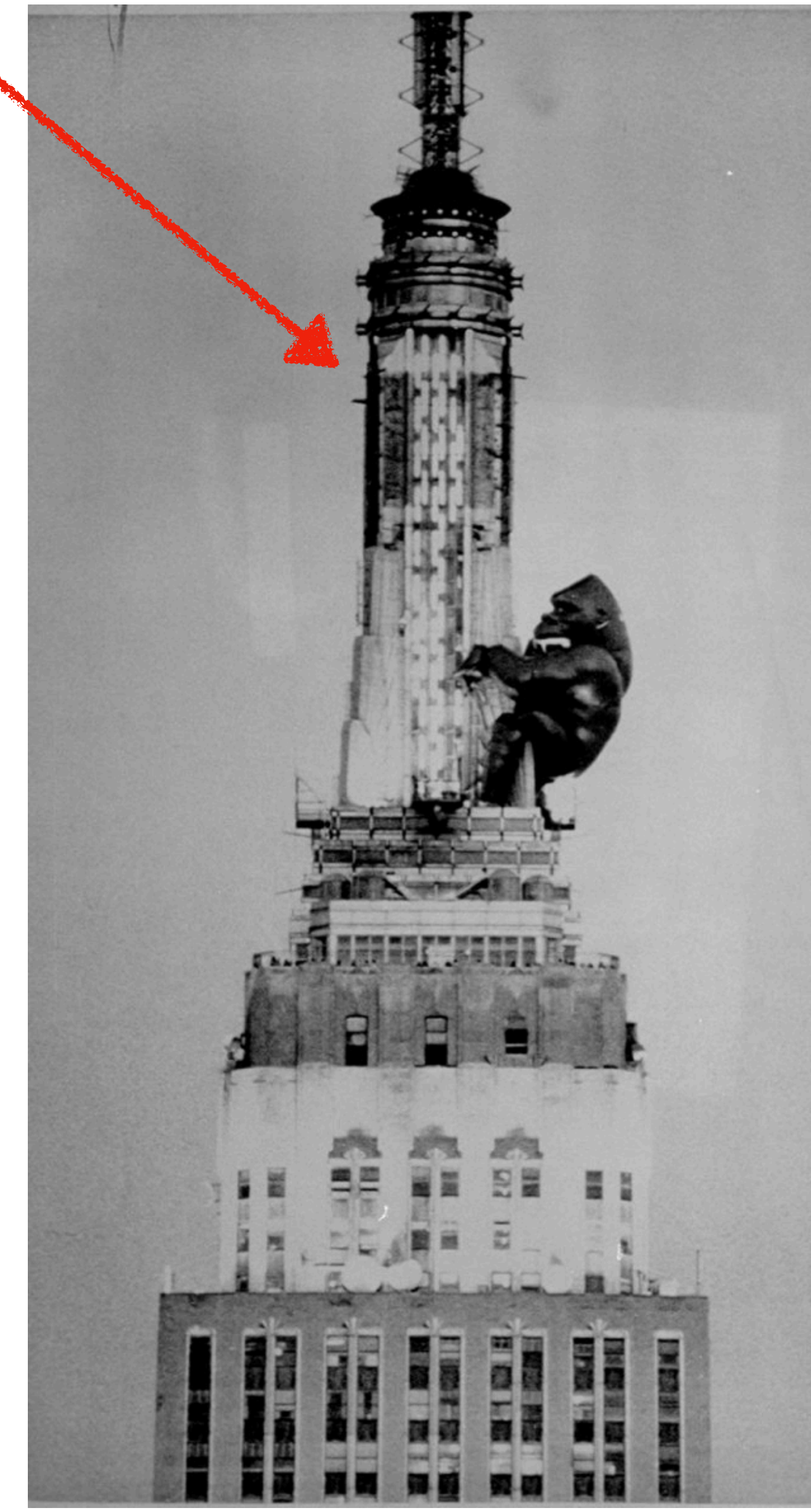
Herrera et al, **In Preparation** (2019)

# TWO APPLICATIONS OF HIGH-DIM ENTANGLEMENT

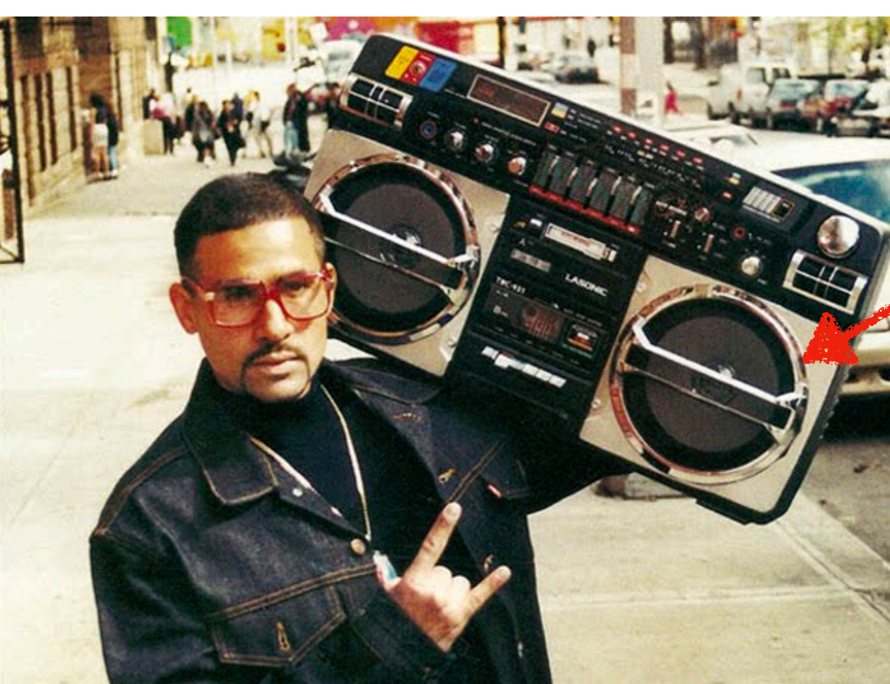
# NOISE-RESISTANT ENTANGLEMENT DISTRIBUTION

# Realistic Quantum Networks

Imperfect Source

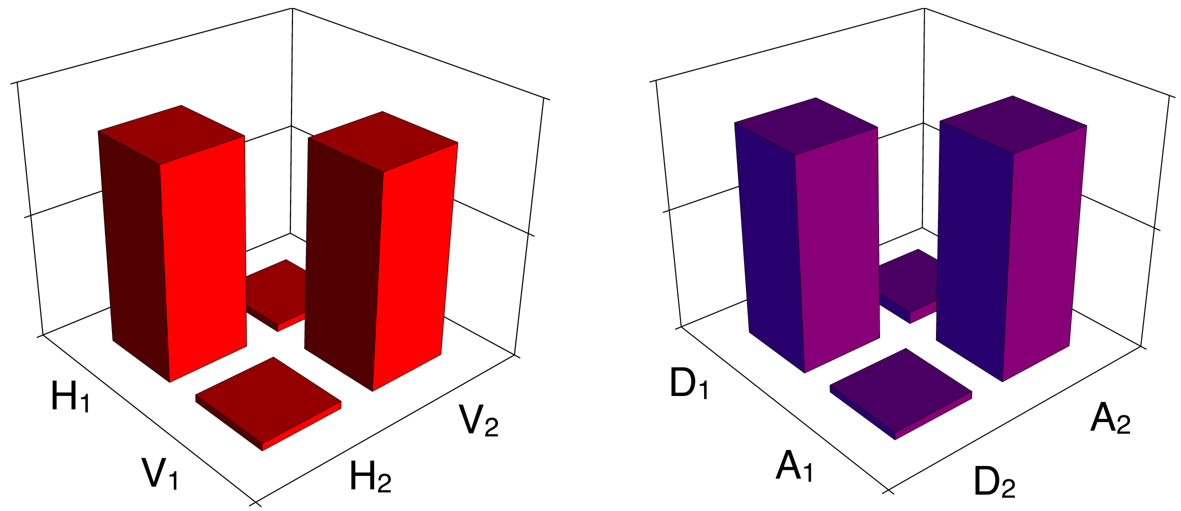


Noisy Channel



Imperfect Detector

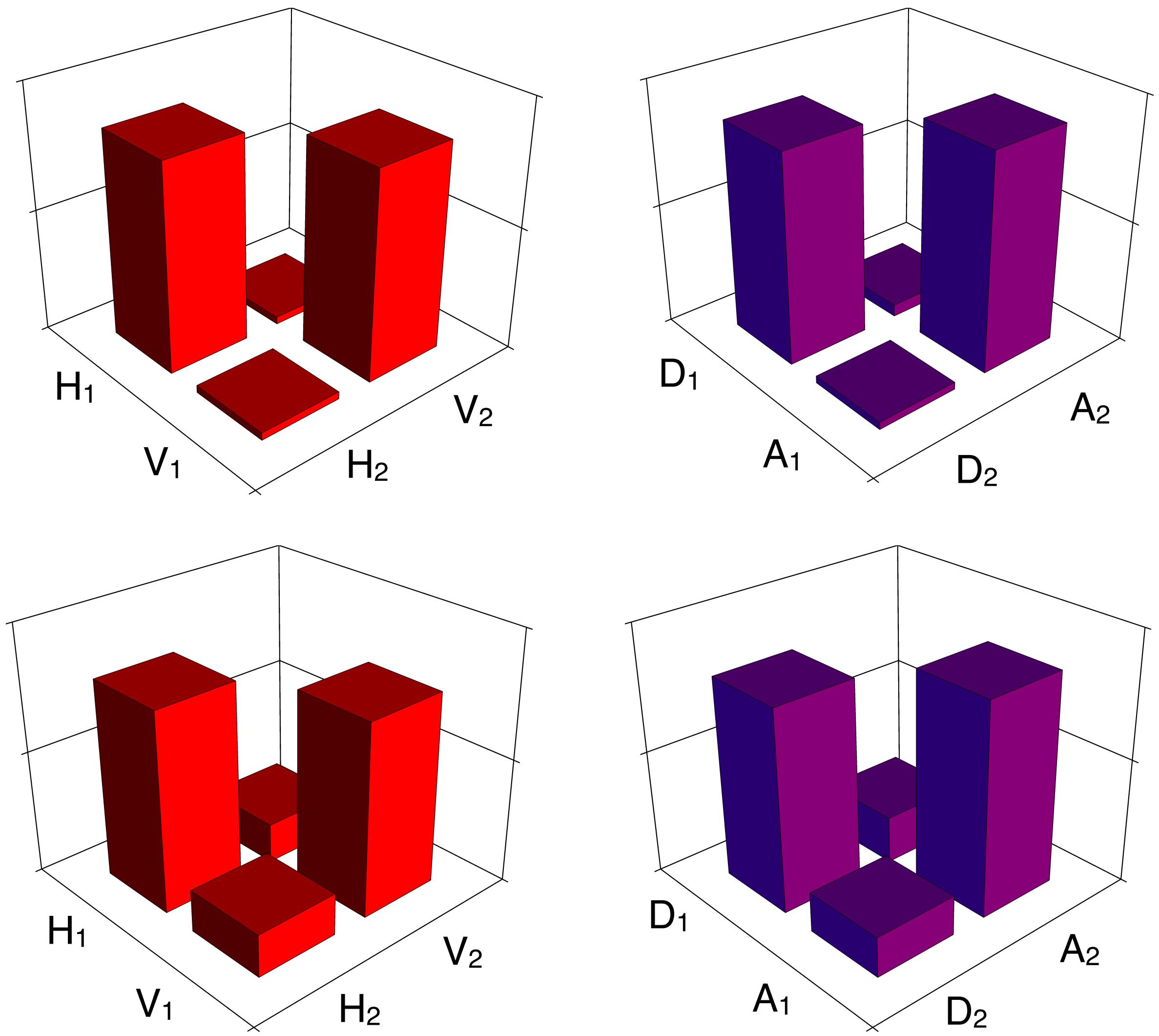
# 2D-Entanglement + Noise = ☹️




$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2]$$

$$F(\rho, \Psi) \geq 0.92$$

# 2D-Entanglement + Noise = ☹️

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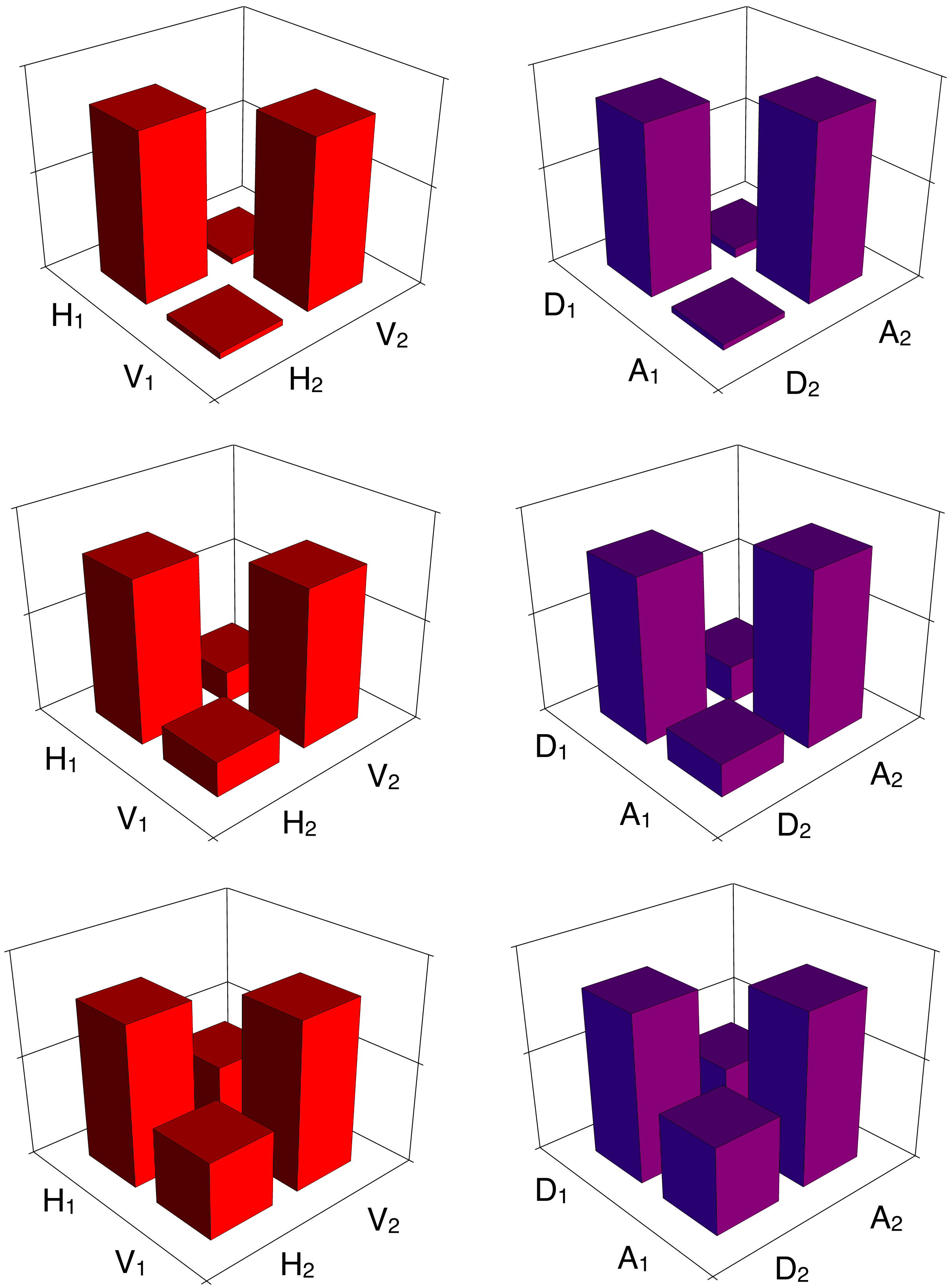
$$\mathrm{F}(\rho, \Psi) \geq 0.65$$

# 2D-Entanglement + Noise = 🙁

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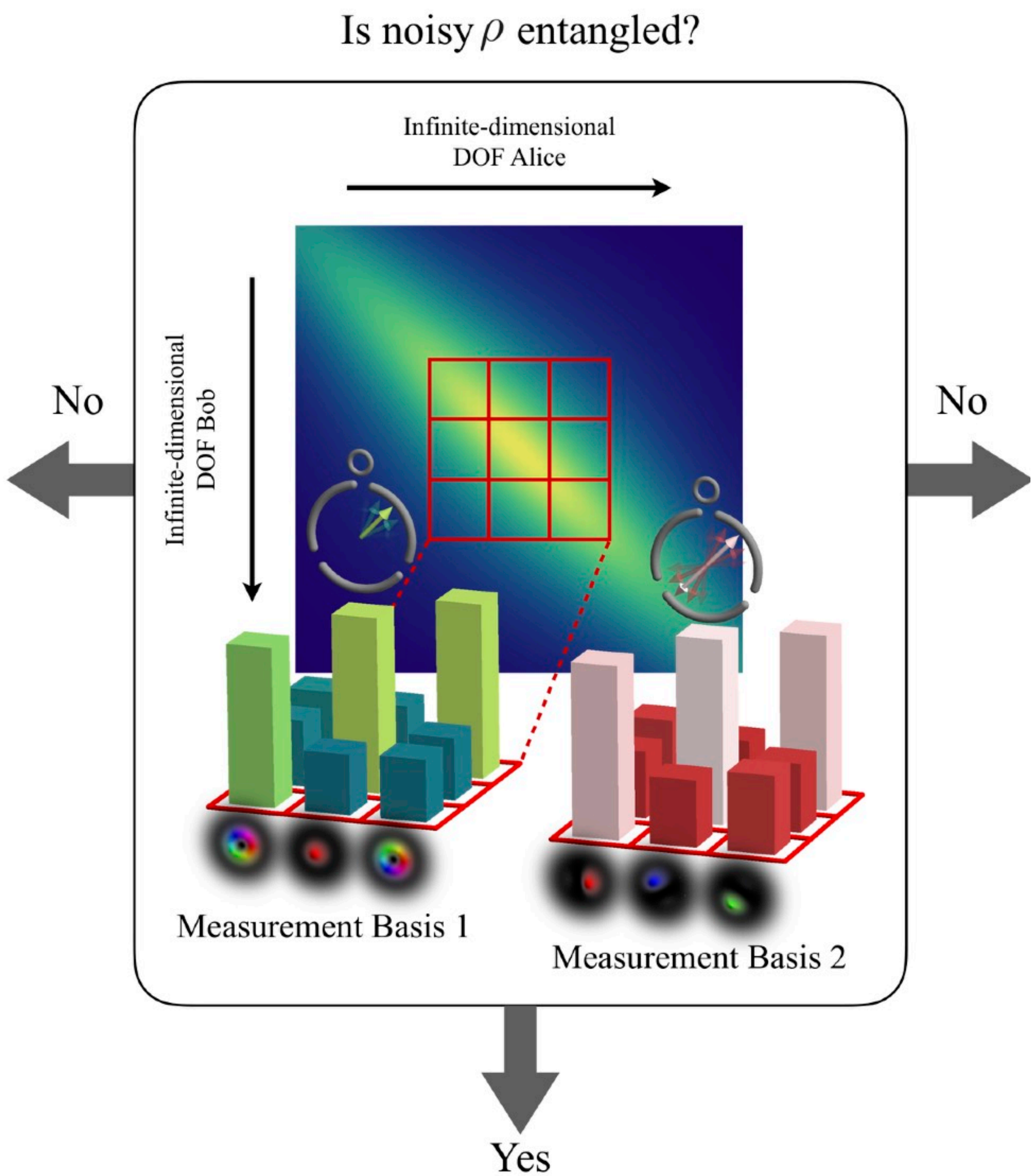
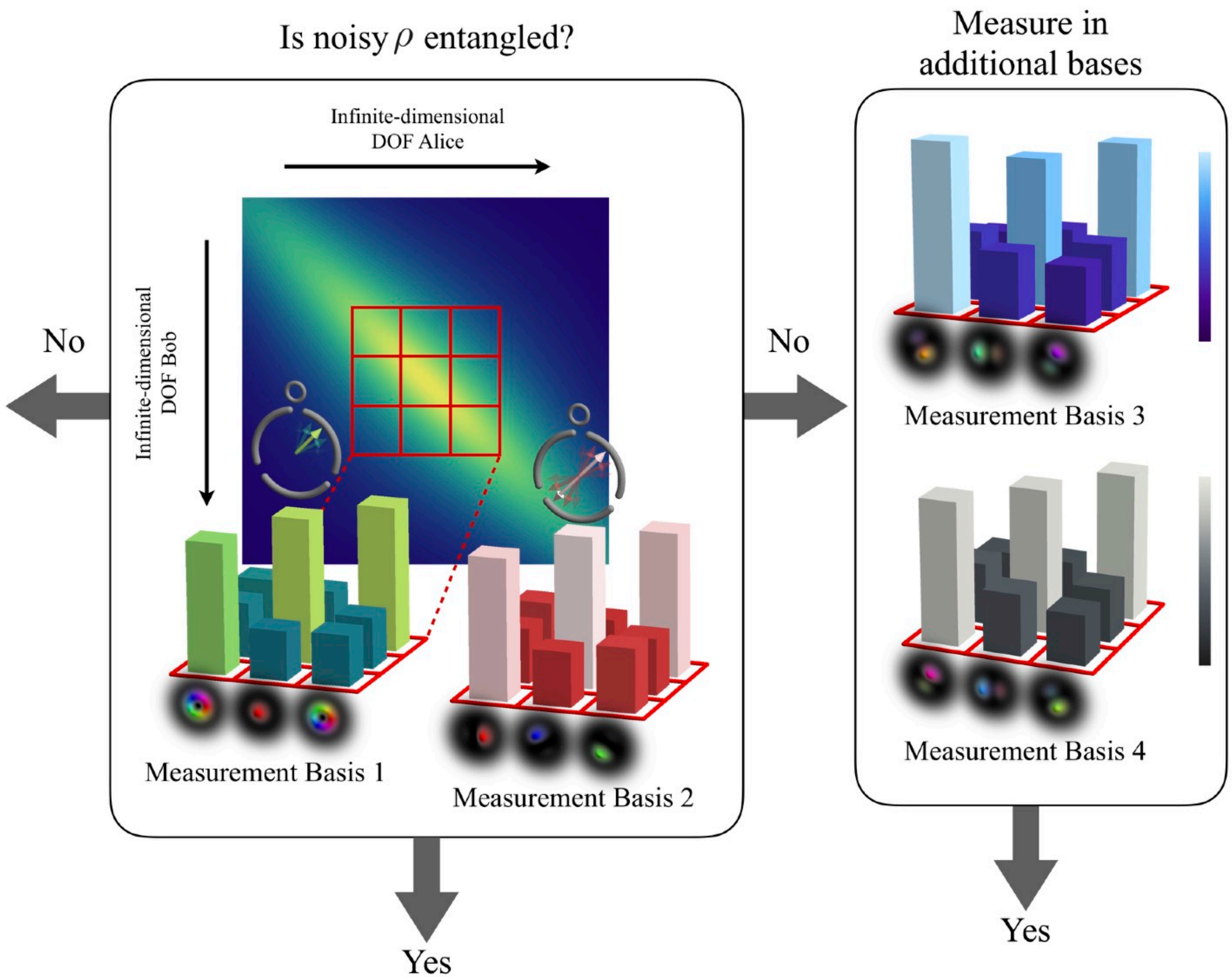
# Pathways to Noise Resistance




S. Ecker, F. Bouchard, et al, arXiv:1904.01552 (2019)

# PATHWAYS TO NOISE RESISTANCE

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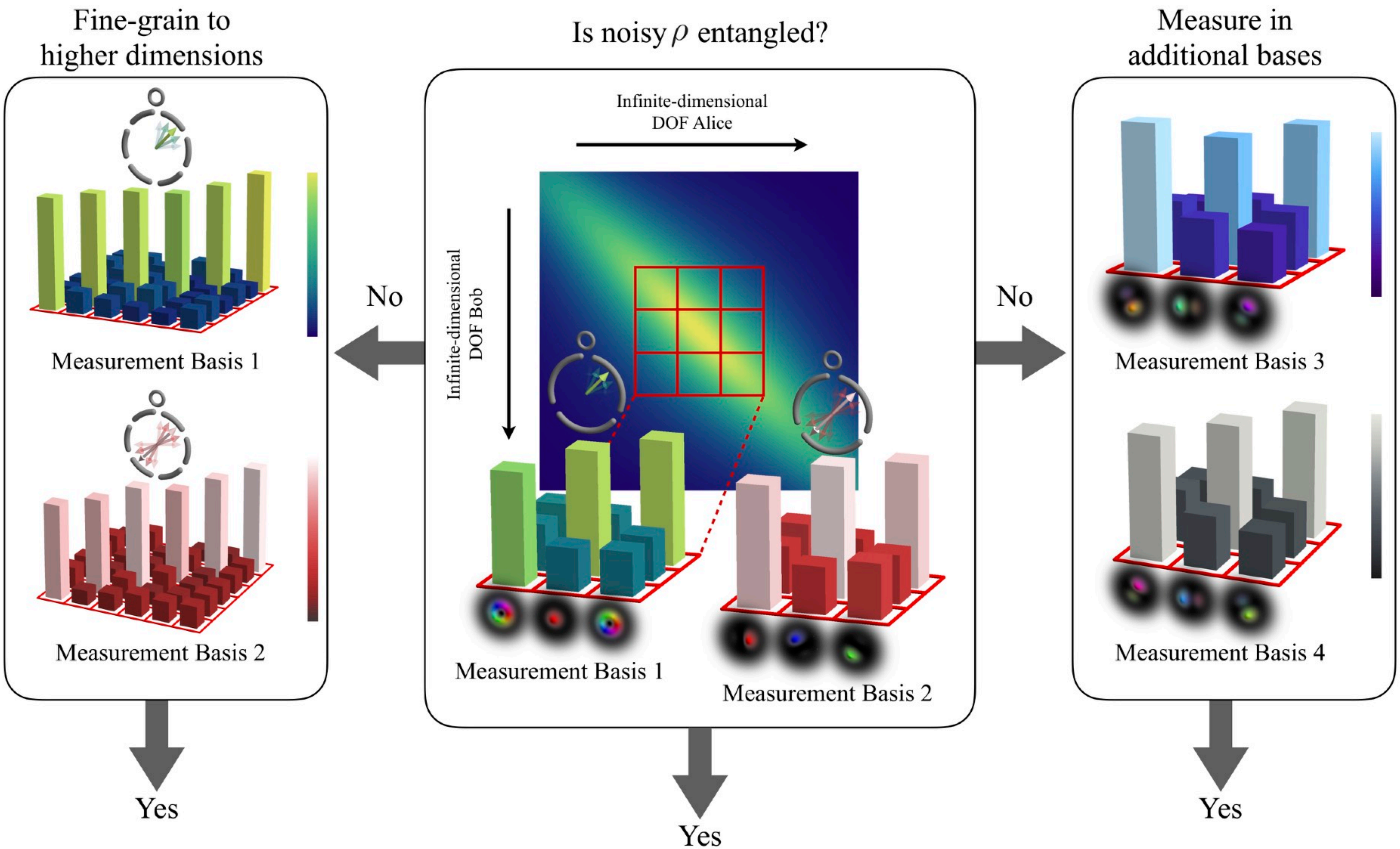


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# HD-Entanglement + Noise = Entanglement!

$$\left|\Phi^{+}\right\rangle = \frac{1}{\sqrt{d}} \sum_{m=0}^{d-1} |mm\rangle$$

$$\rho_{\text{noisy}} = p \left|\Phi^{+}\middle\rangle\middle\langle\Phi^{+}\right| + \frac{1-p}{d^2} \mathbb{1}$$

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**White Noise**

Detector Dark Counts
Background Light
Loss-induced Accidentals

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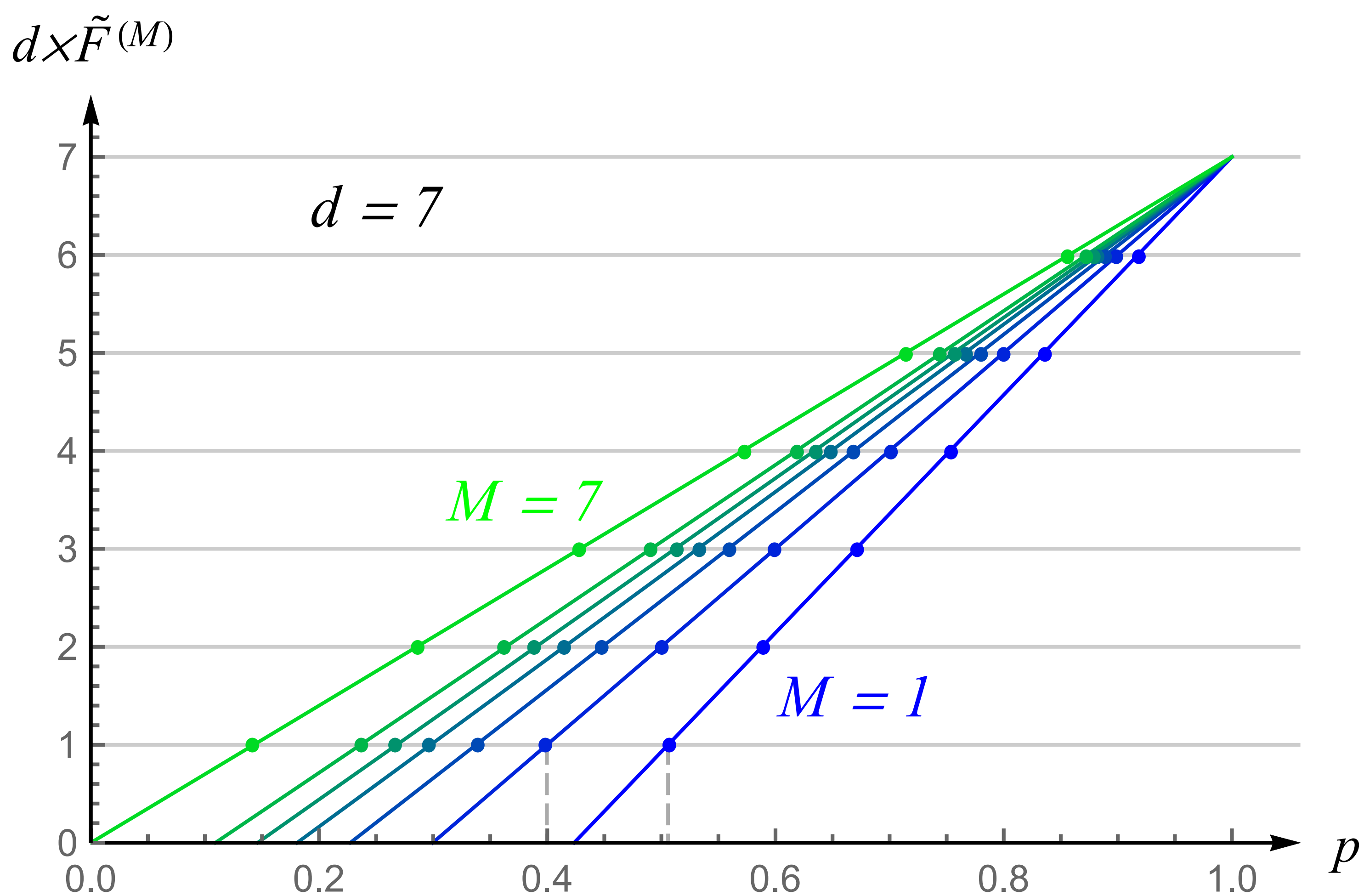


**White Noise**

Detector Dark Counts
Background Light
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Bavaresco, Herrera, Klöckl, Pivoluska, Erker, Friis, **MM**, Huber, **Nat. Phys. 14,** 1032 (2018)

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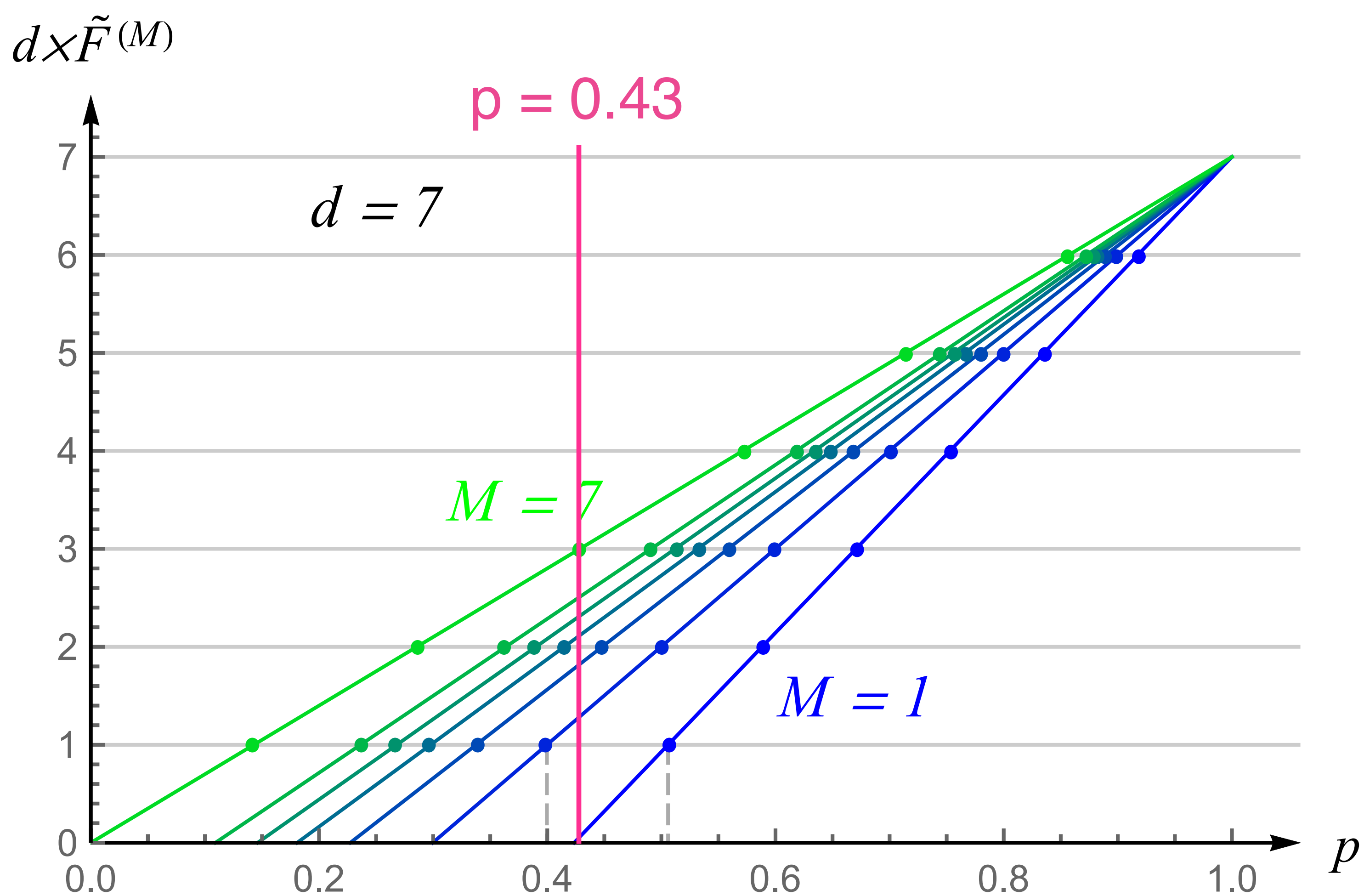


Eg. - For a **7-dim** state with noise level *p = 0.43*,

We can certify **no** entanglement with 1 MUB, but <u>*d* = 4</u> with 7 MUBs!

Bavaresco, Herrera, Klöckl, Pivoluska, Erker, Friis, **MM**, Huber, **Nat. Phys. 14,** 1032 (2018)

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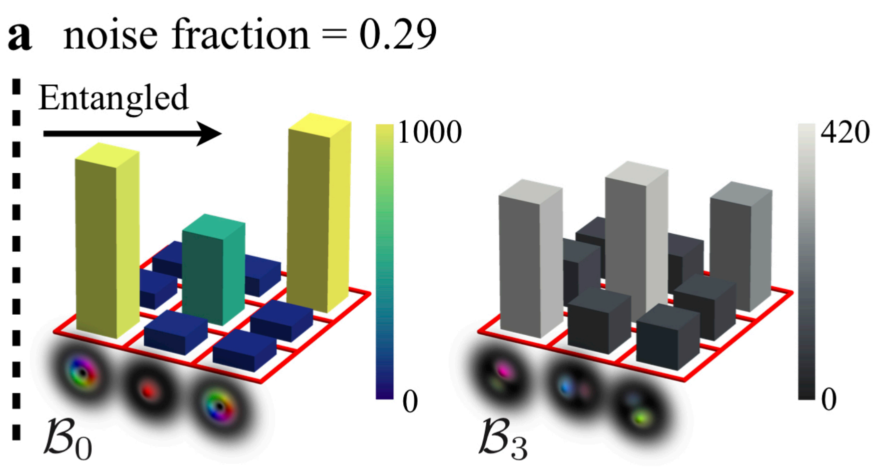


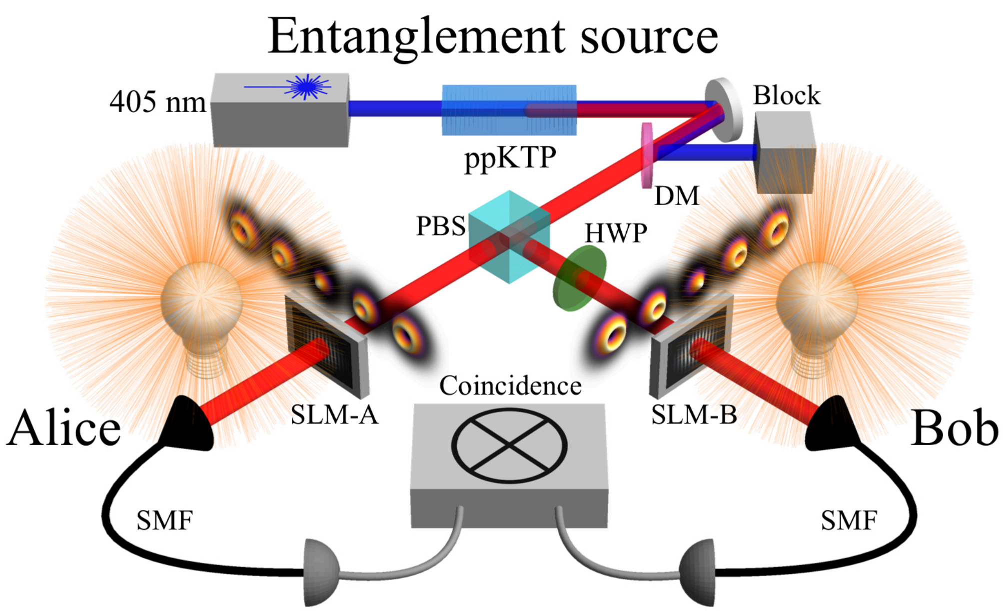


$$\text{noise fraction} = \frac{\#\text{noise counts}}{\#\text{total counts}}$$

S. Ecker, F. Bouchard, et al, **arXiv:1904.01552 (PRX In Press)** (2019)

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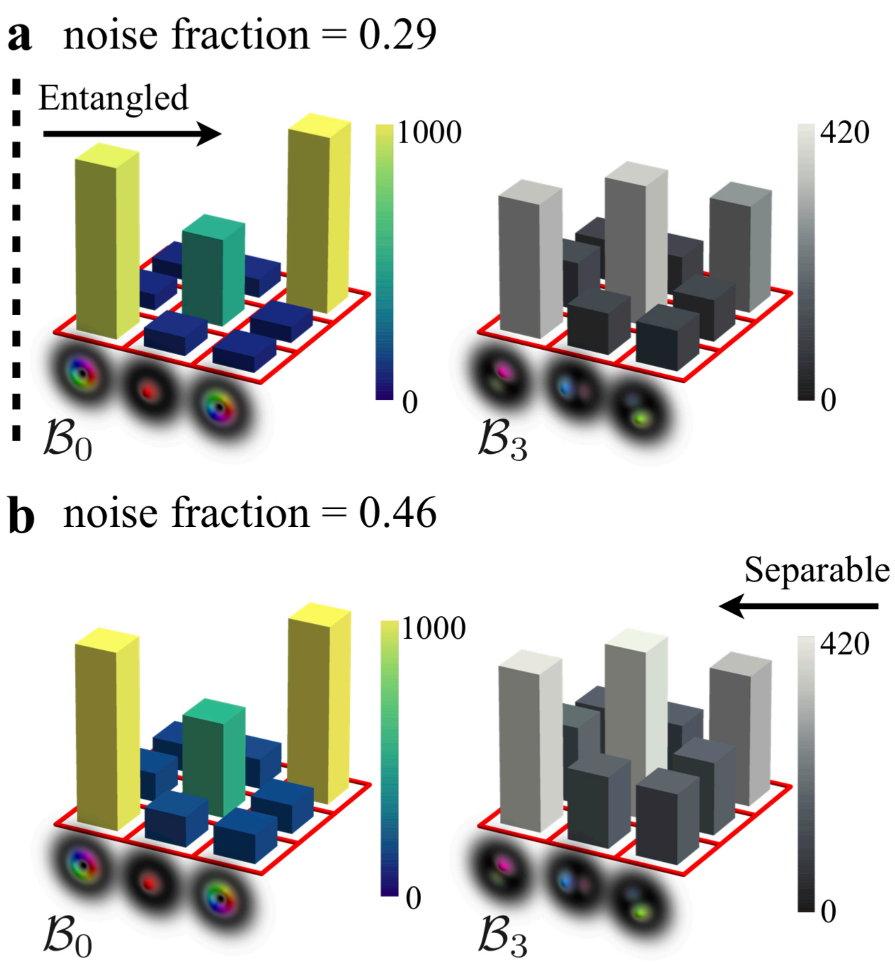


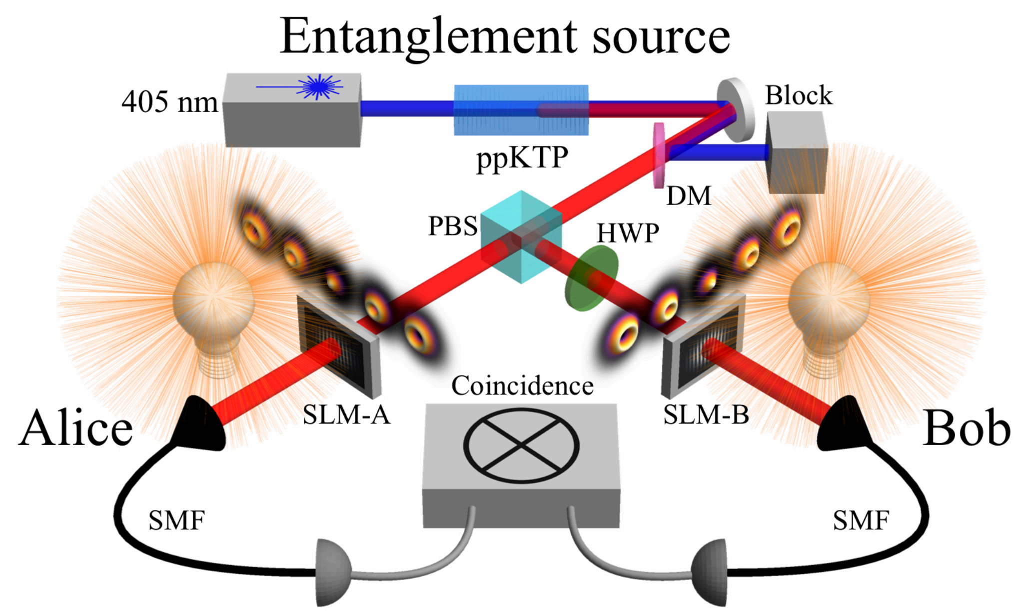


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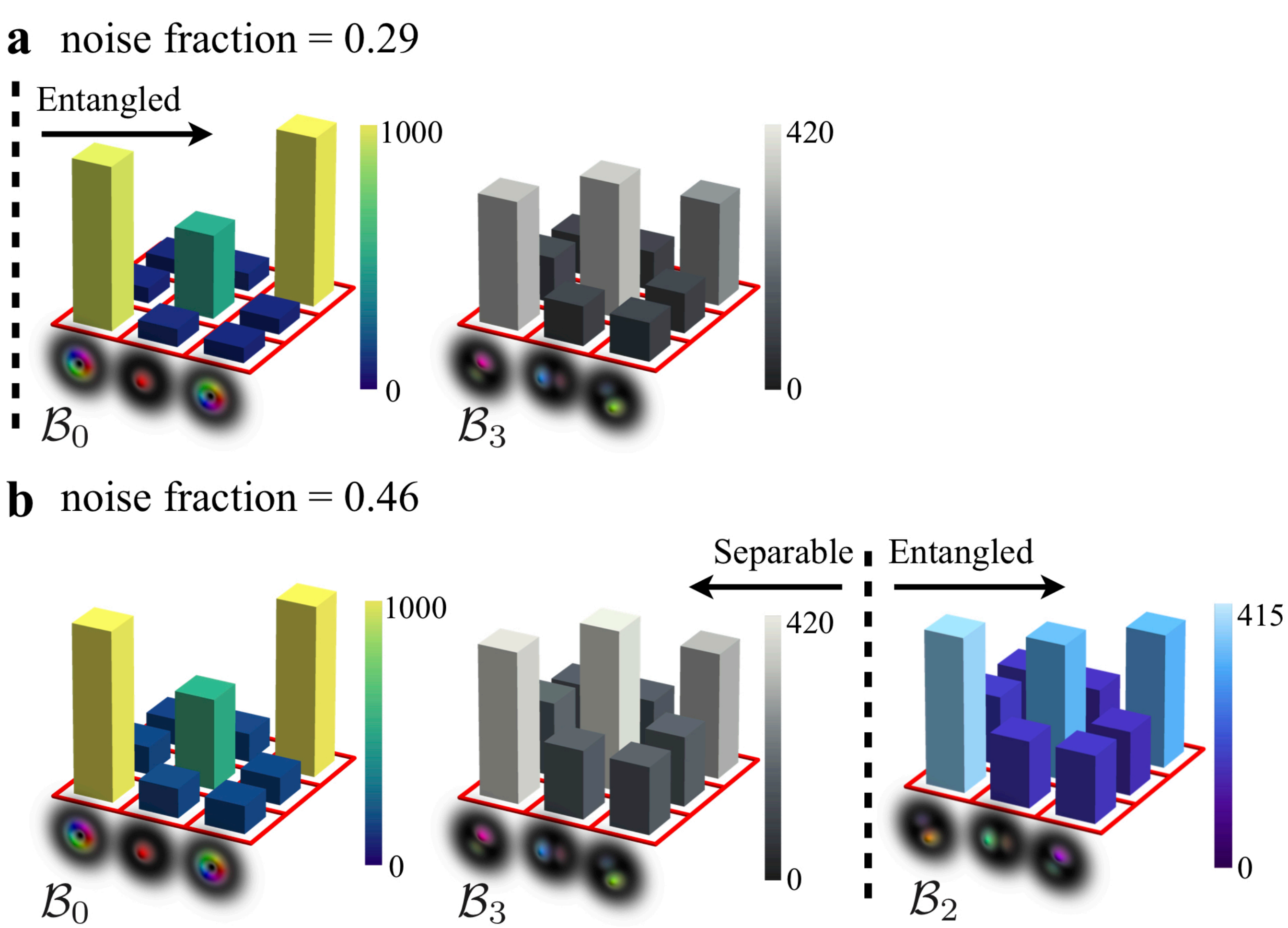


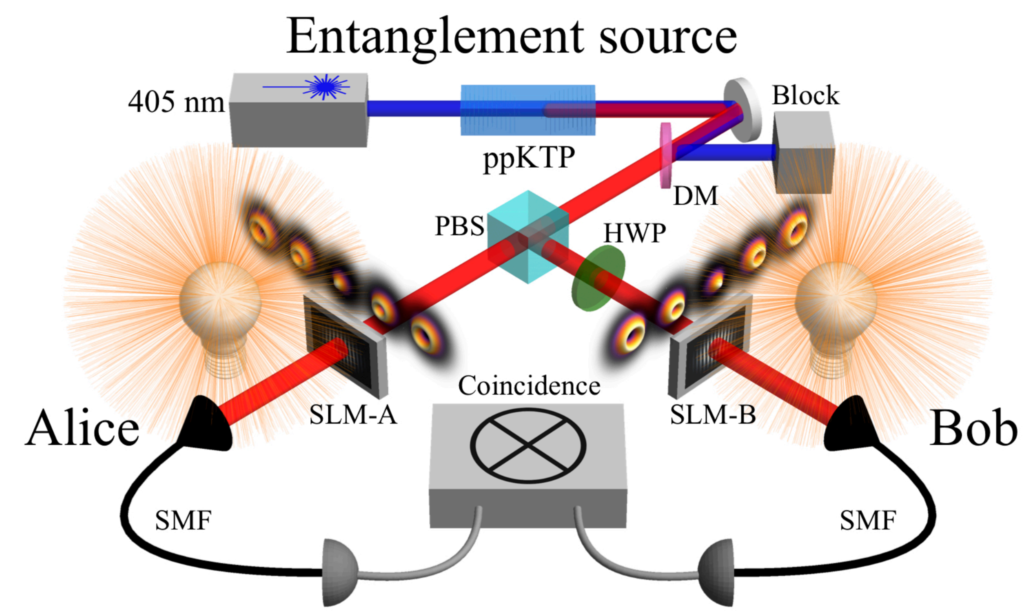


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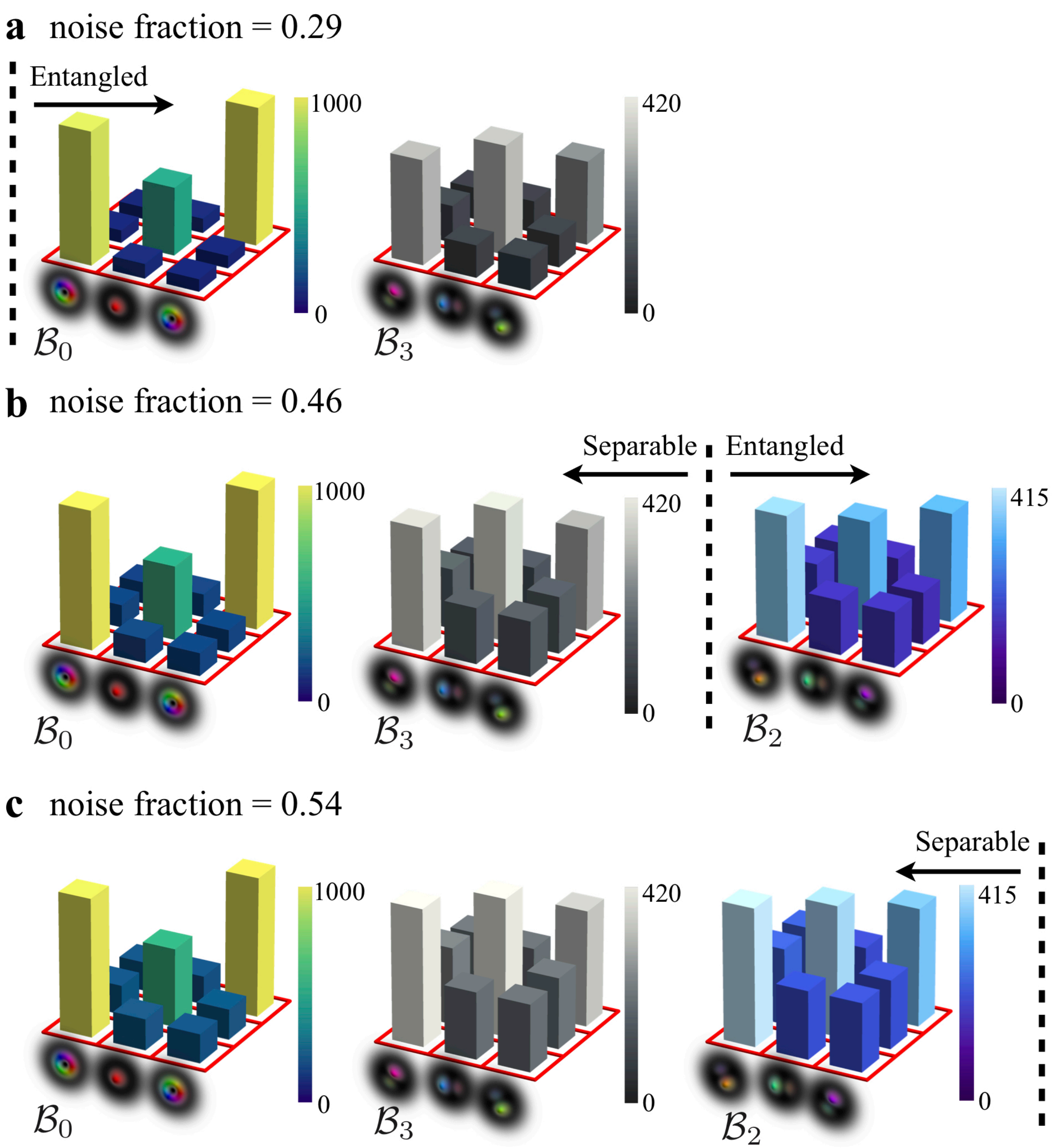


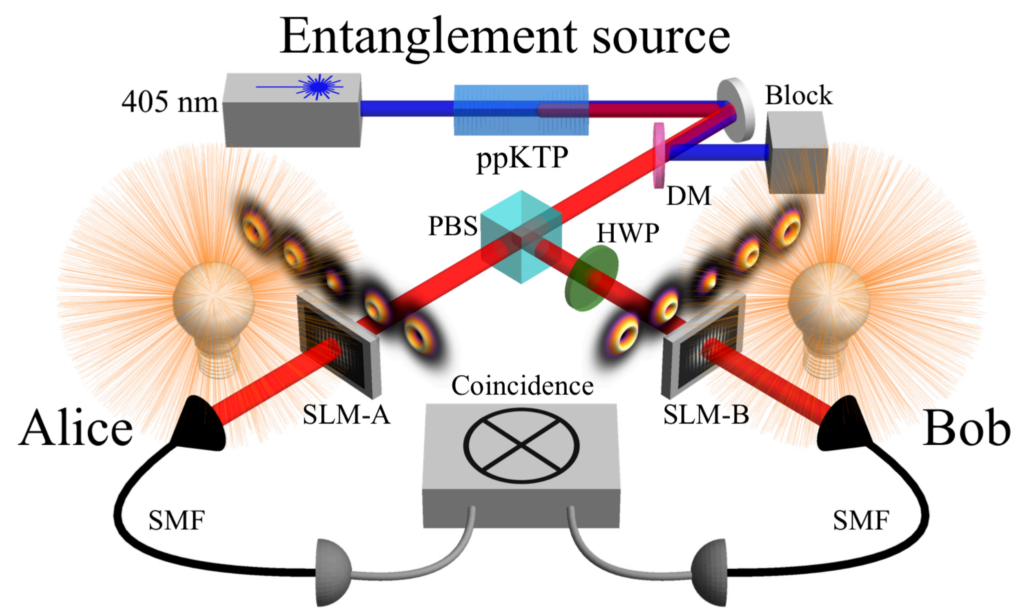


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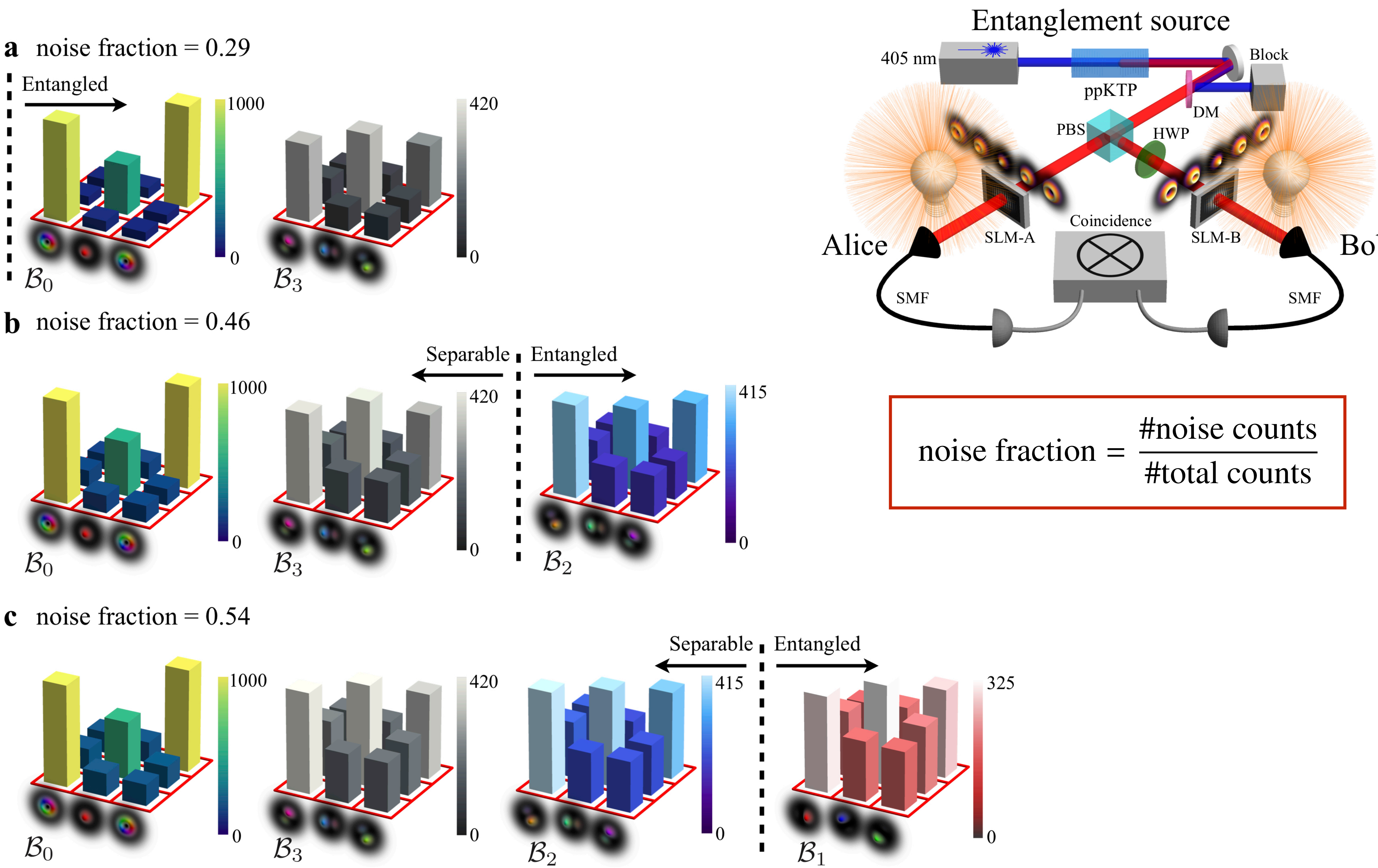


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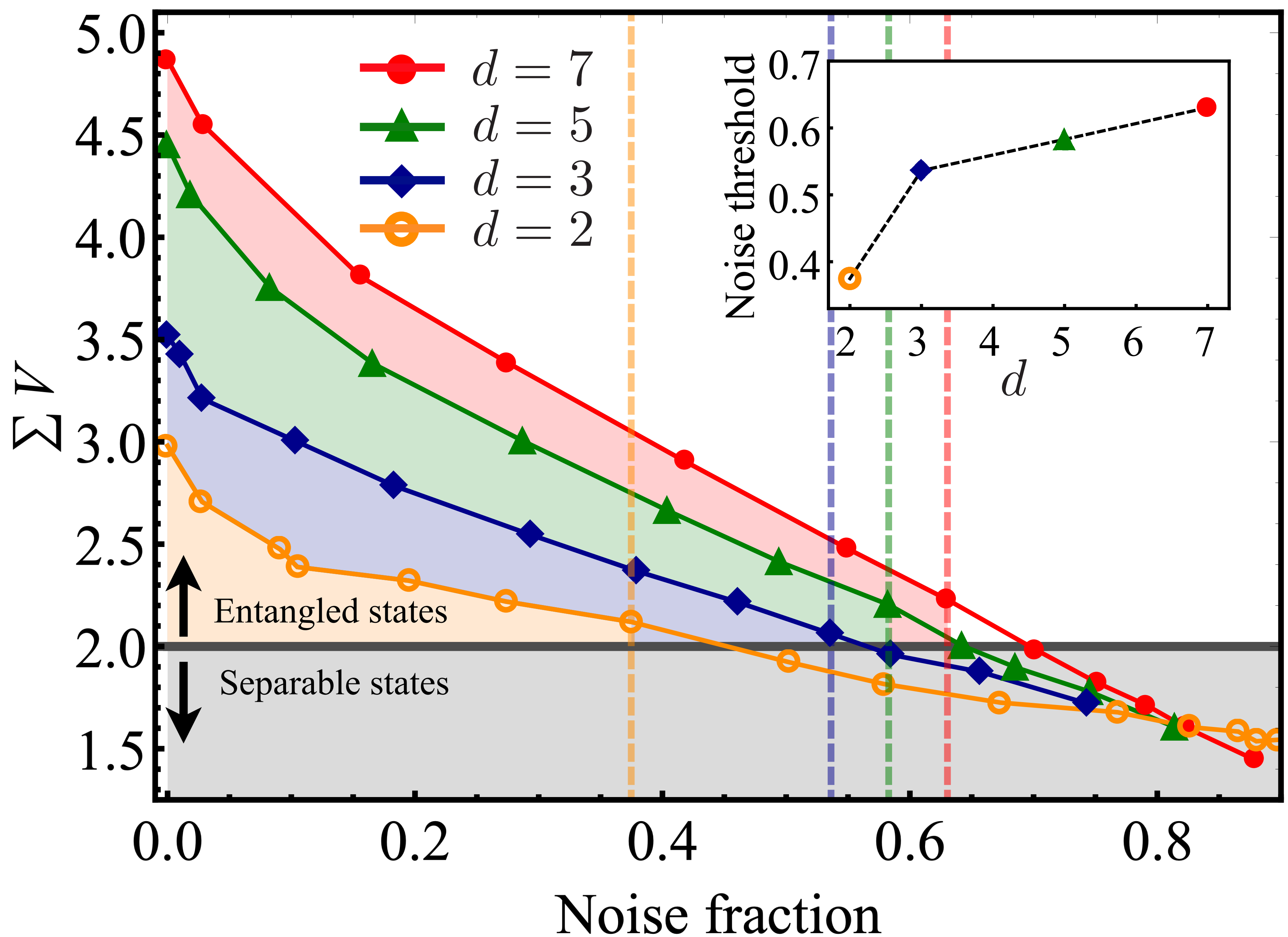


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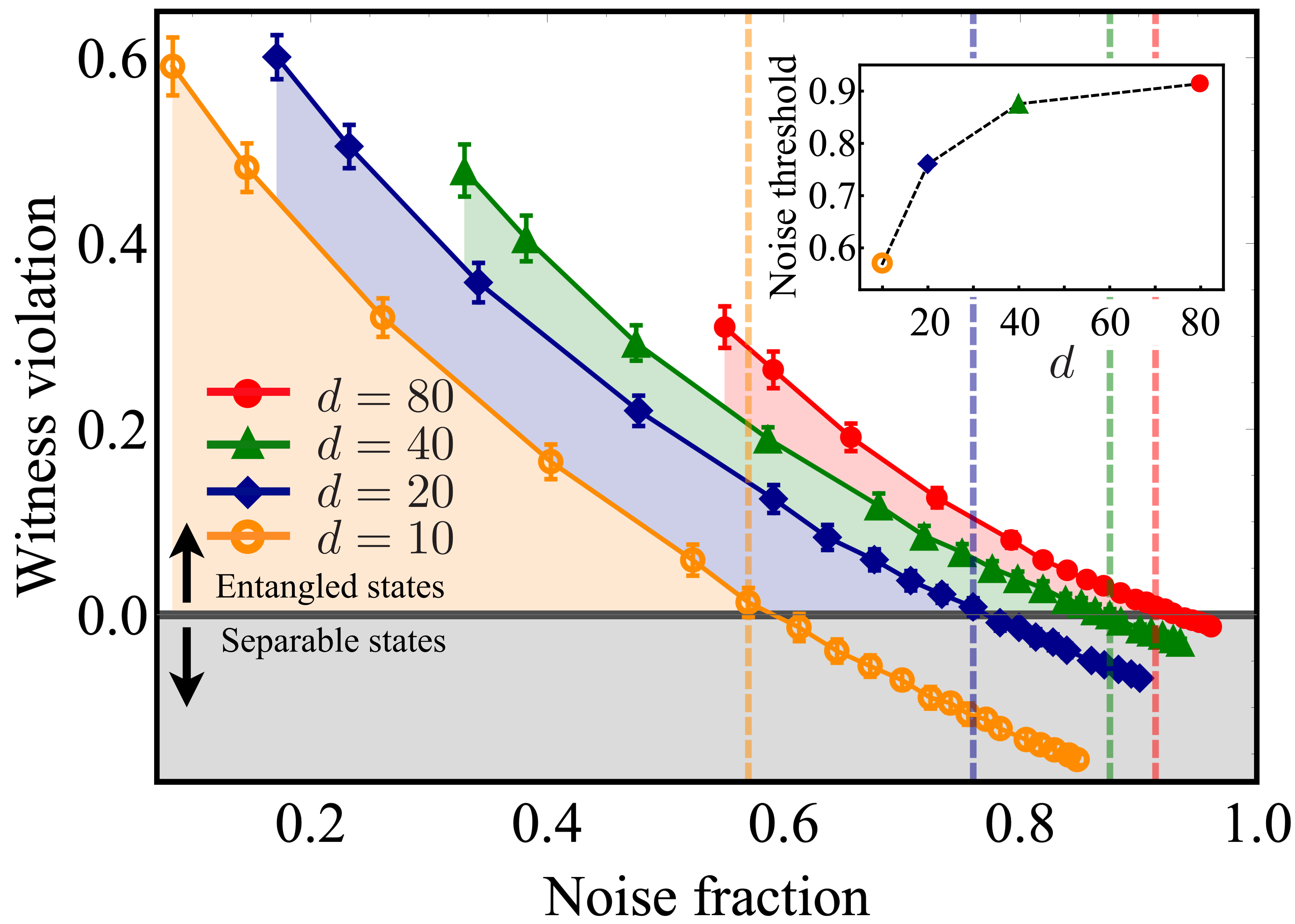


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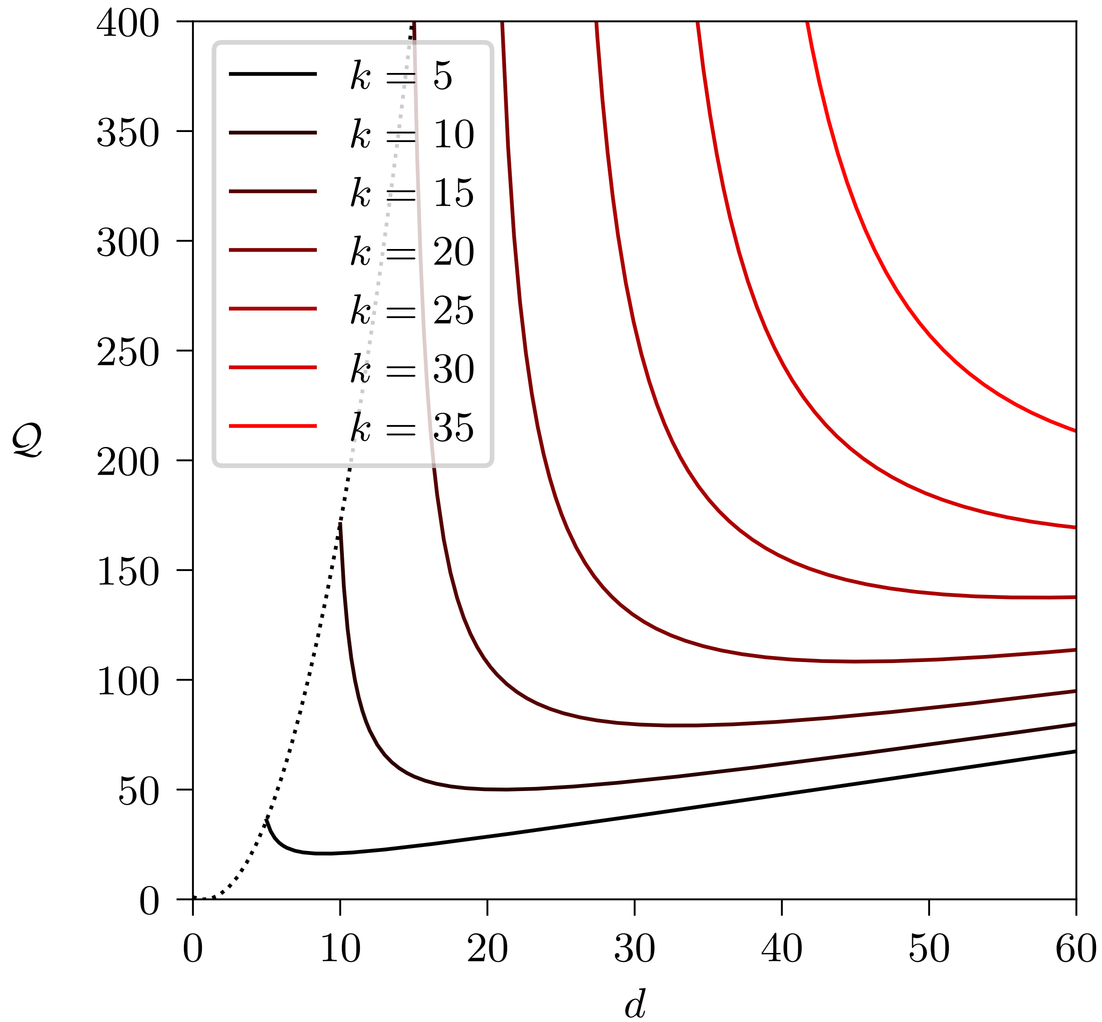
# K-DIM ENTANGLEMENT IN D-DIM HILBERT SPACE




Zhu et al, arXiv:1908.08943 (2019)

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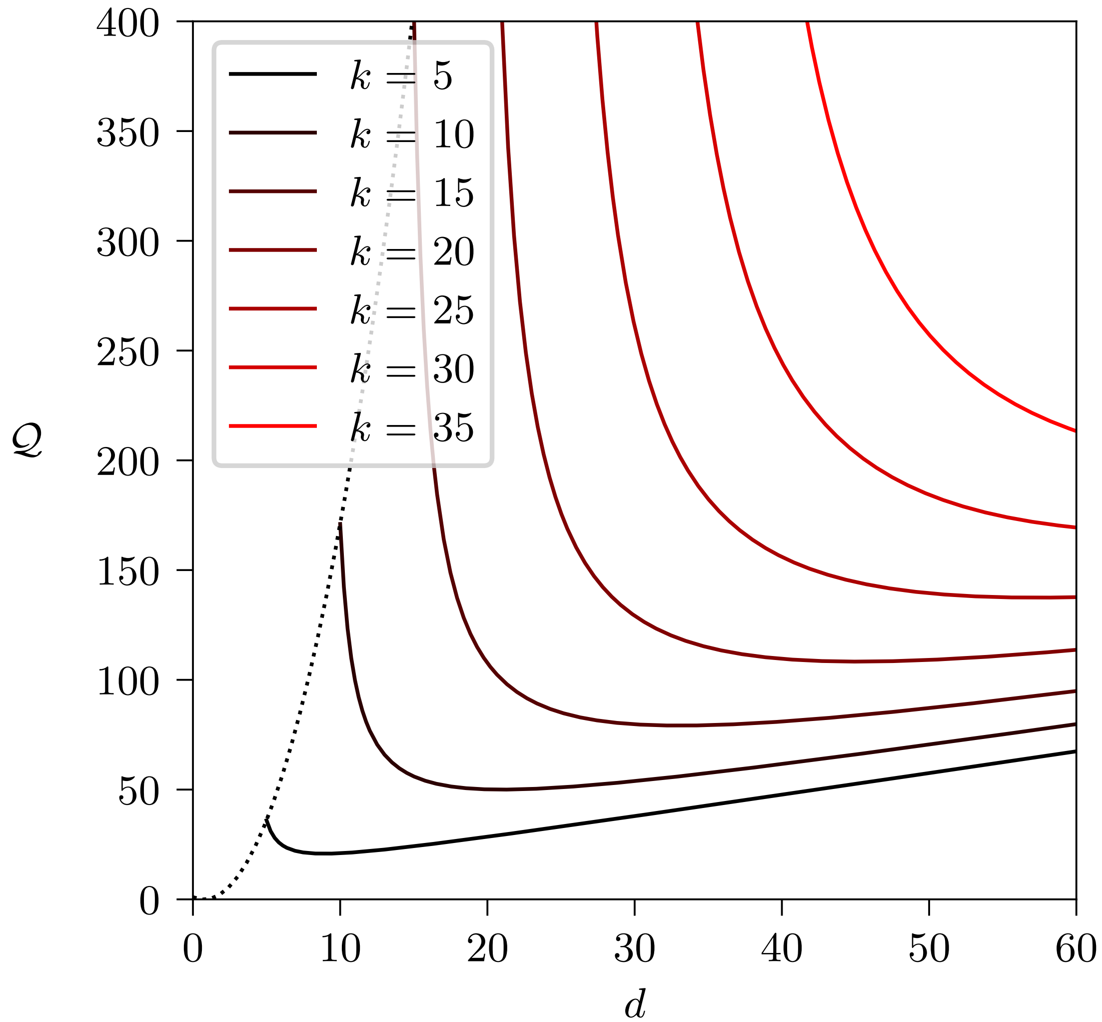


$$Q_{\text{opt}} = 3k + 2\sqrt{2}\sqrt{(k-2)(k-1)} - 4$$

$$d_{\text{opt}} = \sqrt{2}\sqrt{k^2 - 3k + 2} + k - 1$$

Zhu et al, arXiv:1908.08943 (2019)

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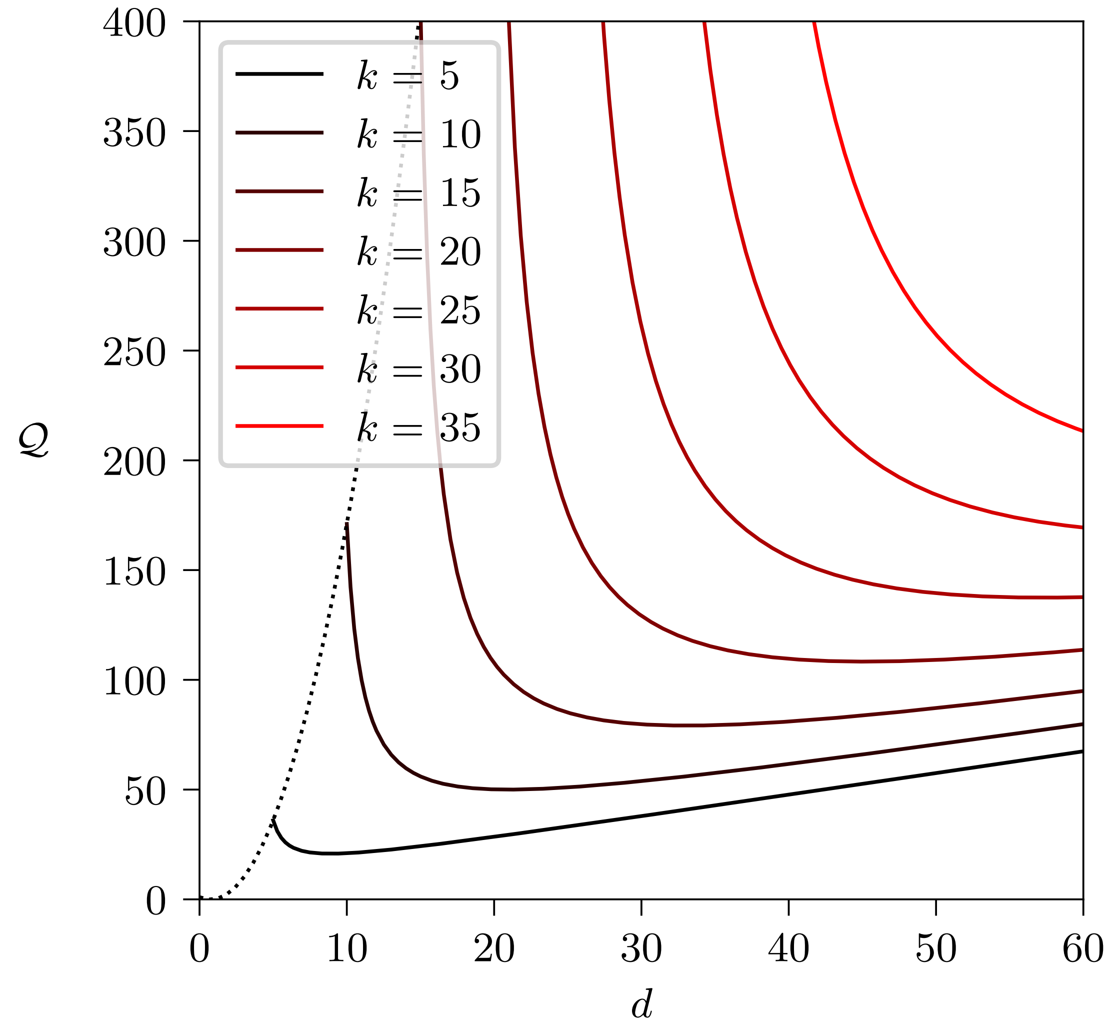


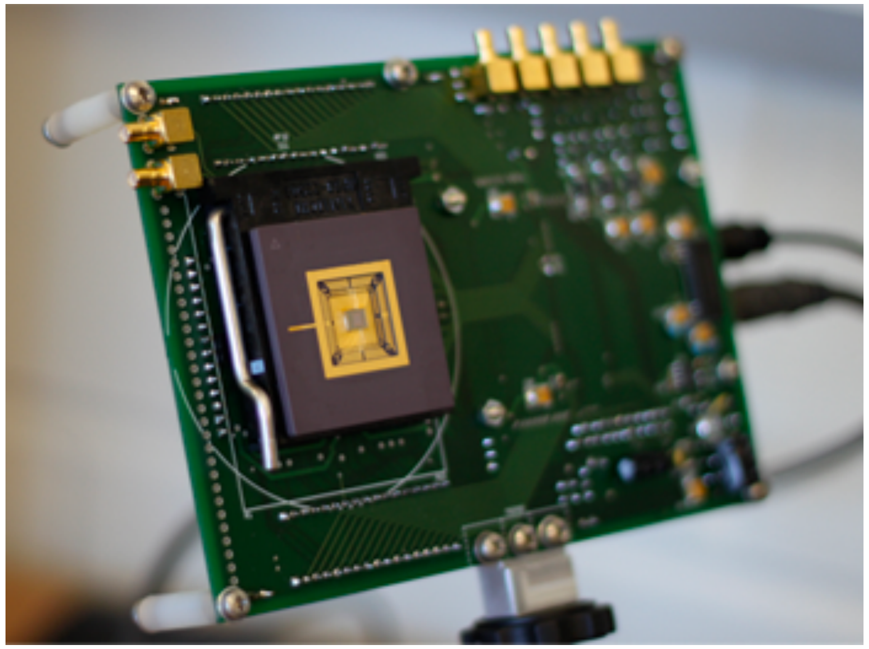

32 x 32 pixel SPAD camera

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Zhu et al, **arXiv:1908.08943** (2019)

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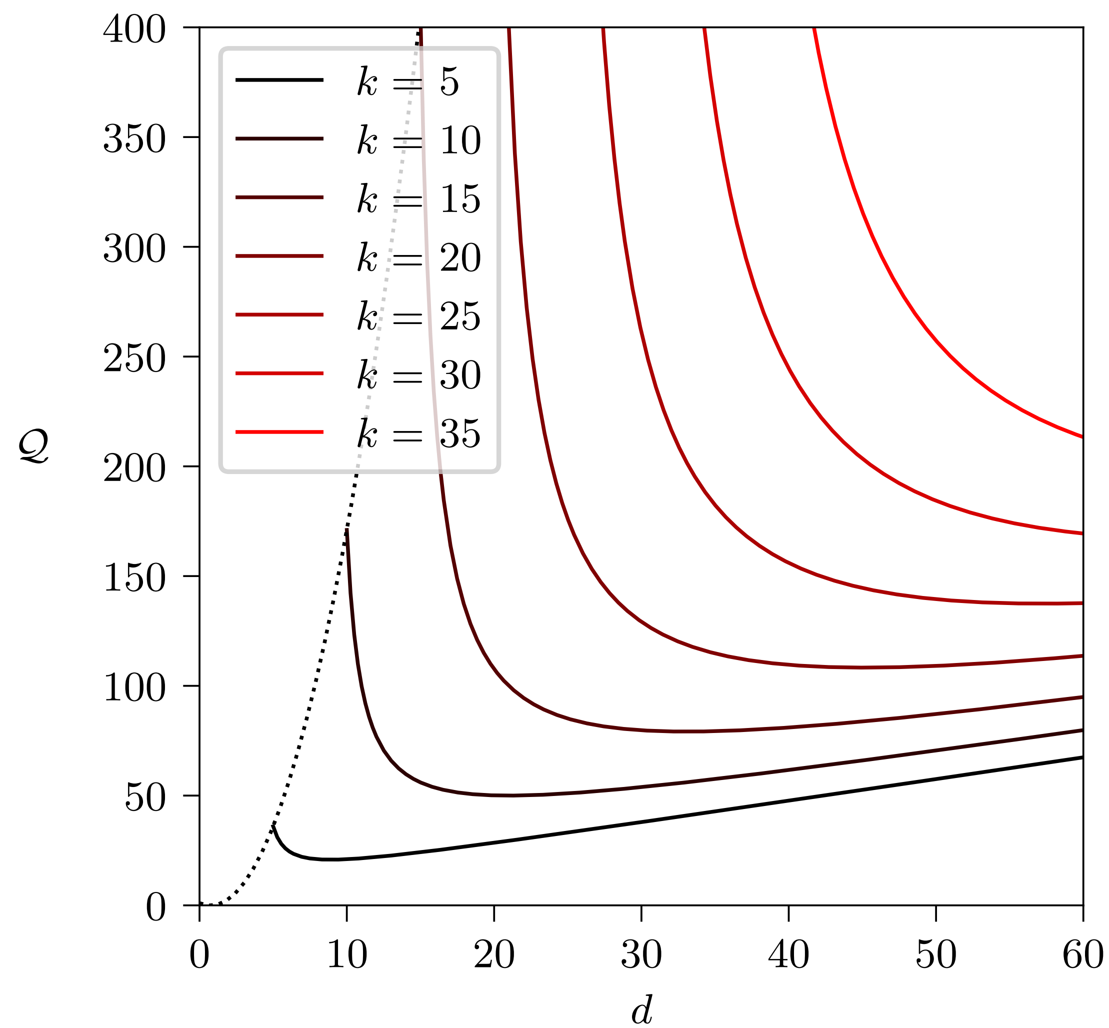


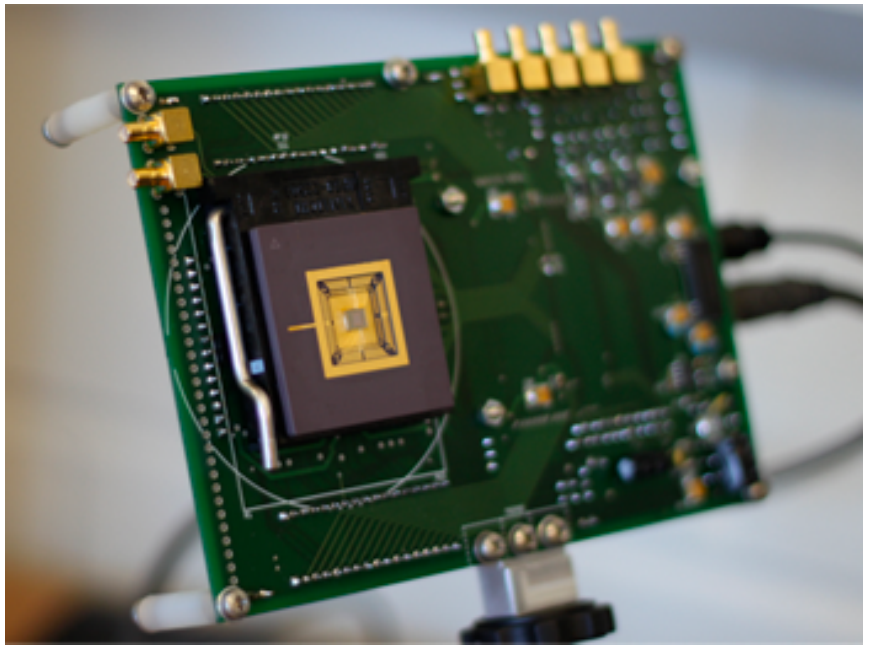

32 x 32 pixel SPAD camera

49 x 49 pixels ⟶ 343 times lower SNR!

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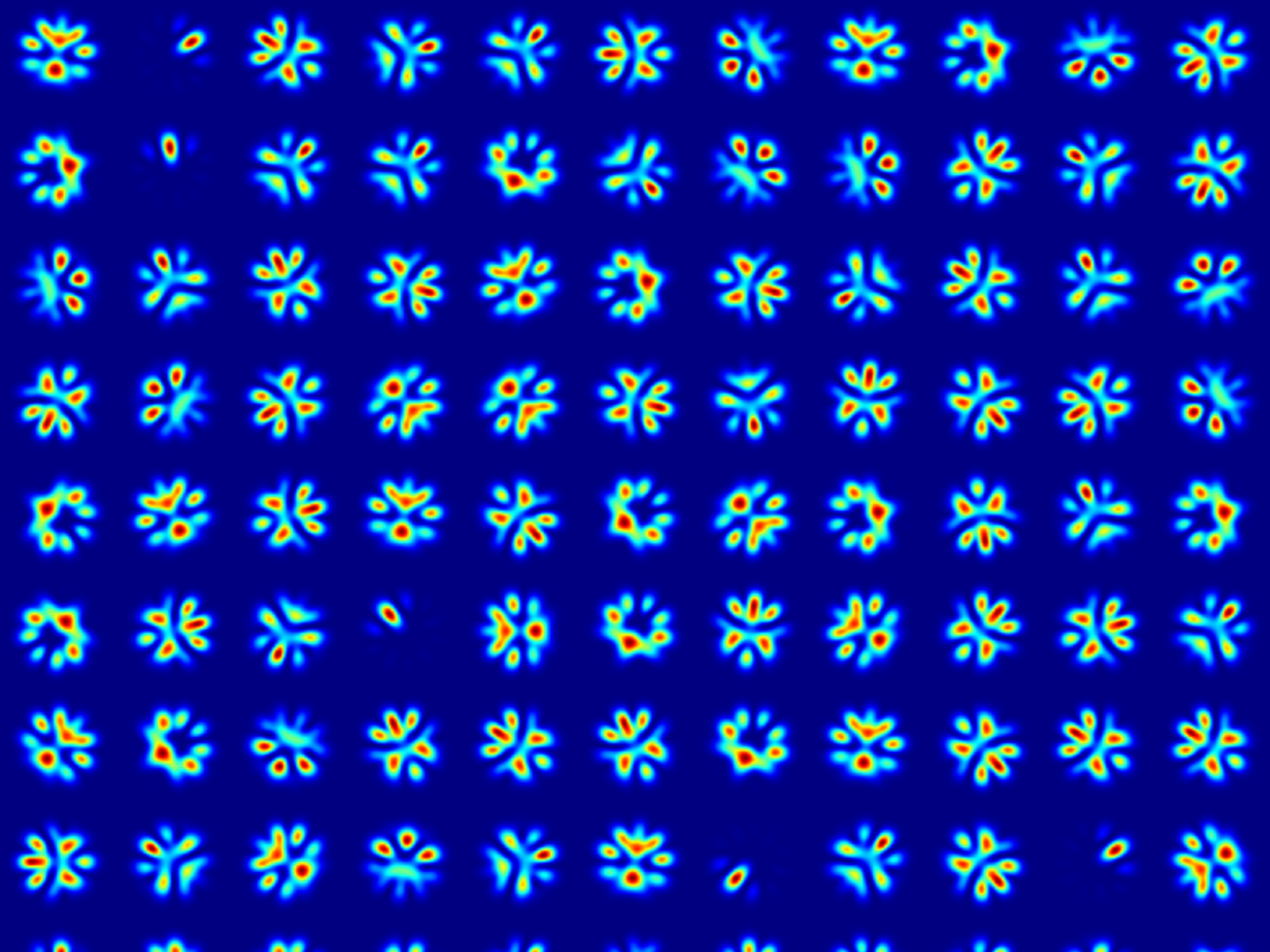


## LECTURE 1

- Introduction / Qudits
- Superposition / Entanglement
- Measuring HD Entanglement
- Noise Resistance

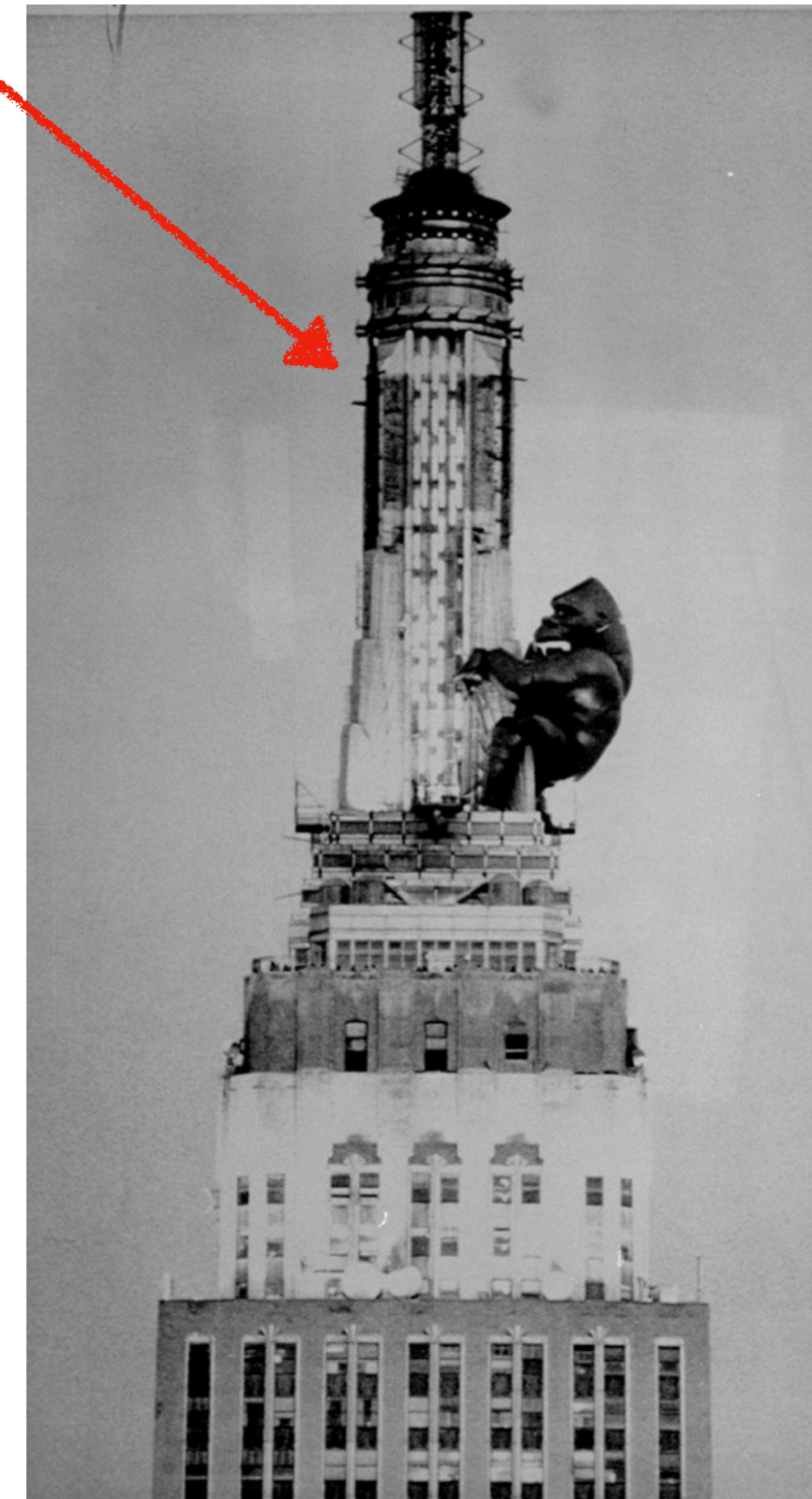
## LECTURE 2

- Unscrambling Entanglement through a Complex Medium
- Multi-photon HD entanglement
- Free-space Quantum Communication

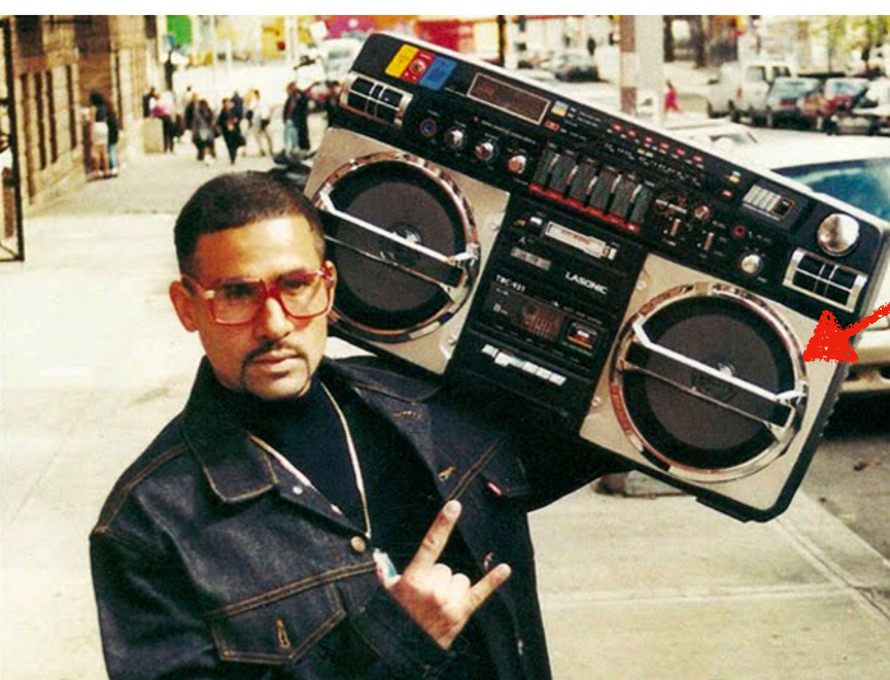
# UNSCRAMBLING ENTANGLEMENT THROUGH A COMPLEX MEDIUM

# Realistic Quantum Networks

Imperfect Source



Noisy Channel



Imperfect Detector

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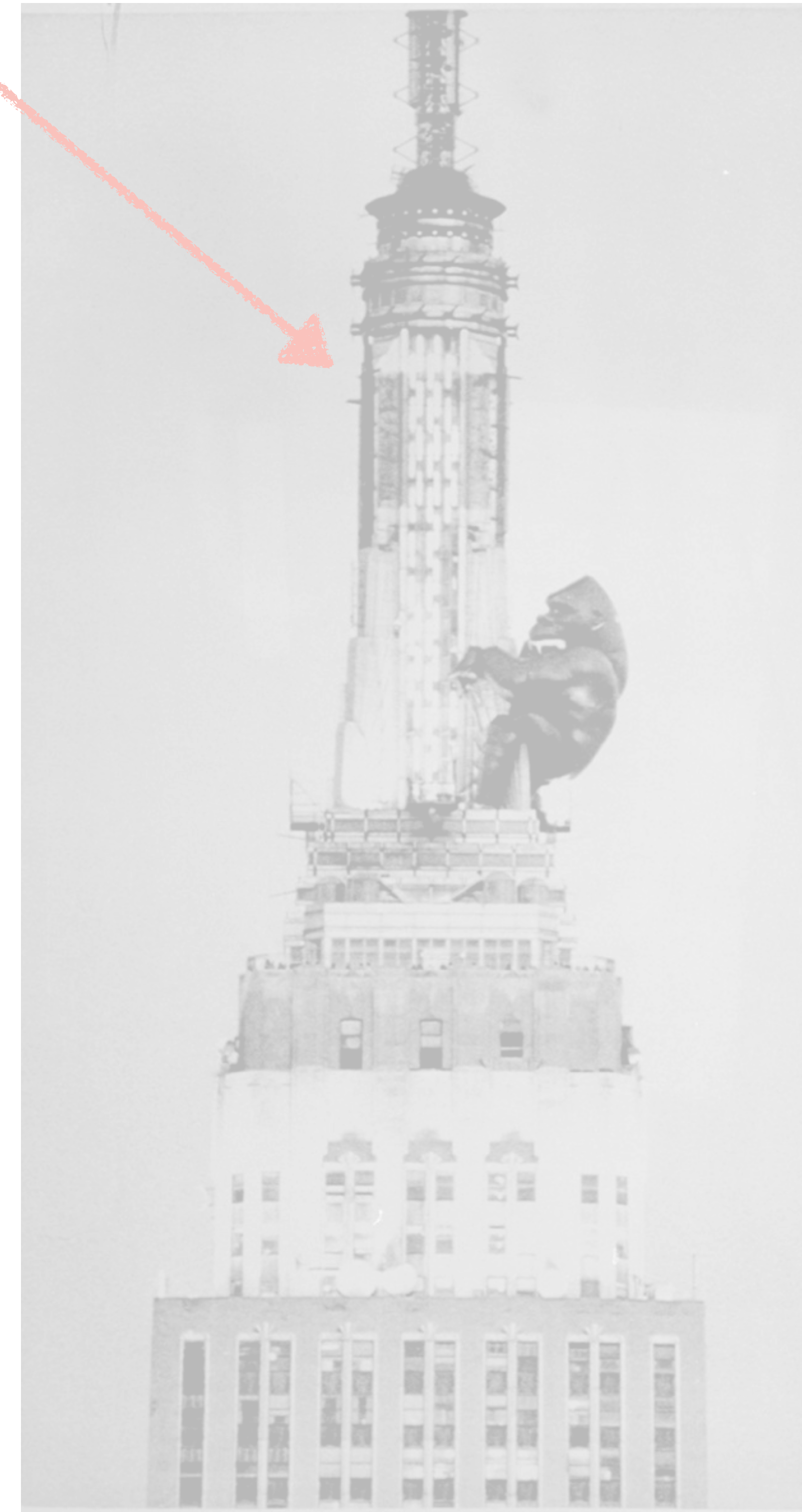
Imperfect Source



Noisy Channel

Imperfect Detector

# Complex Scattering Media

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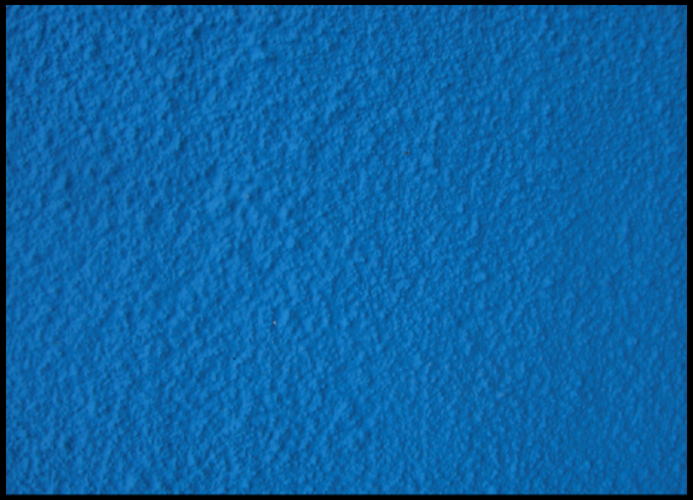
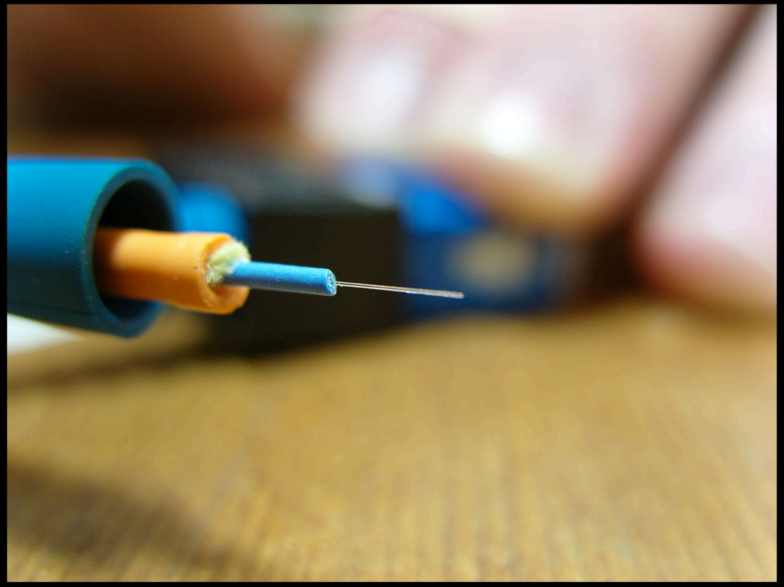




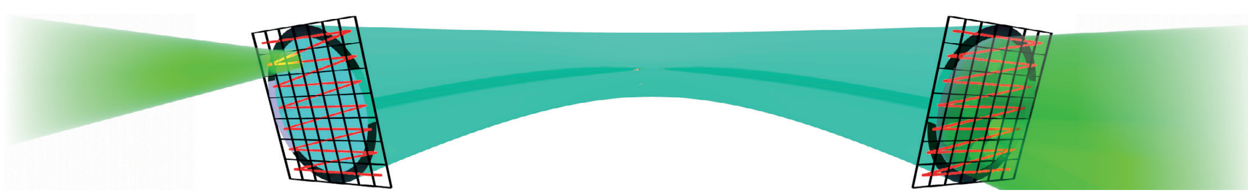
# Classical Comms: SDM

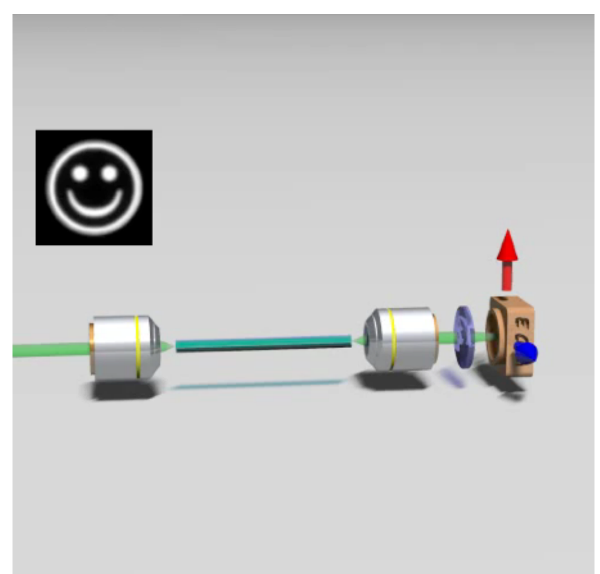
nature photonics
ARTICLES
PUBLISHED ONLINE: 13 JULY 2015 | DOI: 10.1038/NPHOTON.2015.112

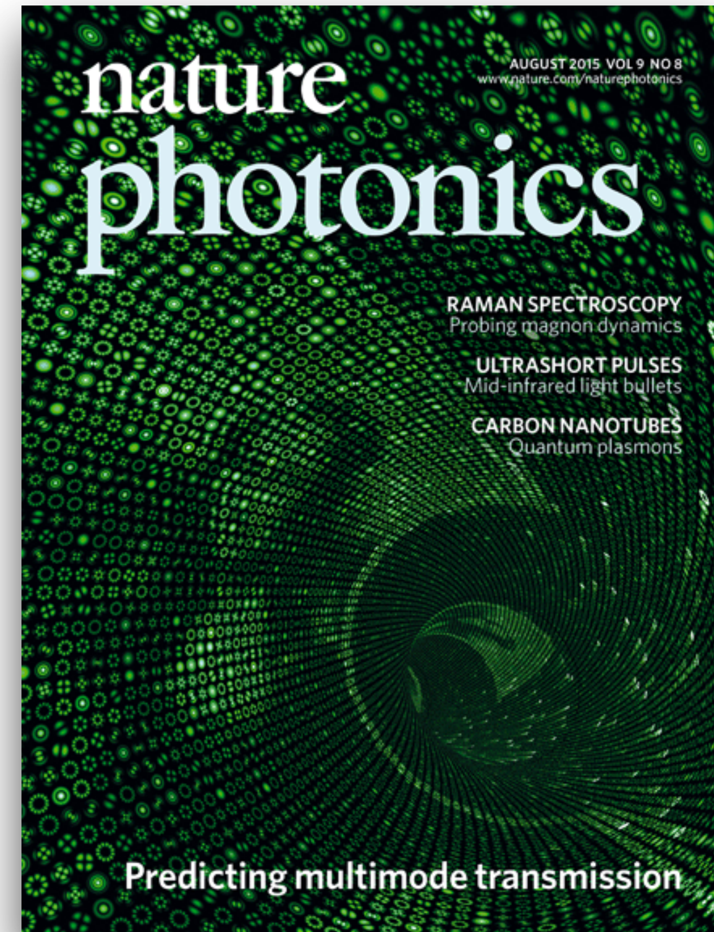
## Seeing through chaos in multimode fibres

**Martin Plöschner[1], Tomáš Tyc[2] and Tomáš Čižmár[1]***

[1]School of Engineering, Physics and Mathematics, College of Art, Science & Engineering, University of Dundee, Nethergate, Dundee DD1 4HN, UK.
[2]Department of Theoretical Physics and Astrophysics, Masaryk University, Kotlarska 2, 61137 Brno, Czech Republic. *e-mail: t.cizmar@dundee.ac.uk








Plöschner, Tyc, Cizmar, Nat. Photonics 9, 529 (2015)

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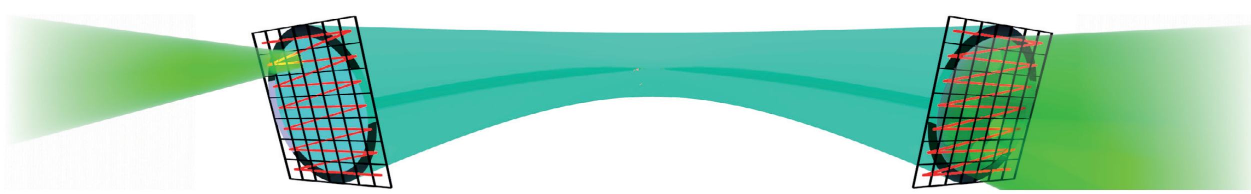
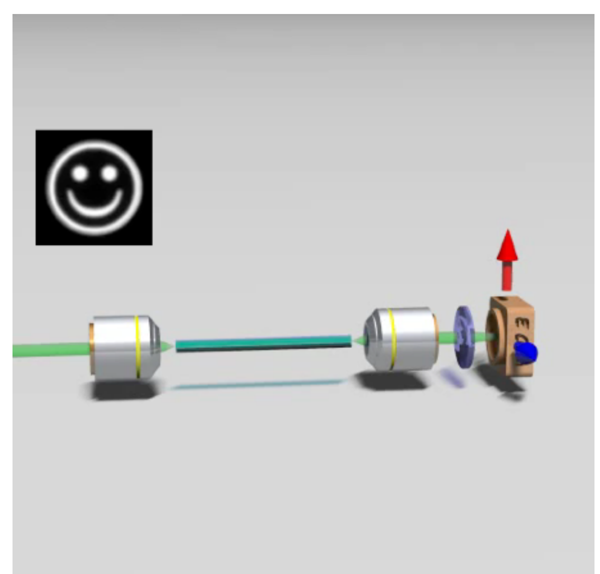
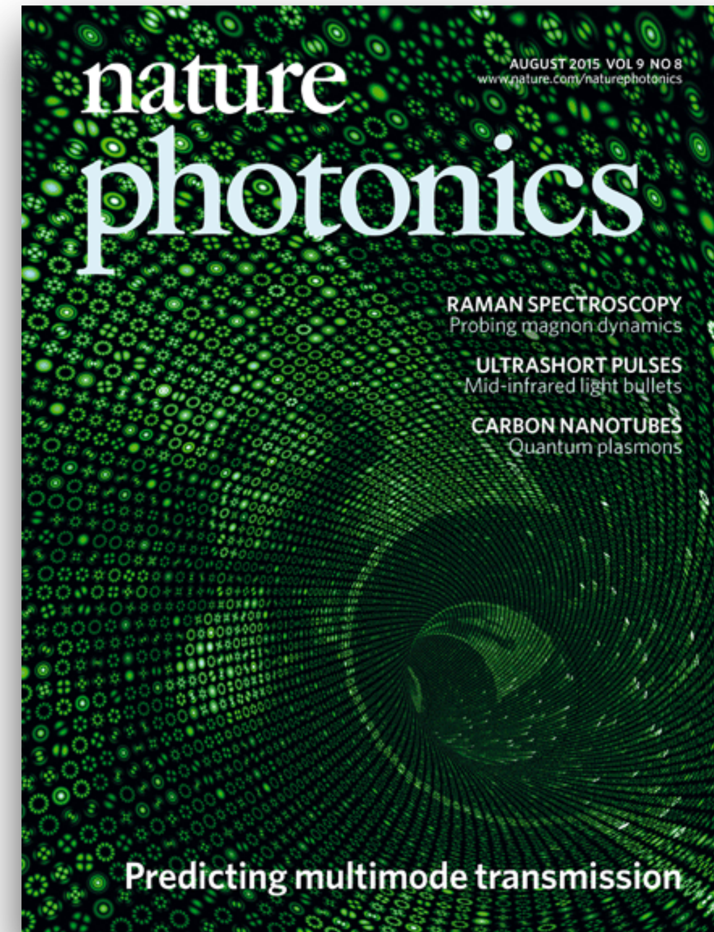
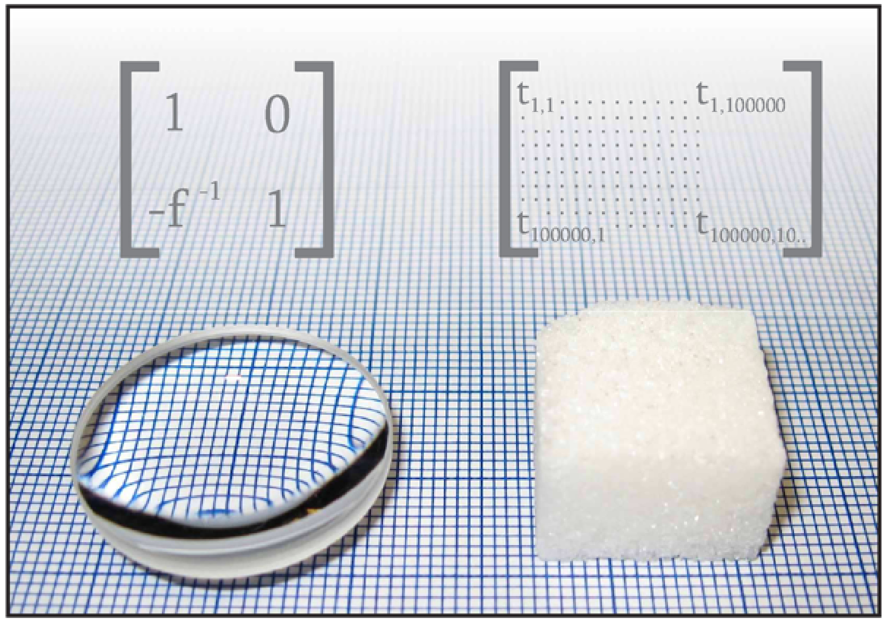







Plöschner, Tyc, Cizmar, **Nat. Photonics** 9, 529 (2015)

# T Measurement: Classical




Spatial Light Modulator (SLM)

Popoff, Lerosey, Carminati, Fink, Boccara, Gigan, **PRL 104,** 100601 (2010)

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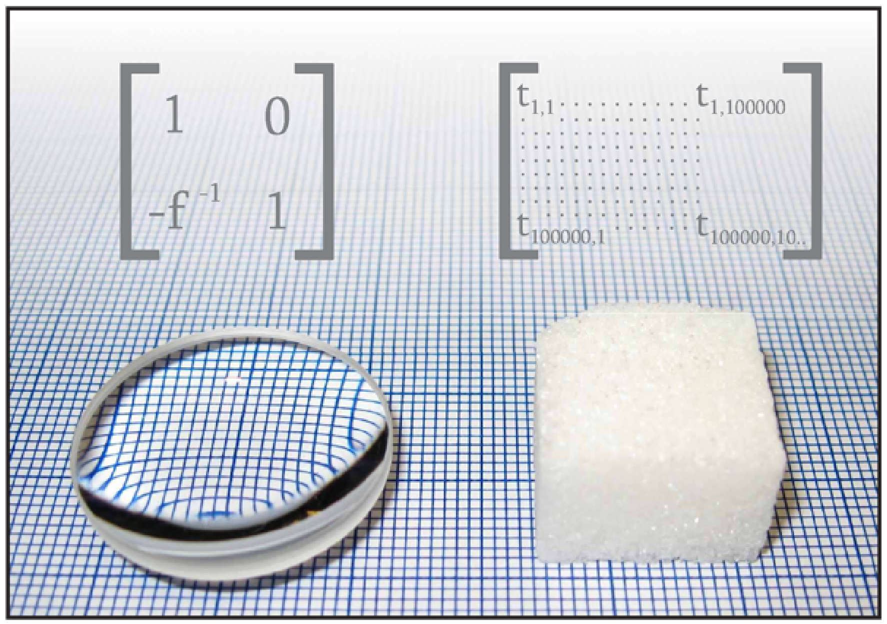


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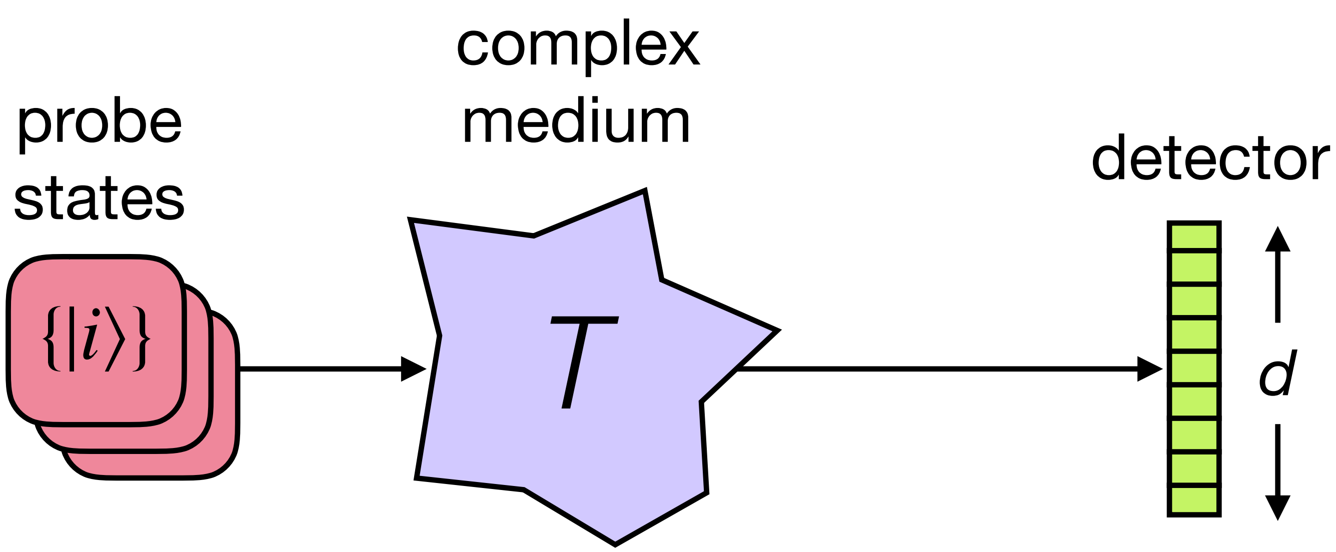


$$T = \sum t_{ij} \, |i\rangle \langle j|$$

$\{|j\rangle\}$

Popoff, Lerosey, Carminati, Fink, Boccara, Gigan, **PRL 104,** 100601 (2010)

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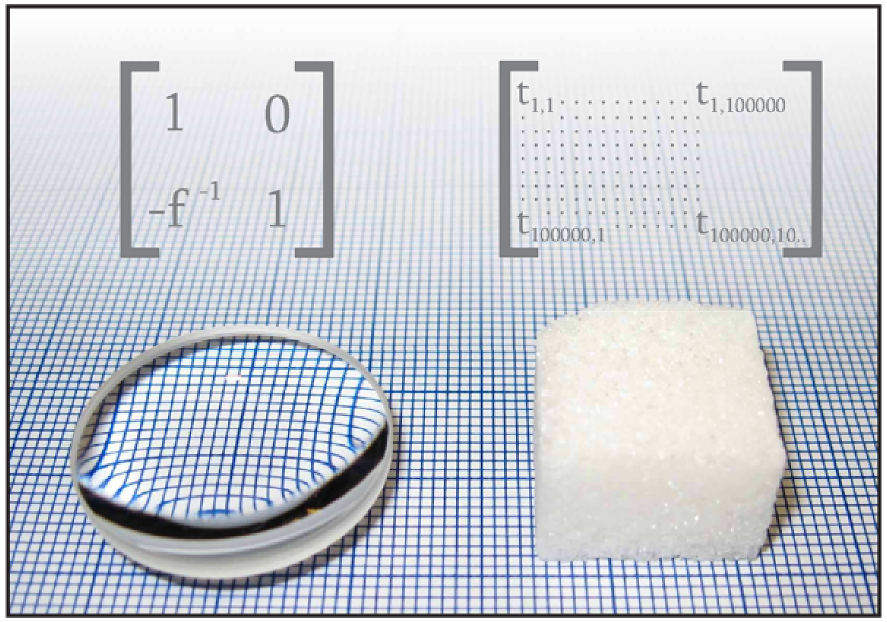



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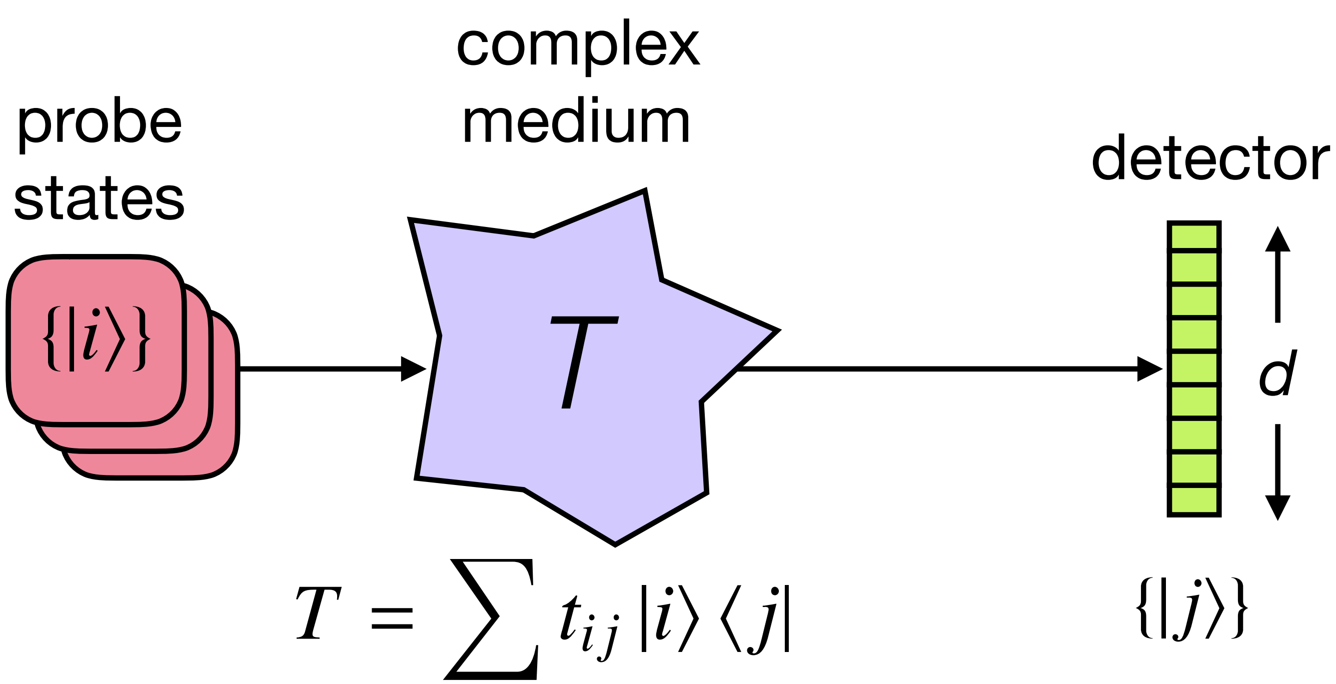


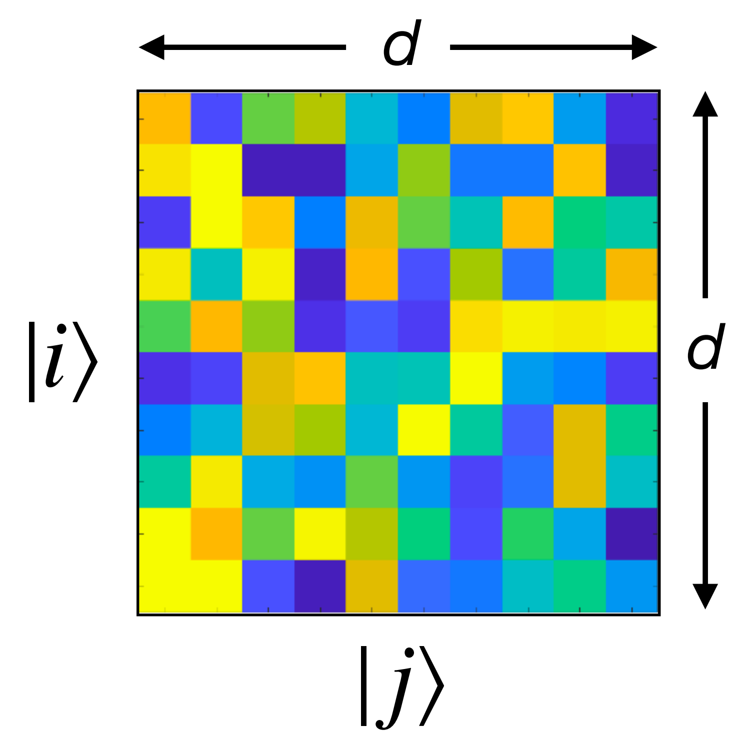


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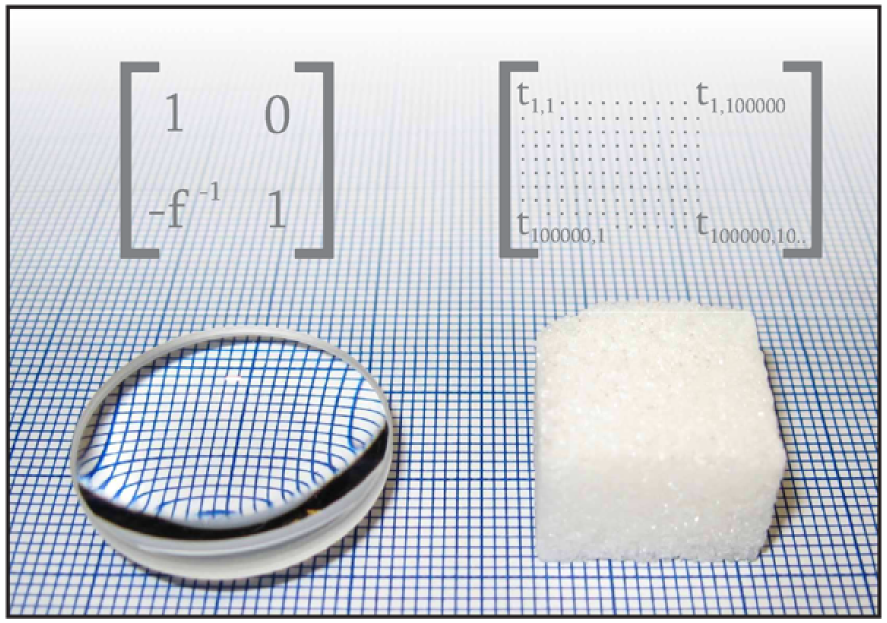
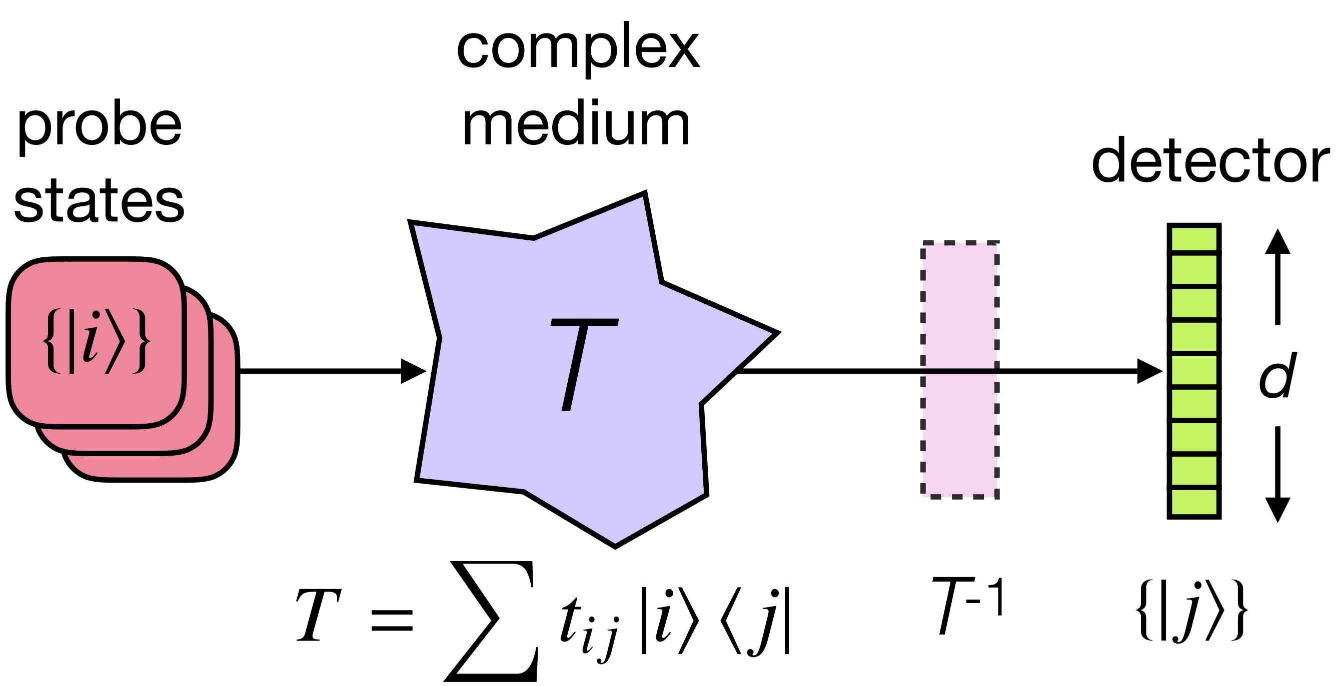
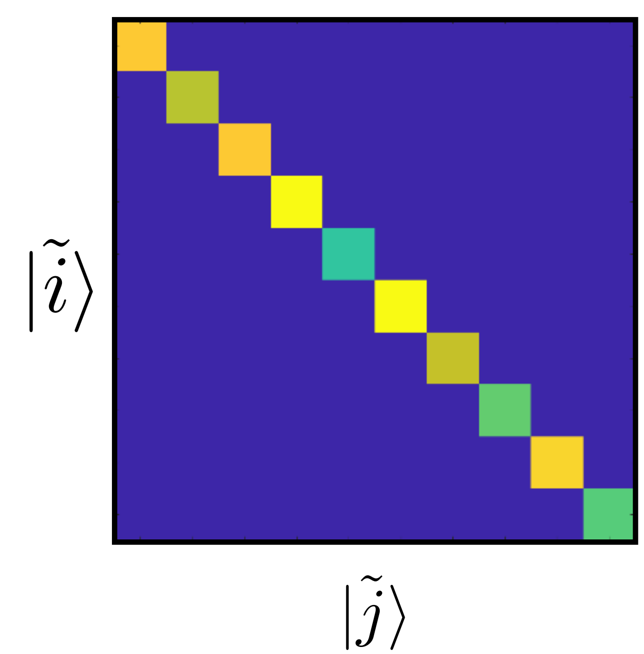



Spatial Light Modulator (SLM)

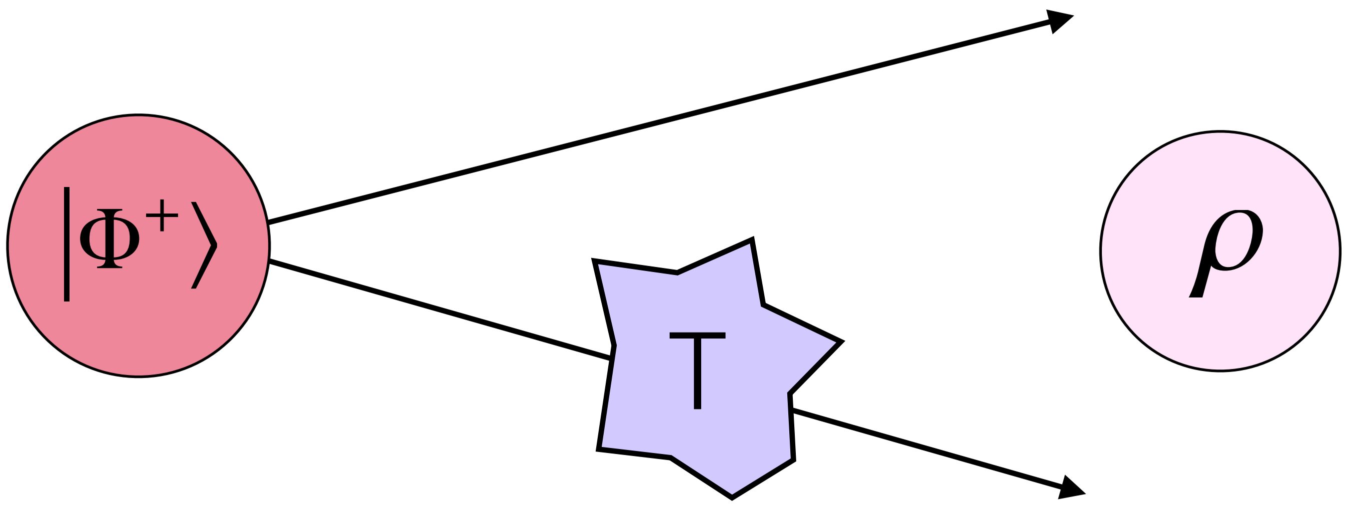



Popoff, Lerosey, Carminati, Fink, Boccara, Gigan, **PRL 104,** 100601 (2010)

# Channel-State Duality

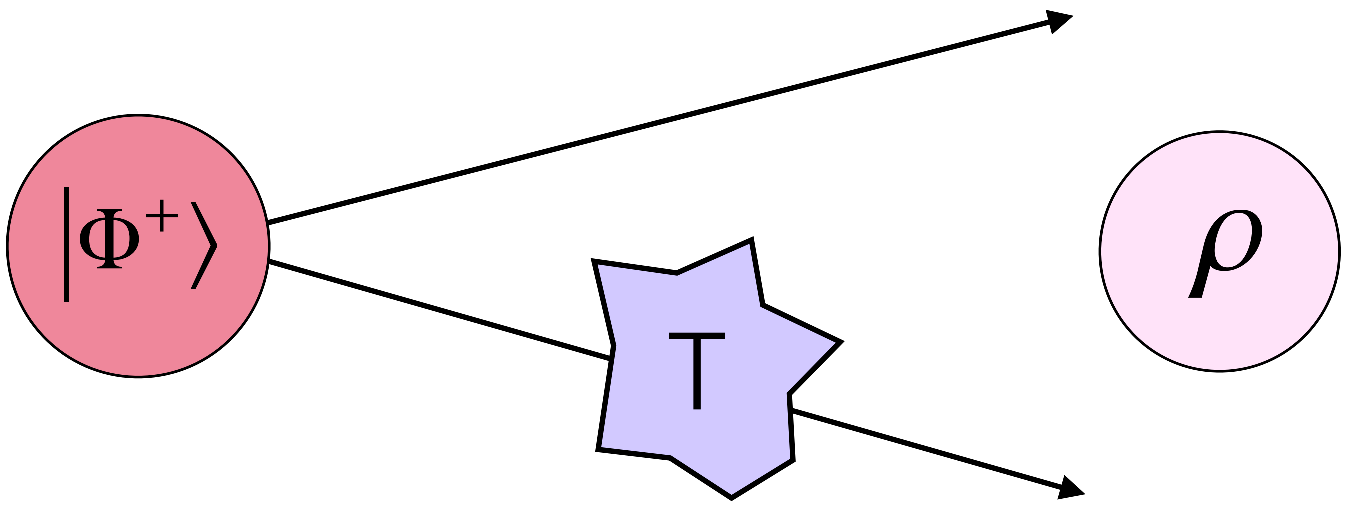
# Channel-State Duality




$$|\Phi^+\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |ii\rangle$$

# Channel-State Duality

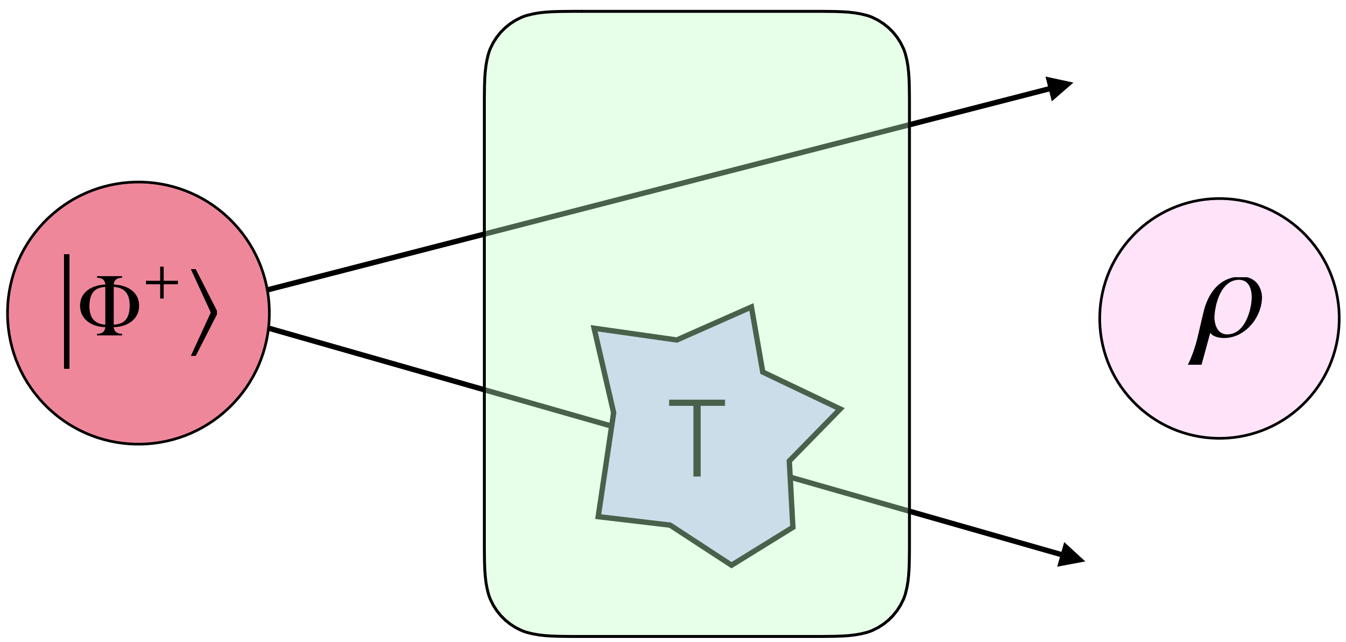
- Statements about **state** ⇔ Statements about <u>*channel*</u>




$$|\Phi^+\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |ii\rangle$$

# CHANNEL-STATE DUALITY

- Statements about **state** ⇔ Statements about *channel*

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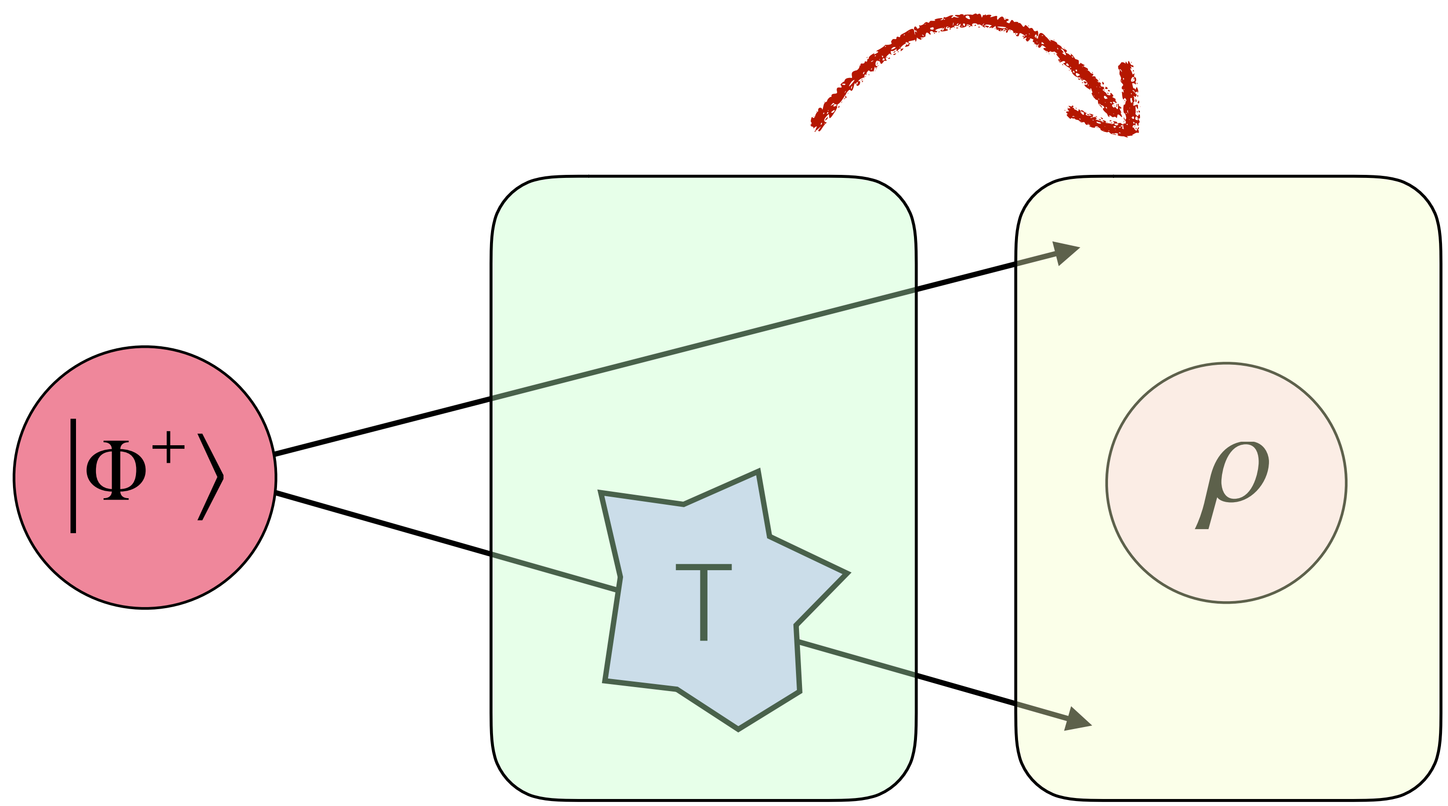
# Channel-State Duality

- Statements about **state** ⇔ Statements about *channel*

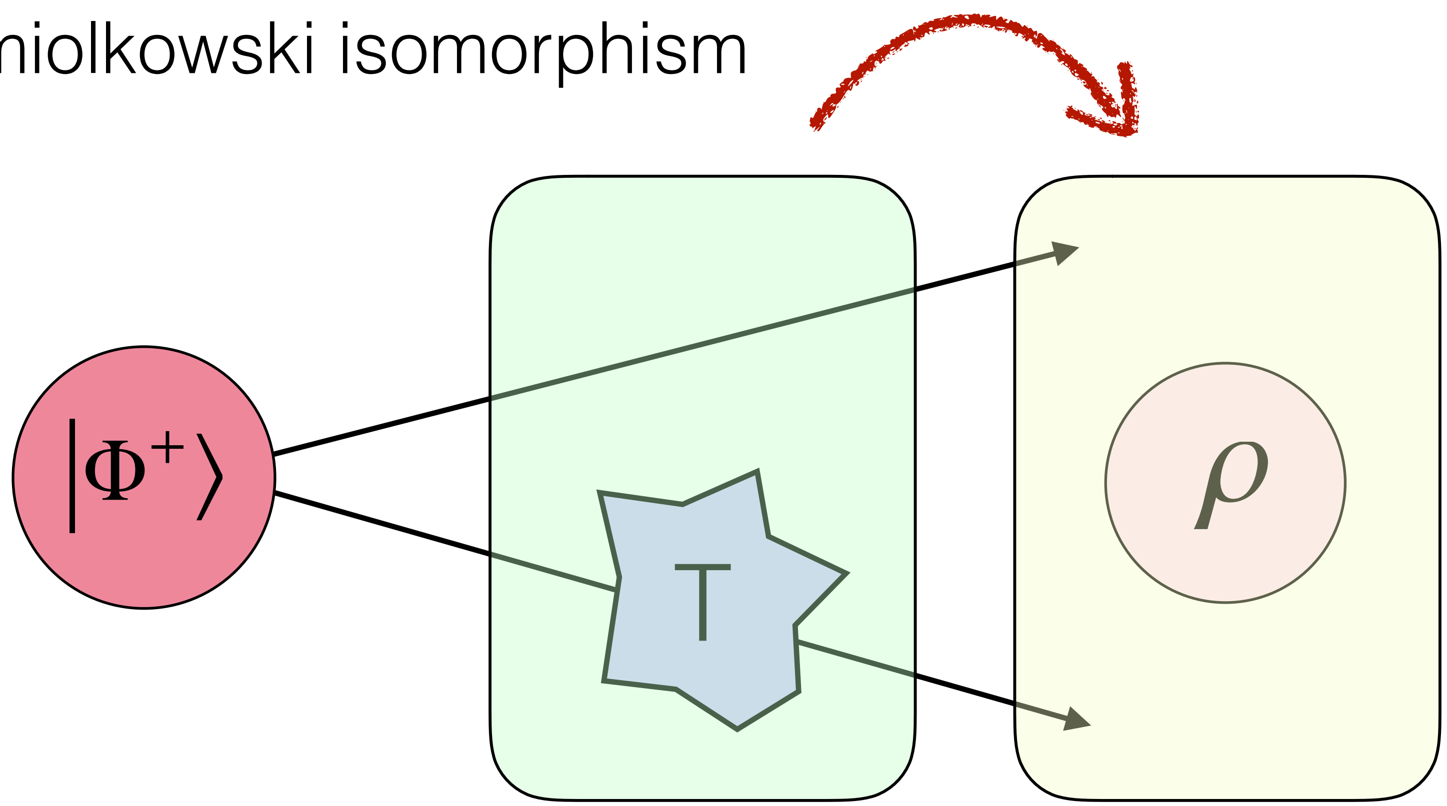



$$|\Phi^+\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |ii\rangle$$

$$\rho = (\mathbb{I} \otimes \mathrm{T}) |\Phi^+\rangle\langle\Phi^+|$$

# CHANNEL-STATE DUALITY

- Statements about **state** ⇔ Statements about <u>*channel*</u>
- Choi-Jamiolkowski isomorphism



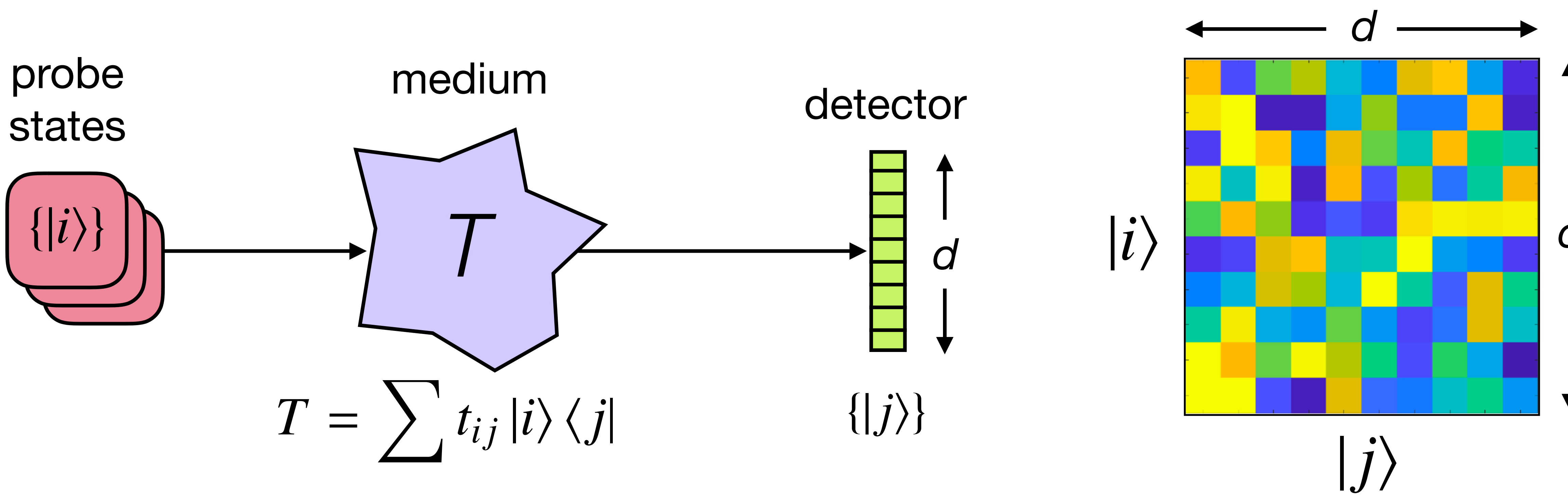

$$\left|\Phi^+\right\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |ii\rangle$$

$$\rho = (\mathbb{I} \otimes \mathrm{T}) \left|\Phi^+\right\rangle \left\langle\Phi^+\right|$$

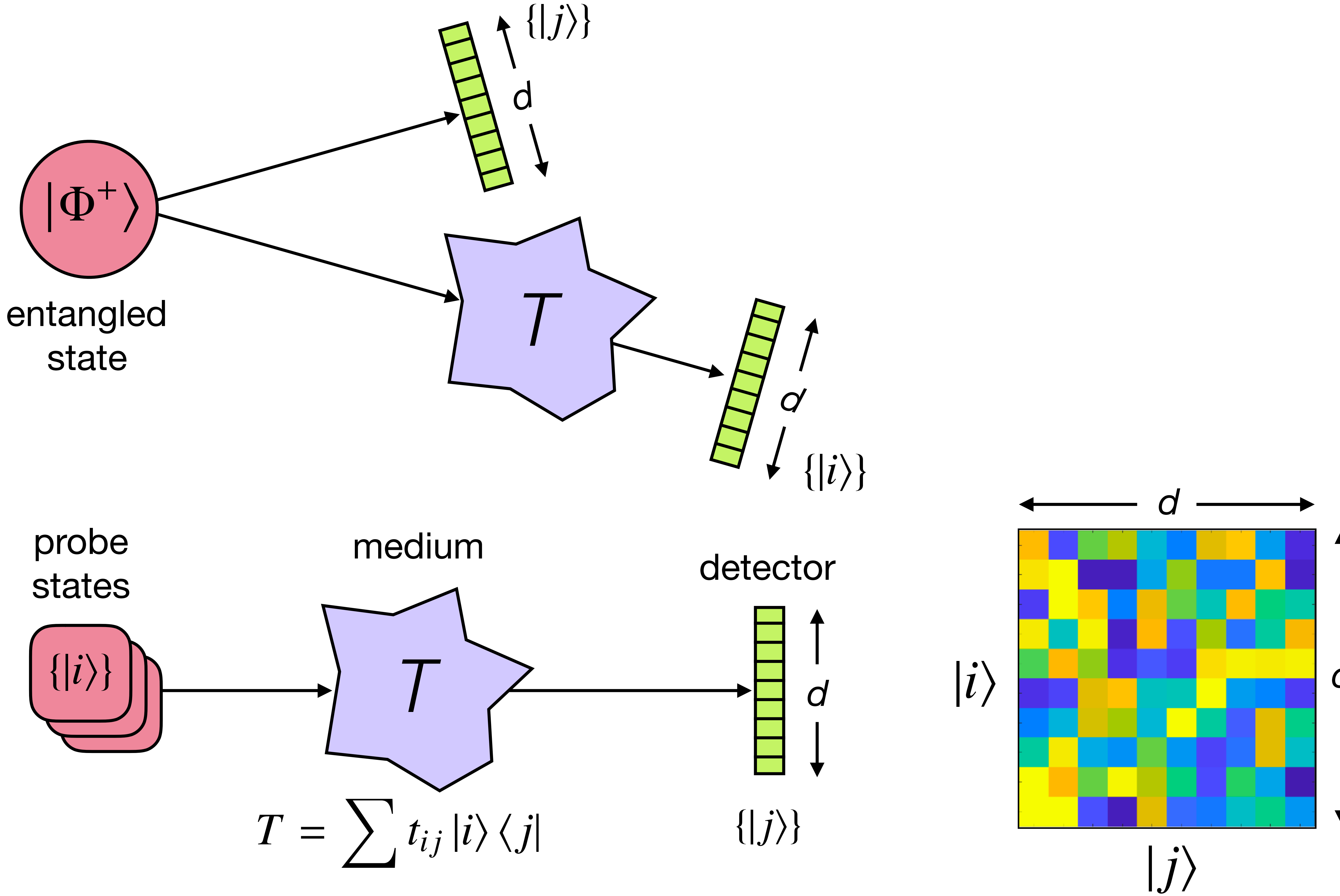
M.D. Choi, **Lin. Alg. Appl. 10**, 285 (1975)

A. Jamiołkowski, **Rep. Math. Phys. 3**, 275 (1972)

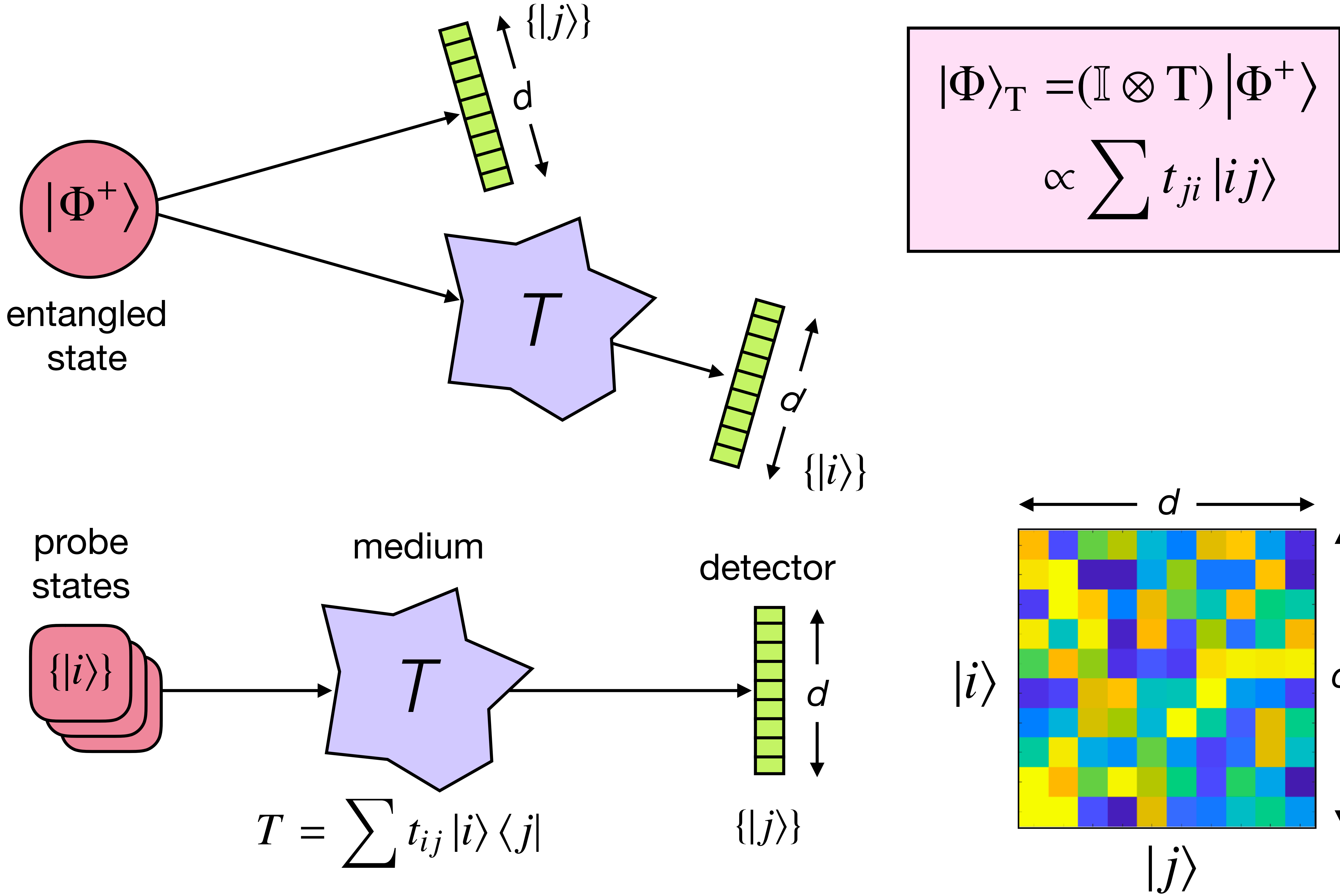
# T Measurement: Quantum



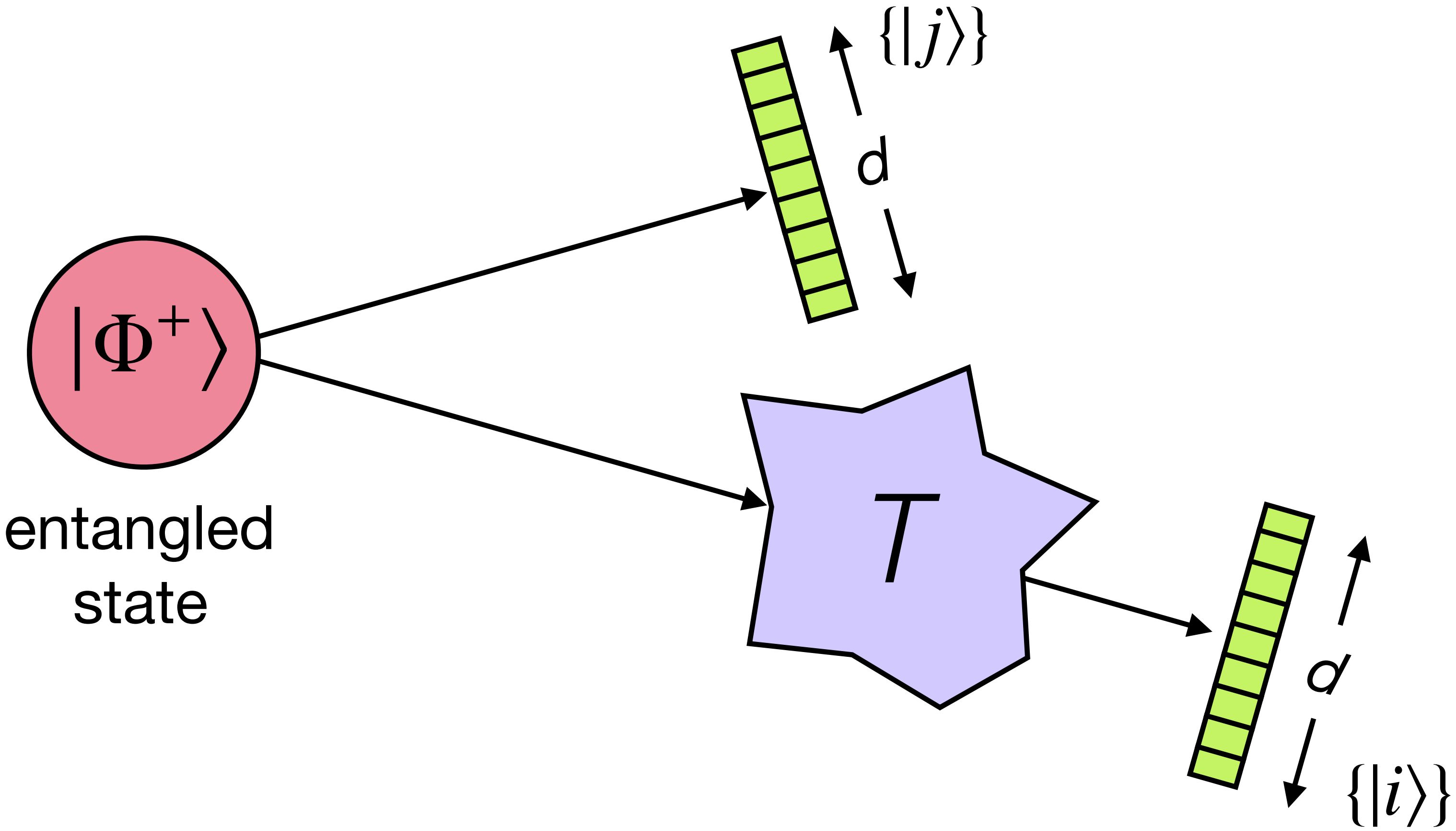
# T Measurement: Quantum



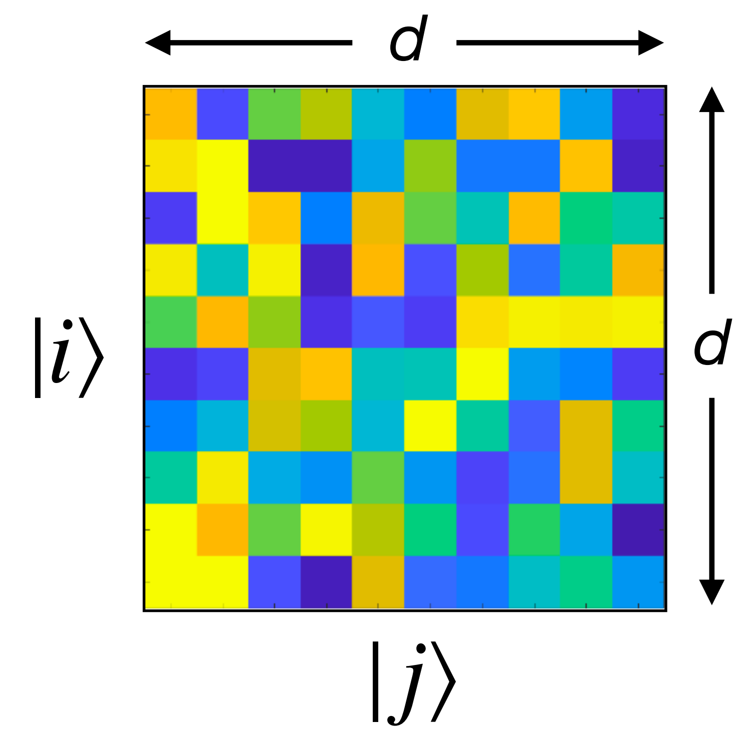
# T MEASUREMENT: QUANTUM



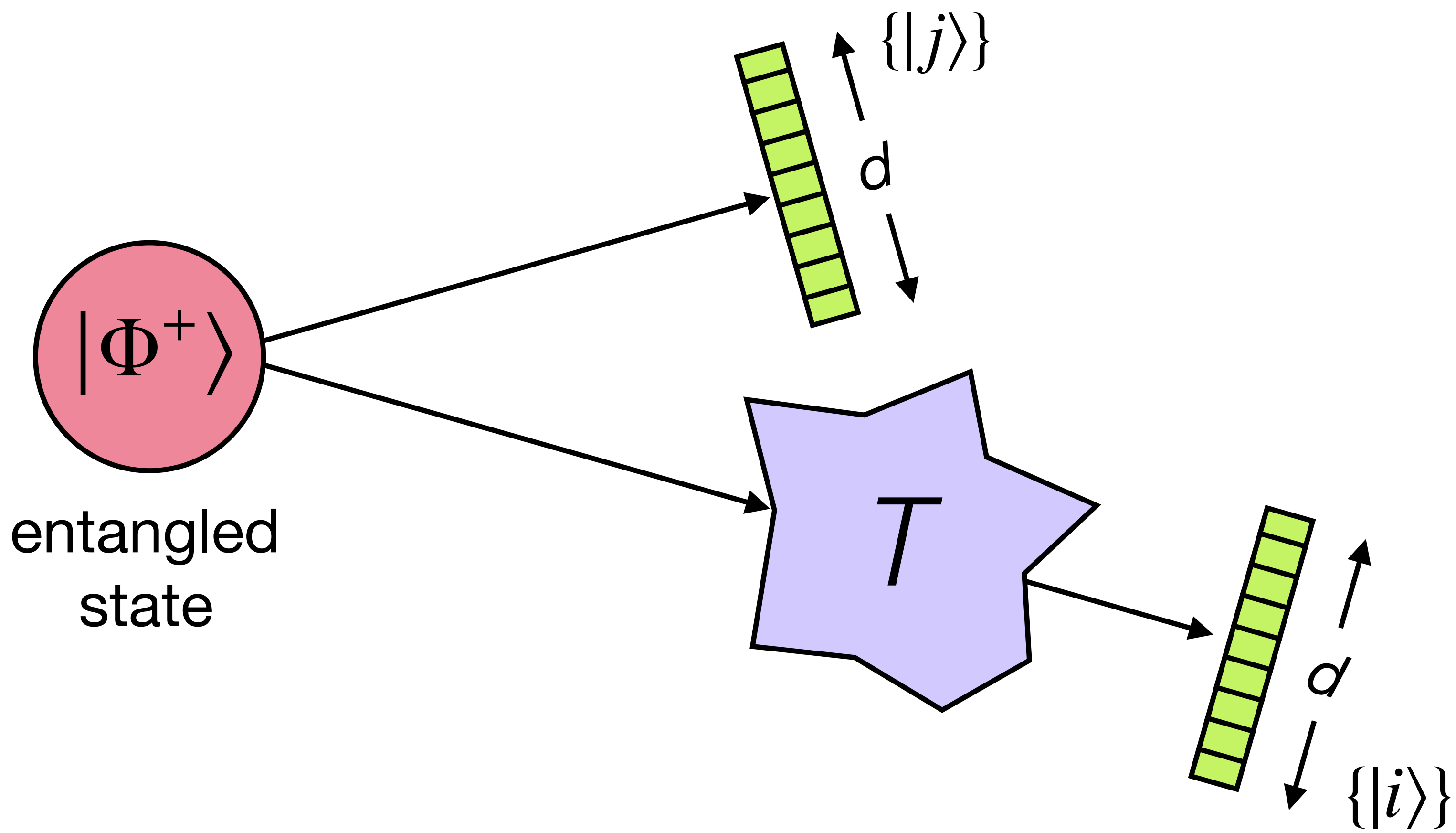
# T Inversion: Unscrambling Bob



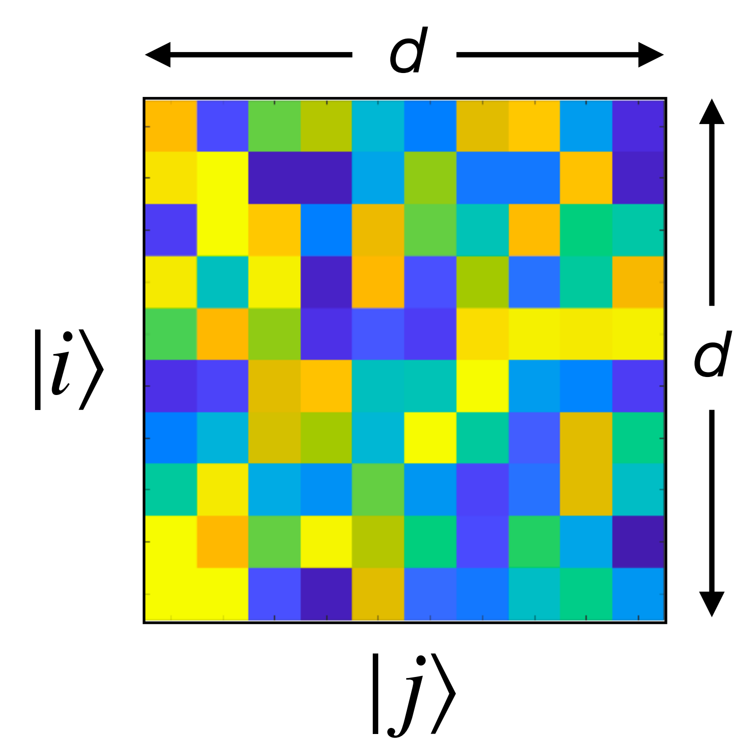

$$|\Phi\rangle_{\mathrm{T}} = (\mathbb{I} \otimes \mathrm{T})\,|\Phi^+\rangle$$
$$\propto \sum t_{ji}\,|i\,j\rangle$$



# T Inversion: Unscrambling Bob

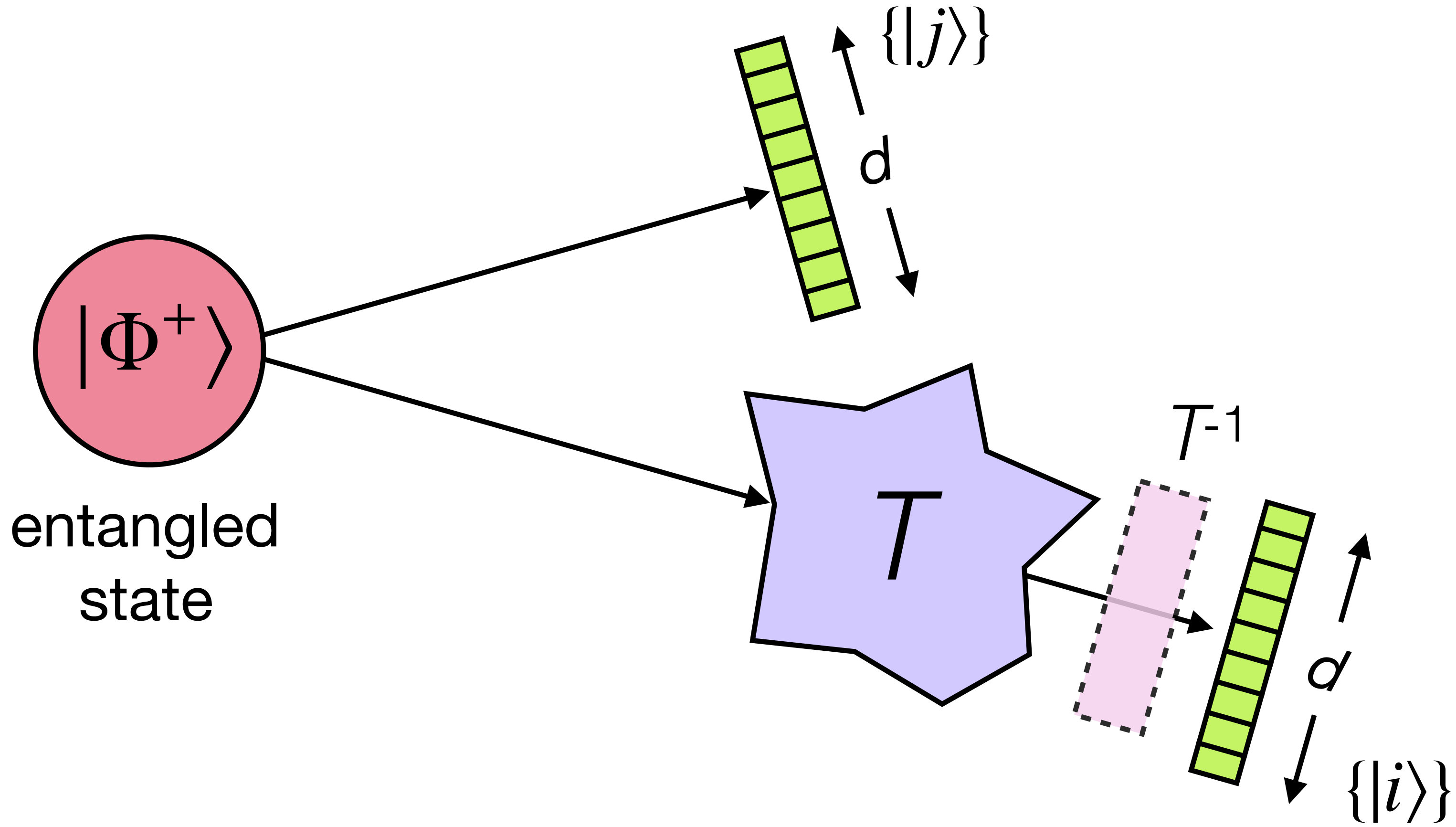



$$|\Phi\rangle_T = (\mathbb{I}\otimes T)\,|\Phi^+\rangle \propto \sum t_{ji}\,|ij\rangle$$




$$|\Phi\rangle_T = (\mathbb{I}\otimes T^{-1}T)\,|\Phi^+\rangle = |\Phi^+\rangle$$

# T Inversion: Unscrambling Bob

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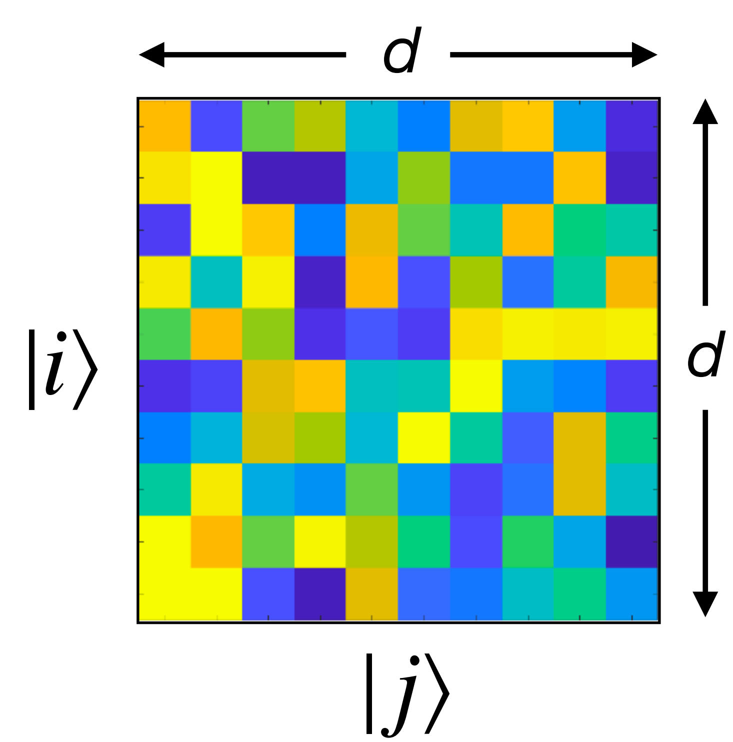
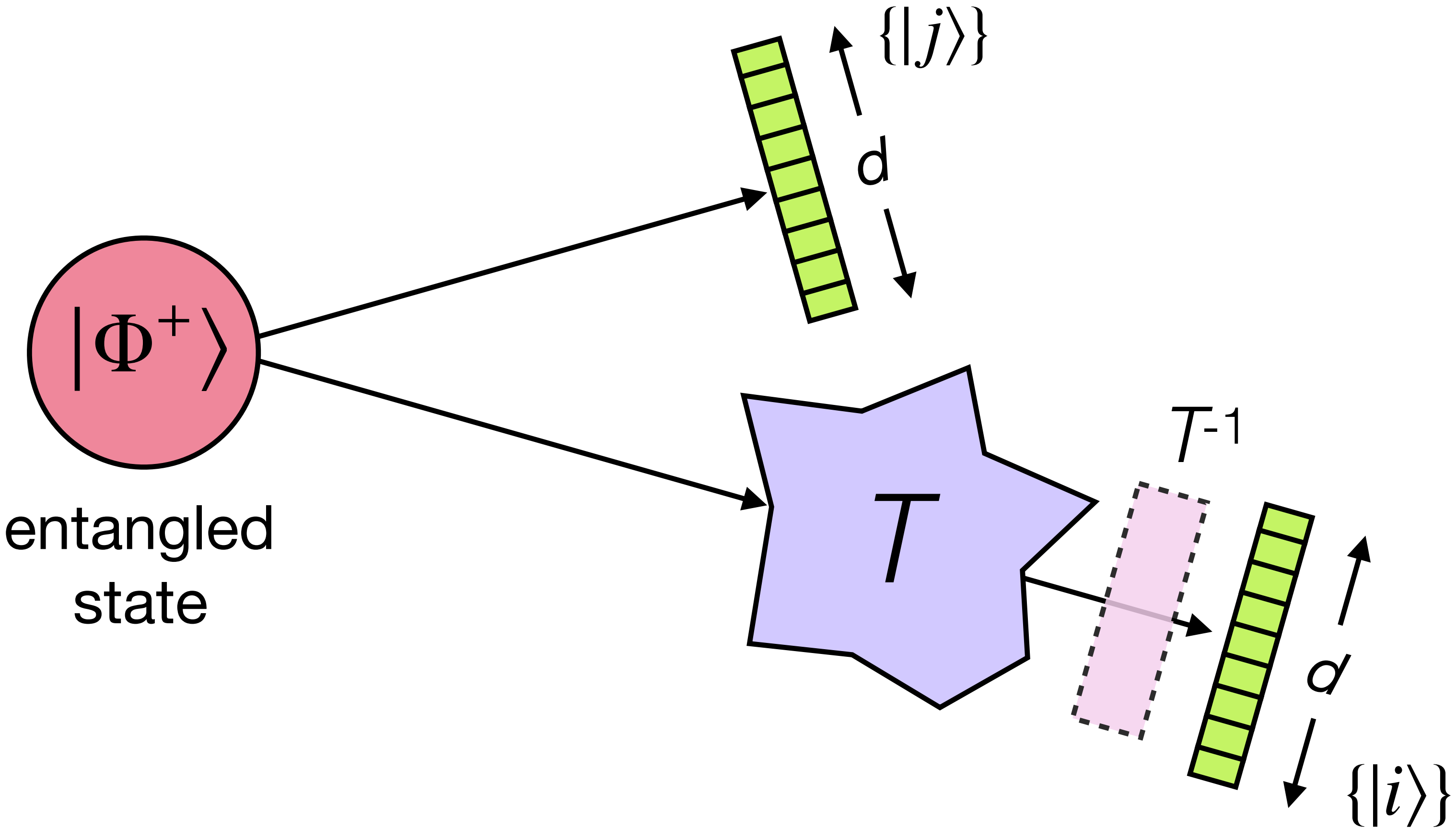
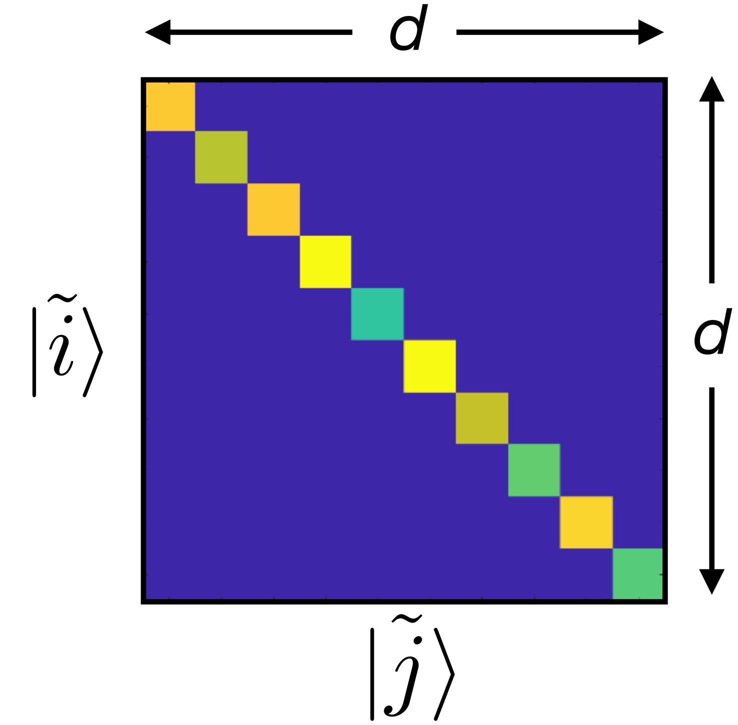


# T Inversion: Unscrambling Bob

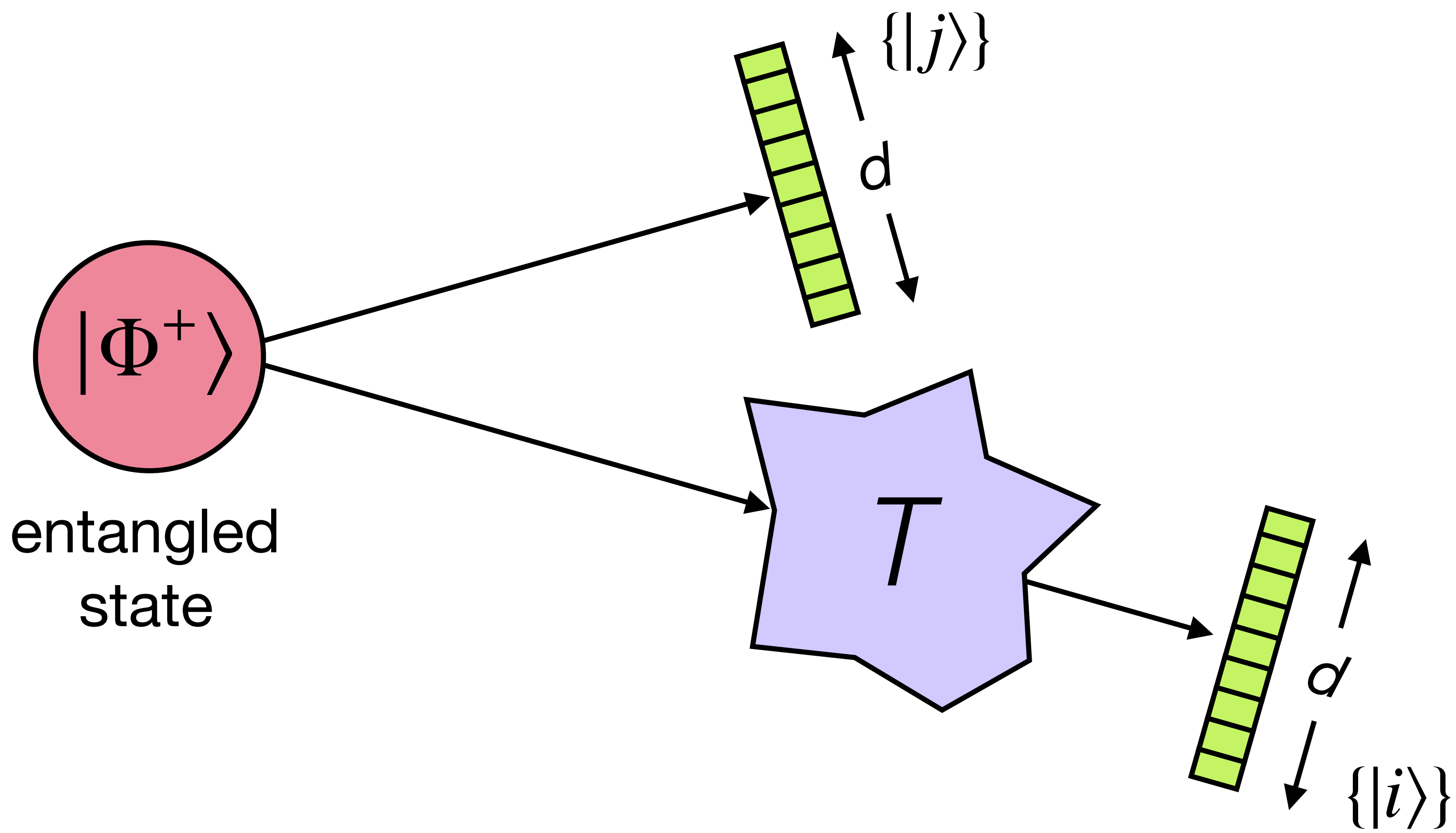



$$|\Phi\rangle_{\mathrm{T}} = (\mathbb{I} \otimes \mathrm{T})\,|\Phi^+\rangle$$
$$\propto \sum t_{ji}\,|i\,j\rangle$$

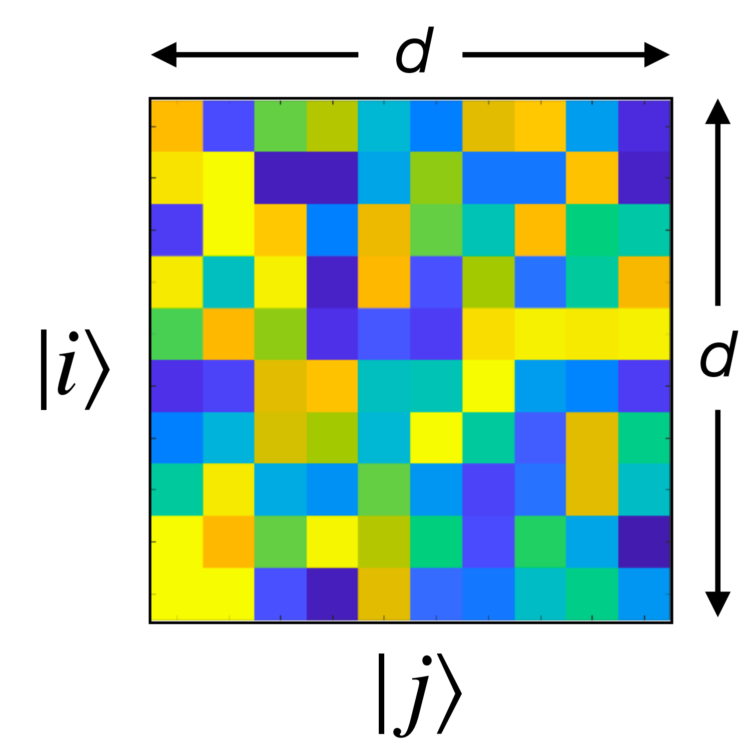
$$|\Phi\rangle_{\mathrm{T}} = (\mathbb{I} \otimes \mathrm{T}^{-1}\mathrm{T})\,|\Phi^+\rangle = |\Phi^+\rangle$$



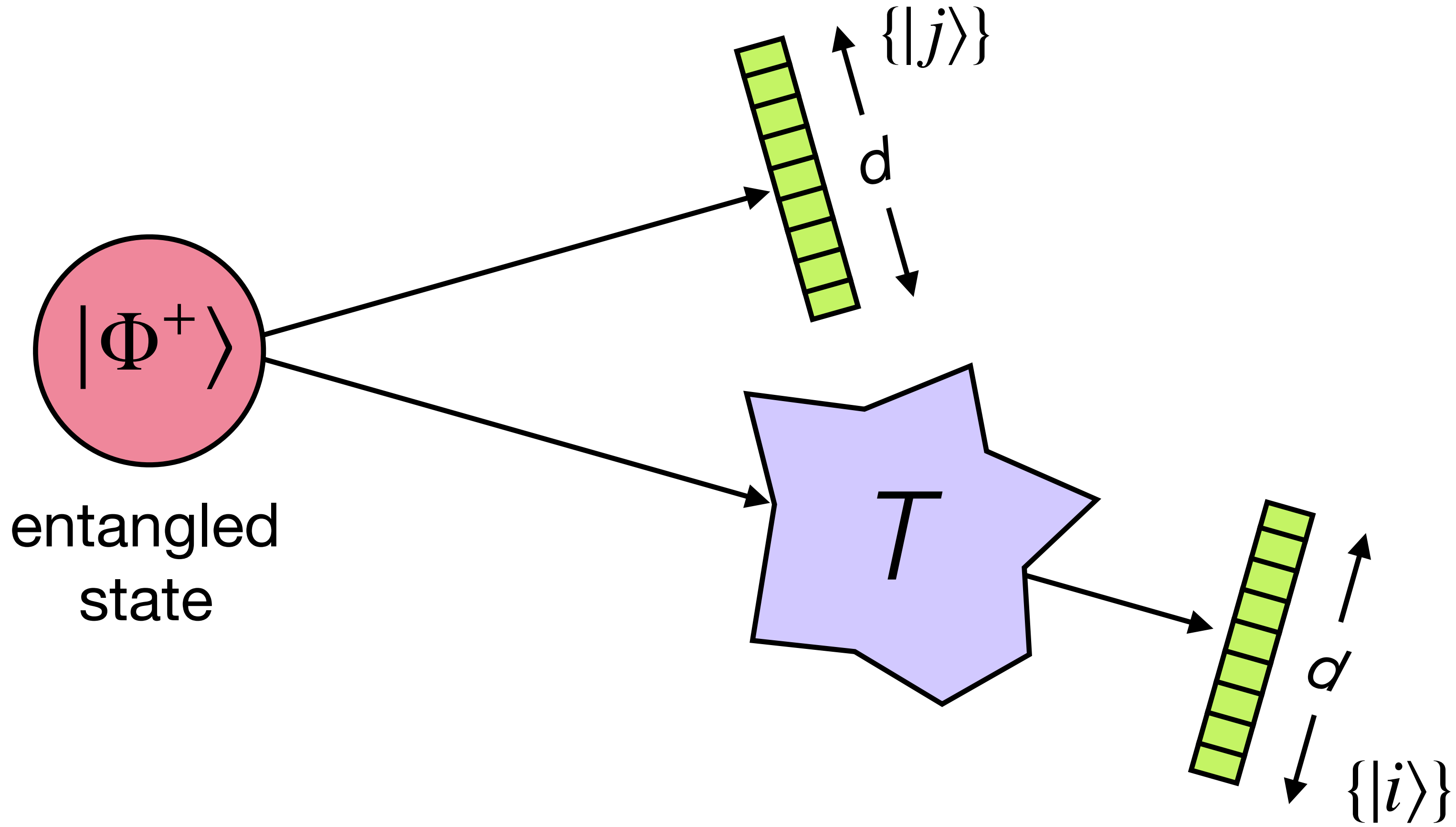
# T Inversion: "Scrambling" Alice




$$
\begin{aligned}
|\Phi\rangle_{\mathrm{T}} &= (\mathbb{I} \otimes \mathrm{T})\,|\Phi^+\rangle \\
&\propto \sum t_{ji}\,|i\,j\rangle
\end{aligned}
$$

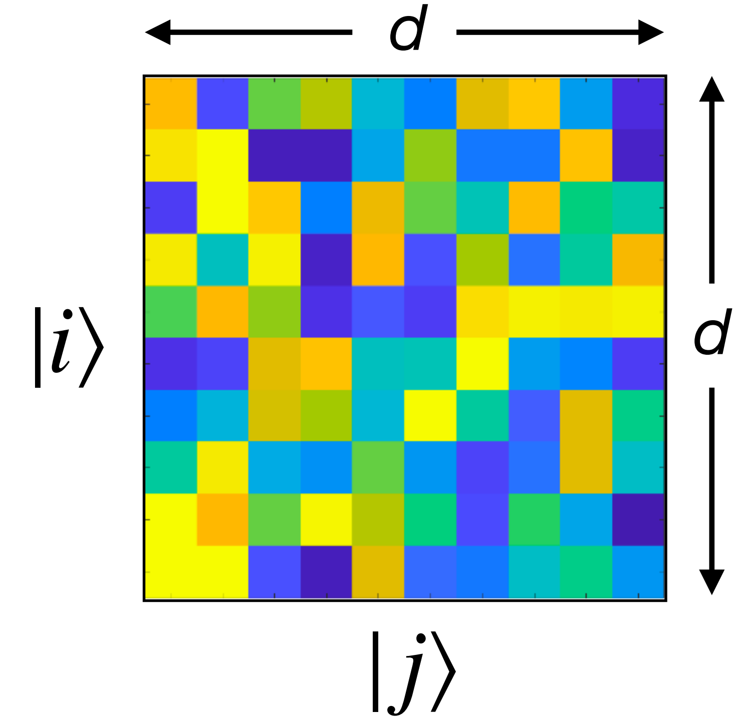


# T Inversion: "Scrambling" Alice

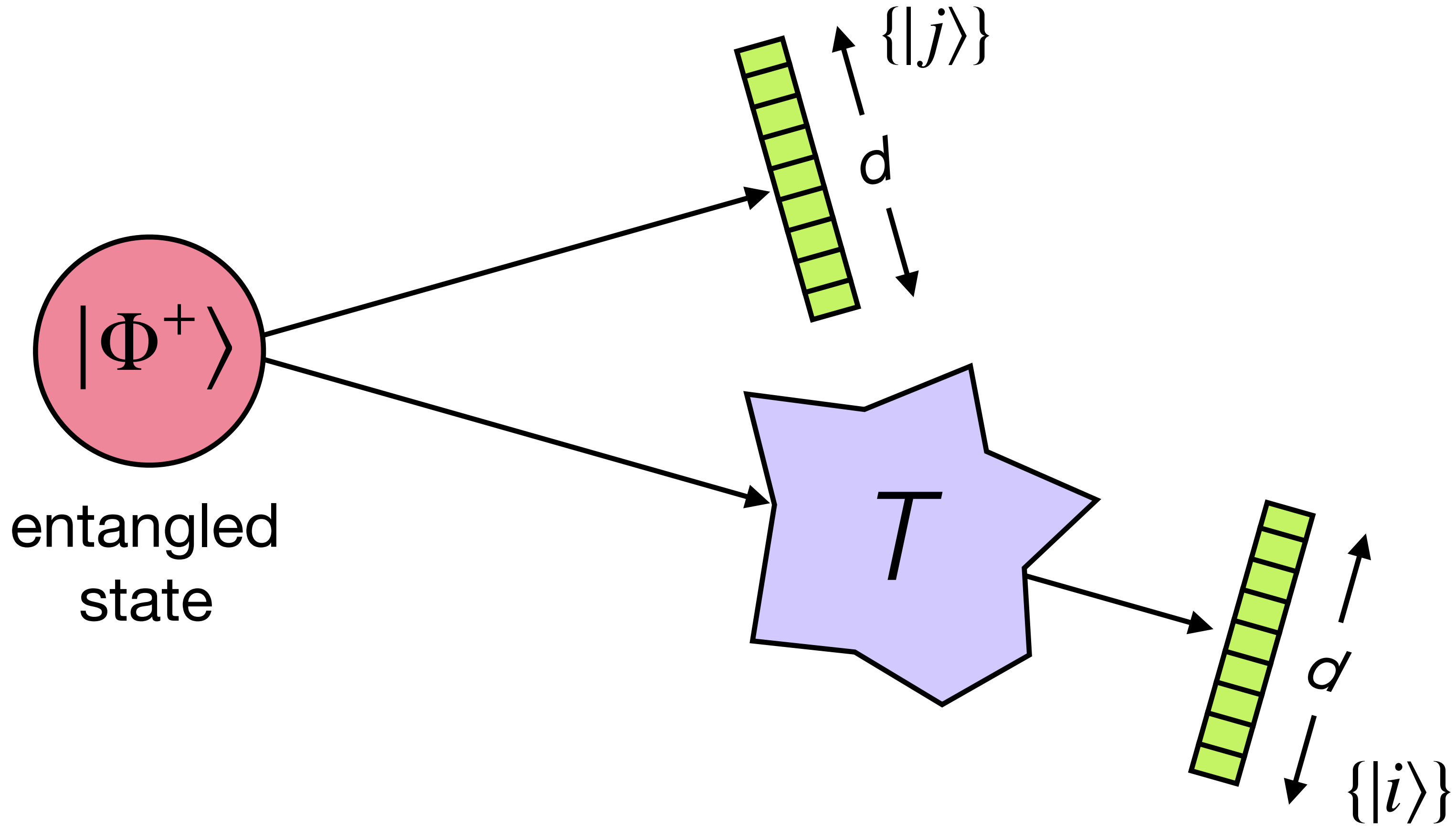



$$|\Phi\rangle_{\mathrm{T}} = (\mathbb{I} \otimes \mathrm{T}) \left|\Phi^+\right\rangle \propto \sum t_{ji} \, |i\, j\rangle$$

$$(A \otimes B) \left|\Phi^+\right\rangle = (B^T A \otimes I) \left|\Phi^+\right\rangle = (I \otimes BA^T) \left|\Phi^+\right\rangle$$



# T Inversion: "Scrambling" Alice




$$|\Phi\rangle_{\mathrm{T}} = (\mathbb{I} \otimes \mathrm{T})\left|\Phi^+\right\rangle$$
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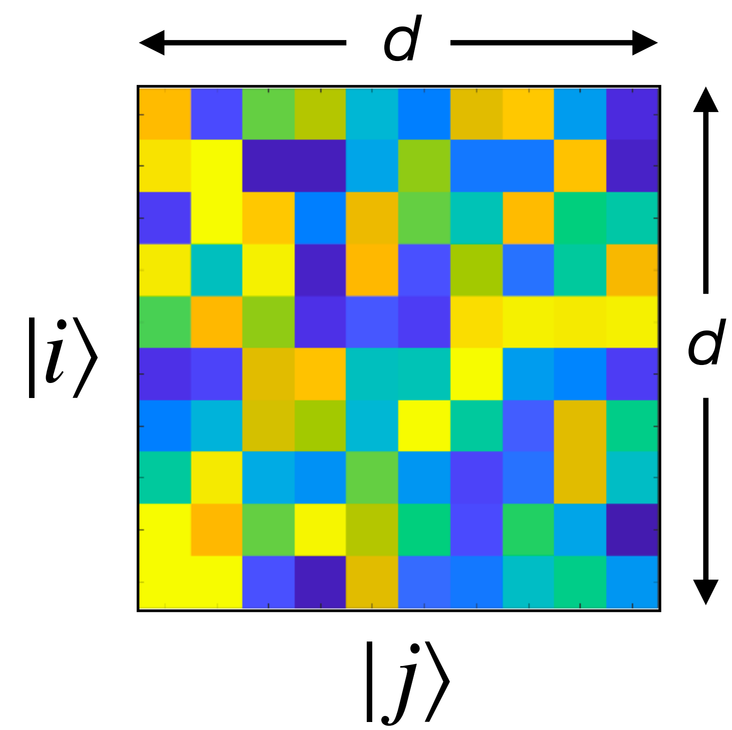
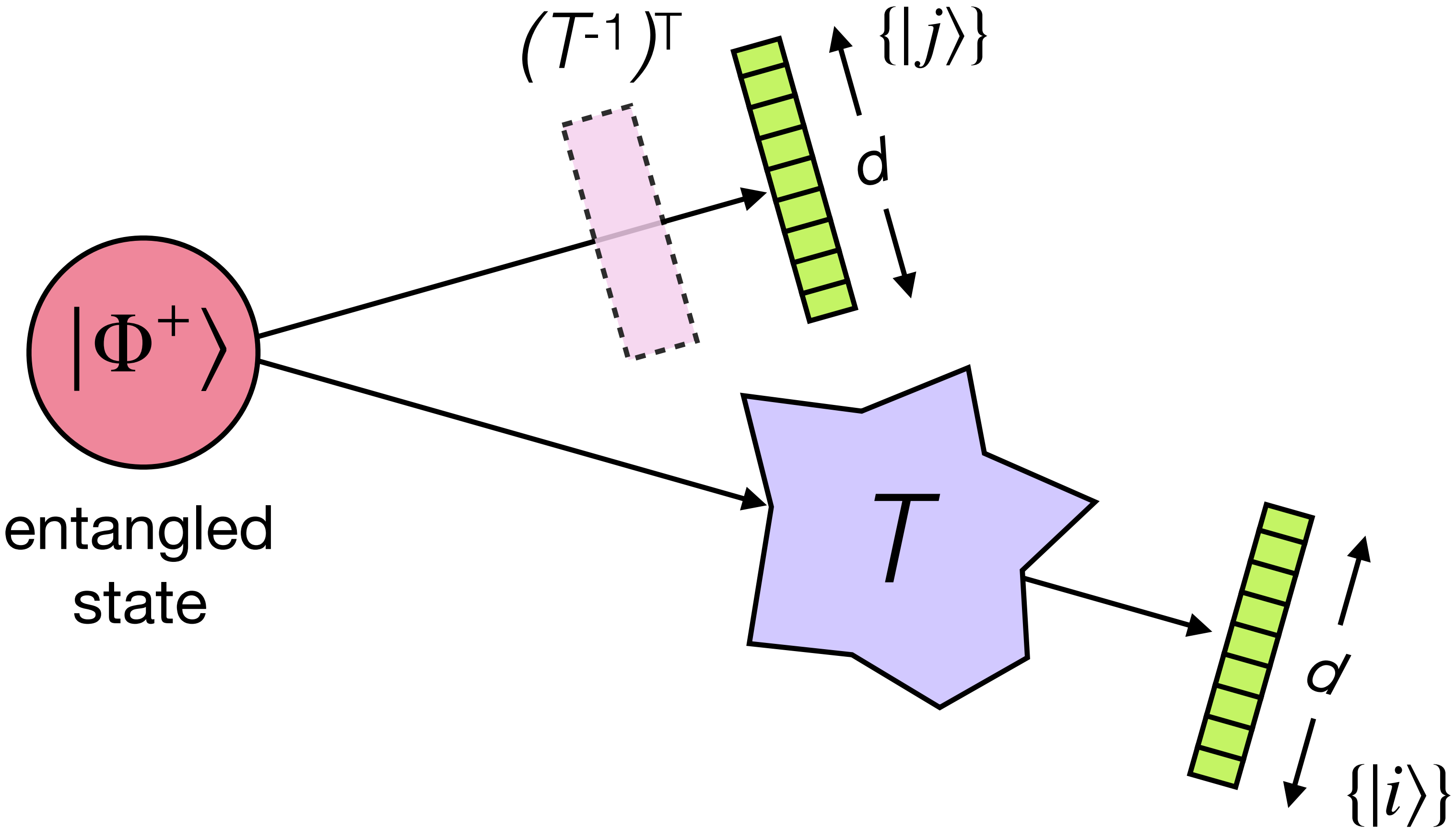



$$|\Phi\rangle_{\mathrm{T}} = ((\mathrm{T}^{-1})^{\mathrm{T}} \otimes \mathrm{T})\left|\Phi^+\right\rangle = \left|\Phi^+\right\rangle$$

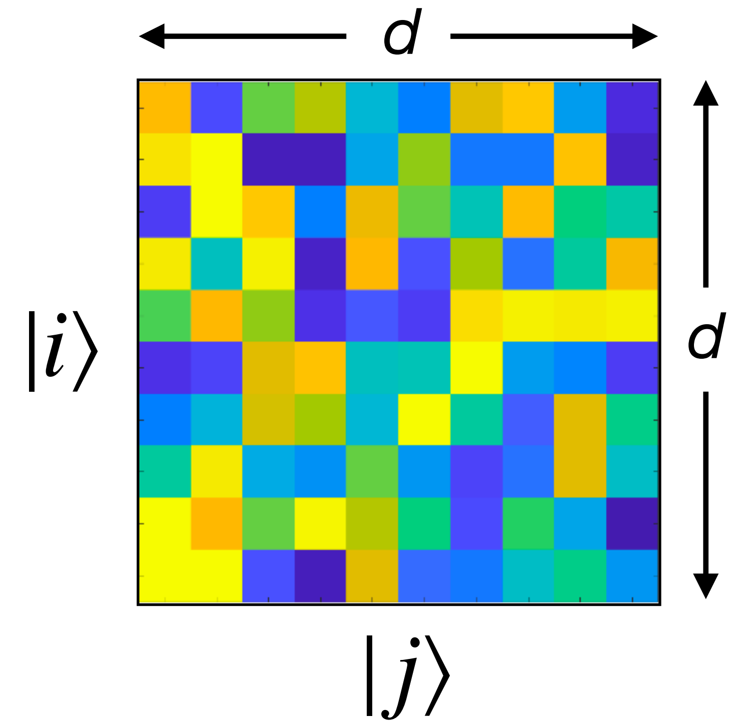
# T INVERSION: "SCRAMBLING" ALICE



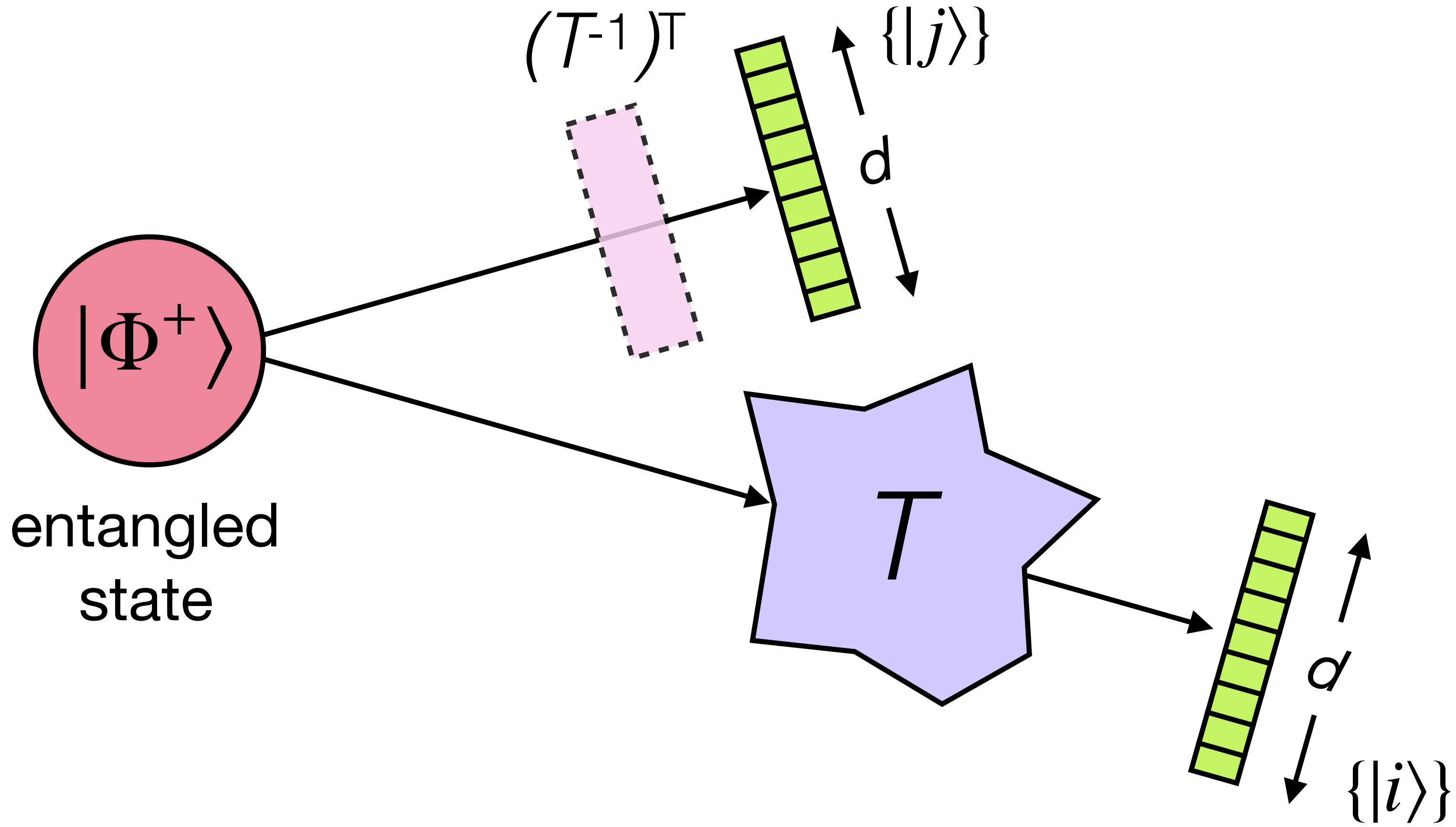

$$|\Phi\rangle_{T} = (\mathbb{I} \otimes T)\,|\Phi^{+}\rangle \propto \sum t_{ji}\,|ij\rangle$$

$$(A \otimes B)\,|\Phi^{+}\rangle = (B^{T}A \otimes I)\,|\Phi^{+}\rangle = (I \otimes BA^{T})\,|\Phi^{+}\rangle$$

$$|\Phi\rangle_{T} = ((T^{-1})^{T} \otimes T)\,|\Phi^{+}\rangle = |\Phi^{+}\rangle$$



# T INVERSION: "SCRAMBLING" ALICE




$$|\Phi\rangle_{\mathrm{T}} = (\mathbb{I} \otimes \mathrm{T}) \left|\Phi^+\right\rangle \propto \sum t_{ji} \, |ij\rangle$$

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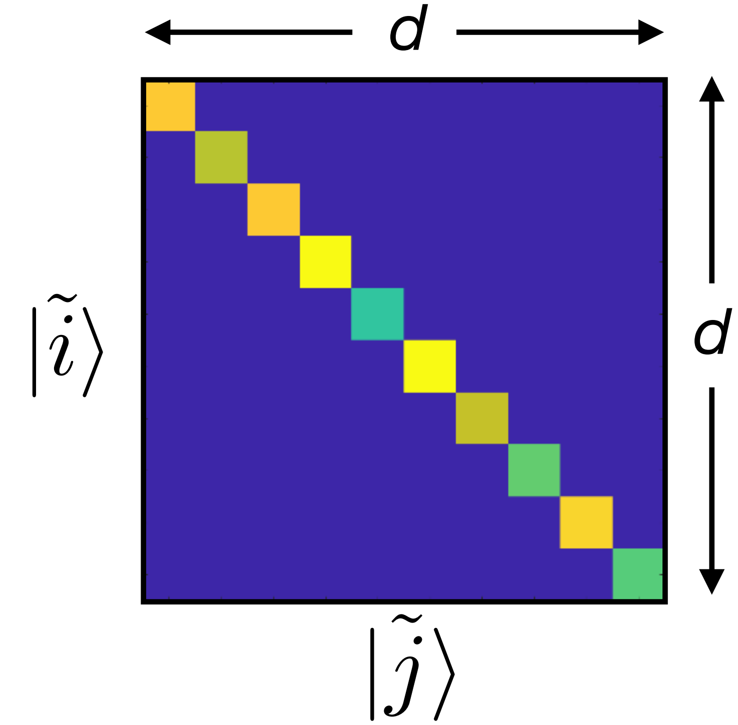
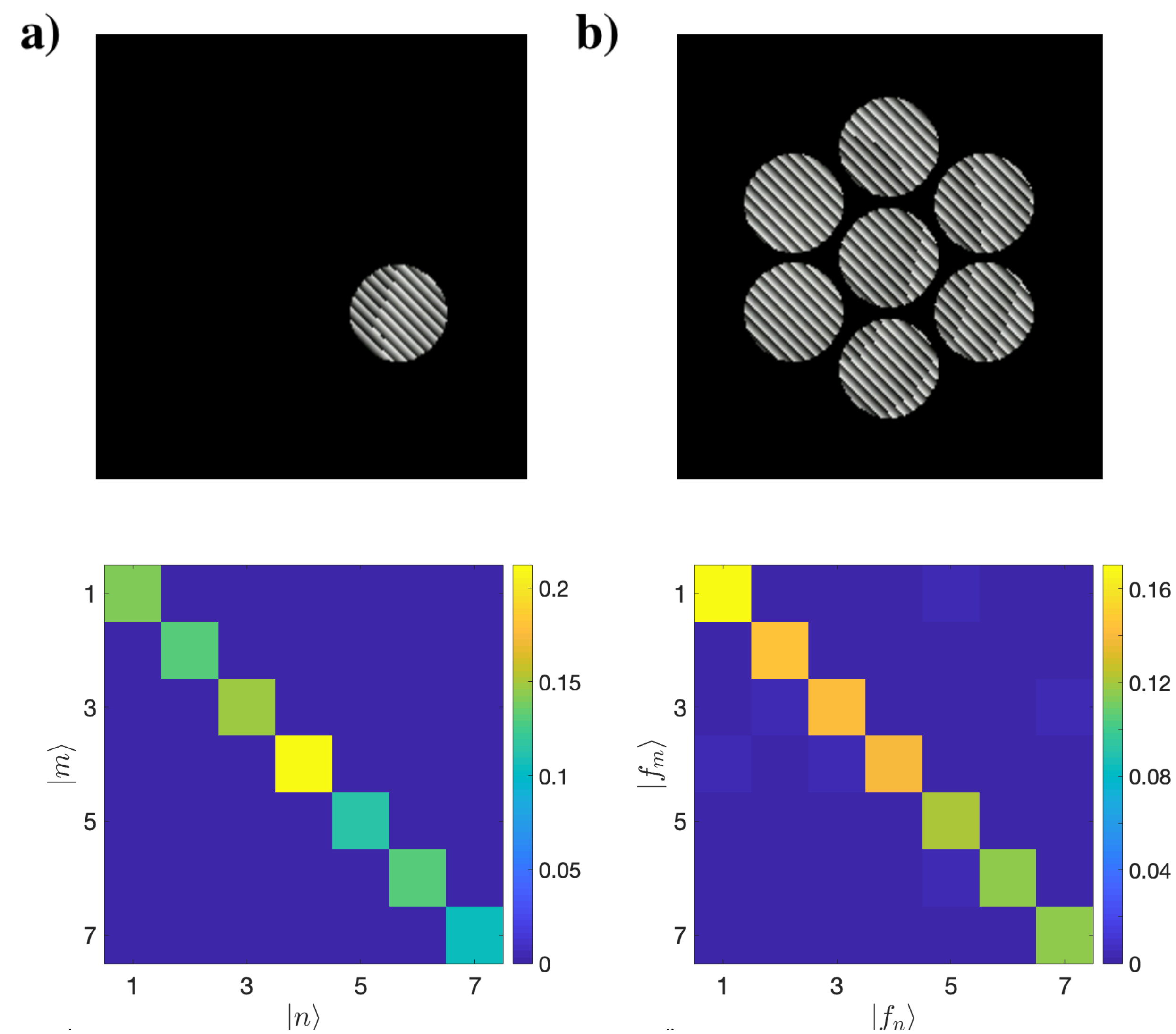
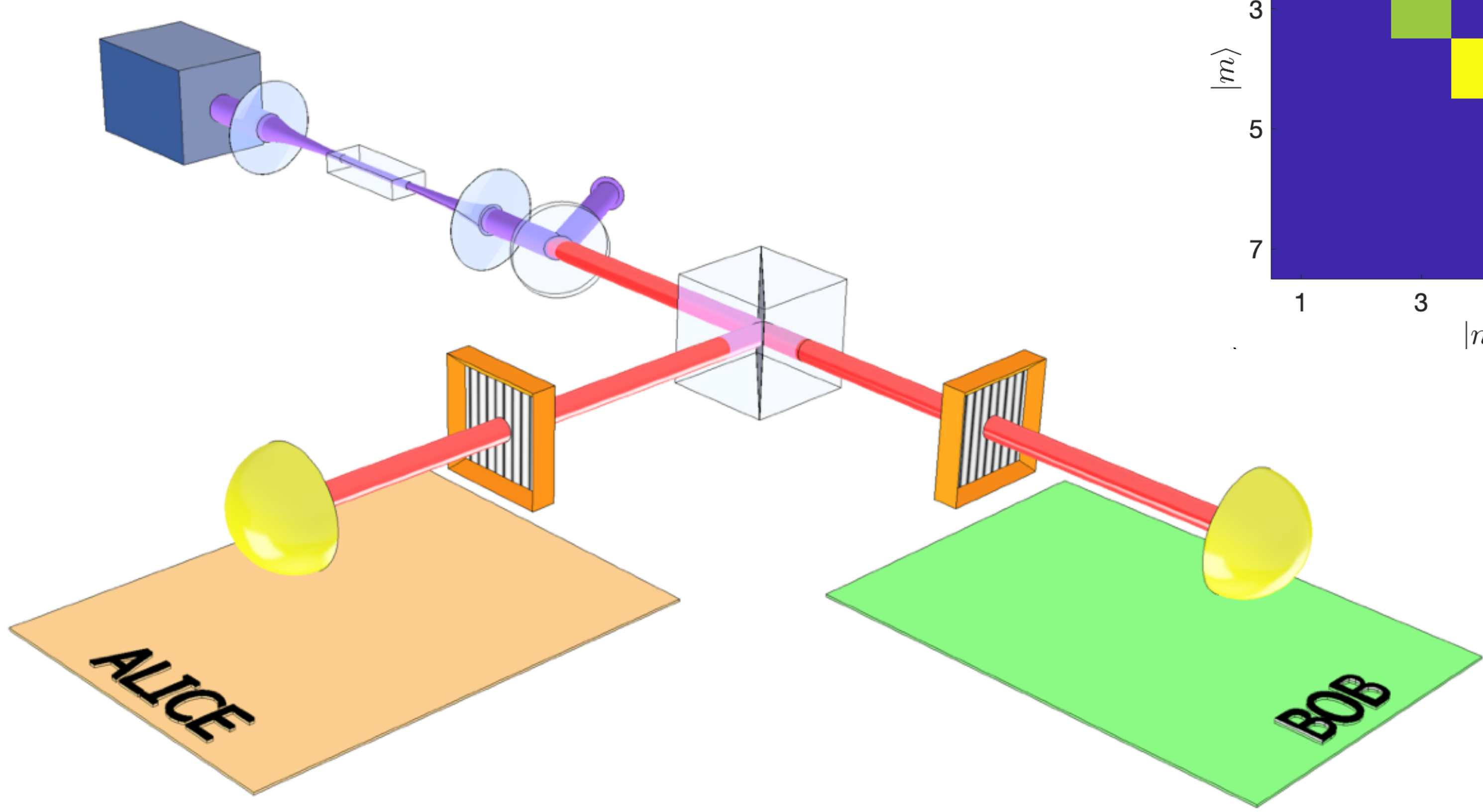


# INITIAL STATE

$$\left|\Phi^{+}\right\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |ii\rangle$$

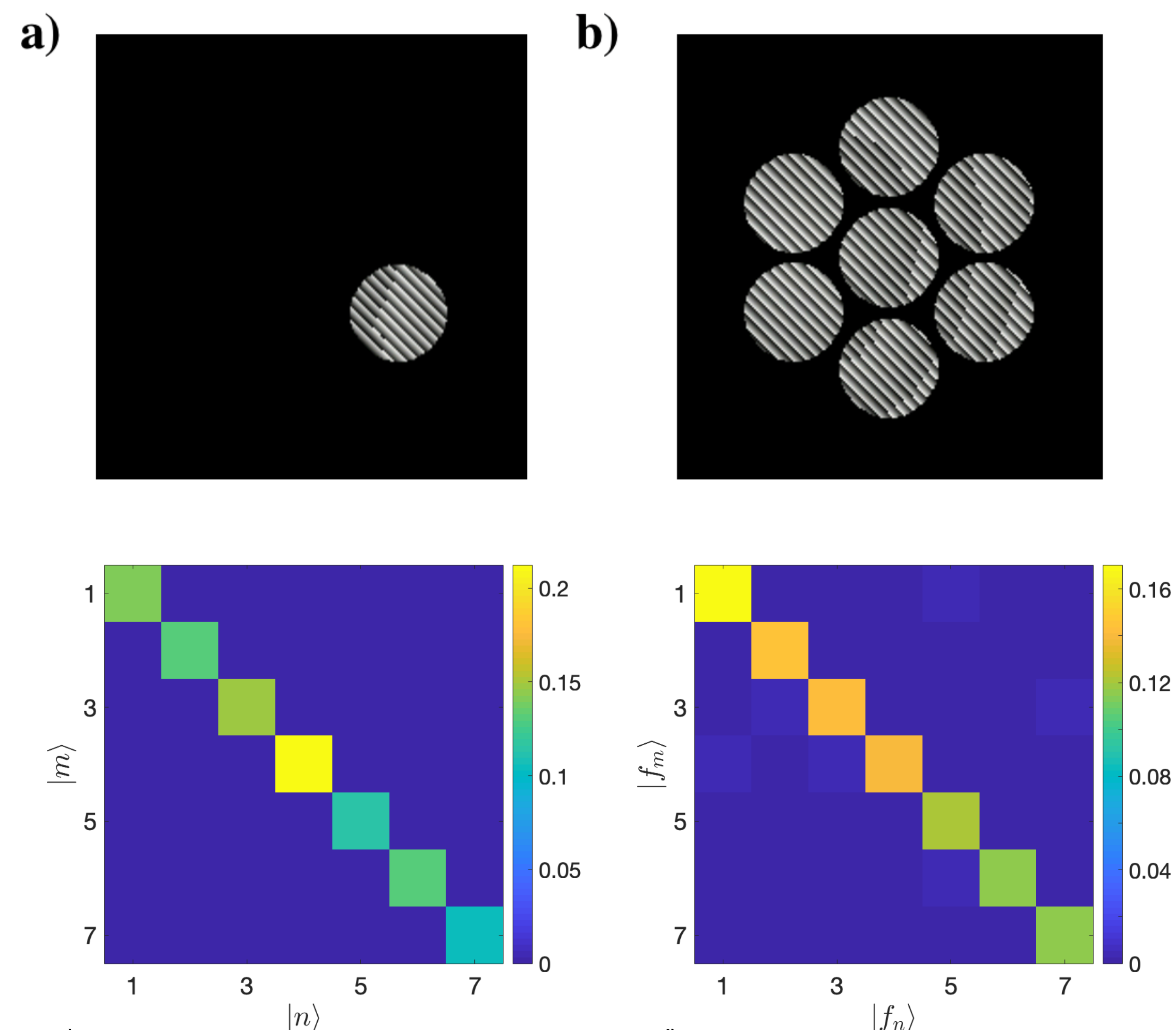


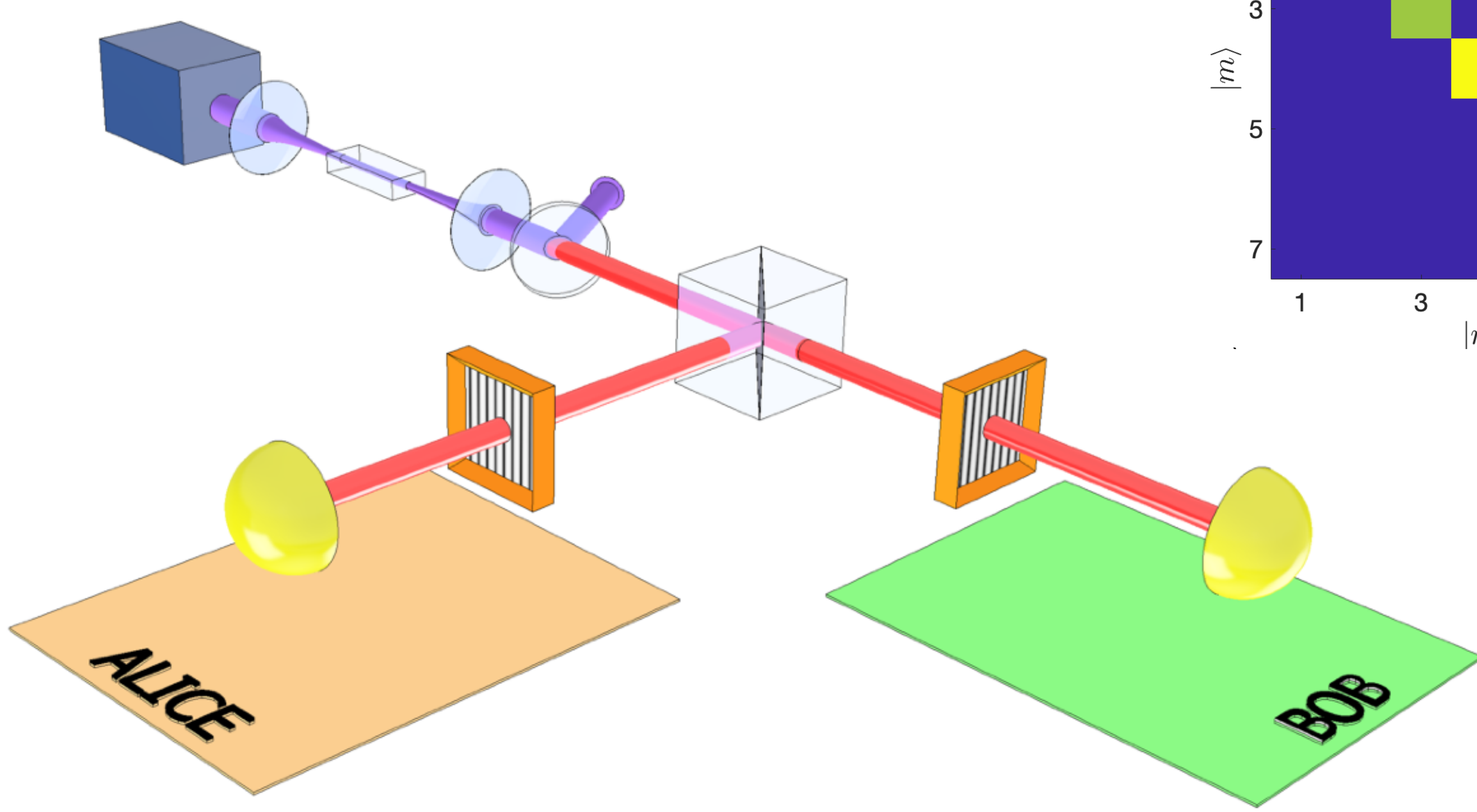




Valencia, Goel, McCutcheon, Defienne, Malik, **arXiv:1910.04490** (2019)

# Initial State

$$|\Phi^{+}\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |ii\rangle$$



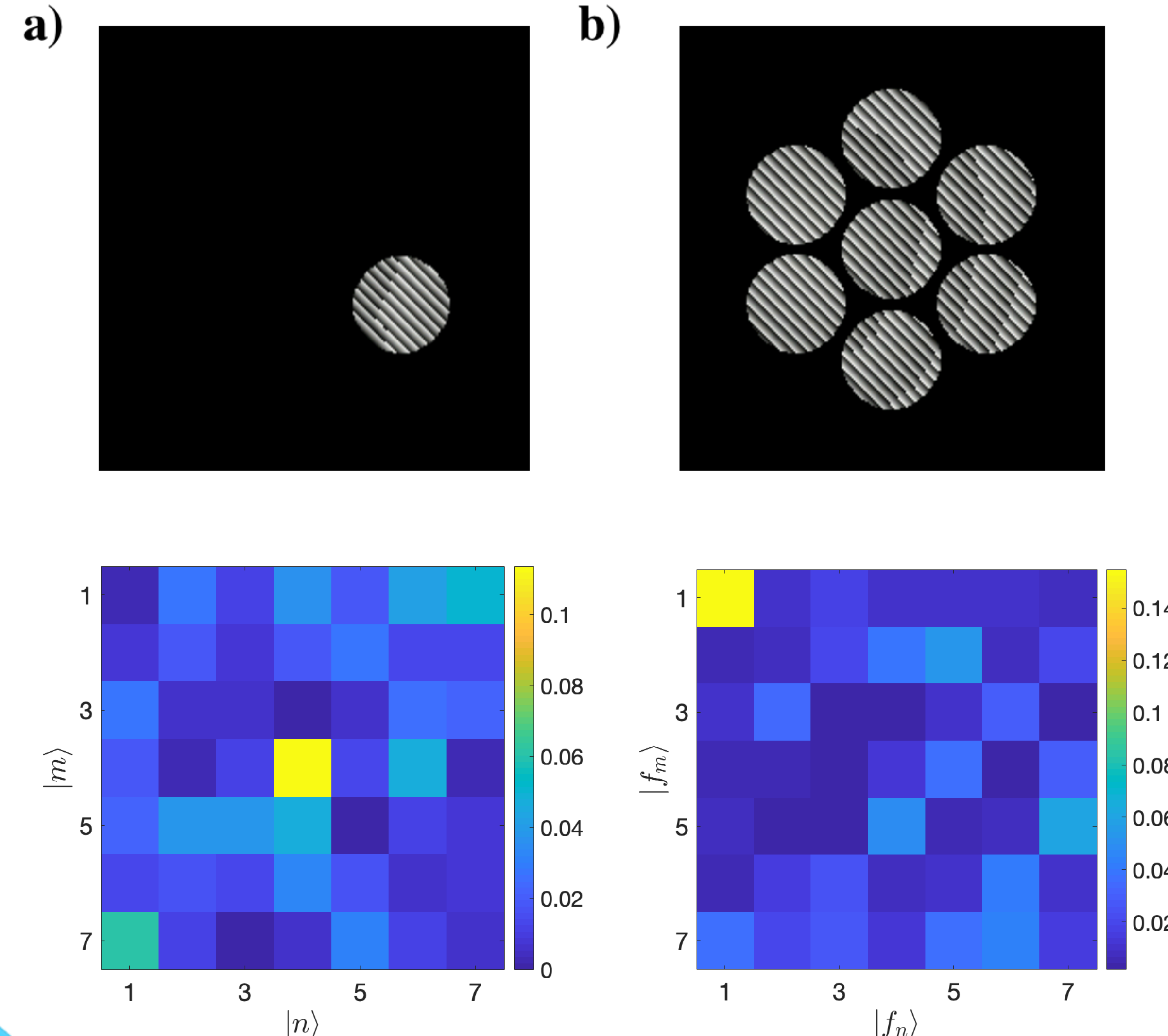



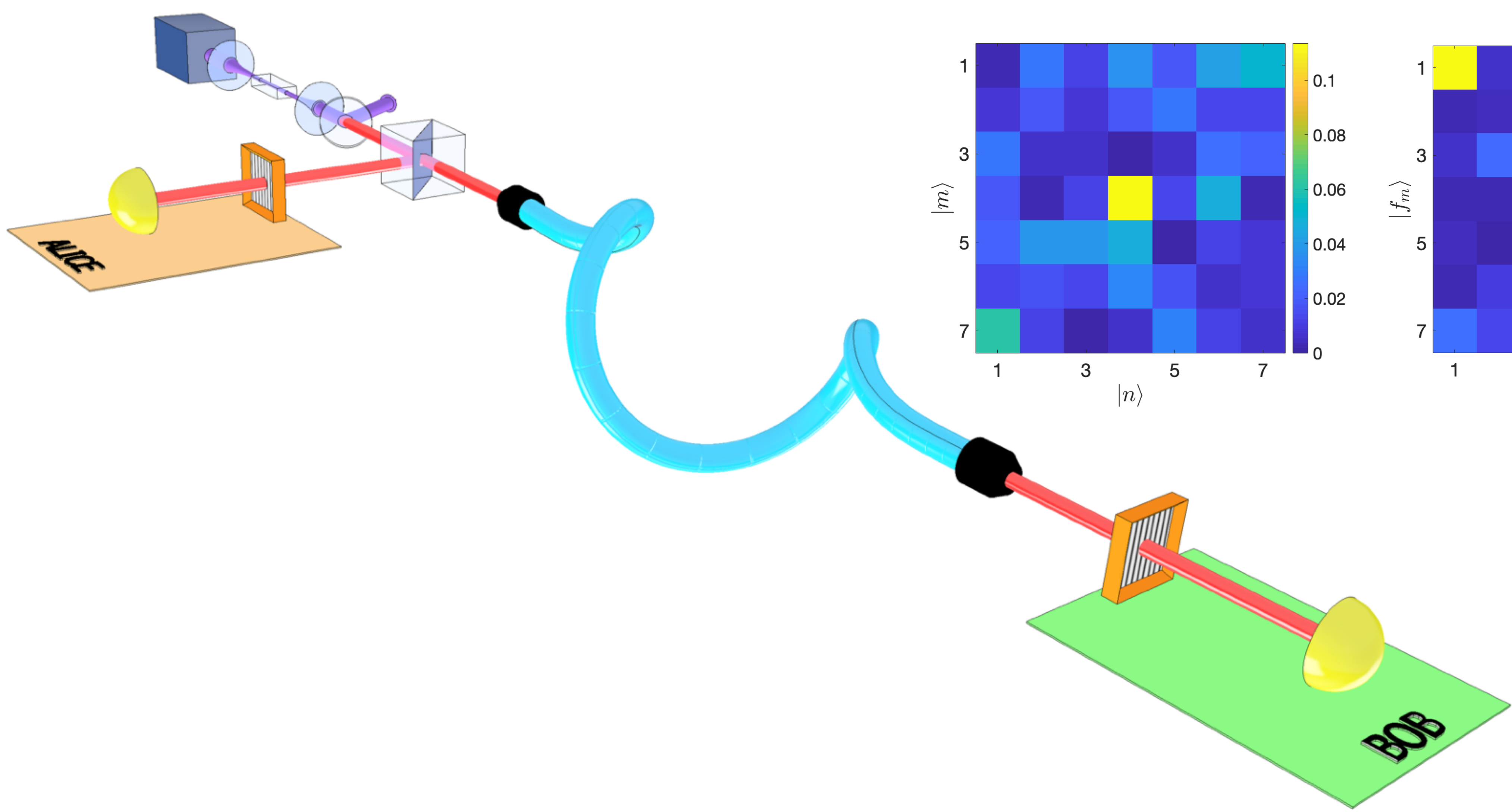

$F \geq 94.12\%$

Valencia, Goel, McCutcheon, Defienne, Malik, **arXiv:1910.04490** (2019)

# Choi State: after Fibre

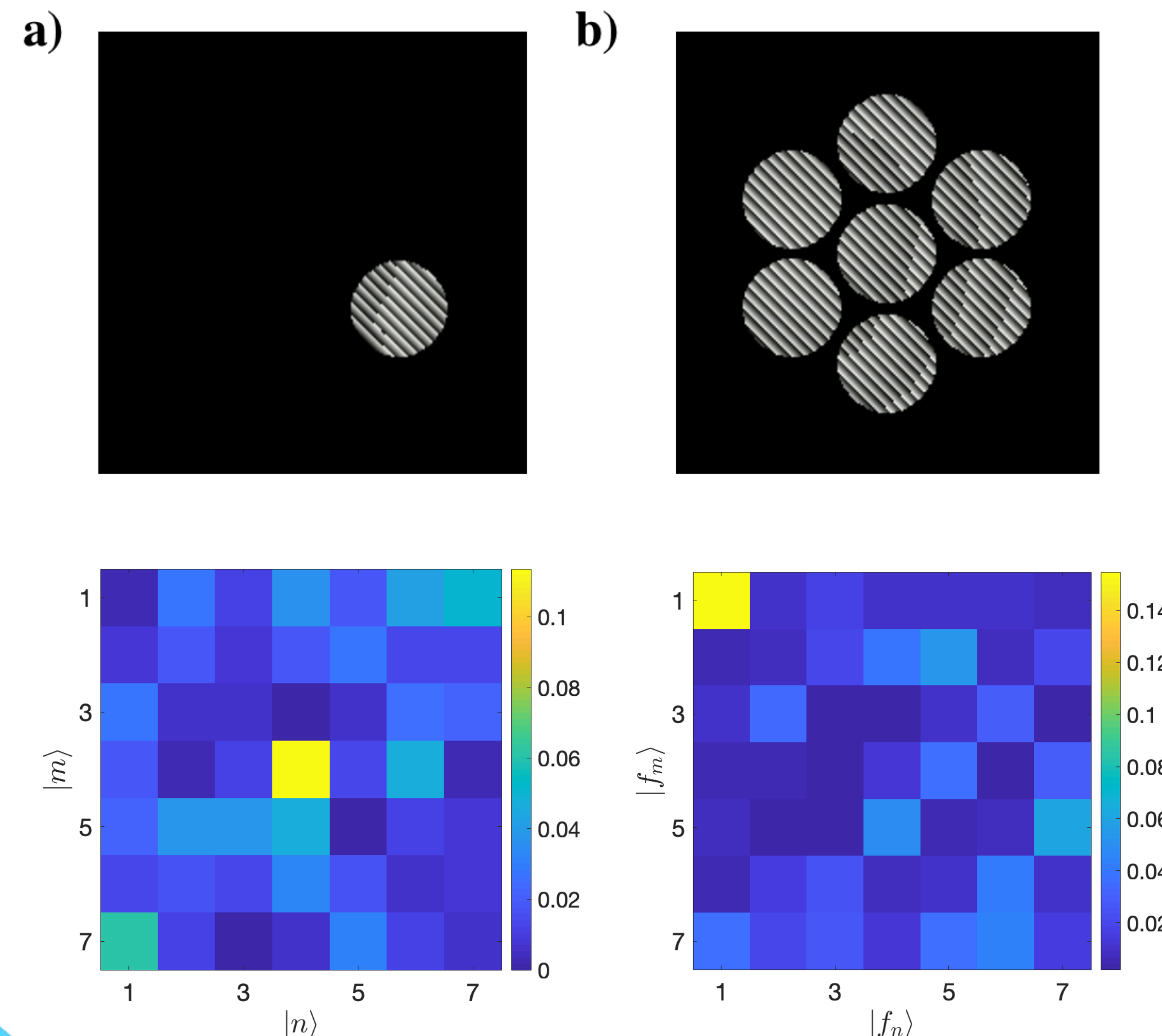
$$|\Phi\rangle_{\mathrm{T}} = \sum_{ij} t_{ji}\,|ij\rangle$$

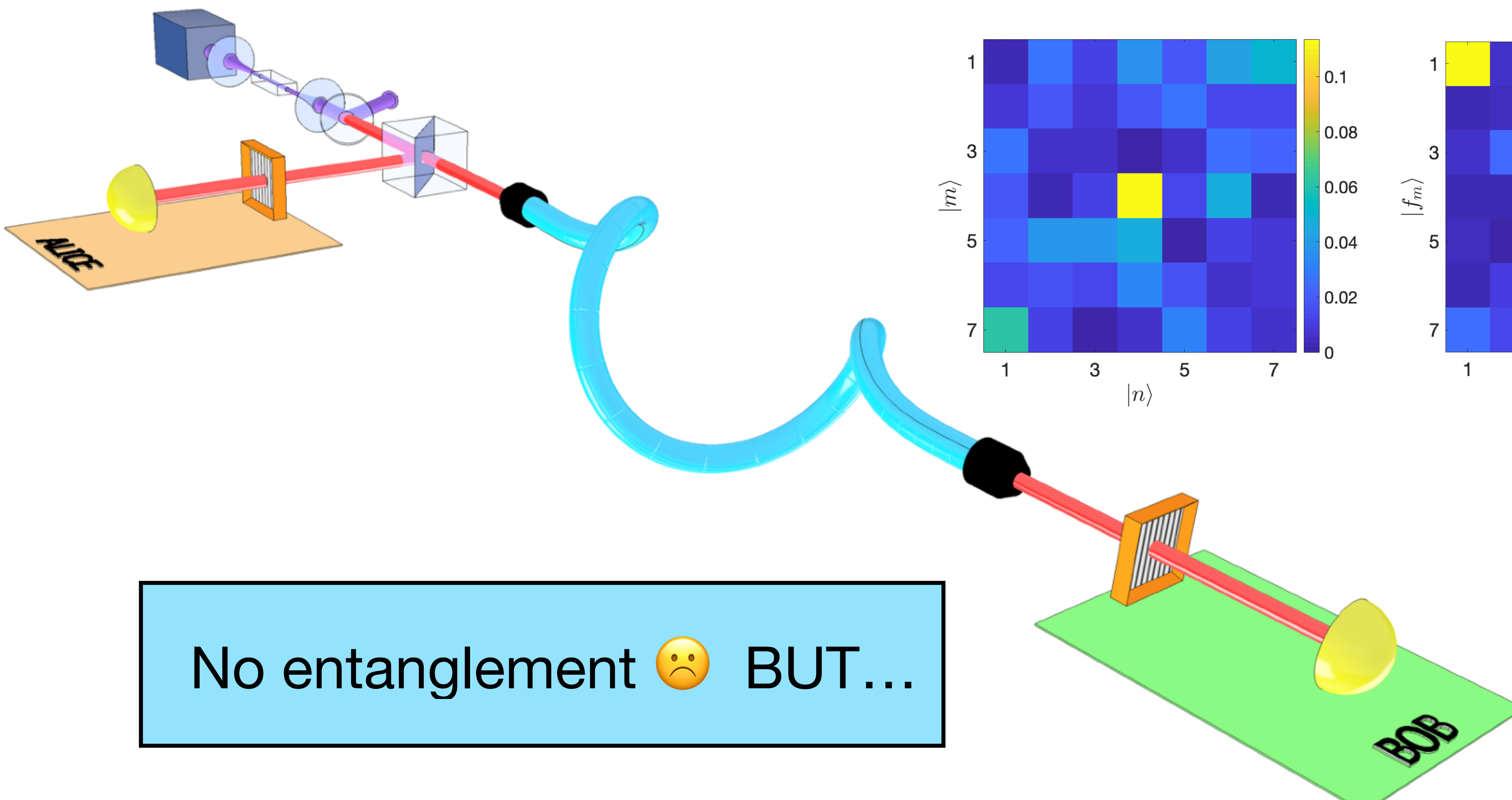


# Choi State: after Fibre

$$|\Phi\rangle_{\mathrm{T}} = \sum_{ij} t_{ji} \, |i\, j\rangle$$




No entanglement ☹ BUT…

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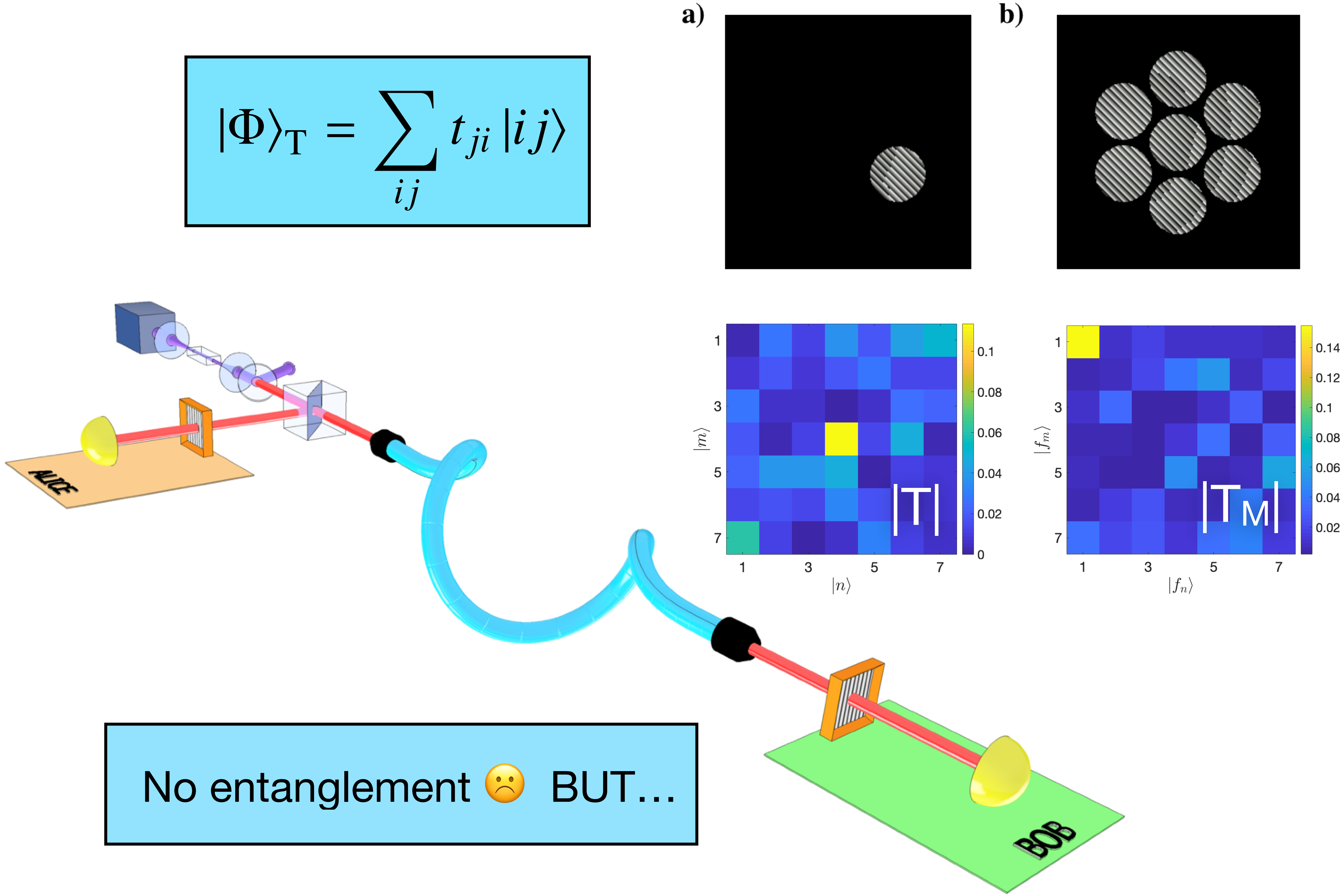
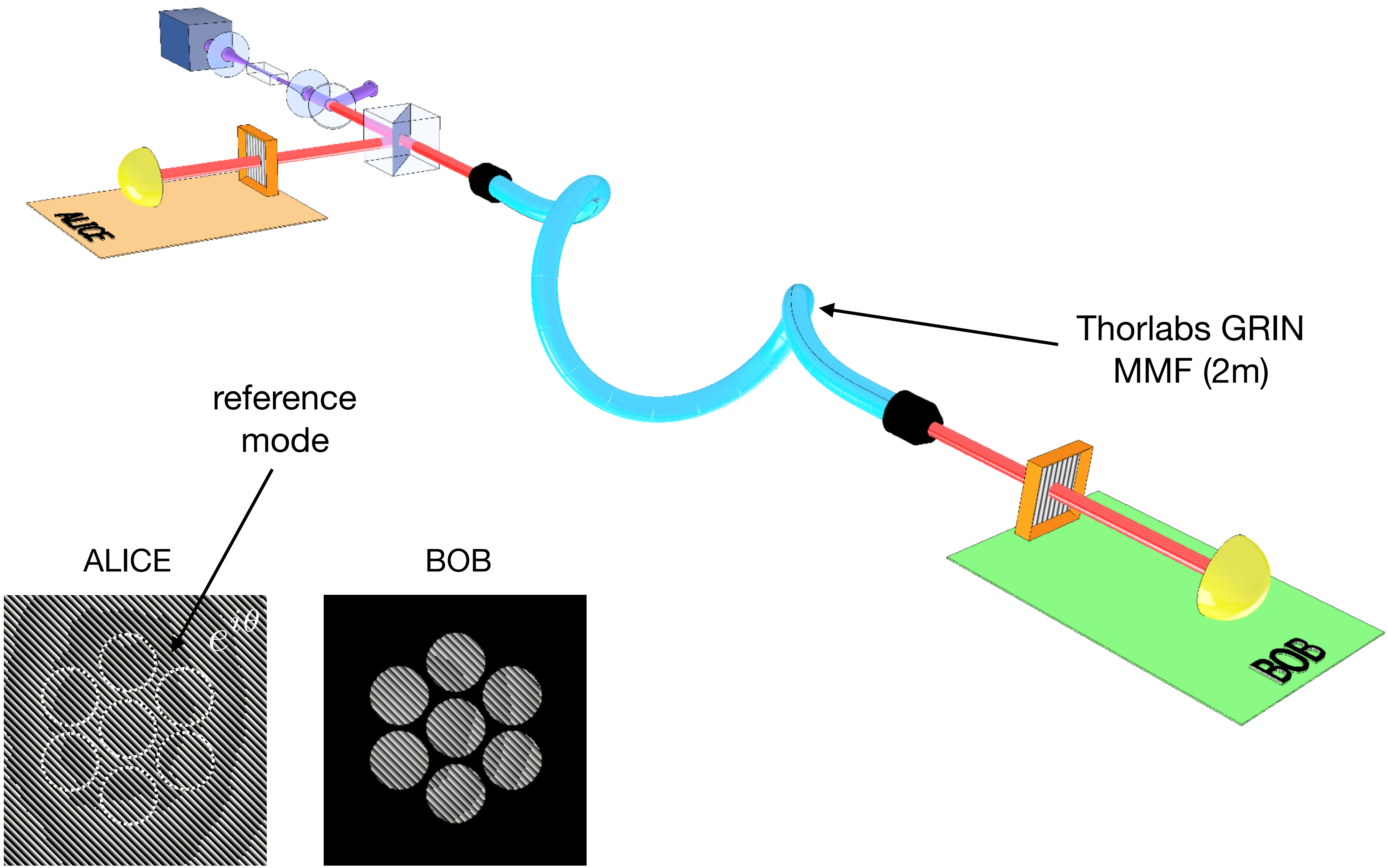
$$|\Phi\rangle_\mathrm{T} = \sum_{ij} t_{ji}\,|ij\rangle$$




No entangled 🙁 BUT…

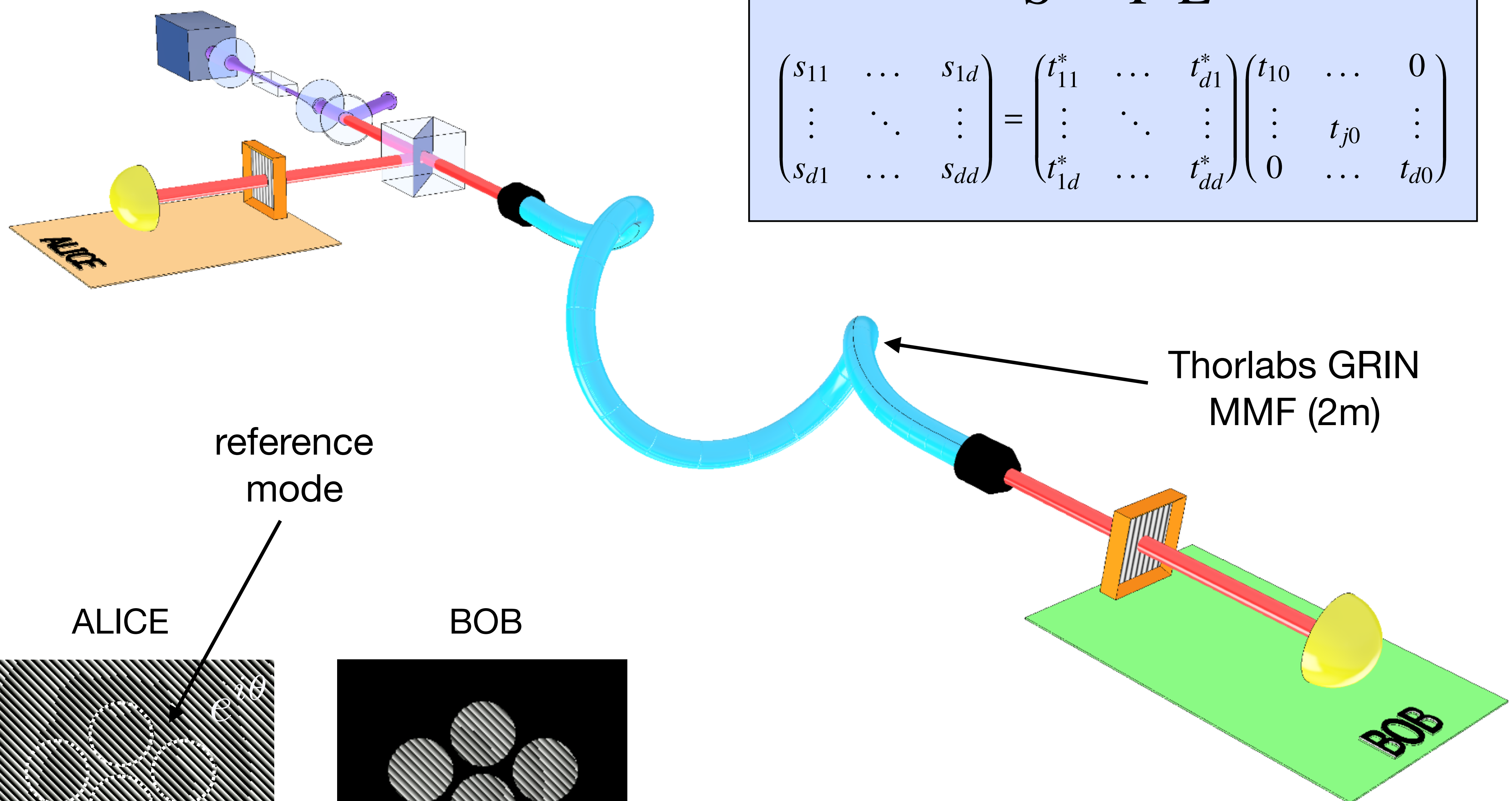
# Complex T Measurement: Phase-Stepping

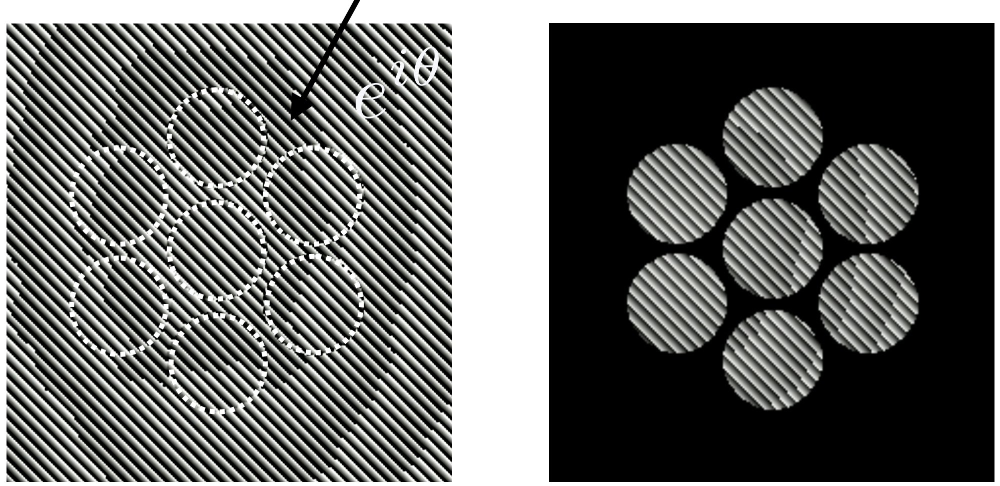


# Complex T Measurement: Phase-Stepping

$$S = T^{\dagger}E$$

$$\begin{pmatrix} s_{11} & \dots & s_{1d} \\ \vdots & \ddots & \vdots \\ s_{d1} & \dots & s_{dd} \end{pmatrix} = \begin{pmatrix} t^*_{11} & \dots & t^*_{d1} \\ \vdots & \ddots & \vdots \\ t^*_{1d} & \dots & t^*_{dd} \end{pmatrix} \begin{pmatrix} t_{10} & \dots & 0 \\ \vdots & t_{j0} & \vdots \\ 0 & \dots & t_{d0} \end{pmatrix}$$

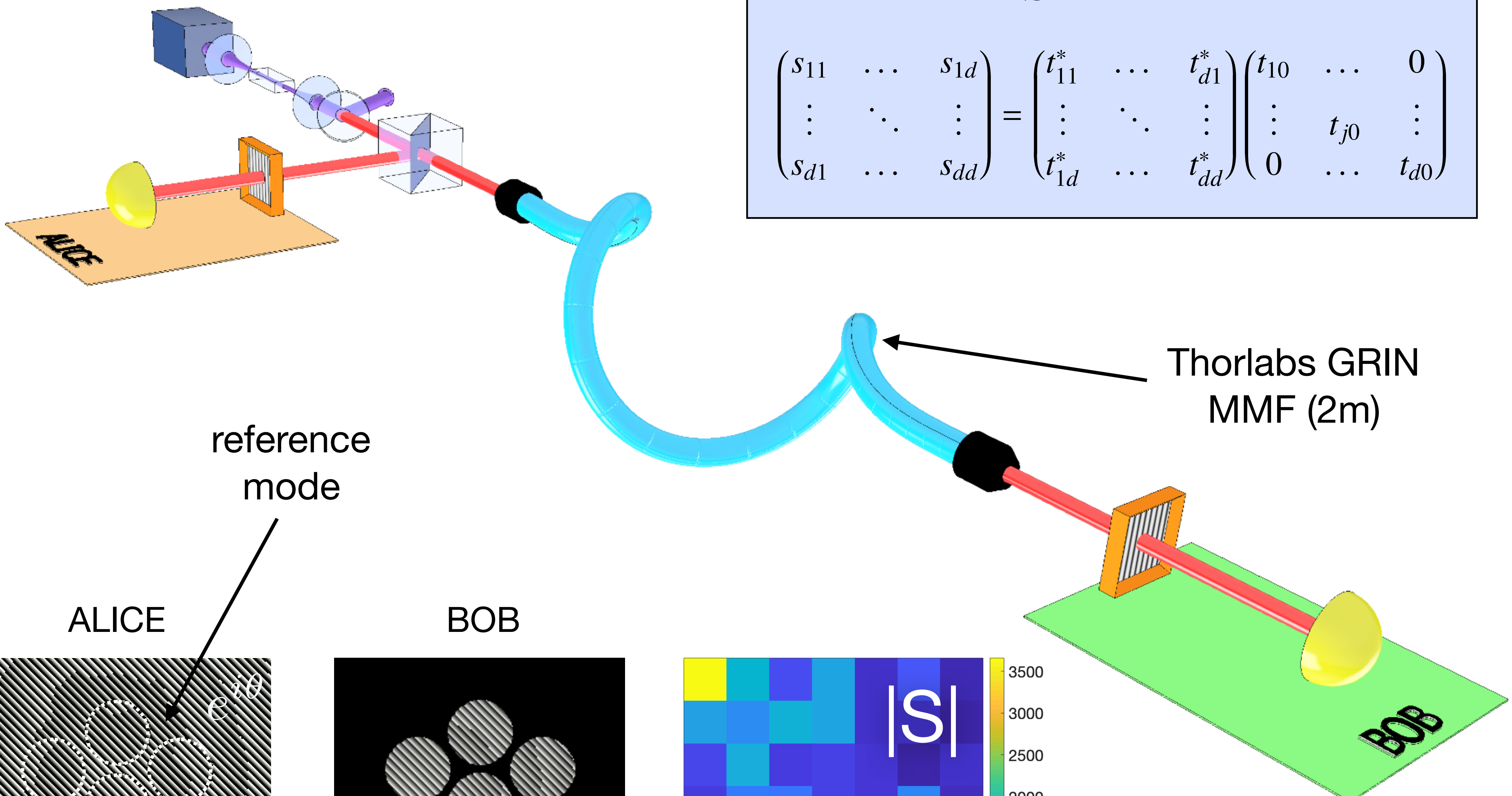


# Complex T Measurement: Phase-Stepping

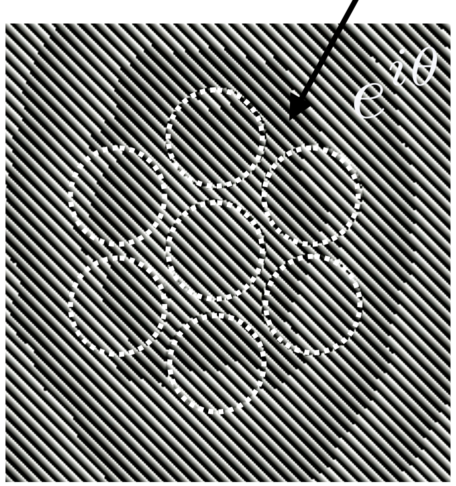
$$\mathrm{S} = \mathrm{T}^{\dagger}\mathrm{E}$$

$$\begin{pmatrix} s_{11} & \dots & s_{1d} \\ \vdots & \ddots & \vdots \\ s_{d1} & \dots & s_{dd} \end{pmatrix} = \begin{pmatrix} t_{11}^{*} & \dots & t_{d1}^{*} \\ \vdots & \ddots & \vdots \\ t_{1d}^{*} & \dots & t_{dd}^{*} \end{pmatrix} \begin{pmatrix} t_{10} & \dots & 0 \\ \vdots & t_{j0} & \vdots \\ 0 & \dots & t_{d0} \end{pmatrix}$$

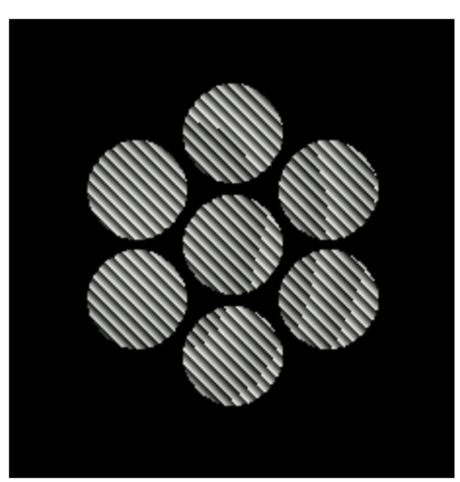


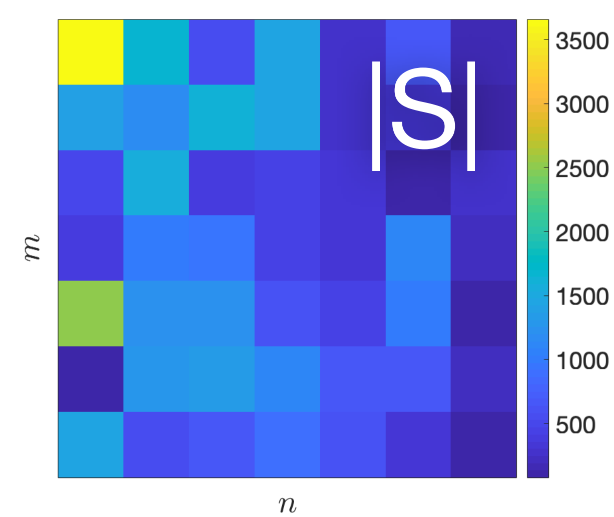

reference mode

ALICE




BOB

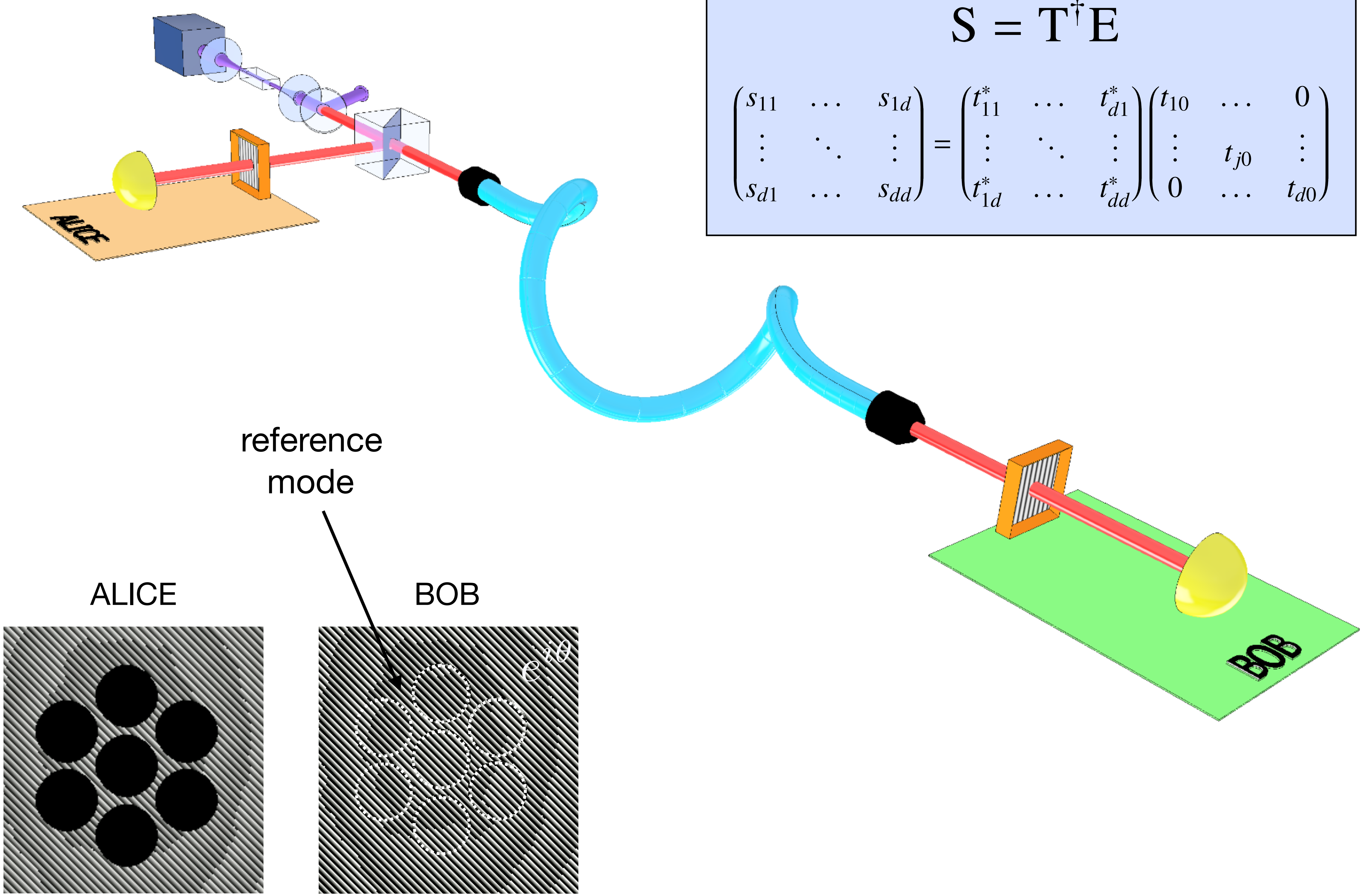




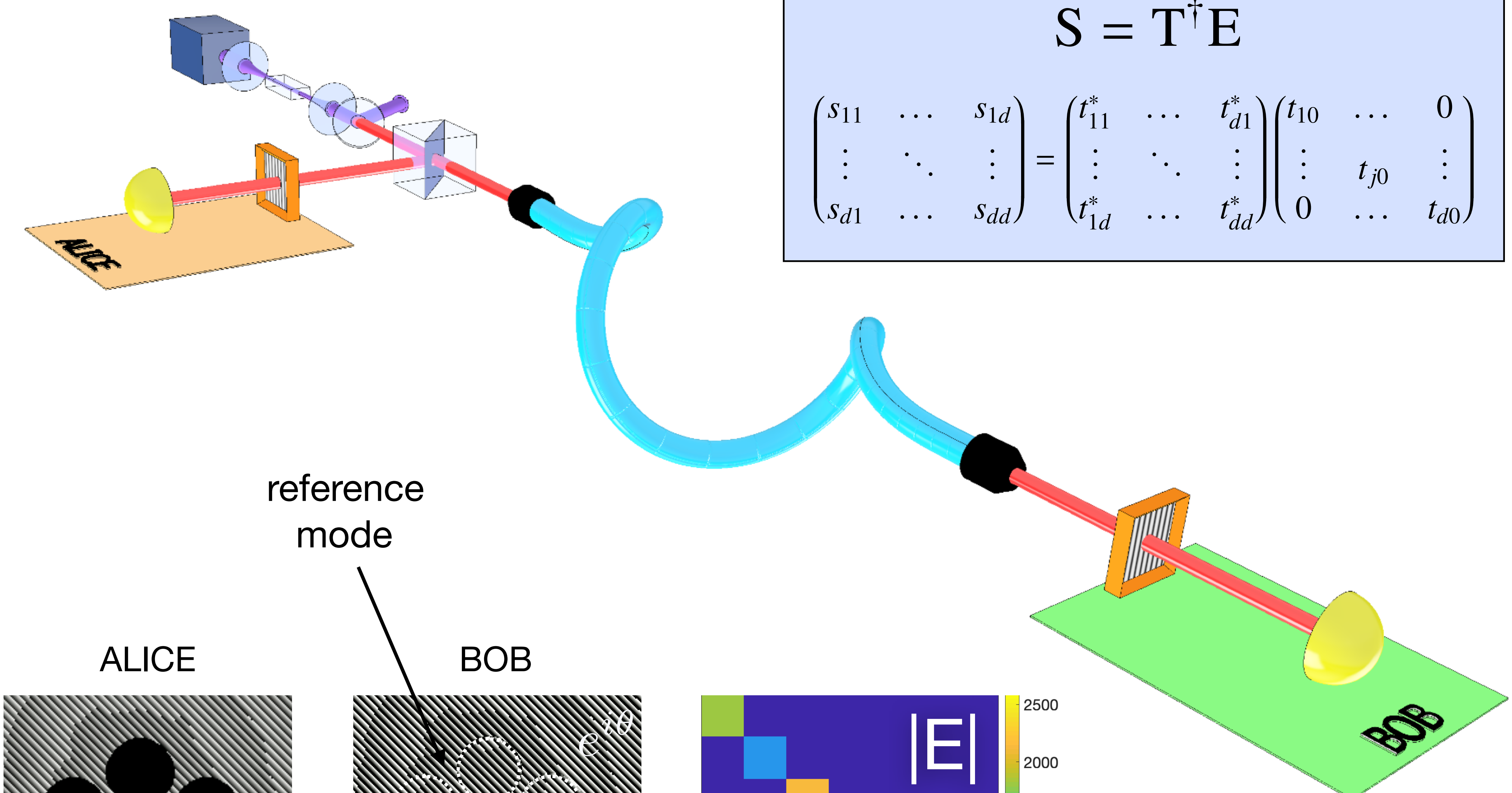
# Reference Measurement

$$\mathbf{S} = \mathbf{T}^{\dagger}\mathbf{E}$$

$$\begin{pmatrix} s_{11} & \dots & s_{1d} \\ \vdots & \ddots & \vdots \\ s_{d1} & \dots & s_{dd} \end{pmatrix} = \begin{pmatrix} t^*_{11} & \dots & t^*_{d1} \\ \vdots & \ddots & \vdots \\ t^*_{1d} & \dots & t^*_{dd} \end{pmatrix} \begin{pmatrix} t_{10} & \dots & 0 \\ \vdots & t_{j0} & \vdots \\ 0 & \dots & t_{d0} \end{pmatrix}$$

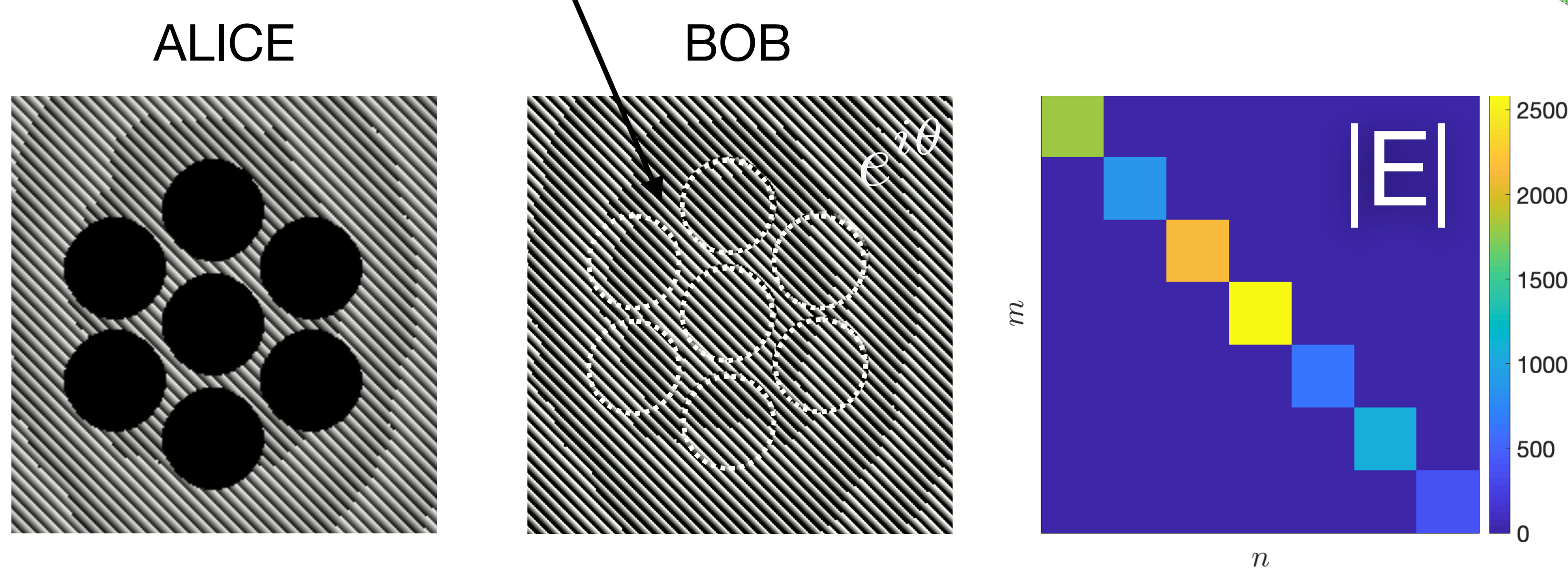


# Reference Measurement




$$\mathbf{S} = \mathbf{T}^{\dagger}\mathbf{E}$$

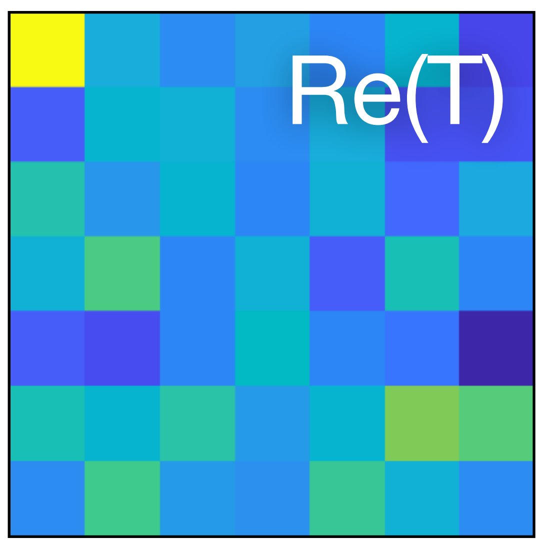
$$\begin{pmatrix} s_{11} & \dots & s_{1d} \\ \vdots & \ddots & \vdots \\ s_{d1} & \dots & s_{dd} \end{pmatrix} = \begin{pmatrix} t^*_{11} & \dots & t^*_{d1} \\ \vdots & \ddots & \vdots \\ t^*_{1d} & \dots & t^*_{dd} \end{pmatrix} \begin{pmatrix} t_{10} & \dots & 0 \\ \vdots & t_{j0} & \vdots \\ 0 & \dots & t_{d0} \end{pmatrix}$$



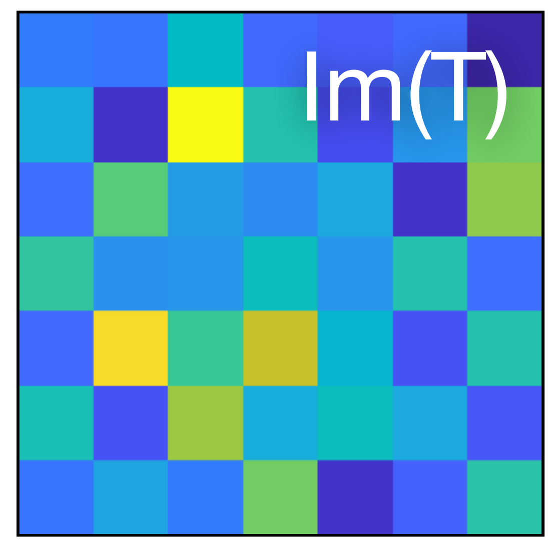
# Transmission Matrix

$$\boxed{\mathbf{T} = (\mathbf{S}\mathbf{E}^{-1})^{\dagger}}$$

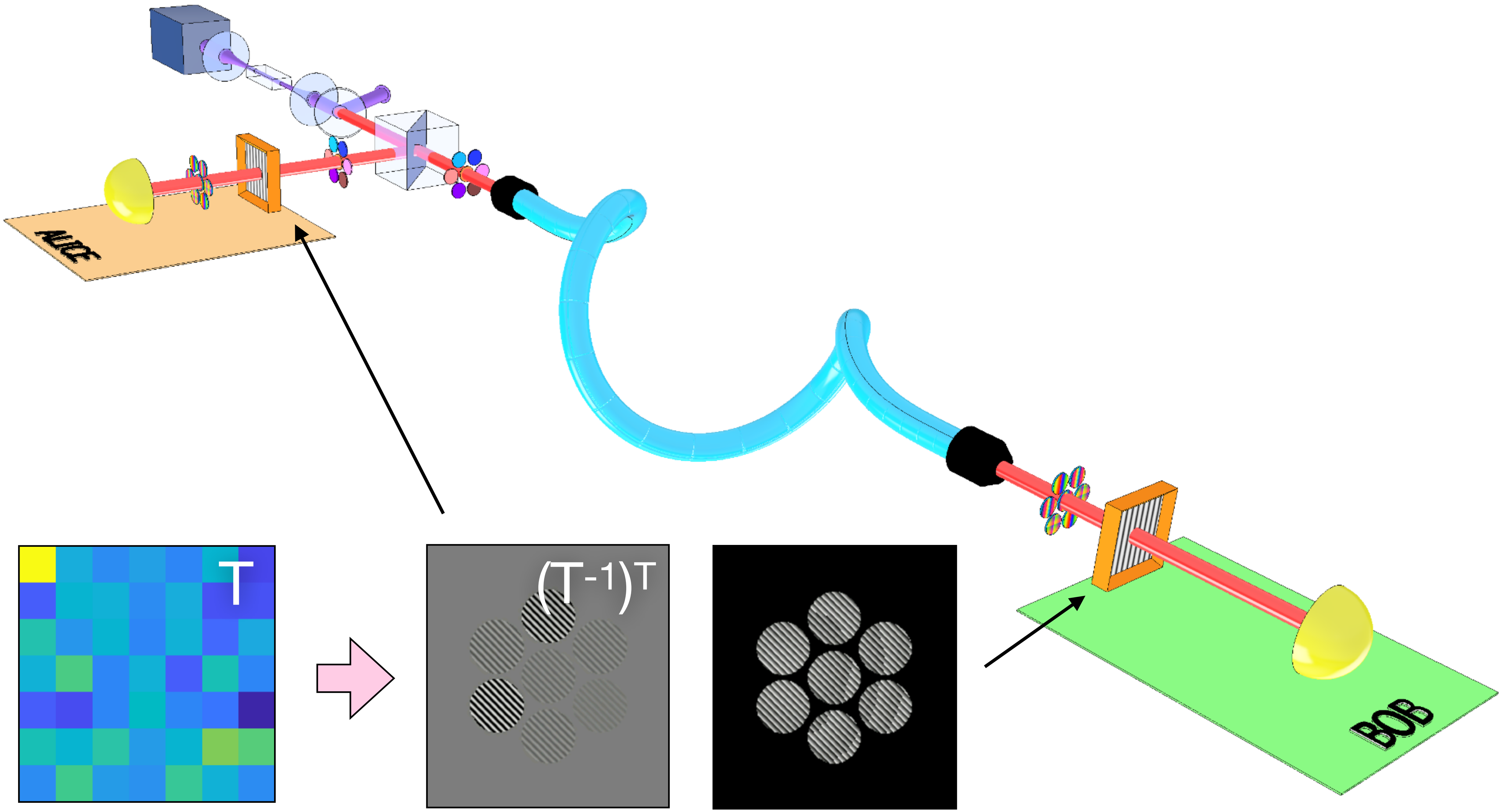
T =




+ i



# “Scrambling” Entanglement

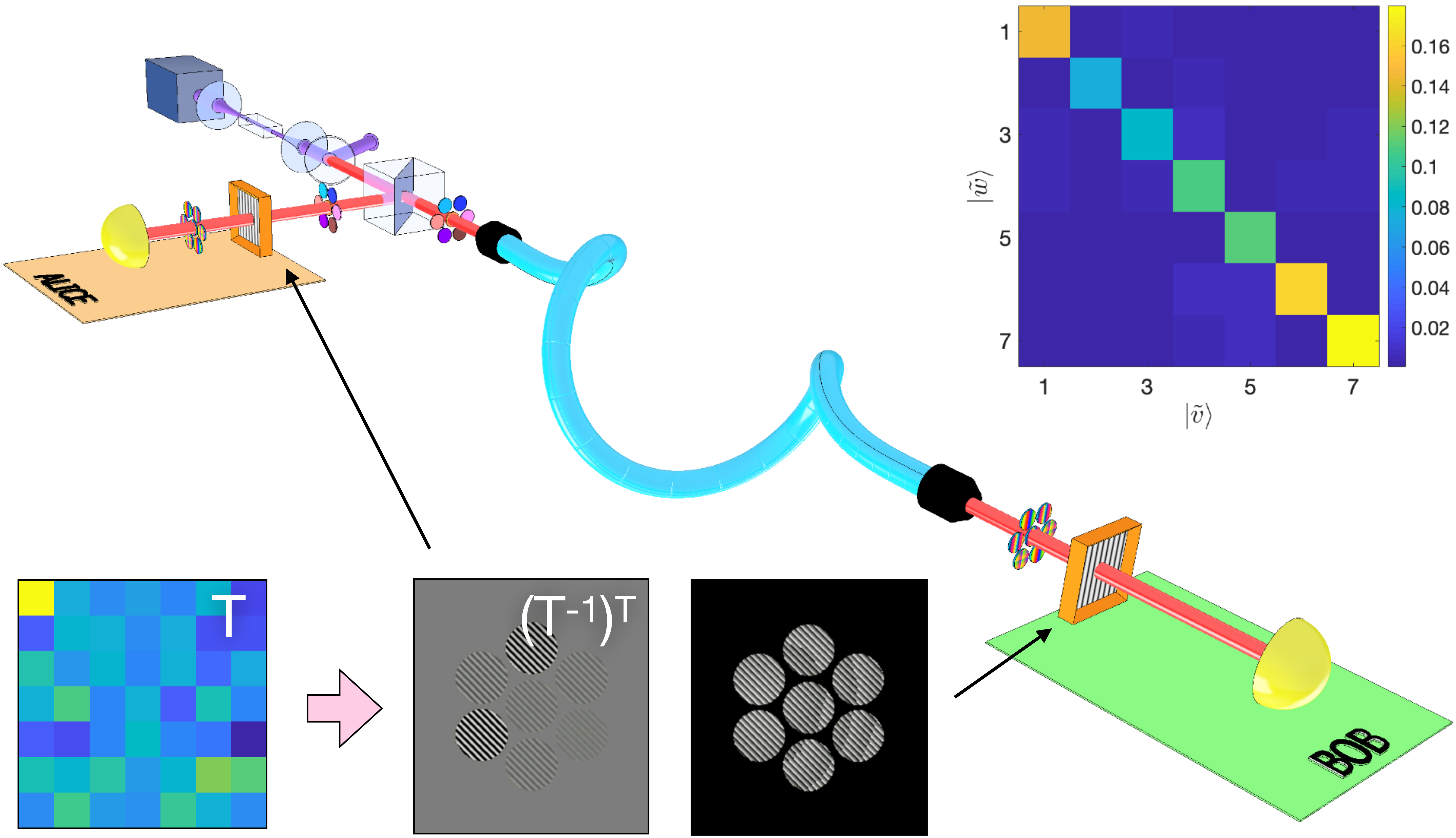



Transmission Matrix

Alice Hologram

Bob Hologram

# “SCRAMBLING” ENTANGLEMENT

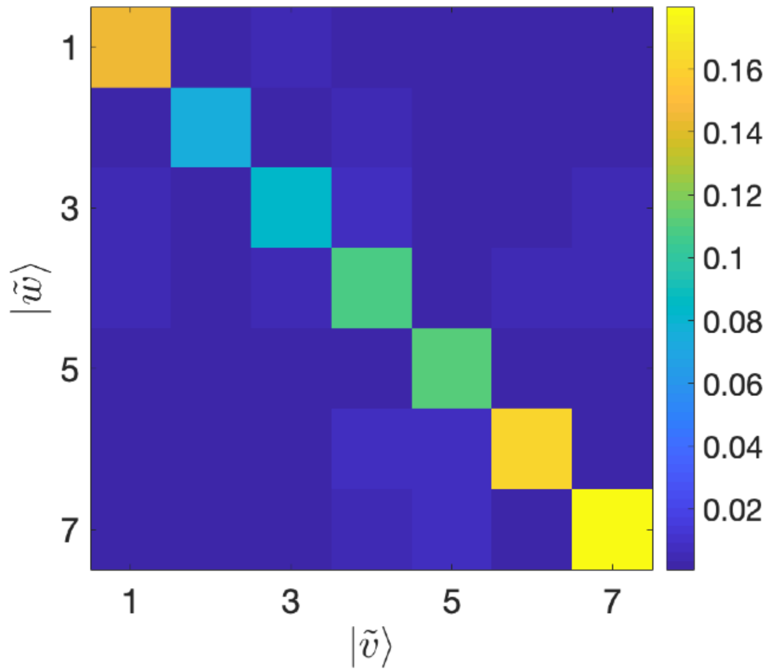


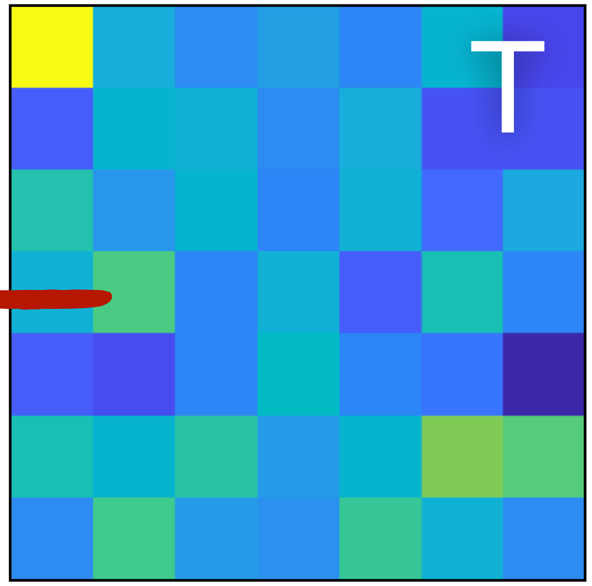

Transmission Matrix

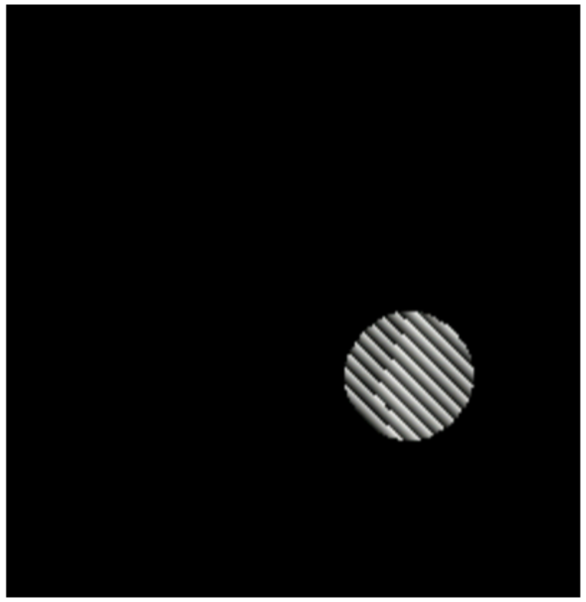
Alice Hologram

Bob Hologram

# MUTUALLY UNBIASED TS

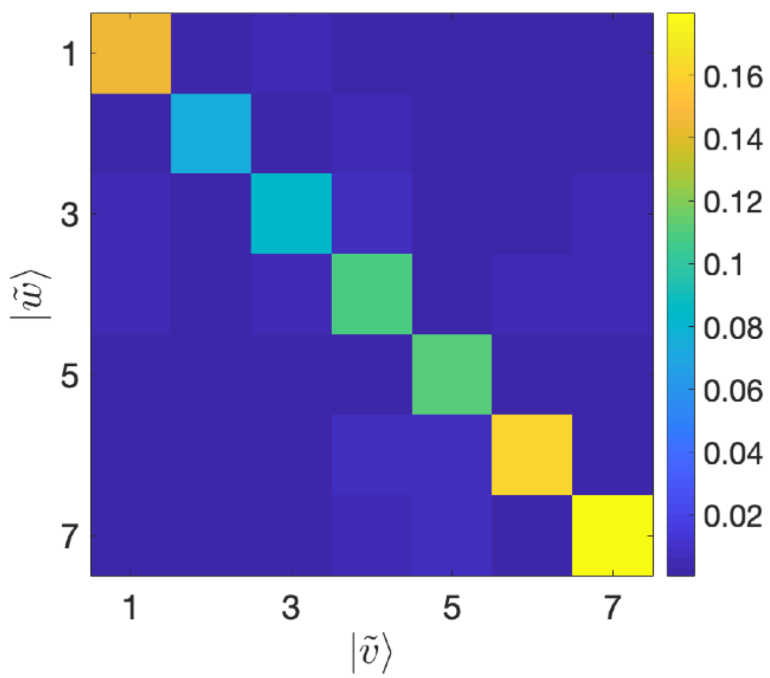


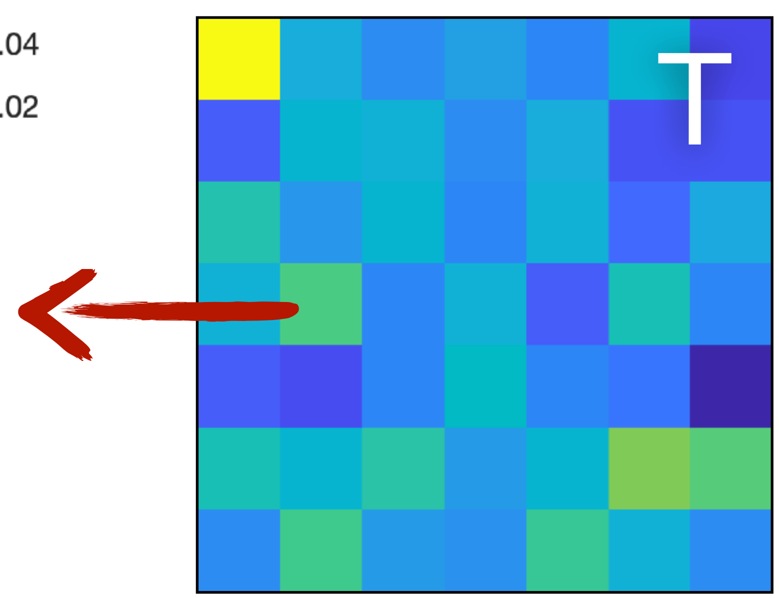



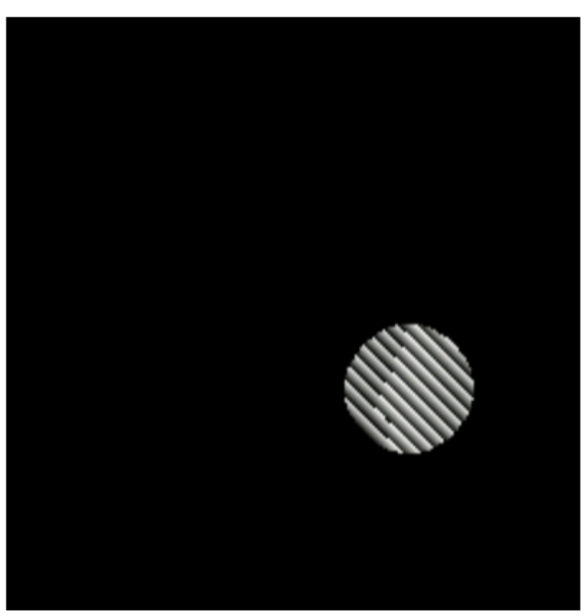



Pixel Basis

# Mutually unbiased Ts



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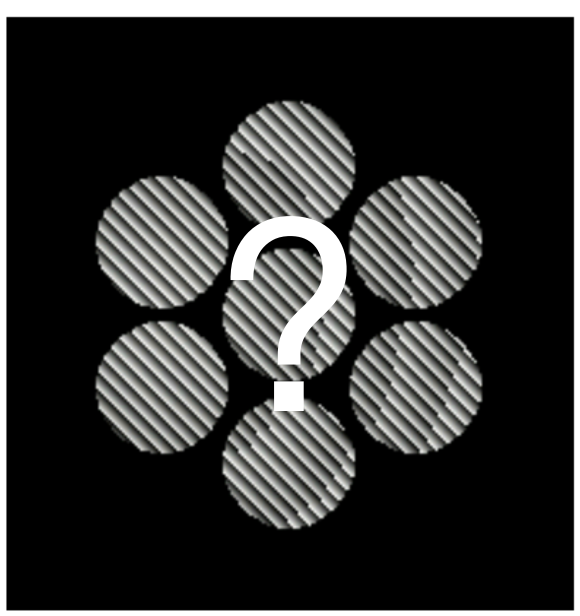
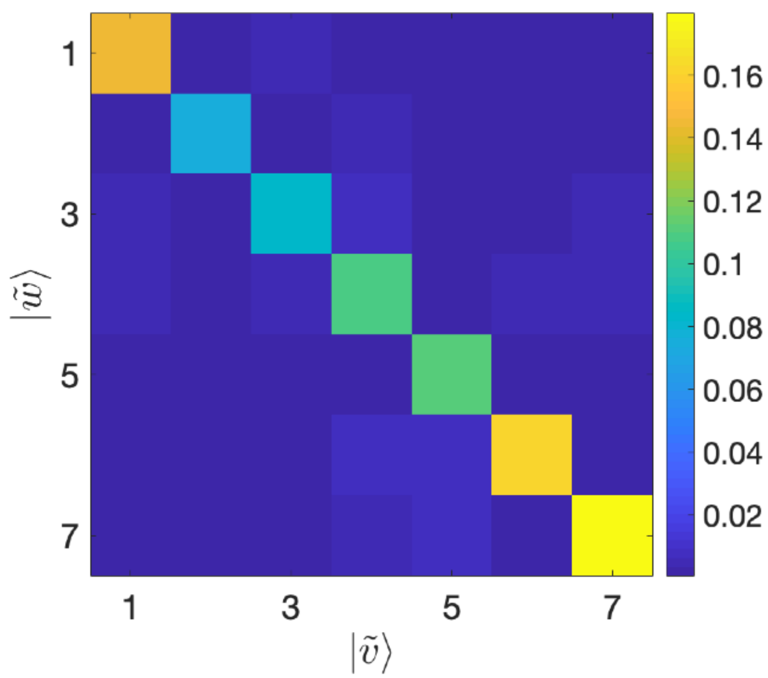
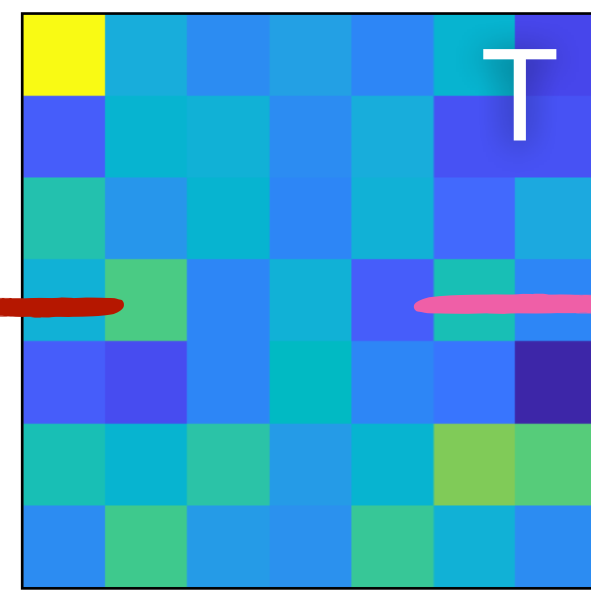



d+1 = 8 MUBs

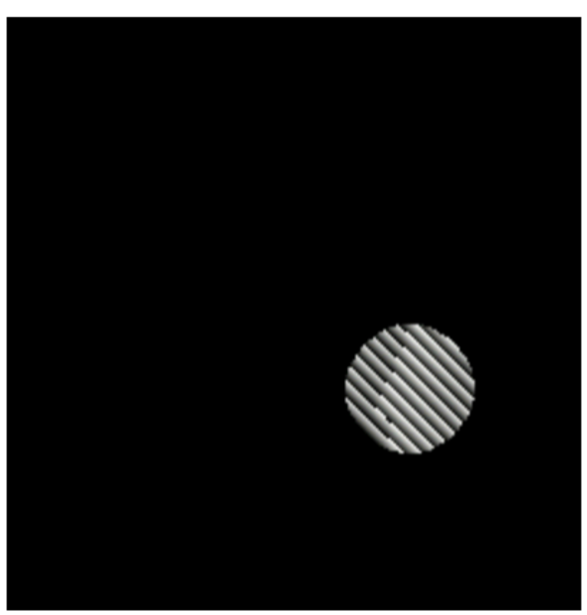
# Mutually unbiased Ts




Rotate the transmission matrix!

$$\boxed{T_{Mi} = M_i^* \, T \, M_i^T}$$



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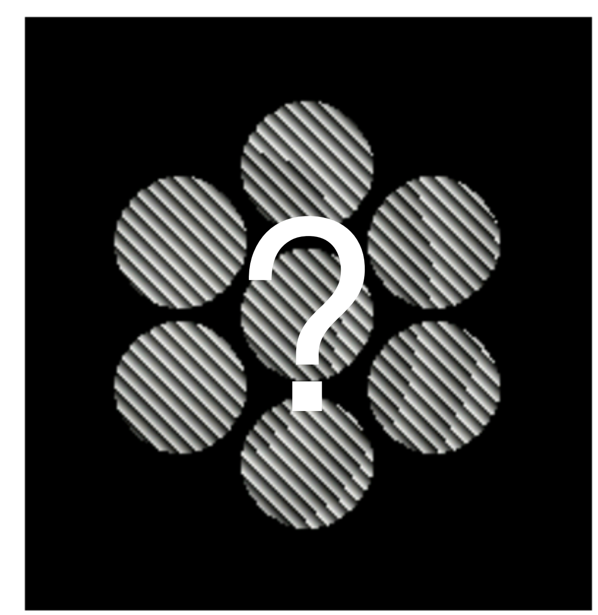
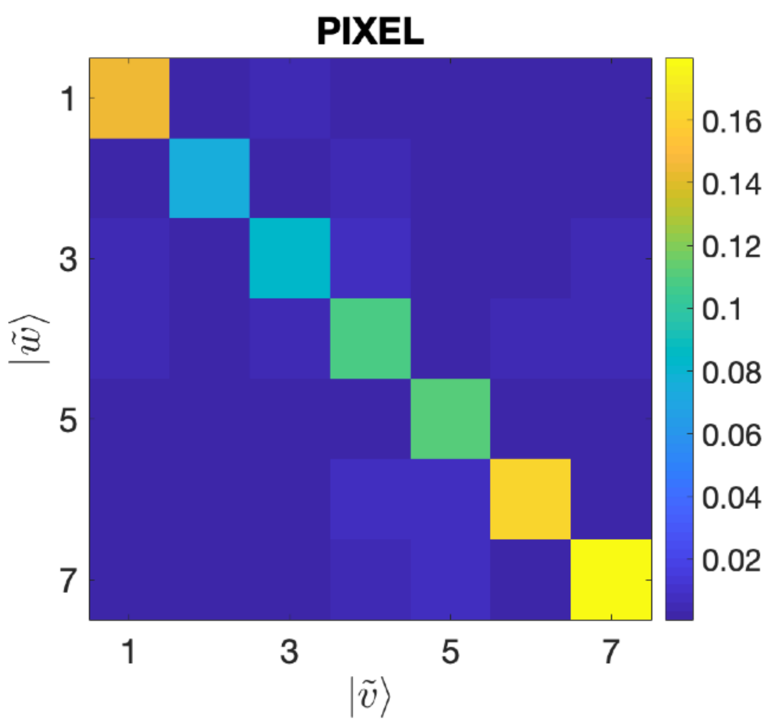



d+1 = 8 MUBs

# MUTUALLY UNBIASED CORRELATIONS




$$\mathrm{T}_{\mathrm{M}i} = \mathrm{M}_i^* \, \mathrm{T} \, \mathrm{M}_i^{\mathrm{T}}$$

Valencia, Goel, McCutcheon, Defienne, Malik, arXiv:1910.04490 (2019)

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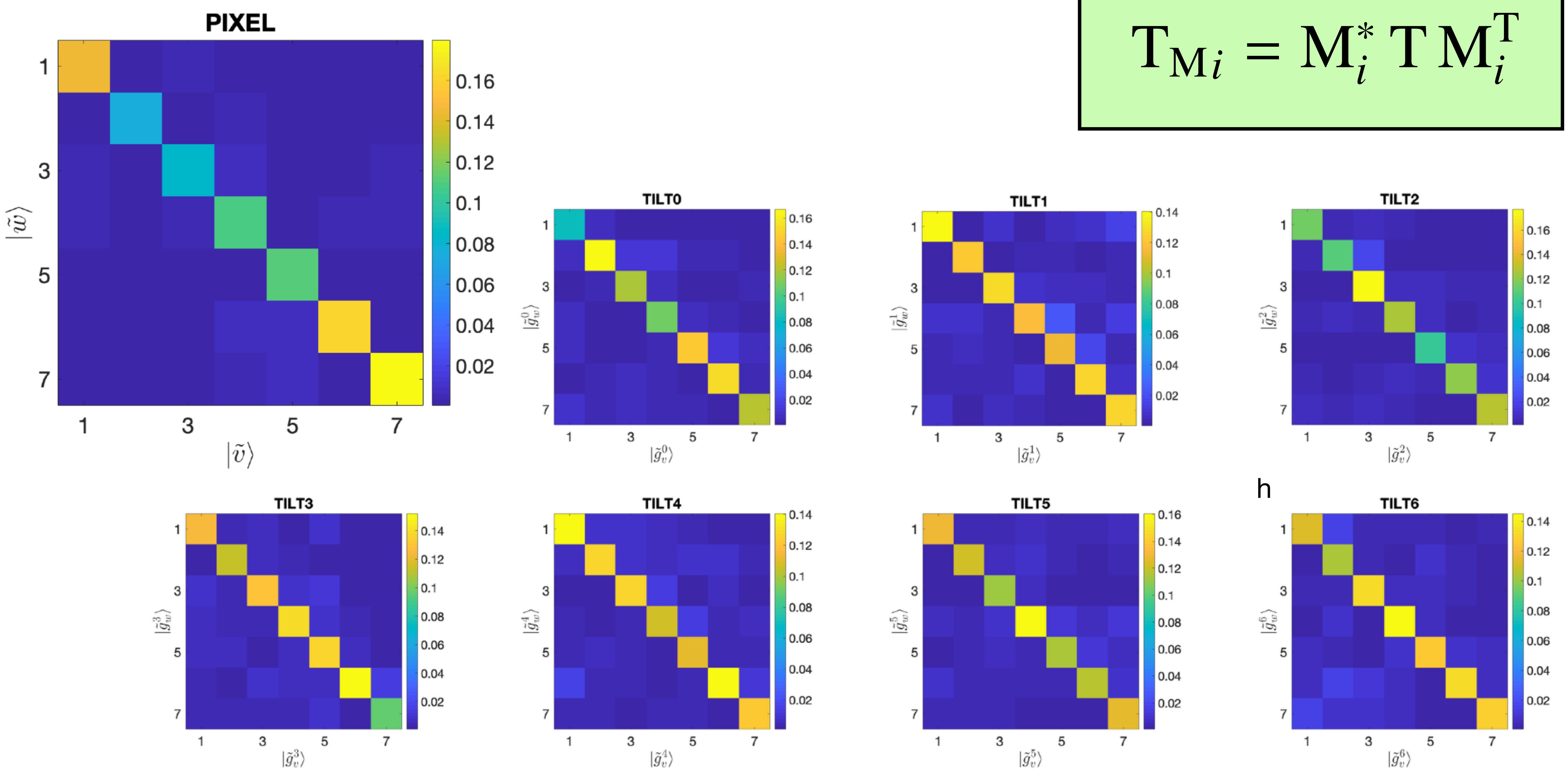
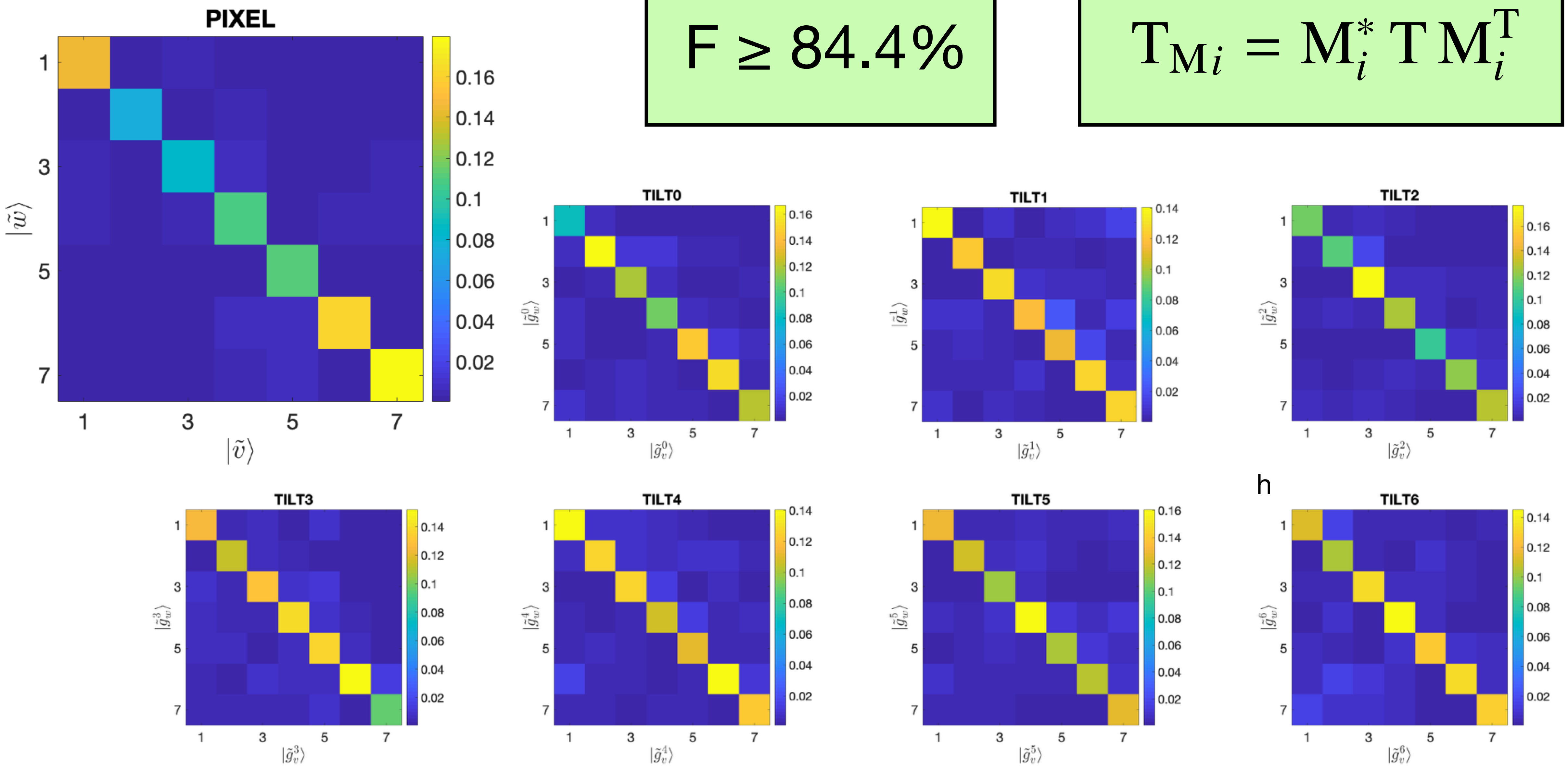



Valencia, Goel, McCutcheon, Defienne, Malik, **arXiv:1910.04490** (2019)

# MUTUALLY UNBIASED CORRELATIONS

$F \geq 84.4\%$

$T_{M_i} = M_i^* \, T \, M_i^T$

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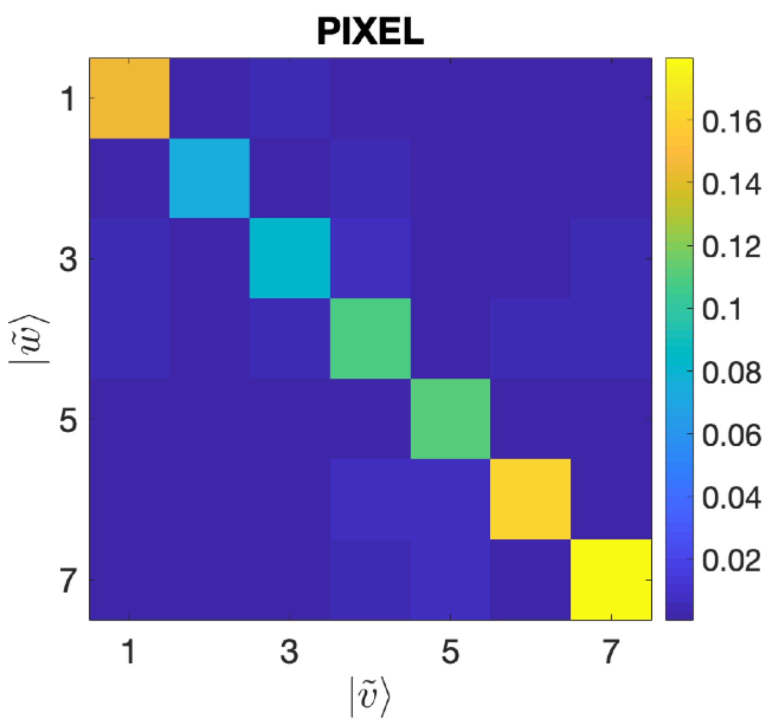


F ≥ 84.4%

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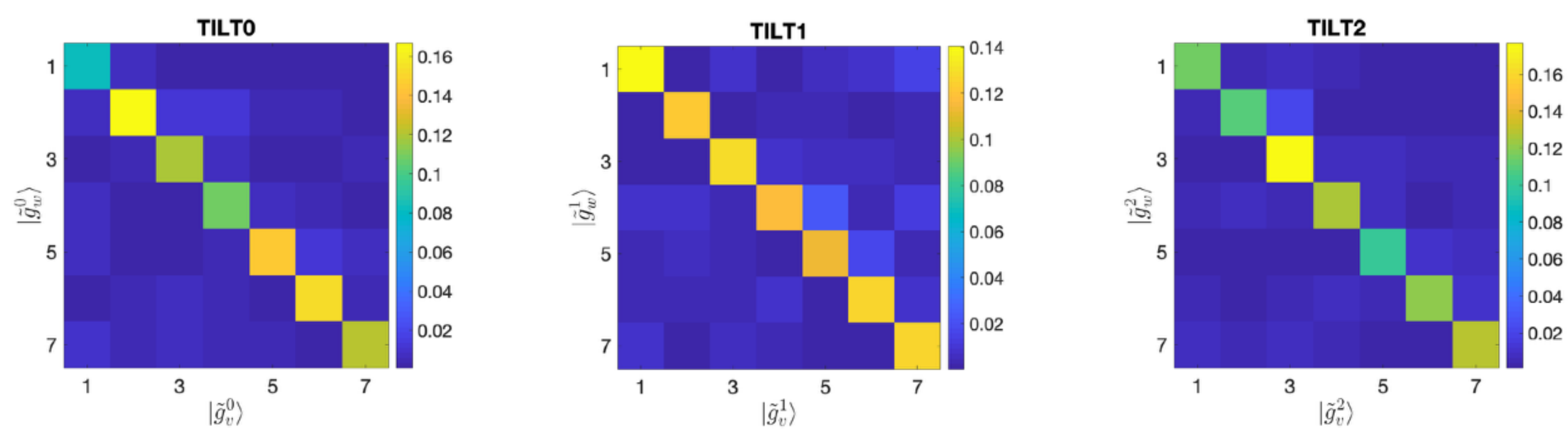


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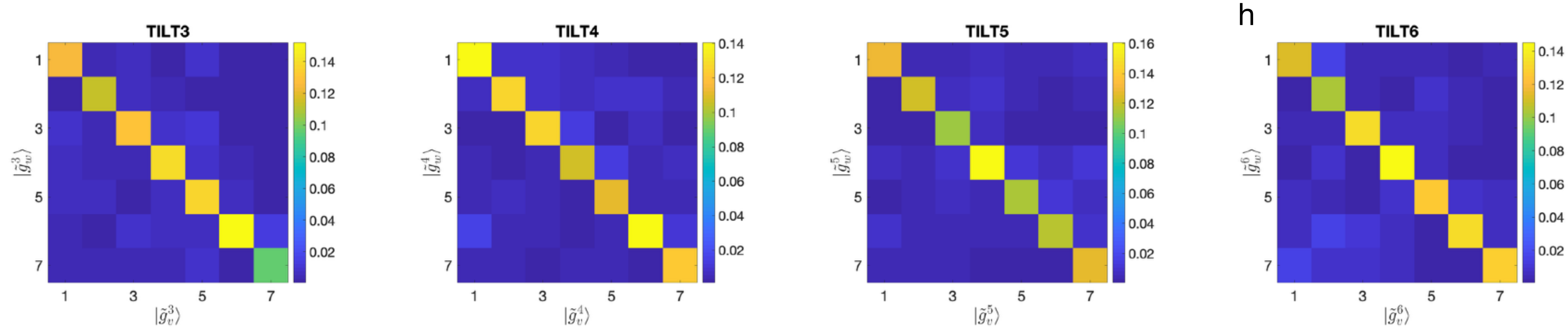


6-dimensional entanglement!

Valencia, Goel, McCutcheon, Defienne, Malik, **arXiv:1910.04490** (2019)

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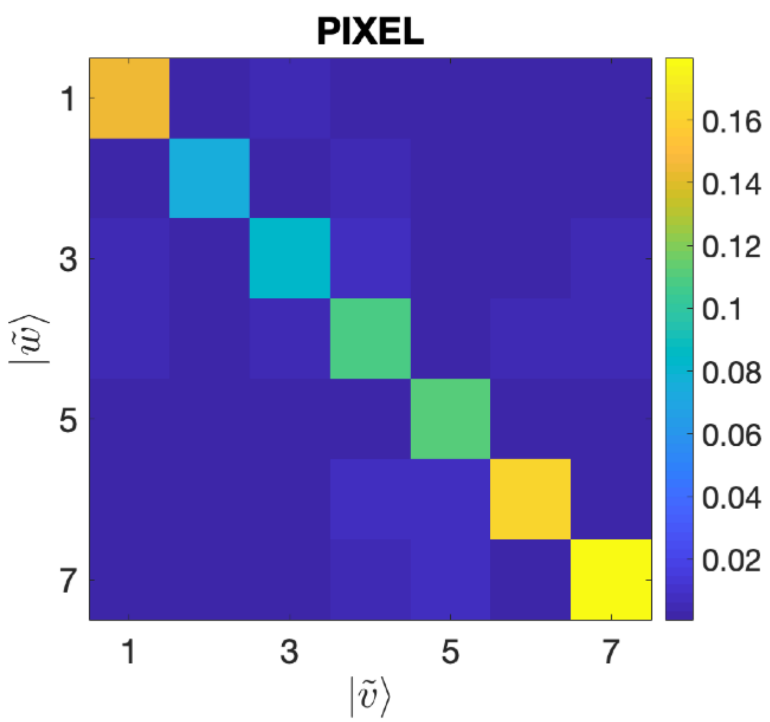


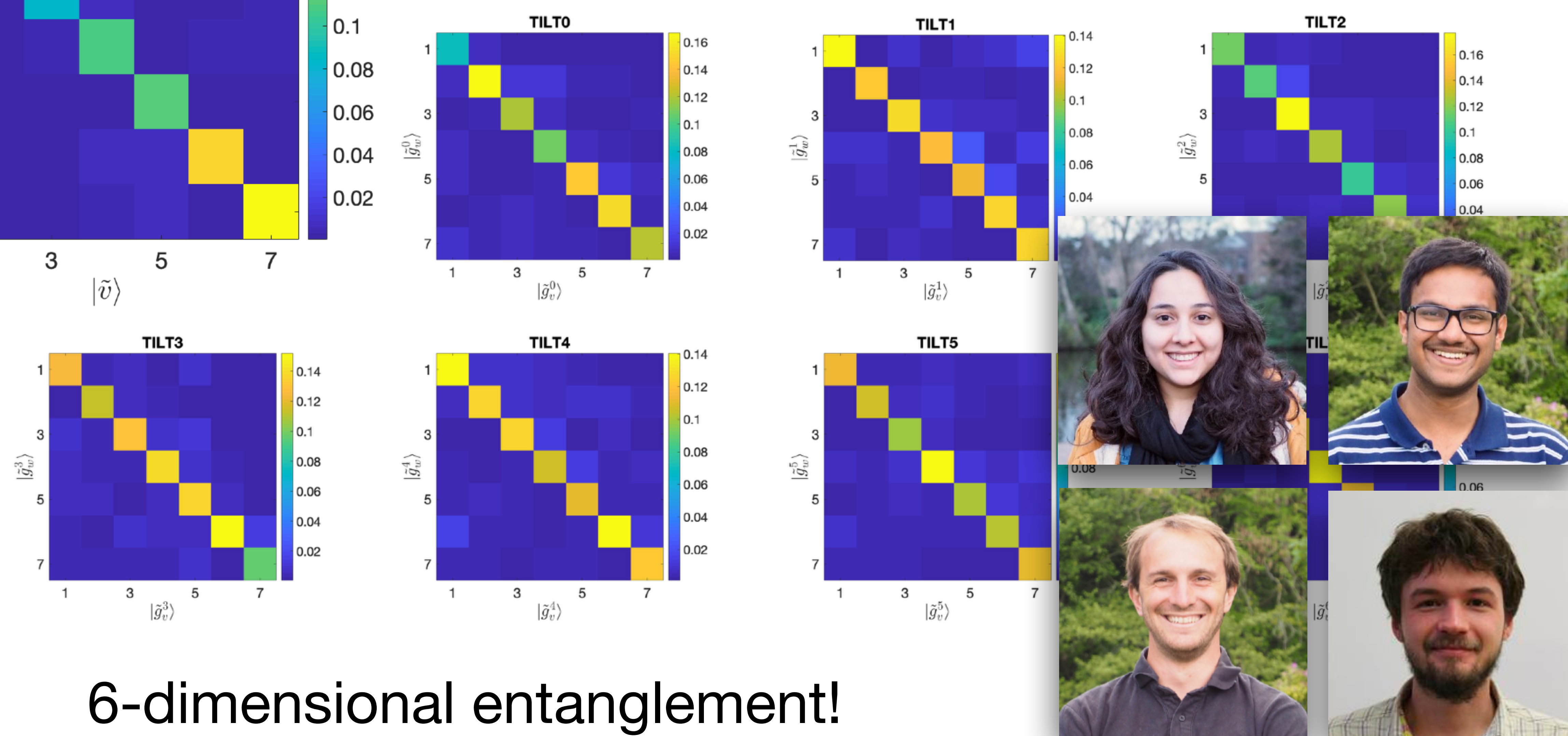


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# THREE *QUDITS*

# ENTANGLEMENT: MILESTONES

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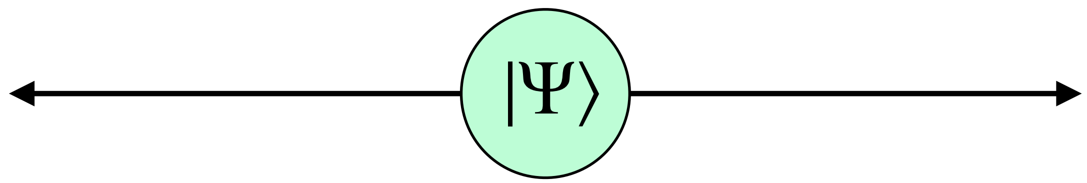
- **1935:** Einstein, Podolsky, Rosen




MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

**Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?**

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

Einstein, Podolsky, Rosen, **Phys. Rev. 47**, 777 (1935)

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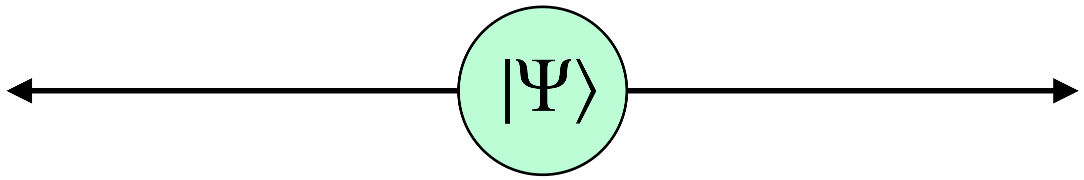
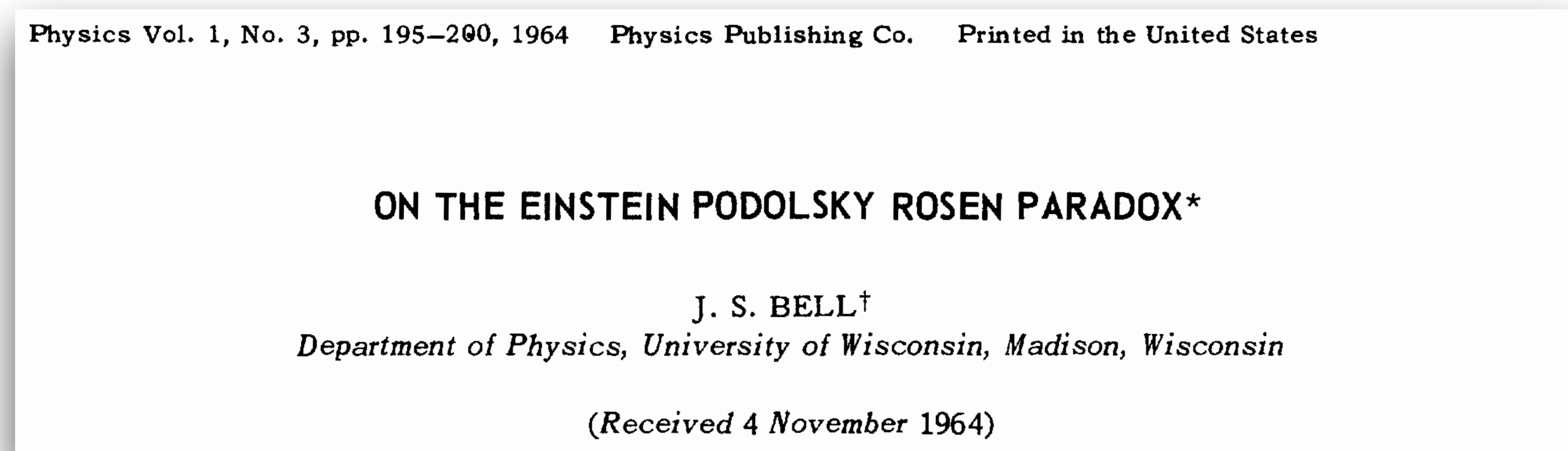



- **1964:** John Bell



Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†

*Department of Physics, University of Wisconsin, Madison, Wisconsin*

(*Received* 4 *November* 1964)


Einstein, Podolsky, Rosen, **Phys. Rev. 47**, 777 (1935)

JS Bell, **Physics 1**, 195 (1964)

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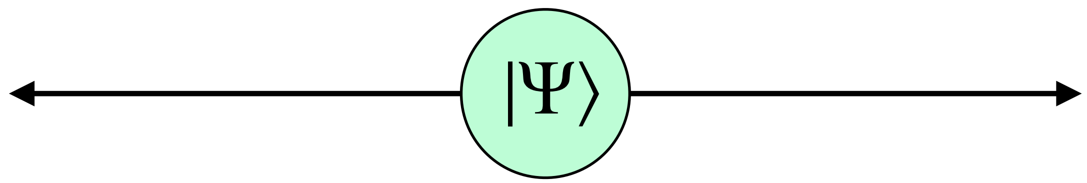


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Einstein, Podolsky, Rosen, **Phys. Rev. 47,** 777 (1935)
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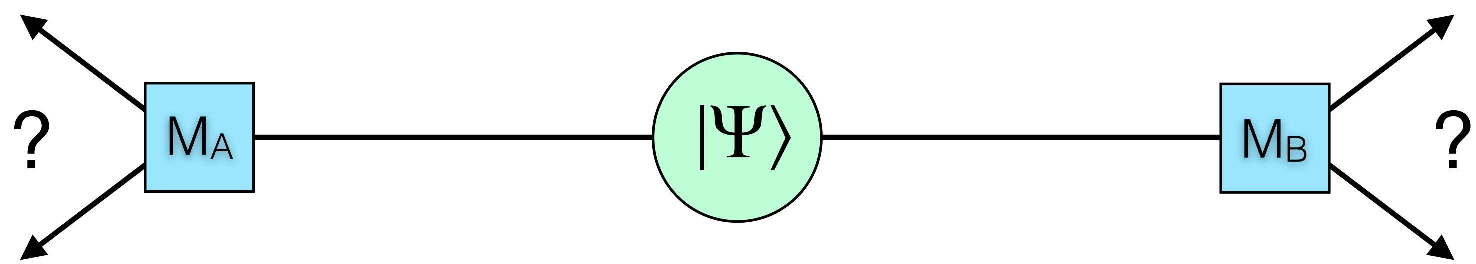


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# Entanglement: Milestones

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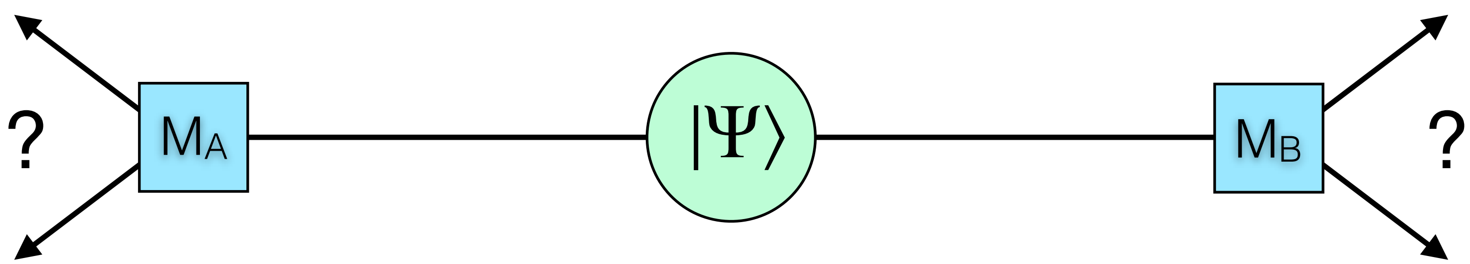


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$$|S| \leq 2$$

Einstein, Podolsky, Rosen, **Phys. Rev. 47,** 777 (1935)
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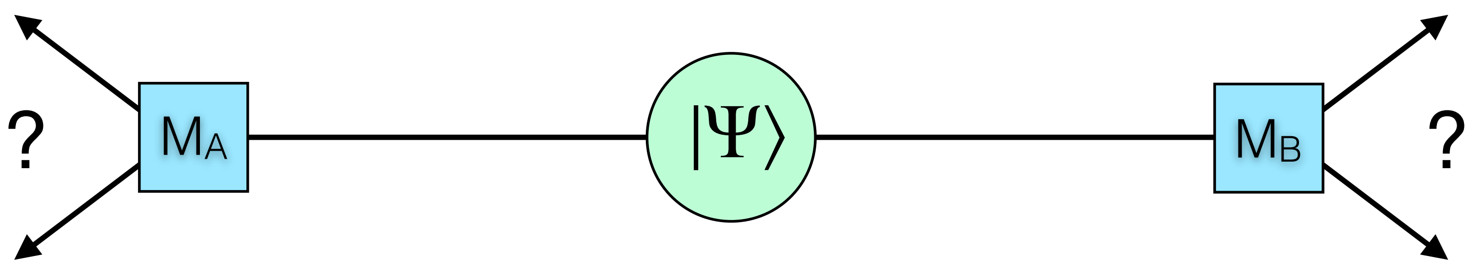


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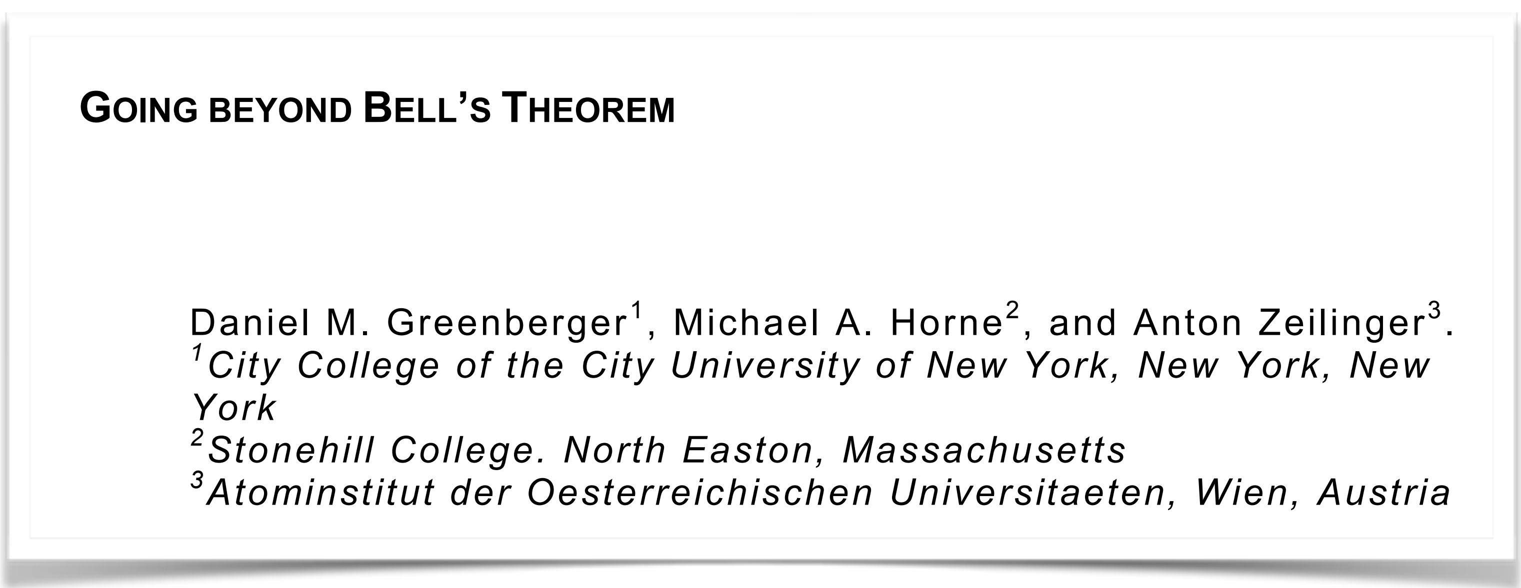

**GOING BEYOND BELL'S THEOREM**

Daniel M. Greenberger[1], Michael A. Horne[2], and Anton Zeilinger[3].
[1]*City College of the City University of New York, New York, New York*
[2]*Stonehill College. North Easton, Massachusetts*
[3]*Atominstitut der Oesterreichischen Universitaeten, Wien, Austria*

Einstein, Podolsky, Rosen, **Phys. Rev. 47**, 777 (1935)
JS Bell, **Physics 1**, 195 (1964)
Greenberger, Horne, Zeilinger, in: **Bell's Theorem, Quantum Theory, Conceptions of the Universe** (1989)

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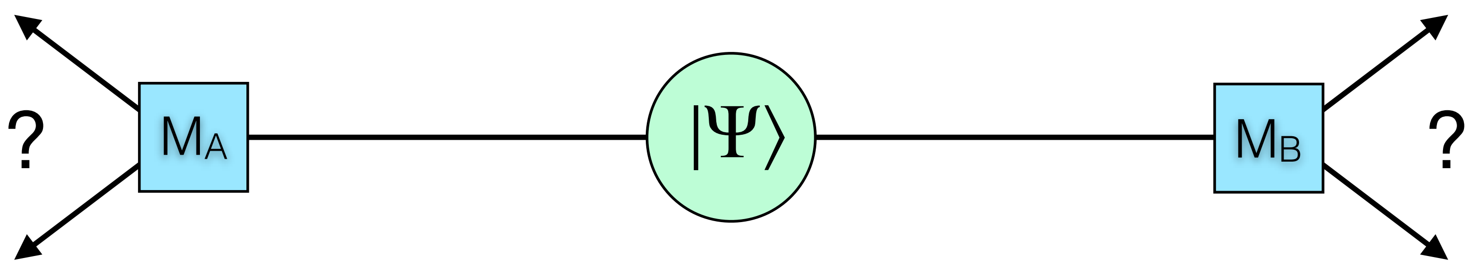


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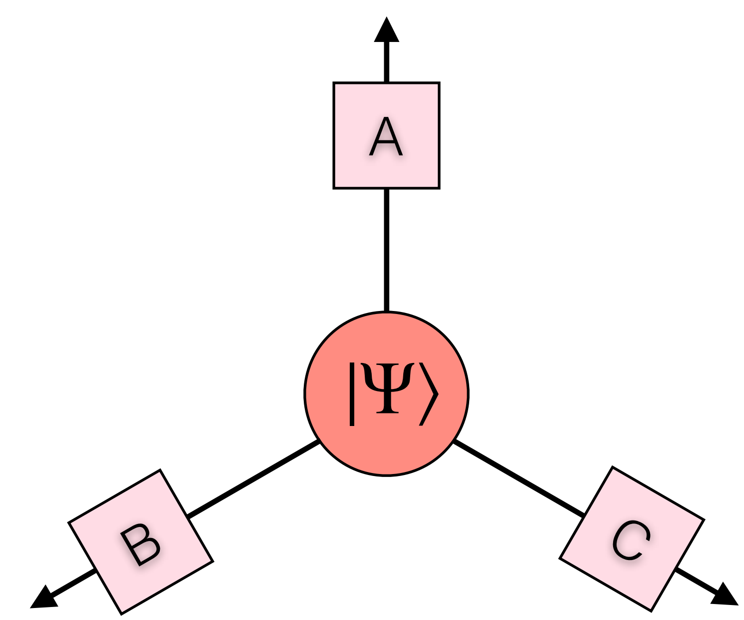


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Einstein, Podolsky, Rosen, **Phys. Rev. 47,** 777 (1935)
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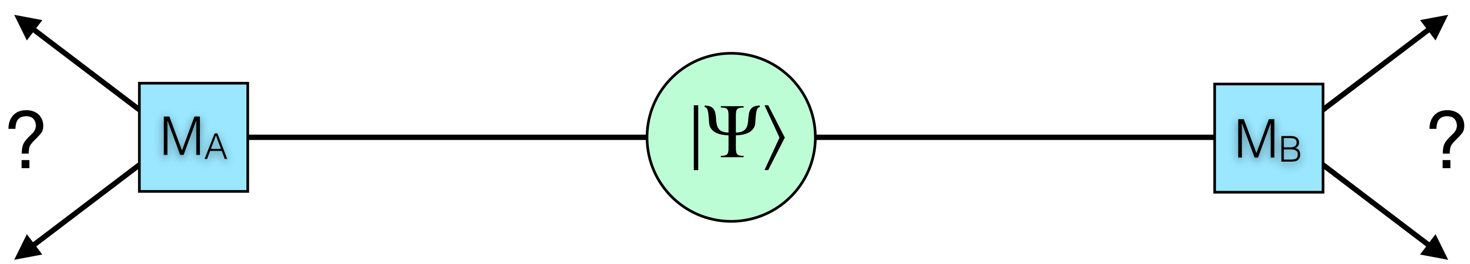


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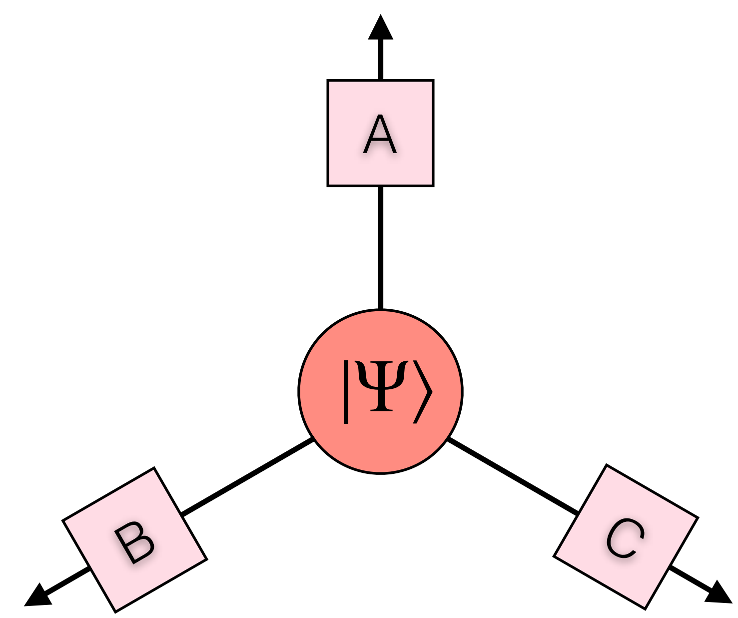
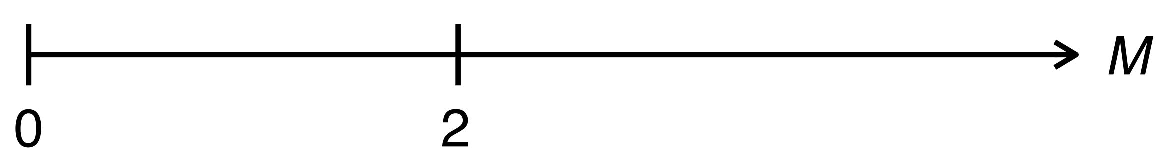



- Mermin parameter, $M = XXX - XYY - YXY - YYX$

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# Entanglement: Milestones

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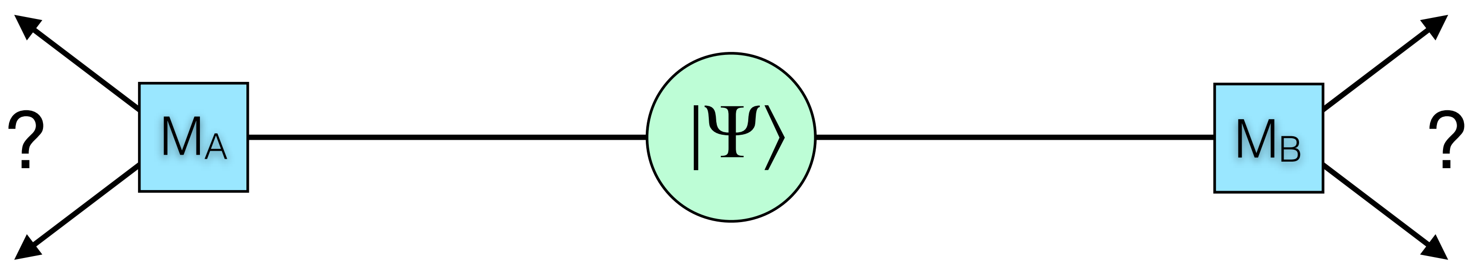


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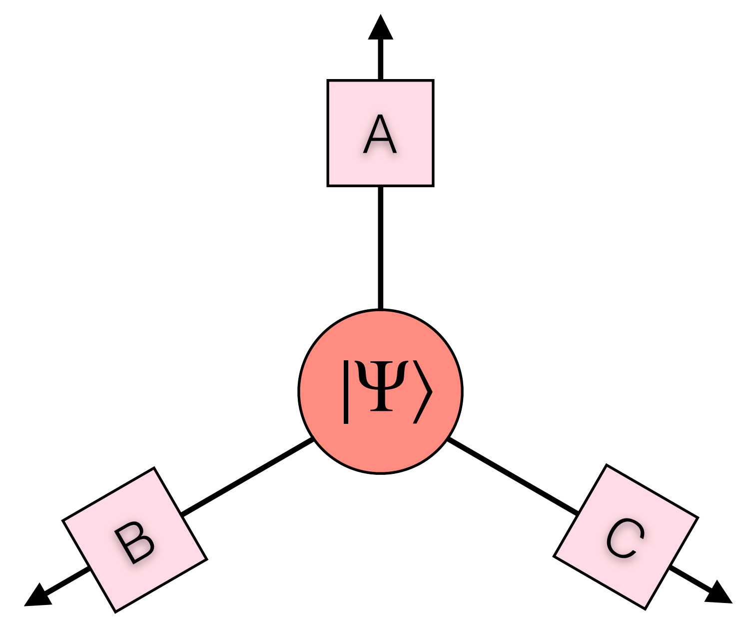


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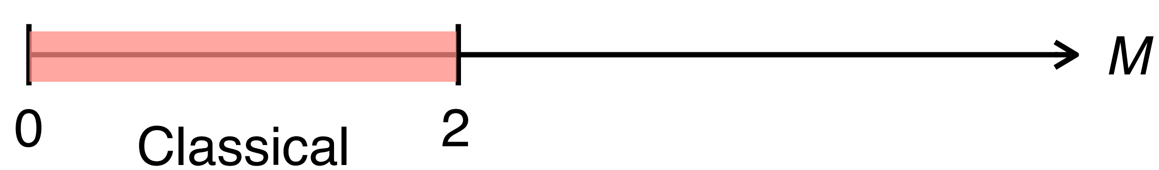


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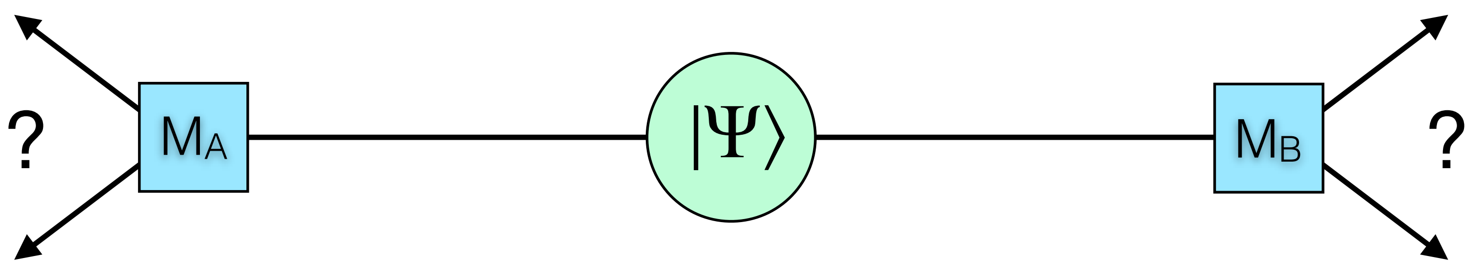


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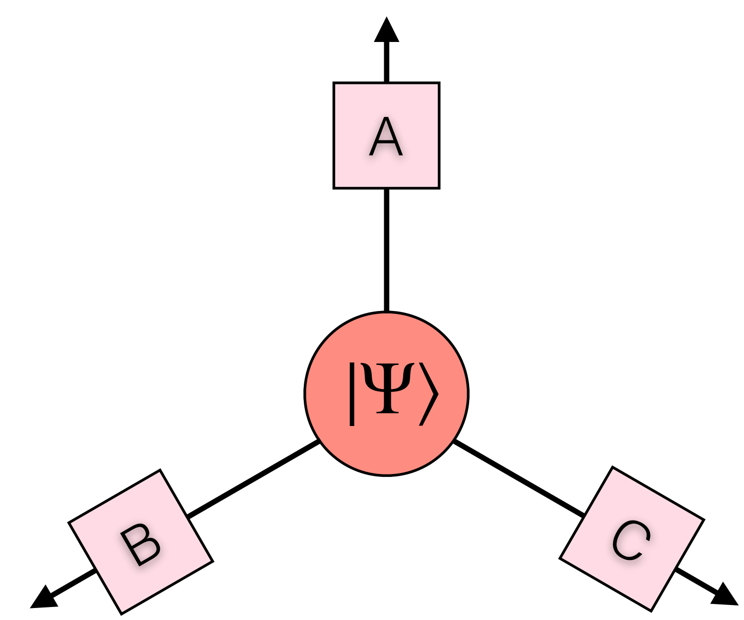


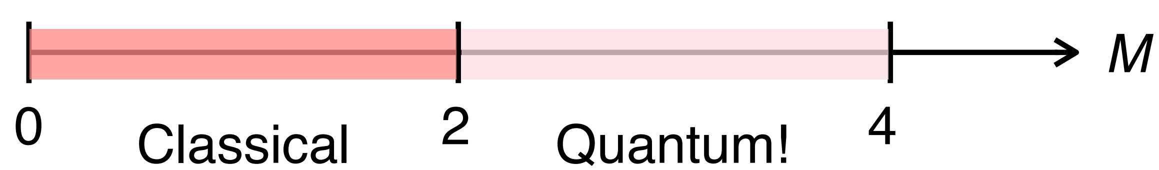


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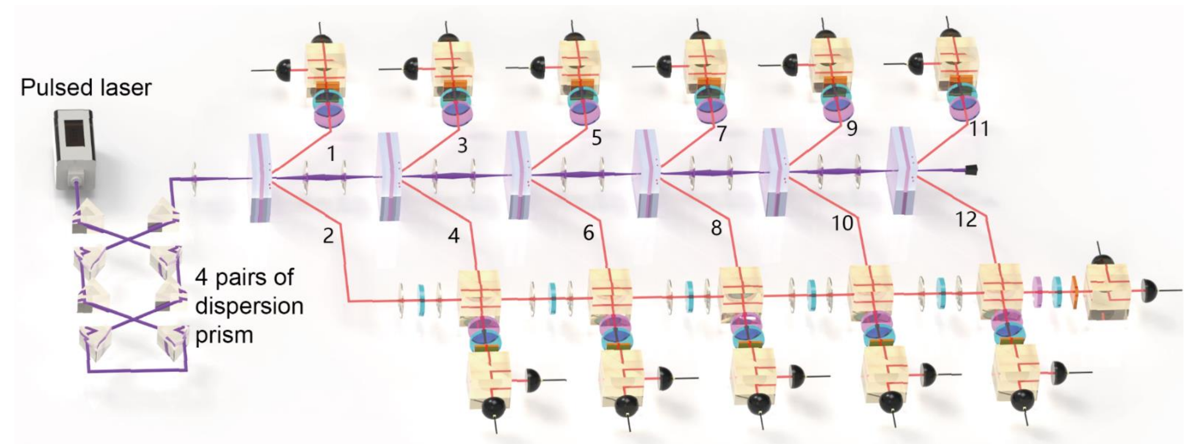
# MULTI-PARTICLE ENTANGLEMENT

# Multi-Particle Entanglement

- 12 photons: USTC China
  - ★ Zhong et al, **Phys. Rev. Lett. 121,** 250505 (2018)



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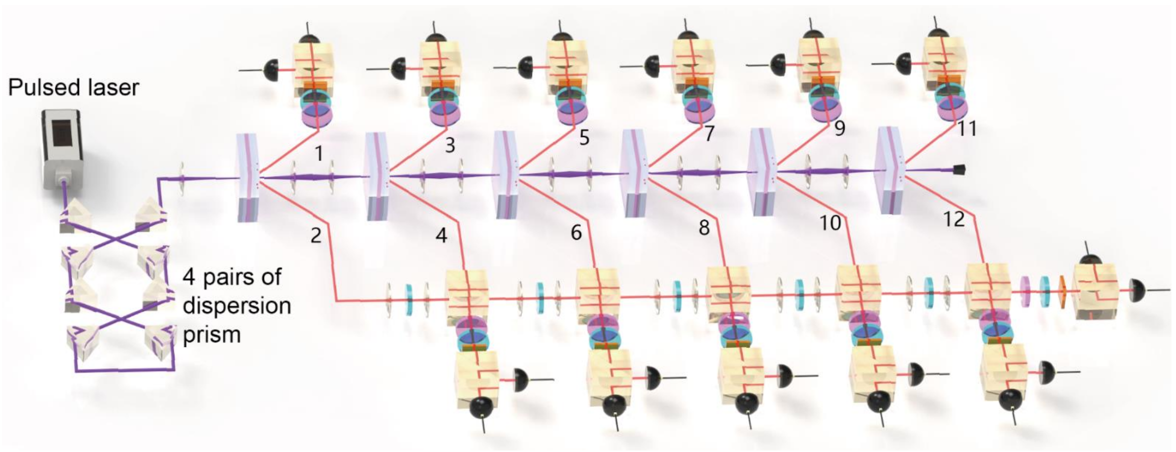


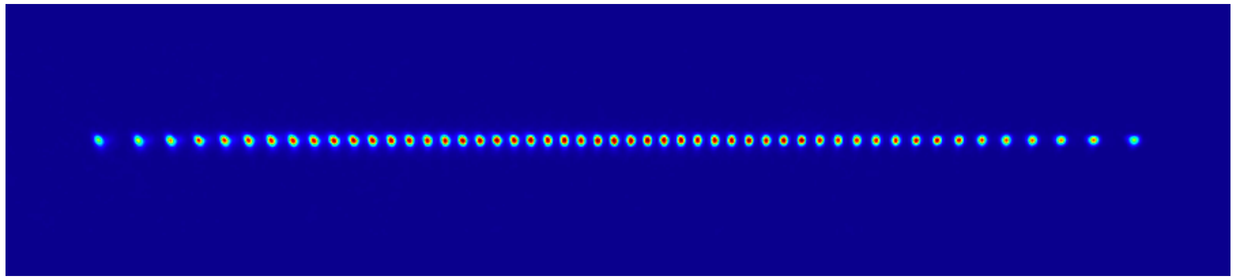

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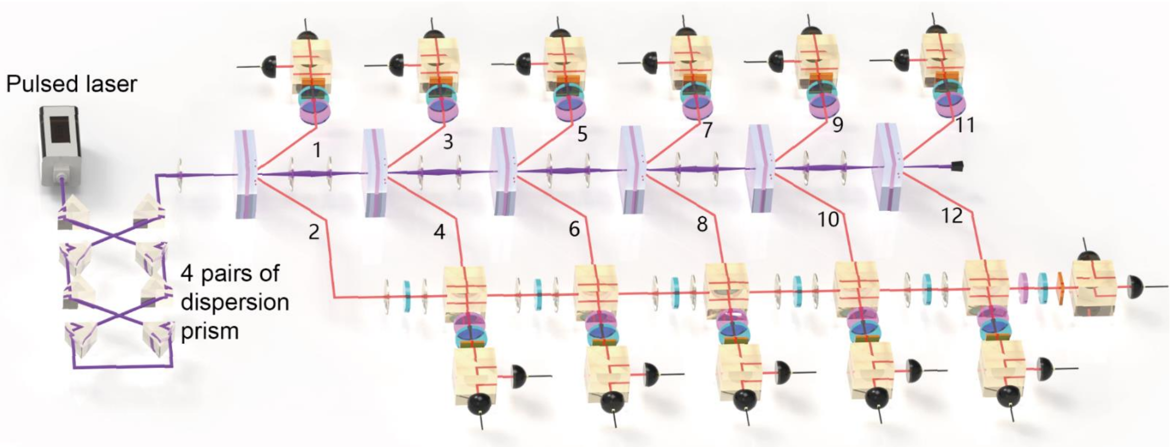


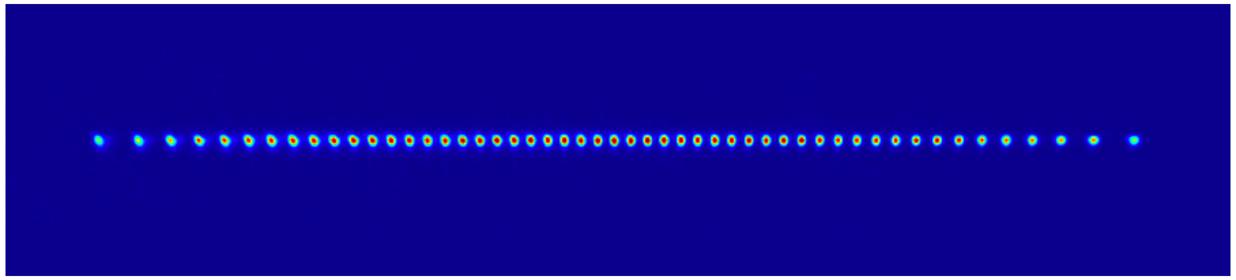

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- 20 SQUIDs: ICQI China
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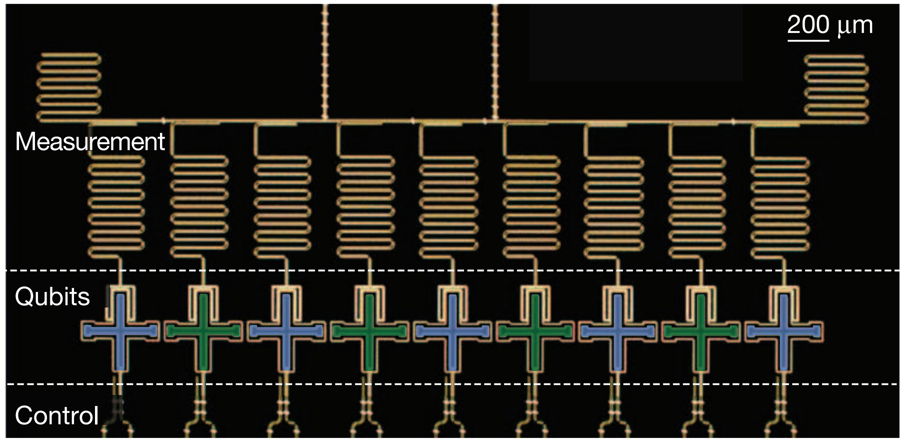

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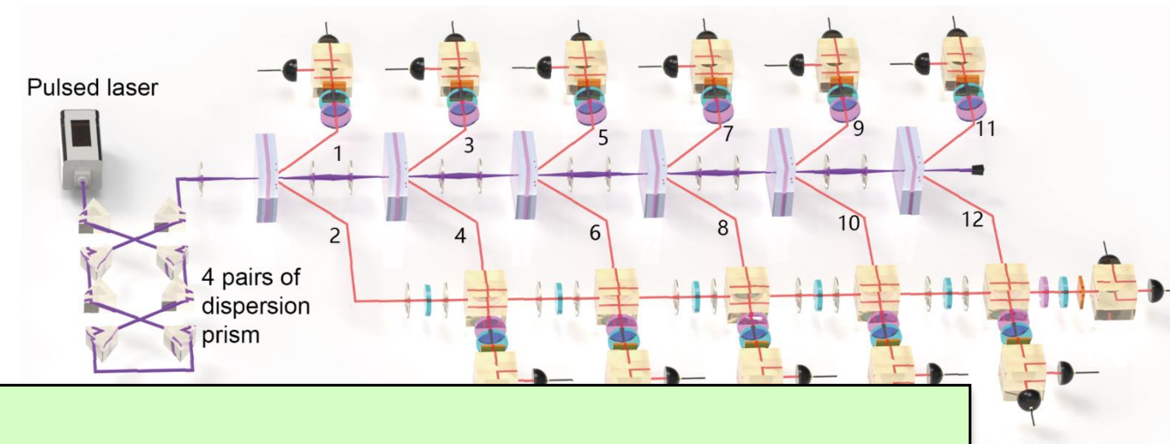


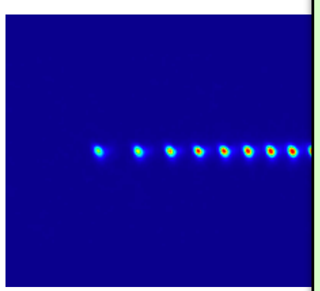

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X 8,

*N* particles entangled in a 2-dimensional Hilbert space:

$$|\Psi\rangle_{\mathrm{GHZ}} = \frac{1}{\sqrt{2}}\left[|0\rangle^{\otimes N} + |1\rangle^{\otimes N}\right]$$

- 20 SQUIDs: ICQI China
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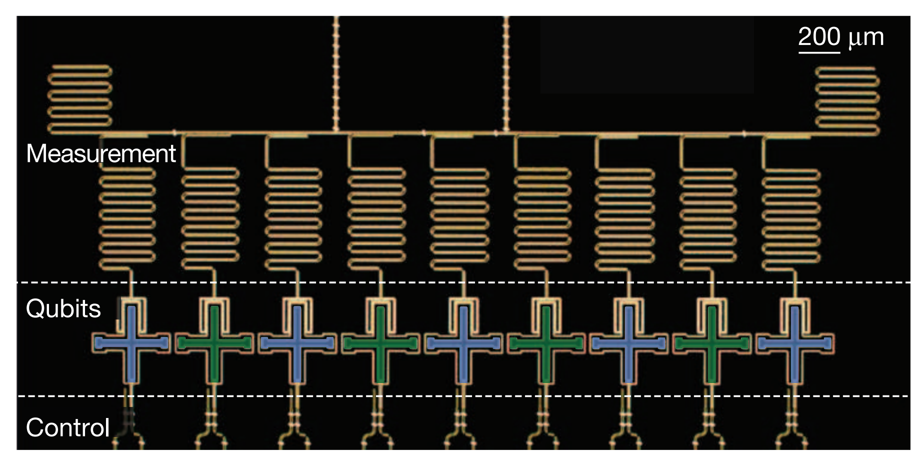
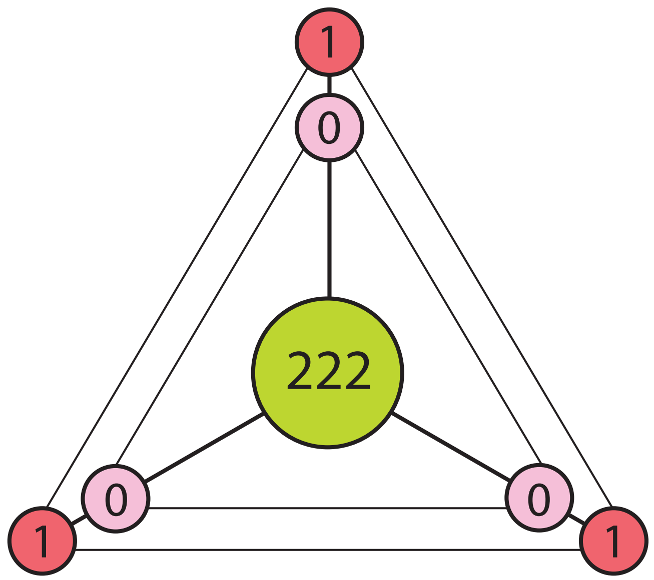


# HIGH-DIMENSIONAL MULTIPARTITE ENTANGLEMENT?

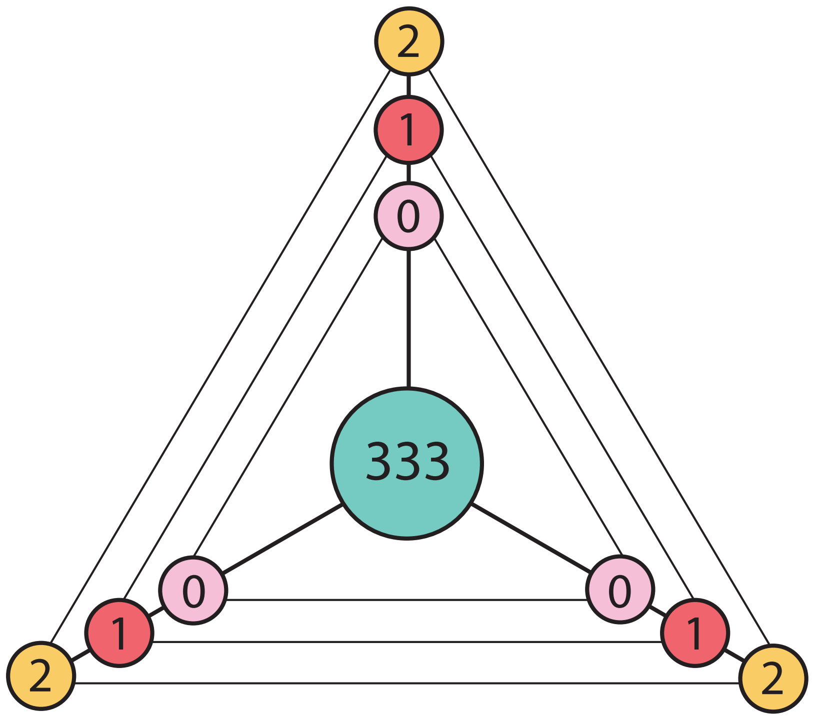
# High-Dimensional Multipartite Entanglement?

$$|\Psi\rangle_{222} = \frac{1}{\sqrt{2}}[|000\rangle + |111\rangle]$$



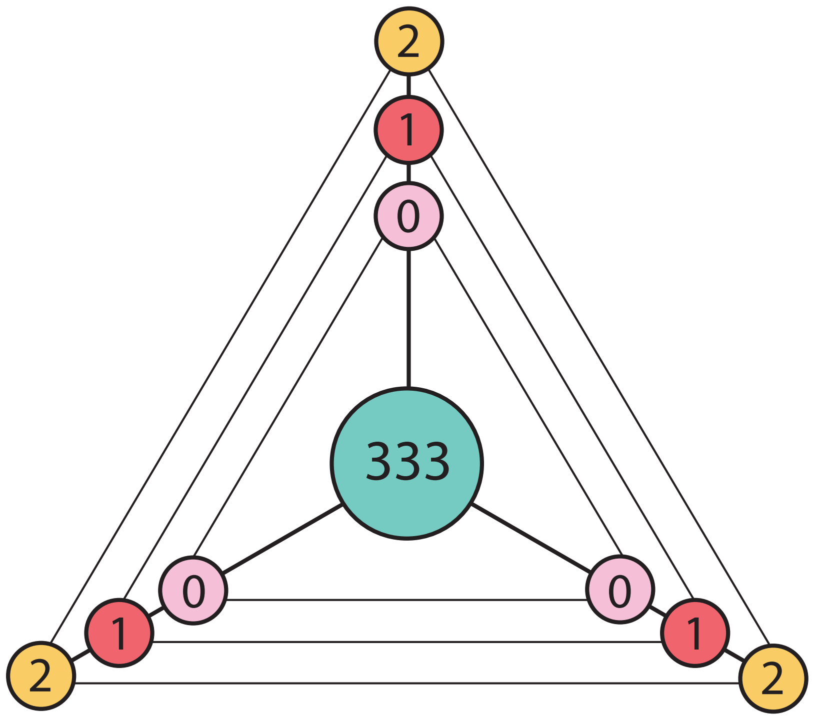
# High-Dimensional Multipartite Entanglement?

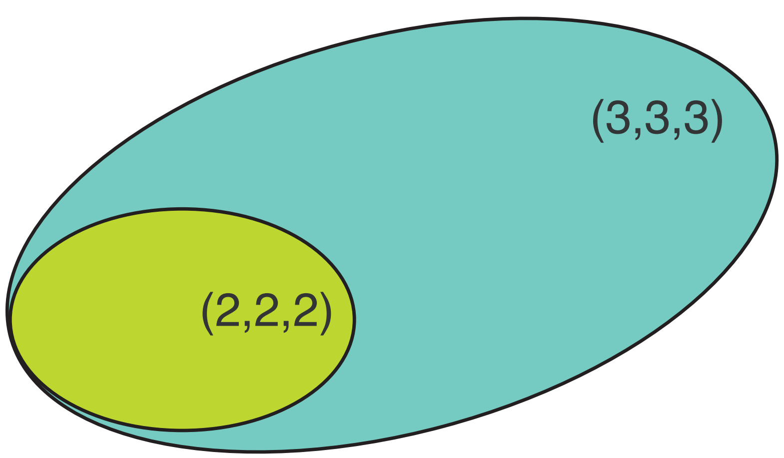
$$|\Psi\rangle_{333} = \frac{1}{\sqrt{3}}[|000\rangle + |111\rangle + |222\rangle]$$



# HIGH-DIMENSIONAL MULTIPARTITE ENTANGLEMENT?

$$|\Psi\rangle_{333} = \frac{1}{\sqrt{3}}[|000\rangle + |111\rangle + |222\rangle]$$




Huber and de Vicente, **Phys. Rev. Lett. 110,** 030501 (2013)

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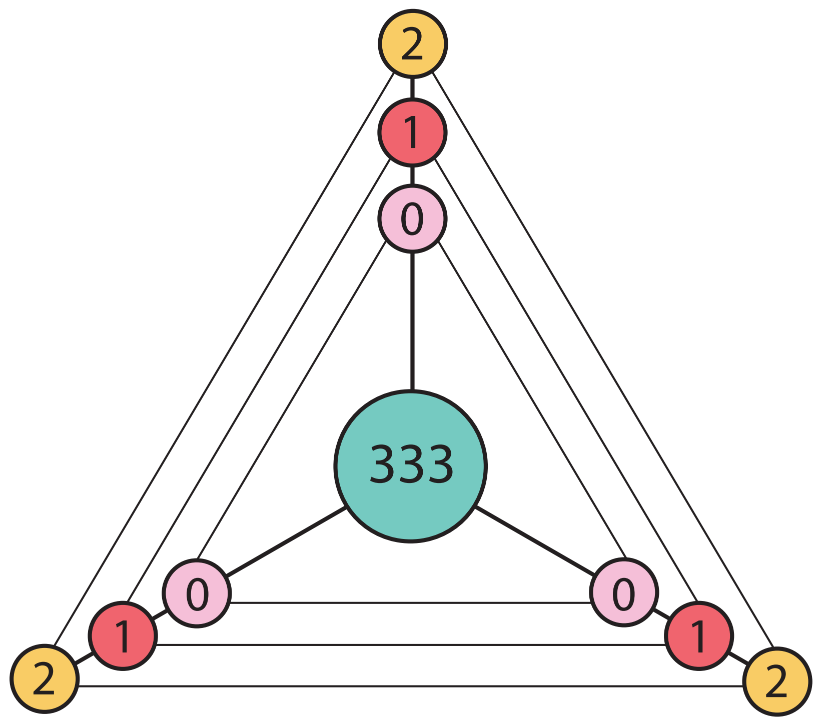


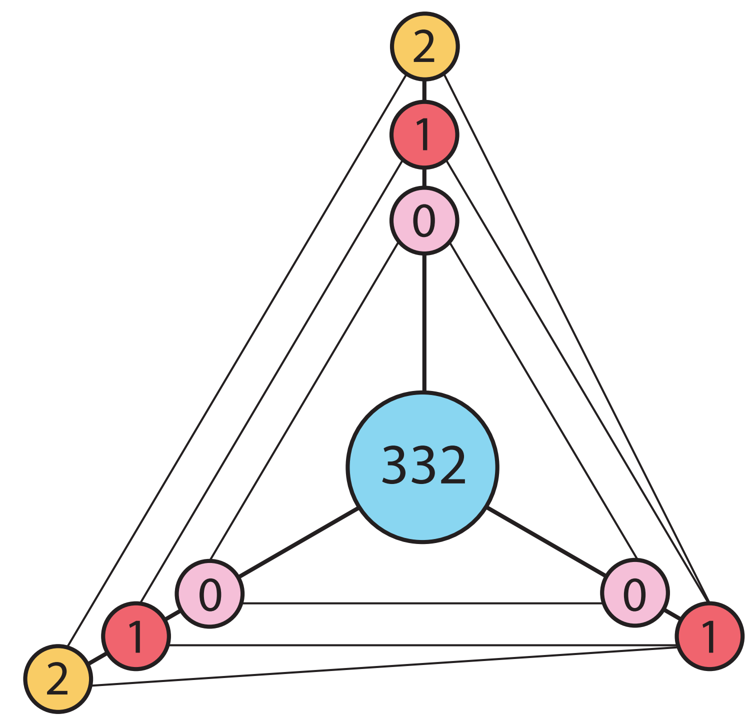


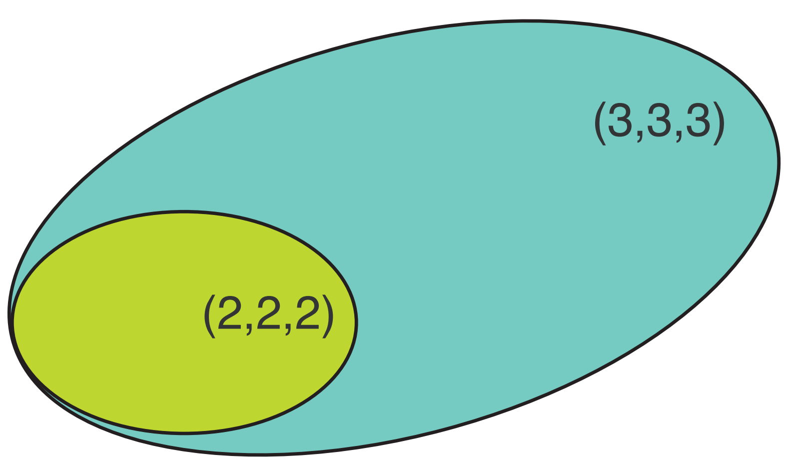


Huber and de Vicente, **Phys. Rev. Lett. 110,** 030501 (2013)

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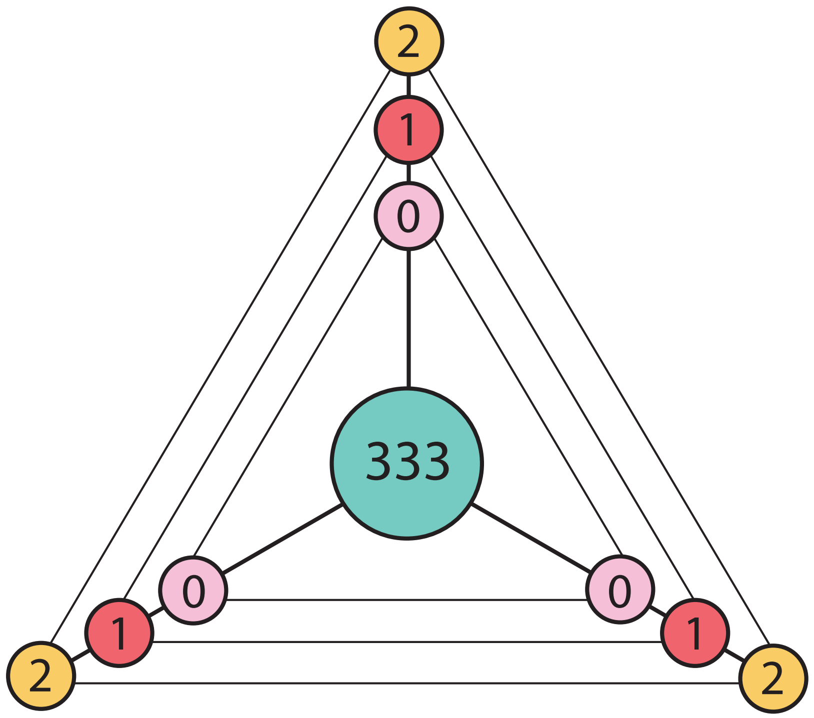


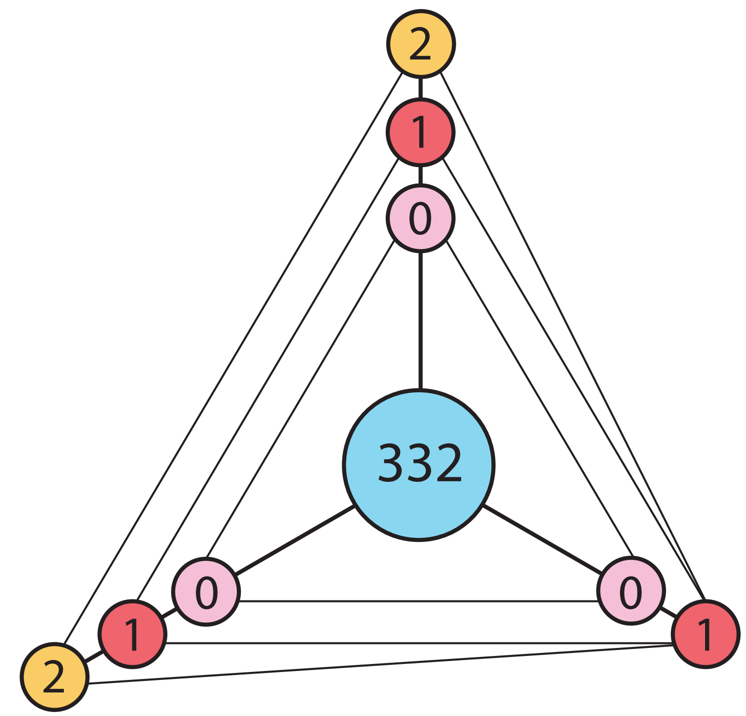


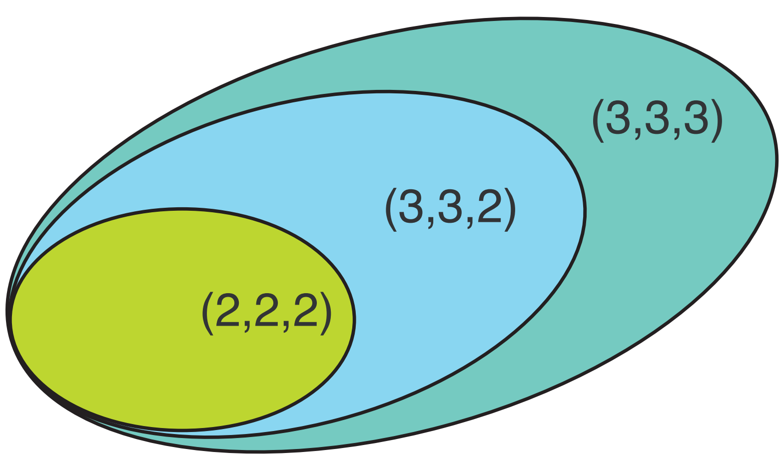


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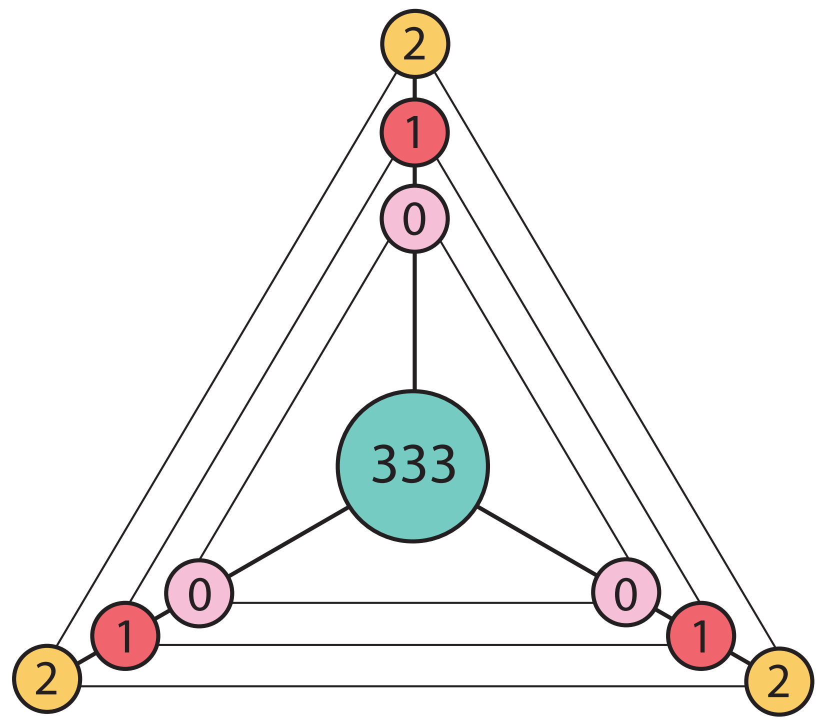


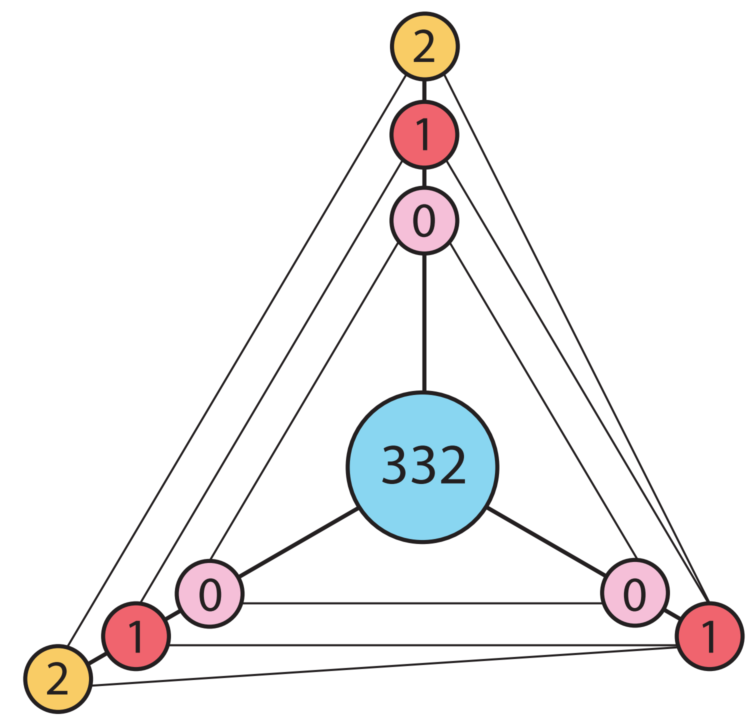


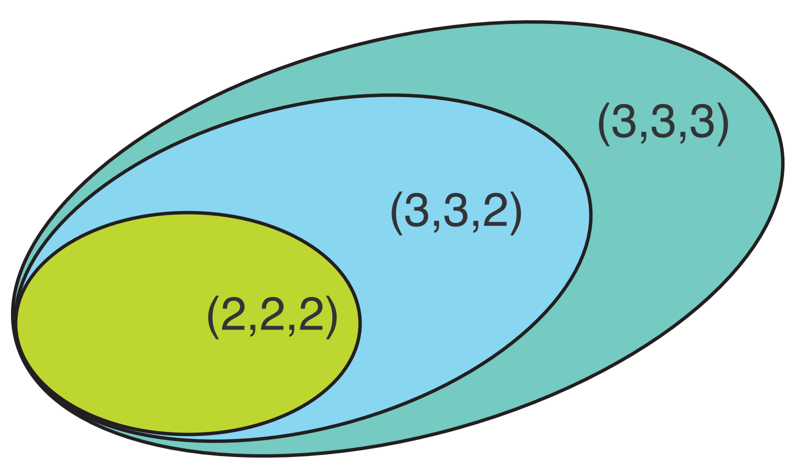


$$(3,3,2)$$

$\mathrm{rank}(\rho_{A|BC}) = 3$

$\mathrm{rank}(\rho_{B|AC}) = 3$

$\mathrm{rank}(\rho_{C|AB}) = 2$

Huber and de Vicente, **Phys. Rev. Lett. 110**, 030501 (2013)

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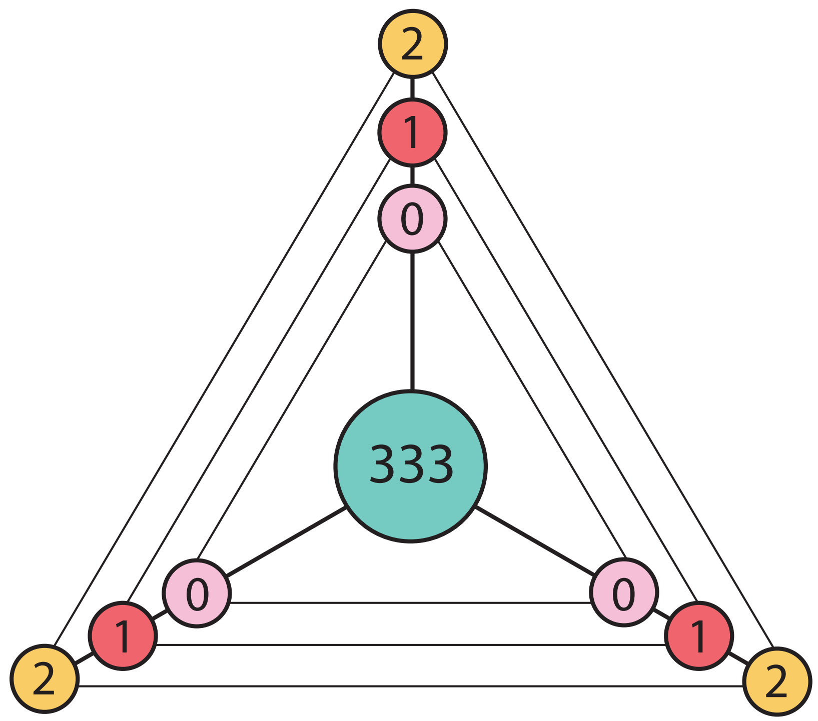


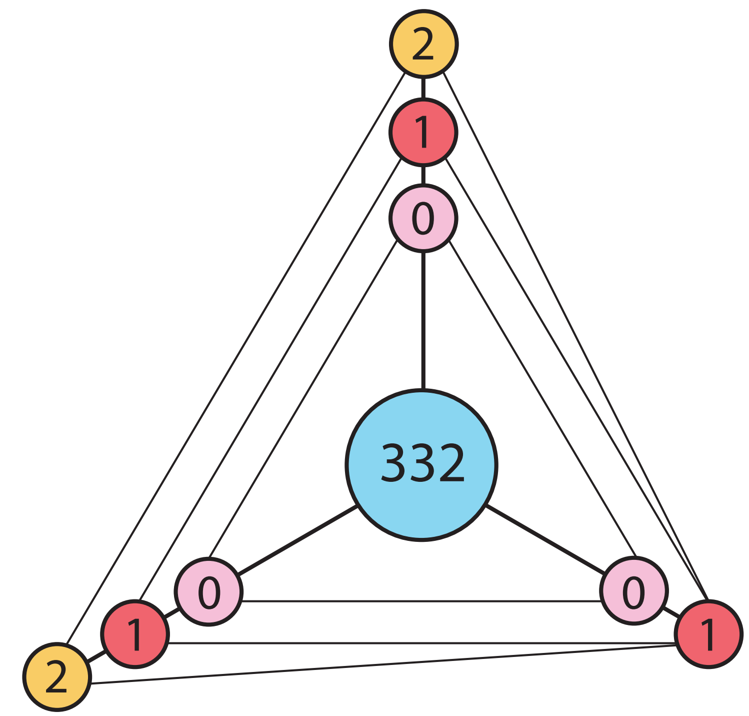


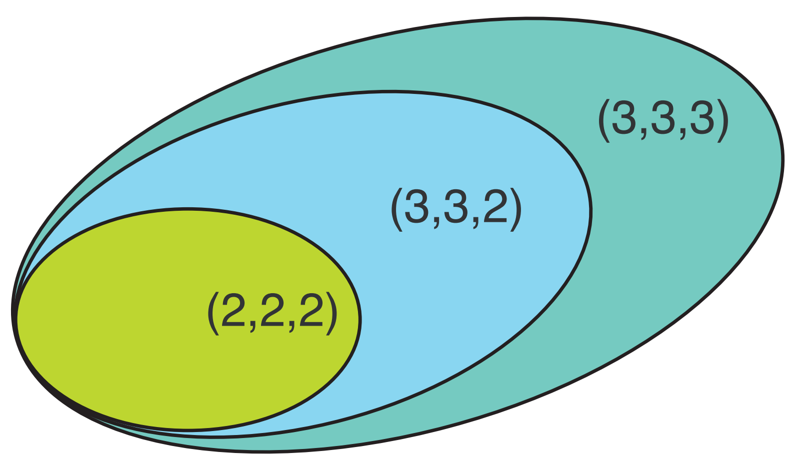


- <u>Asymmetric</u> states: only possible when *N* and *D* are both greater than 2

Huber and de Vicente, **Phys. Rev. Lett. 110,** 030501 (2013)

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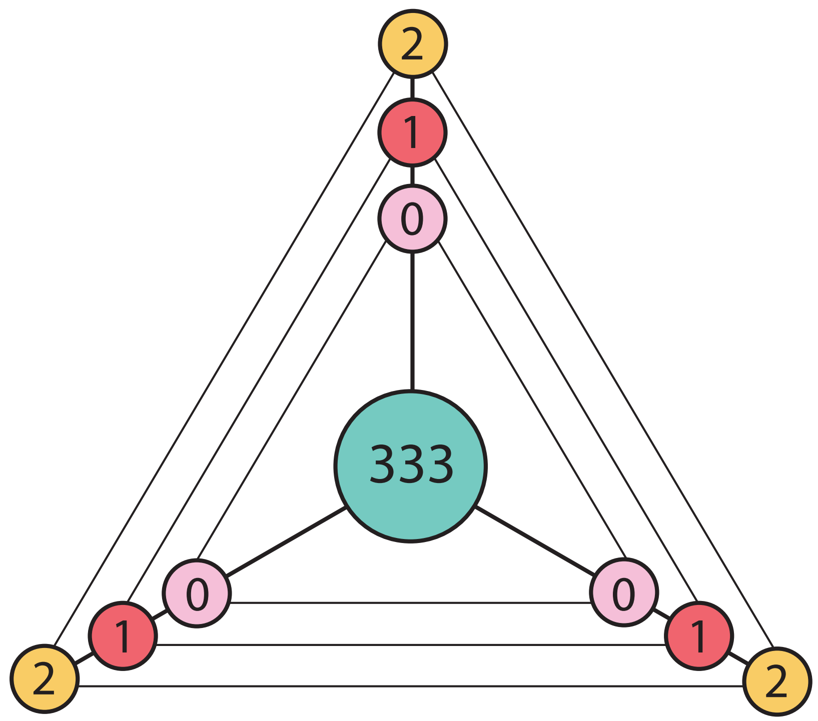


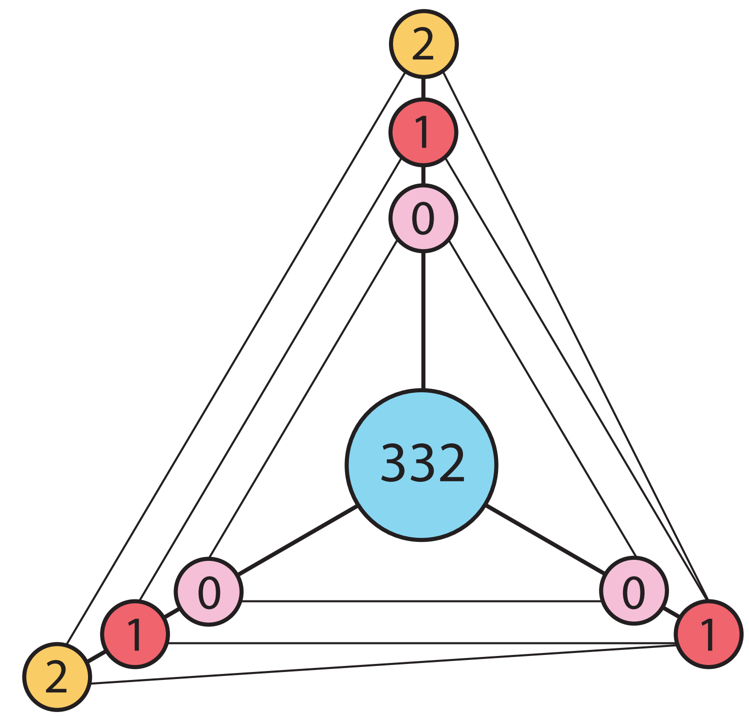


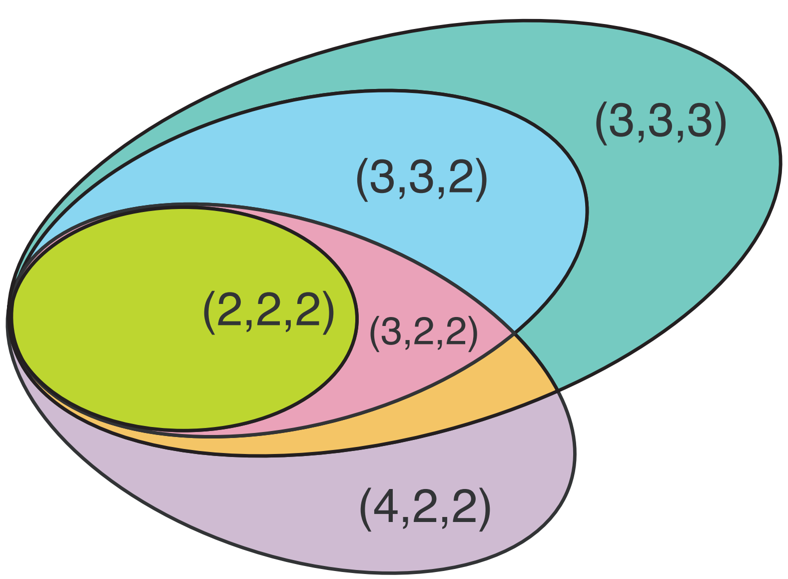


- Asymmetric states: only possible when *N* and *D* are both greater than 2
- Vast family of multi-particle entangled states!

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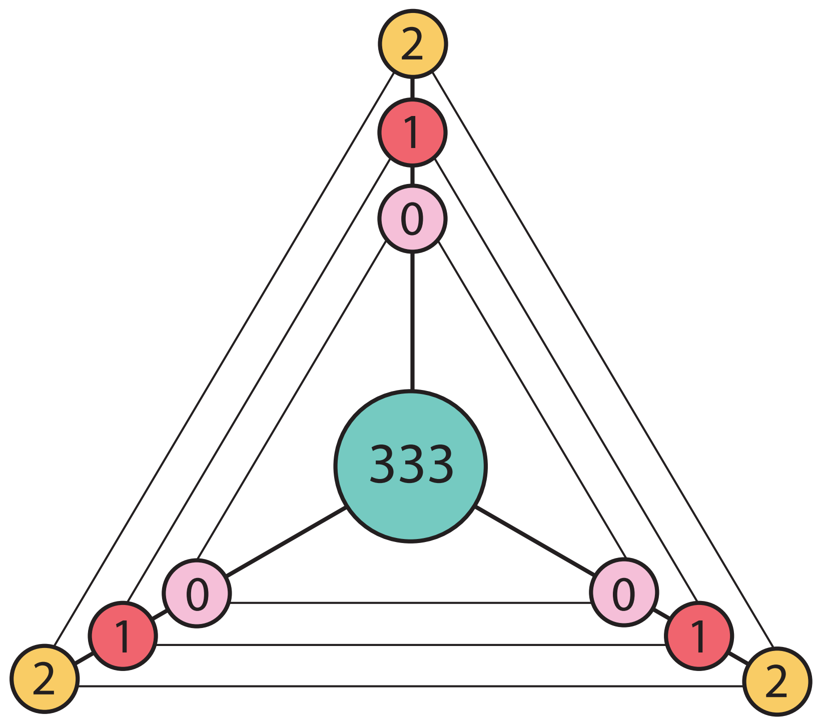


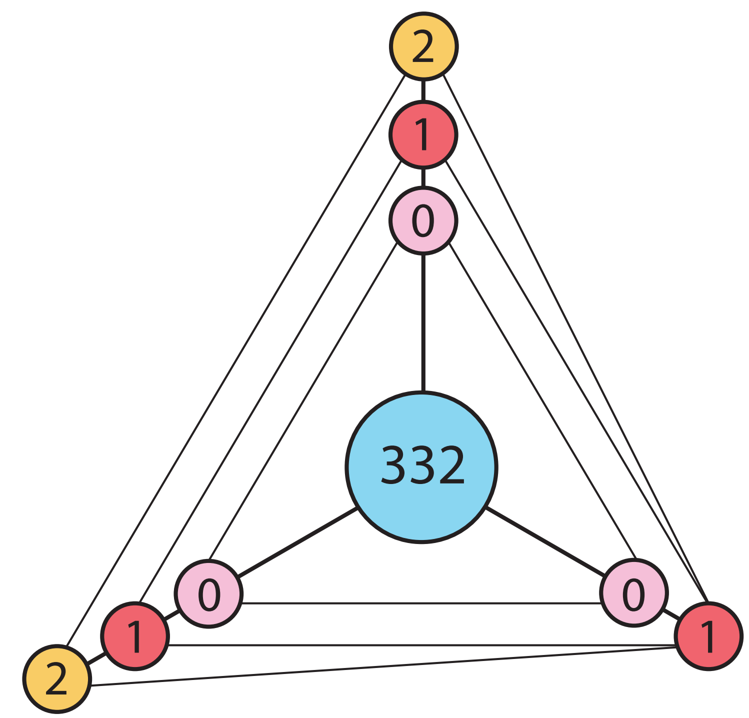


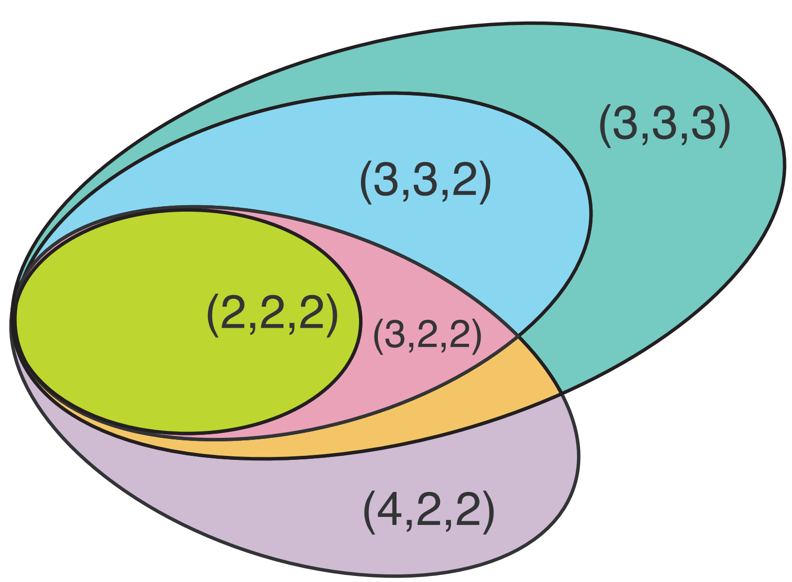


- Asymmetric states: only possible when *N* and *D* are both greater than 2
- Vast family of multi-particle entangled states!
- Creation of 332 and 333 entanglement

Huber and de Vicente, **Phys. Rev. Lett. 110,** 030501 (2013)

# First Multi-photon Experiments

VOLUME 78 21 APRIL 1997 NUMBER 16

## Three-Particle Entanglements from Two Entangled Pairs

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[1]*Institut für Experimentalphysik, Universität Innsbruck, A-6020 Innsbruck, Austria*
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[3]*Instytut Fizyki Teoretycznej i Astrofizyki, Uniwersytet Gdański, PL-80-952 Gdańsk, Poland*
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- Based on erasing "which source" information

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- Based on <u>erasing</u> “which source” information
- Entangled photons from **two** crystals (two pairs)

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- Based on <u>erasing</u> “which source” information
- Entangled photons from **two** crystals (two pairs)
- *“which crystal”* information erased

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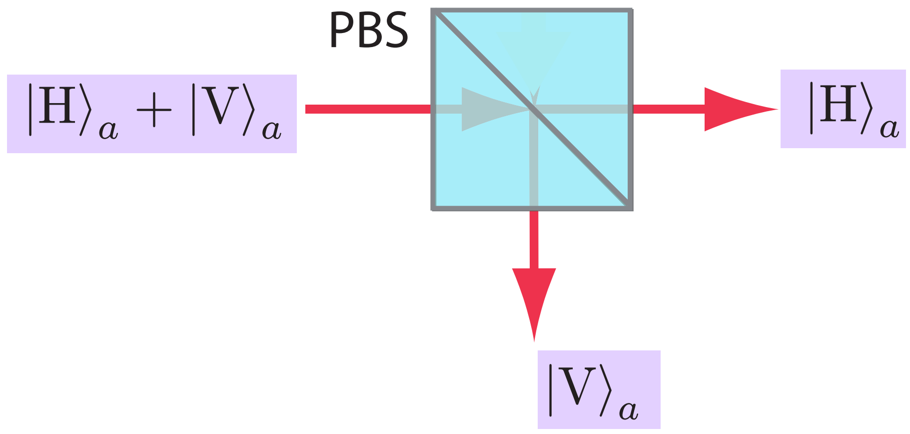

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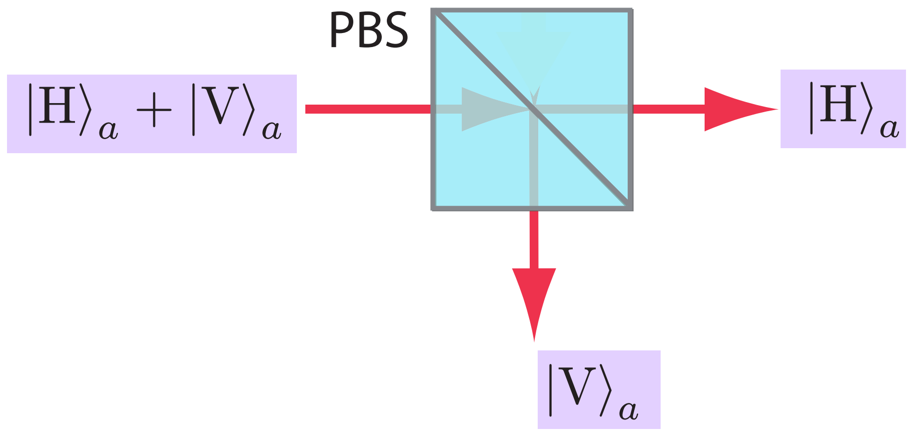


- Polarizing beam splitter (PBS): <u>reflects</u> **V**, *transmits* **H**

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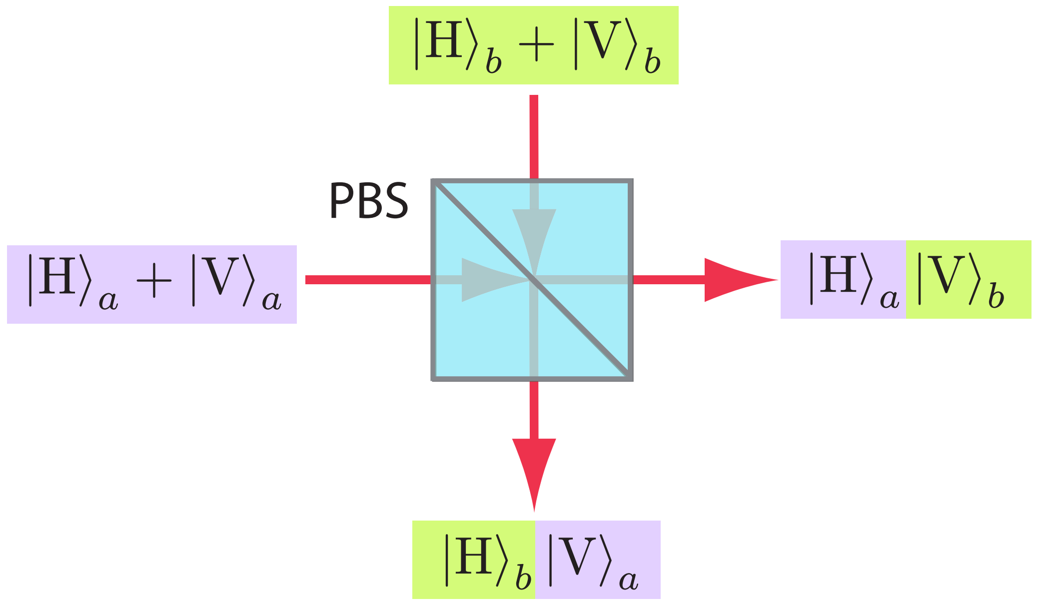
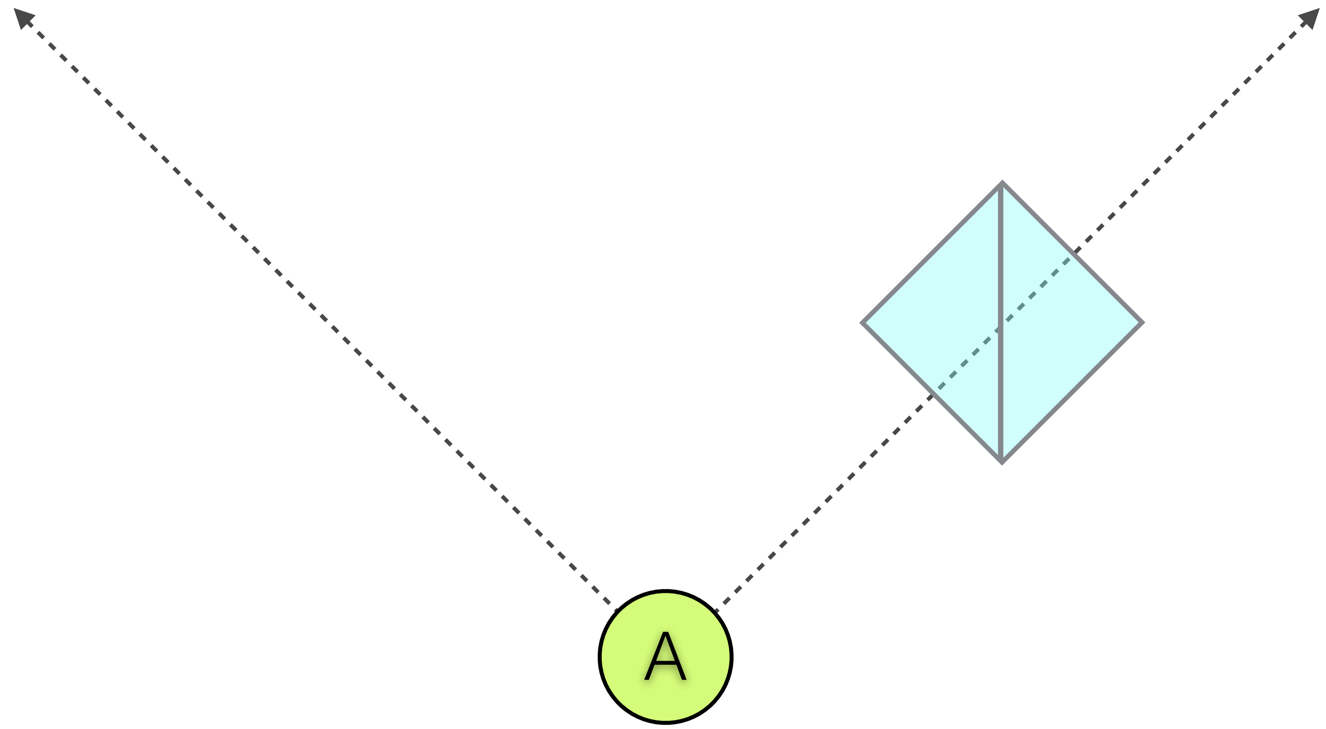


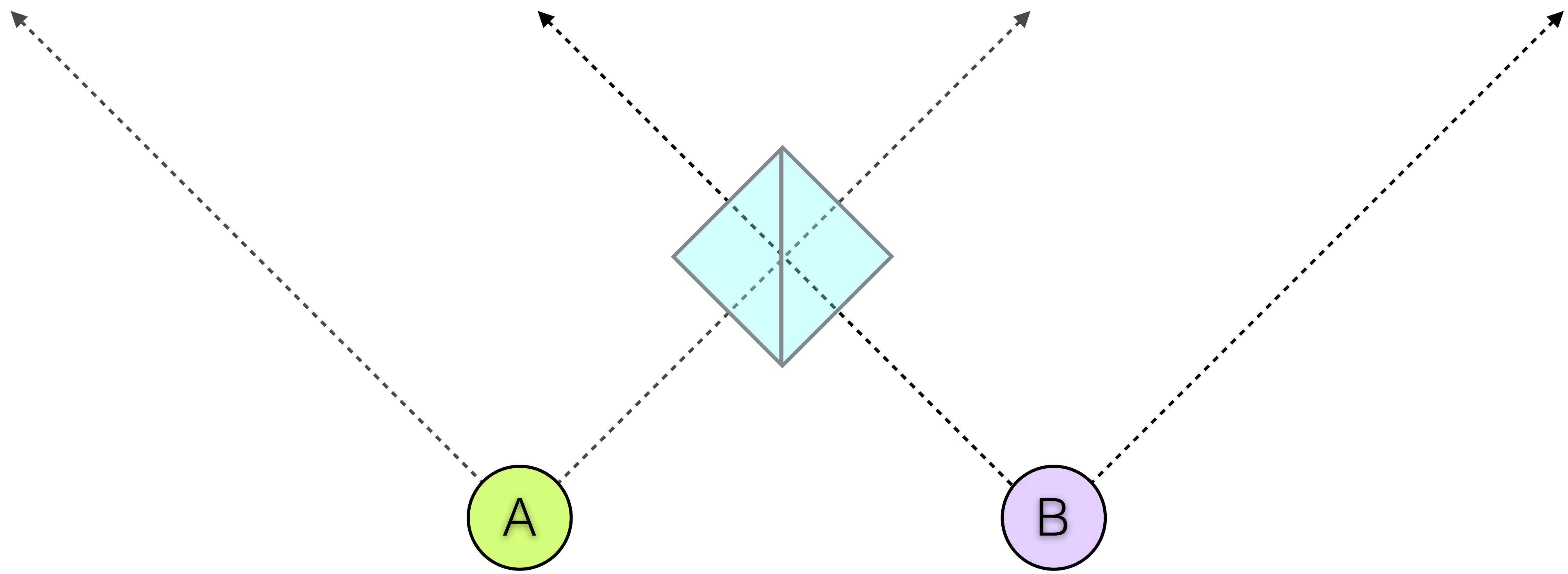

- Polarizing beam splitter (PBS): reflects **V**, *transmits* **H**
- **Mixes** polarizations of two input photons

# First Multi-photon Experiments




$$|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2$$

# First Multi-photon Experiments

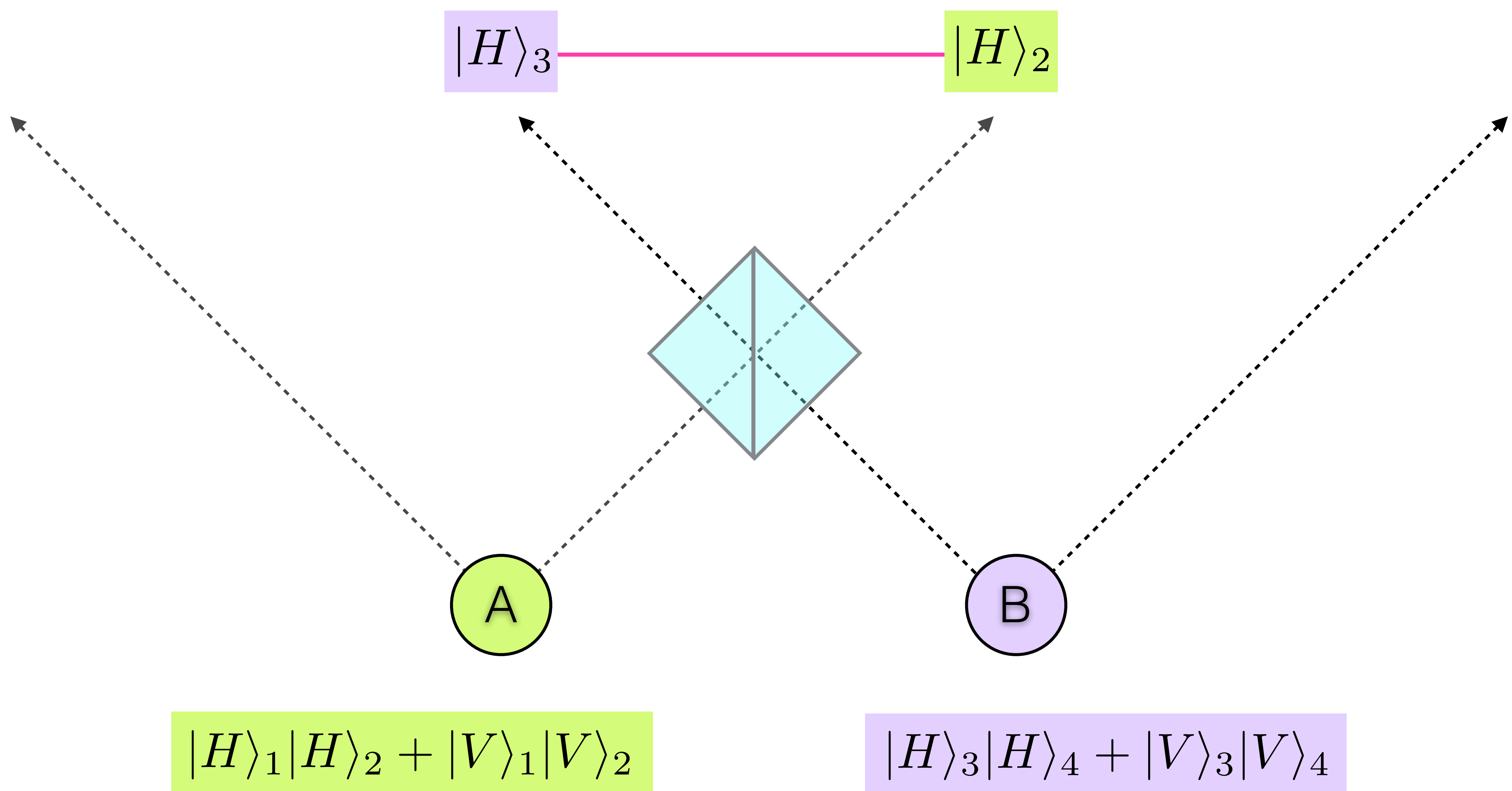



$$|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2$$

$$|H\rangle_3|H\rangle_4 + |V\rangle_3|V\rangle_4$$

Pan et al, **Phys. Rev. Lett. 86,** 4435-4439 (2001)

# First Multi-photon Experiments

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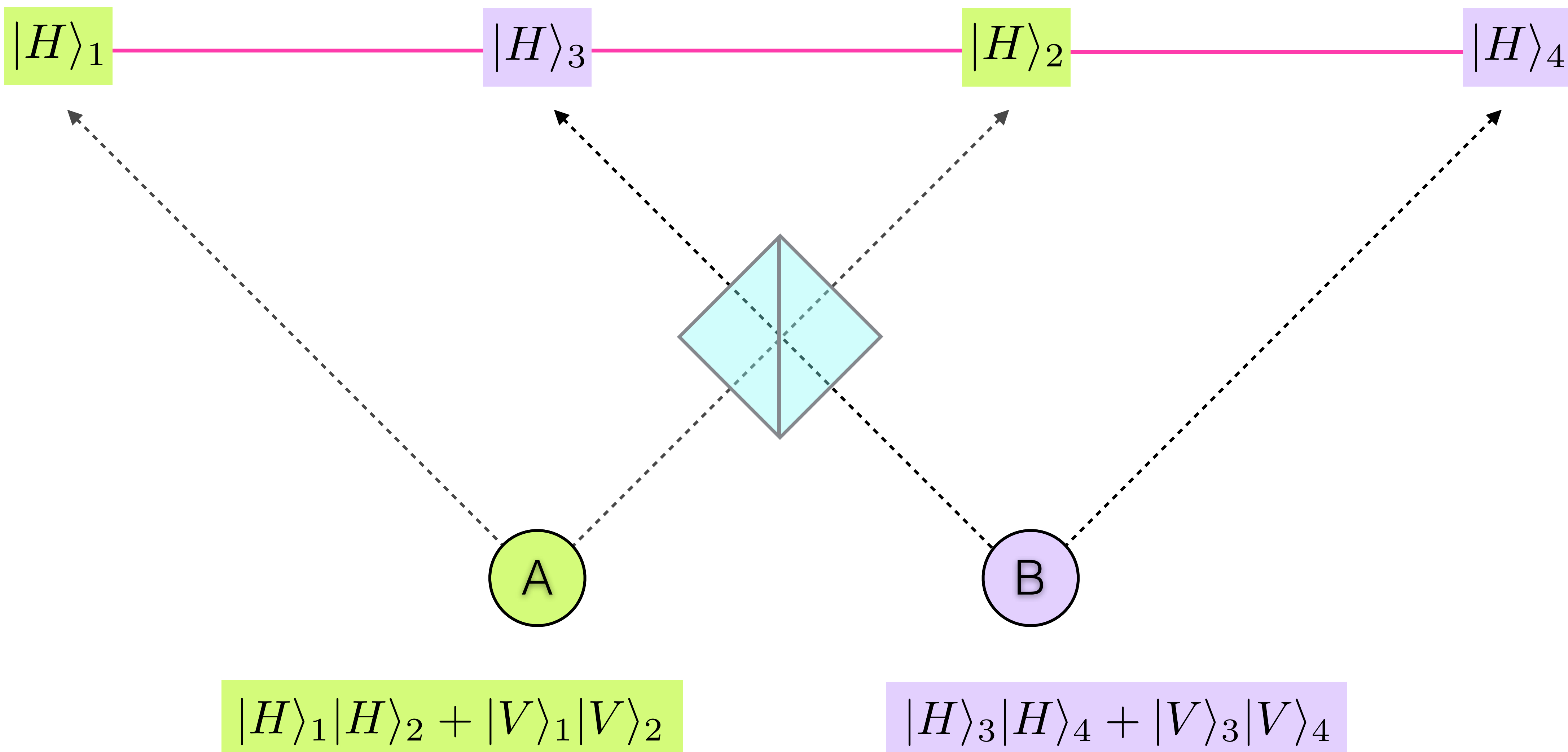


- Coincidences only obtained between same polarizations

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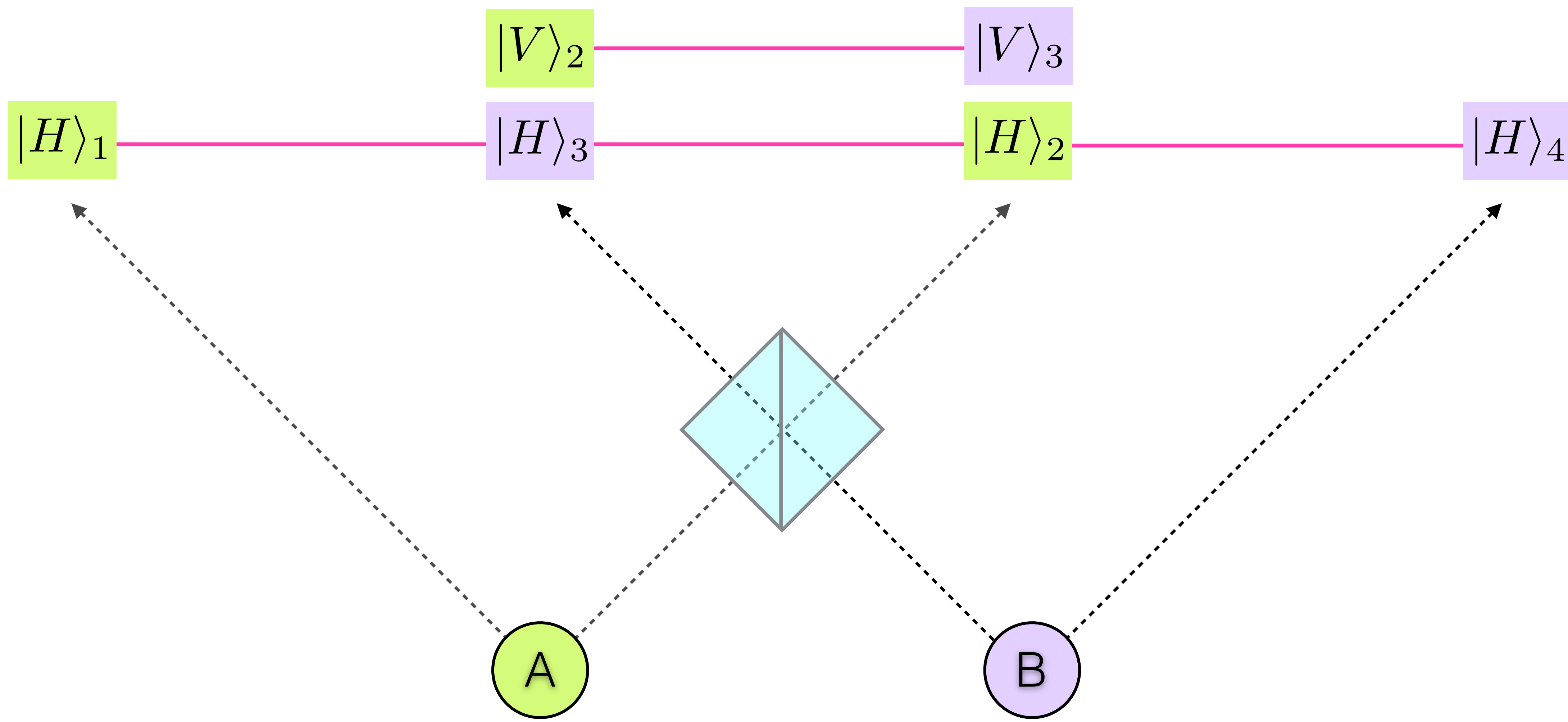


$|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2$

$|H\rangle_3|H\rangle_4 + |V\rangle_3|V\rangle_4$

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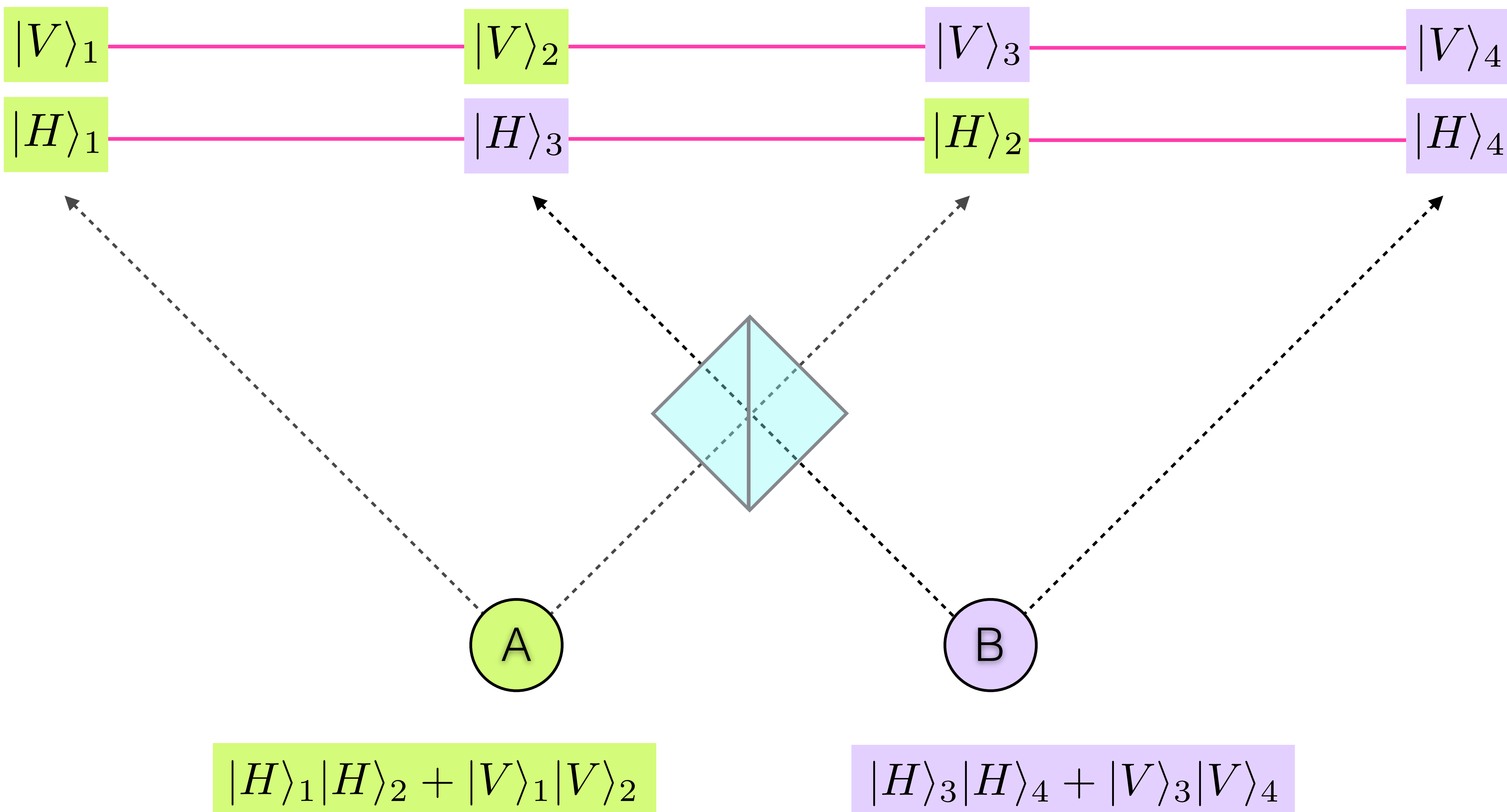


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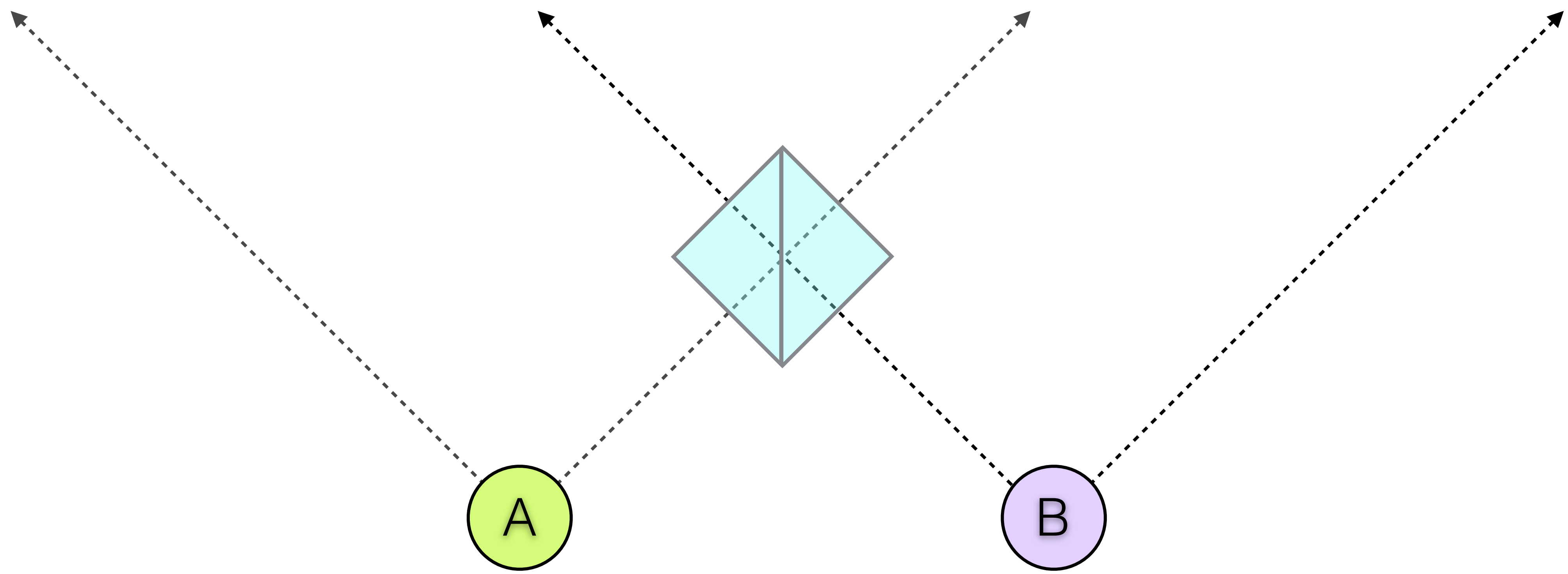
$$|\Psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}[|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4 + |V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4]$$




$$|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2$$

$$|H\rangle_3|H\rangle_4 + |V\rangle_3|V\rangle_4$$

- Coincidences only obtained between same polarizations
- Coherent superposition — four photon GHZ state!

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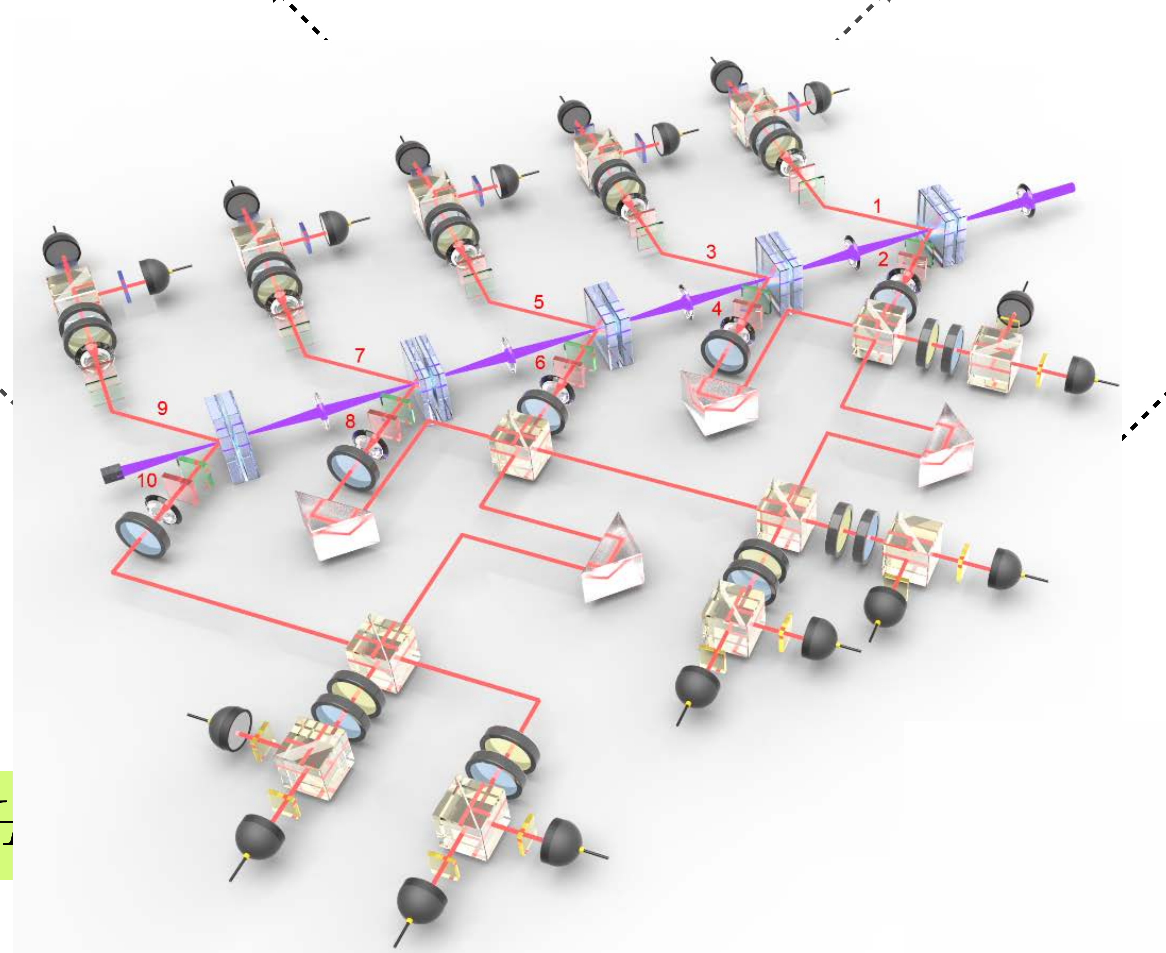
$|H\rangle_1|H$




$\rangle_3|V\rangle_4$

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# A Beam Splitter for OAM

Leach, Padgett, Barnett, Arnold, Courtial, PRL 88, 257901 (2002)

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- Why polarisation? Tools exist!

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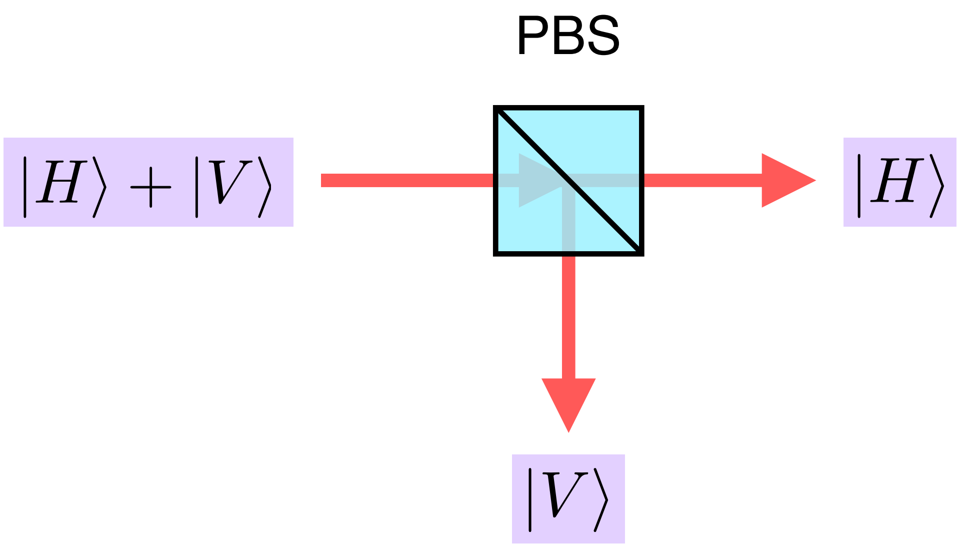


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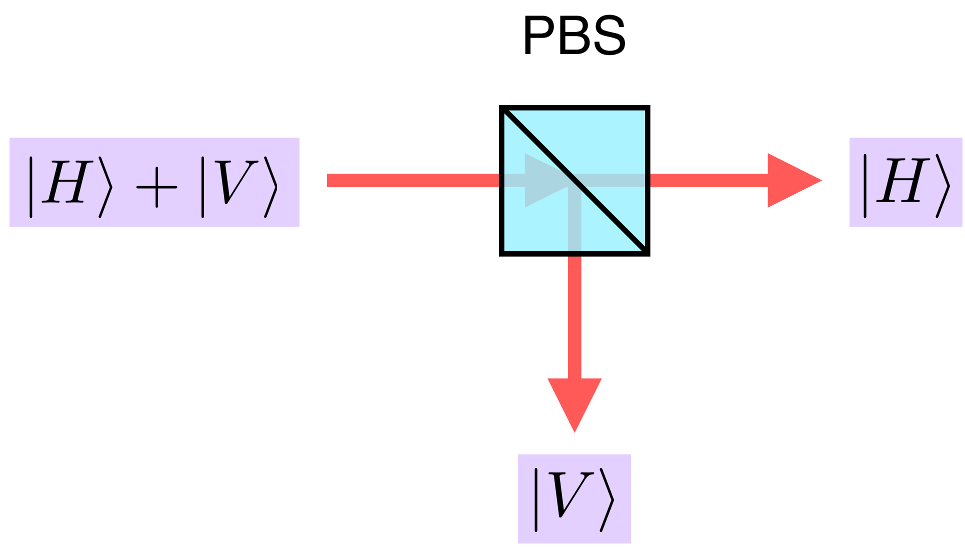


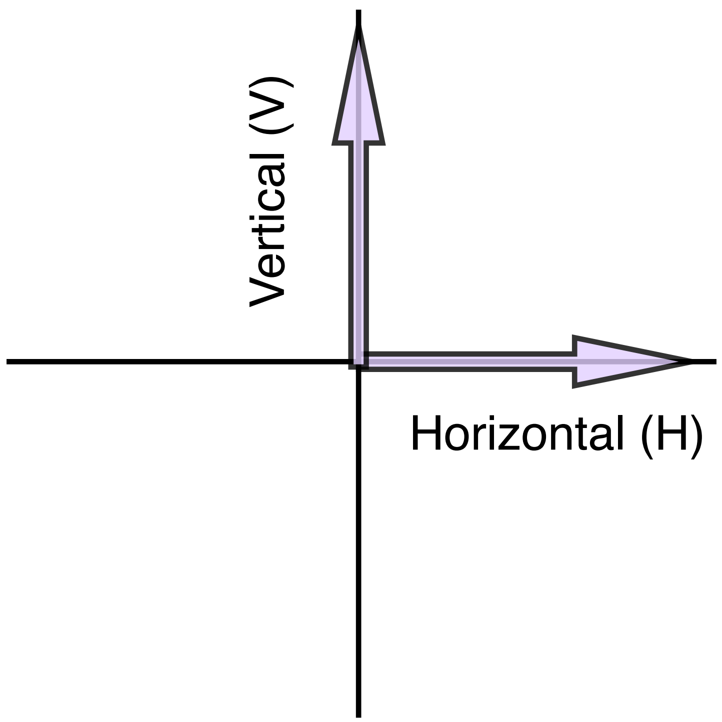


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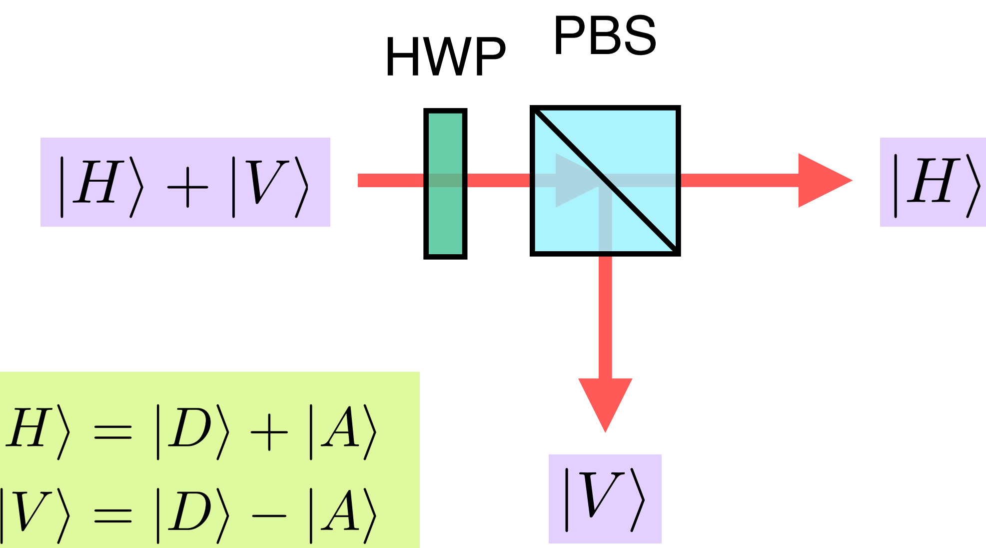
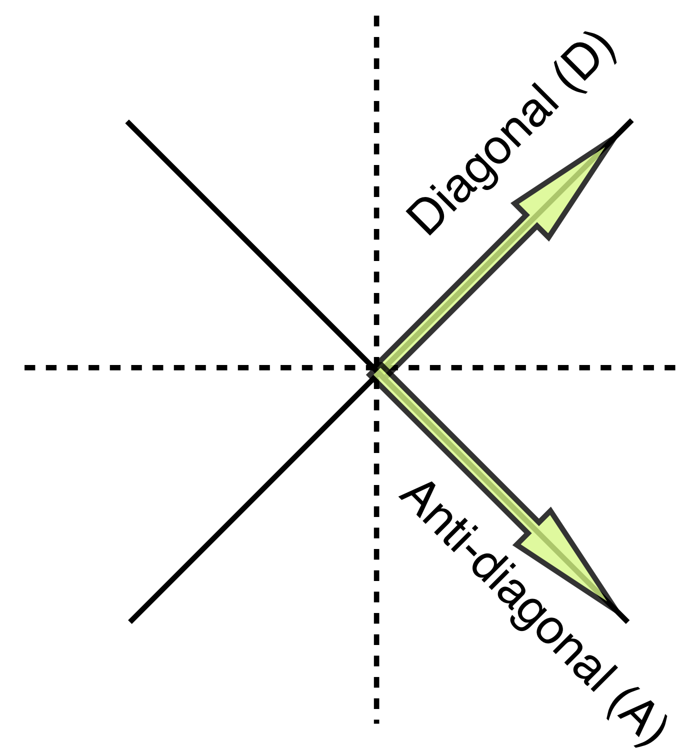



$$|H\rangle = |D\rangle + |A\rangle$$
$$|V\rangle = |D\rangle - |A\rangle$$

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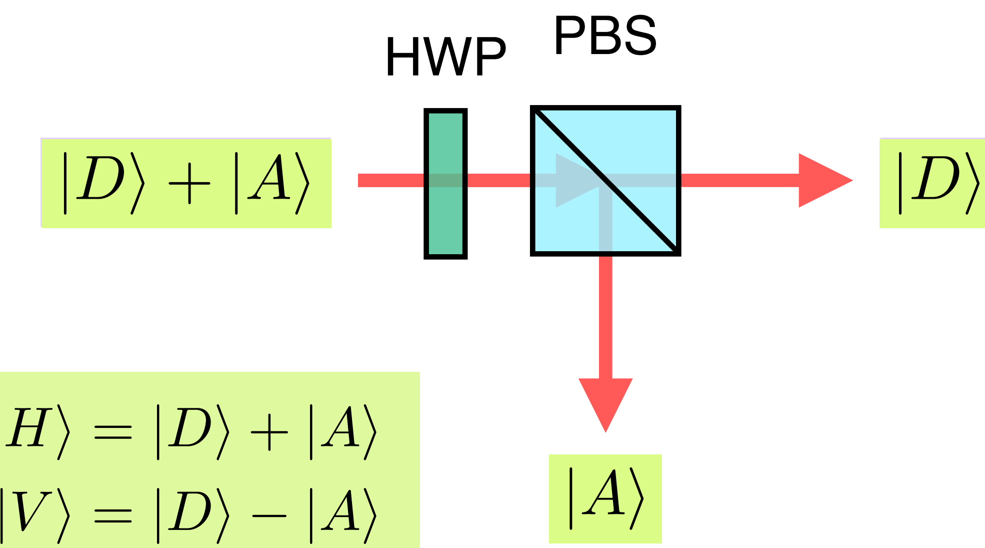


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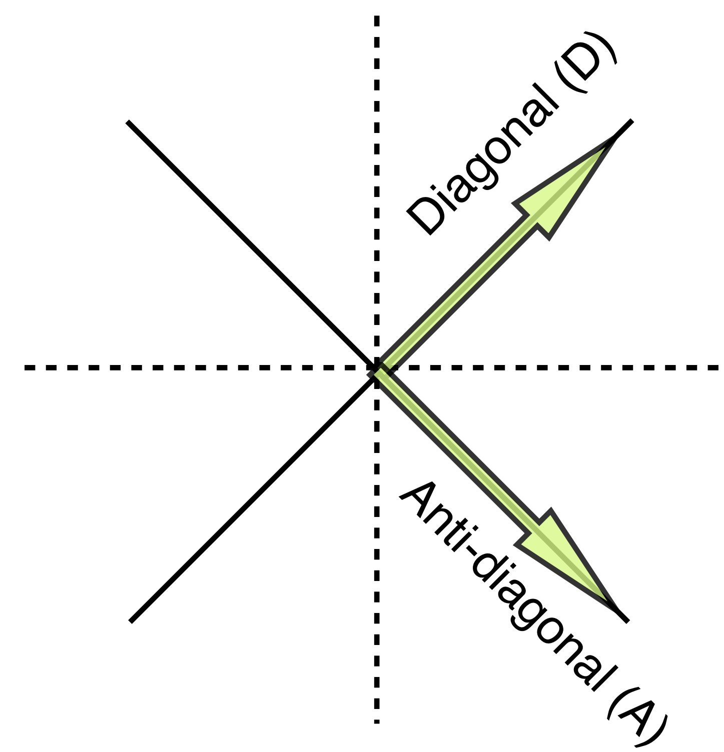


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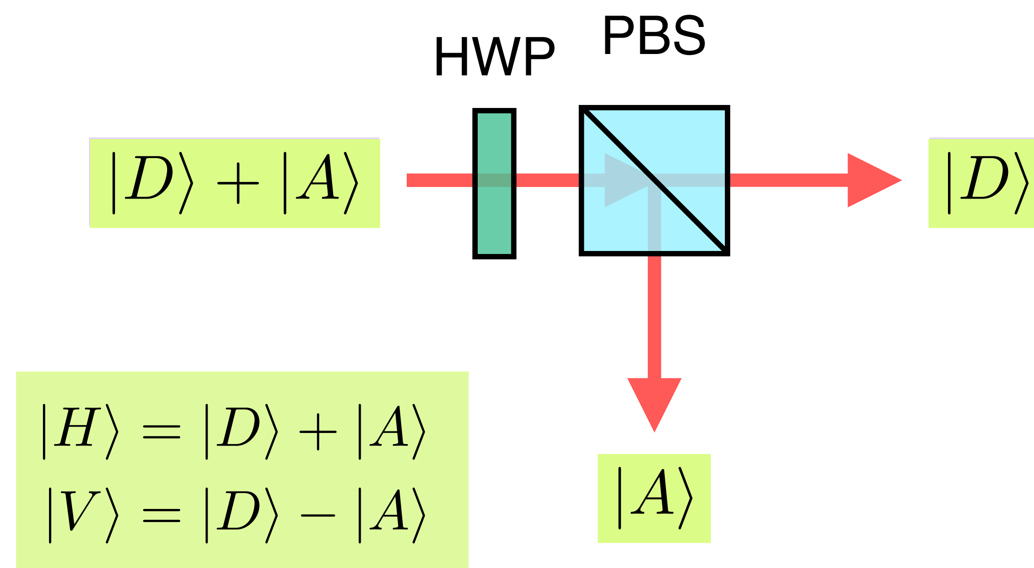


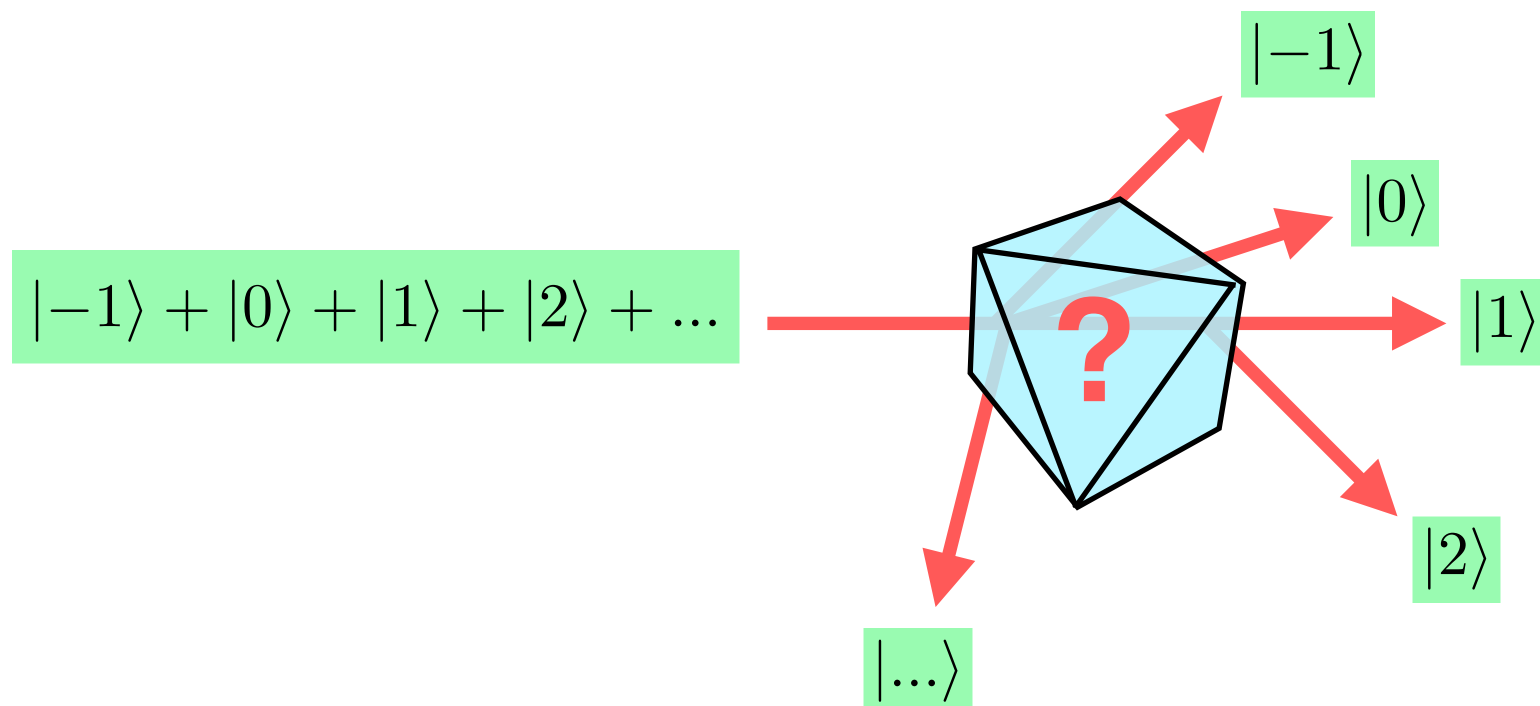


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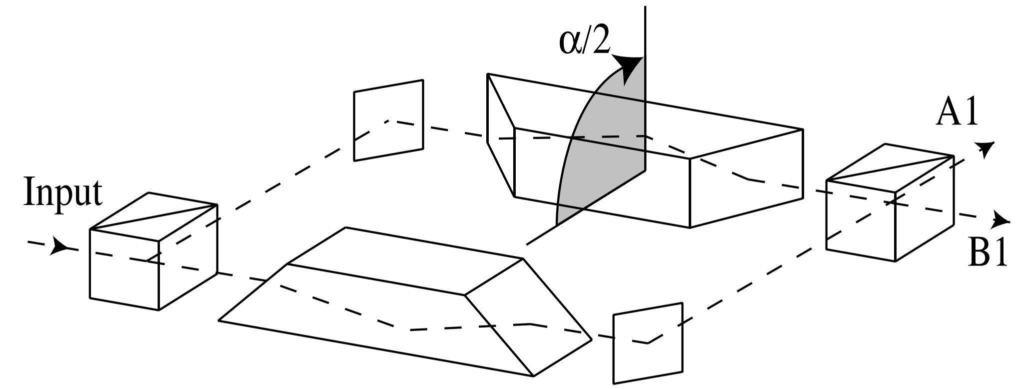
- Interferometer with rotated Dove prism in each arm

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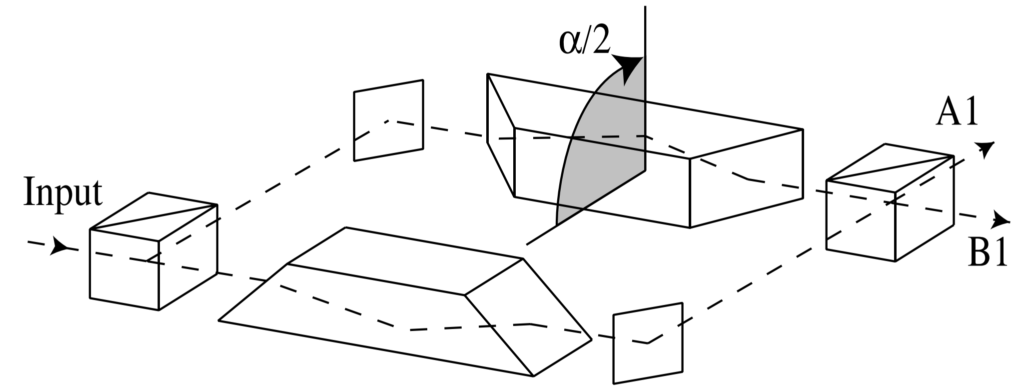


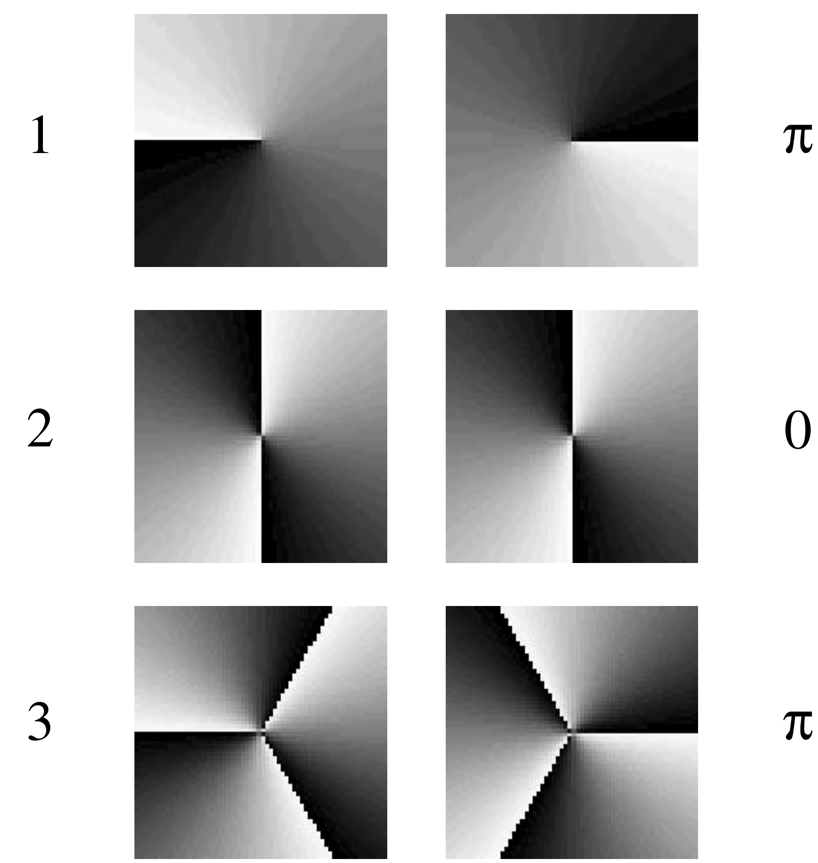


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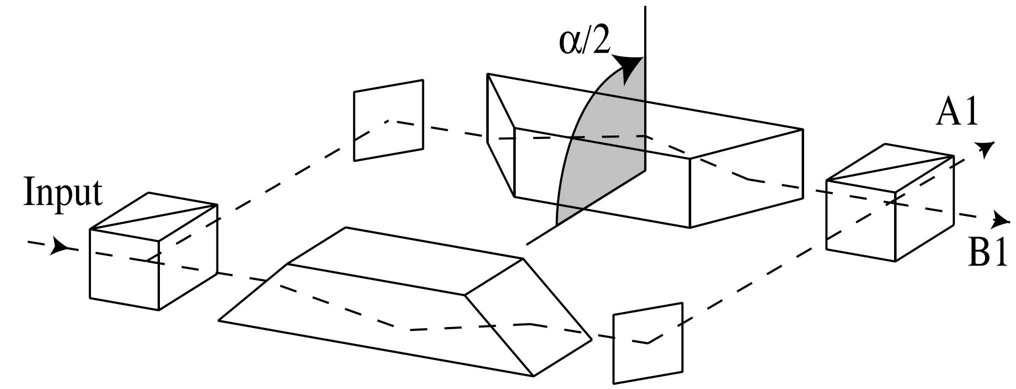


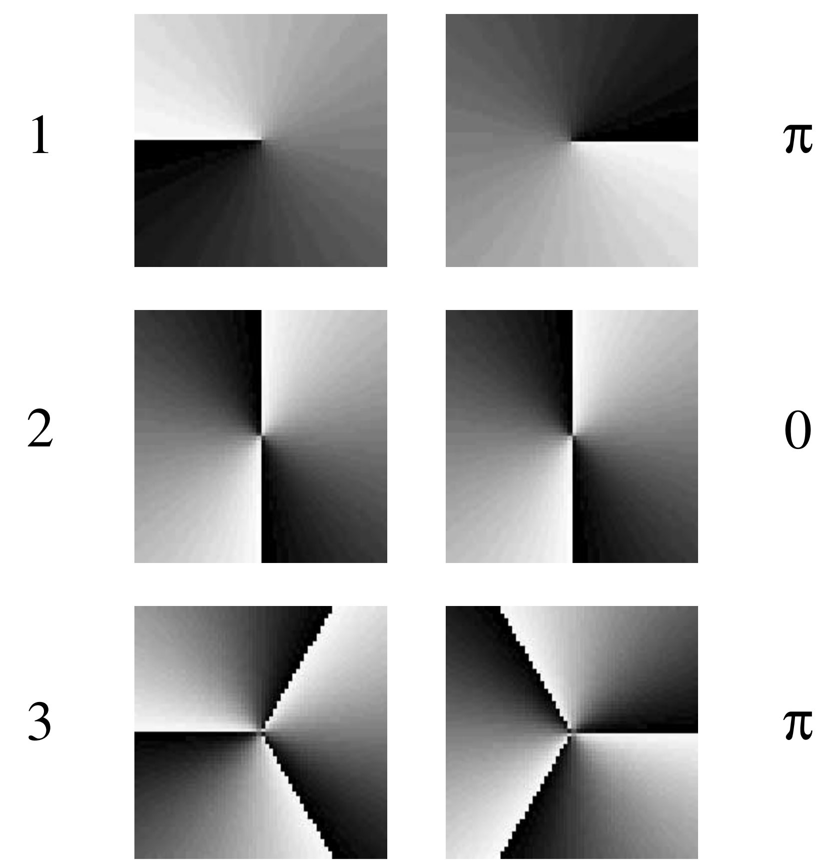


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- Why polarisation? Tools exist!
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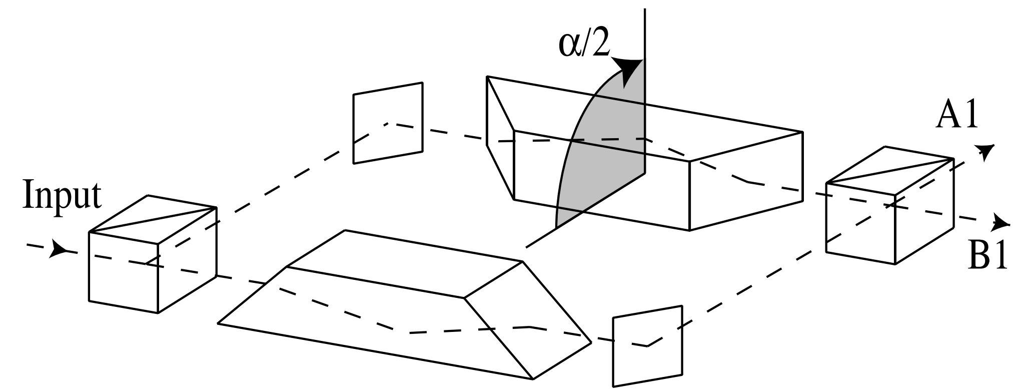
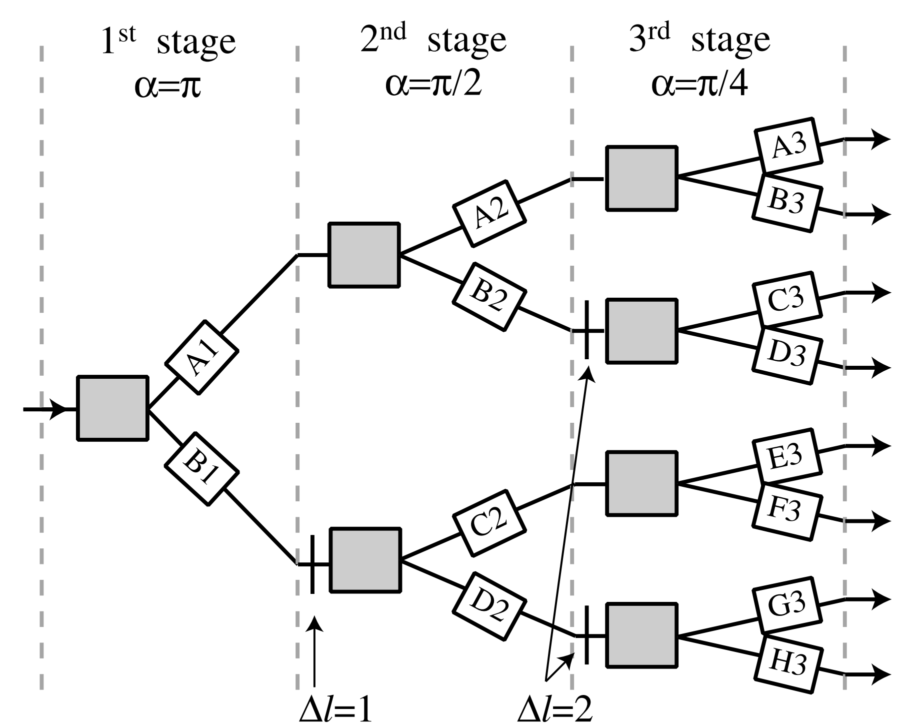
- Interferometer with rotated Dove prism in each arm
- Introduces $\ell$-dependent phase
- Sorts OAM into <u>even</u> and <u>odd</u> values
- Scalable, but not practical



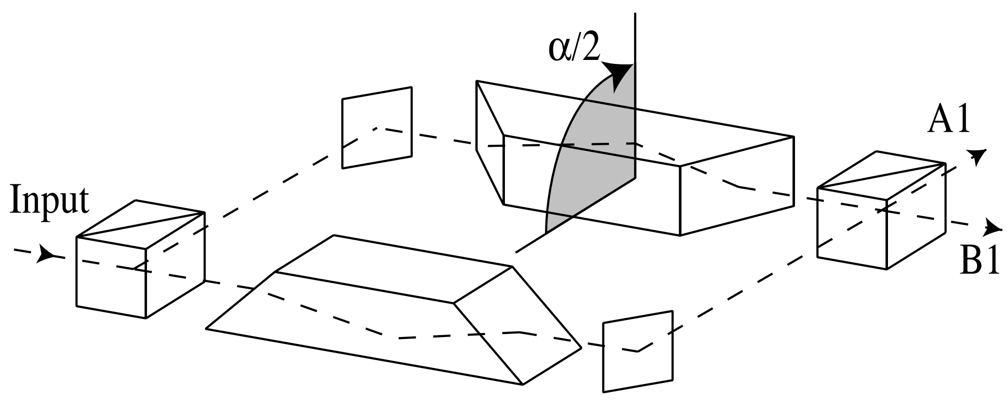

Leach, Padgett, Barnett, Arnold, Courtial, PRL 88, 257901 (2002)

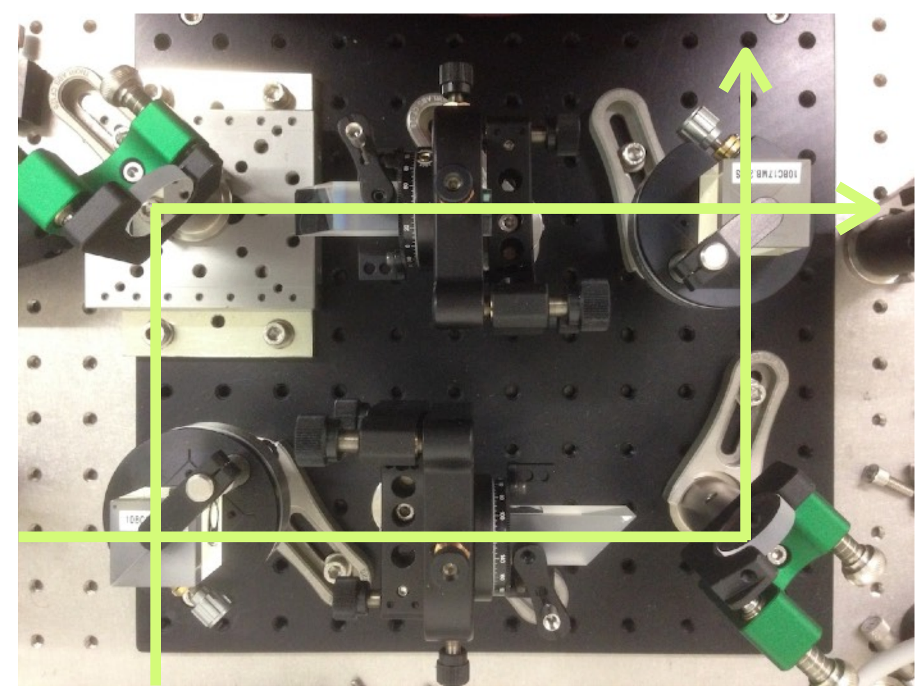
# A Beam Splitter for OAM

Leach, Padgett, Barnett, Arnold, Courtial, **PRL 88**, 257901 (2002)

# A BEAM SPLITTER FOR OAM

- Interferometric sorter: <u>one-input</u>, *two-output* device

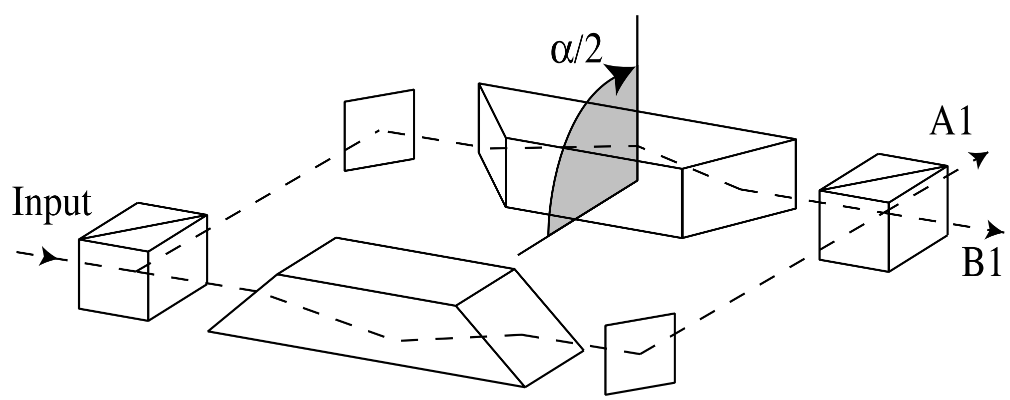


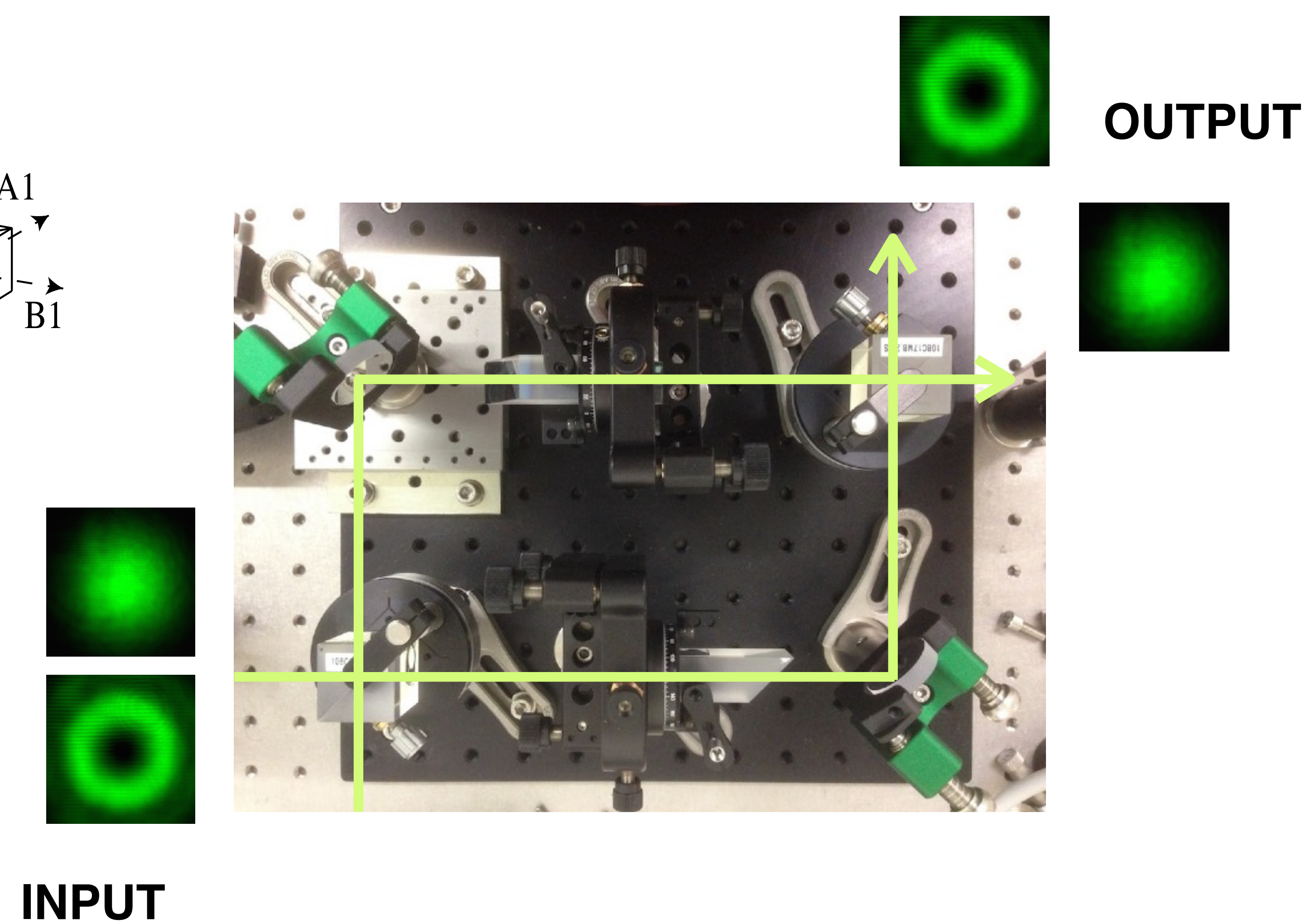



Leach, Padgett, Barnett, Arnold, Courtial, **PRL 88**, 257901 (2002)

# A BEAM SPLITTER FOR OAM

- Interferometric sorter: one-input, *two-output* device

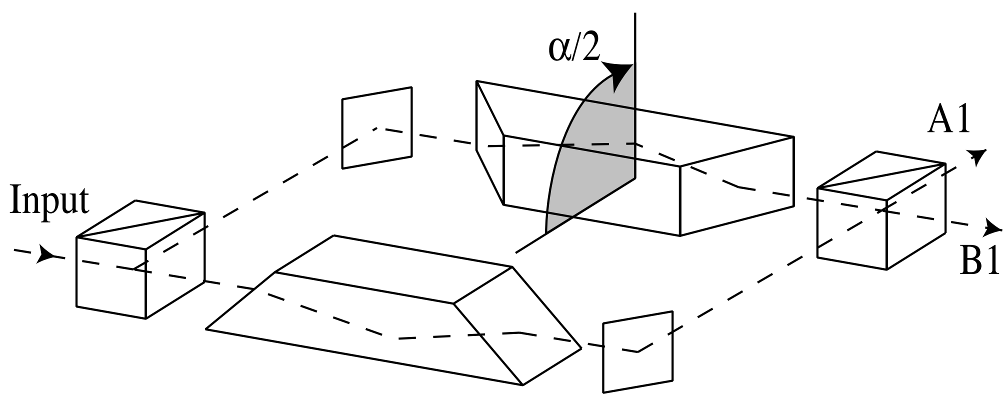


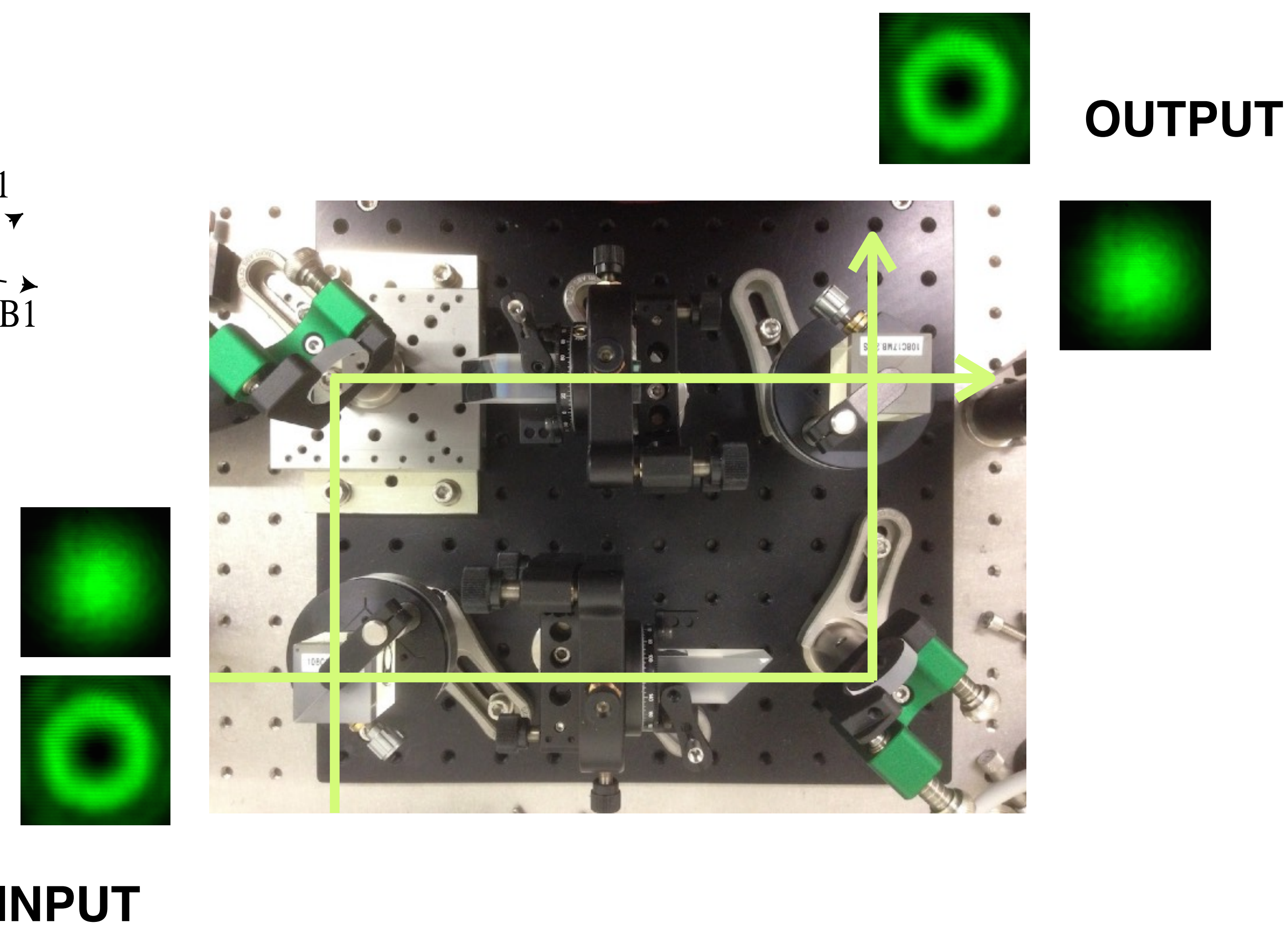




Leach, Padgett, Barnett, Arnold, Courtial, **PRL 88**, 257901 (2002)

# A Beam Splitter for OAM

- Interferometric sorter: <u>one-input</u>, *two-output* device
- Can be used as a **two-input**, *two-output* device

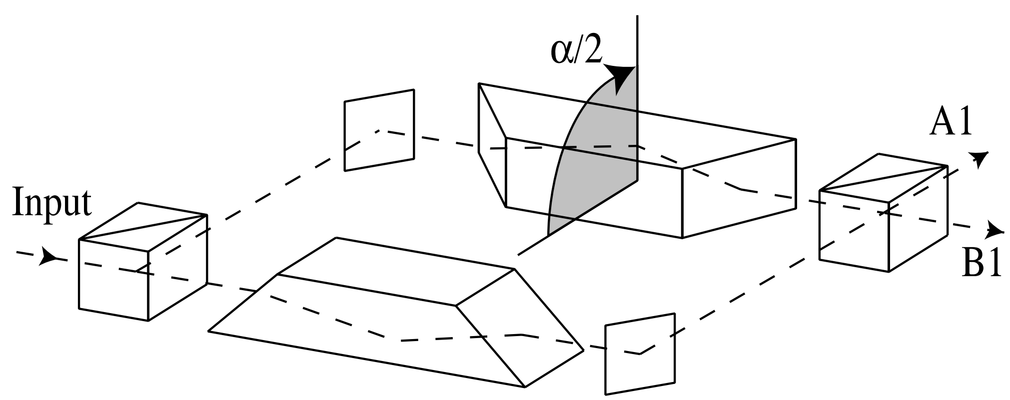


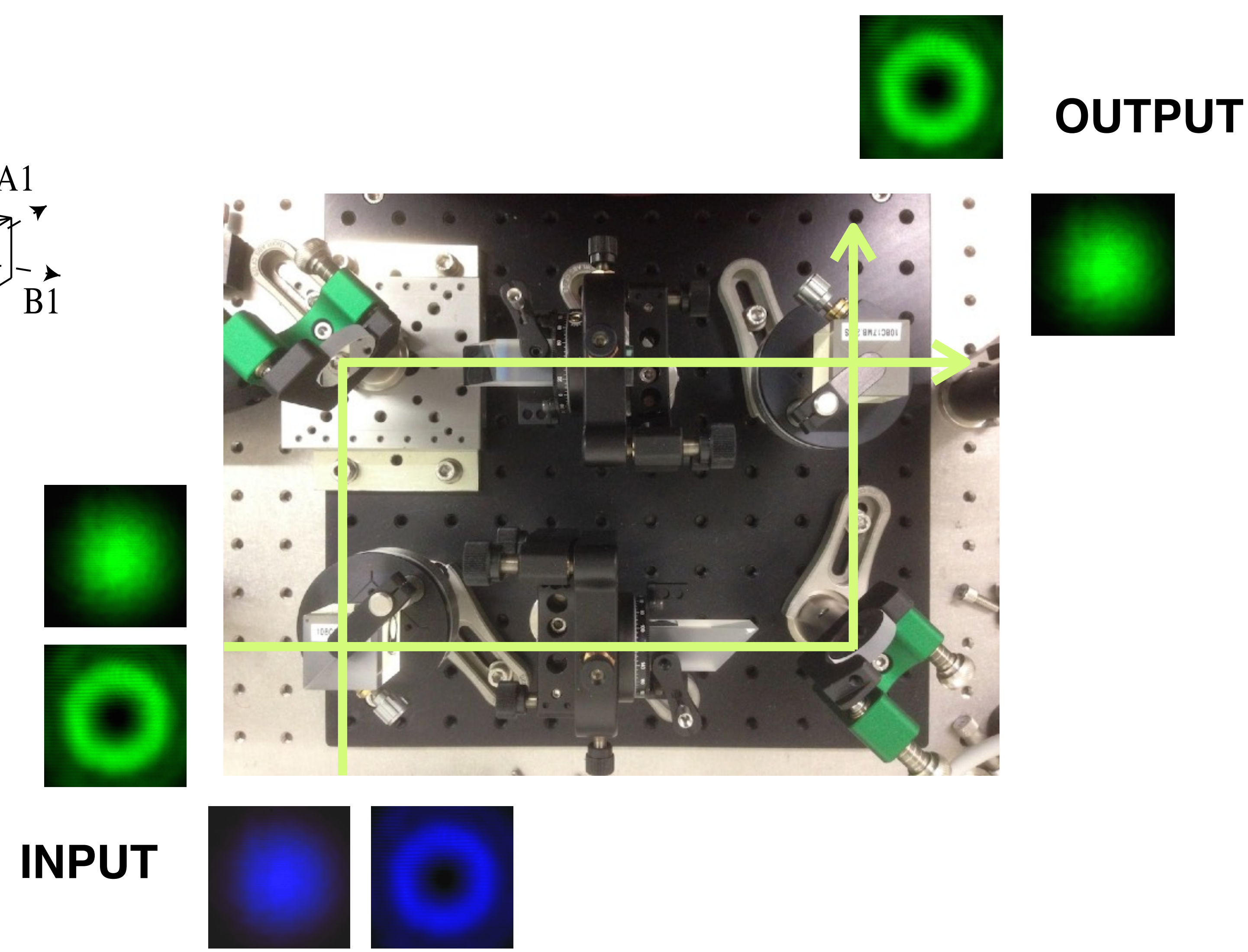




Leach, Padgett, Barnett, Arnold, Courtial, PRL 88, 257901 (2002)

# A Beam Splitter for OAM

- Interferometric sorter: <u>one-input</u>, *two-output* device
- Can be used as a **two-input**, *two-output* device

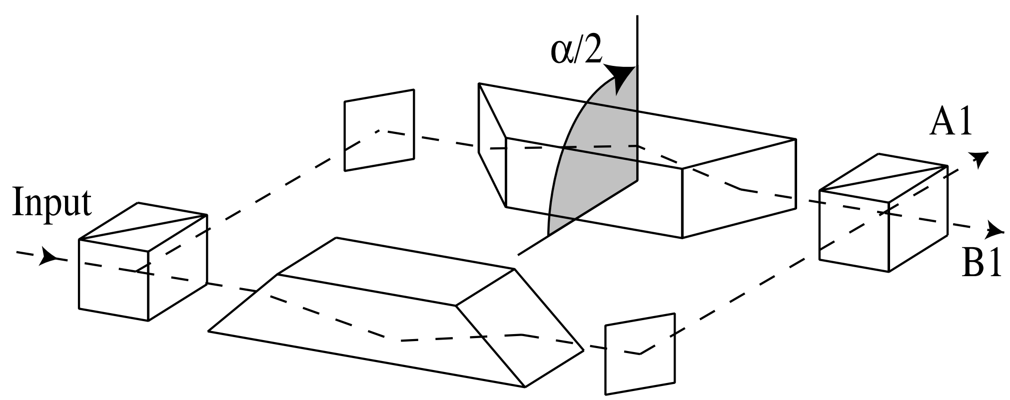


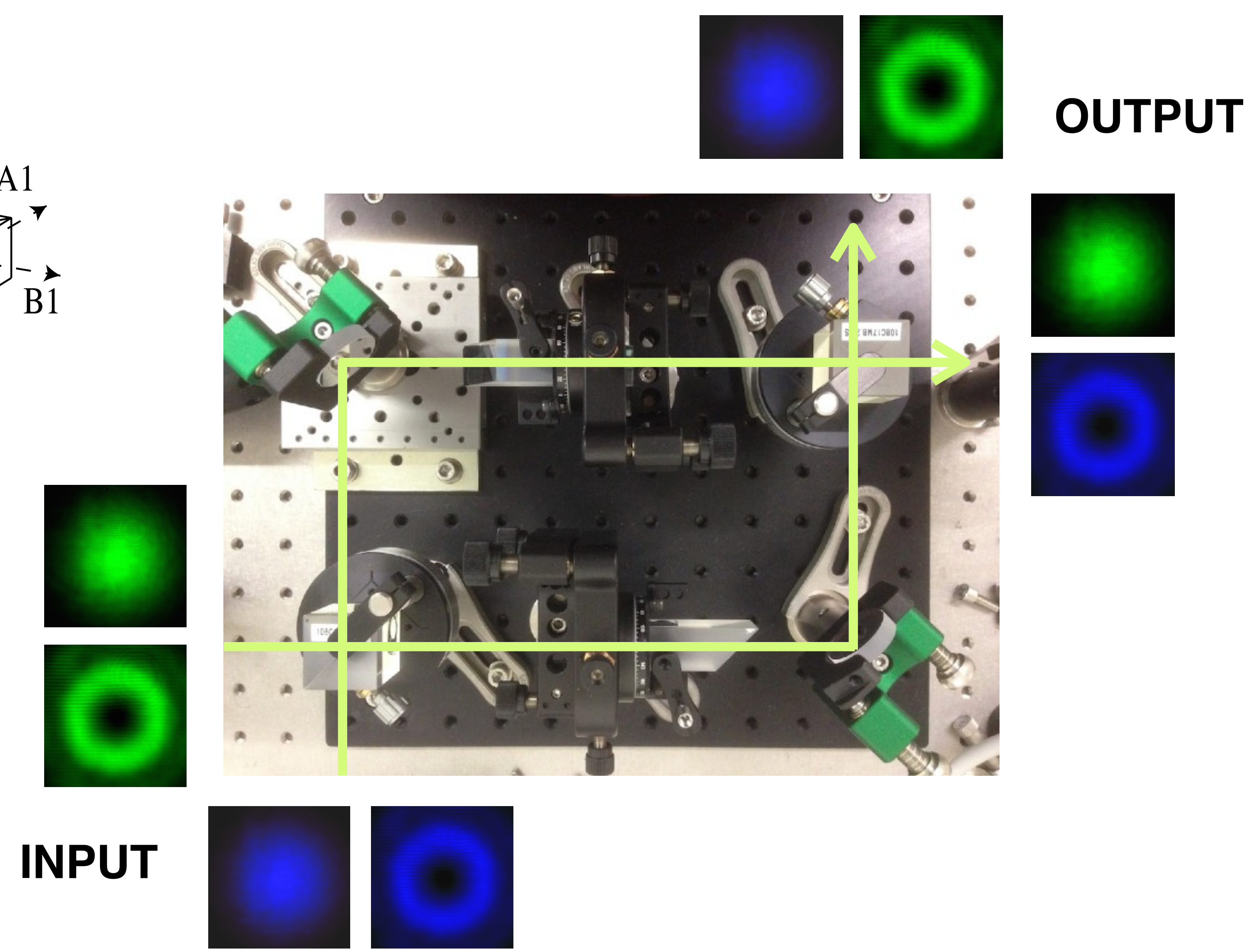




Leach, Padgett, Barnett, Arnold, Courtial, PRL 88, 257901 (2002)

# A BEAM SPLITTER FOR OAM

- Interferometric sorter: one-input, *two-output* device
- Can be used as a **two-input**, *two-output* device

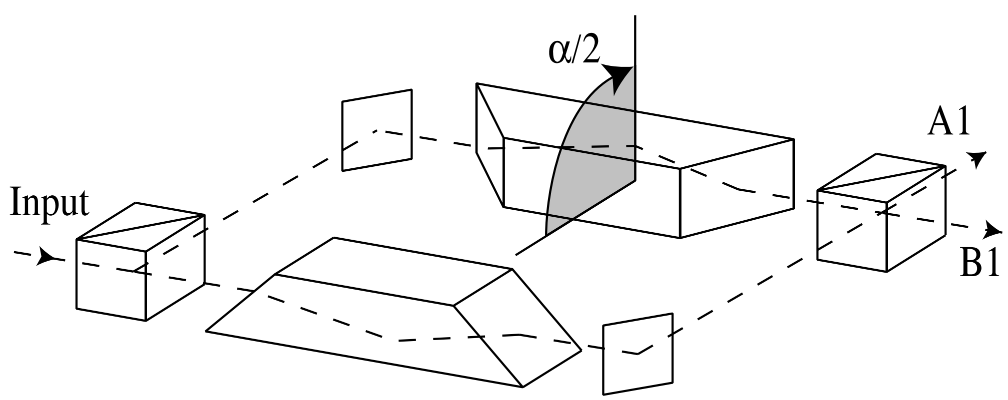


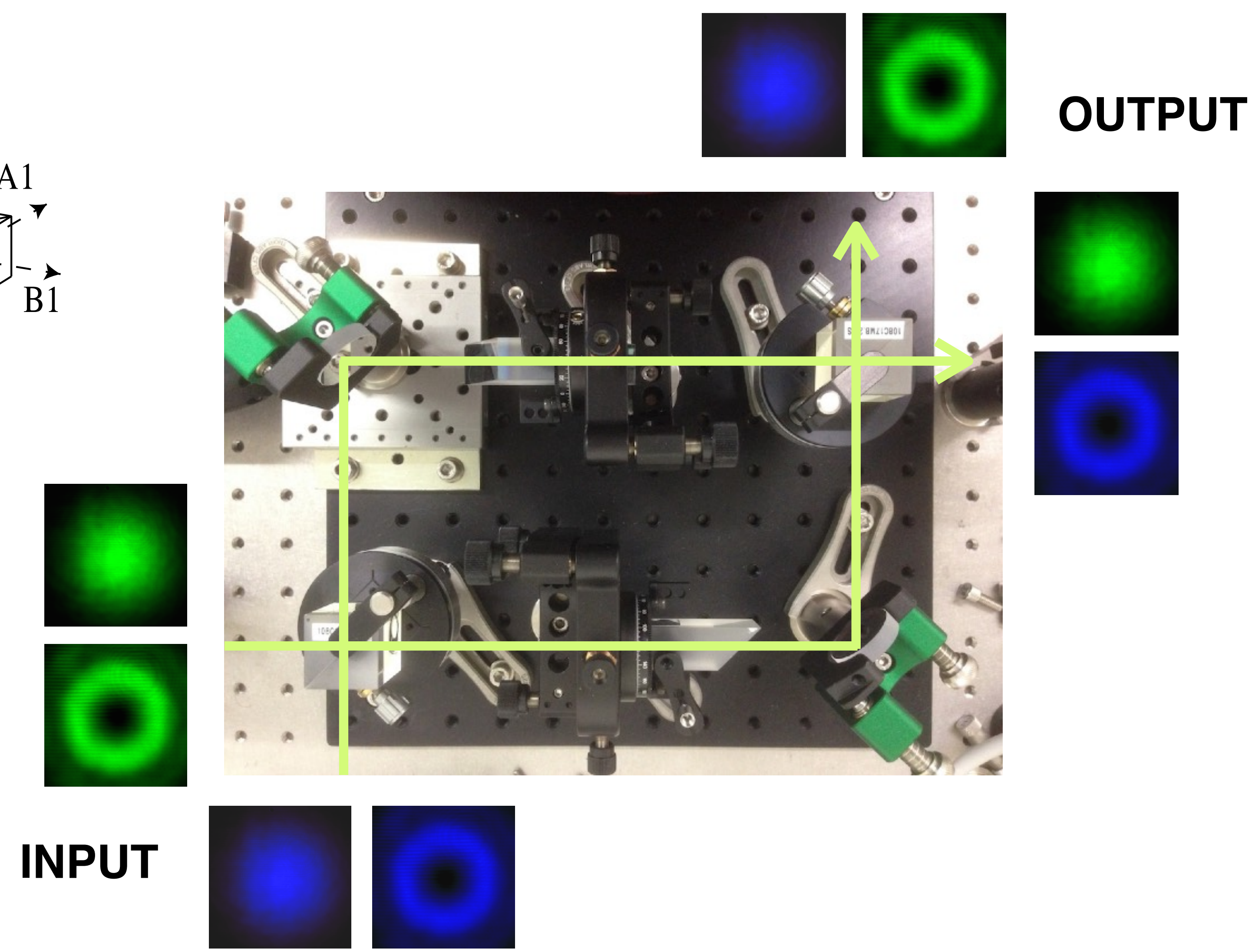




Leach, Padgett, Barnett, Arnold, Courtial, PRL 88, 257901 (2002)

# A Beam Splitter for OAM

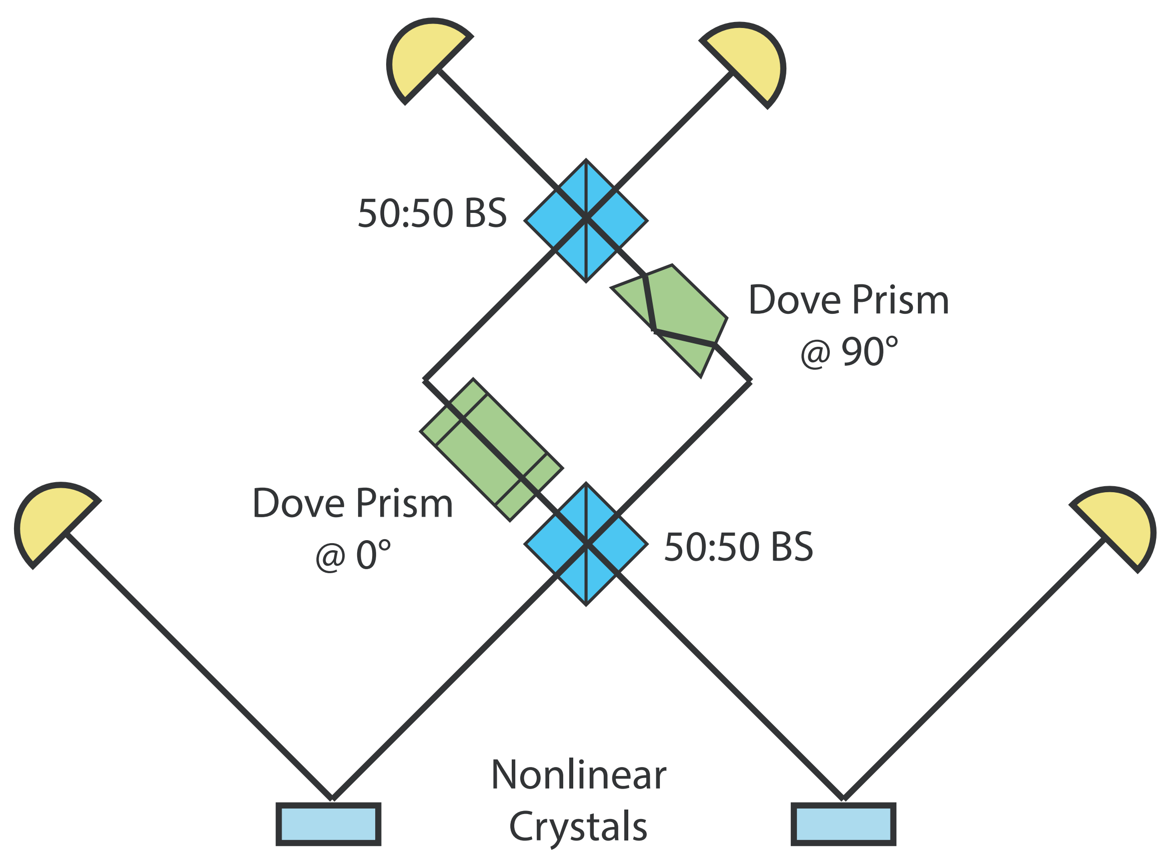
- Interferometric sorter: <u>one-input</u>, *two-output* device
- Can be used as a **two-input**, *two-output* device
- Works like a <u>polarizing beam splitter</u>: reflects **ODD**, transmits **EVEN** modes



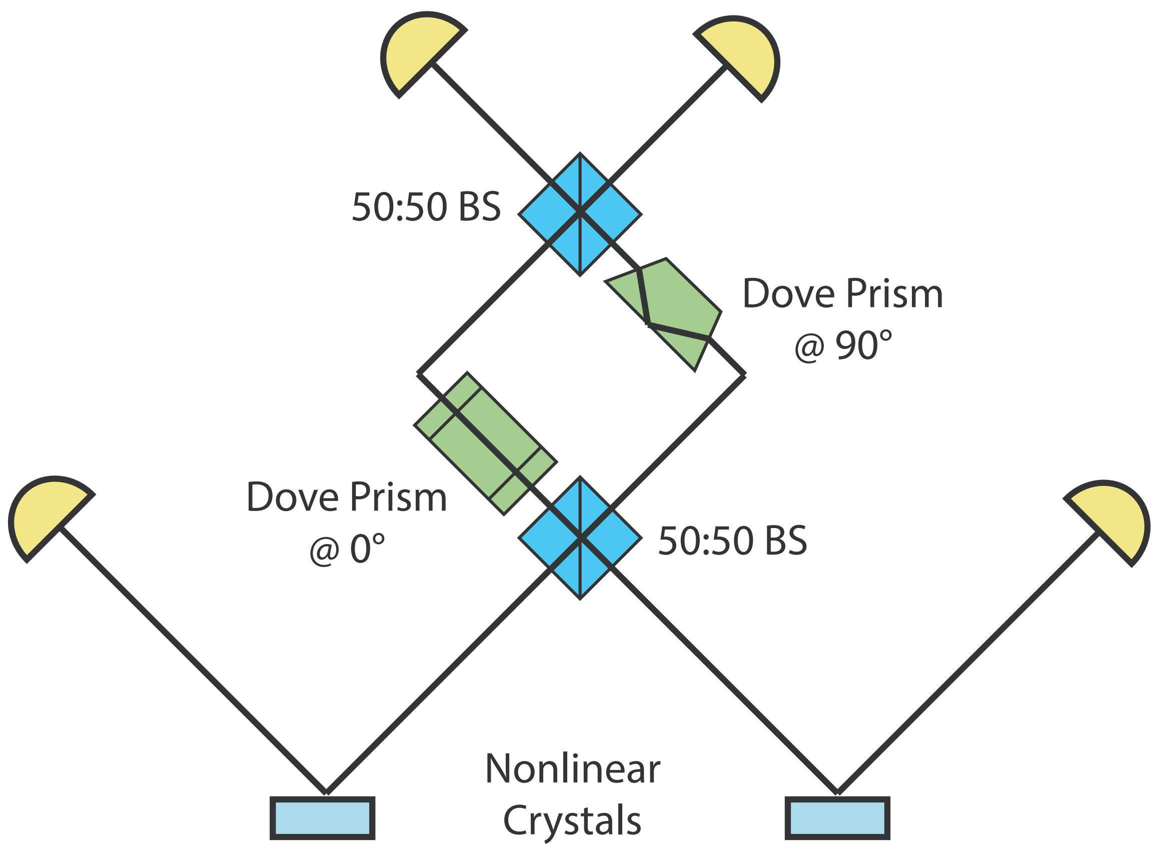

Leach, Padgett, Barnett, Arnold, Courtial, **PRL 88**, 257901 (2002)

# 332 Entanglement




MM, Erhard, Huber, Krenn, Fickler, Zeilinger, Nature Photonics 10, 248 (2016)

# 332 ENTANGLEMENT



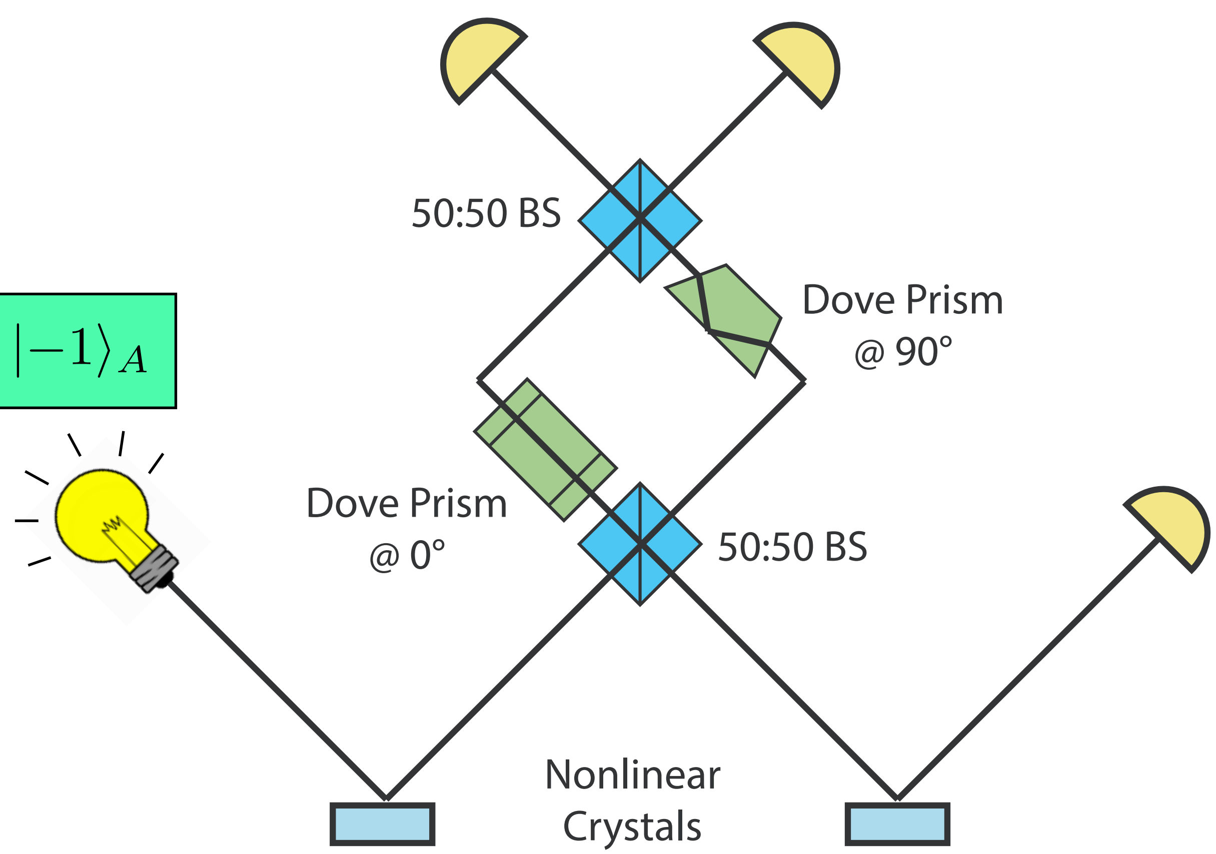

$$|1\rangle_A|-1\rangle_B + |0\rangle_A|0\rangle_B + |-1\rangle_A|1\rangle_B$$

$$|1\rangle_C|-1\rangle_D + |0\rangle_C|0\rangle_D + |-1\rangle_C|1\rangle_D$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 ENTANGLEMENT

$$|0\rangle_A + |-1\rangle_A$$



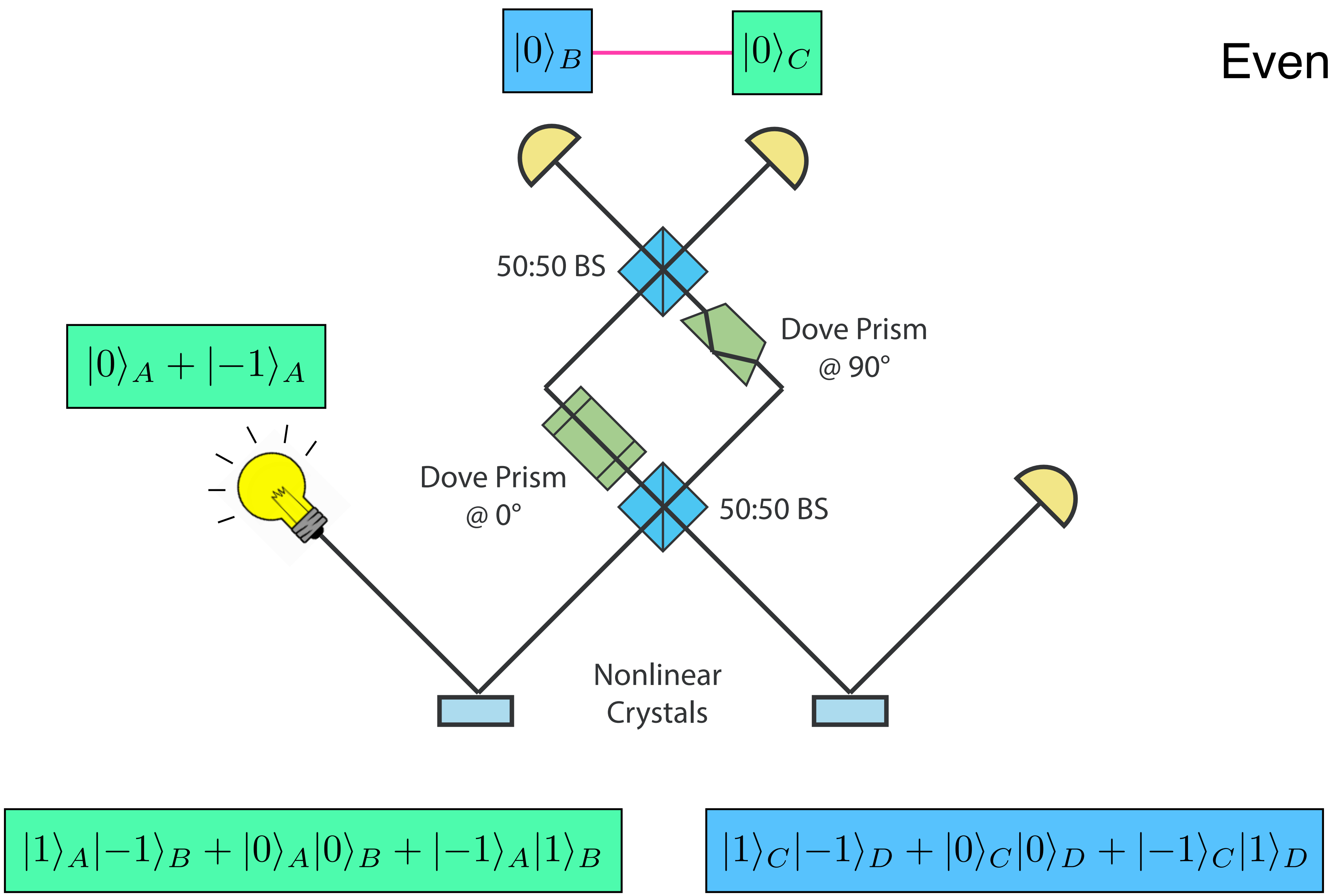

$$|1\rangle_A|-1\rangle_B + |0\rangle_A|0\rangle_B + |-1\rangle_A|1\rangle_B$$

$$|1\rangle_C|-1\rangle_D + |0\rangle_C|0\rangle_D + |-1\rangle_C|1\rangle_D$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 ENTANGLEMENT

Even modes transmitted!

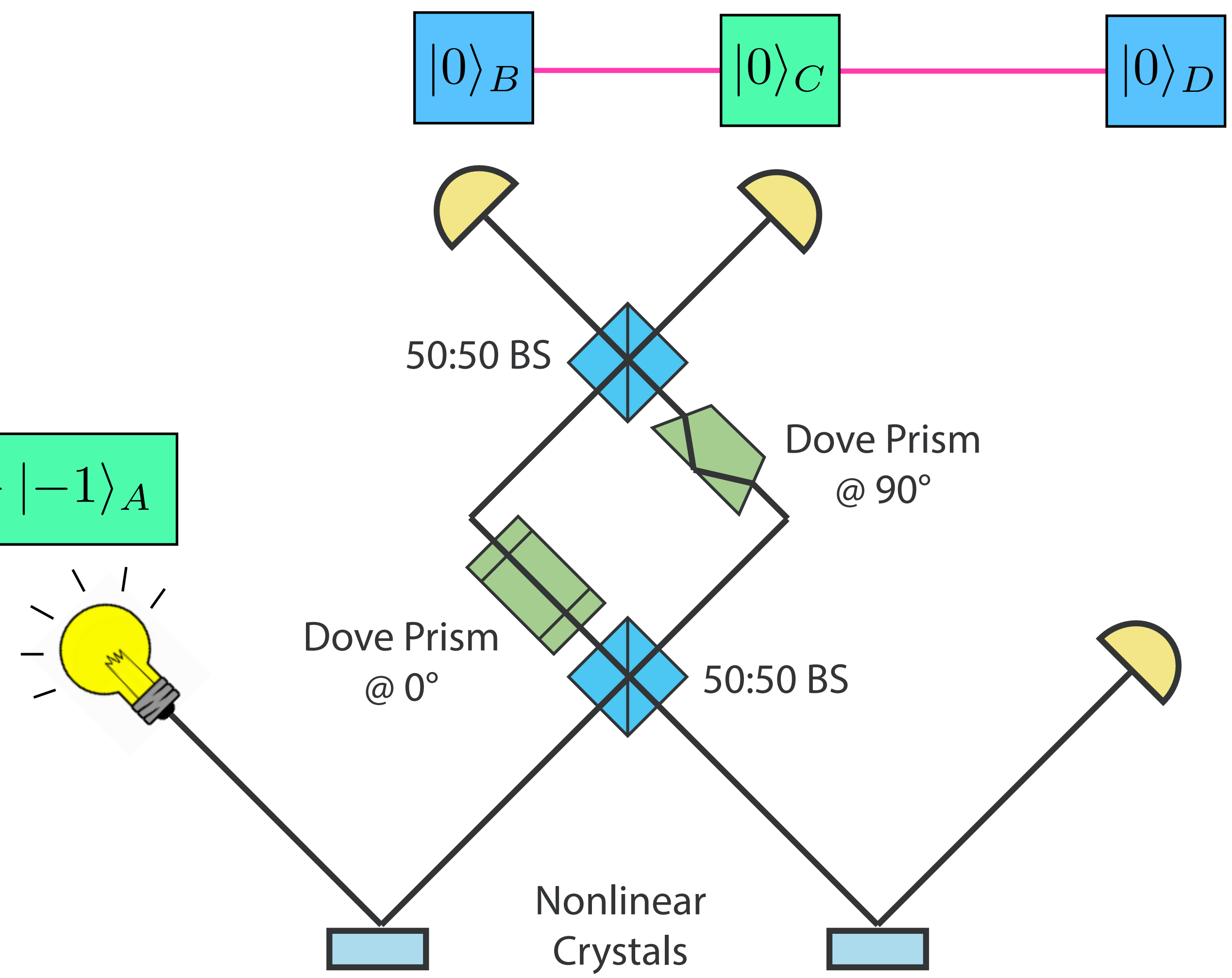



**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 ENTANGLEMENT

**Even** modes transmitted!

$|0\rangle_A + |-1\rangle_A$




$|1\rangle_A|-1\rangle_B + |0\rangle_A|0\rangle_B + |-1\rangle_A|1\rangle_B$

$|1\rangle_C|-1\rangle_D + |0\rangle_C|0\rangle_D + |-1\rangle_C|1\rangle_D$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 Entanglement

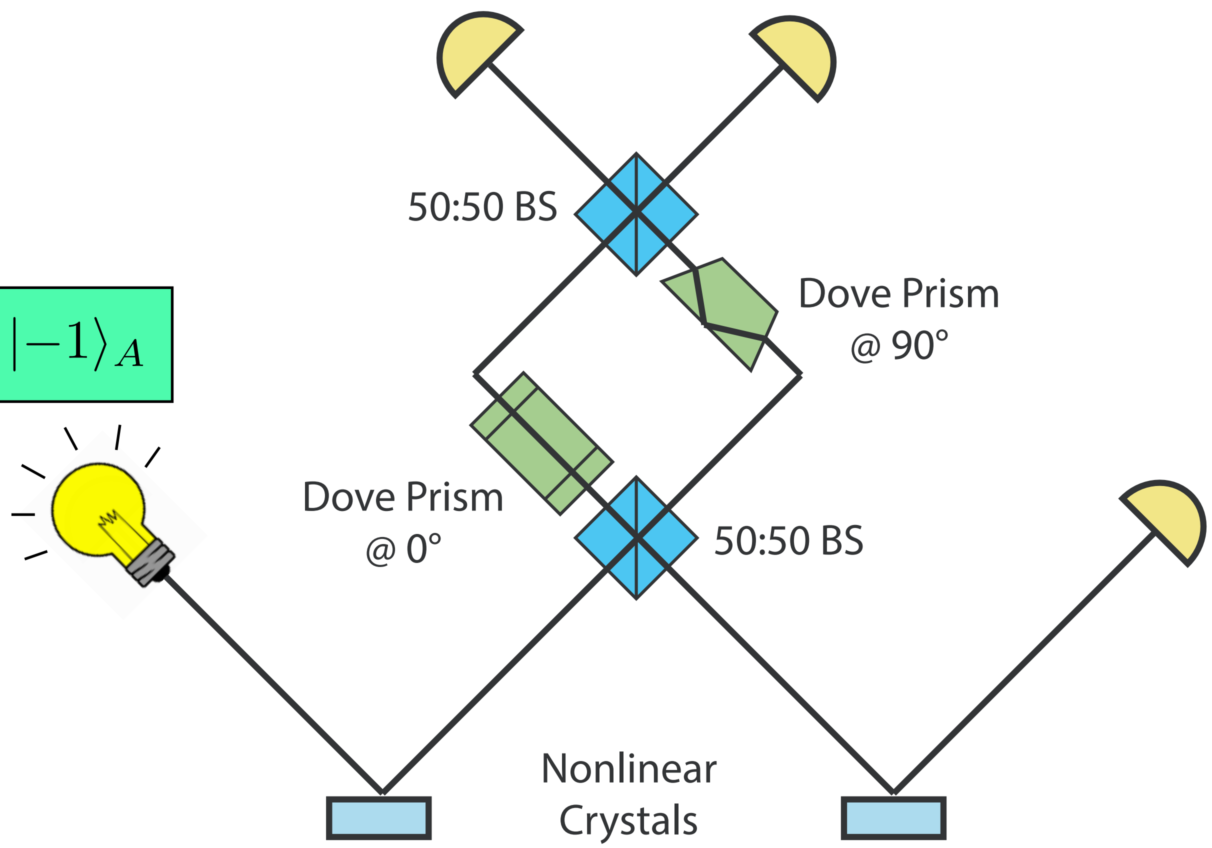
$|1\rangle_B$ — $|1\rangle_C$

$|0\rangle_B$ — $|0\rangle_C$ — $|0\rangle_D$

**Odd** modes reflected!

**Even** modes transmitted!

$|0\rangle_A + |{-1}\rangle_A$

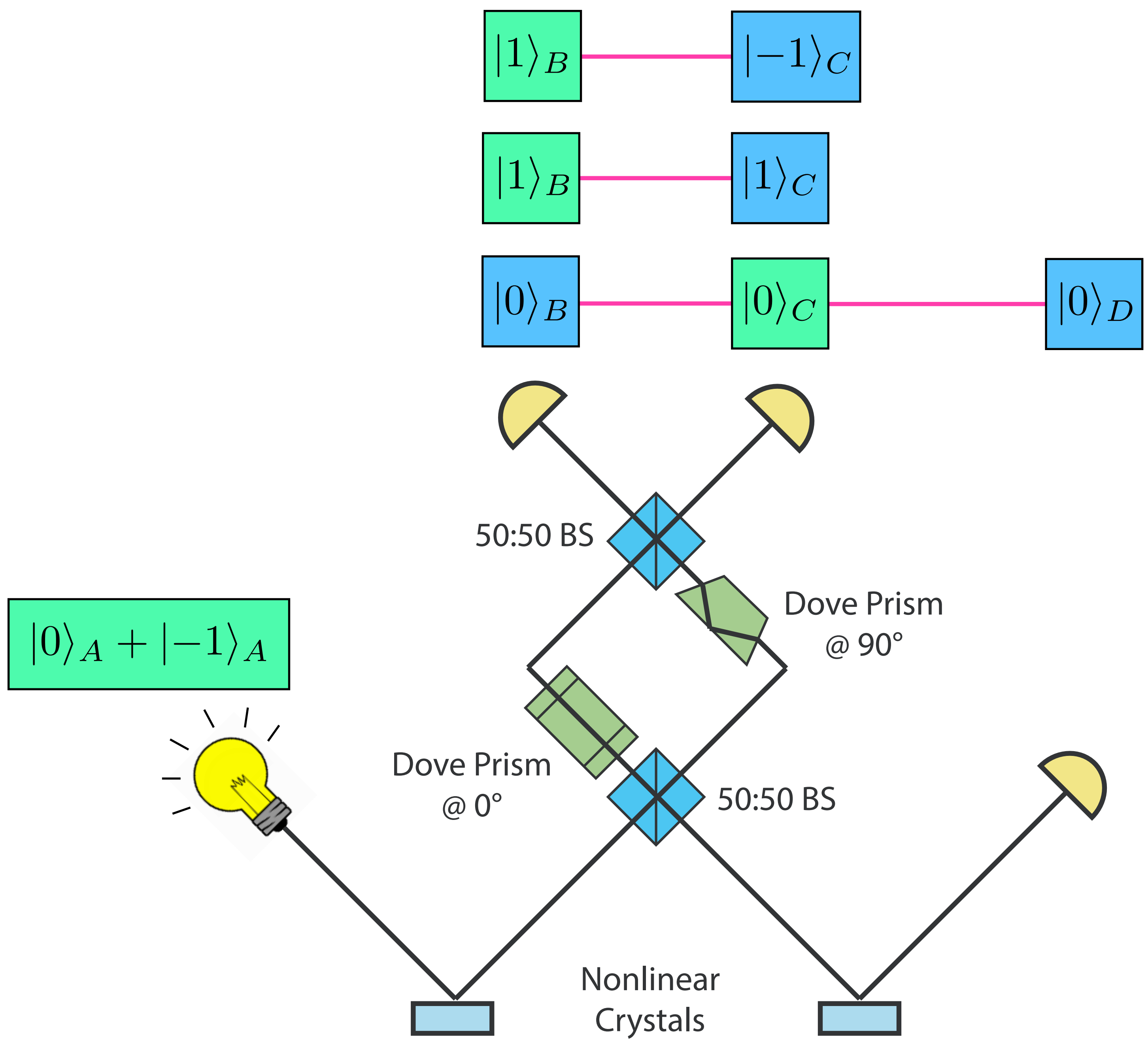



$|1\rangle_A|{-1}\rangle_B + |0\rangle_A|0\rangle_B + |{-1}\rangle_A|1\rangle_B$

$|1\rangle_C|{-1}\rangle_D + |0\rangle_C|0\rangle_D + |{-1}\rangle_C|1\rangle_D$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 ENTANGLEMENT




**Odd** modes reflected!

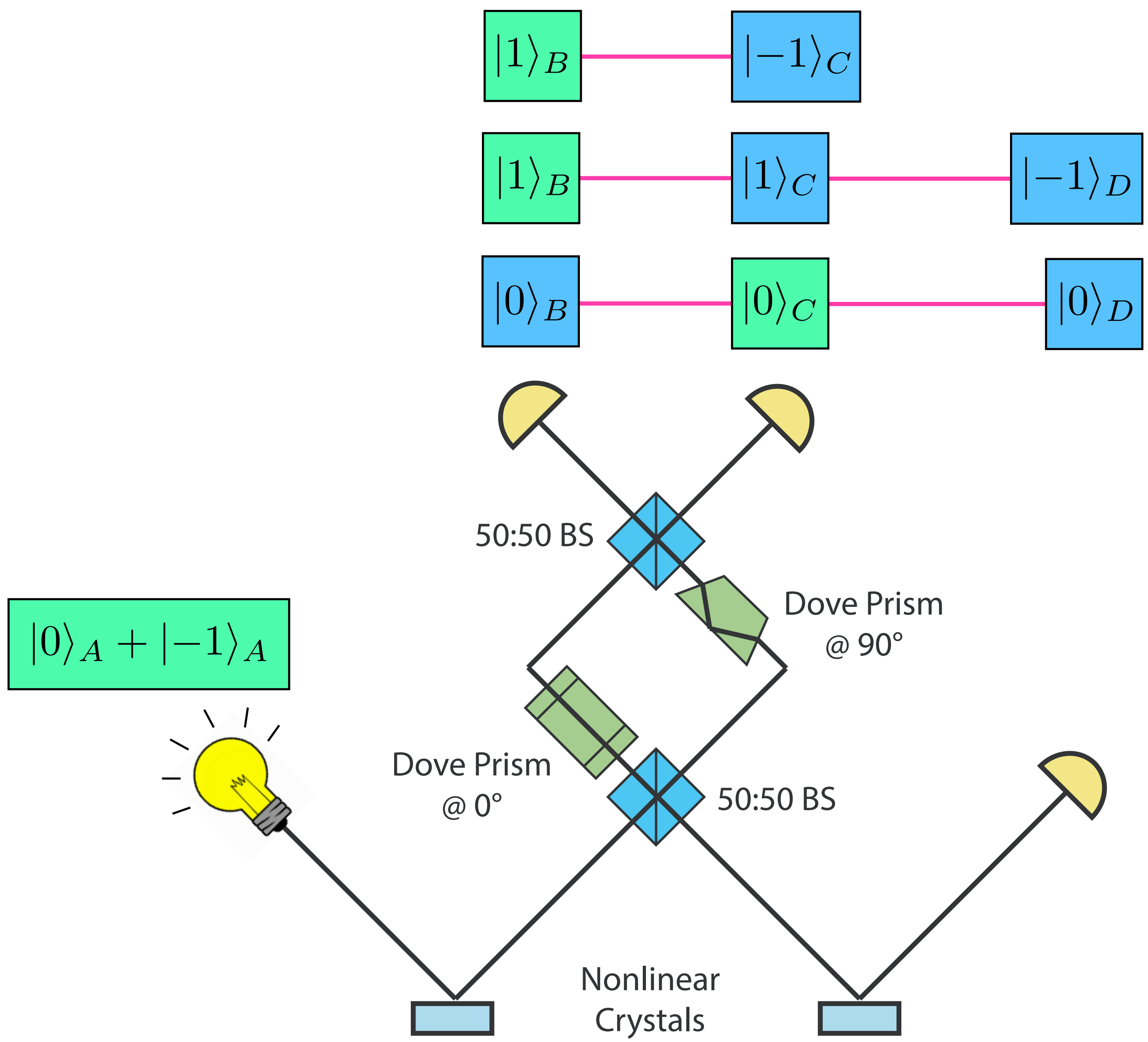
**Even** modes transmitted!

$|1\rangle_A|-1\rangle_B + |0\rangle_A|0\rangle_B + |-1\rangle_A|1\rangle_B$

$|1\rangle_C|-1\rangle_D + |0\rangle_C|0\rangle_D + |-1\rangle_C|1\rangle_D$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 Entanglement




**Odd** modes reflected!

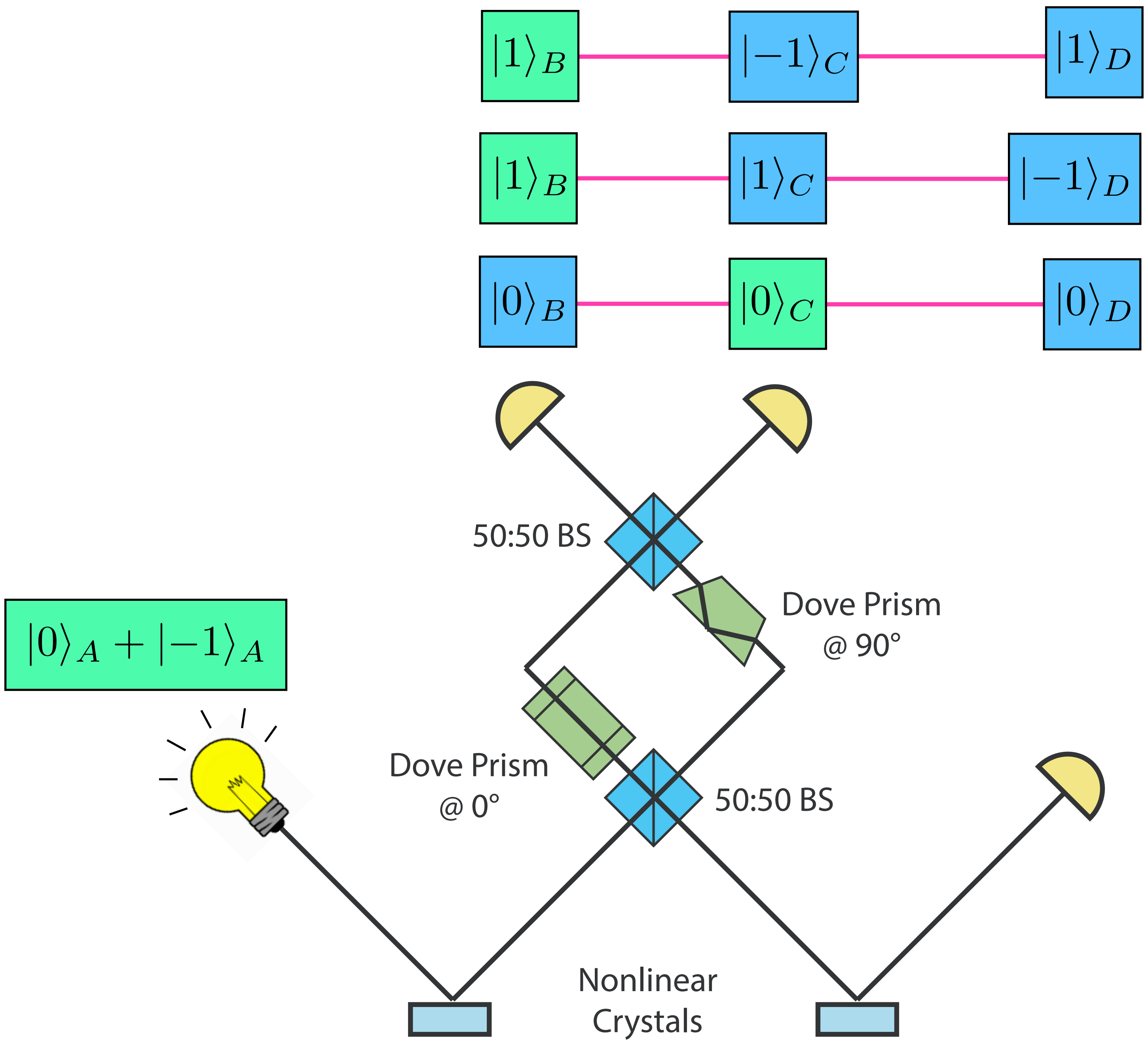
**Even** modes transmitted!

$$|1\rangle_A|-1\rangle_B + |0\rangle_A|0\rangle_B + |-1\rangle_A|1\rangle_B$$

$$|1\rangle_C|-1\rangle_D + |0\rangle_C|0\rangle_D + |-1\rangle_C|1\rangle_D$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 ENTANGLEMENT




**Odd** modes reflected!

**Even** modes transmitted!

$|1\rangle_A|-1\rangle_B + |0\rangle_A|0\rangle_B + |-1\rangle_A|1\rangle_B$

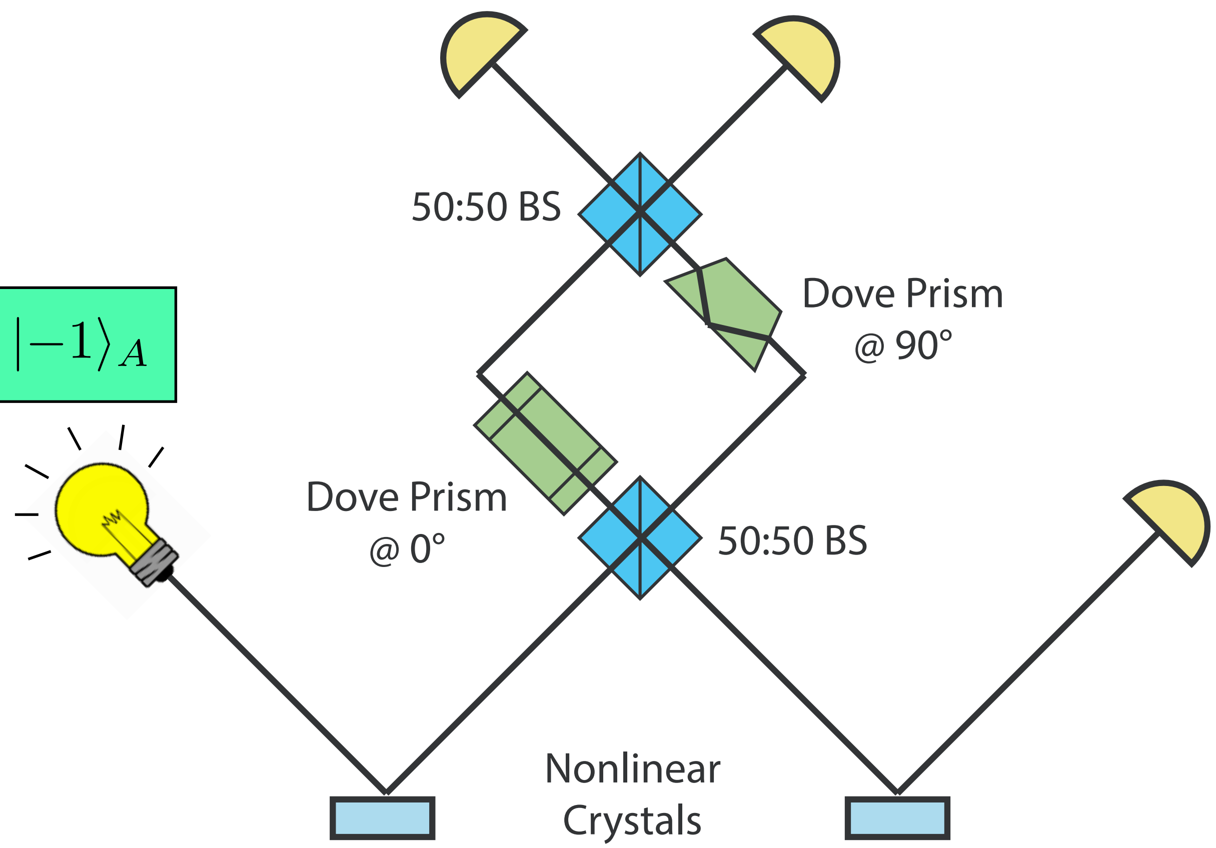
$|1\rangle_C|-1\rangle_D + |0\rangle_C|0\rangle_D + |-1\rangle_C|1\rangle_D$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 Entanglement

$$|\Psi\rangle_{332} \;=\; |1\rangle_B|1\rangle_C|-1\rangle_D + |0\rangle_B|0\rangle_C|0\rangle_D + |1\rangle_B|-1\rangle_C|1\rangle_D$$

$|0\rangle_A + |-1\rangle_A$



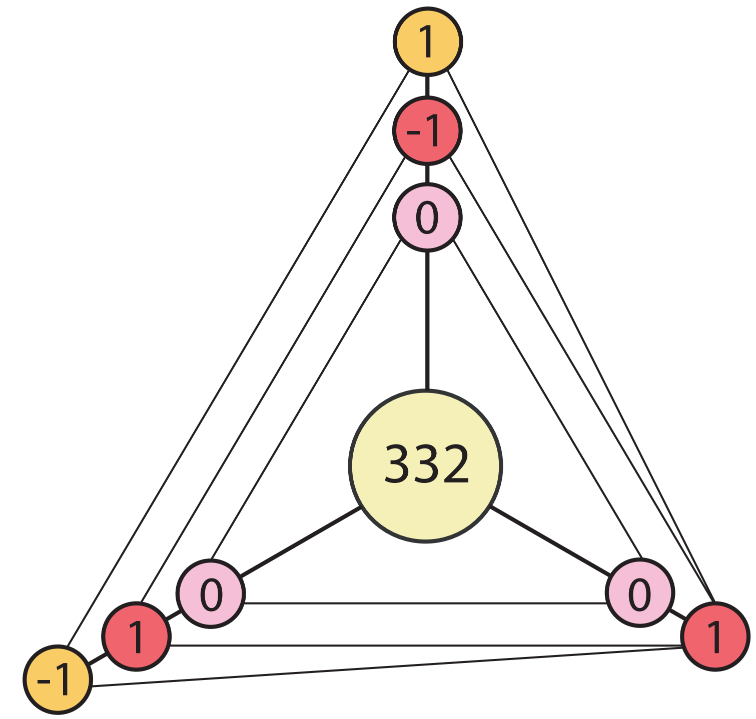

$|1\rangle_A|-1\rangle_B + |0\rangle_A|0\rangle_B + |-1\rangle_A|1\rangle_B$

$|1\rangle_C|-1\rangle_D + |0\rangle_C|0\rangle_D + |-1\rangle_C|1\rangle_D$

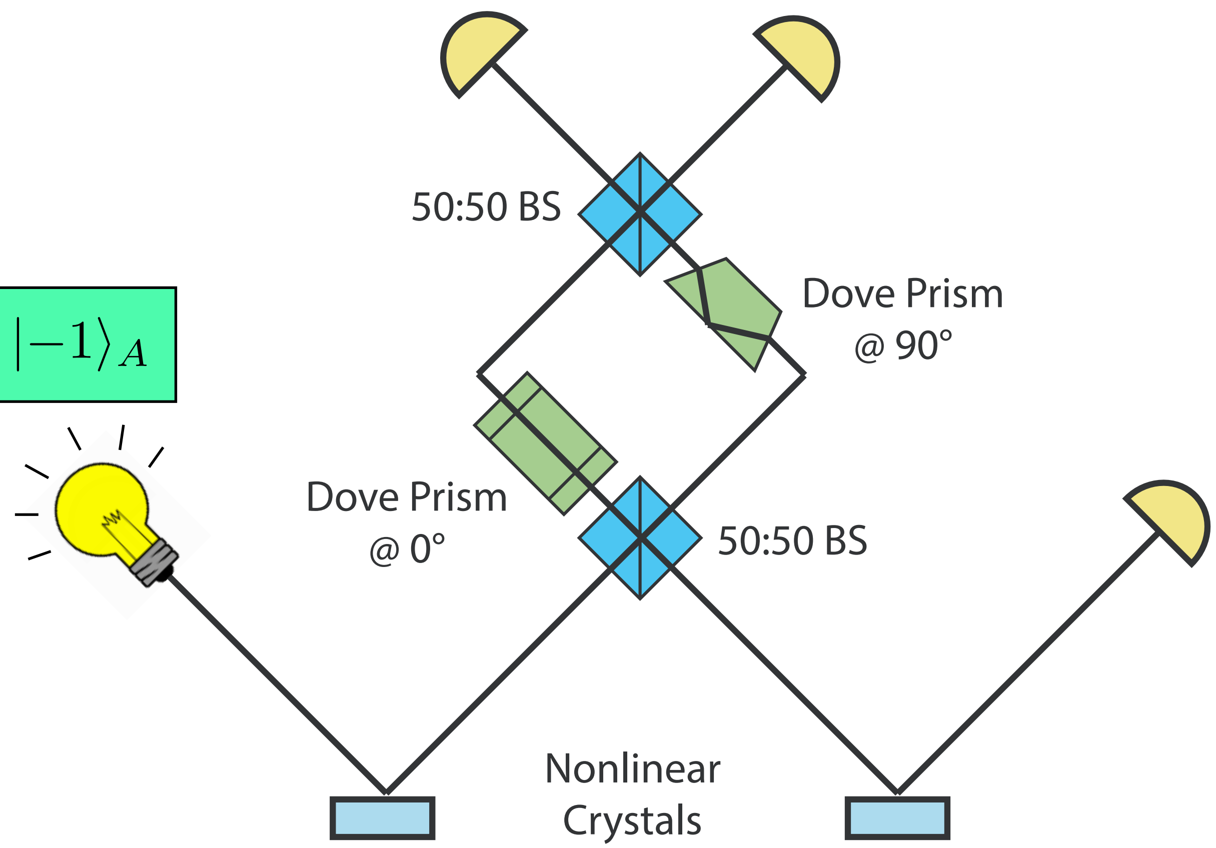
**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 ENTANGLEMENT

$$|\Psi\rangle_{332} \;=\; |1\rangle_B|1\rangle_C|-1\rangle_D + |0\rangle_B|0\rangle_C|0\rangle_D + |1\rangle_B|-1\rangle_C|1\rangle_D$$




$$|0\rangle_A + |-1\rangle_A$$



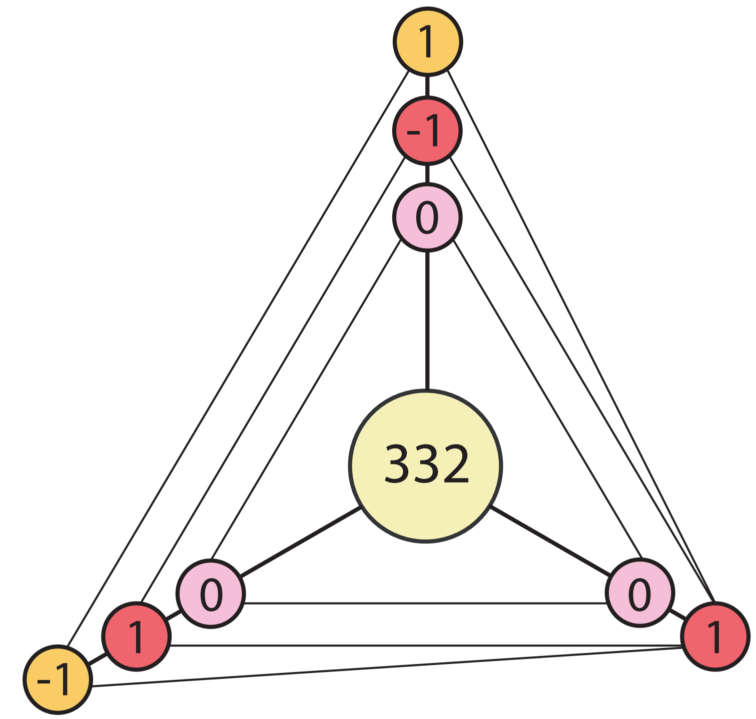

$$|1\rangle_A|-1\rangle_B + |0\rangle_A|0\rangle_B + |-1\rangle_A|1\rangle_B$$

$$|1\rangle_C|-1\rangle_D + |0\rangle_C|0\rangle_D + |-1\rangle_C|1\rangle_D$$

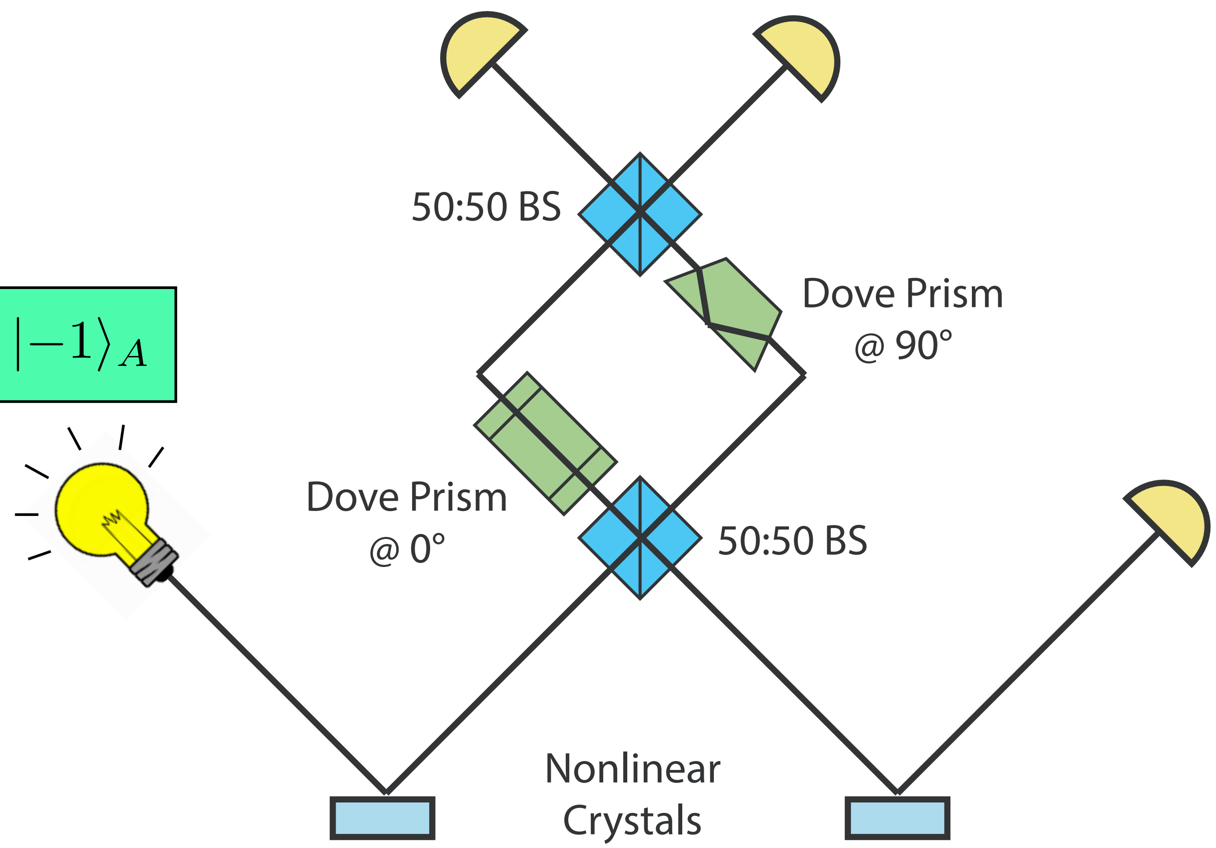
**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 Entanglement

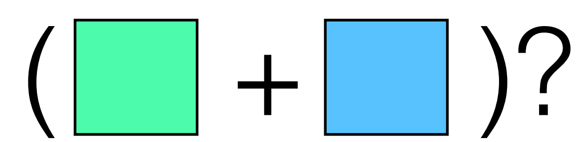
$$|\Psi\rangle_{332} \;=\; |1\rangle_B|1\rangle_C|-1\rangle_D + |0\rangle_B|0\rangle_C|0\rangle_D + |1\rangle_B|-1\rangle_C|1\rangle_D$$




$$|0\rangle_A + |-1\rangle_A$$




Coherent Superposition?



$$|1\rangle_A|-1\rangle_B + |0\rangle_A|0\rangle_B + |-1\rangle_A|1\rangle_B$$

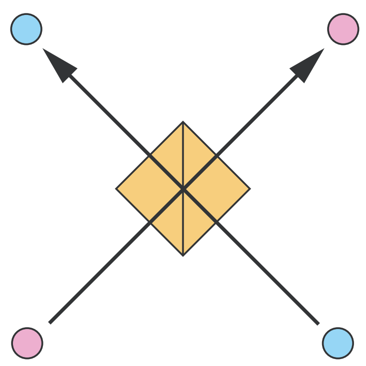
$$|1\rangle_C|-1\rangle_D + |0\rangle_C|0\rangle_D + |-1\rangle_C|1\rangle_D$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10**, 248 (2016)

# TWO-PHOTON INTERFERENCE

Hong, Ou, Mandel, **Phys. Rev. Lett. 59,** 2044 (1987)

# Two-Photon Interference



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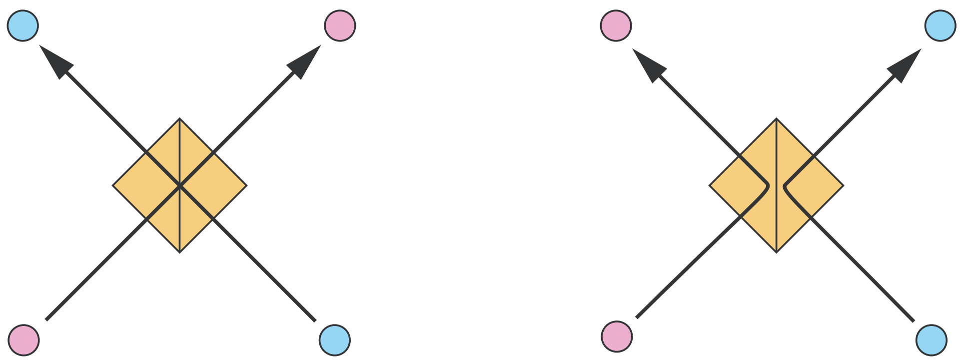
# Two-Photon Interference



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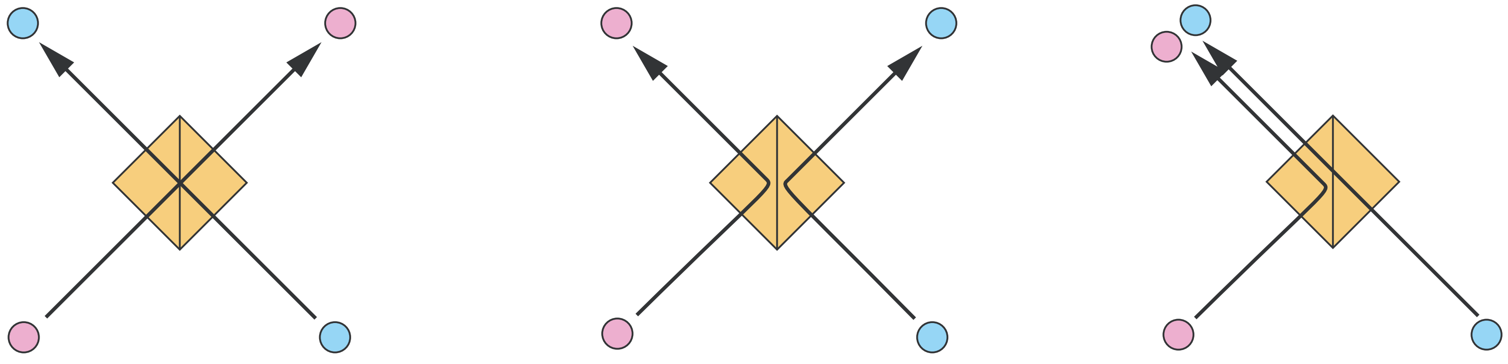

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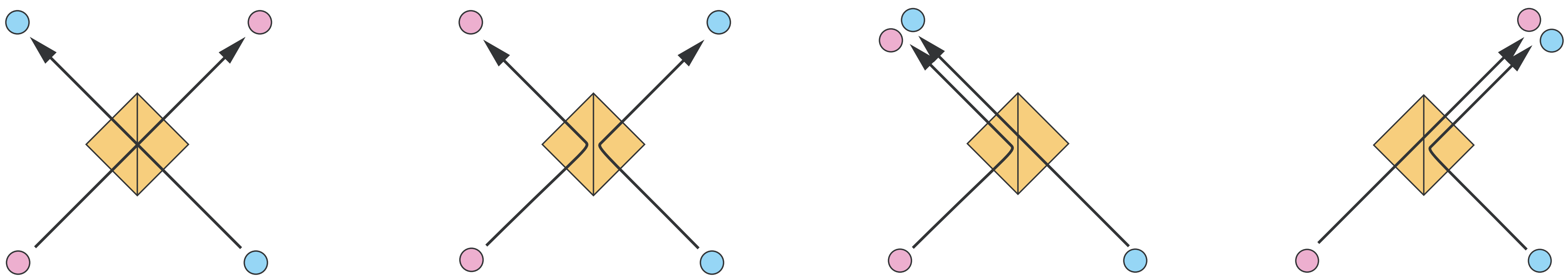

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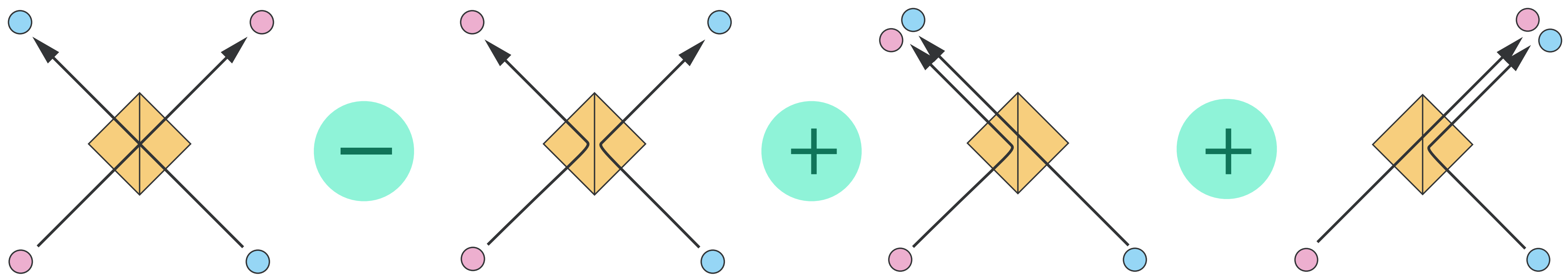

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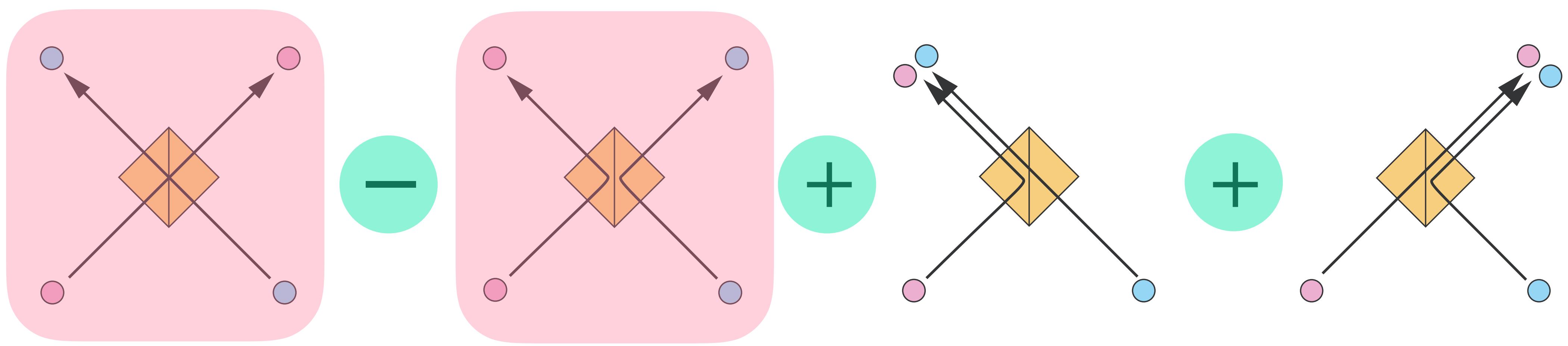


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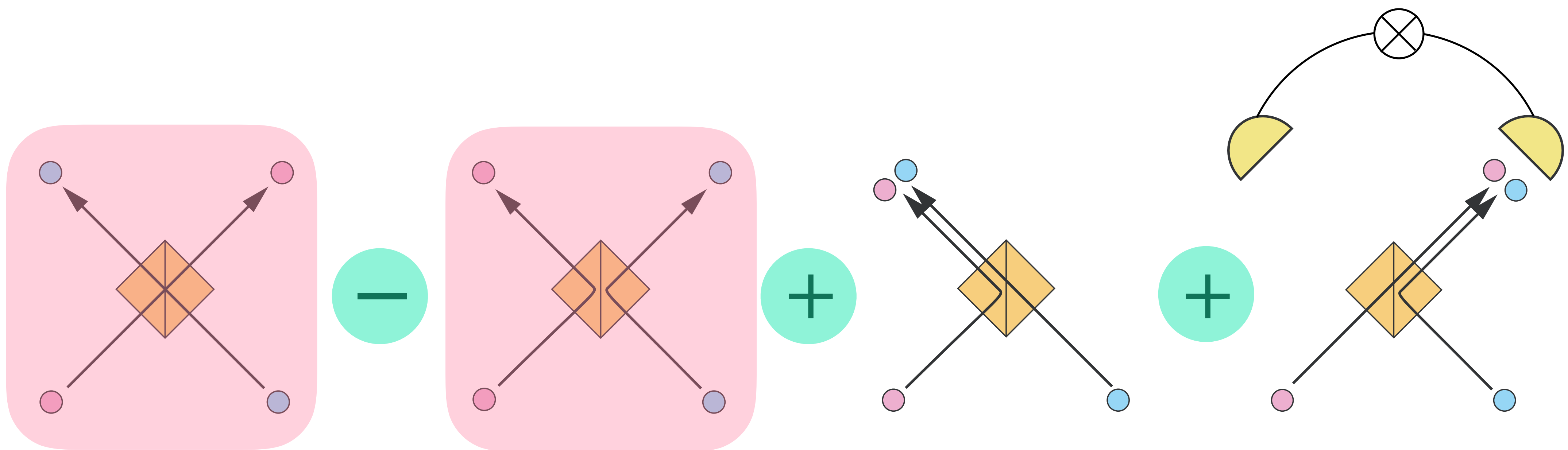

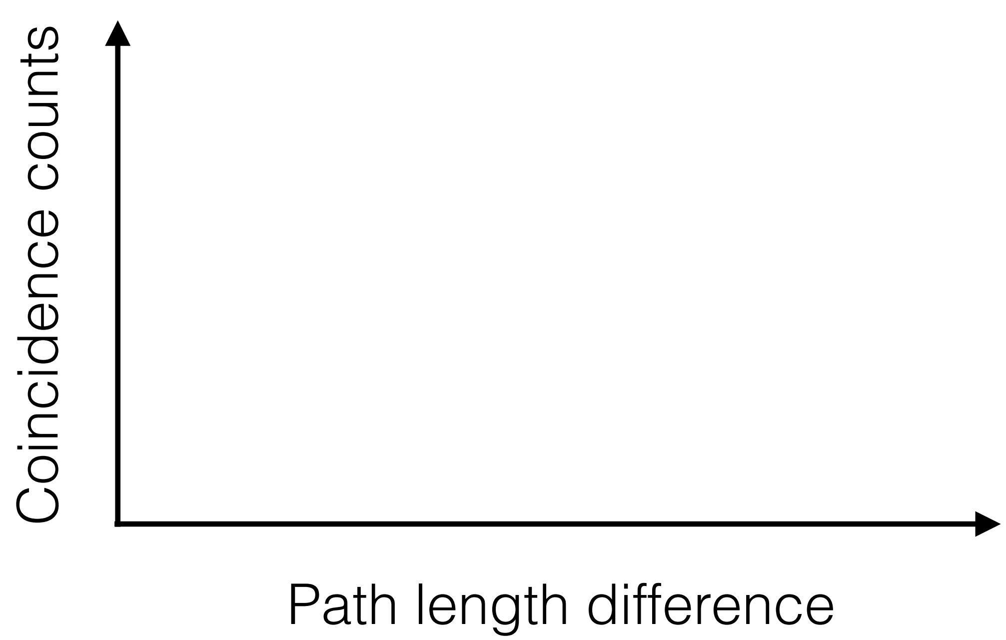


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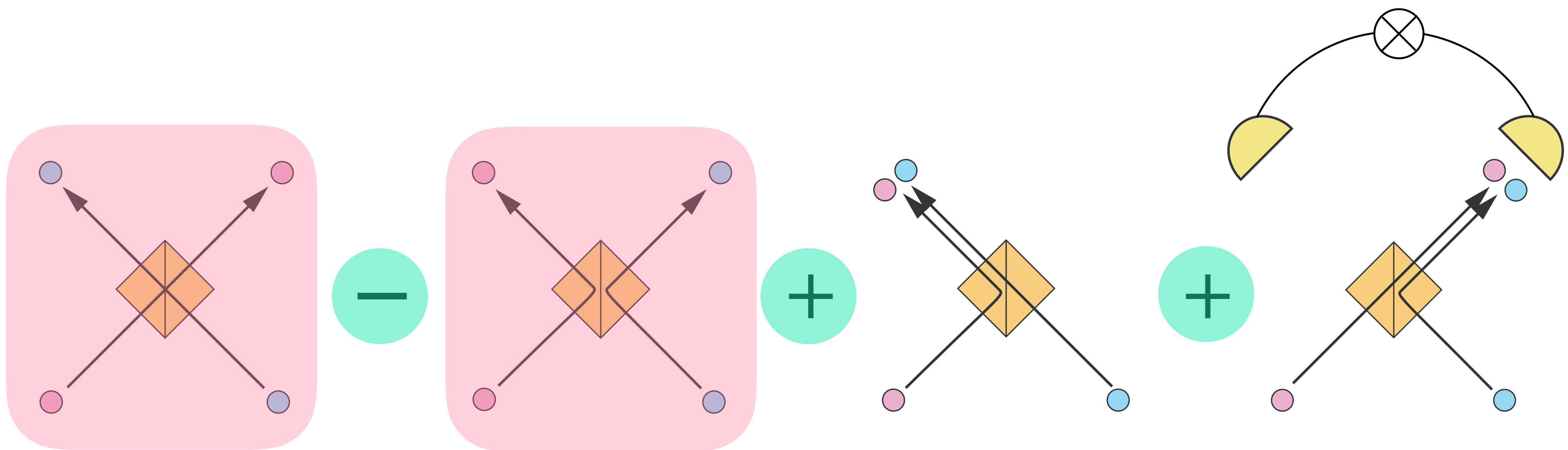

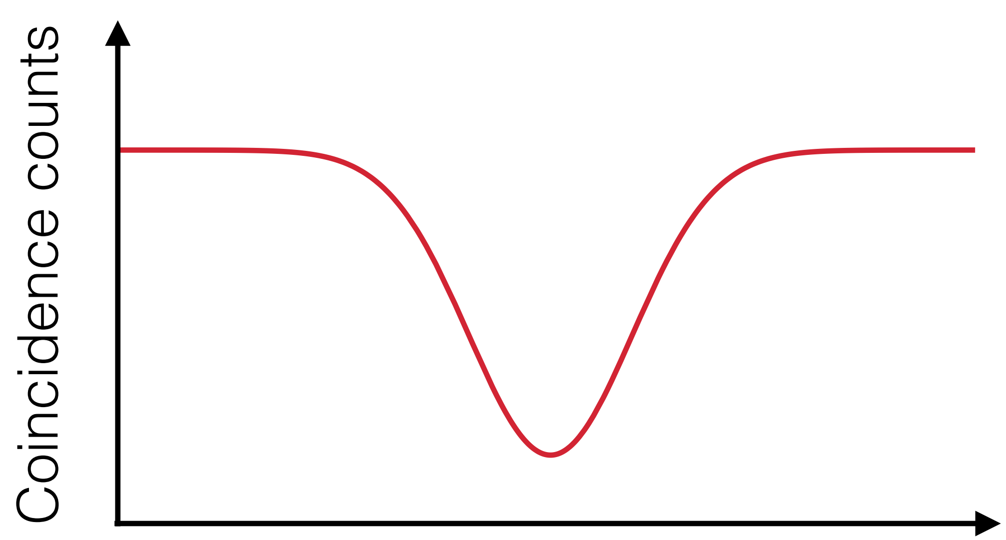


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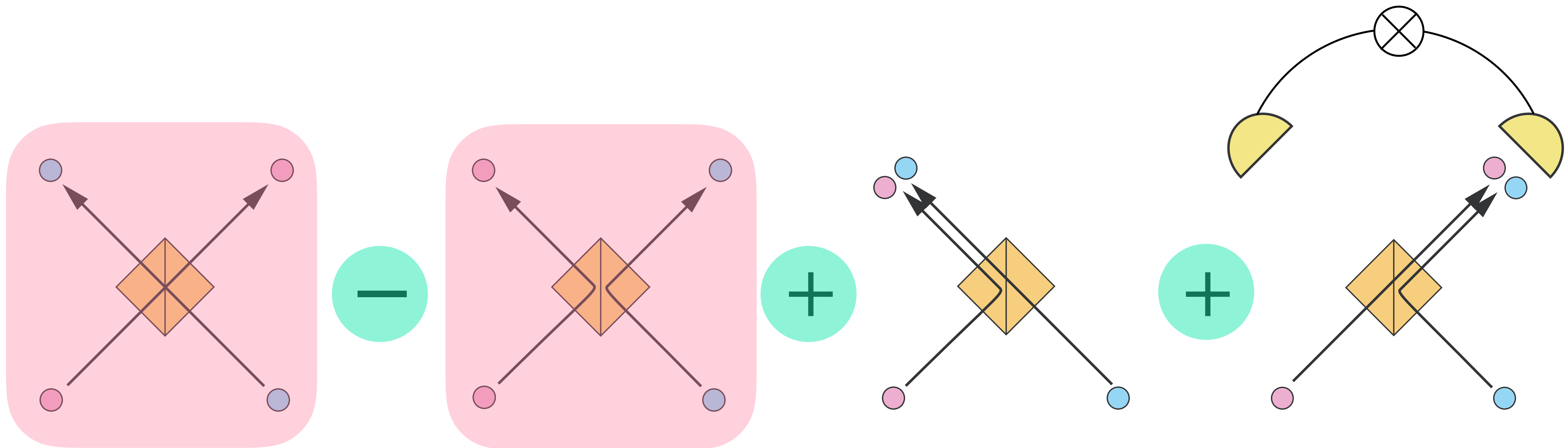
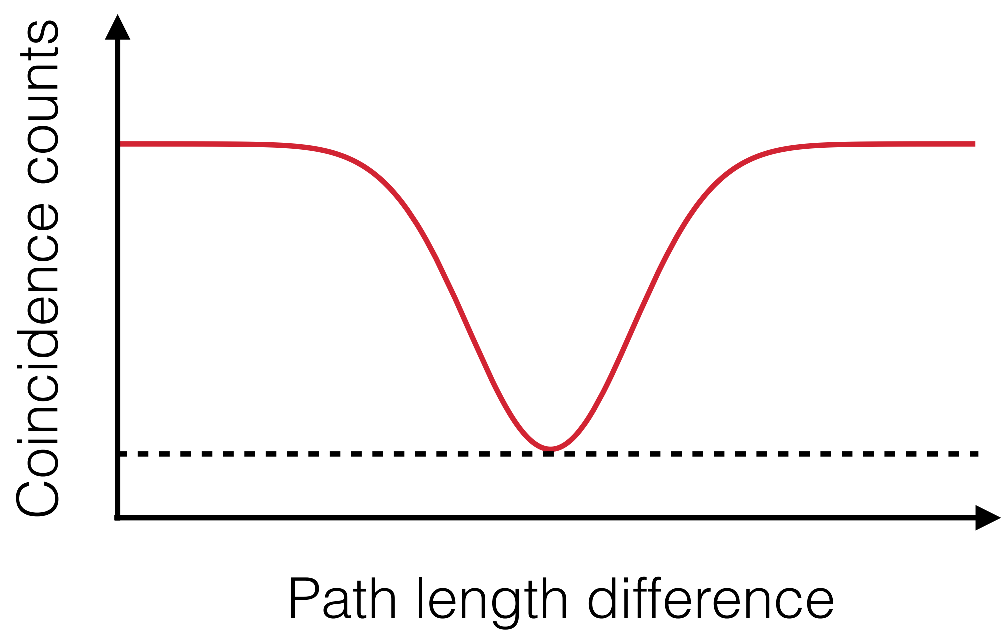
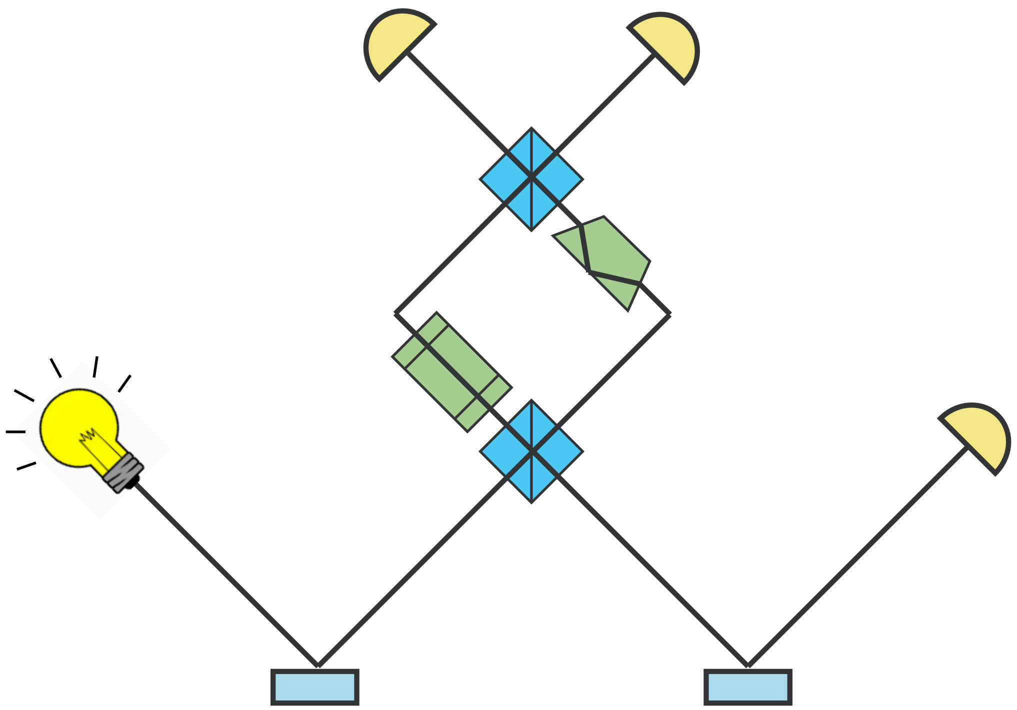


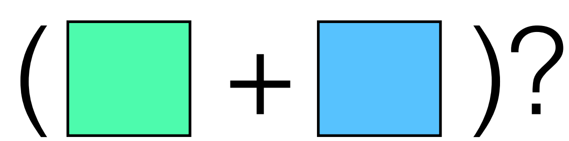



Hong, Ou, Mandel, **Phys. Rev. Lett. 59,** 2044 (1987)

# Coherent Superposition



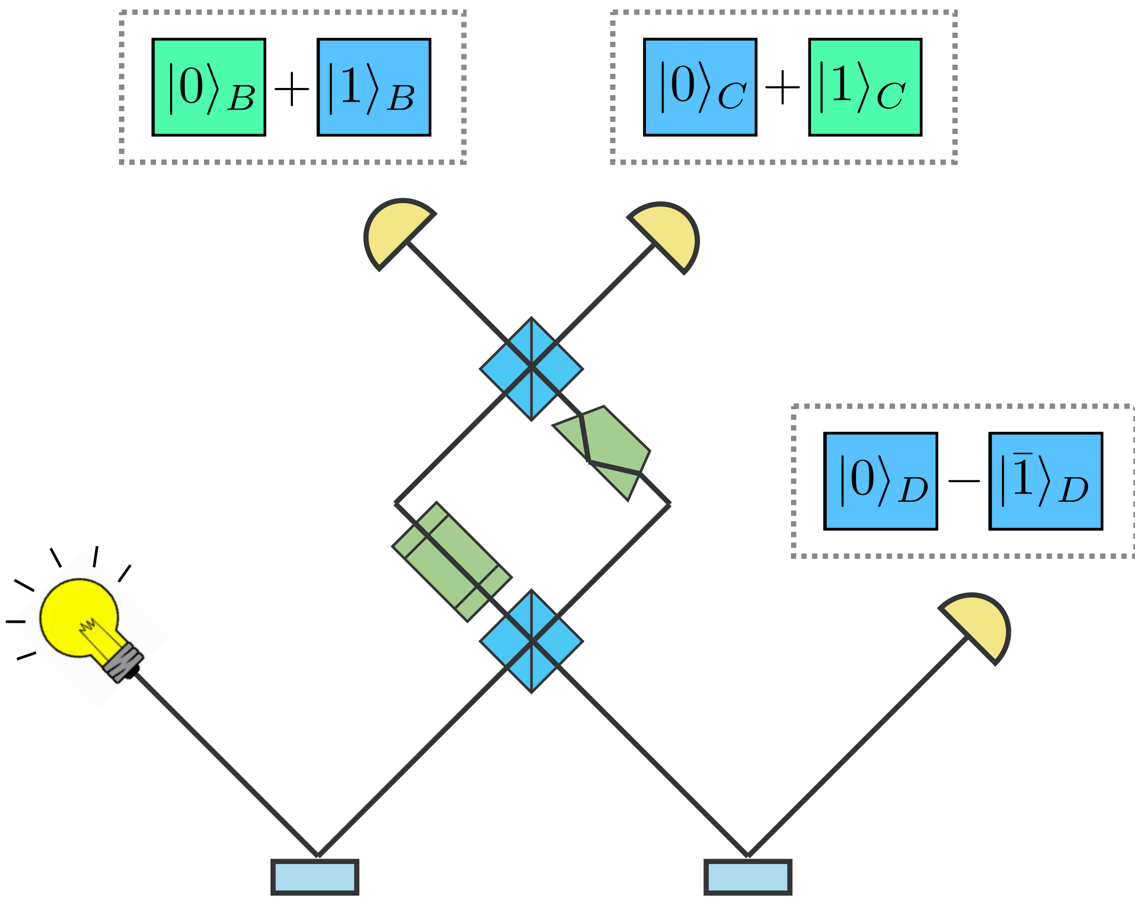
Coherent Superposition?



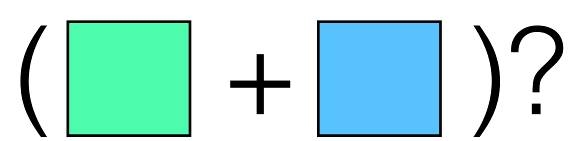
$$|\Psi\rangle_{332} = \boxed{|1\rangle_B|1\rangle_C|-1\rangle_D} + \boxed{|0\rangle_B|0\rangle_C|0\rangle_D} + \boxed{|1\rangle_B|-1\rangle_C|1\rangle_D}$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# Coherent Superposition




Coherent Superposition?

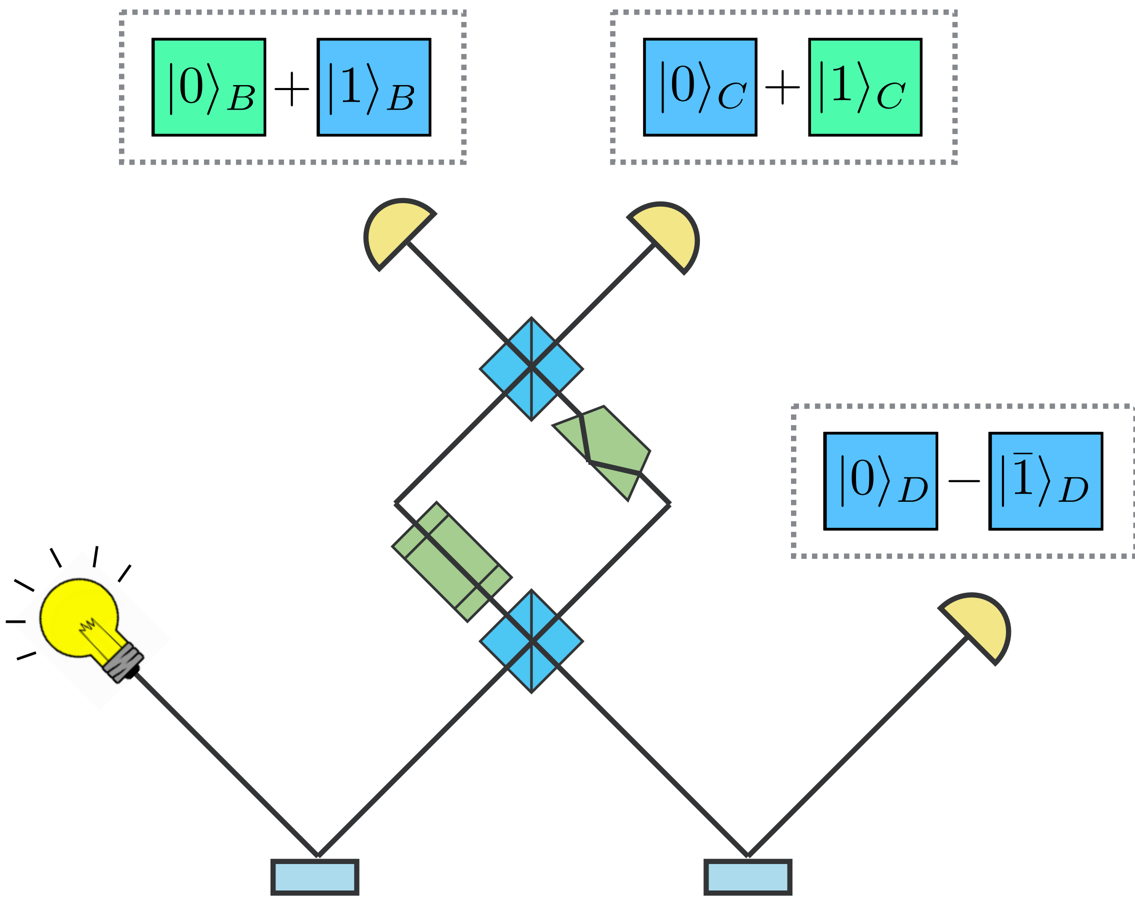


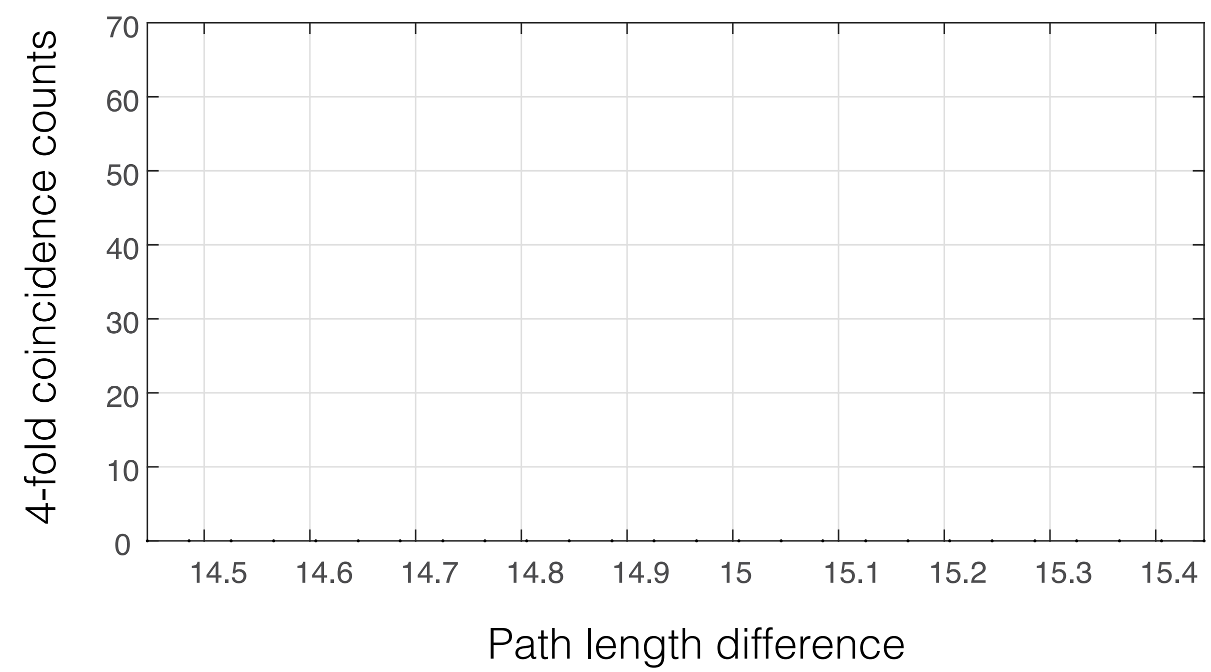

- Superposition measurements of OAM

$$|\Psi\rangle_{332} \;=\; |1\rangle_B|1\rangle_C|-1\rangle_D + |0\rangle_B|0\rangle_C|0\rangle_D$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# Coherent Superposition



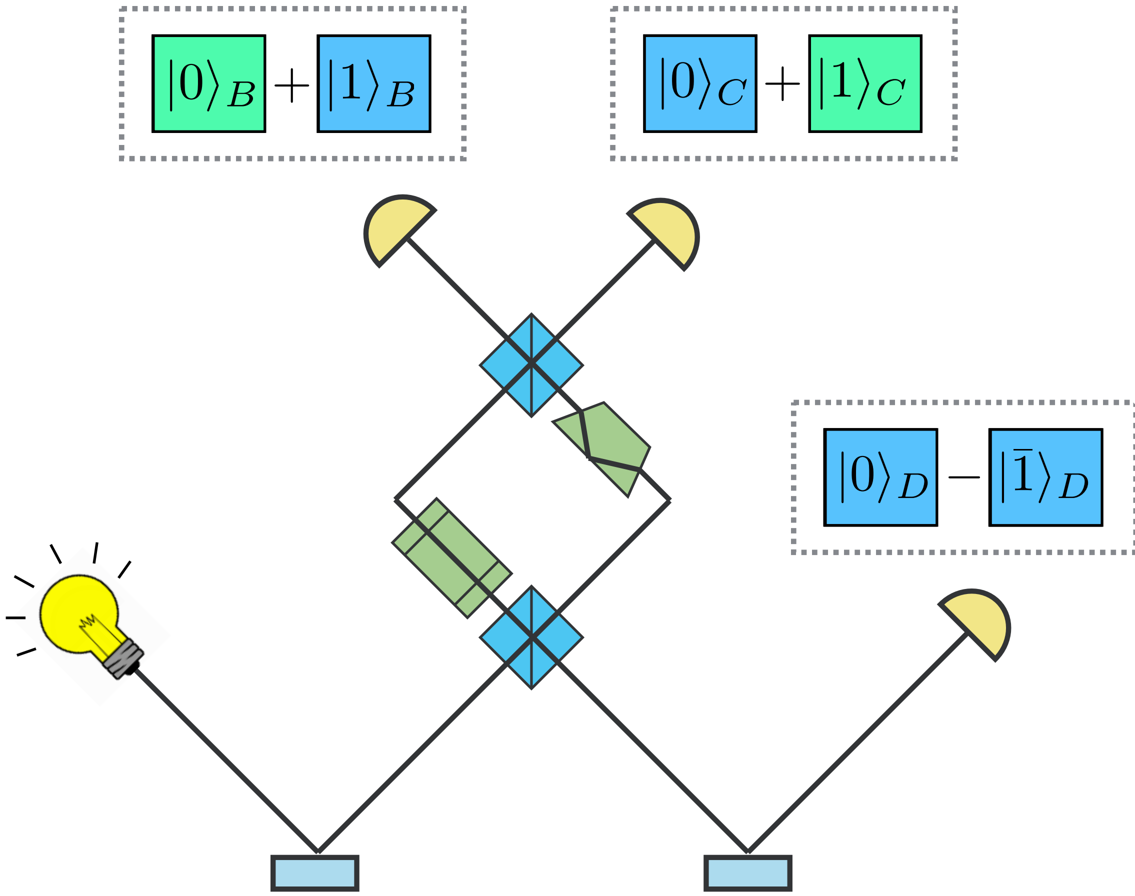



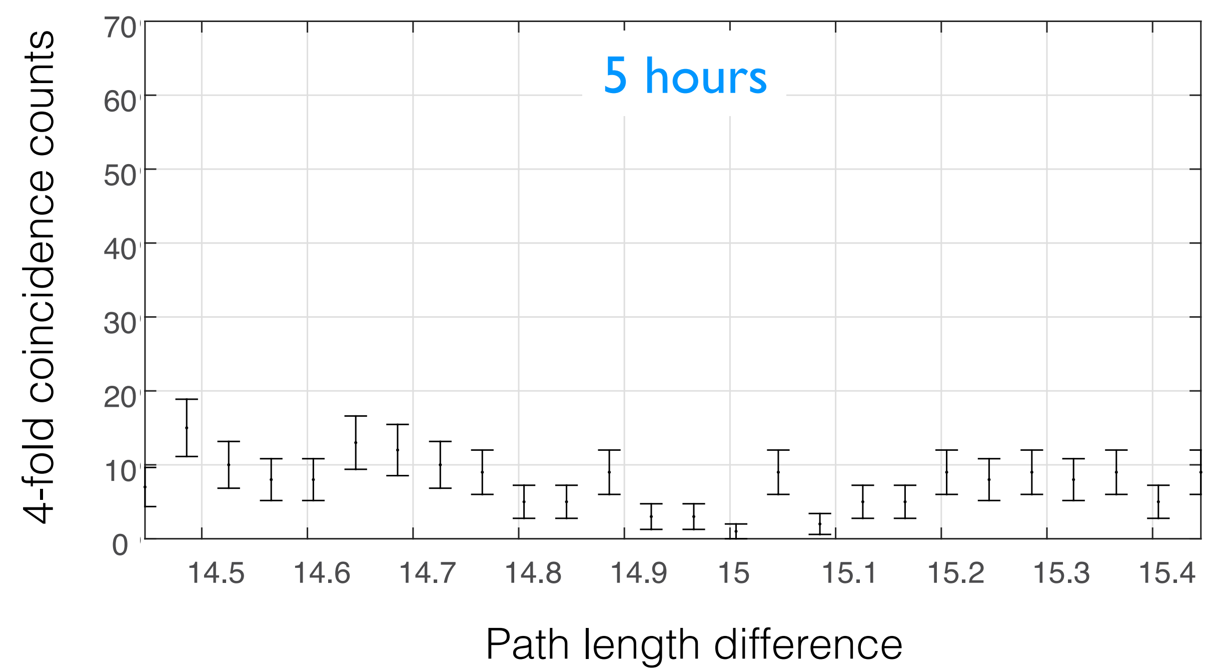

- Superposition measurements of OAM
- Look for two-photon interference from **independent** crystals

$$|\Psi\rangle_{332} = \boxed{|1\rangle_B|1\rangle_C|-1\rangle_D} + \boxed{|0\rangle_B|0\rangle_C|0\rangle_D}$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# Coherent Superposition



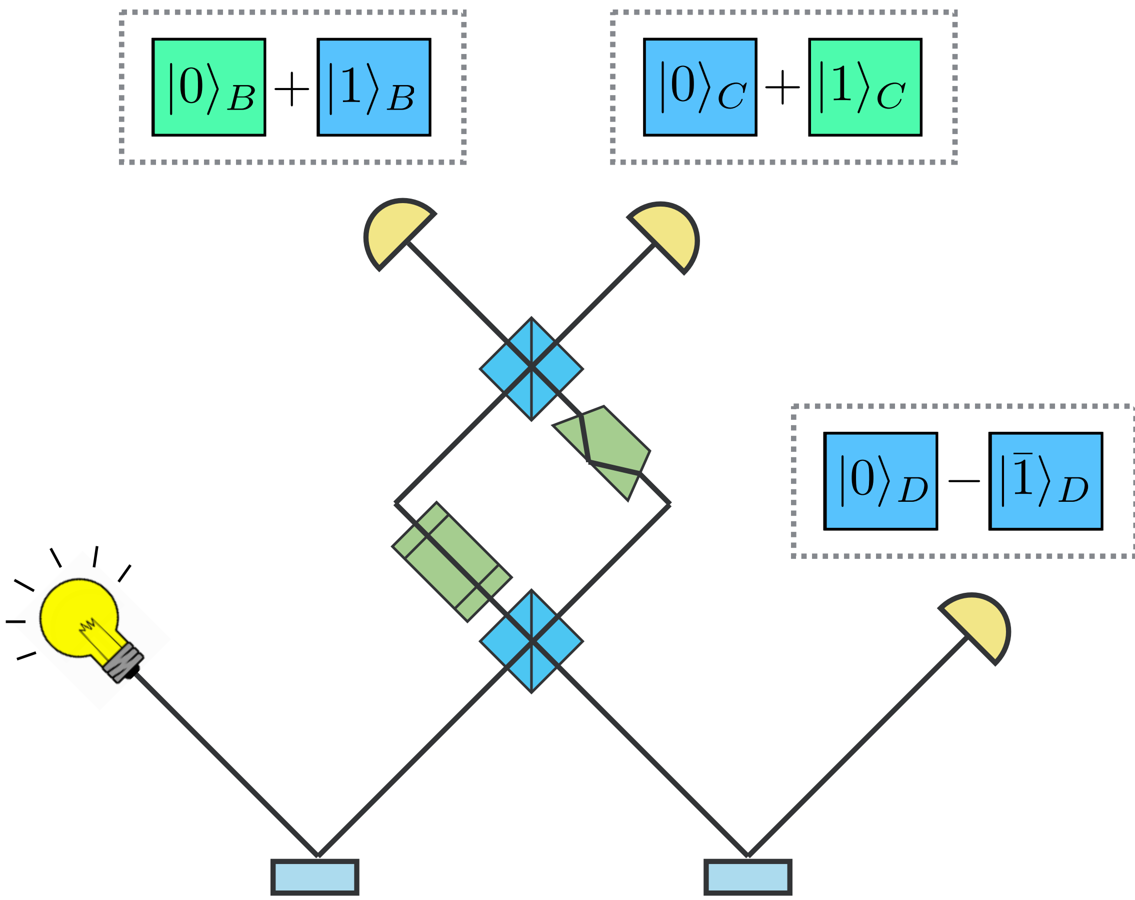



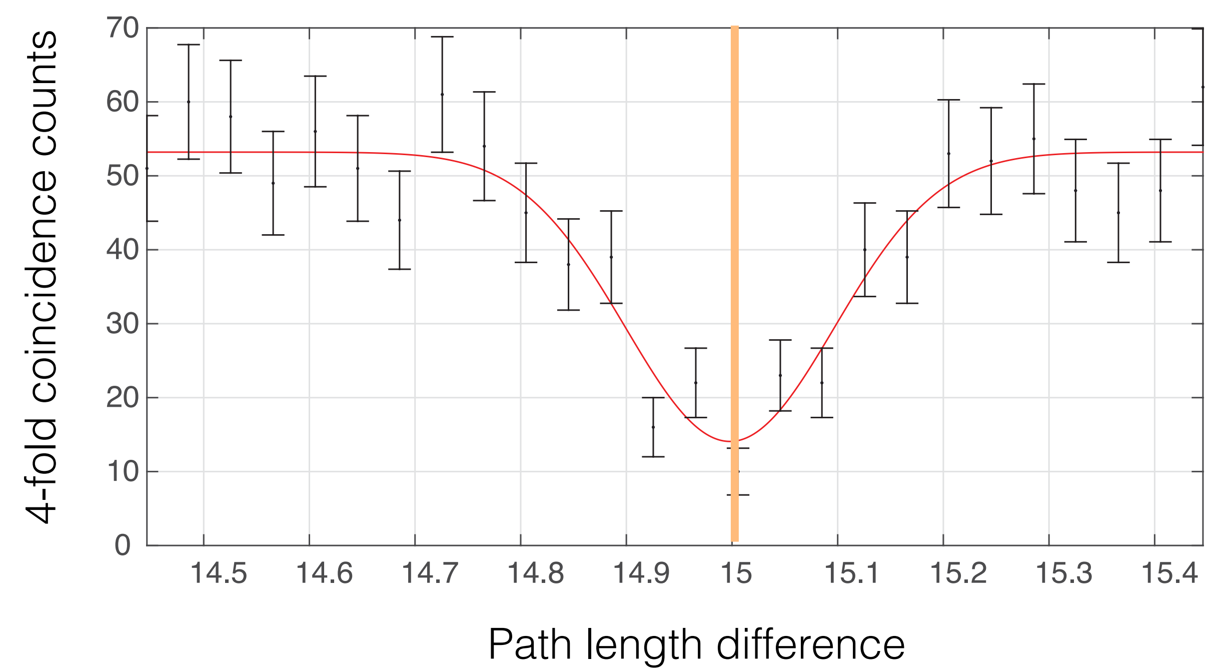

- Superposition measurements of OAM
- Look for two-photon interference from **independent** crystals

$$|\Psi\rangle_{332} = \boxed{|1\rangle_B|1\rangle_C|-1\rangle_D} + \boxed{|0\rangle_B|0\rangle_C|0\rangle_D}$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# Coherent Superposition

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$$|\Psi\rangle_{332} = \boxed{|1\rangle_B|1\rangle_C|-1\rangle_D} + \boxed{|0\rangle_B|0\rangle_C|0\rangle_D}$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

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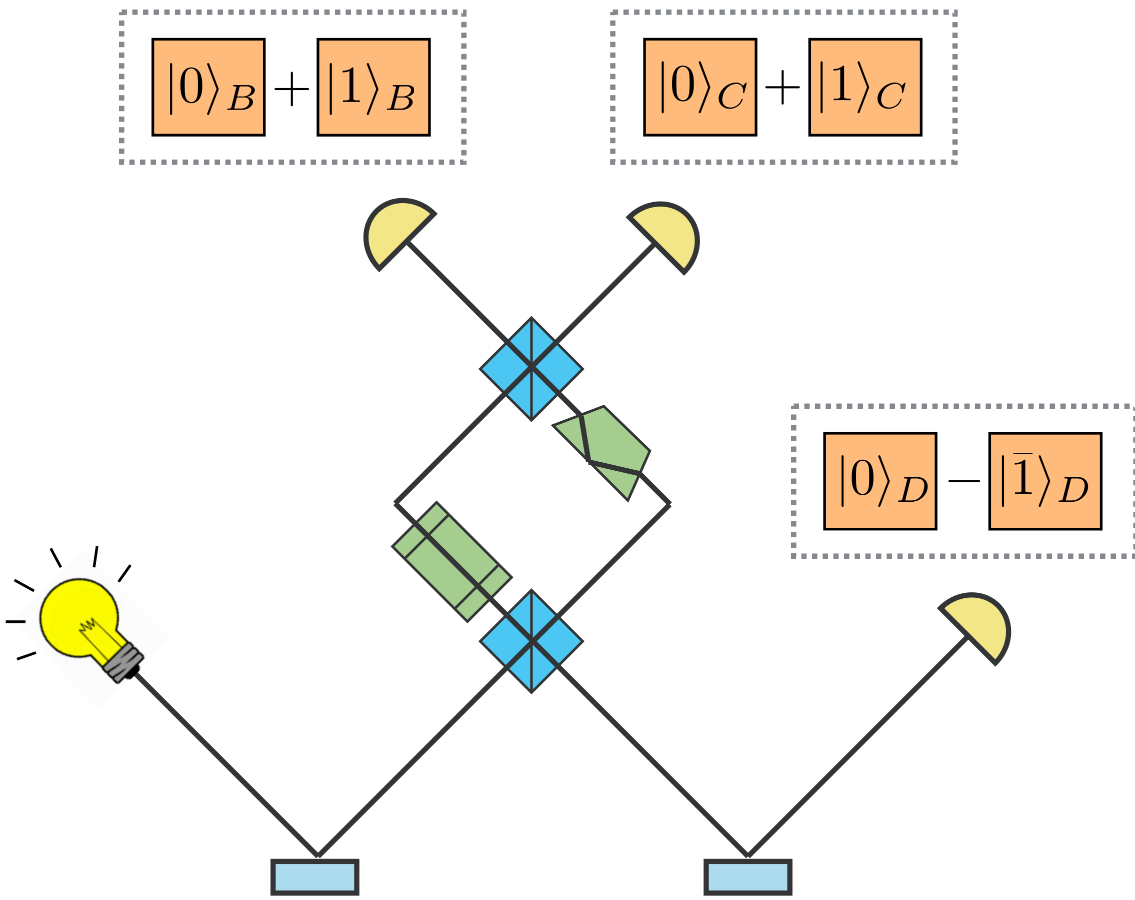
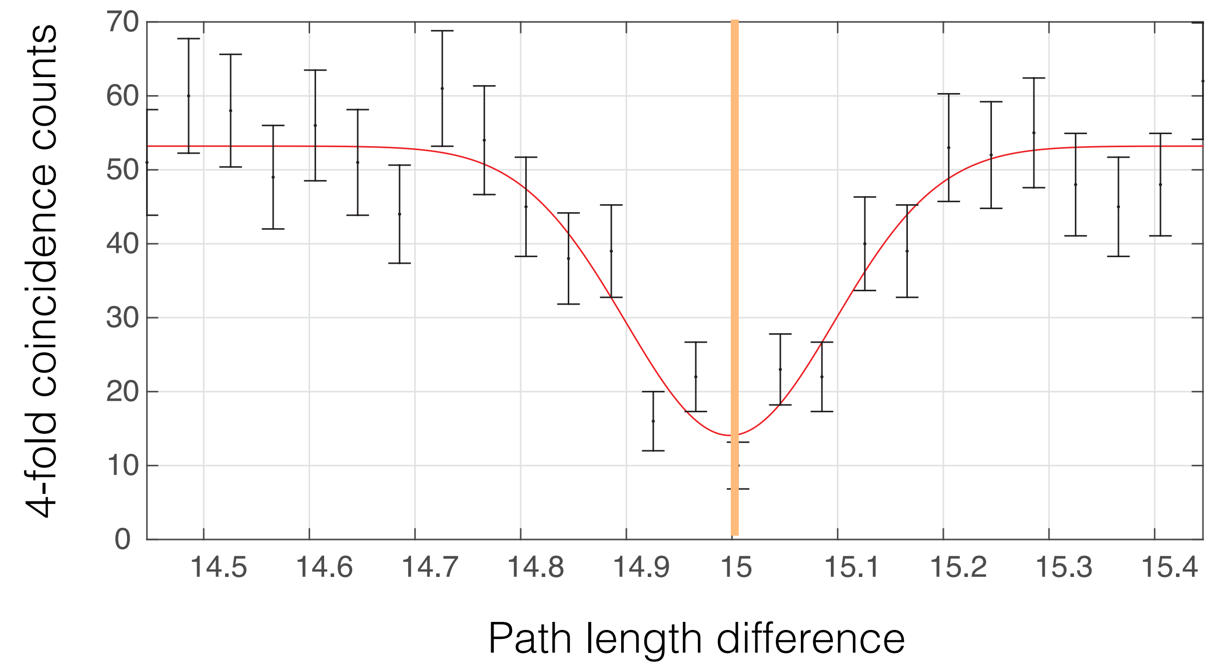


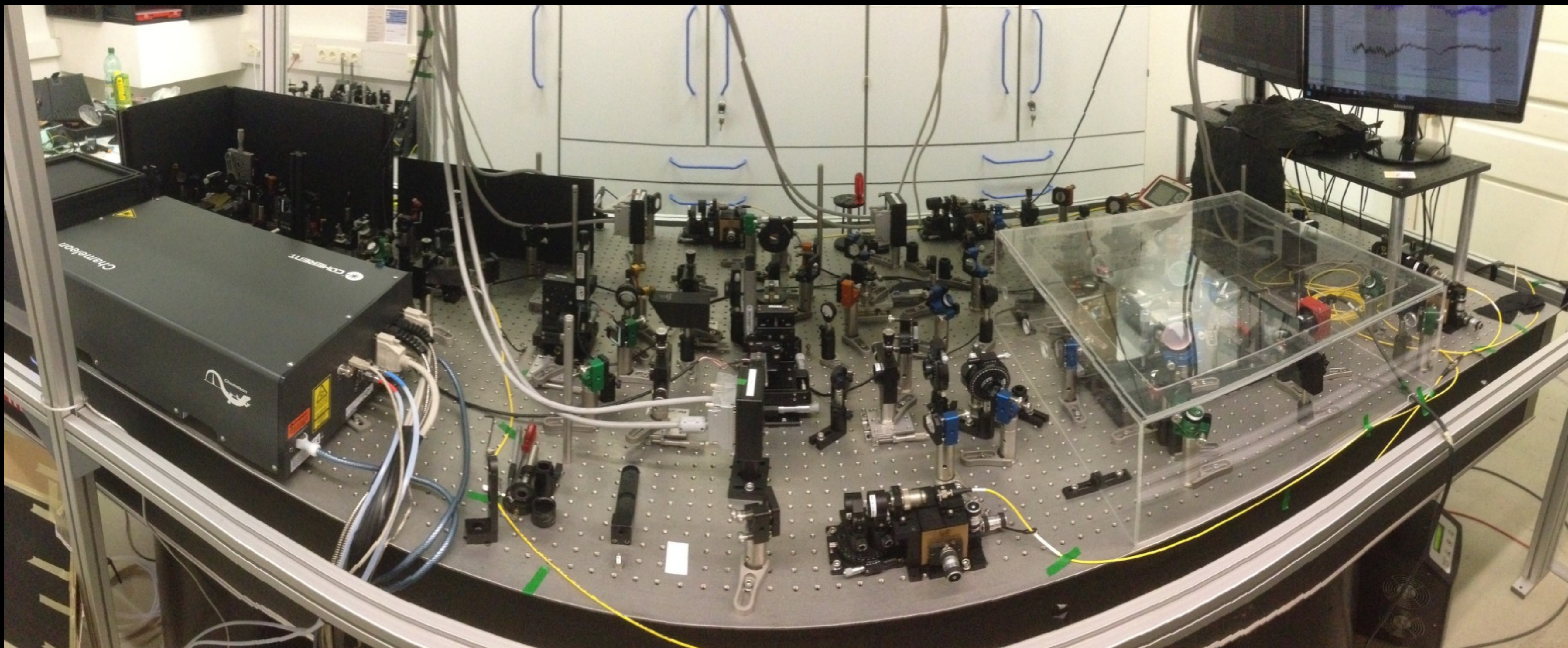




- Superposition measurements of OAM
- Look for two-photon interference from **independent** crystals
- Appearance of interference dip indicates **coherent superposition**:

$$|\Psi\rangle_{332} = \boxed{|1\rangle_B|1\rangle_C|-1\rangle_D} + \boxed{|0\rangle_B|0\rangle_C|0\rangle_D} + \boxed{|1\rangle_B|-1\rangle_C|1\rangle_D}$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# Experimental Setup



# Experimental Details

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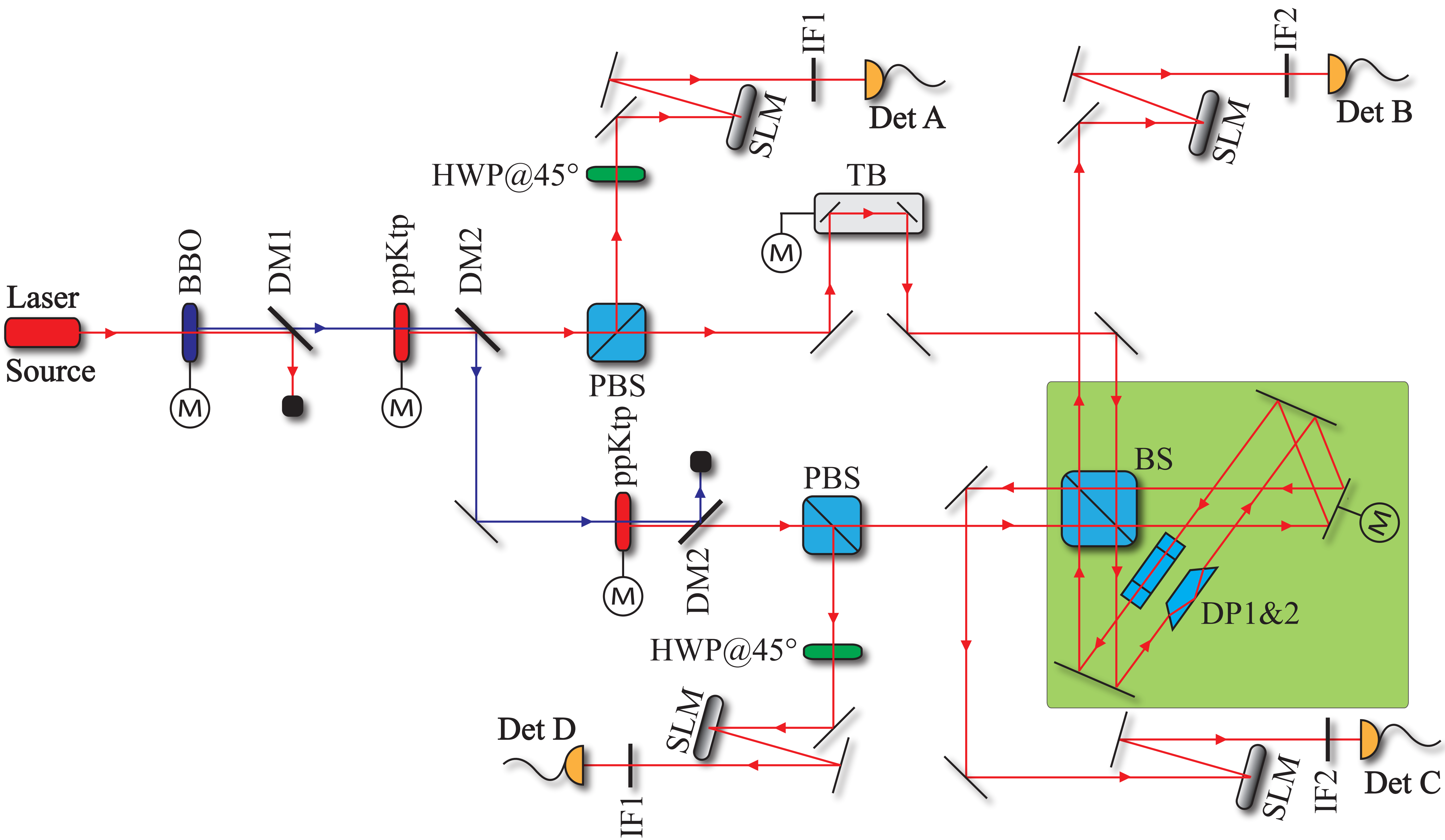

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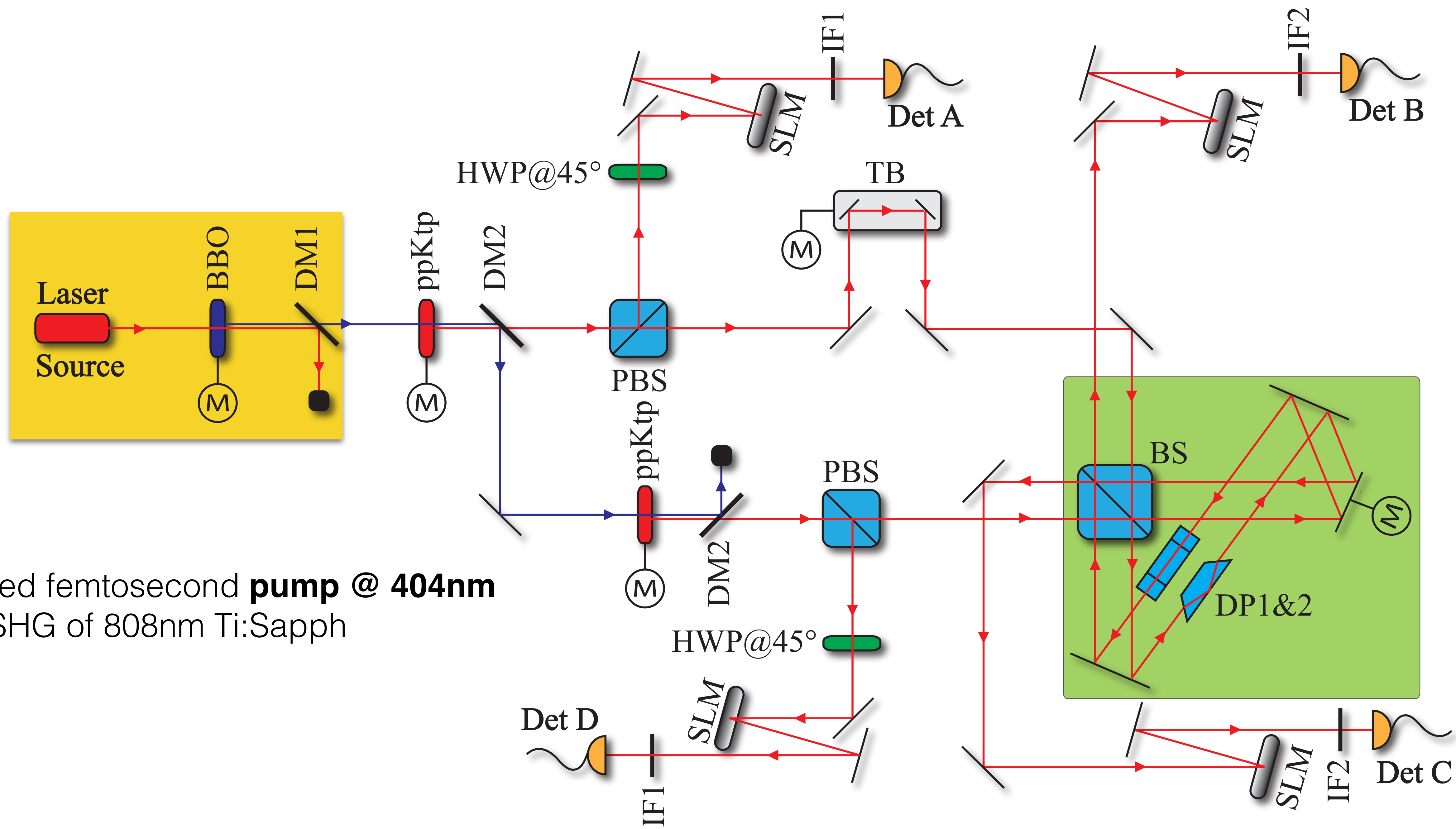
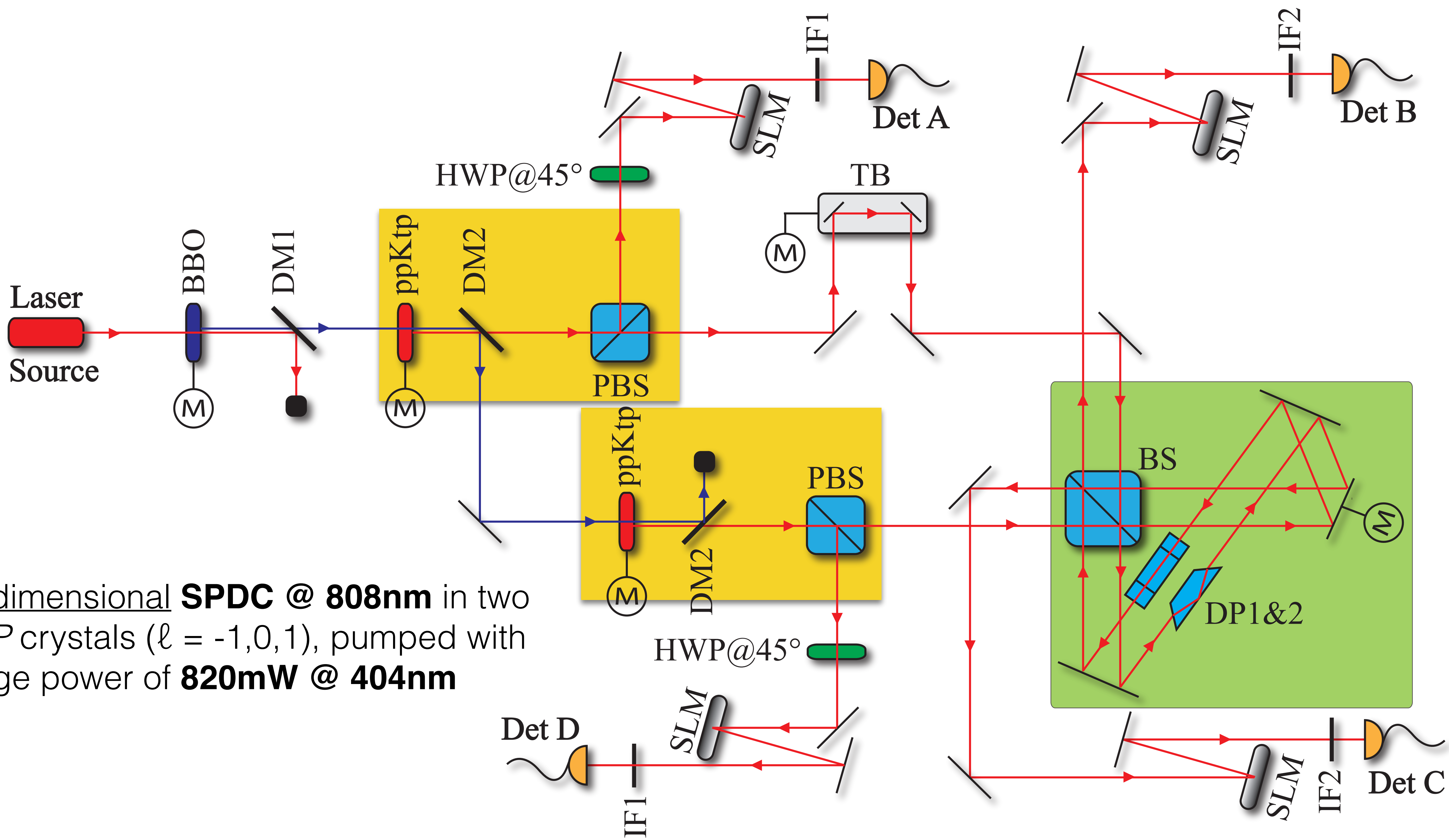


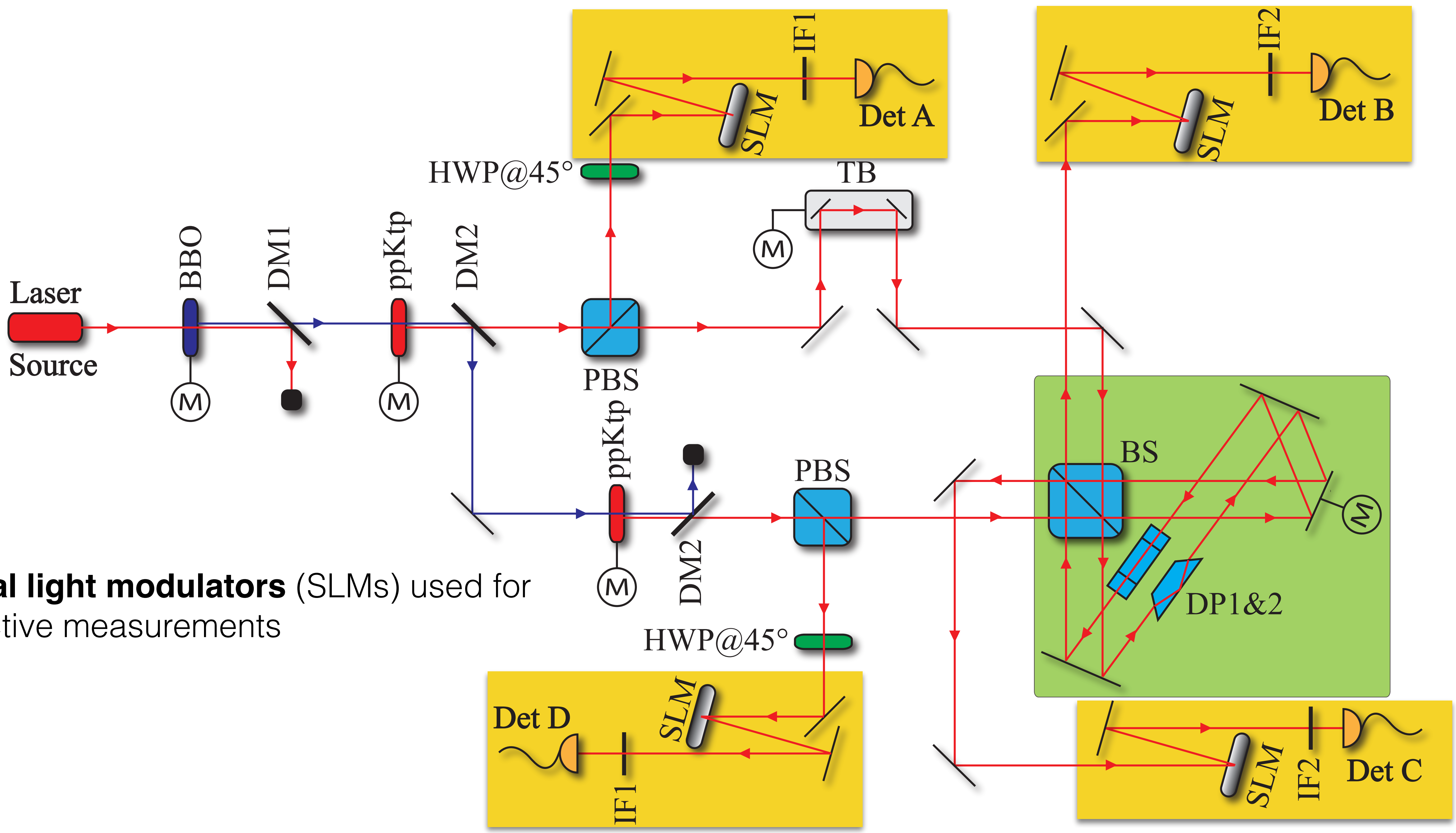

- Pulsed femtosecond **pump @ 404nm** via SHG of 808nm Ti:Sapph

# Experimental Details



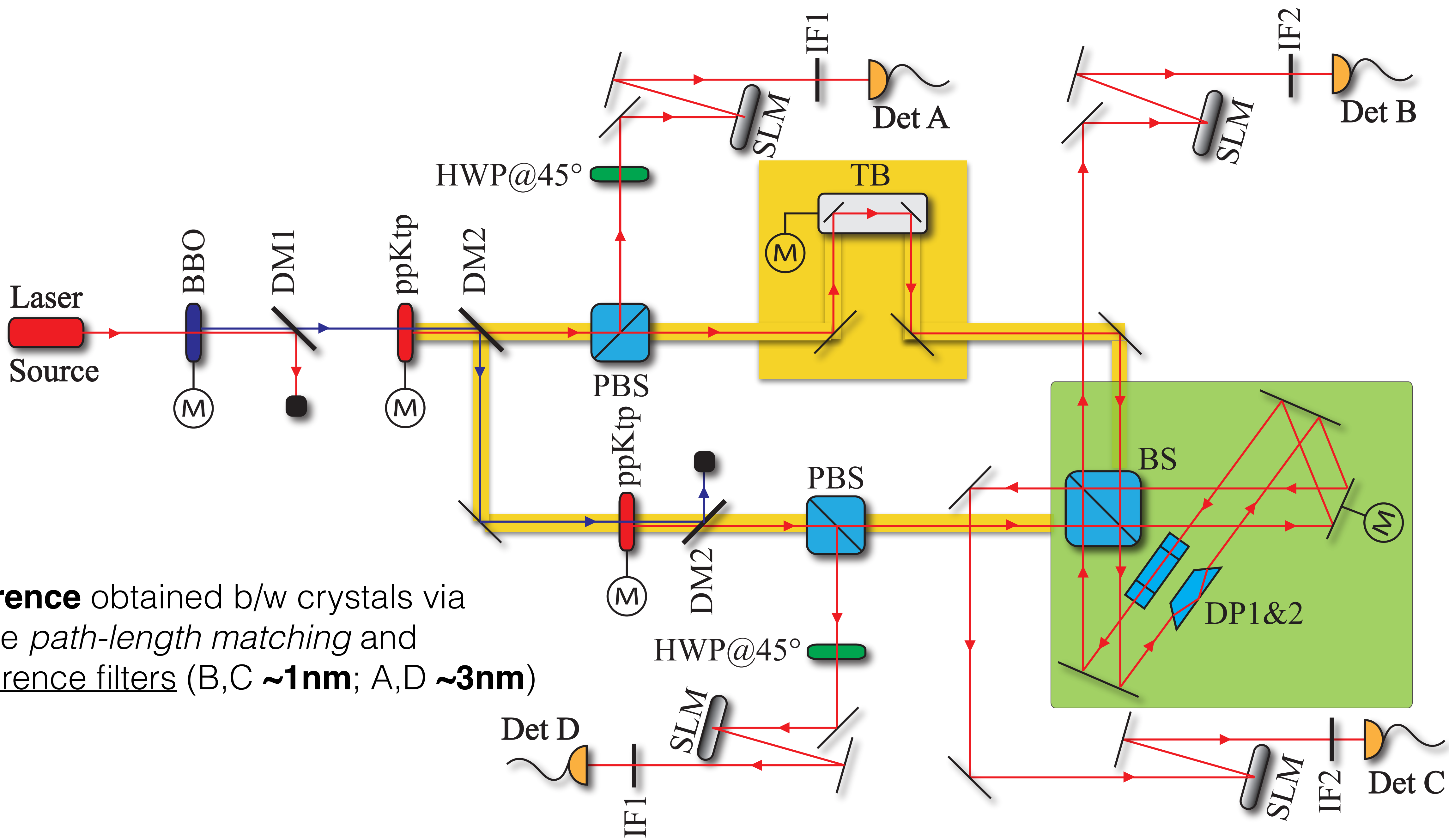

- High-dimensional **SPDC @ 808nm** in two *PPKTP* crystals (ℓ = -1,0,1), pumped with average power of **820mW @ 404nm**

# Experimental Details



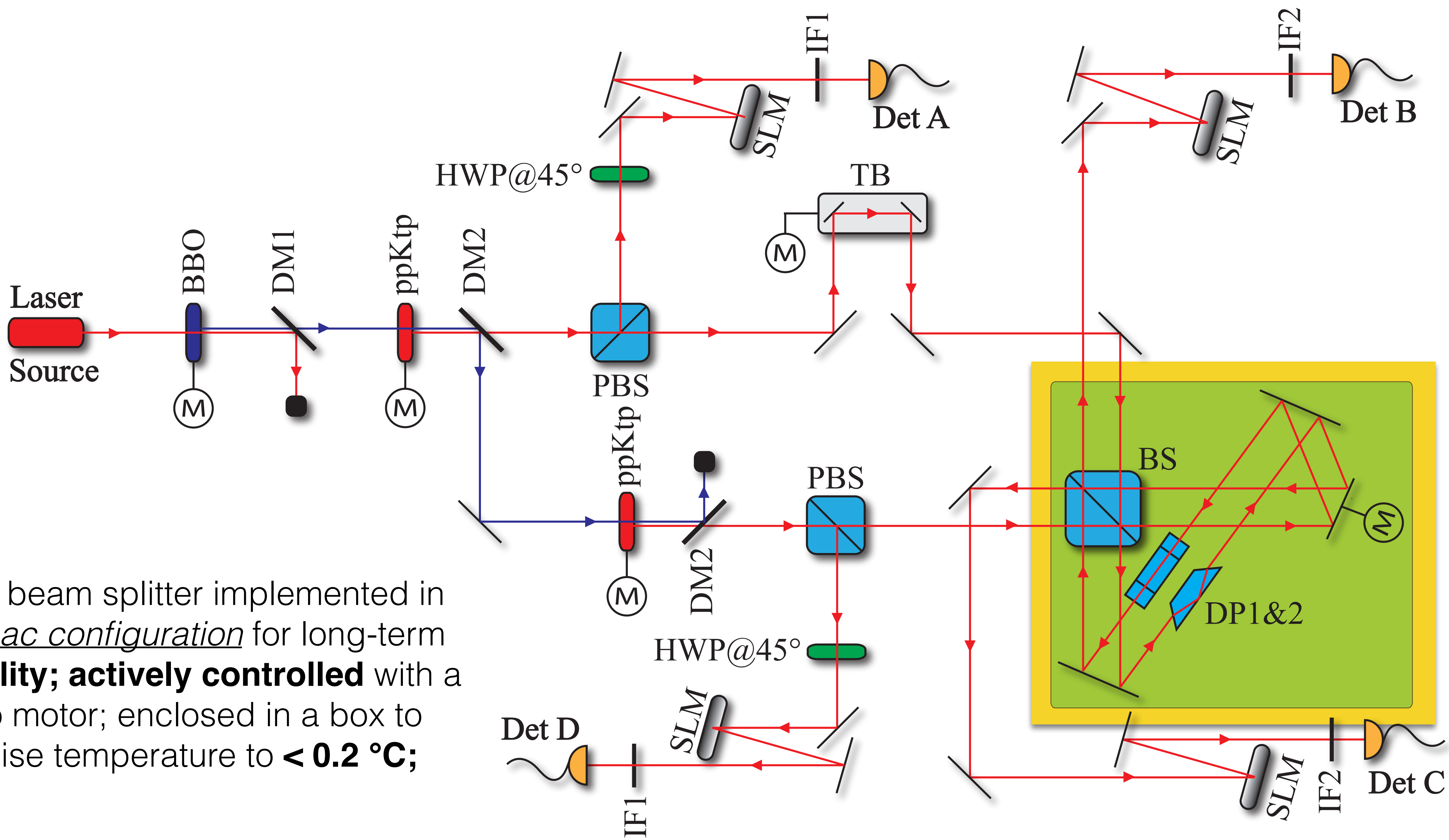

- **Spatial light modulators** (SLMs) used for projective measurements

# EXPERIMENTAL DETAILS



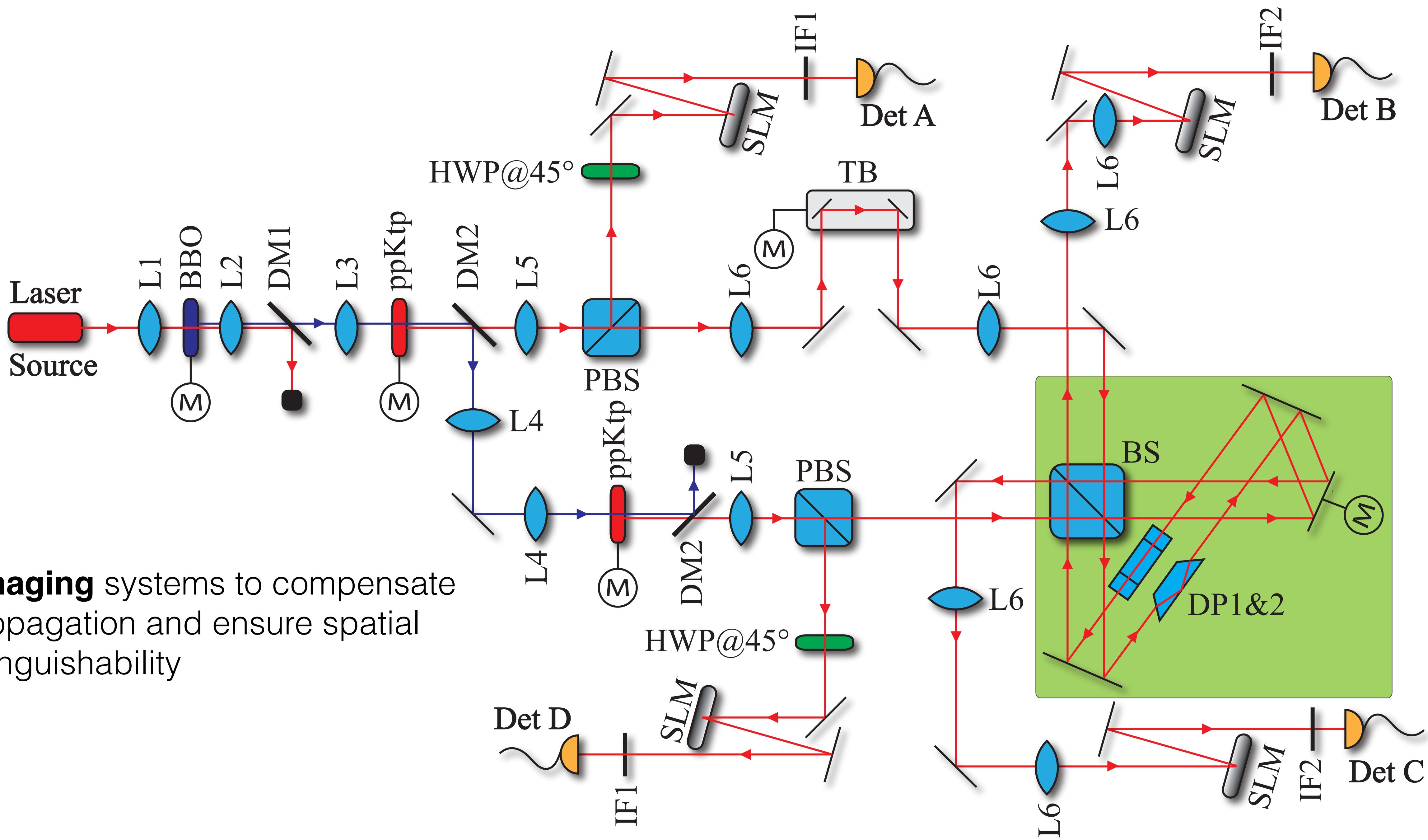

- **Coherence** obtained b/w crystals via precise *path-length matching* and <u>interference filters</u> (B,C **~1nm**; A,D **~3nm**)

# Experimental Details




- OAM beam splitter implemented in *Sagnac configuration* for long-term **stability; actively controlled** with a piezo motor; enclosed in a box to stabilise temperature to **< 0.2 °C;**

# Experimental Details




- **4f - Imaging** systems to compensate for propagation and ensure spatial indistinguishability

# VERIFYING ENTANGLEMENT

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- Full reconstruction of our (large) state is prohibitive

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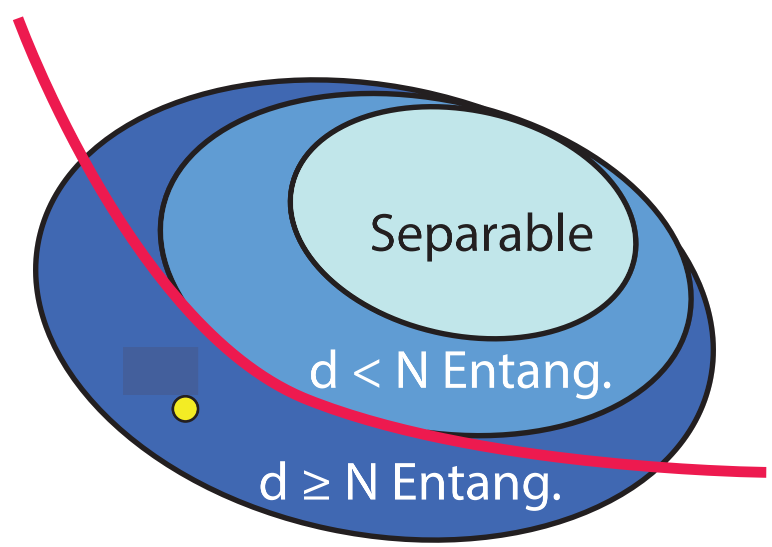
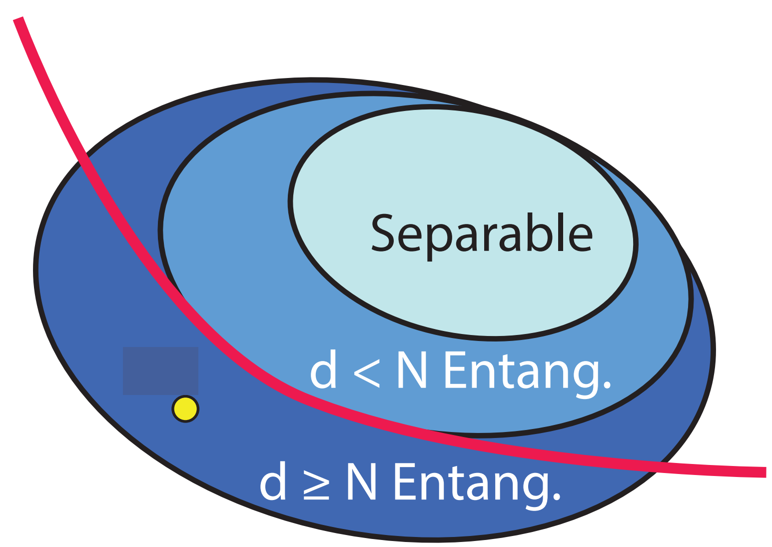
- Entanglement Witness!



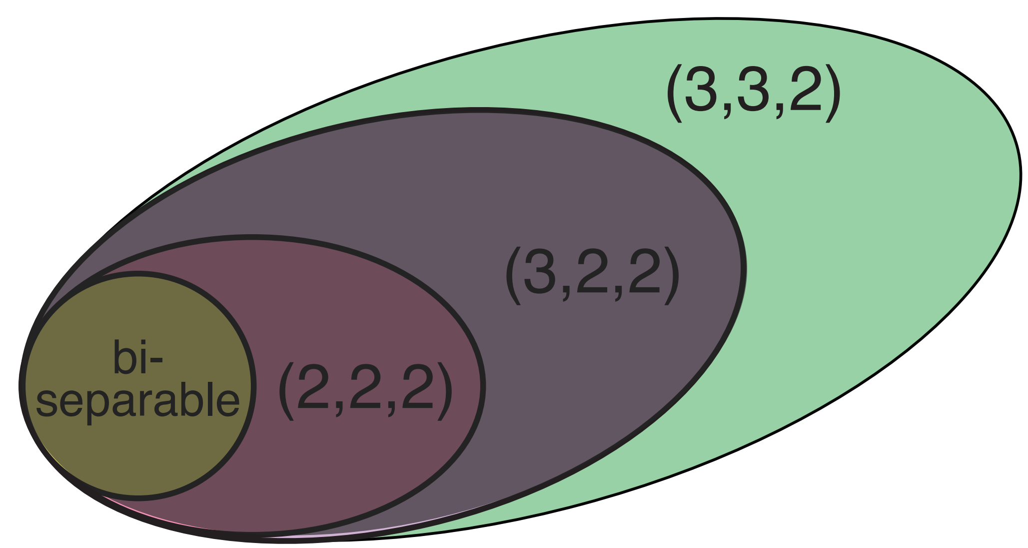

Entanglement Witness (W)

# VERIFYING ENTANGLEMENT

- Full reconstruction of our (large) state is prohibitive
- Entanglement Witness!
- Prove: measured state **cannot** be decomposed into <u>entangled states</u> of *smaller dimensionality structure* (e.g. - 322, 222, 22)




Entanglement Witness (W)

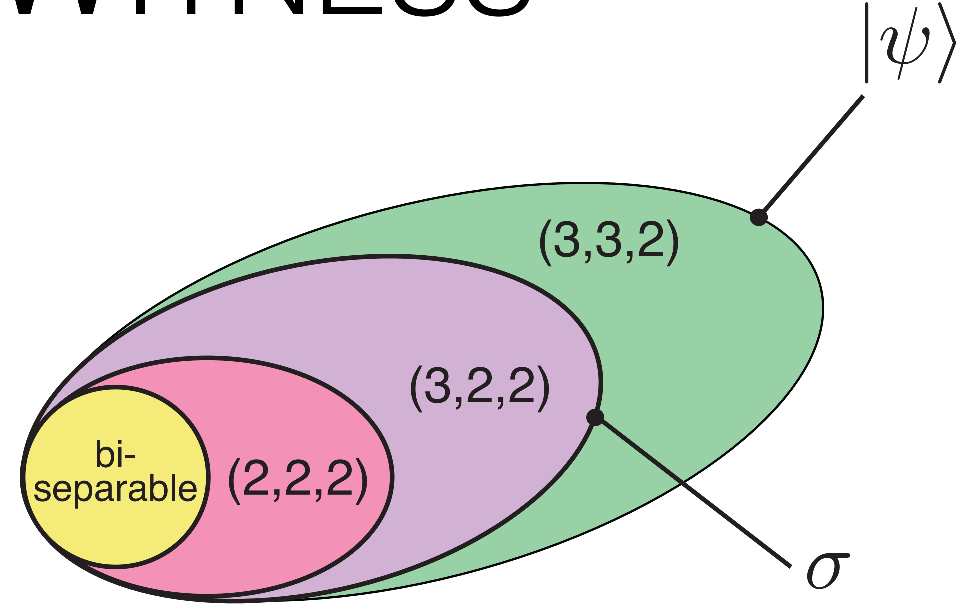



Huber and Vicente, **Phys. Rev. Lett. 110,** 030501 (2013)

# 332 Entanglement Witness

# 332 ENTANGLEMENT WITNESS

- Best achievable fidelity of a <u>322 state</u> (***σ***) with an *ideal* <u>332 state</u> (Ψ):




MM, Erhard, Huber, Krenn, Fickler, Zeilinger, Nature Photonics 10, 248 (2016)

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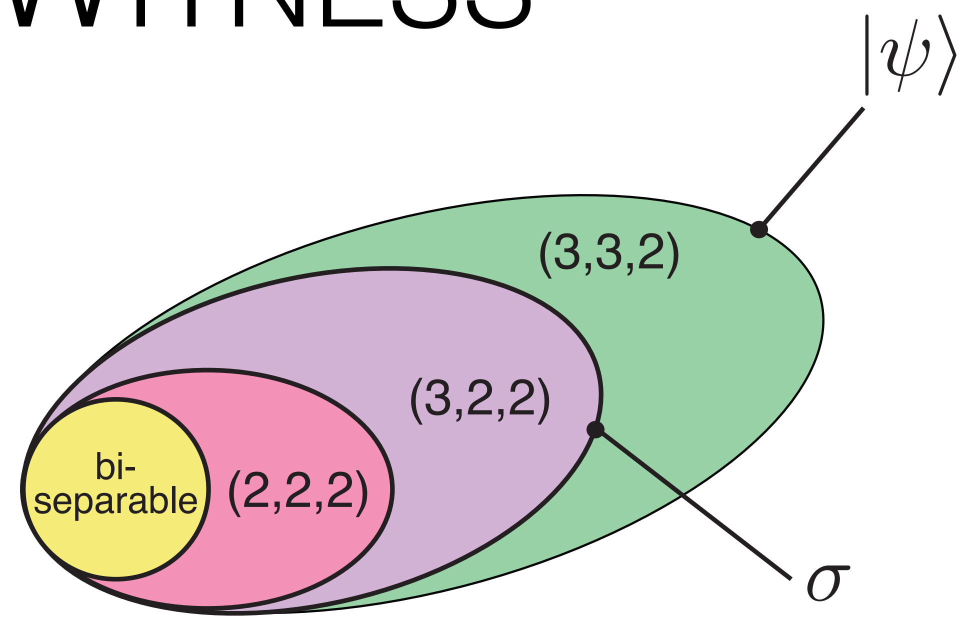
$$F_{max} := \max_{\sigma \in (322)} \mathrm{Tr}(\sigma|\Psi\rangle\langle\Psi|) = 0.6\bar{6}$$




MM, Erhard, Huber, Krenn, Fickler, Zeilinger, Nature Photonics 10, 248 (2016)

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- Best achievable fidelity of a 322 state ($\sigma$) with an *ideal* 332 state ($\Psi$):

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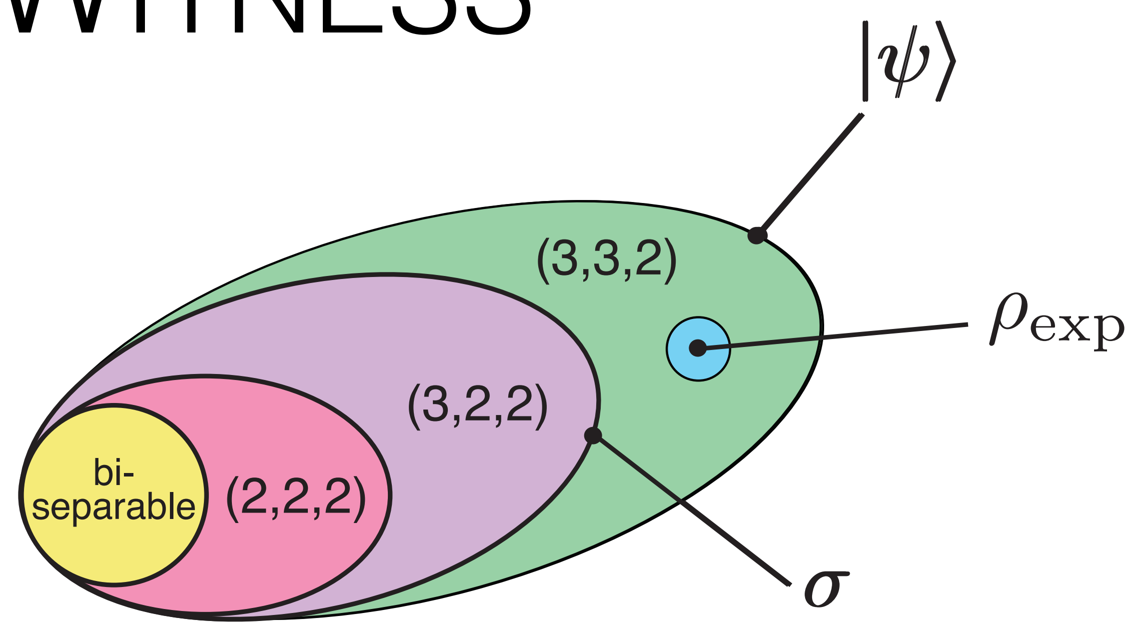
- Measured fidelity of generated state ($\rho_{exp}$) with an *ideal* 332 state ($\Psi$)




MM, Erhard, Huber, Krenn, Fickler, Zeilinger, Nature Photonics 10, 248 (2016)

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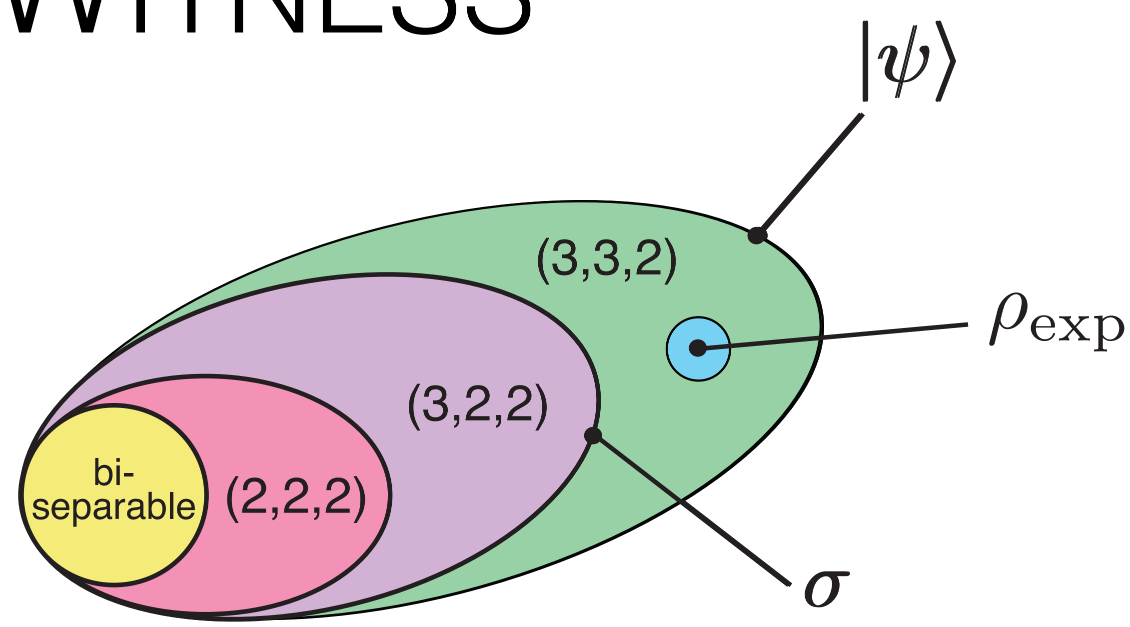
- Density matrix elements: **18** diagonal, **3** off-diagonal




**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

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32 measurements!

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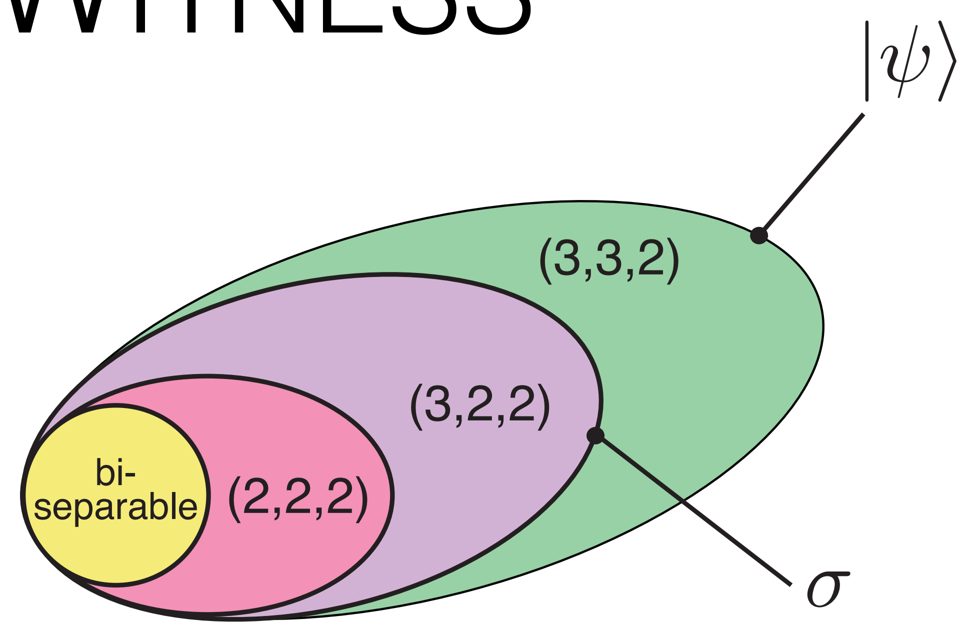


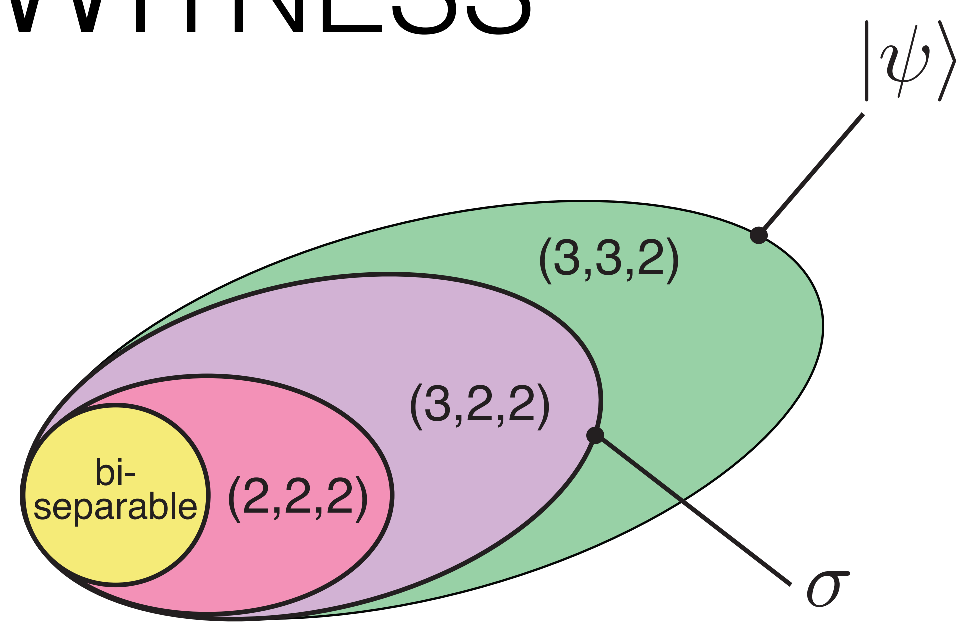

$$|\Psi\rangle_{332} = \frac{1}{\sqrt{3}}[|000\rangle + |111\rangle + |221\rangle]$$

MM, Erhard, Huber, Krenn, Fickler, Zeilinger, Nature Photonics 10, 248 (2016)

# 332 ENTANGLEMENT WITNESS

- Best achievable fidelity of a 322 state ($\sigma$) with an *ideal* 332 state ($\Psi$):

$$F_{max} := \max_{\sigma \in (322)} \mathrm{Tr}(\sigma|\Psi\rangle\langle\Psi|) = 0.6\bar{6}$$

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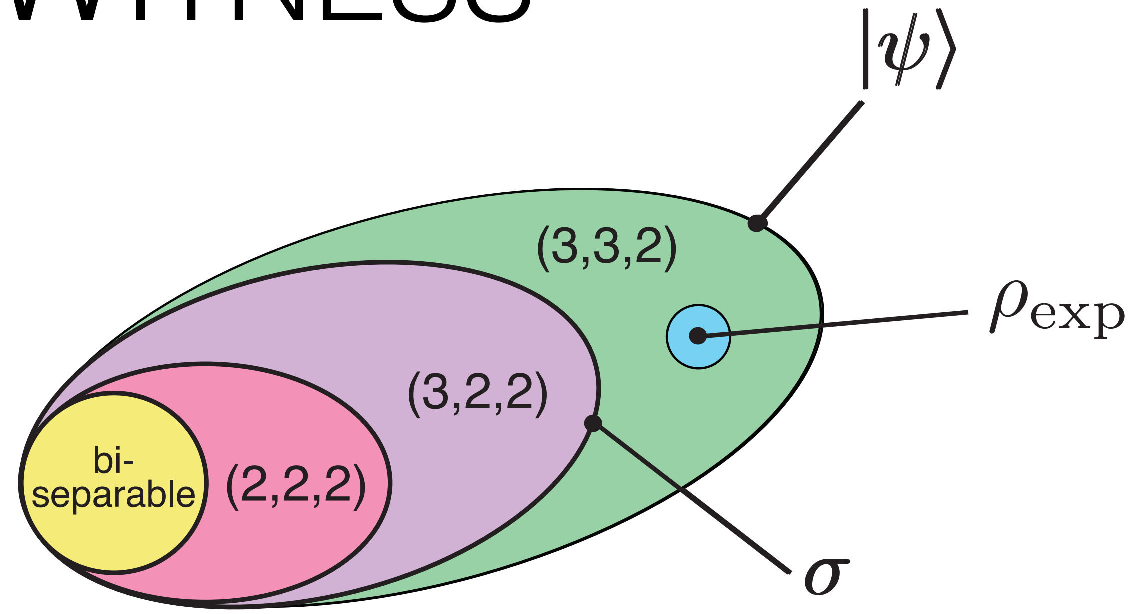
# 332 ENTANGLEMENT WITNESS

- Best achievable fidelity of a 322 state ($\sigma$) with an *ideal* 332 state ($\Psi$):

$$F_{max} := \max_{\sigma\in(322)} \mathrm{Tr}(\sigma|\Psi\rangle\langle\Psi|) = 0.6\bar{6}$$

- Measured fidelity of generated state ($\rho_{exp}$) with an *ideal* 332 state ($\Psi$)

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MM, Erhard, Huber, Krenn, Fickler, Zeilinger, Nature Photonics 10, 248 (2016)

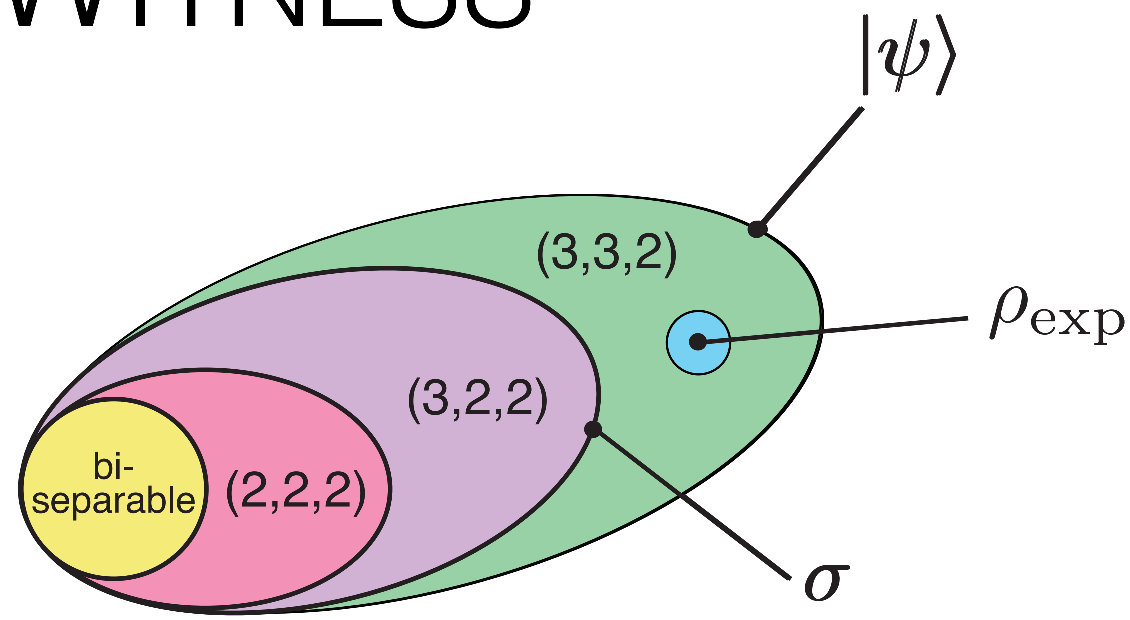
# 332 Entanglement Witness

- Best achievable fidelity of a <u>322 state</u> (*σ*) with an *ideal* <u>332 state</u> (Ψ):

$$F_{max} := \max_{\sigma \in (322)} \mathrm{Tr}(\sigma|\Psi\rangle\langle\Psi|) = 0.6\bar{6}$$

- Measured fidelity of <u>generated state</u> ($\rho_{exp}$) with an *ideal* <u>332 state</u> (Ψ)

- Density matrix elements: **18** diagonal, **3** off-diagonal




$$|\Psi\rangle_{332} = \frac{1}{\sqrt{3}}[|000\rangle + |111\rangle + |221\rangle]$$

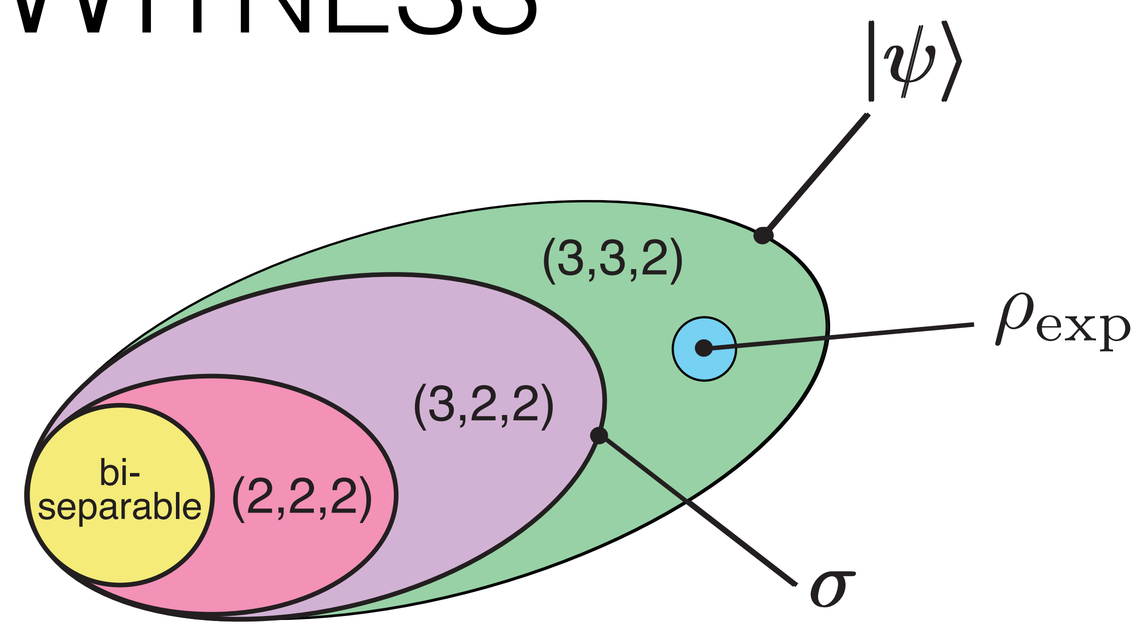
**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

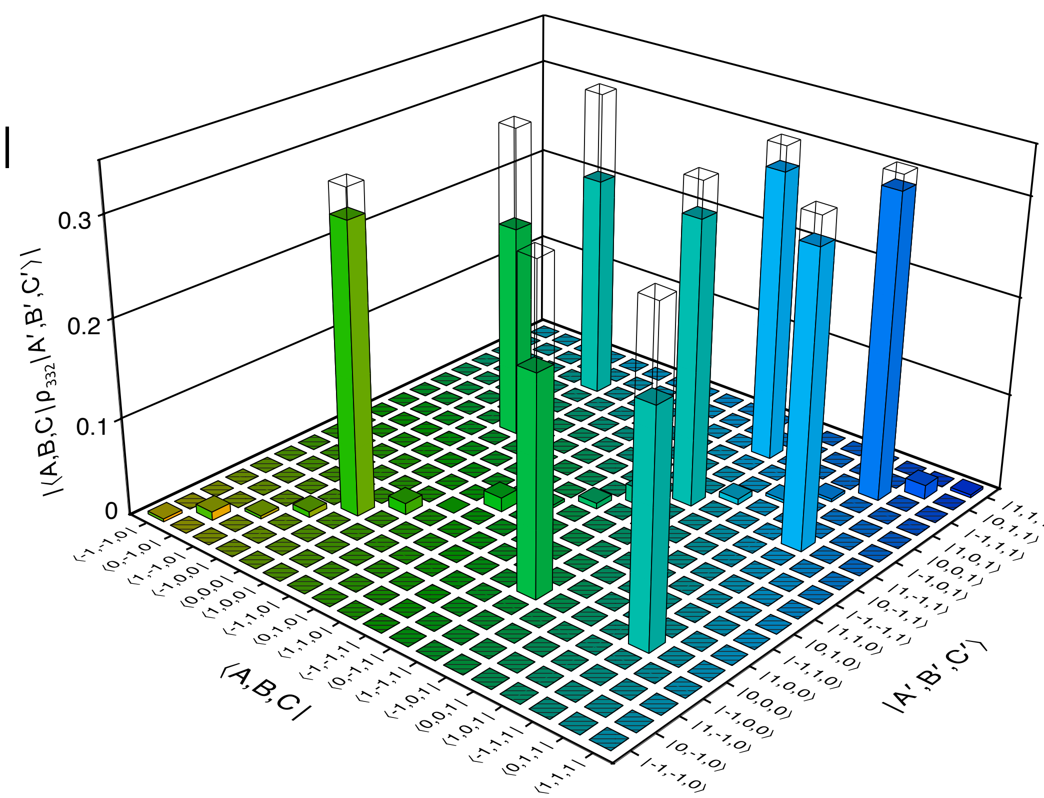
# 332 ENTANGLEMENT WITNESS

- Best achievable fidelity of a 322 state ($\sigma$) with an *ideal* 332 state ($\Psi$):

$$F_{max} := \max_{\sigma \in (322)} \mathrm{Tr}(\sigma|\Psi\rangle\langle\Psi|) = 0.6\bar{6}$$

- Measured fidelity of generated state ($\rho_{exp}$) with an *ideal* 332 state ($\Psi$)
- Density matrix elements: **18** diagonal, **3** off-diagonal
- After 80 hours of data-taking

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**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 Entanglement Witness

- Best achievable fidelity of a <u>322 state</u> (**$\sigma$**) with an *ideal* <u>332 state</u> (Ψ):

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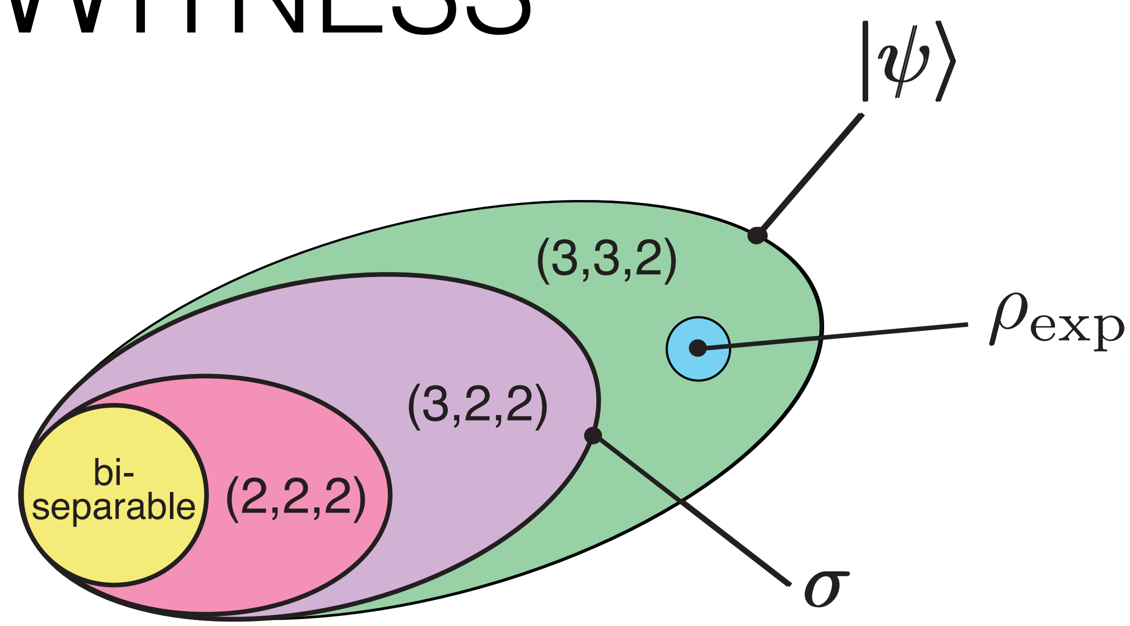
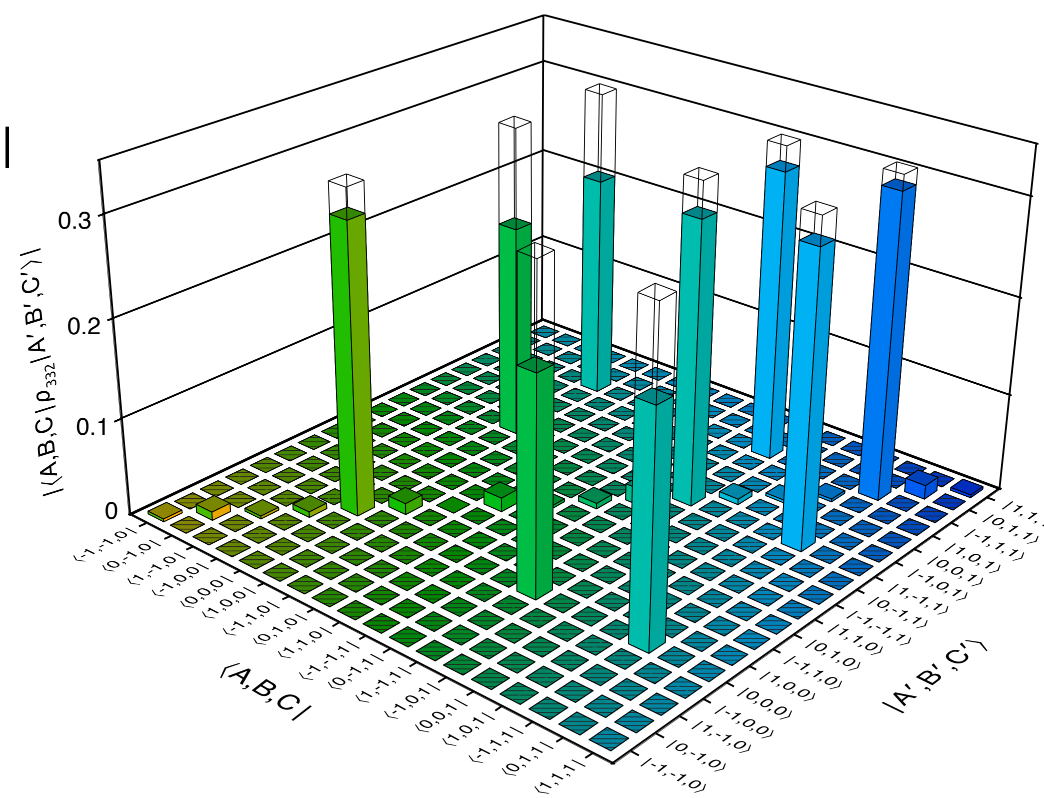



- Measured fidelity of <u>generated state</u> (**$\rho_{exp}$**) with an *ideal* <u>332 state</u> (Ψ)
- Density matrix elements: **18** diagonal, **3** off-diagonal
- After 80 hours of data-taking

$$F_{exp} := \mathrm{Tr}(\rho_{exp}|\Psi\rangle\langle\Psi|) = 0.801 \pm 0.018$$




$$|\Psi\rangle_{332} = \frac{1}{\sqrt{3}}[|000\rangle + |111\rangle + |221\rangle]$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 ENTANGLEMENT WITNESS

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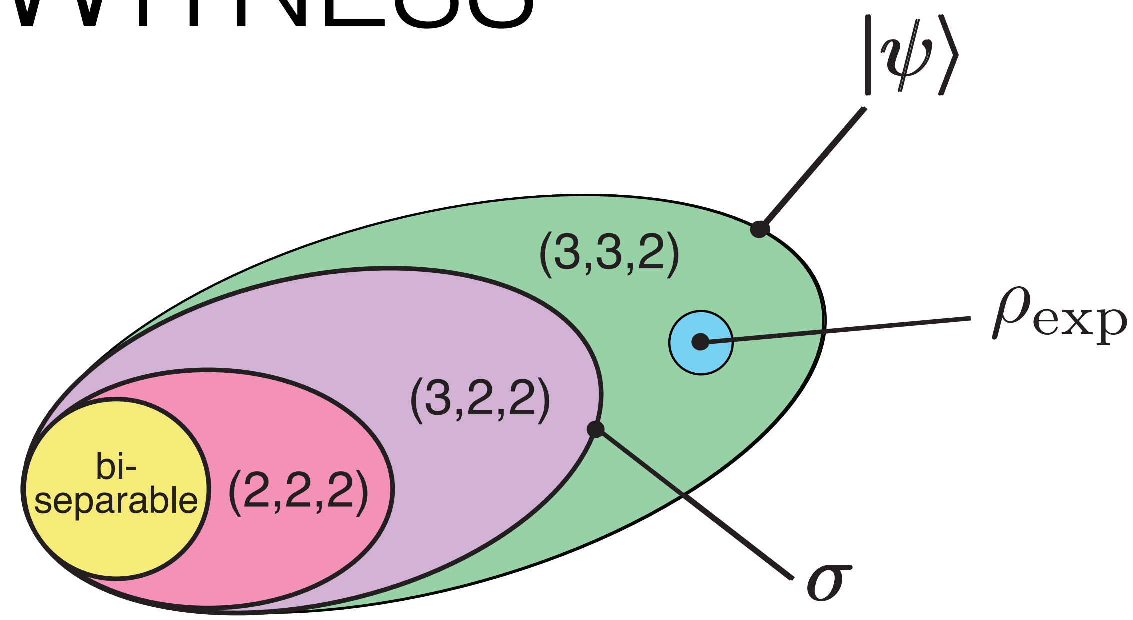


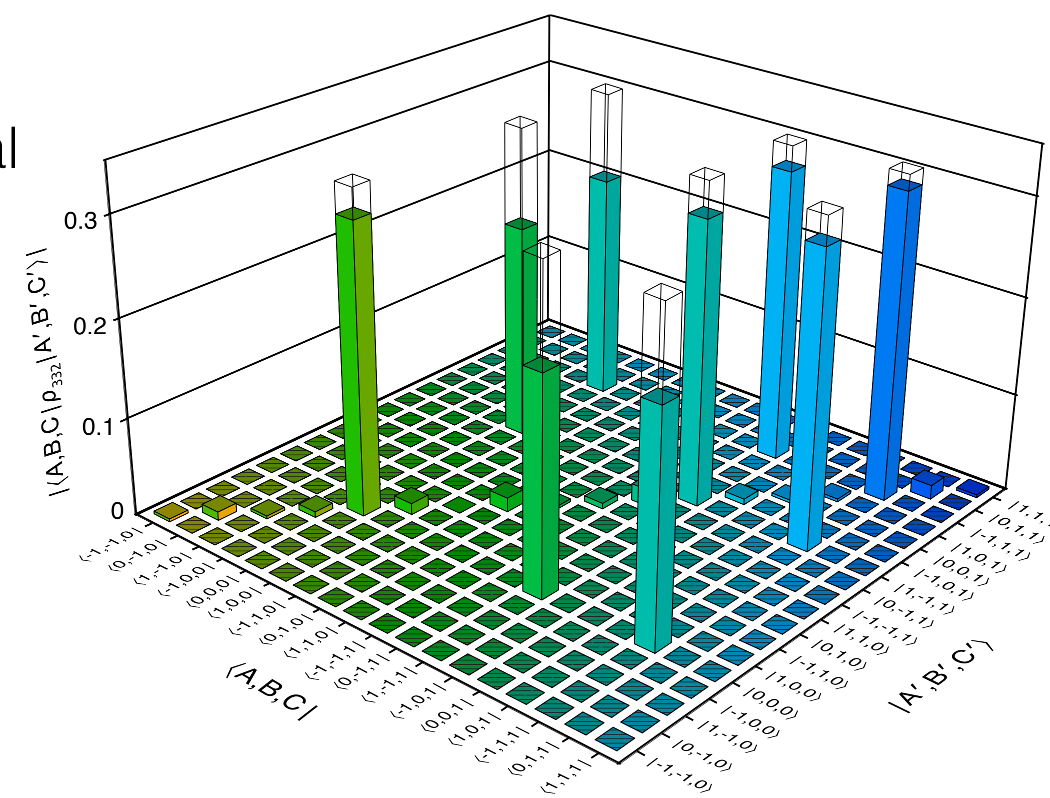

- Measured fidelity of <u>generated state</u> (**$\rho_{exp}$**) with an *ideal* <u>332 state</u> ($\Psi$)

- Density matrix elements: **18** diagonal, **3** off-diagonal

- After 80 hours of data-taking

$$F_{exp} := \mathrm{Tr}(\rho_{exp}|\Psi\rangle\langle\Psi|) = 0.801 \pm 0.018$$

$$F_{exp} > F_{max}$$



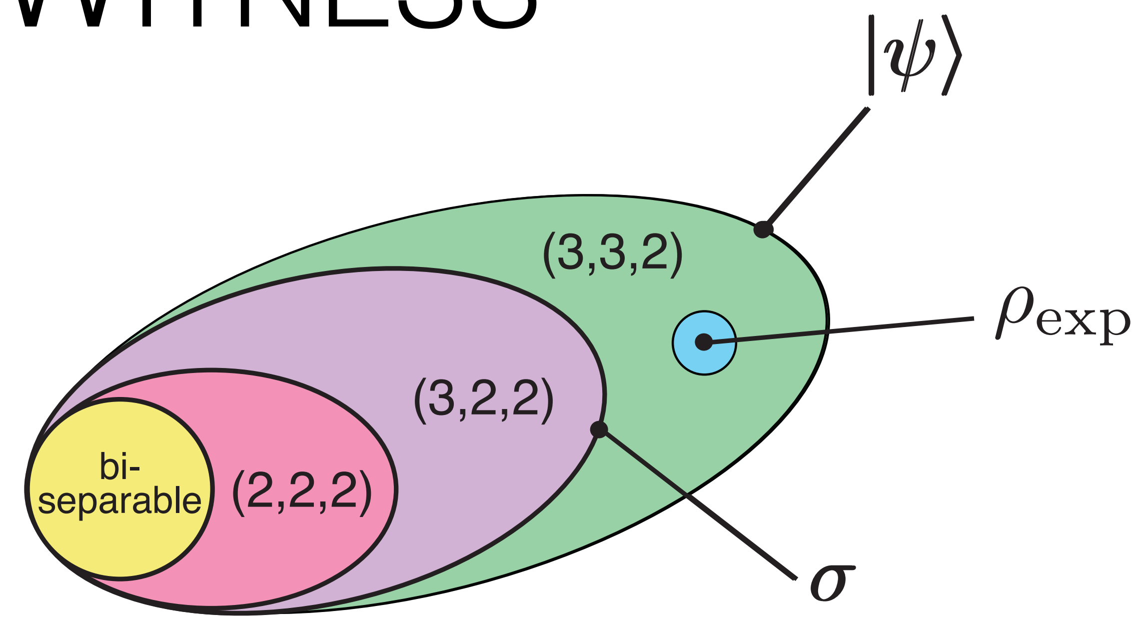

$$|\Psi\rangle_{332} = \frac{1}{\sqrt{3}}[|000\rangle + |111\rangle + |221\rangle]$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# 332 ENTANGLEMENT WITNESS

- Best achievable fidelity of a <u>322 state</u> (***σ***) with an *ideal* <u>332 state</u> (Ψ):

$$F_{max} := \max_{\sigma \in (322)} \mathrm{Tr}(\sigma|\Psi\rangle\langle\Psi|) = 0.6\bar{6}$$



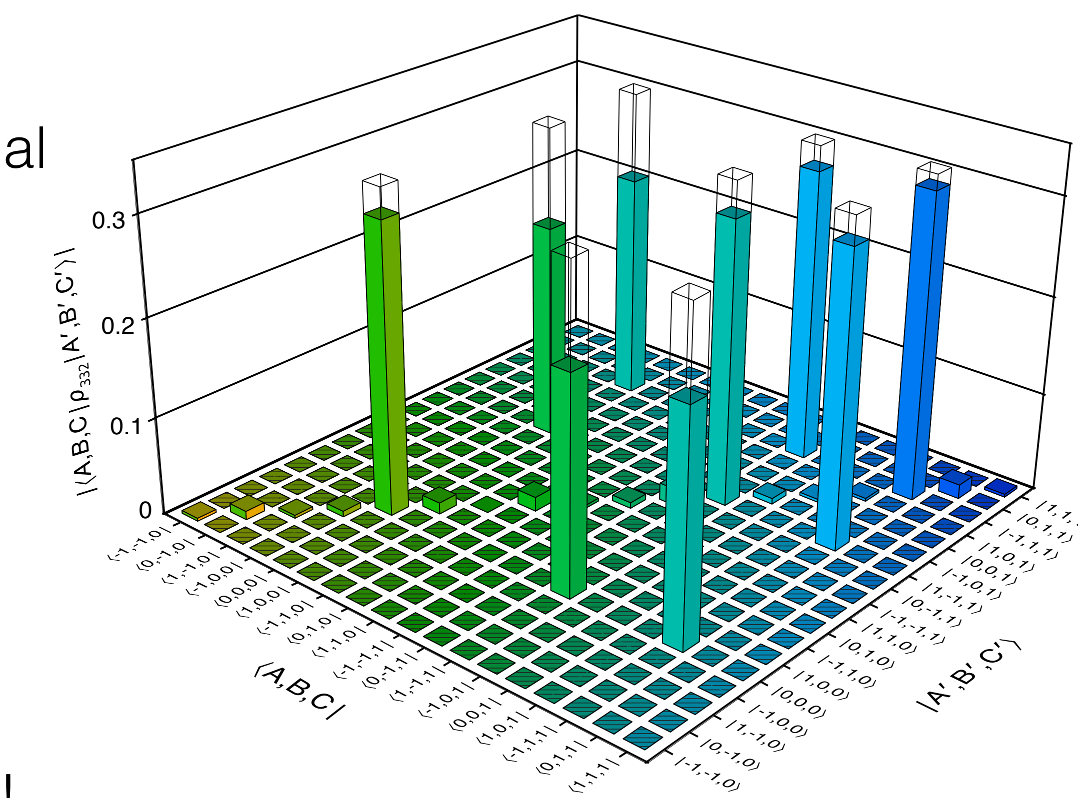

- Measured fidelity of <u>generated state</u> (***ρ**<sub></sub>*$_{exp}$) with an *ideal* <u>332 state</u> (Ψ)

- Density matrix elements: **18** diagonal, **3** off-diagonal

- After 80 hours of data-taking

$$F_{exp} := \mathrm{Tr}(\rho_{exp}|\Psi\rangle\langle\Psi|) = 0.801 \pm 0.018$$

$$F_{exp} > F_{max}$$

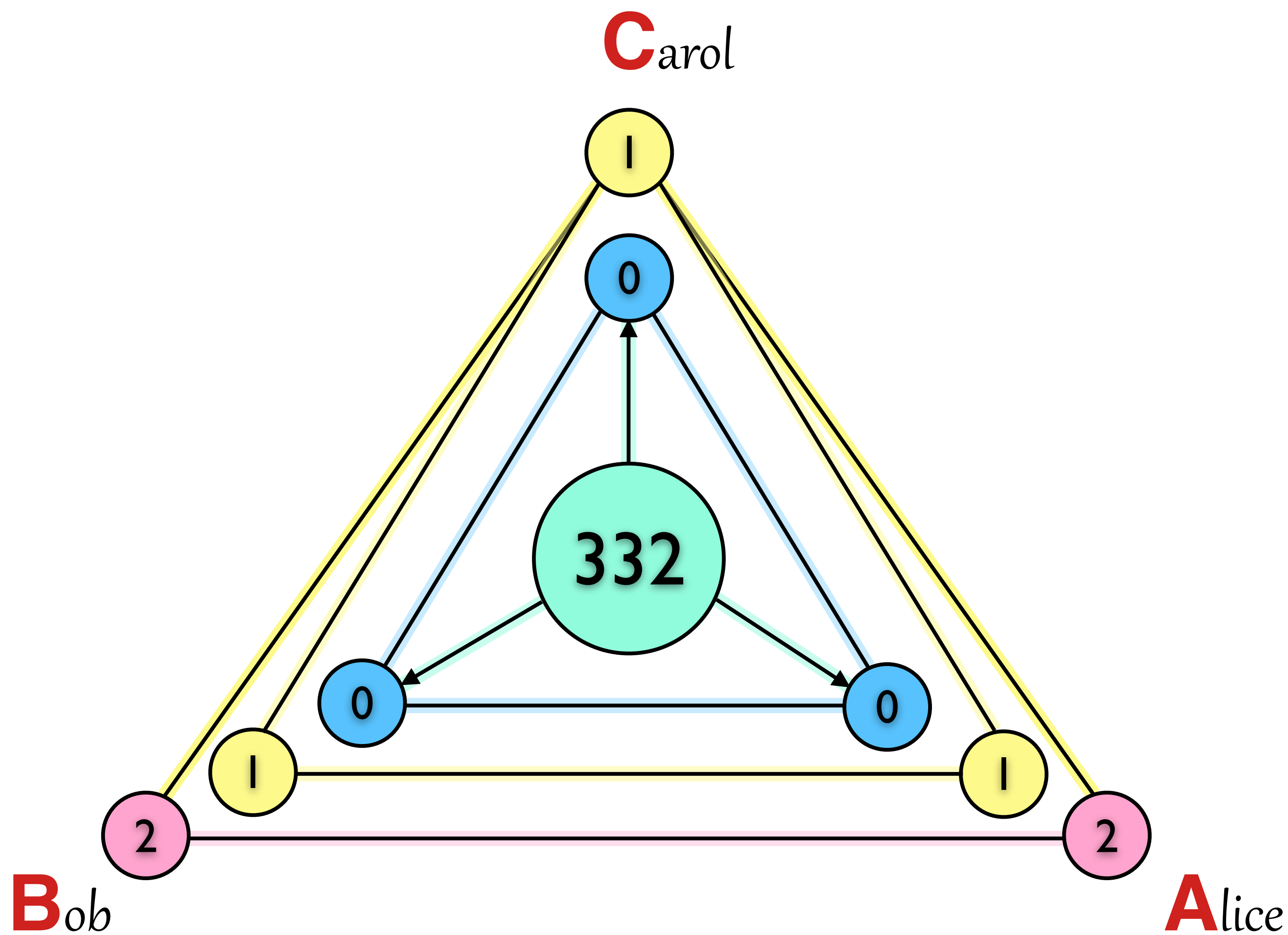



- *First* <u>high-dimensional</u> multipartite entangled state!

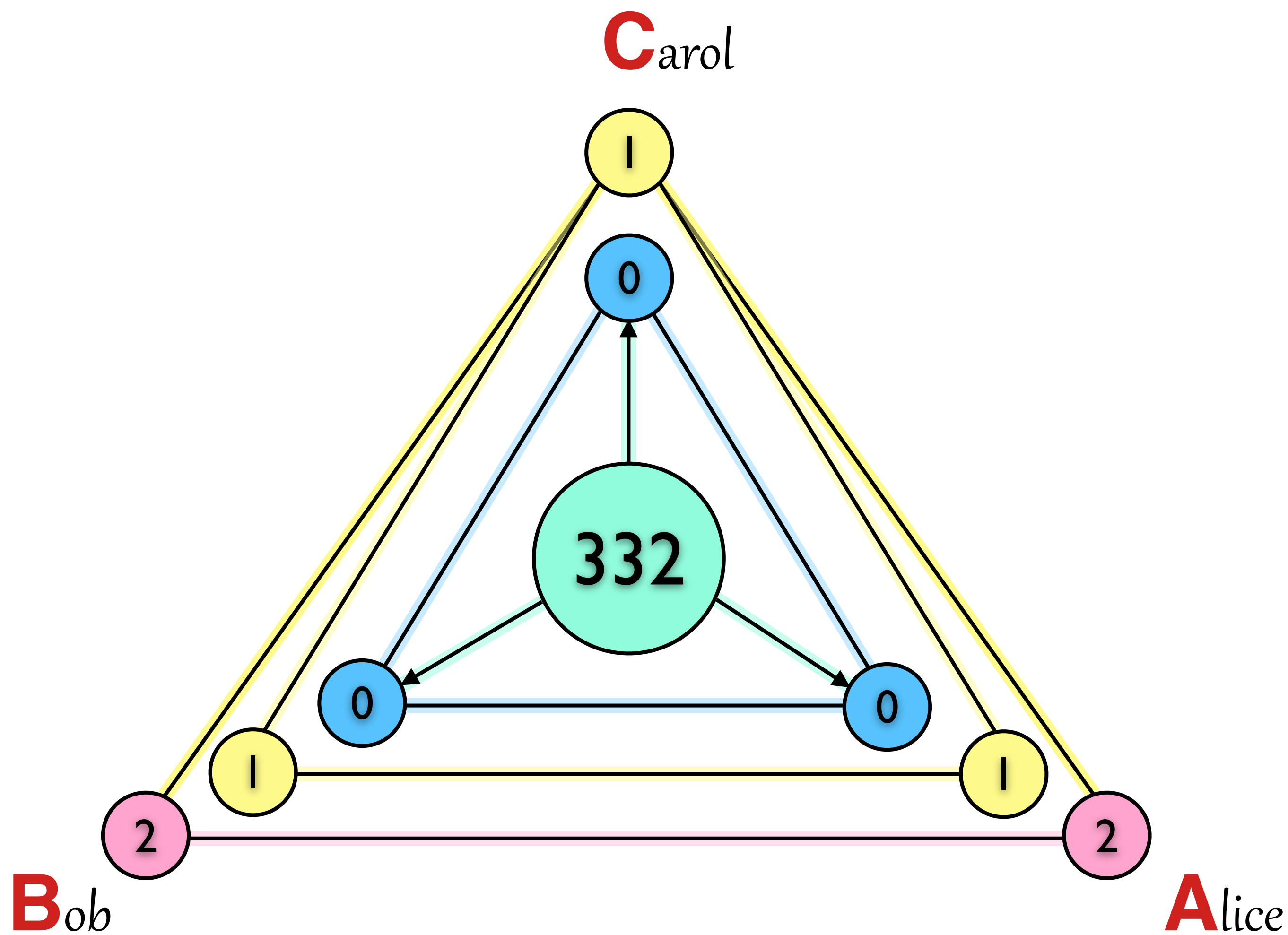
$$|\Psi\rangle_{332} = \frac{1}{\sqrt{3}}[|000\rangle + |111\rangle + |221\rangle]$$

**MM**, Erhard, Huber, Krenn, Fickler, Zeilinger, **Nature Photonics 10,** 248 (2016)

# LAYERED QUANTUM CRYPTOGRAPHY



# LAYERED QUANTUM CRYPTOGRAPHY




Pivoluska, Huber, and **MM,** PRA 97, 032312 (2018)

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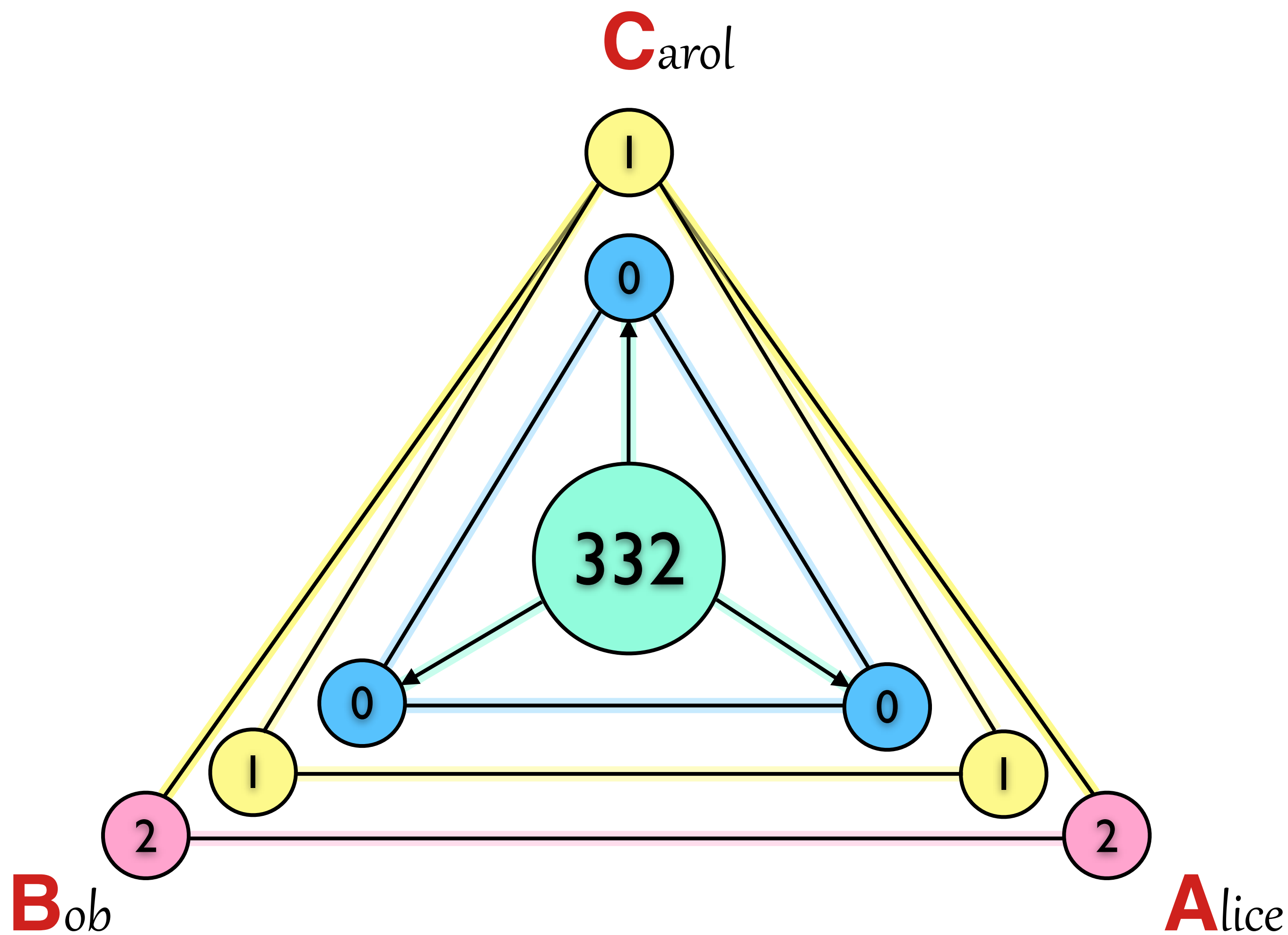
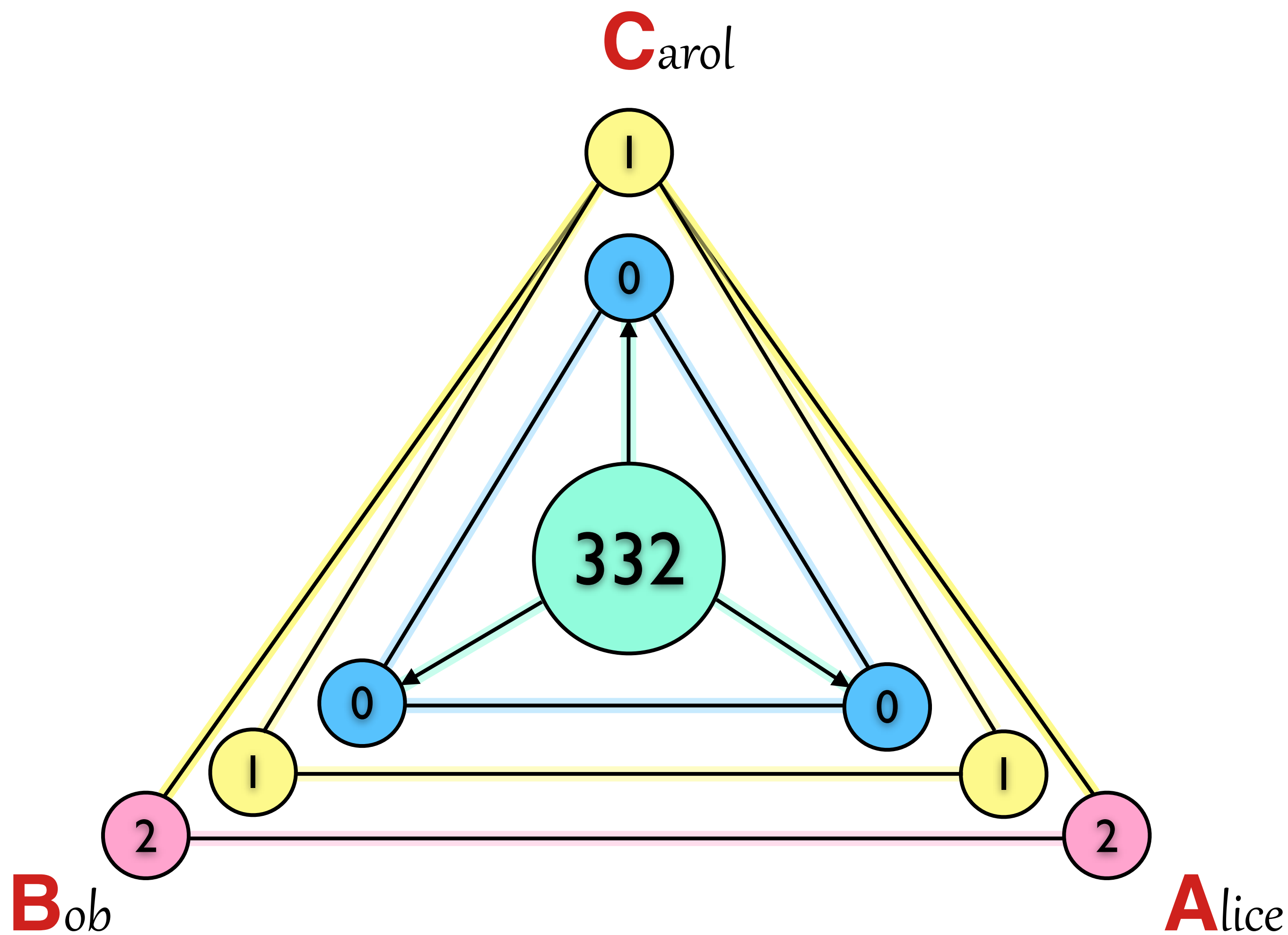



| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **A** | 0 | 1 | 0 | 2 | 0 | 0 | 2 | 0 | 0 | 1 | 2 | 0 | 2 | 0 | 2 | 1 | 0 | ··· |
| **B** | 0 | 1 | 0 | 2 | 0 | 0 | 2 | 0 | 0 | 1 | 2 | 0 | 2 | 0 | 2 | 1 | 0 | ··· |
| **C** | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | ··· |

Pivoluska, Huber, and **MM,** PRA 97, 032312 (2018)

# Layered Quantum Cryptography



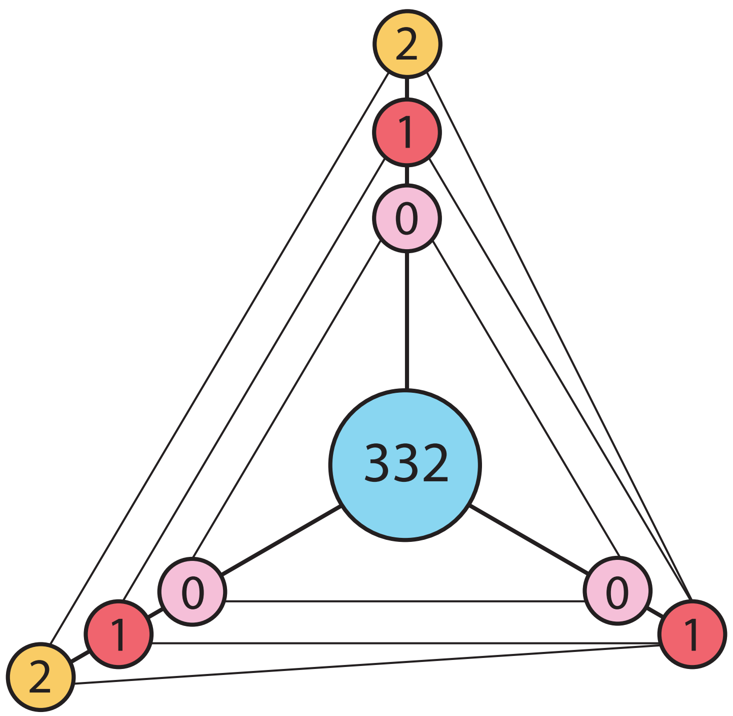

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **A** | 0 | 1 | 0 | 2 | 0 | 0 | 2 | 0 | 0 | 1 | 2 | 0 | 2 | 0 | 2 | 1 | 0 | ··· |
| **B** | 0 | 1 | 0 | 2 | 0 | 0 | 2 | 0 | 0 | 1 | 2 | 0 | 2 | 0 | 2 | 1 | 0 | ··· |
| **C** | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | ··· |

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **A B C** | I | T | H | A | P | P | E | N | S | T | O | M | O | R | R | O | W |
| **A B** | | I | | N | | | V | | | I | E | | N | | N | A | |

Pivoluska, Huber, and **MM,** PRA 97, 032312 (2018)

# The Quest for 3D GHZ

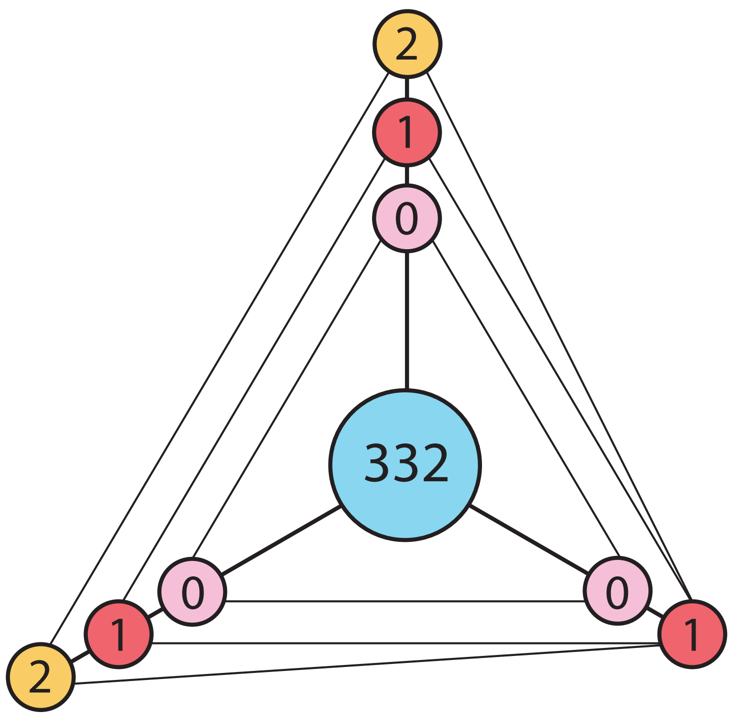
# The Quest for 3D GHZ

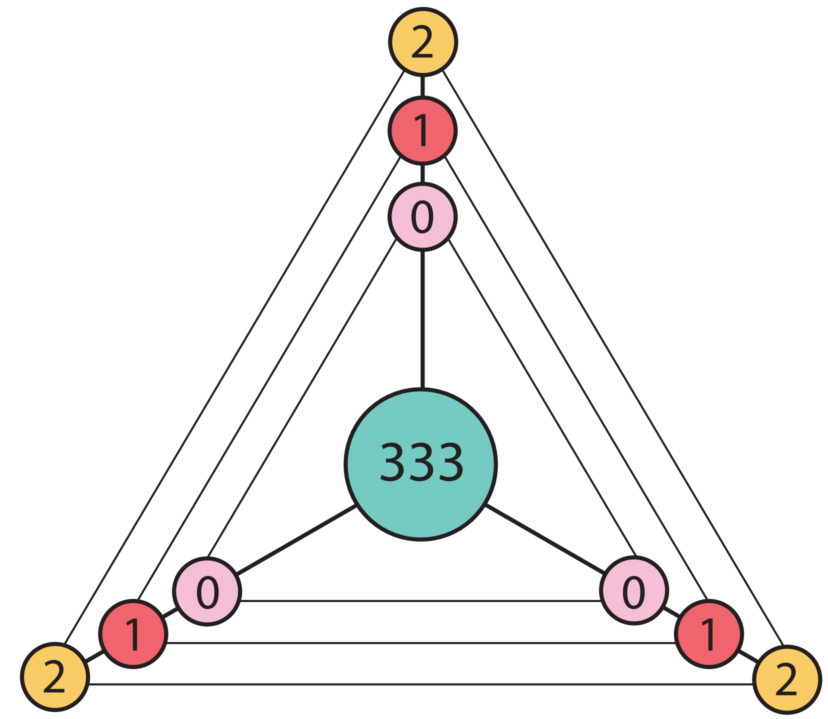



$$|\Psi\rangle_{332} = \frac{1}{\sqrt{3}}[|000\rangle + |111\rangle + |221\rangle]$$

✓

# The Quest for 3D GHZ




$$|\Psi\rangle_{332} = \frac{1}{\sqrt{3}}[|000\rangle + |111\rangle + |221\rangle]$$

✓

$$|\Psi\rangle_{333} = \frac{1}{\sqrt{3}}[|000\rangle + |111\rangle + |222\rangle]$$

?

# Automated Search for new Quantum Experiments

Mario Krenn,[1,2,*] Mehul Malik,[1,2] Robert Fickler,[1,2,†] Radek Lapkiewicz,[1,2,‡] and Anton Zeilinger[1,2]

PRL 116, 090405 (2016) PHYSICAL REVIEW LETTERS week ending 4 MARCH 2016

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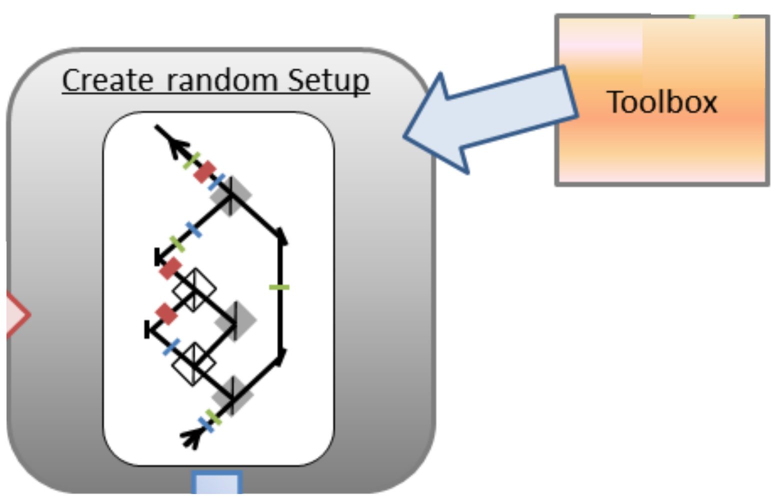


Krenn, **MM**, Fickler, Lapkiewicz, Zeilinger, **Phys. Rev. Lett. 116,** 090405 (2016)

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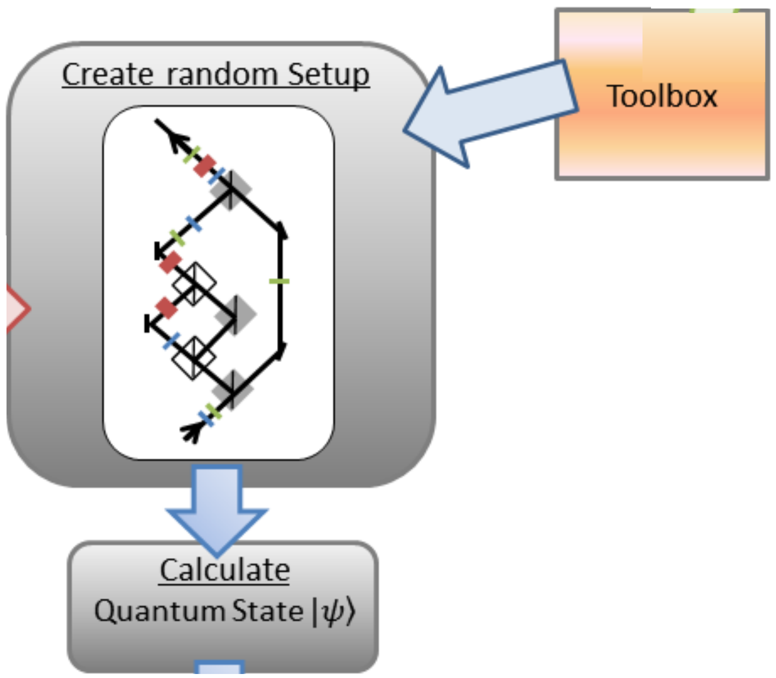


Krenn, MM, Fickler, Lapkiewicz, Zeilinger, Phys. Rev. Lett. 116, 090405 (2016)

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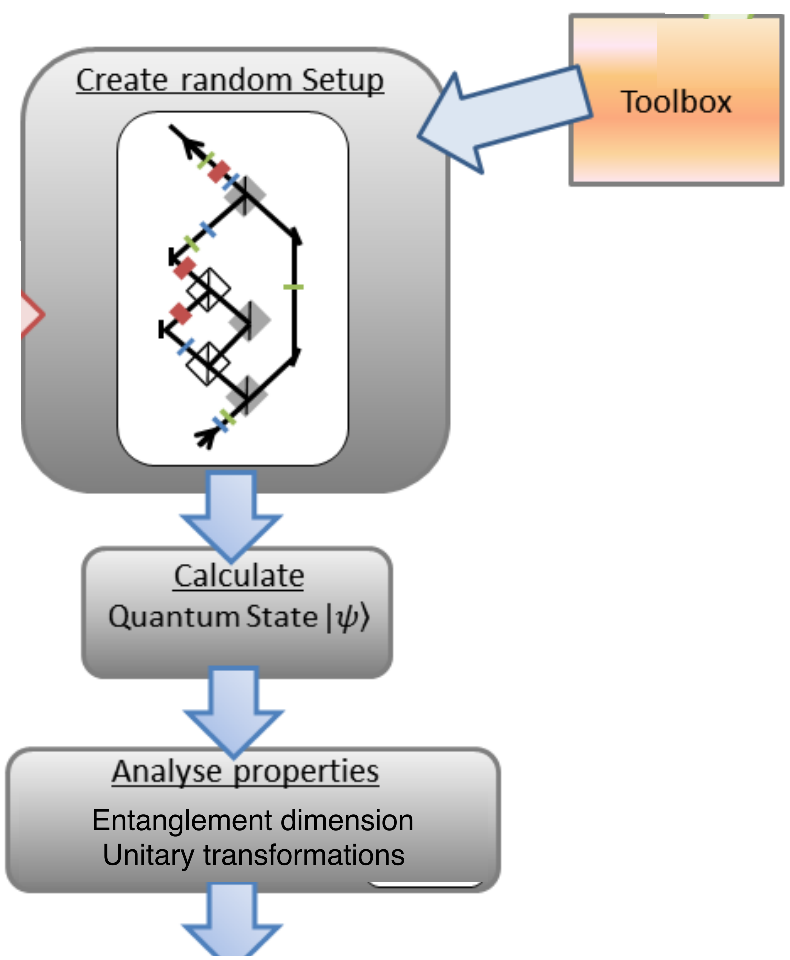


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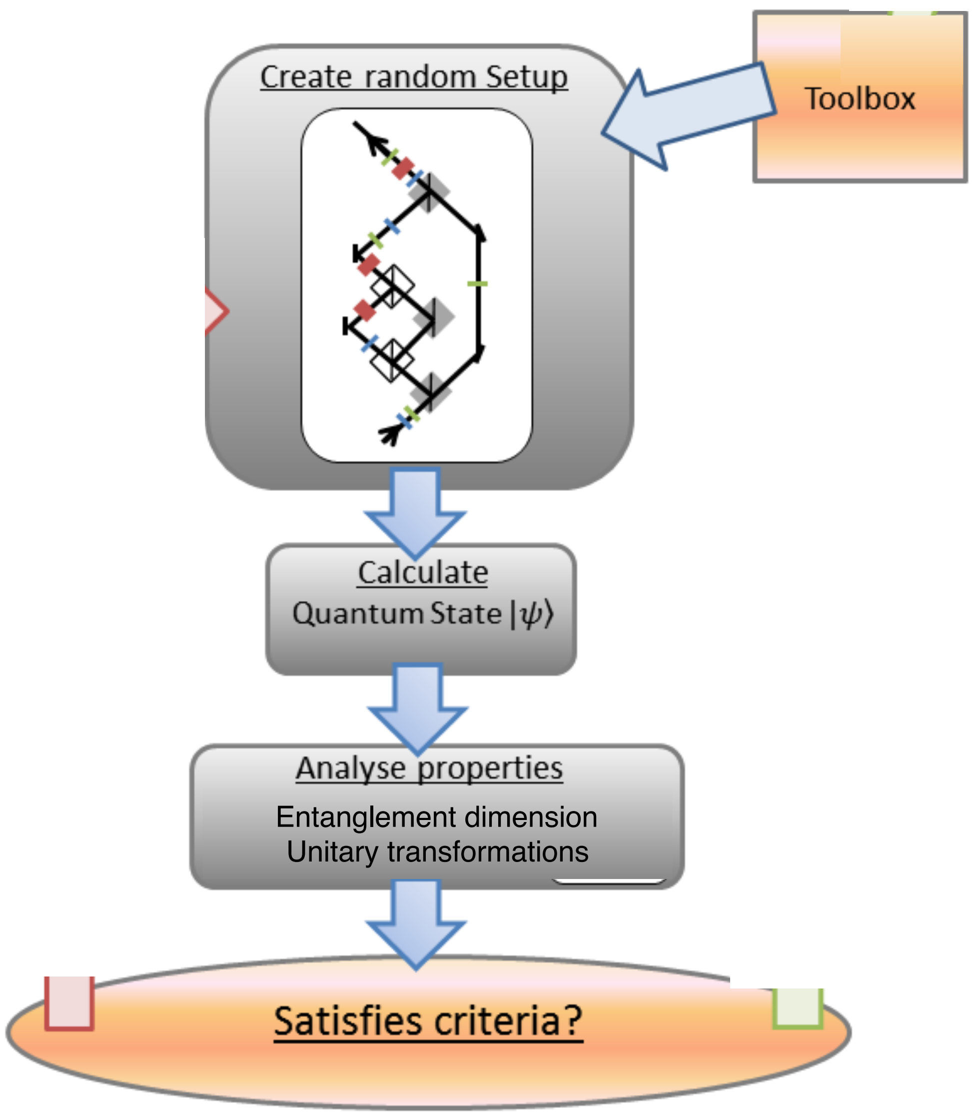


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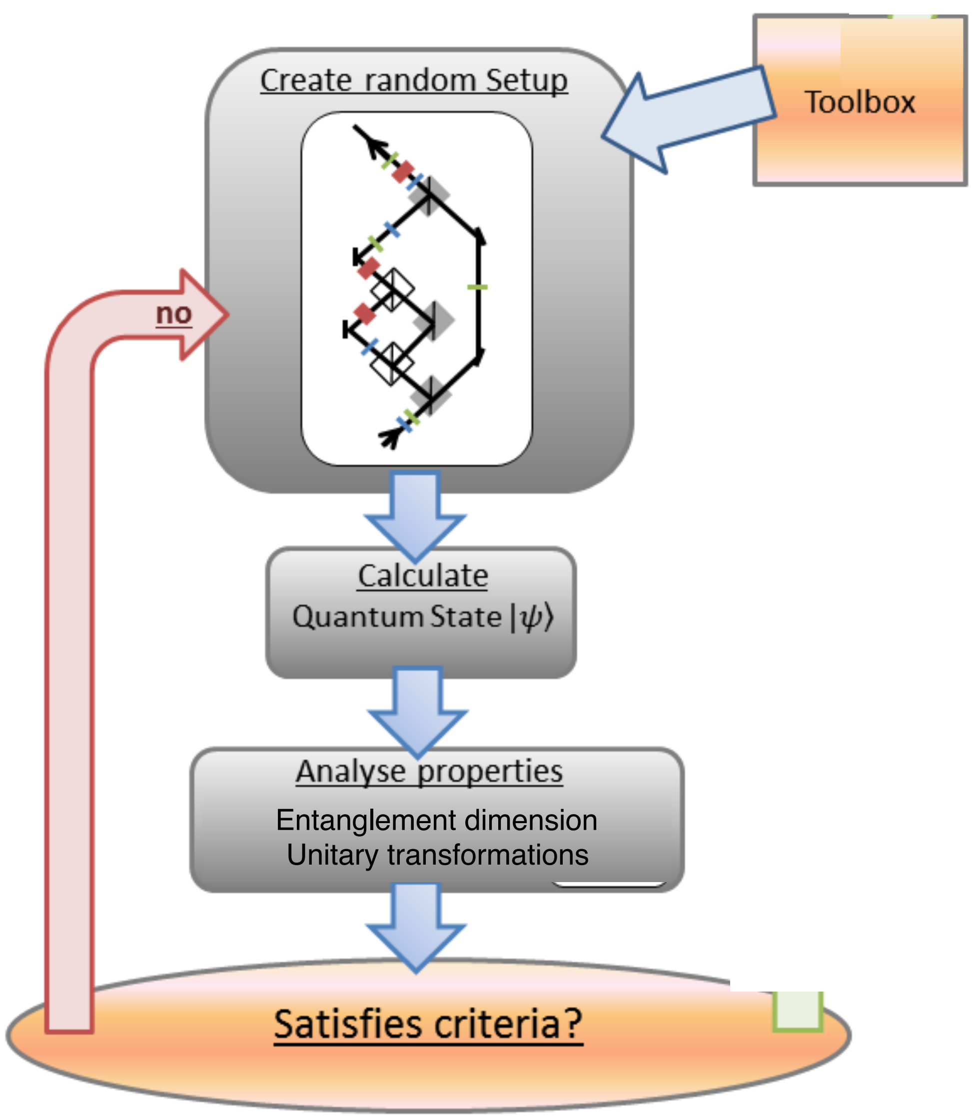


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PRL 116, 090405 (2016) PHYSICAL REVIEW LETTERS week ending 4 MARCH 2016

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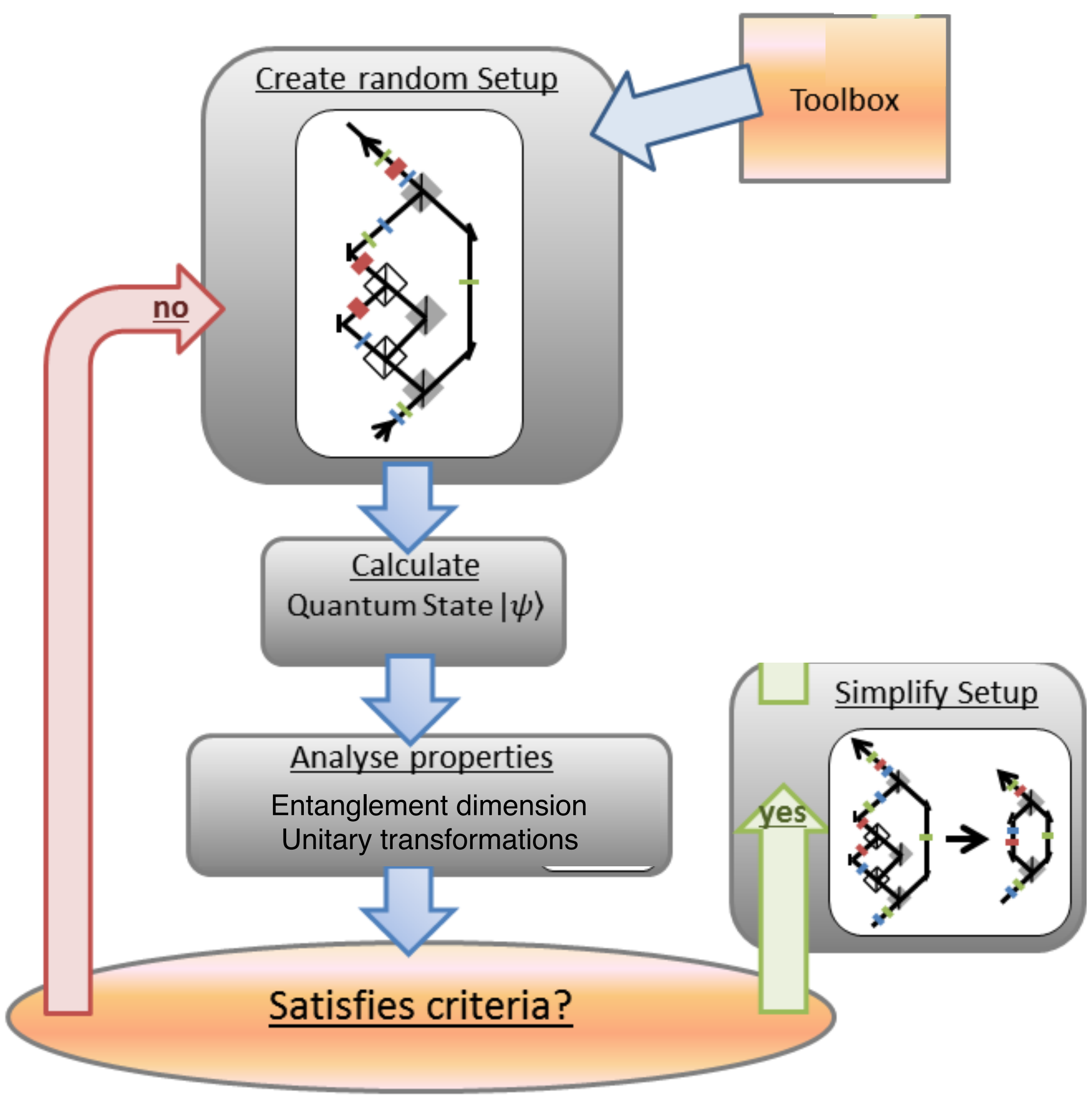


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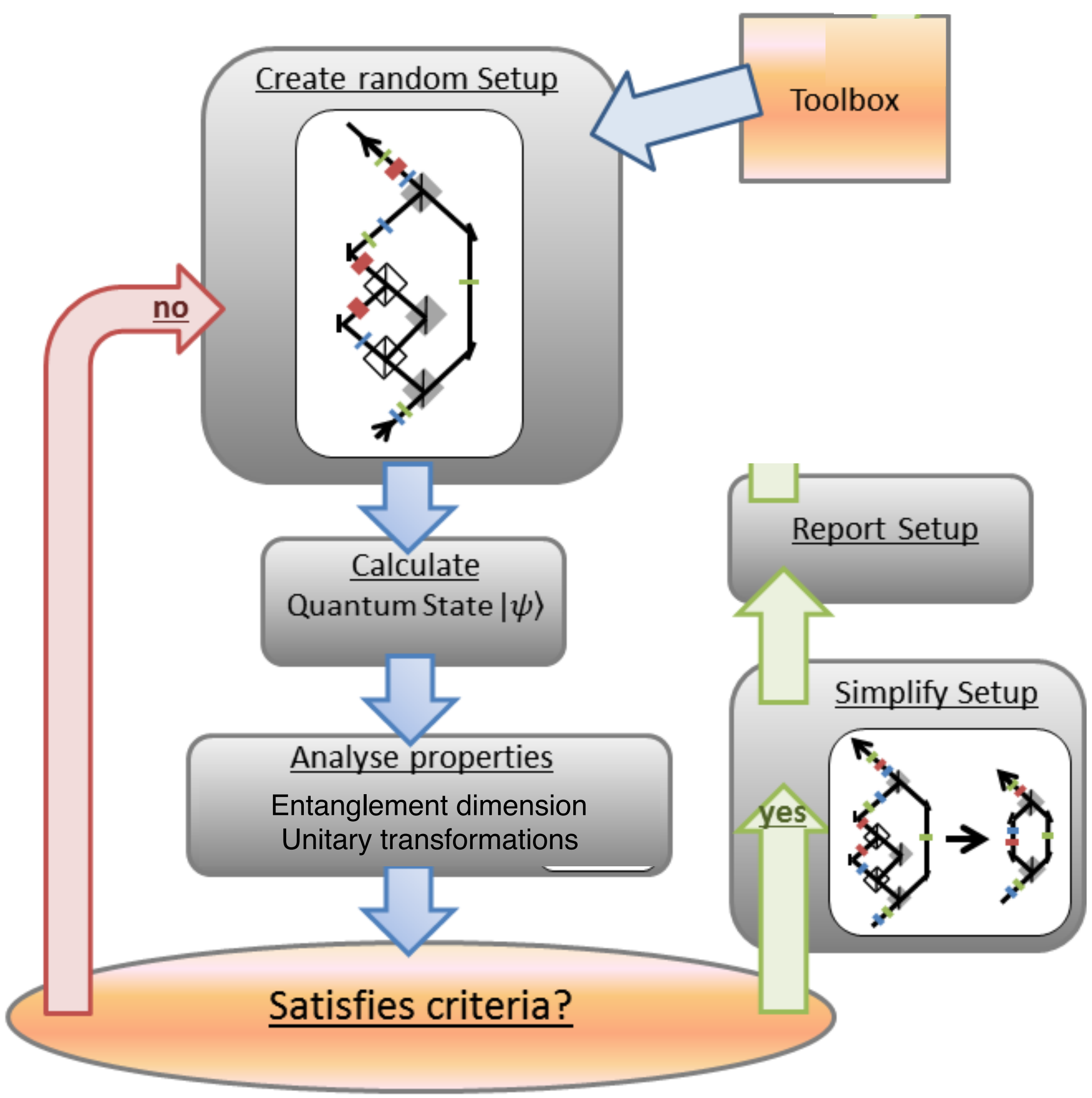


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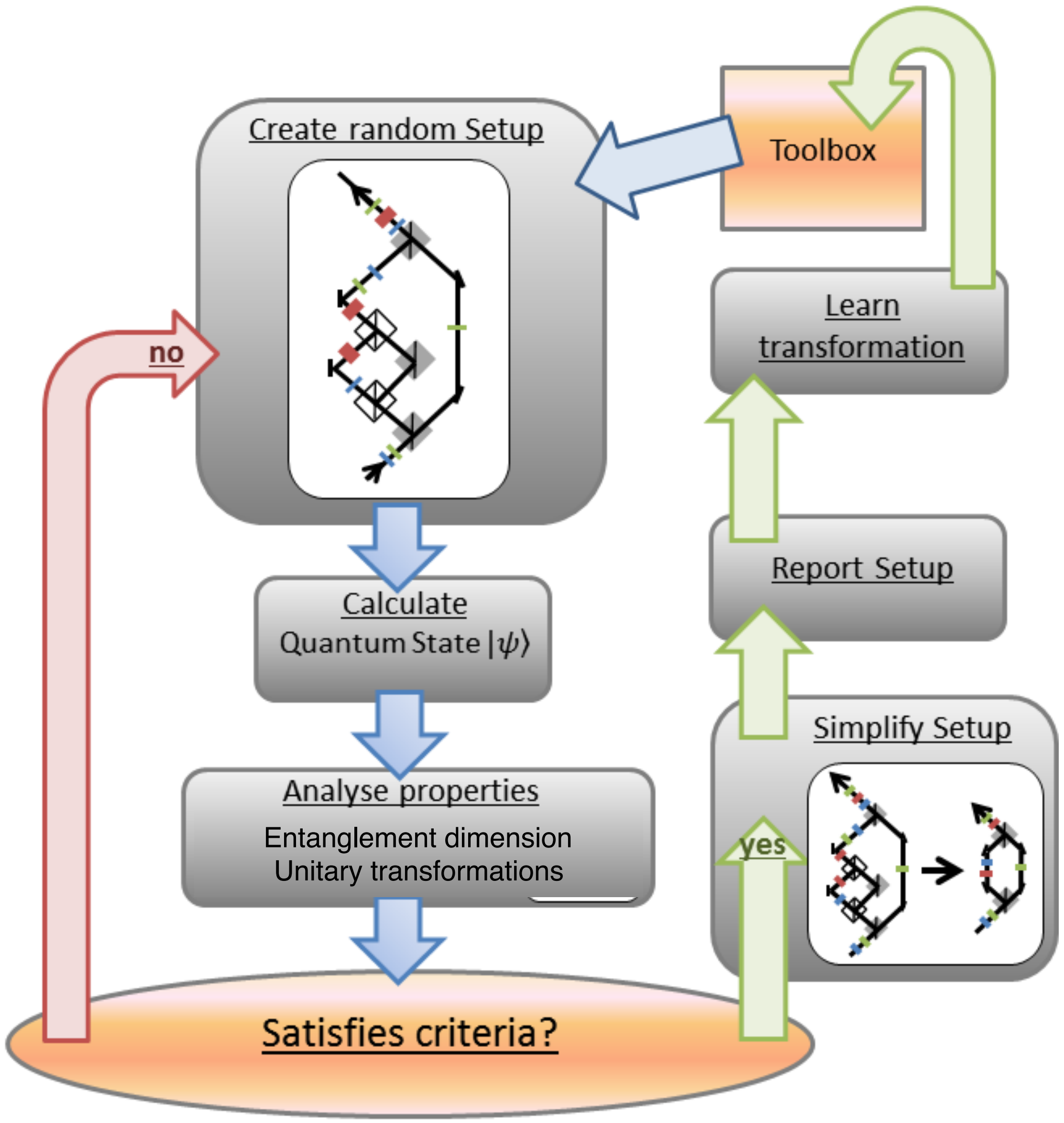


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PRL **116,** 090405 (2016) PHYSICAL REVIEW LETTERS week ending 4 MARCH 2016

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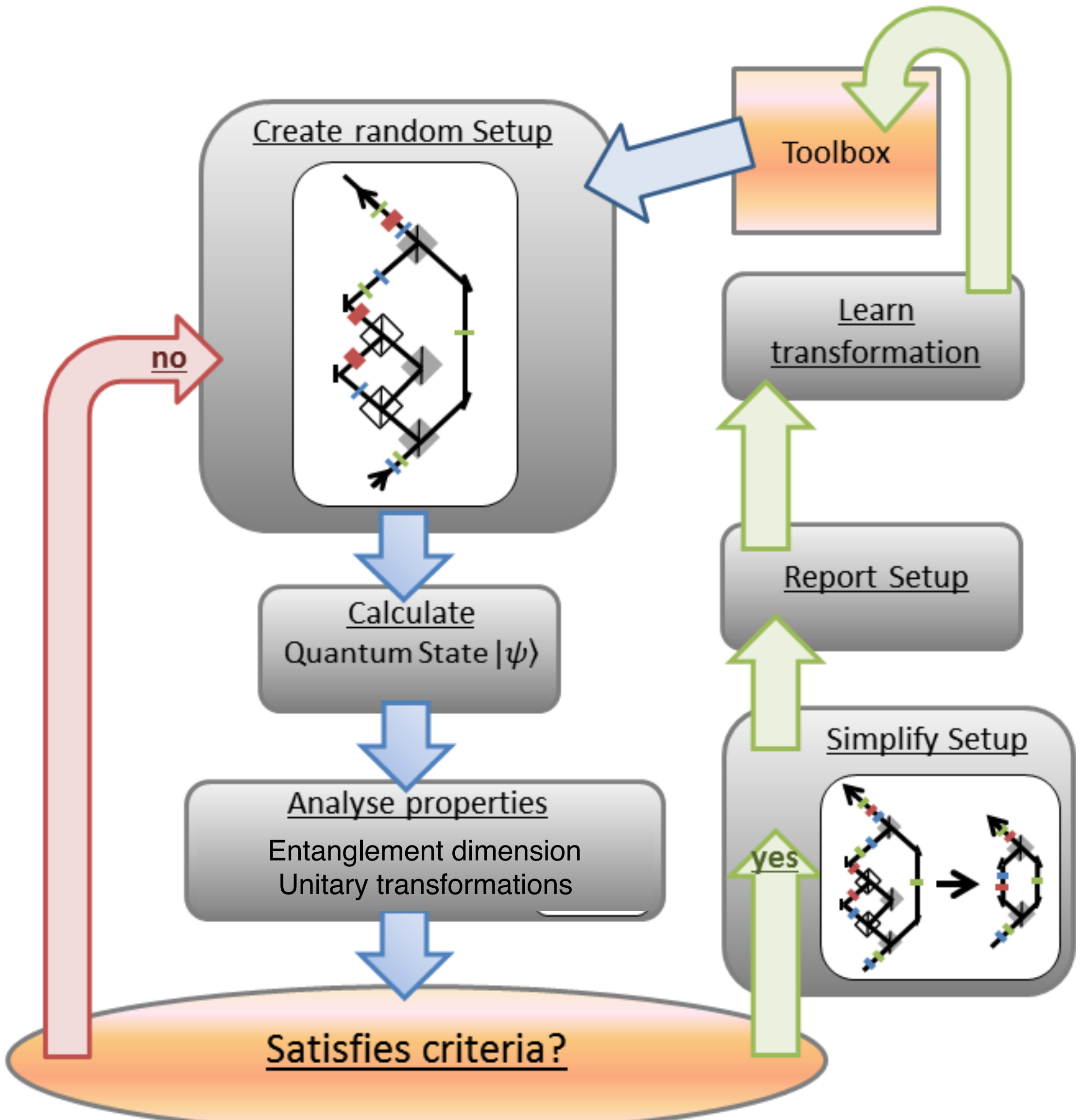


| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2,2,2 | 3,2,2 | 4,2,2 | ~~5,2,2~~ | ~~6,2,2~~ | ~~7,2,2~~ | ~~8,2,2~~ | ~~9,2,2~~ | ~~10,2,2~~ | ~~11,2,2~~ | ~~12,2,2~~ |
| | 3,3,2 | 4,3,2 | 5,3,2 | 6,3,2 | ~~7,3,2~~ | ~~8,3,2~~ | ~~9,3,2~~ | ~~10,3,2~~ | ~~11,3,2~~ | ~~12,3,2~~ |
| | 3,3,3 | 4,3,3 | 5,3,3 | 6,3,3 | 7,3,3 | 8,3,3 | 9,3,3 | ~~10,3,3~~ | ~~11,3,3~~ | ~~12,3,3~~ |
| | | 4,4,2 | 5,4,2 | 6,4,2 | 7,4,2 | 8,4,2 | ~~9,4,2~~ | ~~10,4,2~~ | ~~11,4,2~~ | ~~12,4,2~~ |
| | | 4,4,3 | 5,4,3 | 6,4,3 | 7,4,3 | 8,4,3 | 9,4,3 | 10,4,3 | 11,4,3 | 12,4,3 |
| | | 4,4,4 | 5,4,4 | 6,4,4 | 7,4,4 | 8,4,4 | 9,4,4 | 10,4,4 | 11,4,4 | 12,4,4 |
| | | | 5,5,2 | 6,5,2 | 7,5,2 | 8,5,2 | 9,5,2 | 10,5,2 | ~~11,5,2~~ | ~~12,5,2~~ |
| | | | 5,5,3 | 6,5,3 | 7,5,3 | 8,5,3 | 9,5,3 | 10,5,3 | 11,5,3 | 12,5,3 |
| | | | 5,5,4 | 6,5,4 | 7,5,4 | 8,5,4 | 9,5,4 | 10,5,4 | 11,5,4 | 12,5,4 |
| | | | 5,5,5 | 6,5,5 | 7,5,5 | 8,5,5 | 9,5,5 | 10,5,5 | 11,5,5 | 12,5,5 |
| | | | | 6,6,2 | 7,6,2 | 8,6,2 | 9,6,2 | 10,6,2 | 11,6,2 | 12,6,2 |
| | | | | 6,6,3 | 7,6,3 | 8,6,3 | 9,6,3 | 10,6,3 | 11,6,3 | 12,6,3 |
| | | | | 6,6,4 | 7,6,4 | 8,6,4 | 9,6,4 | 10,6,4 | 11,6,4 | 12,6,4 |
| | | | | 6,6,5 | 7,6,5 | 8,6,5 | 9,6,5 | 10,6,5 | 11,6,5 | 12,6,5 |
| | | | | 6,6,6 | 7,6,6 | 8,6,6 | 9,6,6 | 10,6,6 | 11,6,6 | 12,6,6 |
| | | | | | 7,7,2 | 8,7,2 | 9,7,2 | 10,7,2 | 11,7,2 | 12,7,2 |
| | | | | | 7,7,3 | 8,7,3 | 9,7,3 | 10,7,3 | 11,7,3 | 12,7,3 |
| | | | | | 7,7,4 | 8,7,4 | 9,7,4 | 10,7,4 | 11,7,4 | 12,7,4 |
| | | | | | 7,7,5 | 8,7,5 | 9,7,5 | 10,7,5 | 11,7,5 | 12,7,5 |
| | | | | | 7,7,6 | 8,7,6 | 9,7,6 | 10,7,6 | 11,7,6 | 12,7,6 |
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| | | | | | | 8,8,3 | 9,8,3 | 10,8,3 | 11,8,3 | 12,8,3 |
| | | | | | | 8,8,4 | 9,8,4 | 10,8,4 | 11,8,4 | 12,8,4 |
| | | | | | | 8,8,5 | 9,8,5 | 10,8,5 | 11,8,5 | 12,8,5 |
| | | | | | | 8,8,6 | 9,8,6 | 10,8,6 | 11,8,6 | 12,8,6 |
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| | | | | | | | 9,9,2 | 10,9,2 | 11,9,2 | 12,9,2 |
| | | | | | | | 9,9,3 | 10,9,3 | 11,9,3 | 12,9,3 |
| | | | | | | | 9,9,4 | 10,9,4 | 11,9,4 | 12,9,4 |
| | | | | | | | 9,9,5 | 10,9,5 | 11,9,5 | 12,9,5 |
| | | | | | | | 9,9,6 | 10,9,6 | 11,9,6 | 12,9,6 |

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| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2,2,2 | 3,2,2 | 4,2,2 | ~~5,2,2~~ | ~~6,2,2~~ | ~~7,2,2~~ | ~~8,2,2~~ | ~~9,2,2~~ | ~~10,2,2~~ | ~~11,2,2~~ | ~~12,2,2~~ |
| | 3,3,2 | 4,3,2 | 5,3,2 | 6,3,2 | ~~7,3,2~~ | ~~8,3,2~~ | ~~9,3,2~~ | ~~10,3,2~~ | ~~11,3,2~~ | ~~12,3,2~~ |
| | 3,3,3 | 4,3,3 | 5,3,3 | 6,3,3 | 7,3,3 | 8,3,3 | 9,3,3 | ~~10,3,3~~ | ~~11,3,3~~ | ~~12,3,3~~ |
| | | 4,4,2 | 5,4,2 | 6,4,2 | 7,4,2 | 8,4,2 | ~~9,4,2~~ | ~~10,4,2~~ | ~~11,4,2~~ | ~~12,4,2~~ |
| | | 4,4,3 | 5,4,3 | 6,4,3 | 7,4,3 | 8,4,3 | 9,4,3 | 10,4,3 | 11,4,3 | 12,4,3 |
| | | 4,4,4 | 5,4,4 | 6,4,4 | 7,4,4 | 8,4,4 | 9,4,4 | 10,4,4 | 11,4,4 | 12,4,4 |
| | | | 5,5,2 | 6,5,2 | 7,5,2 | 8,5,2 | 9,5,2 | 10,5,2 | ~~11,5,2~~ | ~~12,5,2~~ |
| | | | 5,5,3 | 6,5,3 | 7,5,3 | 8,5,3 | 9,5,3 | 10,5,3 | 11,5,3 | 12,5,3 |
| | | | 5,5,4 | 6,5,4 | 7,5,4 | 8,5,4 | 9,5,4 | 10,5,4 | 11,5,4 | 12,5,4 |
| | | | 5,5,5 | 6,5,5 | 7,5,5 | 8,5,5 | 9,5,5 | 10,5,5 | 11,5,5 | 12,5,5 |
| | | | | 6,6,2 | 7,6,2 | 8,6,2 | 9,6,2 | 10,6,2 | 11,6,2 | 12,6,2 |
| | | | | 6,6,3 | 7,6,3 | 8,6,3 | 9,6,3 | 10,6,3 | 11,6,3 | 12,6,3 |
| | | | | 6,6,4 | 7,6,4 | 8,6,4 | 9,6,4 | 10,6,4 | 11,6,4 | 12,6,4 |
| | | | | 6,6,5 | 7,6,5 | 8,6,5 | 9,6,5 | 10,6,5 | 11,6,5 | 12,6,5 |
| | | | | 6,6,6 | 7,6,6 | 8,6,6 | 9,6,6 | 10,6,6 | 11,6,6 | 12,6,6 |
| | | | | | 7,7,2 | 8,7,2 | 9,7,2 | 10,7,2 | 11,7,2 | 12,7,2 |
| | | | | | 7,7,3 | 8,7,3 | 9,7,3 | 10,7,3 | 11,7,3 | 12,7,3 |
| | | | | | 7,7,4 | 8,7,4 | 9,7,4 | 10,7,4 | 11,7,4 | 12,7,4 |
| | | | | | 7,7,5 | 8,7,5 | 9,7,5 | 10,7,5 | 11,7,5 | 12,7,5 |
| | | | | | 7,7,6 | 8,7,6 | 9,7,6 | 10,7,6 | 11,7,6 | 12,7,6 |
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| | | | | | | | 9,9,2 | 10,9,2 | 11,9,2 | 12,9,2 |
| | | | | | | | 9,9,3 | 10,9,3 | 11,9,3 | 12,9,3 |
| | | | | | | | 9,9,4 | 10,9,4 | 11,9,4 | 12,9,4 |
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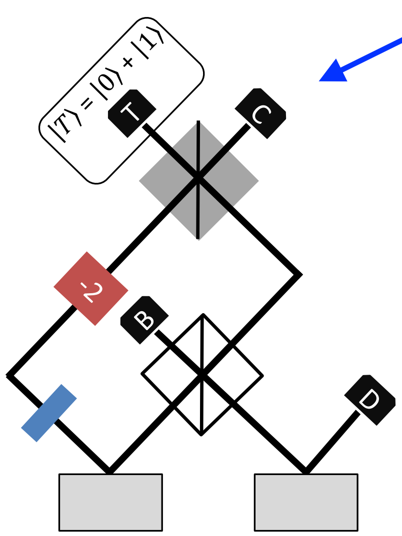


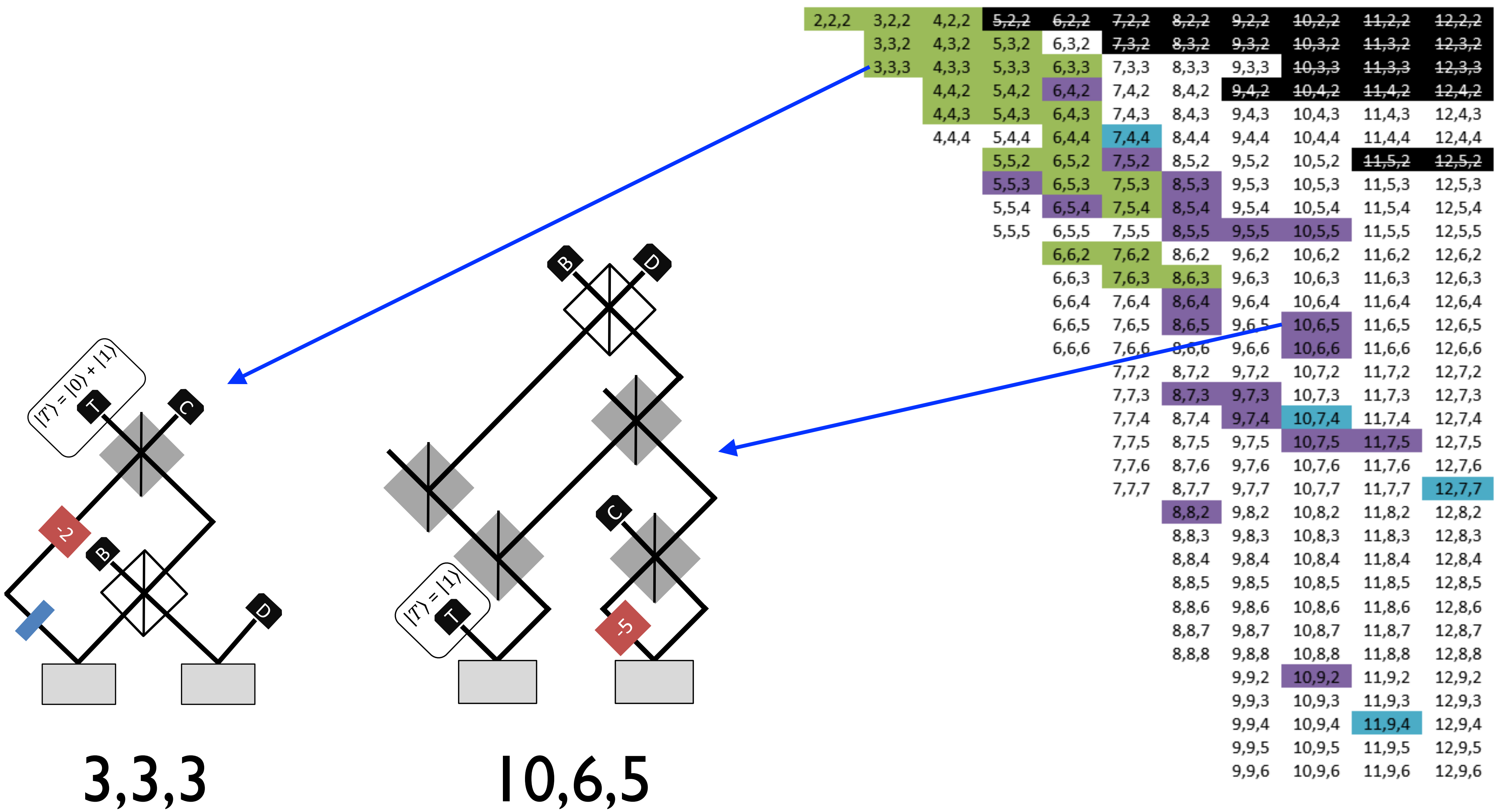

3,3,3

Krenn, **MM**, Fickler, Lapkiewicz, Zeilinger, **Phys. Rev. Lett. 116,** 090405 (2016)

PRL **116,** 090405 (2016) PHYSICAL REVIEW LETTERS week ending 4 MARCH 2016

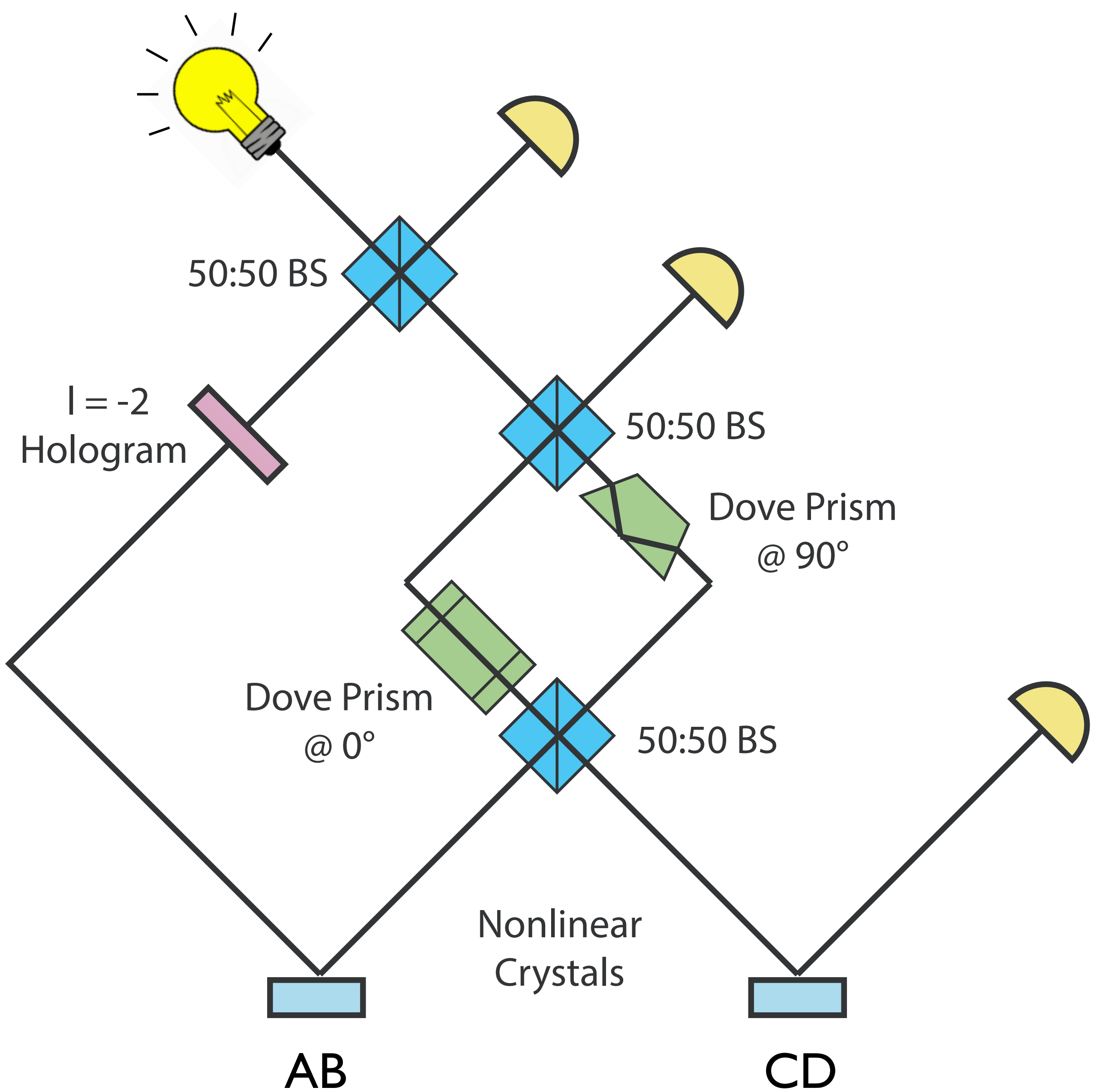
# Automated Search for new Quantum Experiments

Mario Krenn,[1,2,*] Mehul Malik,[1,2] Robert Fickler,[1,2,†] Radek Lapkiewicz,[1,2,‡] and Anton Zeilinger[1,2]



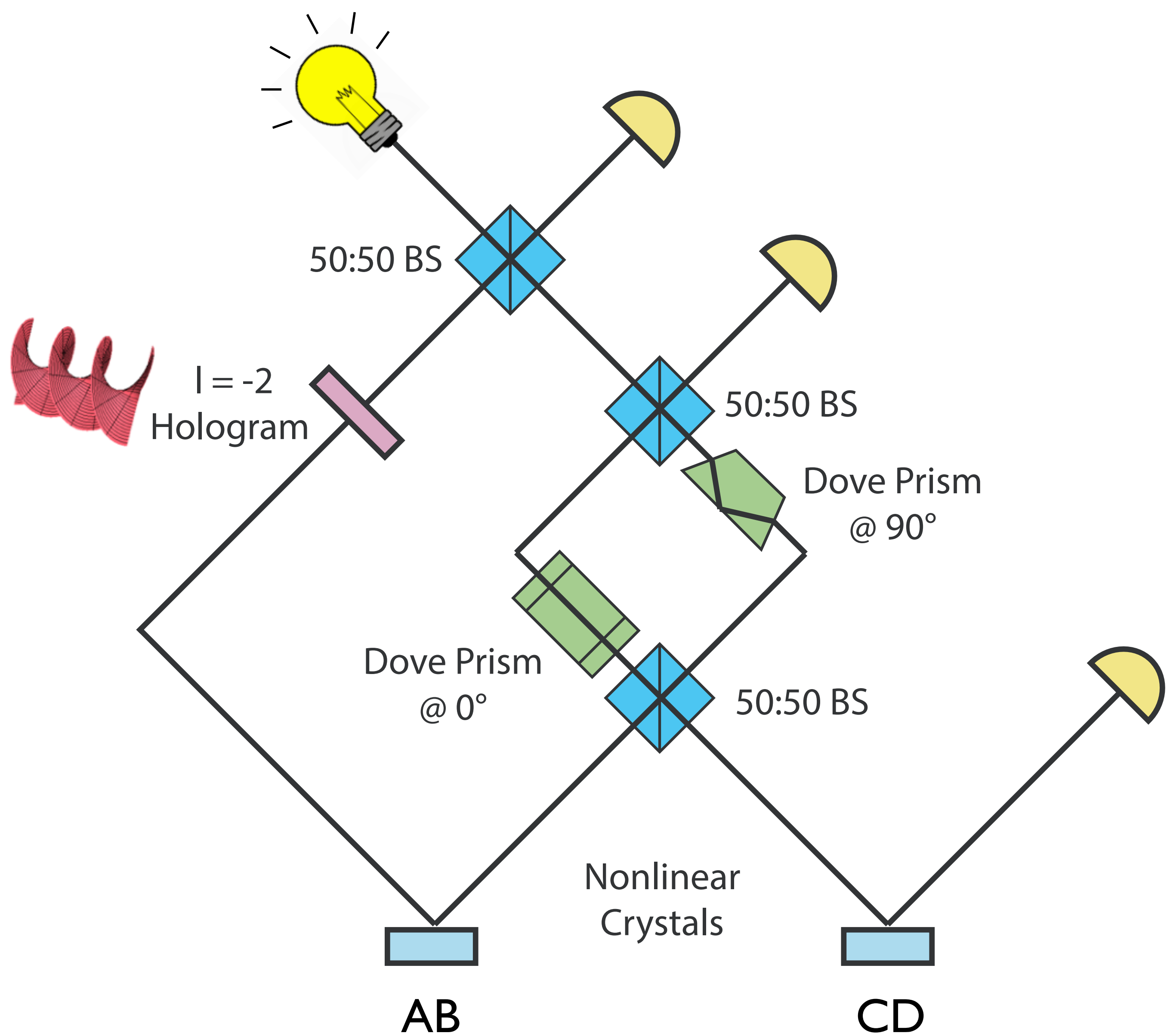

Krenn, **MM**, Fickler, Lapkiewicz, Zeilinger, **Phys. Rev. Lett. 116,** 090405 (2016)

# 333 ENTANGLEMENT




Erhard, **MM**, Krenn and Zeilinger, **Nature Photonics 12,** 759 (2018)

# 333 ENTANGLEMENT




Erhard, **MM**, Krenn and Zeilinger, **Nature Photonics 12,** 759 (2018)

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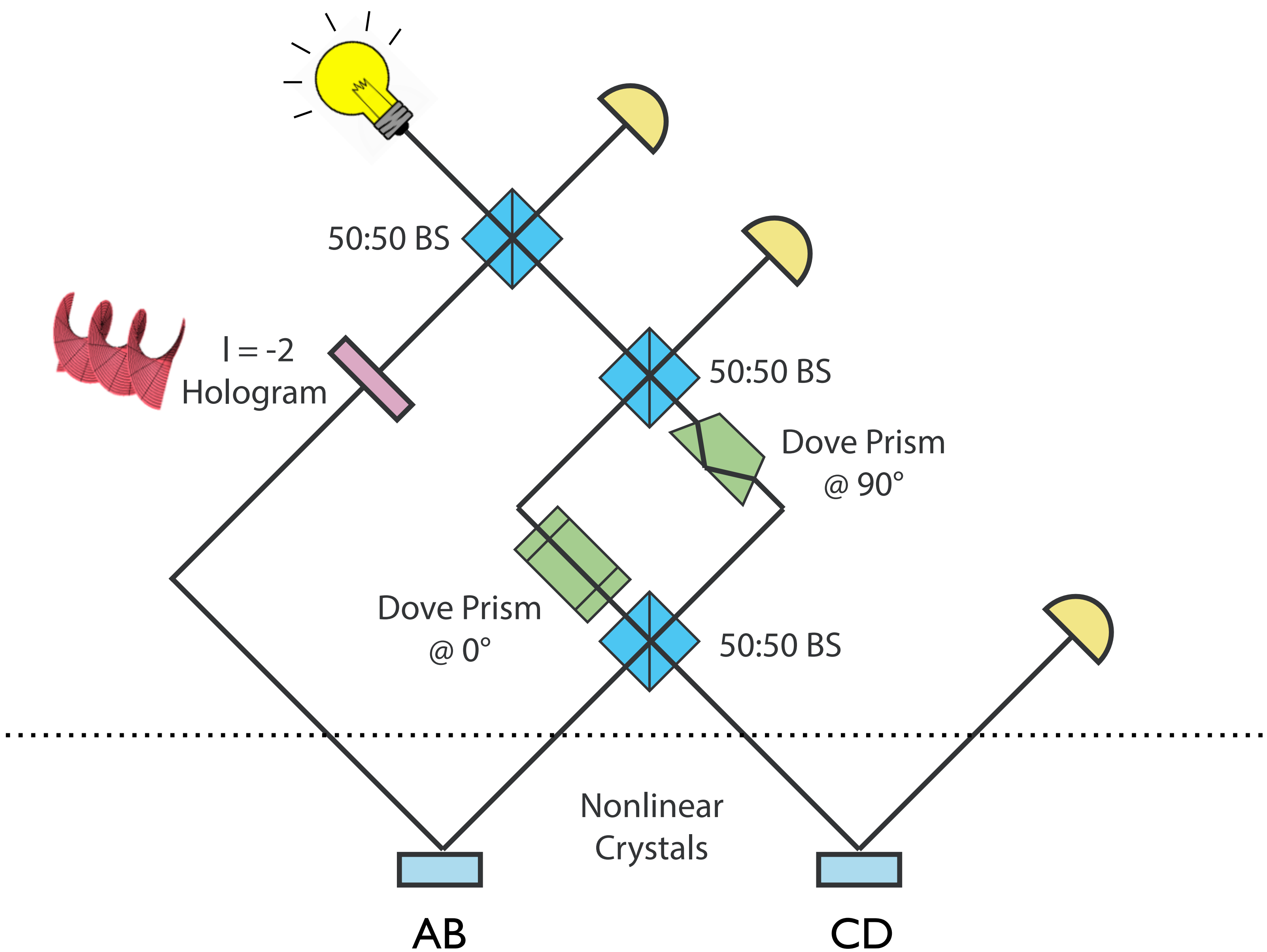
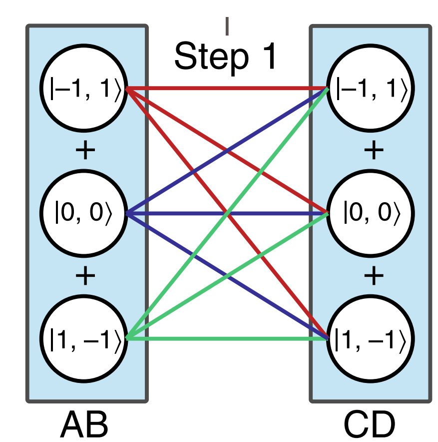
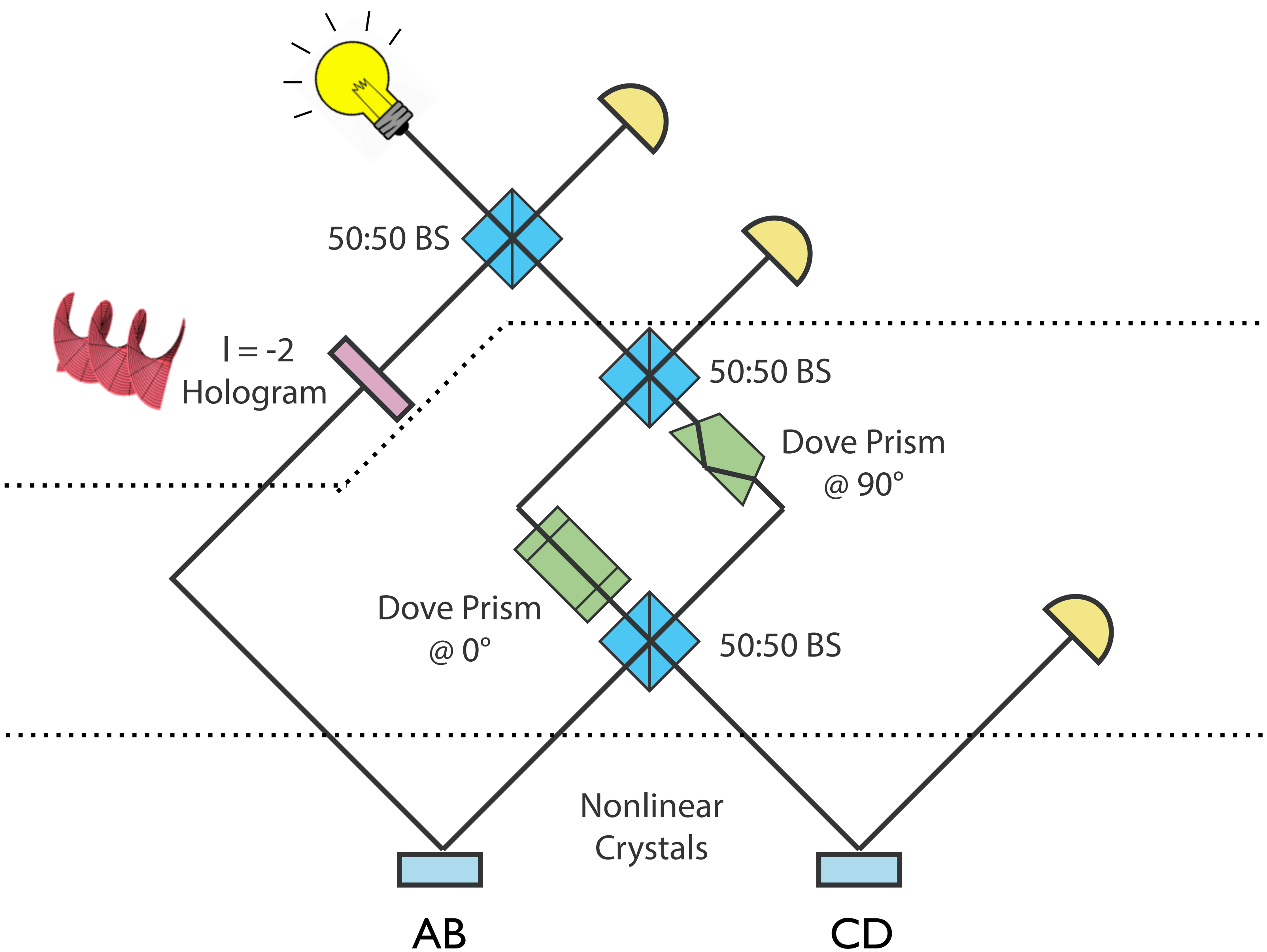
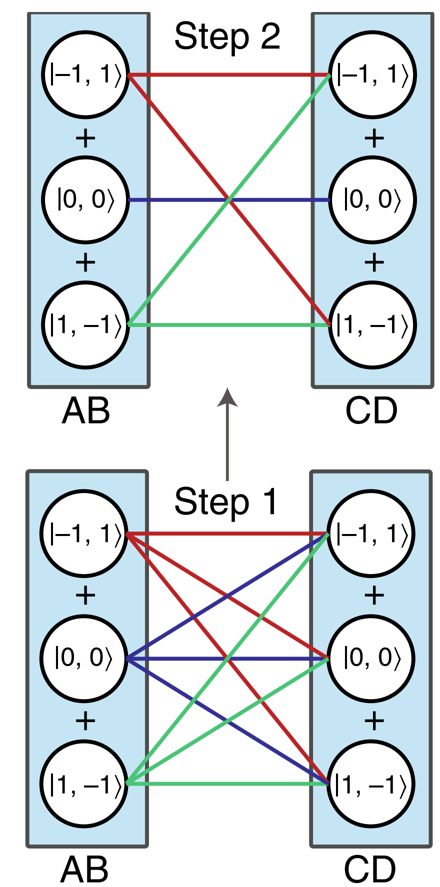



Erhard, **MM**, Krenn and Zeilinger, **Nature Photonics 12,** 759 (2018)

# 333 Entanglement

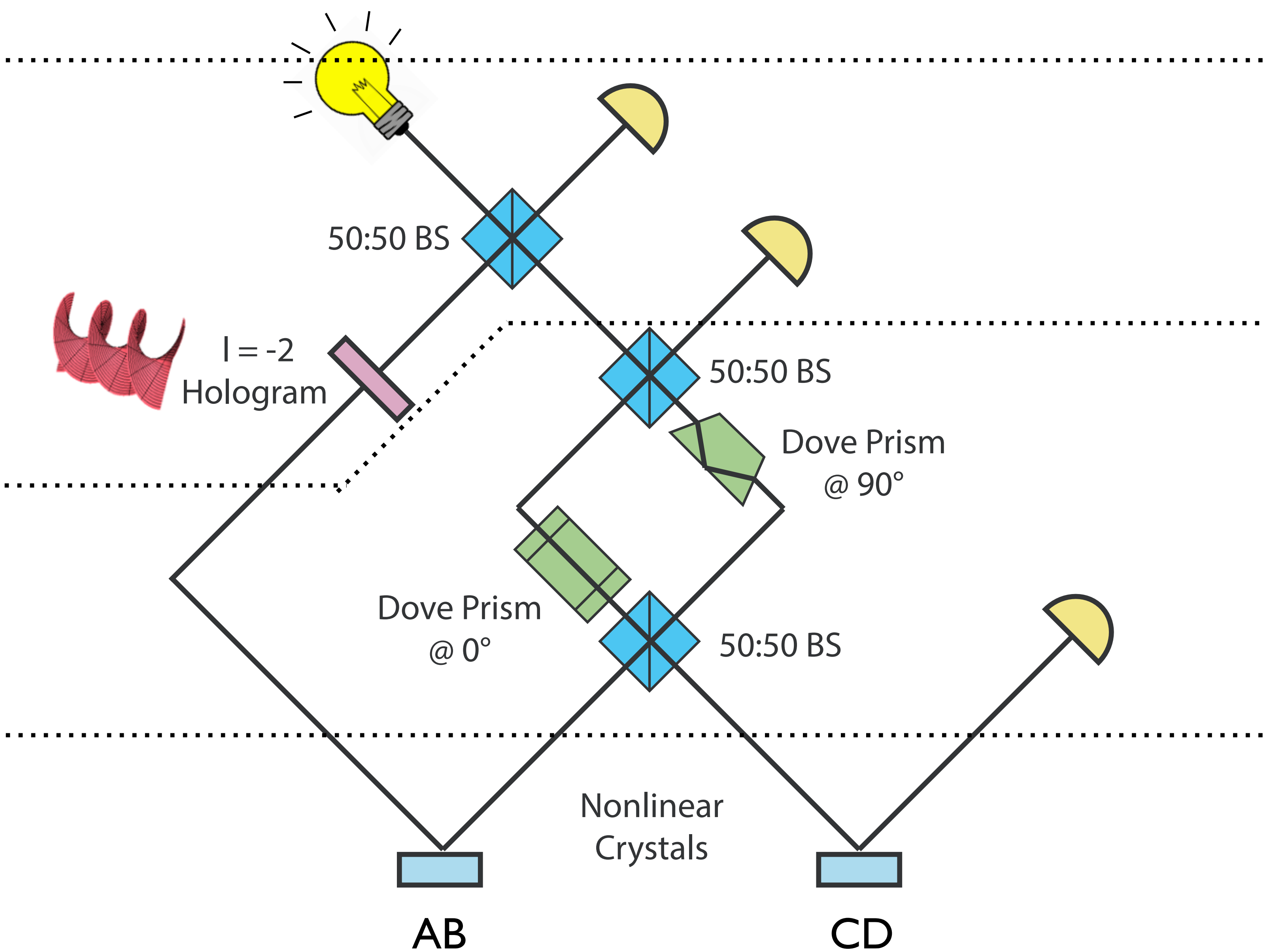


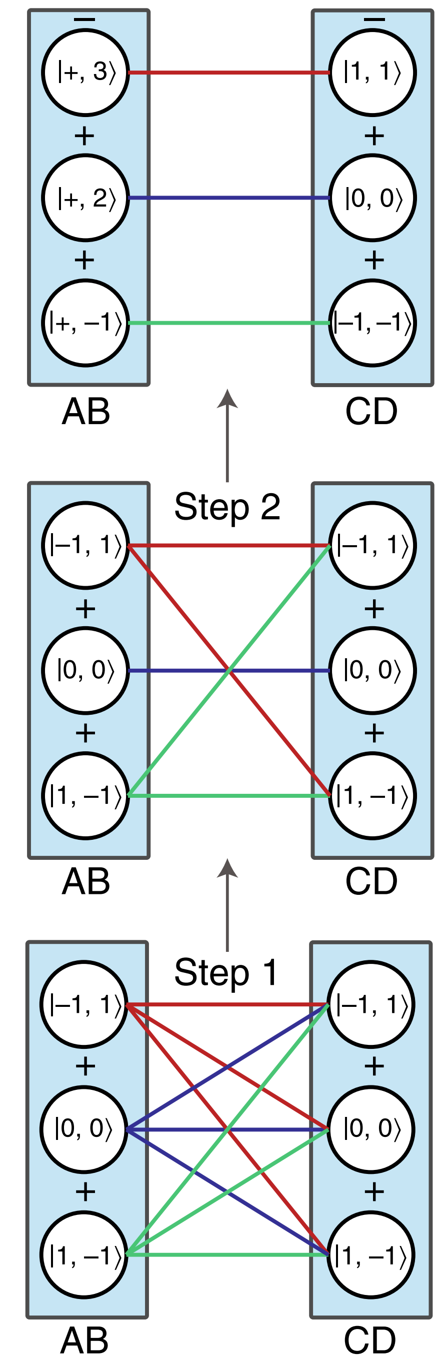




Erhard, **MM**, Krenn and Zeilinger, **Nature Photonics 12,** 759 (2018)

# 333 ENTANGLEMENT

$$|\Psi\rangle_{333} = \frac{1}{\sqrt{3}}[|-3,-1,-1\rangle + |-2,0,0\rangle - |1,1,1\rangle]$$




Erhard, **MM**, Krenn and Zeilinger, **Nature Photonics 12,** 759 (2018)

# Multi-Mode HOM Interference

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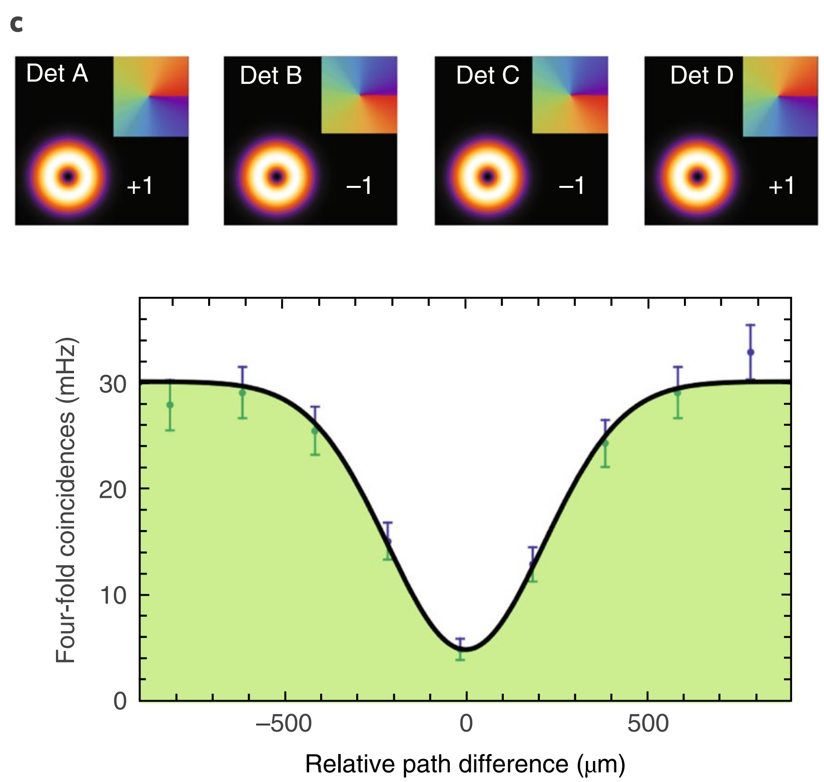
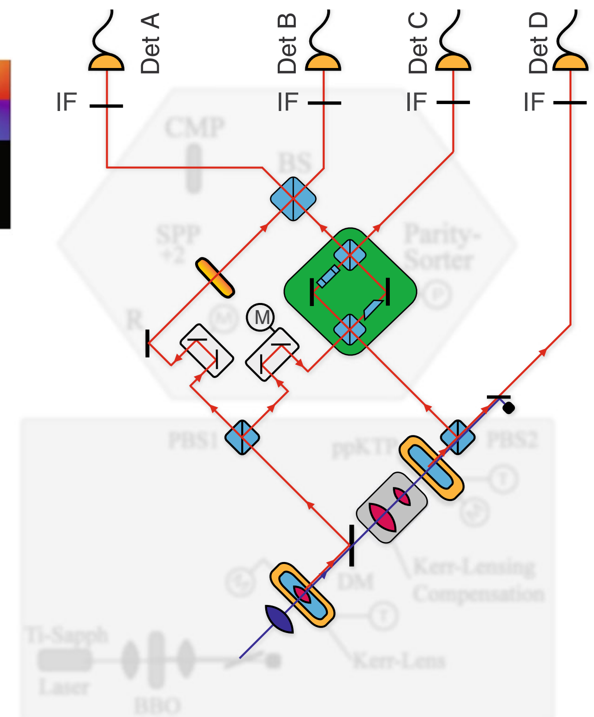
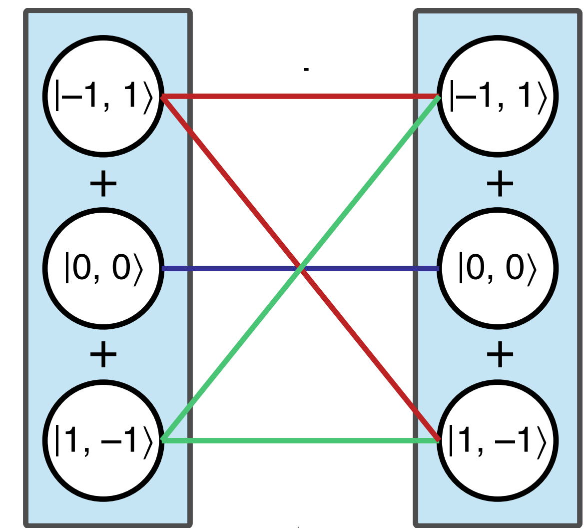
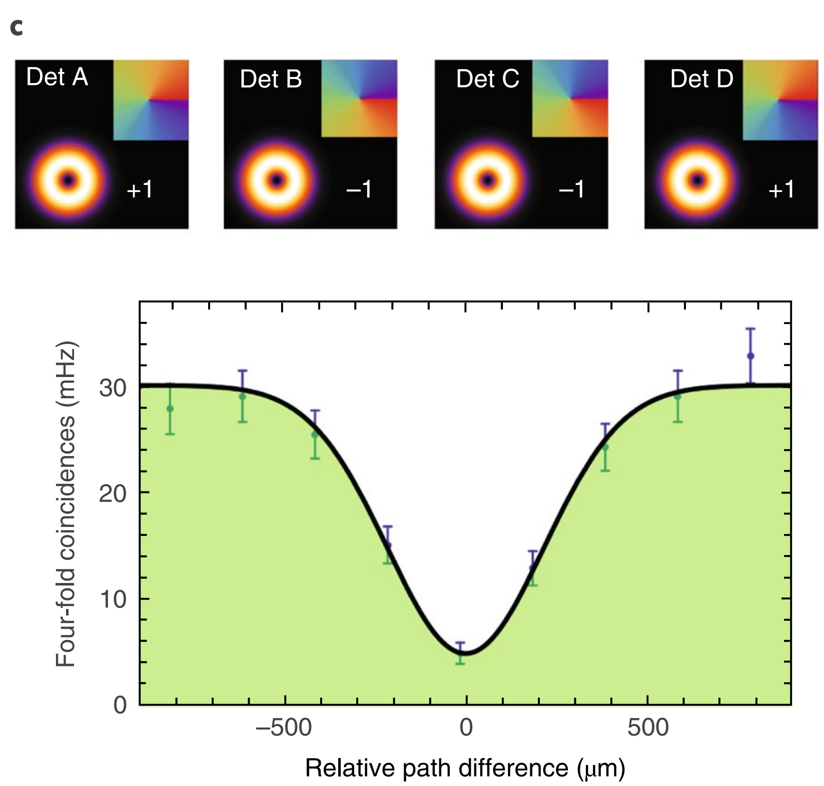
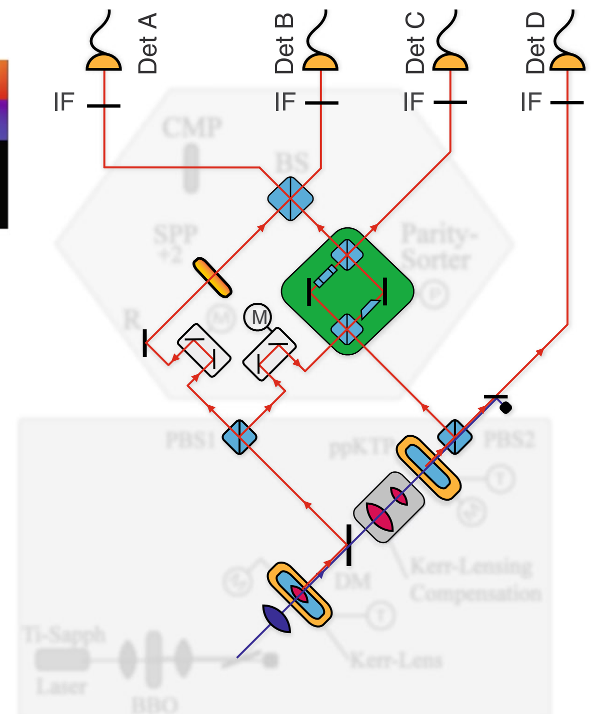


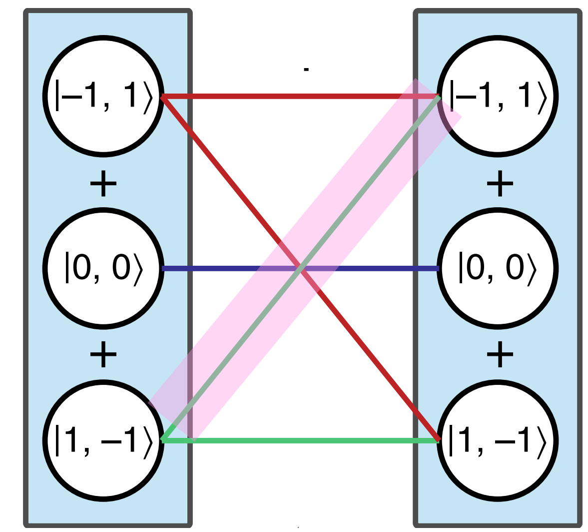



# MULTI-MODE HOM INTERFERENCE

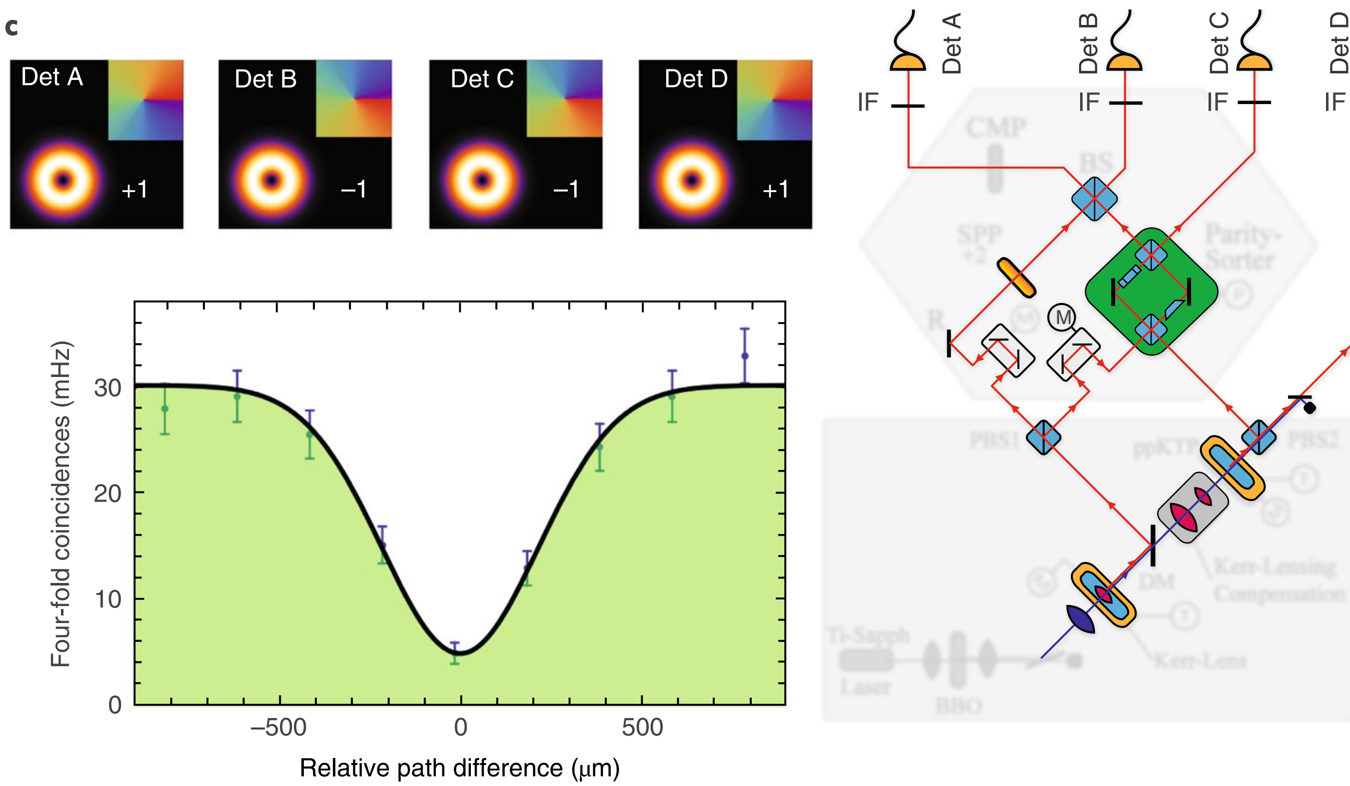


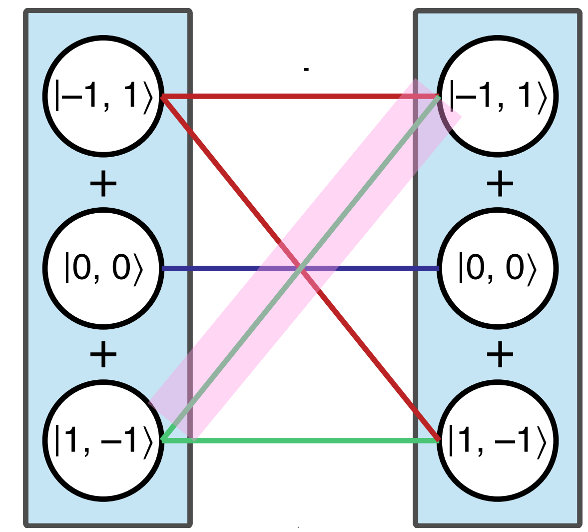



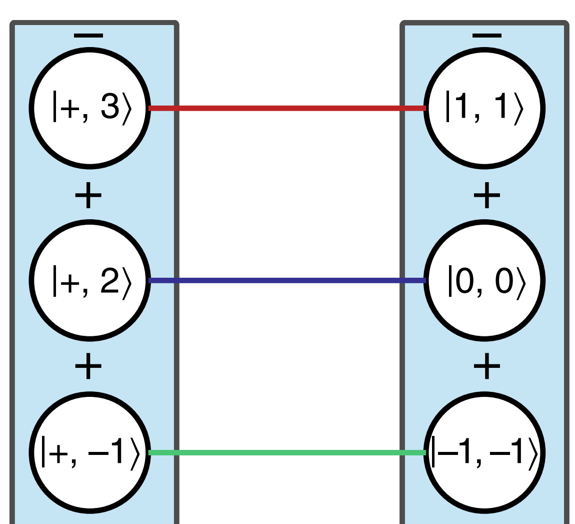



# Multi-Mode HOM Interference

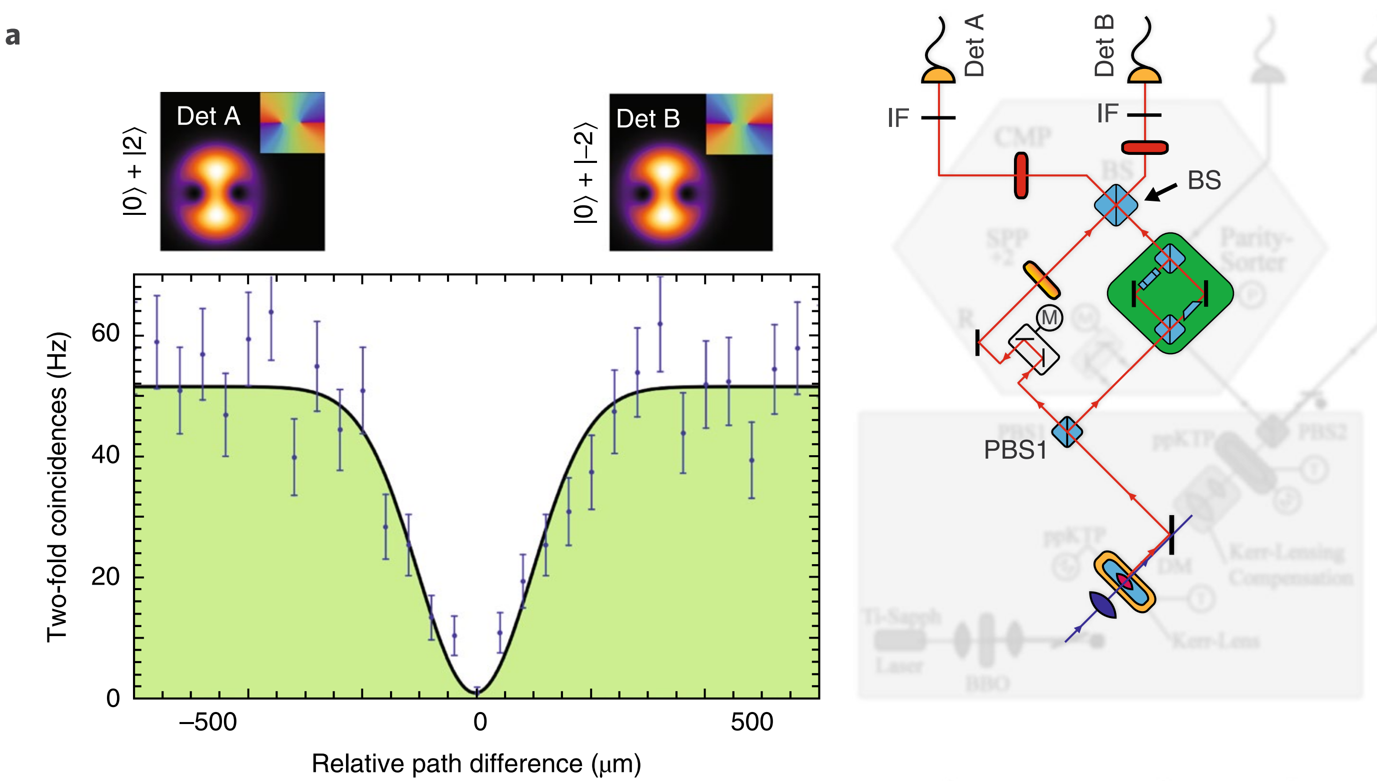
c




a



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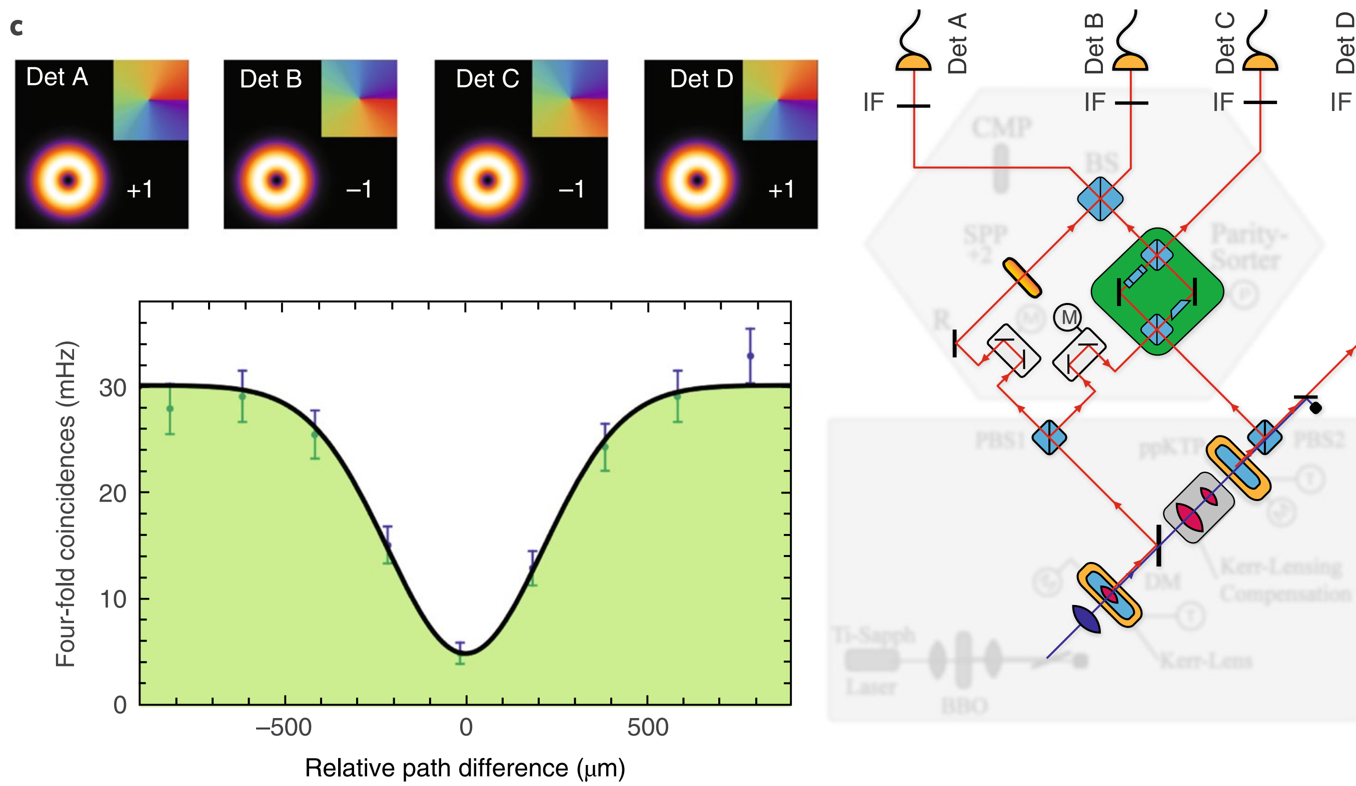
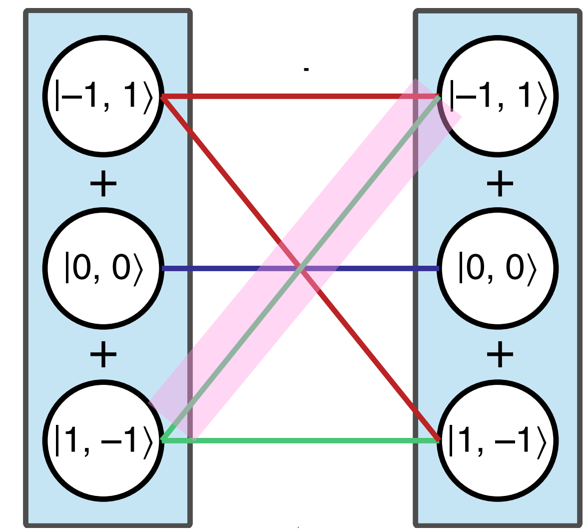
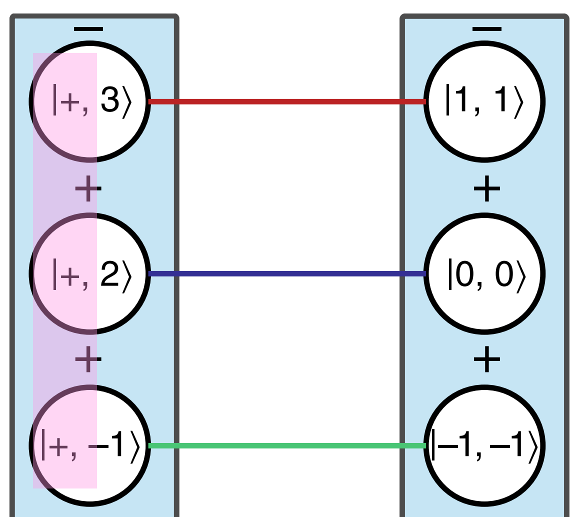
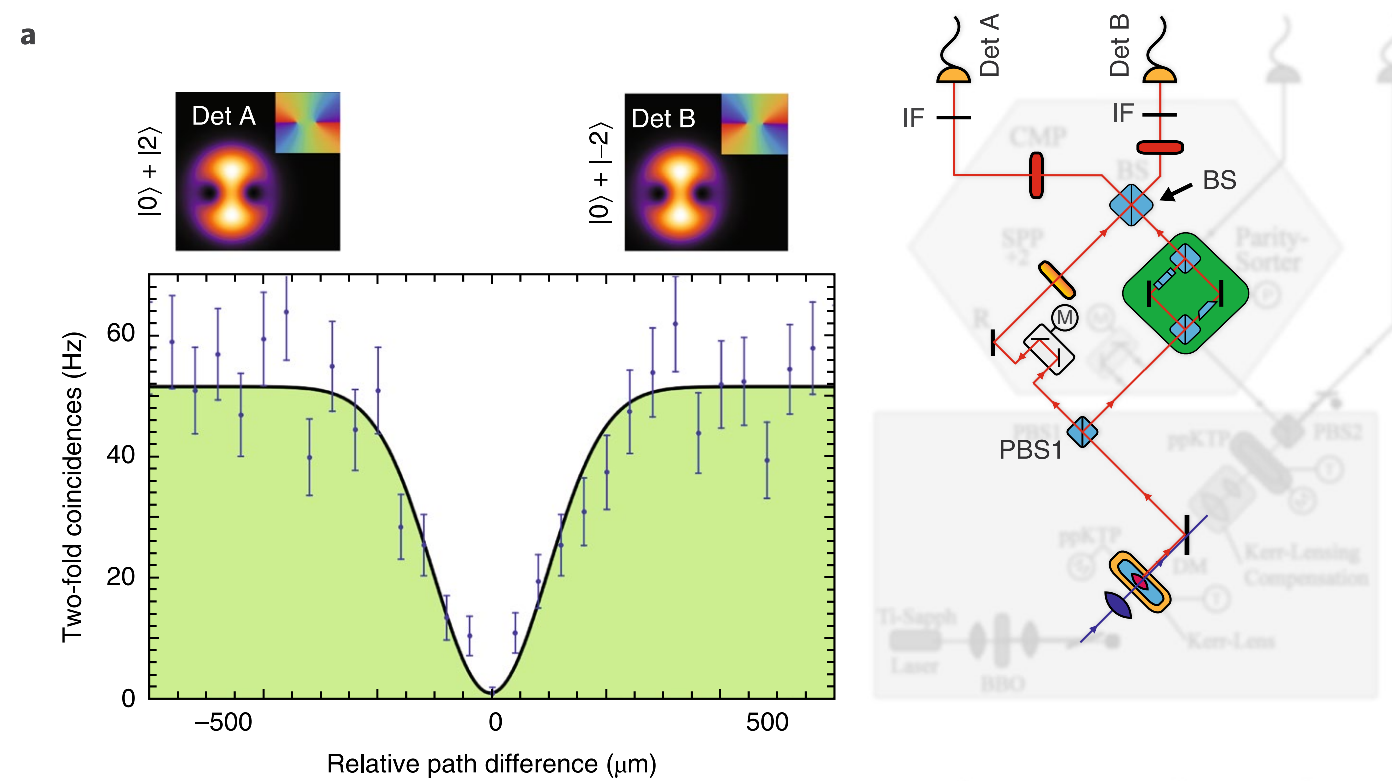
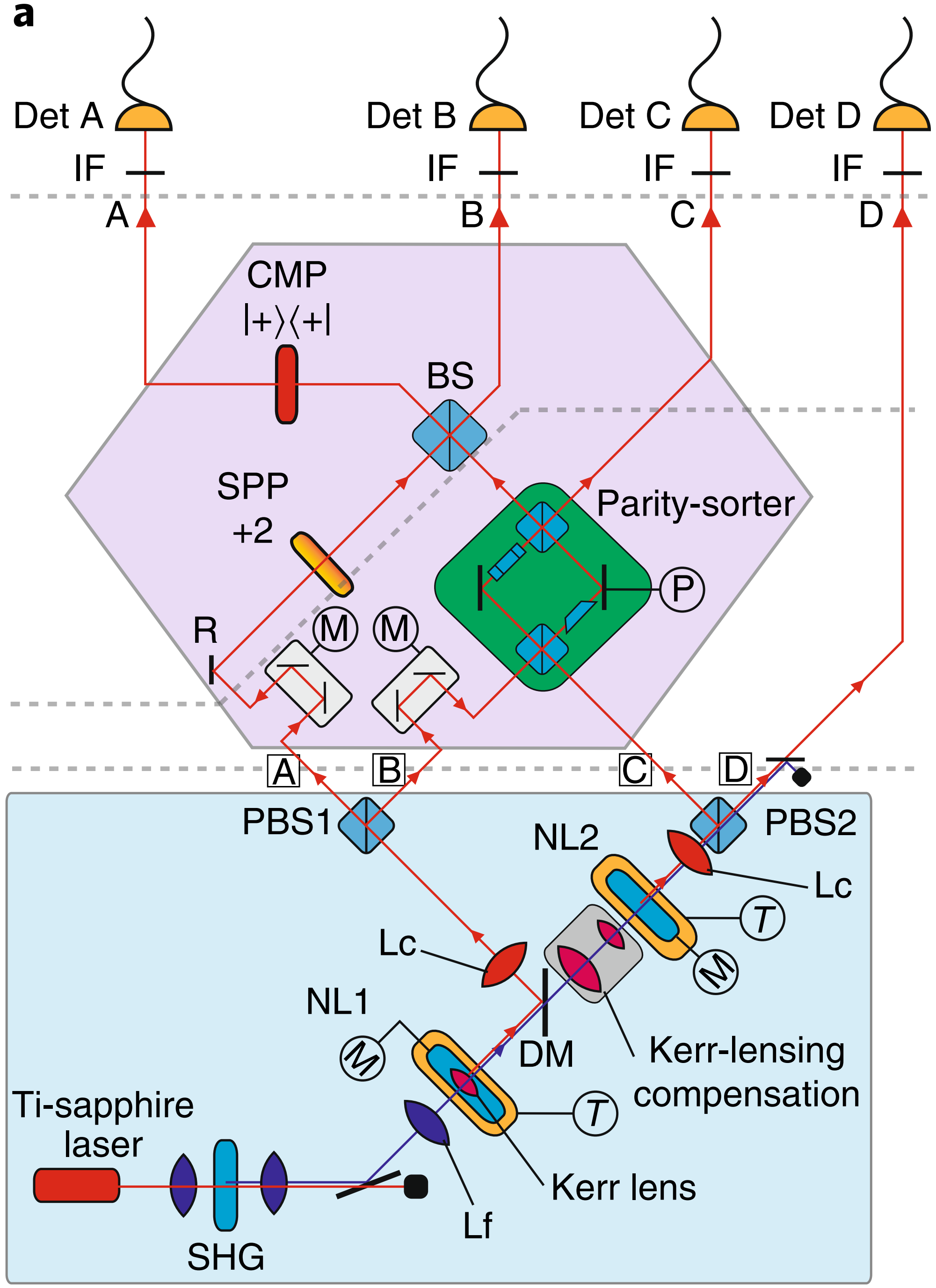


**a**



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compensation
Ti-sapphire
laser
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Kerr lens
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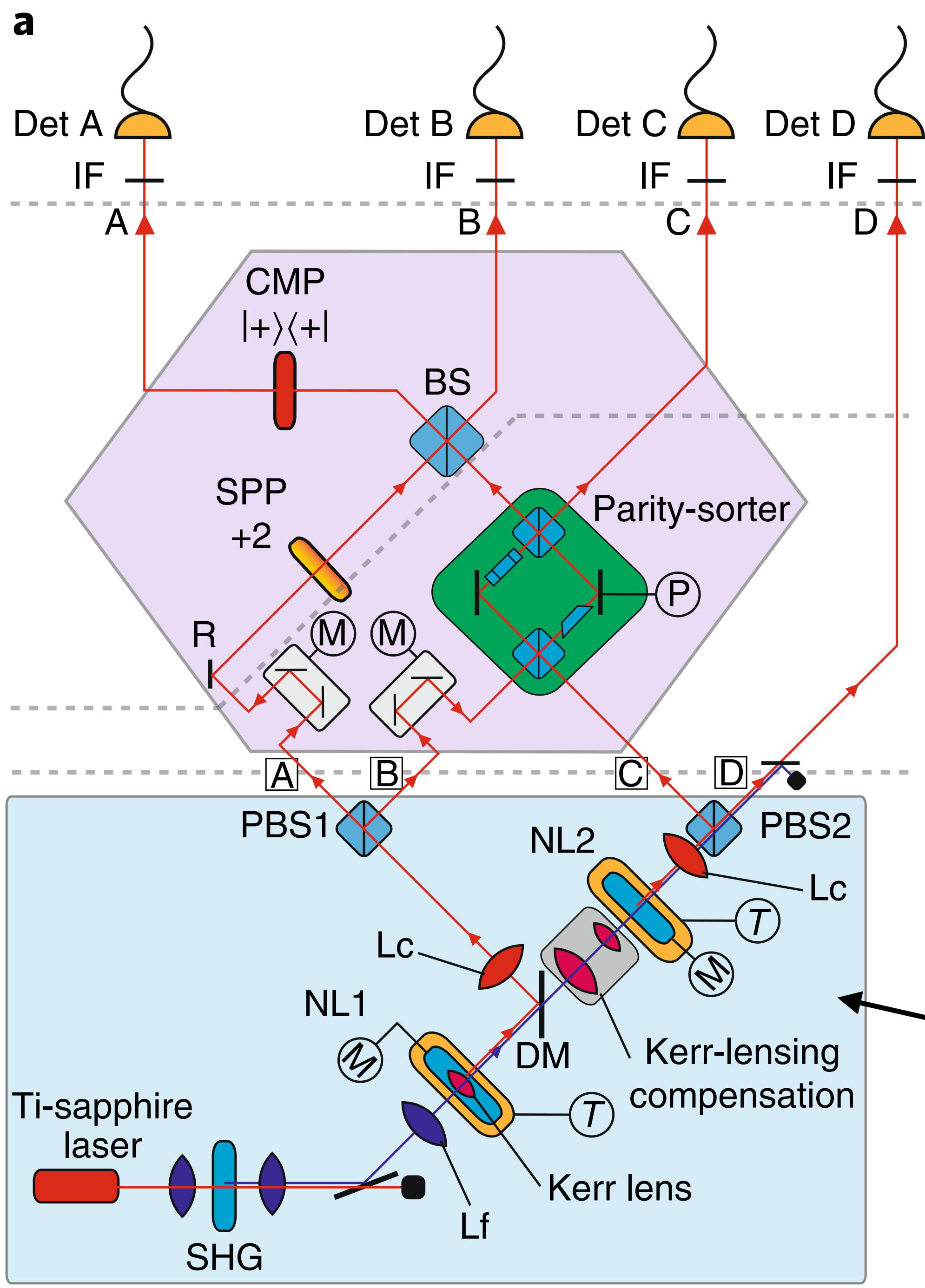
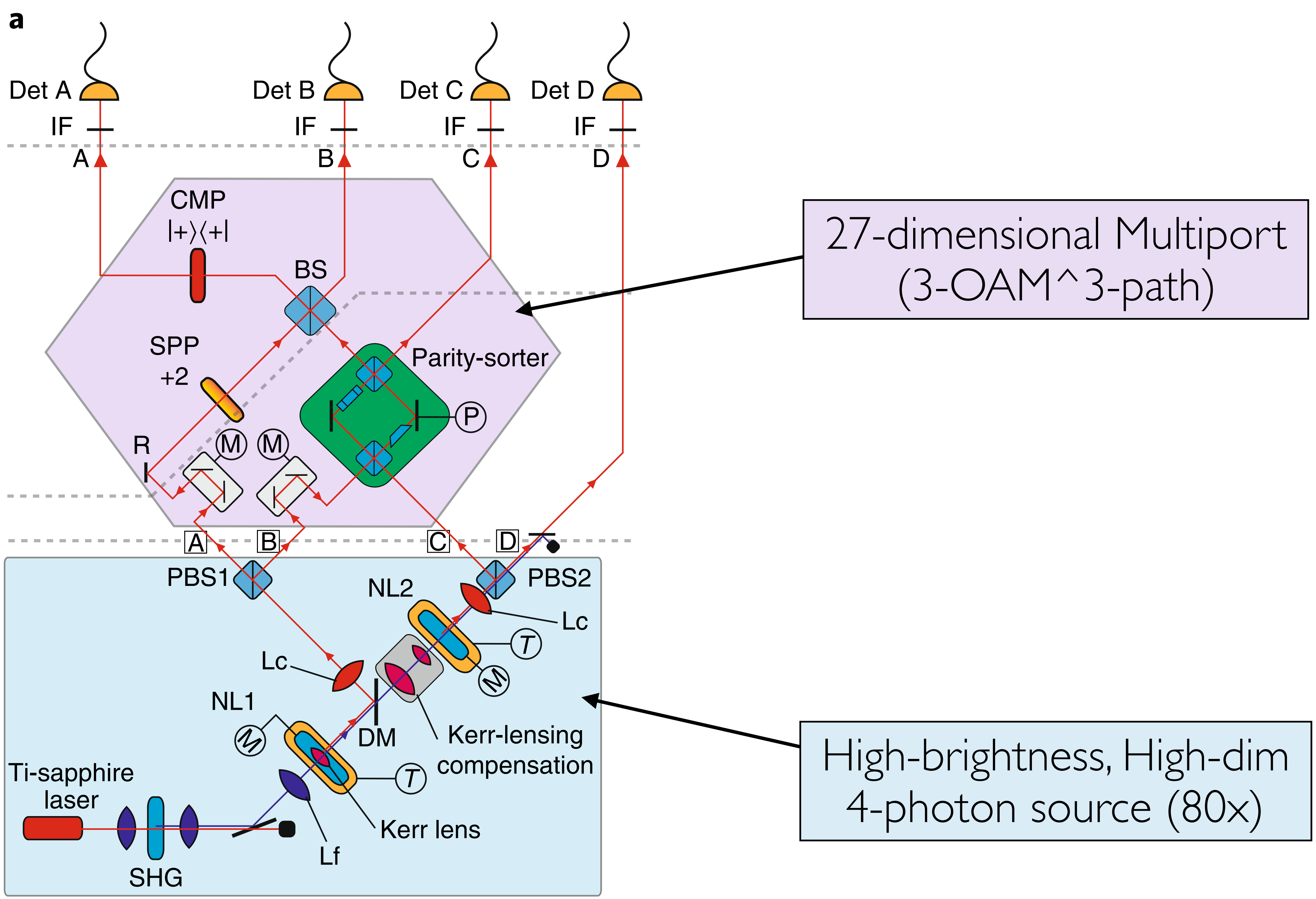
SHG




High-brightness, High-dim
4-photon source (80x)


a
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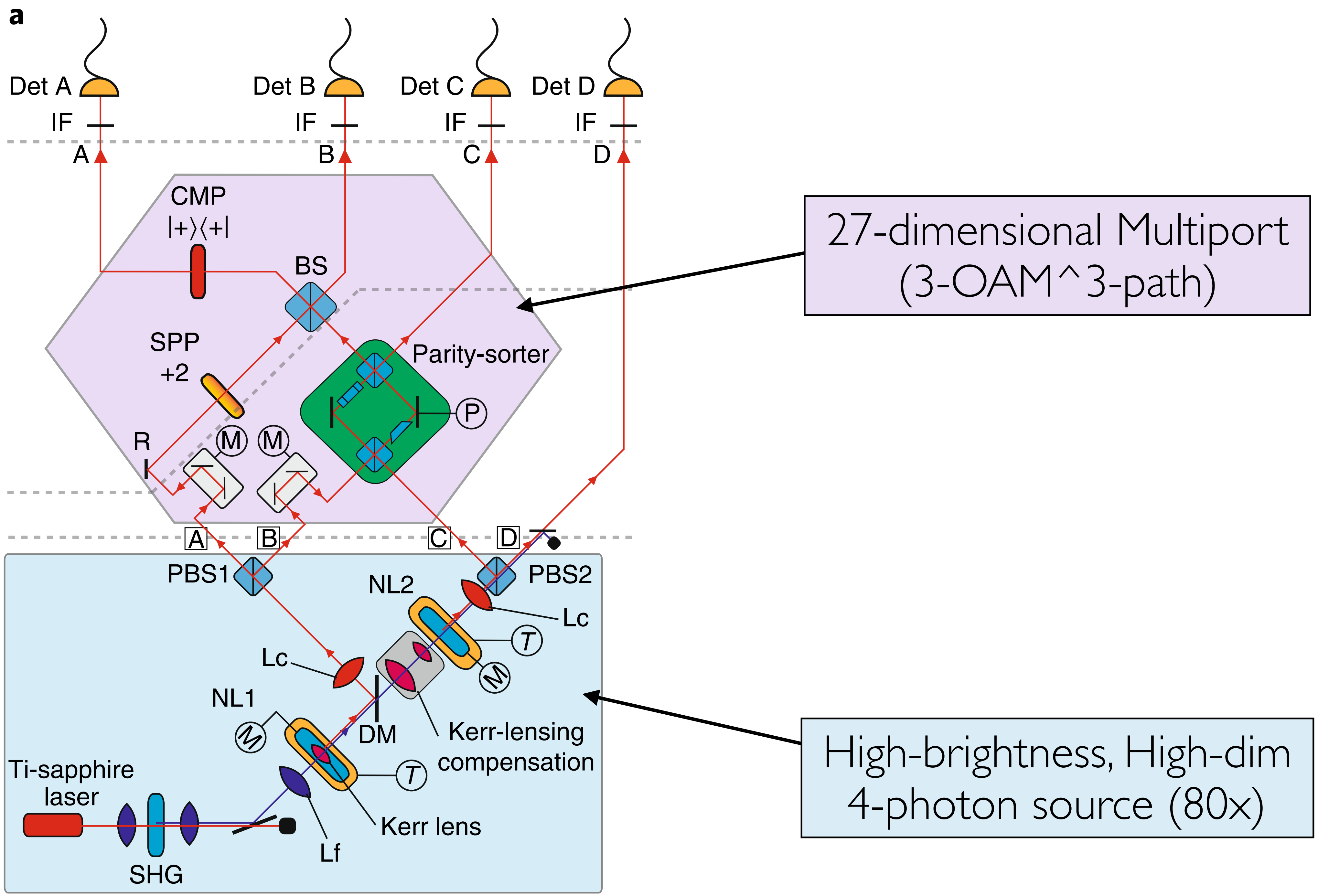
PBS2
NL2
Lc
T
M
Lc
NL1
M
DM
Kerr-lensing
compensation
Ti-sapphire
laser
T
Kerr lens
Lf
SHG
27-dimensional Multiport
(3-OAM^3-path)
High-brightness, High-dim
4-photon source (80x)



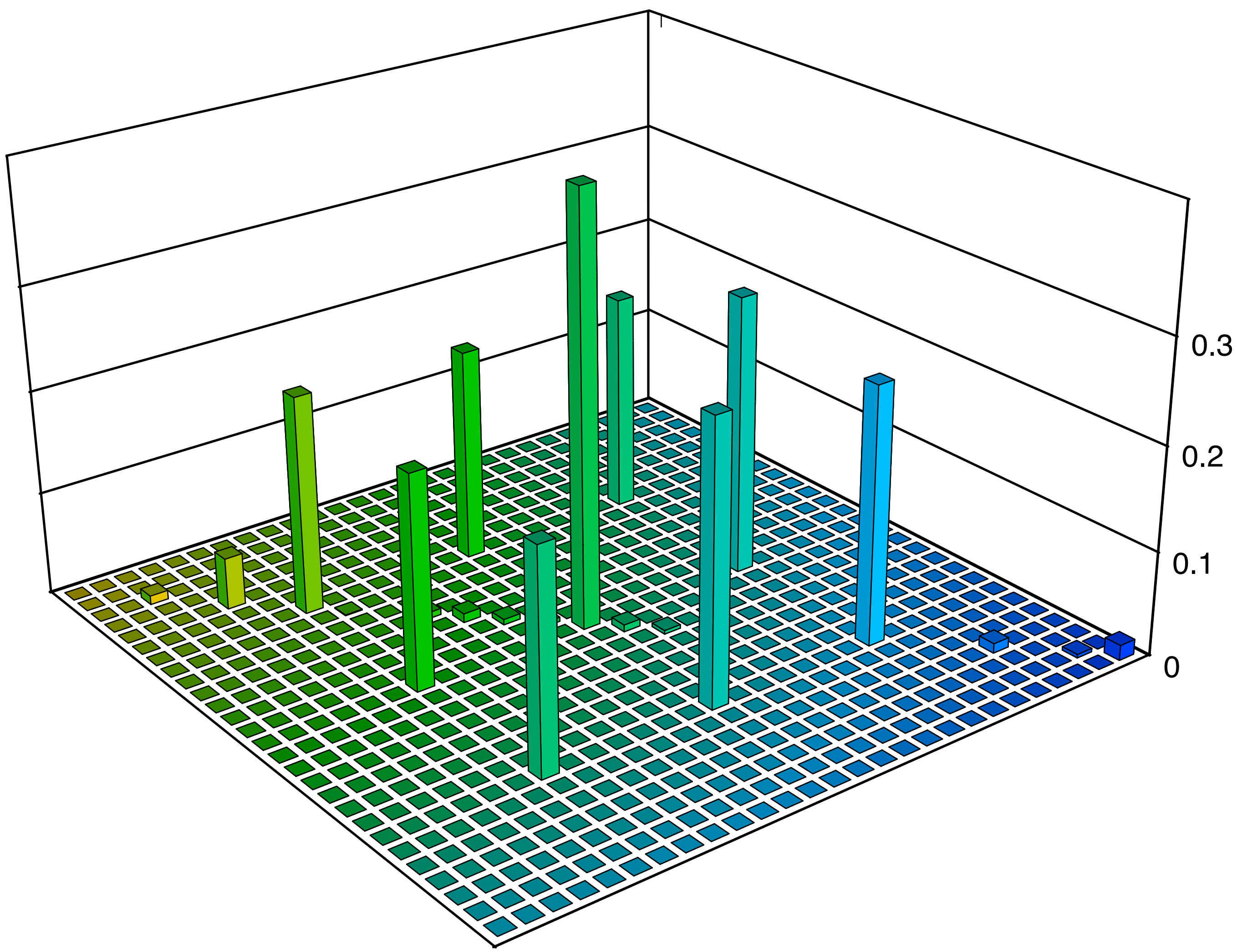

**c** Multiport transformations

| | | |
|---|---|---|
| $\vert 1\rangle_A \rightarrow i\vert 1\rangle_B - \vert +\rangle_A$ | $\vert 1\rangle_B \rightarrow \vert 1\rangle_C$ | $\vert 1\rangle_C \rightarrow i\vert -1\rangle_C$ |
| $\vert 0\rangle_A \rightarrow i\vert 2\rangle_B$ | $\vert 0\rangle_B \rightarrow i\vert +\rangle_A - \vert 0\rangle_B$ | $\vert 0\rangle_C \rightarrow i\vert 0\rangle_C$ |
| $\vert -1\rangle_A \rightarrow i\vert 3\rangle_B$ | $\vert -1\rangle_B \rightarrow \vert -1\rangle_C$ | $\vert -1\rangle_C \rightarrow \vert +\rangle_A + i\vert 1\rangle_B$ |

# VERIFYING 333 ENTANGLEMENT

$$|\Psi\rangle_{333} = \frac{1}{\sqrt{3}}[|{-3}, -1, -1\rangle + |{-2}, 0, 0\rangle - |1, 1, 1\rangle]$$

# VERIFYING 333 ENTANGLEMENT




- **27** *diagonal*, **3** *off-diagonal* elements measured

$$|\Psi\rangle_{333} = \frac{1}{\sqrt{3}}[|-3,-1,-1\rangle + |-2,0,0\rangle - |1,1,1\rangle]$$

Erhard, **MM**, Krenn and Zeilinger, Nature Photonics 12, 759 (2018)

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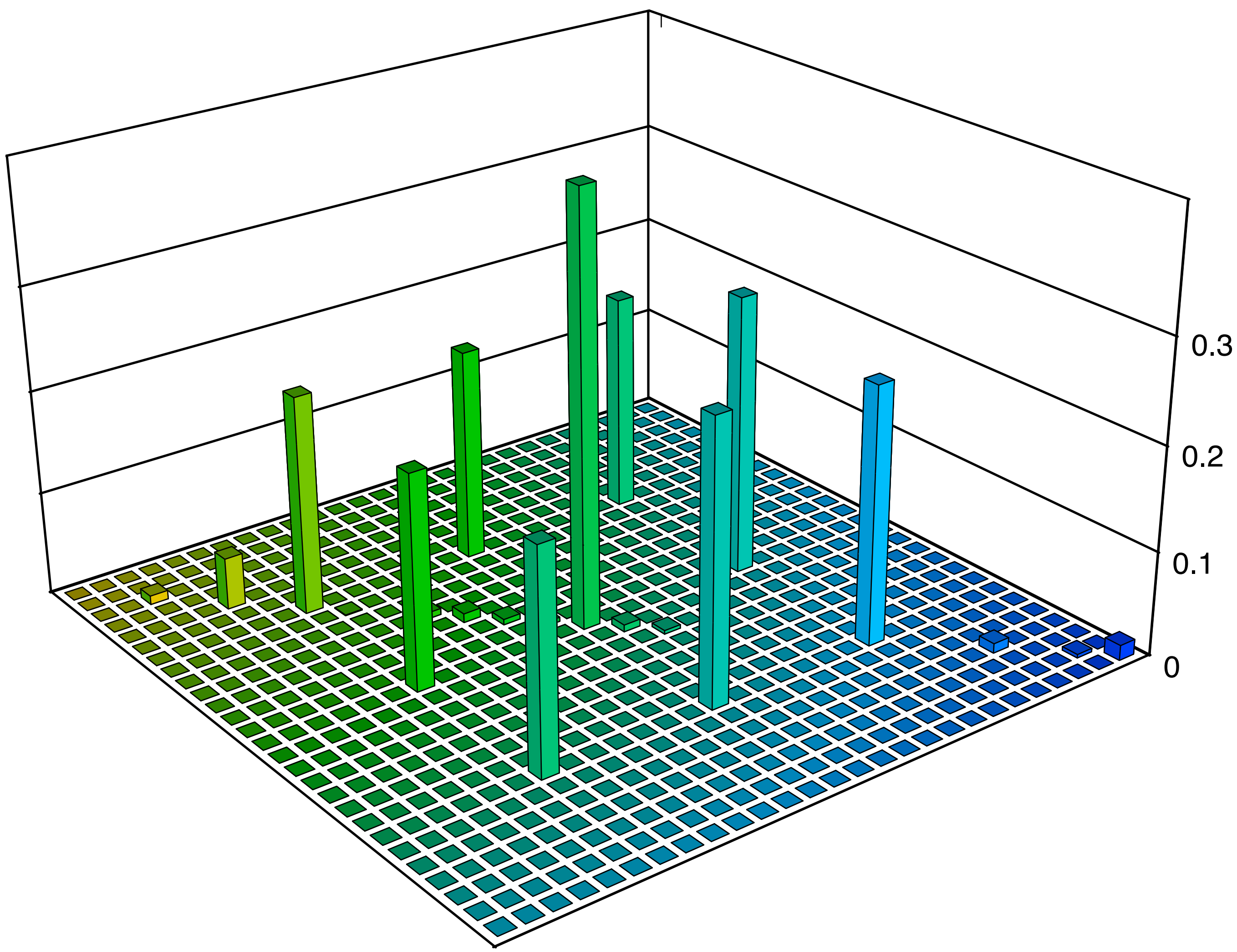


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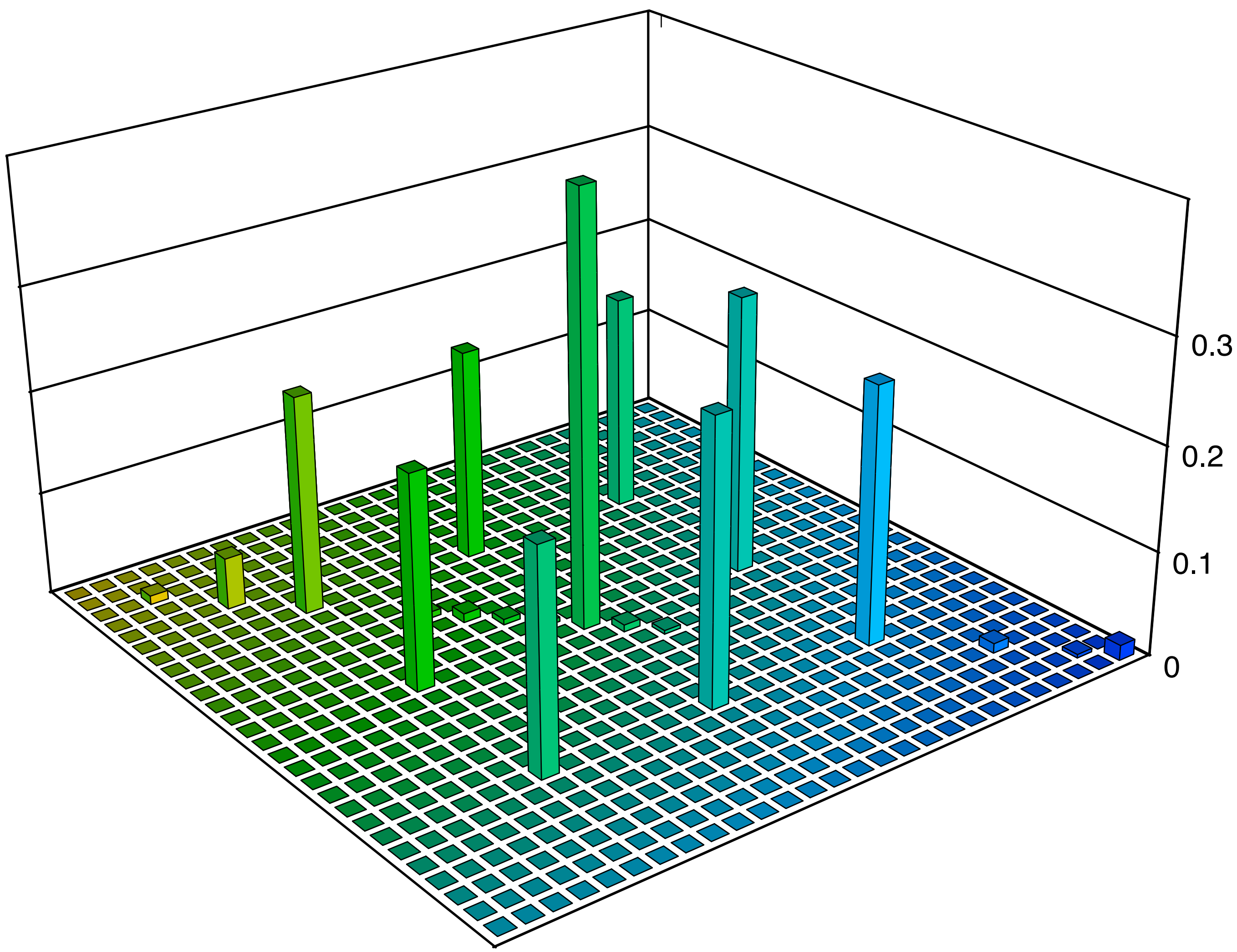


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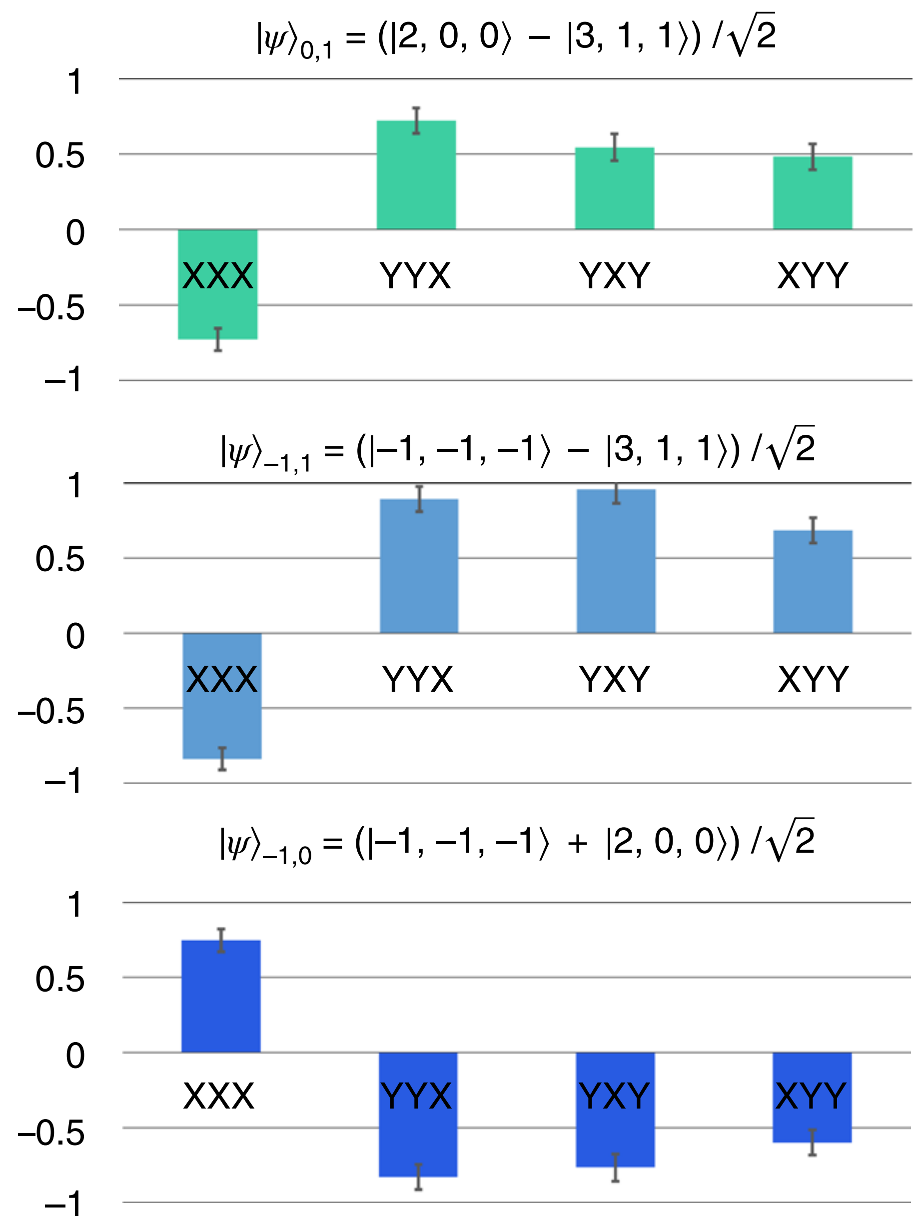


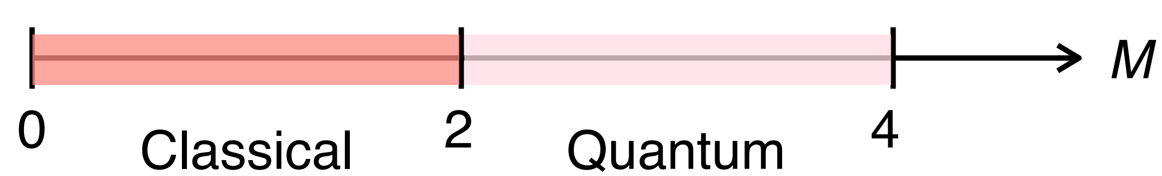


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Erhard, **MM**, Krenn and Zeilinger, **Nature Photonics 12,** 759 (2018)

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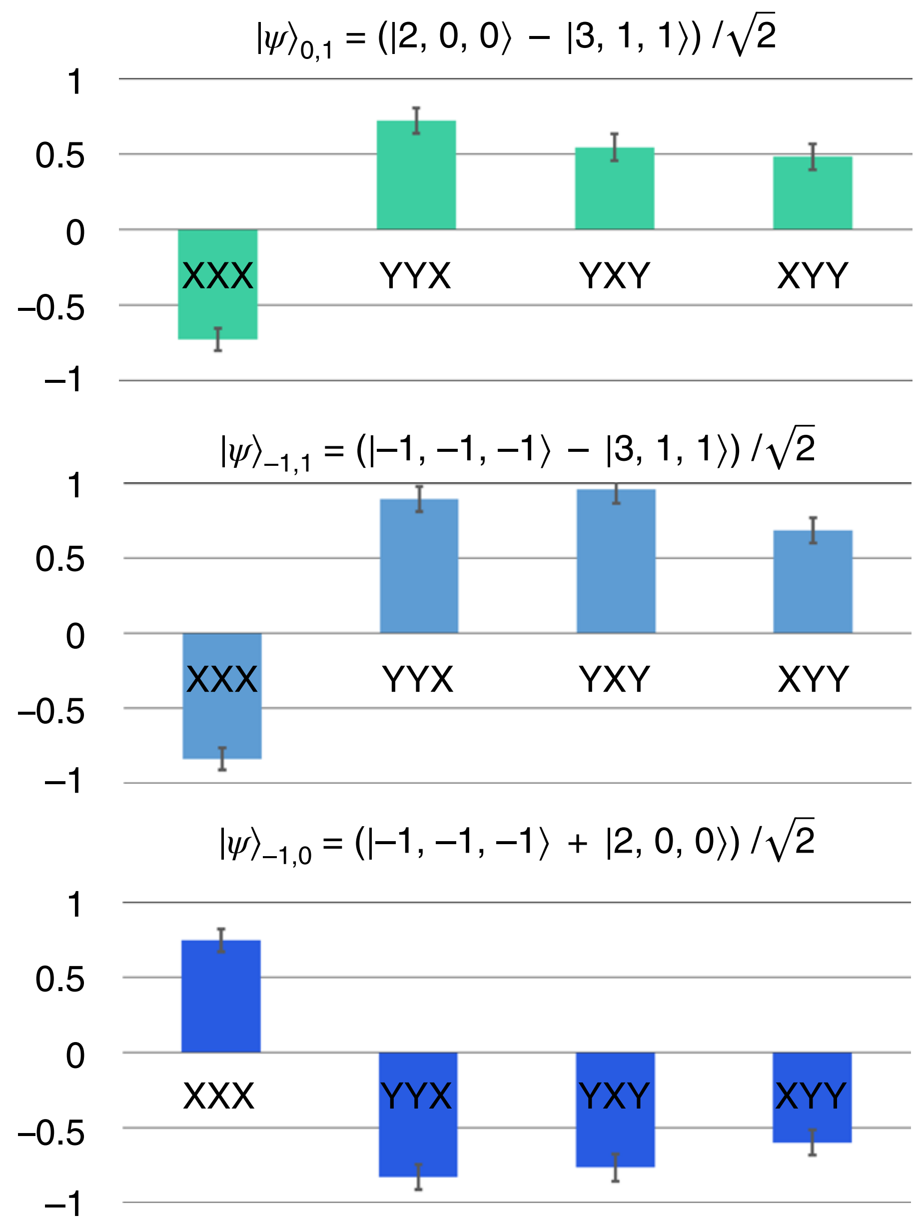


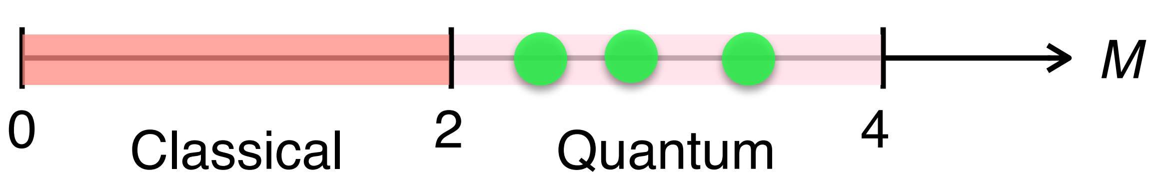


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Erhard, **MM**, Krenn and Zeilinger, **Nature Photonics 12,** 759 (2018)

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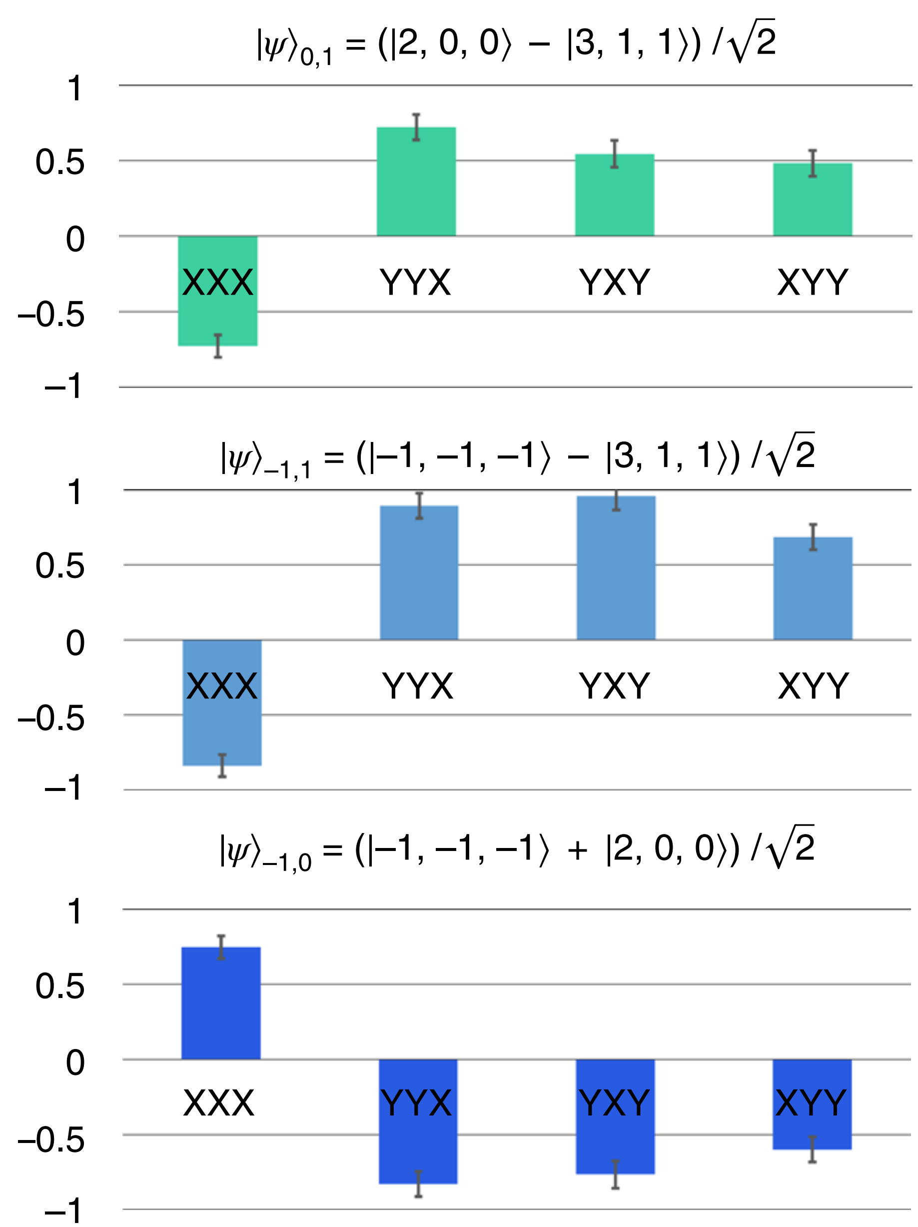


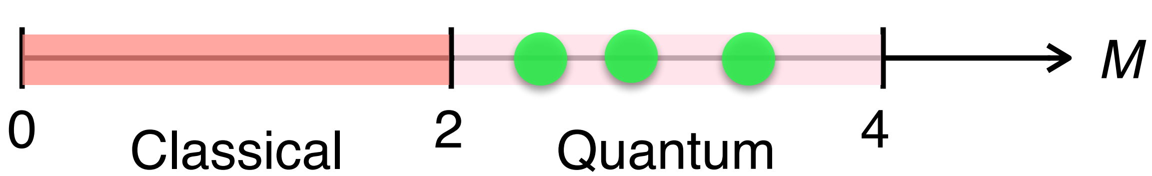


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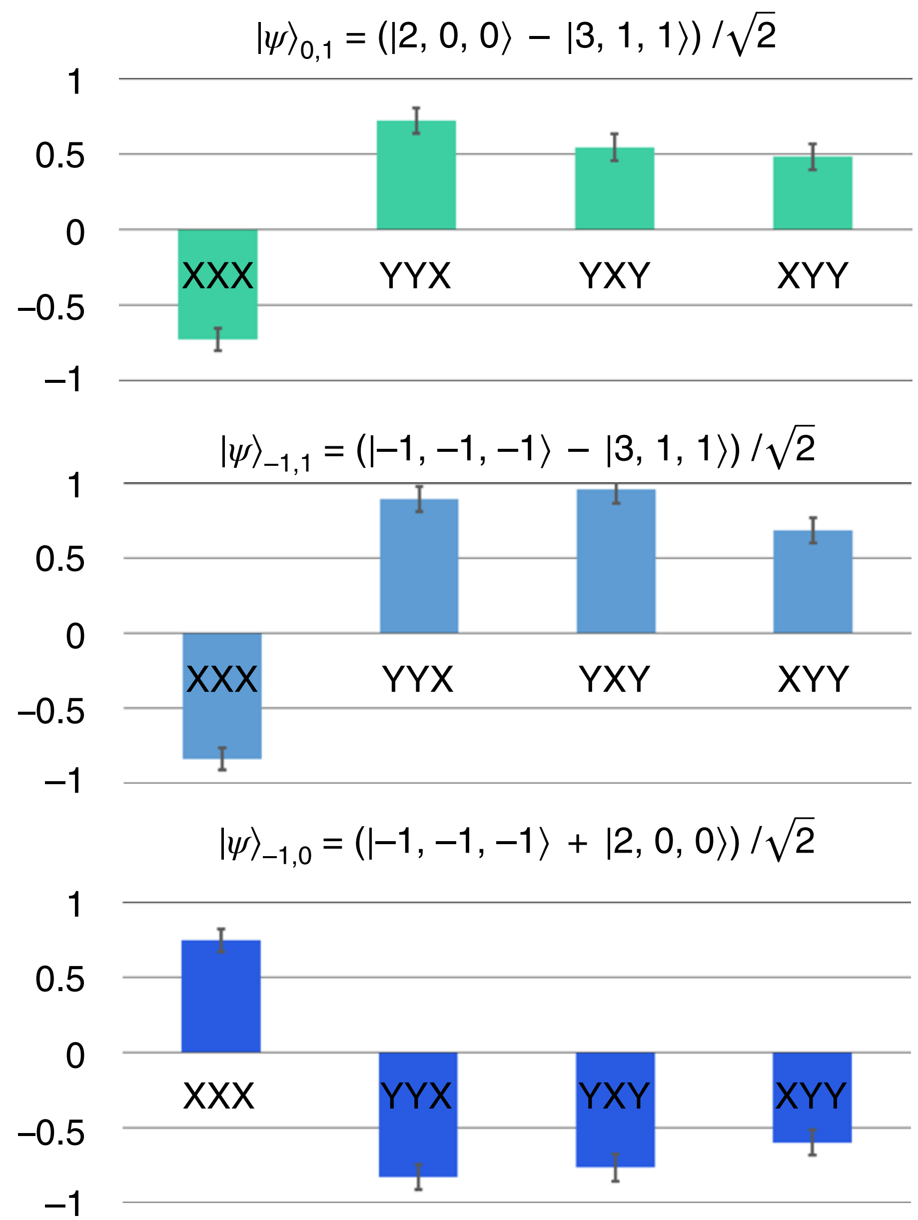


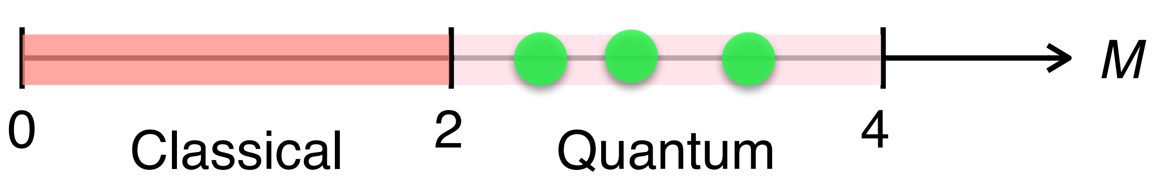


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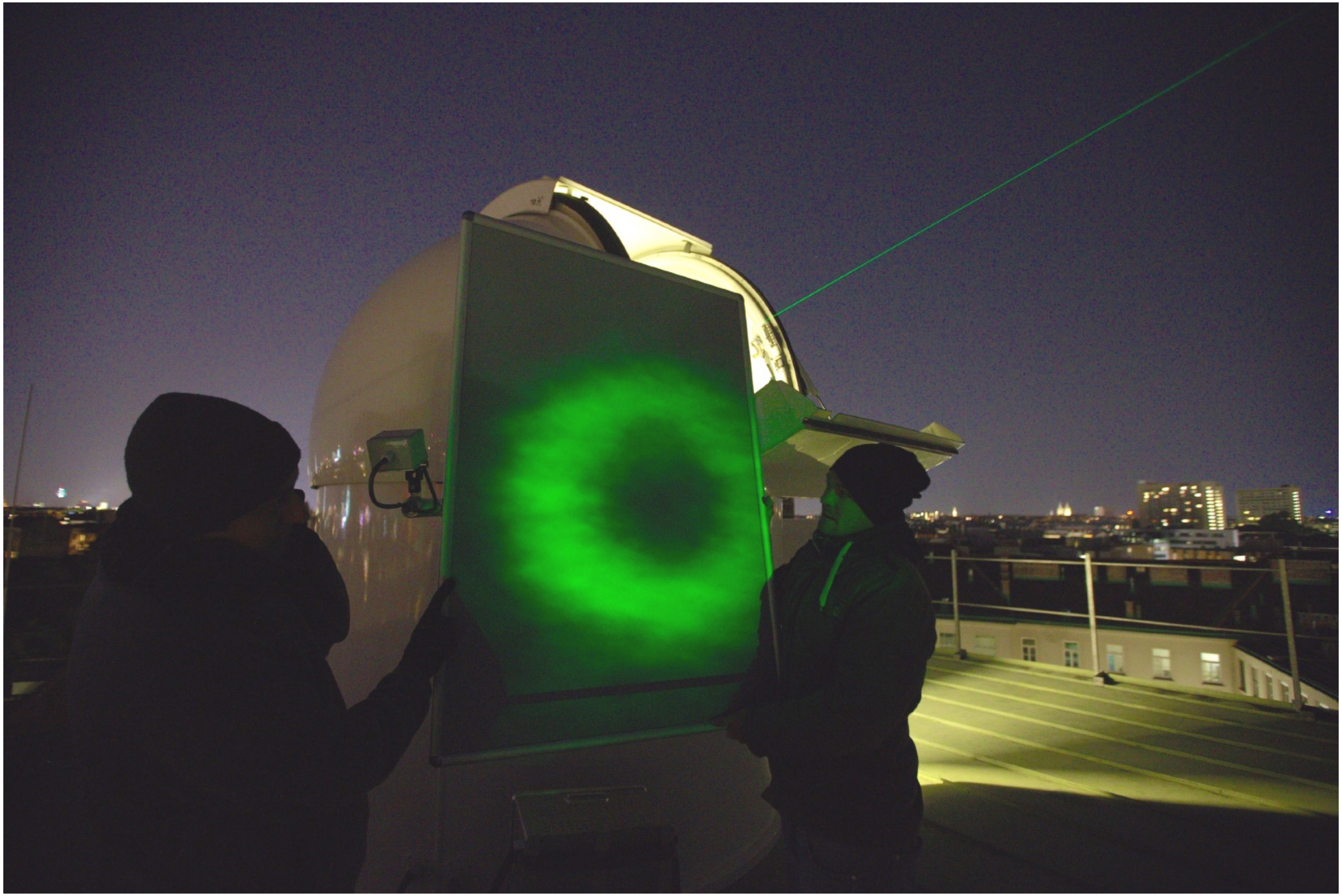
Erhard, **MM**, Krenn and Zeilinger, **Nature Photonics 12,** 759 (2018)

# OAM OUTSIDE THE LAB

# Large-Scale Experiments with Classical Light



Krenn, Fickler, Fink, Handsteiner, MM, Scheidl, Ursin, Zeilinger, New J. Phys. 16, 113028 (2014)

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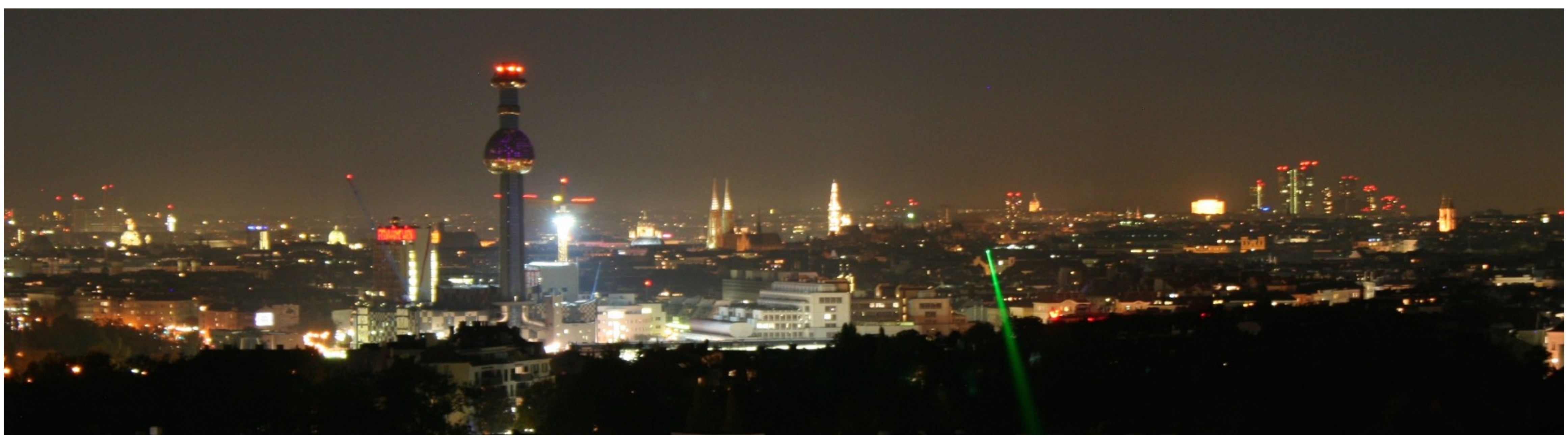
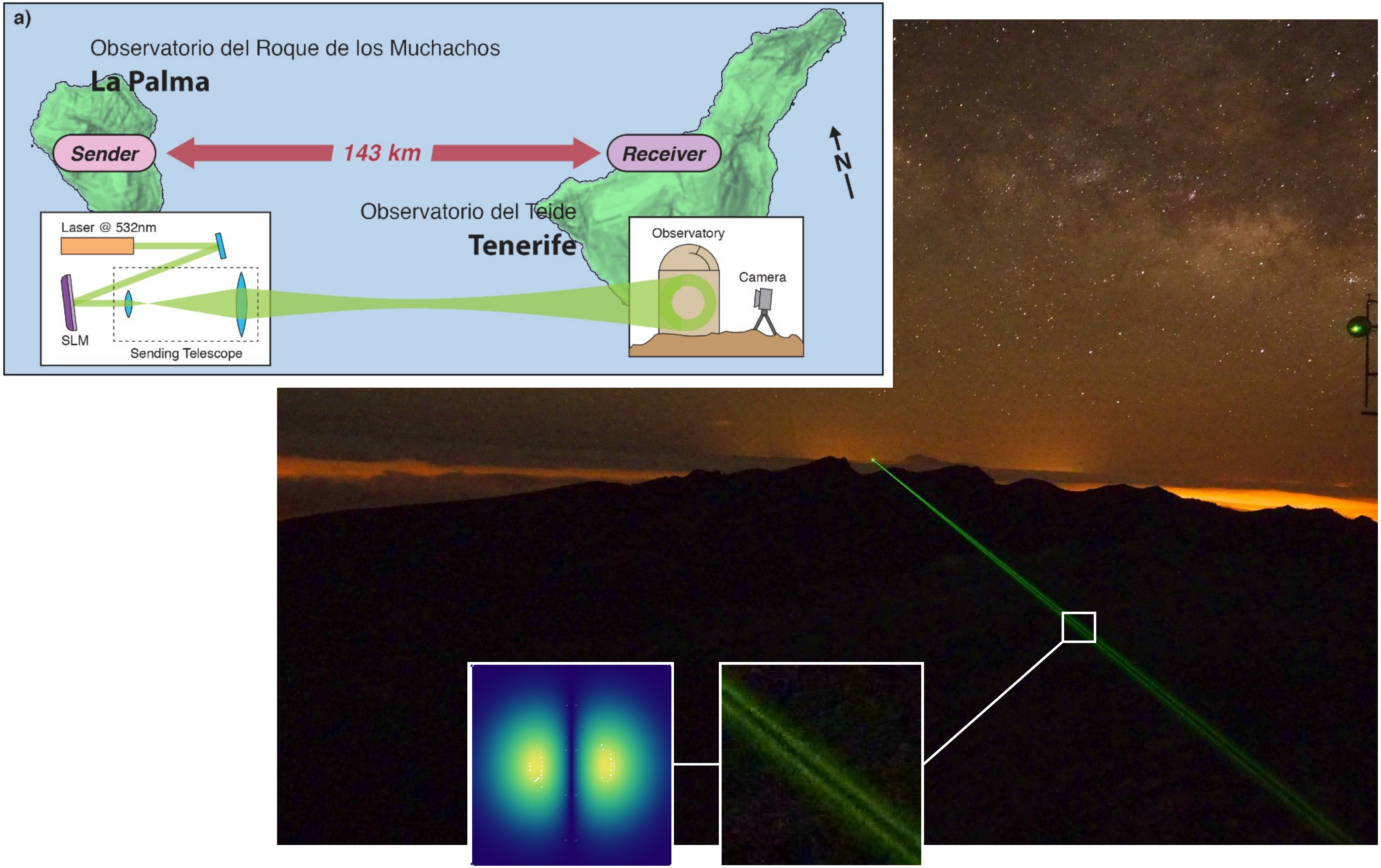


Krenn, Fickler, Fink, Handsteiner, MM, Scheidl, Ursin, Zeilinger, New J. Phys. 16, 113028 (2014)

# LARGE-SCALE EXPERIMENTS WITH CLASSICAL LIGHT




Krenn, Fickler, Fink, Handsteiner, Ursin, **MM**, Zeilinger, **PNAS 113,** 13648 (2016)

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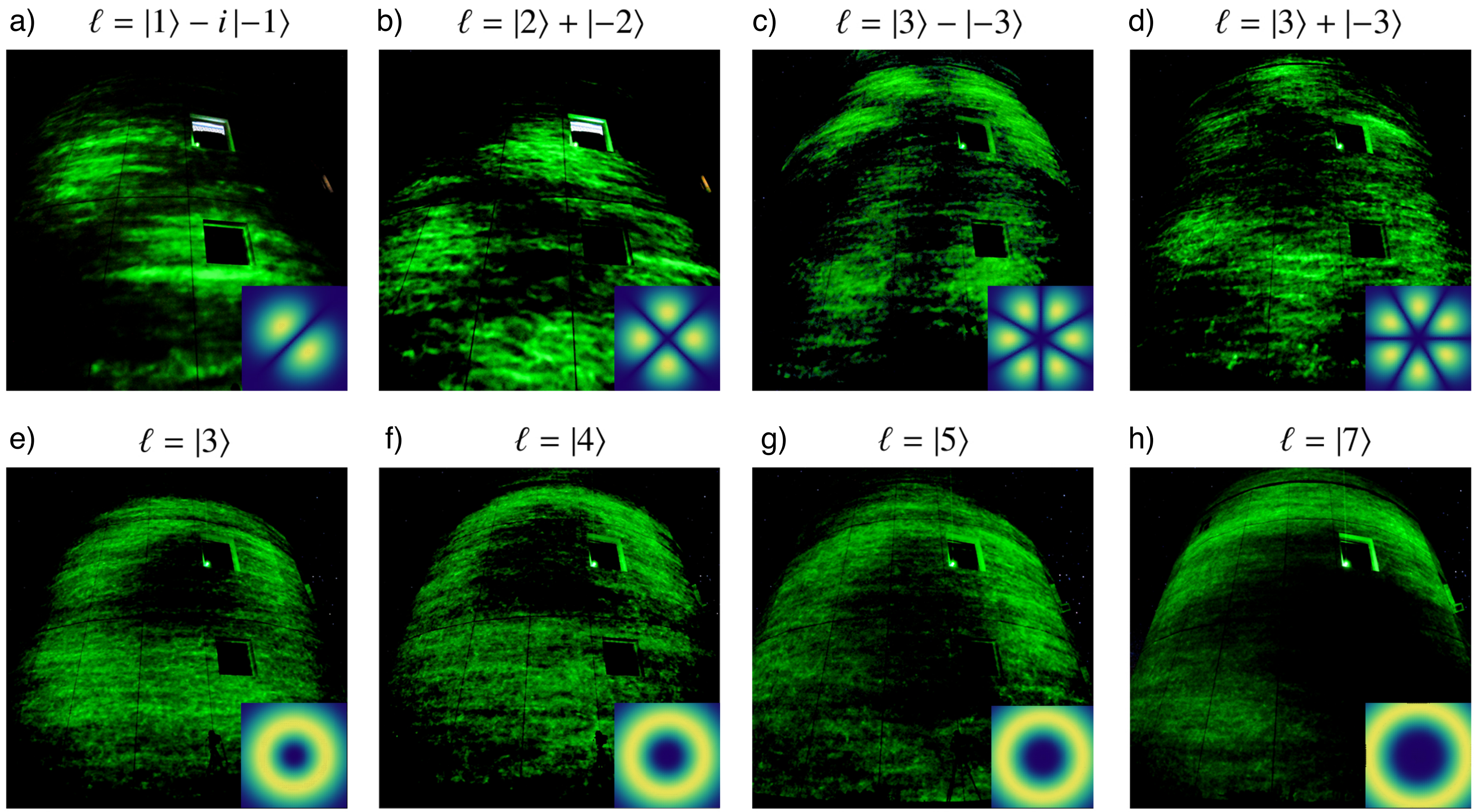


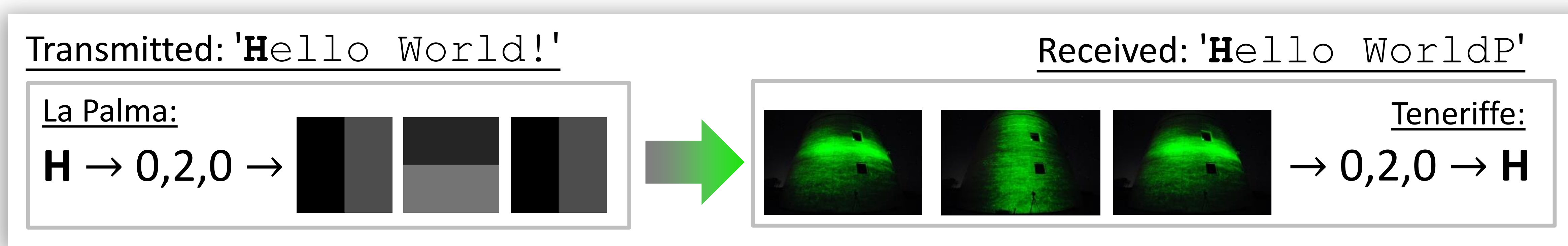


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- Theory: Certification Strategies, Non-Locality Tests

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