$$|\psi_\lambda\rangle = e^{-iG\lambda}|\psi_0\rangle = U_\lambda|\psi_0\rangle \qquad G = a^\dagger a,\ \hat{p},\ \hat{L}$$

$$\bar{C}[\{\pi\}] = \int d\lambda\, z(\lambda) \int d\gamma\, p(\gamma|\lambda)\, c(\gamma,\lambda)$$

$$\gamma = \gamma(x) \quad d\gamma = \frac{d\gamma}{dx}dx \qquad z(\lambda) \text{ broad } \simeq \text{uniform}$$

$$p(\gamma|\lambda) = Tr[\pi_\gamma \rho_\lambda]$$

covariance of $p(\gamma|\lambda)$

$$p(\gamma|\lambda+\lambda') = p(\gamma-\lambda'|\lambda)$$

$$Tr[\pi_\gamma \rho_{\lambda+\lambda'}] = Tr[\pi_{\gamma-\lambda'} \rho_\lambda]$$

$$\parallel$$

$$Tr[\pi_\gamma U_{\lambda'} \rho_\lambda U^\dagger_{\lambda'}] = Tr[U^\dagger_{\lambda'} \pi_\gamma U_{\lambda'} \rho_\lambda]$$

$$\pi_{\gamma-\lambda'} = U^\dagger_{\lambda'} \pi_\gamma U_{\lambda'} \qquad \boxed{\pi_\gamma = U_\gamma X_0 U_\gamma^\dagger} \qquad X_0 \geq 0, \quad \sum_\gamma \pi_\gamma = I$$

$$\boxed{c(\gamma,\lambda) = c(\gamma-\lambda)} \qquad \text{e.g. } (\gamma-\lambda)^2$$

$$\begin{aligned}
C[\{\pi\}] &= \iint d\lambda\, d\gamma\, c(\gamma-\lambda)\, Tr[U_\lambda \rho_0 U_\lambda^\dagger U_\gamma X_0 U_\gamma^\dagger]\, z(\lambda) \\
&= \int d\lambda\, z(\lambda) \int d\gamma\, c(\gamma-\lambda)\, Tr[\rho_0 \pi_{\gamma-\lambda}] \\
&= \int dx\, c(x)\, Tr[\rho_0 \pi_x] = Tr\left[\rho_0 \int dx\, c(x) \pi_x\right]
\end{aligned}$$

Now, thanks to a theorem by Holevo we may focus on $C(x) = -\delta(x)$. The corresponding solution is valid also for a broad class of <u>even</u> cost function (and also periodic)

$$C[\{\pi\}] \equiv C[X_0] = -\mathrm{Tr}[\rho_0 X_0] = -\langle\psi_0|X_0|\psi_0\rangle$$

We are thus left with the problem of maximizing

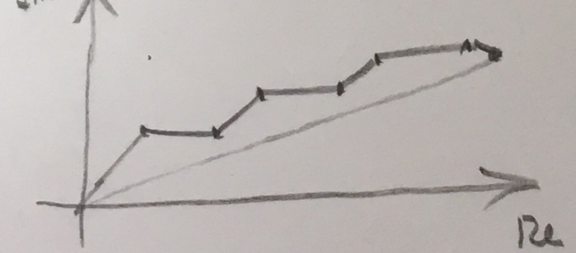
$$\langle\psi_0|X_0|\psi_0\rangle = \sum_{nk} \langle\psi_0|g_n\rangle\langle g_n|X_0|g_k\rangle\langle g_k|\psi_0\rangle$$

$$\downarrow$$

$$\sum_s x_s |\langle\psi_0|\chi_s\rangle|^2 \longrightarrow X_0 = |\chi_0\rangle\langle\chi_0|$$

$$= \left|\sum_n \langle\psi_0|g_n\rangle\langle g_n|\chi_0\rangle\right|^2$$

modulus of the sum of complex numbers



max if they are all reals and with maximum modulus

$$|\langle g_n|\chi_0\rangle| = 1$$

$$\arg\langle g_n|\chi_0\rangle = -\arg\langle\psi_0|g_n\rangle \equiv -\vartheta_n$$

$$\Pi_\gamma = e^{-i\hat{G}\gamma} X_0 \, e^{i\hat{G}\gamma}$$

$$= \sum_{mk} e^{-i\gamma(g_m - g_k)} \langle g_m | X_0 | g_k \rangle \; |g_m\rangle\langle g_k|$$

$$\Pi = \int \Pi_\gamma \, d\gamma = \sum_m \underbrace{\langle g_m | X_0 | g_m \rangle}_{=1} \; |g_m\rangle\langle g_m|$$

$$|X_0\rangle = \sum_m e^{-i\vartheta_m} |g_m\rangle \qquad \vartheta_m = \arg \langle \psi_0 | g_m \rangle$$

$$\Pi_\gamma = \sum_{mk} \underbrace{e^{-i\gamma(g_m - g_k)}}_{\text{covariance}} \; \underbrace{e^{-i(\vartheta_m - \vartheta_k)}}_{\text{initial state}} \; \underbrace{|g_m\rangle\langle g_k|}_{\text{generator}}$$