Quantum simulation by quantum annealing

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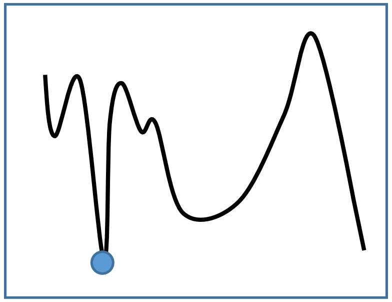
Quantum Annealing

Original target:

To solve classical combinatorial optimization problems.



Method:

Appropriate control of coefficients of the transverse-field Ising model

$$H(t) = -\sum_{i,j} J_{ij}(t)\sigma_i^z\sigma_j^z - \Gamma(t)\sum_i \sigma_i^x$$

An additional target:

Quantum simulation (experiment of quantum systems)

Recent examples of quantum simulation by quantum annealing

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Static (equilibrium) properties

- Spin glass in 3 dimensions — *Harris et al., Science (2018)*
- Kosterlitz-Thouless transition — *King et al., Nature (2018)* ⟵ To be reviewed
- Spin ice — *King et al., arXiv (2020)*
- Shastry-Sutherland model — *Kairys et al., arXiv (2020)* ⟵ To be reviewed
- Field theory — *Abel et al., arXiv (2020)*
- Z_2 lattice gauge theory — *Zhou et al., arXiv (2020)*
- Griffiths-McCoy singularity — *Nishimura et al., Phys. Rev. A (2020)* ⟵ To be explained

Dynamical (non-equilibrium) properties

- Kibble-Zurek mechanism — *Gardas et al., Sci. Rep. (2018)* *1d*
  - *Weinberg et al., Phys. Rev. Lett. (2020)* *2d*
- Generalized Kibble-Zurek — *Bando et al., Phys. Rev. Res. (2020)* *1d* ⟵ To be explained

Review (1)

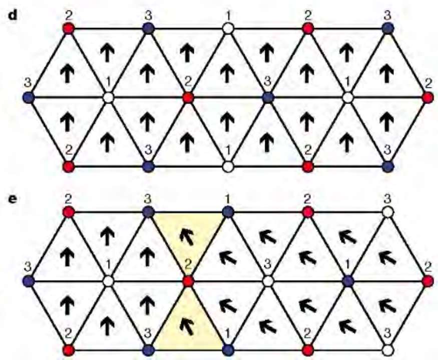
Quantum simulation of the Kostertlitz-Thouless transition

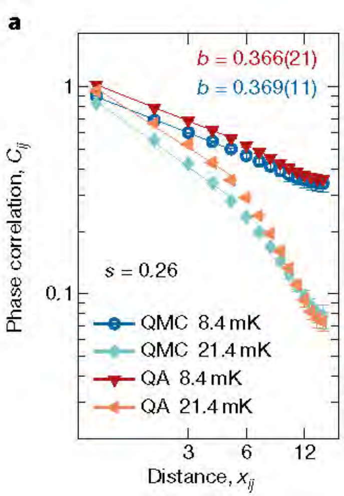
A. D. King et al., *Nature* ***560***, *456 (2018)*

Kosterlitz-Thouless transition

A. D. King et al., Nature ***560****, 456 (2018)*

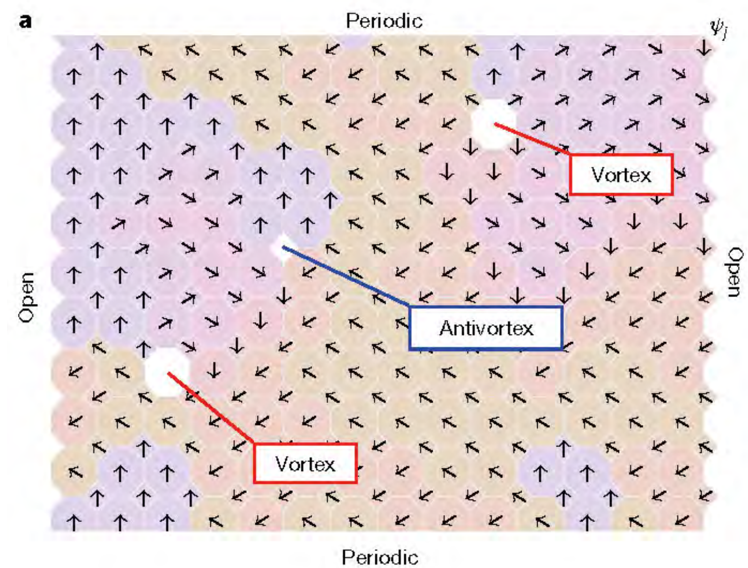
Representation of Z_6 model by the frustrated Ising model embedded on the Chimera graph of D-Wave

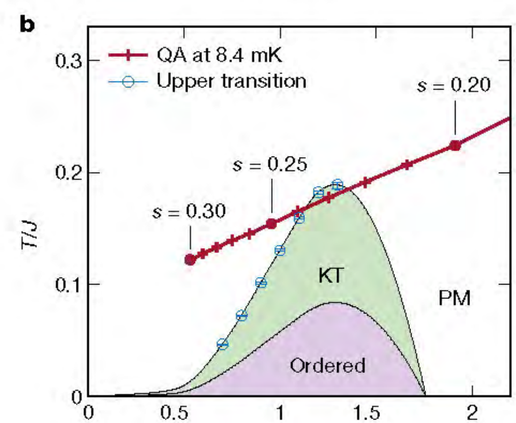




Power decay of correlation, a hallmark of the KT transition

Observation of vortex-antivortex pairs in some parameter range





Review (2)

Quantum simulation of the Shastry-Sutherland model

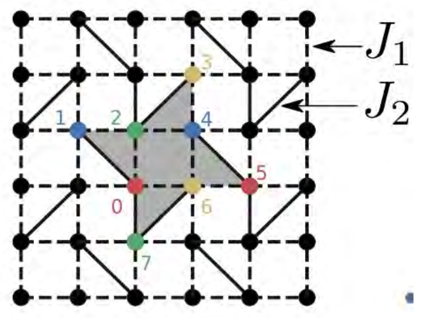
P. Kairys, A. D. King, I. Ozfidan, K. Boothby, J. Raymond, A. Banerjee, T. S. Humble,
arXiv:2003.01019

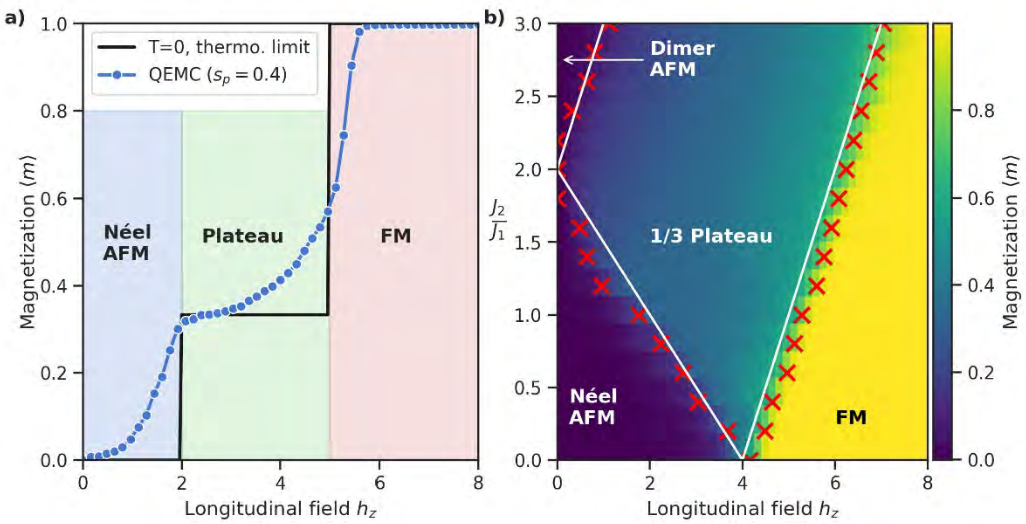
Shastry-Sutherland model

P. Kairys et al., arXiv:2003.01019

Ground-state phase diagram of a frustrated classical Ising model on the square lattice

$$H = J_1 \sum_{\langle i,j \rangle} \sigma^z_{(i)} \sigma^z_{(j)} + J_2 \sum_{\langle\langle i,j \rangle\rangle} \sigma^z_{(i)} \sigma^z_{(j)} + h_z \sum_i \sigma^z_{(i)}$$





Magnetization plateau observed.
Phase diagram confirmed.

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Our contribution (1)

Quantum simulation of the generalized Kibble-Zurek mechanism

Y. Bando, Y. Susa, H. Oshiyama, N. Shibata, M. Ohzeki, F. Gómez-Ruiz, D. A. Lidar, S. Suzuki, A. del Campo, and H. Nishimori, *Phys. Rev. Res.* **2**, *033369 (2020)*

cf. Talk by Y. Bando in Session 3 for additional information

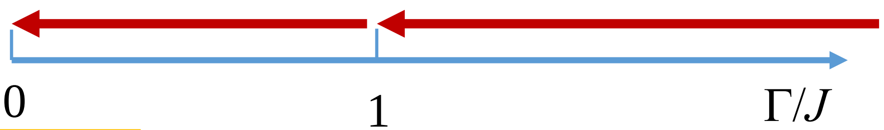
Supported by IARPA QEO / DARPA QAFS

Kibble-Zurek mechanism

Kibble (1976), Zurek (1985)

Quantum phase transition of the 1d transverse-field Ising model

$$H = -J\sum_{i=1}^{L}\sigma_i^z\sigma_{i+1}^z - \Gamma\sum_{i=1}^{L}\sigma_i^x \quad (J>0)$$

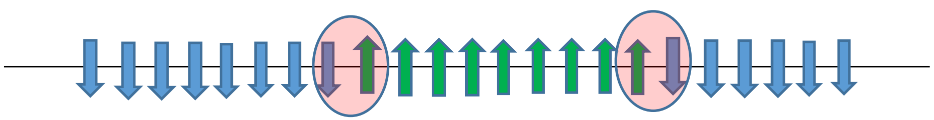


$|\uparrow_1\uparrow_2\uparrow_3\cdots\uparrow_{L-1}\uparrow_L\rangle$

$|\downarrow_1\downarrow_2\downarrow_3\cdots\downarrow_{L-1}\downarrow_L\rangle$

$|\uparrow_1\uparrow_2\uparrow_3\cdots\uparrow_L\rangle + |\uparrow_1\uparrow_2\uparrow_3\cdots\downarrow_L\rangle + \cdots + |\downarrow_1\downarrow_2\downarrow_3\cdots\downarrow_L\rangle$

Defect (kink)



When we change Γ/J at a finite speed, the system goes out of equilibrium (diabatic). Defects are created in the final state at $\Gamma/J=0$.

Problem: Set a quantitative measure of diabaticity.
How many defects (n) are created as a function of the annealing time (t_a)?

n as a function of t_a

Prediction of the Kibble-Zurek theory

$$\frac{n}{L} = \rho \propto (t_a)^{-\frac{d\nu}{1+\nu z}}$$

d: system dimension
n: average of the number of kinks
L: chain length
t_a: elapsed time =1/ (speed of parameter change)

$$\xi \sim (\Gamma - \Gamma_c)^{-\nu}$$
$$\tau \sim (\Gamma - \Gamma_c)^{-\nu z}$$

1d ferromagnetic transverse-field Ising model

$\rho \propto t_a^{-0.5}$ Isolated system: $\nu = z = 1$

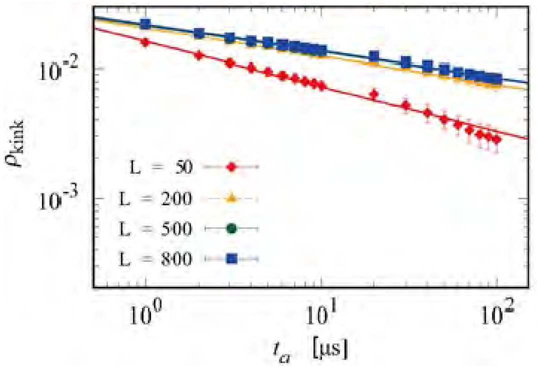
$\rho \propto t_a^{-0.28}$ Under bosonic environment: $\nu = 0.63$, $z = 1.98$ (*Werner et al., 2005*)

$$H = -J\sum_{i=1}^{N_x} \sigma_i^z \sigma_{i+1}^z - \Delta \sum_{i=1}^{N_x} \sigma_i^x + \sum_{i,k}\{C_k(a_{i,k}^\dagger + a_{i,k})\sigma_i^z + \omega_{i,k} a_{i,k}^\dagger a_{i,k}\},$$

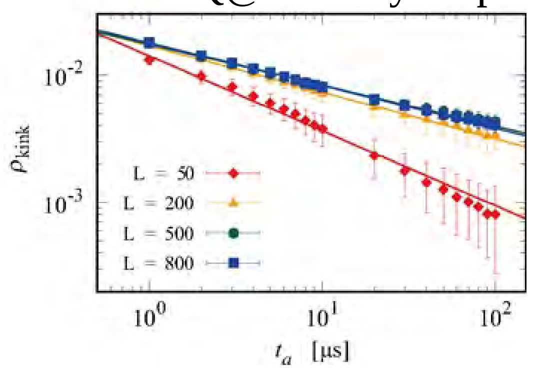
Result

Average number of kinks n

DW2000Q@NASA : exponent 0.20



DW2000Q@Burnaby: exponent 0.34

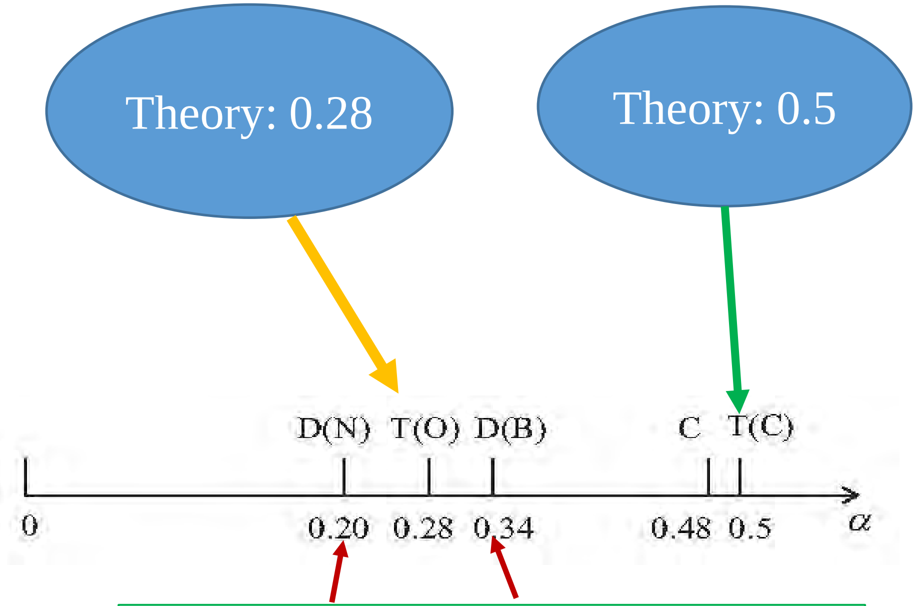


Data for L=50 are excluded because they don't satisfy the condition for KZ theory to be valid, n>1.

Exponent for the average n

Under bosonic environment

Isolated system



Experiment 0.20(NASA), 0.34(Burnaby)

$$H = -J\sum_{i=1}^{N_x}\sigma_i^z\sigma_{i+1}^z - \Delta\sum_{i=1}^{N_x}\sigma_i^x + \sum_{i,k}\{C_k(a_{i,k}^\dagger + a_{i,k})\sigma_i^z + \omega_{i,k}a_{i,k}^\dagger a_{i,k}\},$$

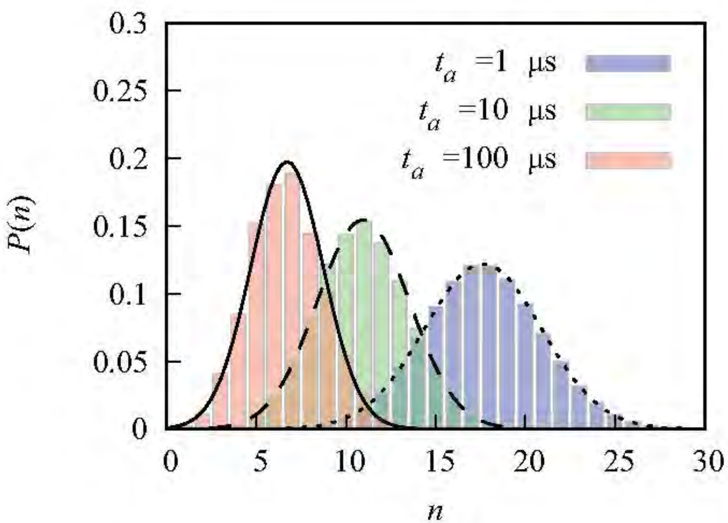
Deviations would come from non-universal effects.

Result

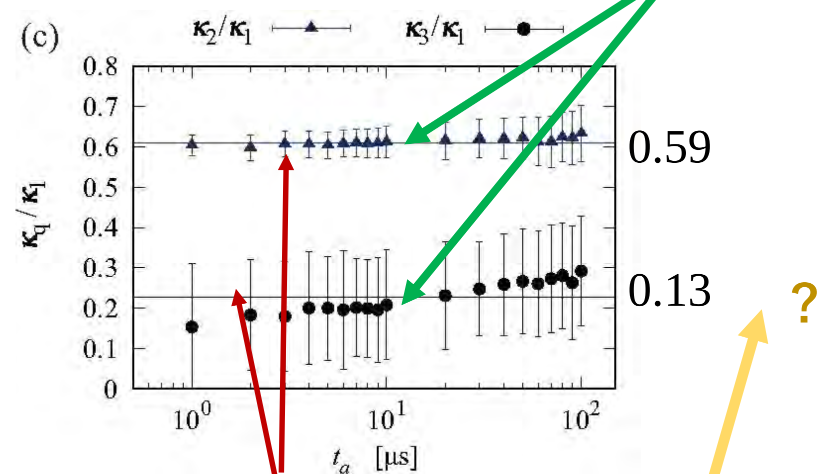
Generalized KZ theory on the distribution of *n*

Adolfo del Campo (2018)

Distribution of the number of kinks



Theoretical prediction for isolated system

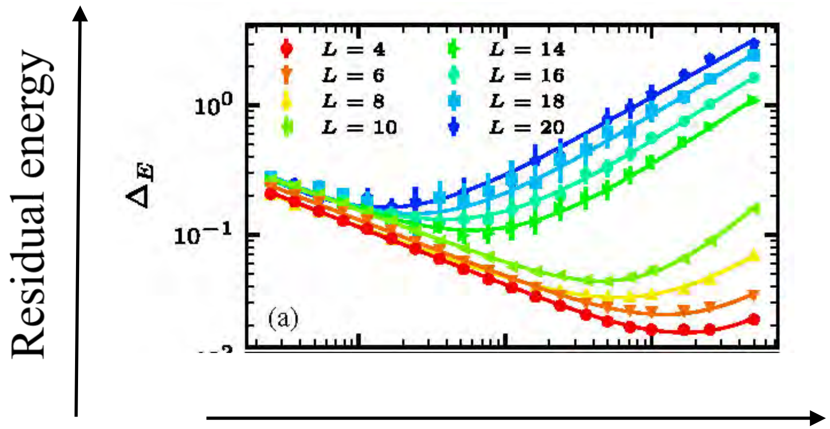


Experiments ▲,●

No theory for systems under bosonic environment

- Our quantum simulation (experiment) predicts that the theorical prediction for isolated system should remain valid even under bosonic environment.
- Prompting theoretical development for confirmation.
- This is (probably) the first time that quantum simulation has gone ahead of theory.

Related study
2*d* square lattice



Speed of change v =(annealing time t_a)$^{-1}$
(the other way round from our notation)

Weinberg et al.,
PRL ***124****, 090502(2020)*

$$f(v) = a_L v^{\alpha} + b_L v^{-\beta}$$

Kibble-Zurek

Experiment α=0.74 ± 0.02

Theory for isolated system

α=0.77

Bath

$$V_{\text{noise}}(t) = \lambda \sum_i \eta_i(t) \sigma_i^x,$$

Confirmed qualitatively for 1d numerically

Necessary:
- Theory for 2d system under bosonic environment.
- To understand why 1d has no minimum but 2d has.

Our contribution (2)

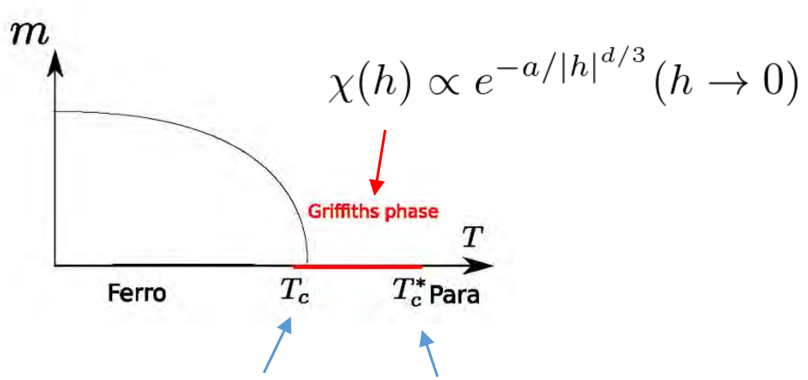
Quantum simulation of the Griffiths-McCoy singularity

Kohji Nishimura, H. Nishimori, Helmut G. Katzgraber,
Phys. Rev. A (to be published), arXiv:2006.16219

Supported by IARPA QEO / DARPA QAFS

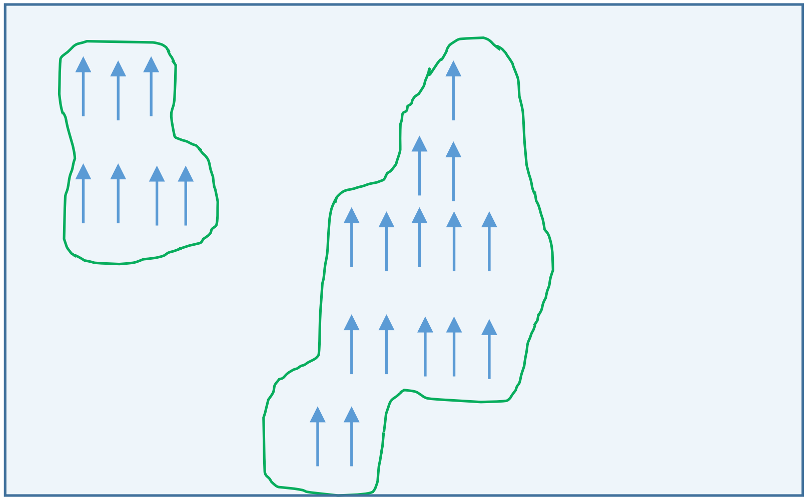
Griffiths-McCoy singularity

Problem: Does the Griffiths-McCoy singularity exist on the Chimera graph?



Tc for randomly diluted system

Tc for non-diluted system



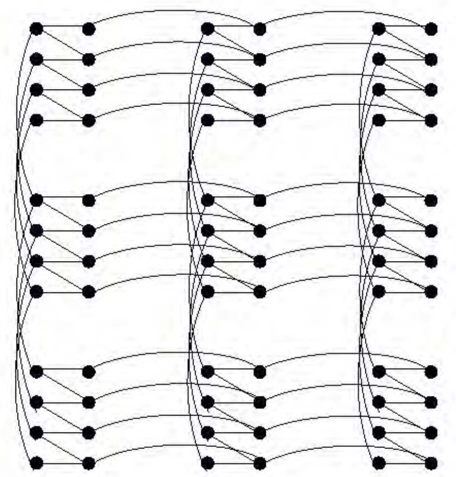
Ising ferromagnet on a diluted lattice

- Large (but very rare) clusters respond very strongly to an external field even in the paramagnetic phase for T_c(diluted) < T < T_c(non-diluted). → (Weak) singularity in χ at h=0.
- This is enhanced in low-dimensional quantum spin systems, resulting in divergence of non-linear susceptibility.

GM singularity on diluted Chimera graph

- Numerical and experimental studies show evidence of the Griffiths-McCoy singularity in low-dimensional quantum magnets.
- But studies of the 2d randomly-diluted ferromagnet have been rare. Mostly spin glass.
- There has been no study for the Chimera graph.
- There has been no study by quantum simulation on quantum device.

Regularly-diluted Chimera graph with smaller connectivity
→ Stronger Griffiths-McCoy singularity if randomness is introduced.



Randomly diluted ferromagnet

$$P(J_{ij}) = \frac{1}{6}\sum_{k=0}^{5}\delta(J_{ij}+0.2k),$$

Analysis and result

Non-linear susceptibility
(stronger singularity than linear)

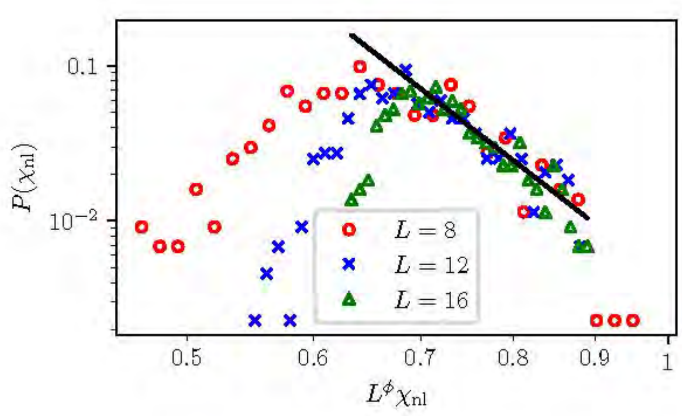
$$\chi_{\rm nl} = \left.\frac{\partial^3 m}{\partial h^3}\right|_{h=0} = -(\langle m^4\rangle - 3\langle m^2\rangle^2)$$

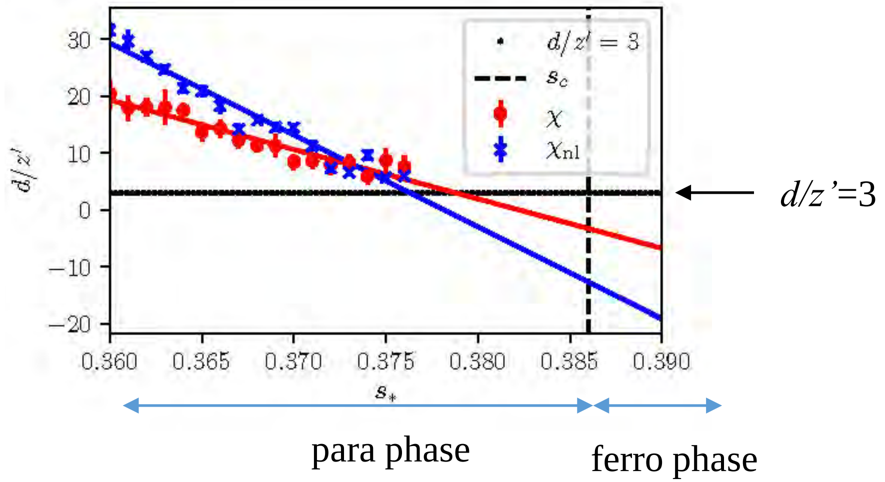
Distribution over samples and local environments

$$P(\chi_{\rm nl}) \sim \chi_{\rm nl}^{\;-(d/(3z')+1)}$$

Average diverges if $d/z' < 3$

$$\langle\chi_{\rm nl}\rangle = \int^{\infty} \chi_{\rm nl} P(\chi_{\rm nl}) d\chi_{\rm nl} \sim \int^{\infty} \chi_{\rm nl}^{-d/(3z')} d\chi_{\rm nl}$$





- Quantum simulation has shown: non-linear susceptibility is likely to diverge in the para phase.
- Consistent with the existence of the Griffiths-McCoy singularity on the Chimera graph.
- Backed up by classical simulations ("quantum" Monte Carlo)

Conclusion

- Non-equilibrium (dynamical) phenomenon of Kibble-Zurek mechanism and its generalization.
  Quantum experiment has gone ahead of theory: A theory for qubits under environment should be developed.
- Equilibrium (static) phenomenon of the Griffiths-McCoy singularity.
  Nonlinear susceptibility has been shown to be likely to diverge even in the paramagnetic phase on the Chimera graph.
- These result motivate further quantum simulations on the existing and future quantum annealers for discoveries/verifications of new/existing physics.