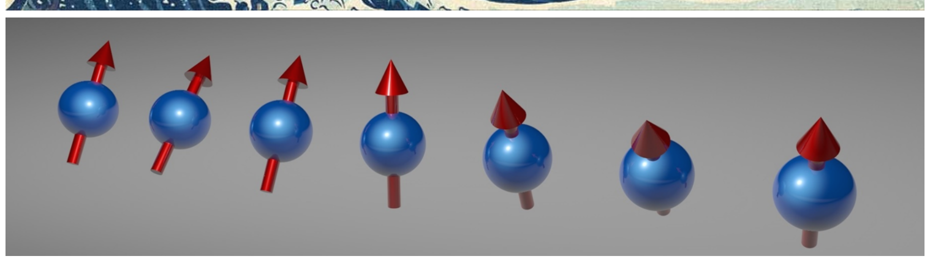
Logarithmic anomalies of spin transport in non-integrable magnets



UNIVERSITEIT GENT

fwo Research Foundation Flanders
Opening new horizons

Jacopo De Nardis

Collaboration with: E. Ilievski, M. Medenjak, B. Doyon, D. Bernard, C. Karrasch, R. Vasseur, S. Gopalakrishnan

THE NON-EQUILIBRIUM PROBLEM: DIFFUSION

Fick's law

$$\partial_t \rho(x,t) = \mathfrak{D} \Delta_x \rho(x,t)$$

How to derive it from microscopic reversible dynamics?

Exactly solvable Hamiltonian models exhibiting diffusion?

Where and when it fails?

Interacting integrable spin chains

INTEGRABLE MODELS: SUPER-DIFFUSION

$$(\mathfrak{D}C)_{ij} = \int dt \left(\int dx \langle j_i(x,t) j_j(0,0) \rangle - \mathcal{D}_{ij} \right)$$

Interacting (integrable)

$$(\mathfrak{D}C)_{ij} > 0$$

De Nardis, Bernard, Doyon, 2018

Gopalakrishnan, Huse, Khemani, Vasseur, 2018

De Nardis, Bernard, Doyon, 2019

Gopalakrishnan, Vasseur, 2019

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Some integrable chains at finite temperature

$$(\mathfrak{D}C)_{00} = \infty$$

Ilievski, De Nardis, Medenjak, Prosen, 2018

De Nardis, Bernard, Doyon, 2019

Gopalakrishnan, Vasseur, 2019

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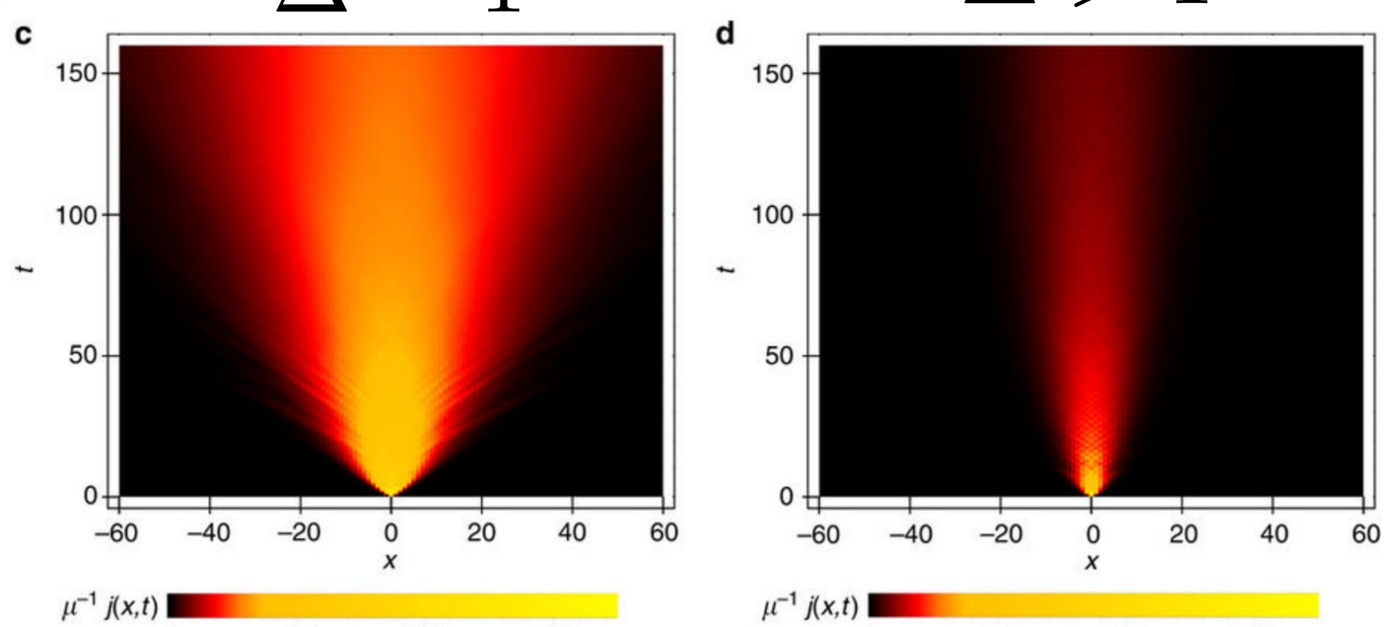
Some integrable chains at finite temperature

$$(\mathfrak{D}C)_{00} = \infty$$

Quantum integrable

$$H_{\mathrm{XXZ}} = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + (\Delta - 1) S_j^z S_{j+1}^z$$

$\Delta = 1$ $\Delta > 1$



$\mathfrak{D}_{\mathrm{spin}} \sim t^{1/3}$ $\mathfrak{D}_{\mathrm{spin}} \to \mathrm{const}$

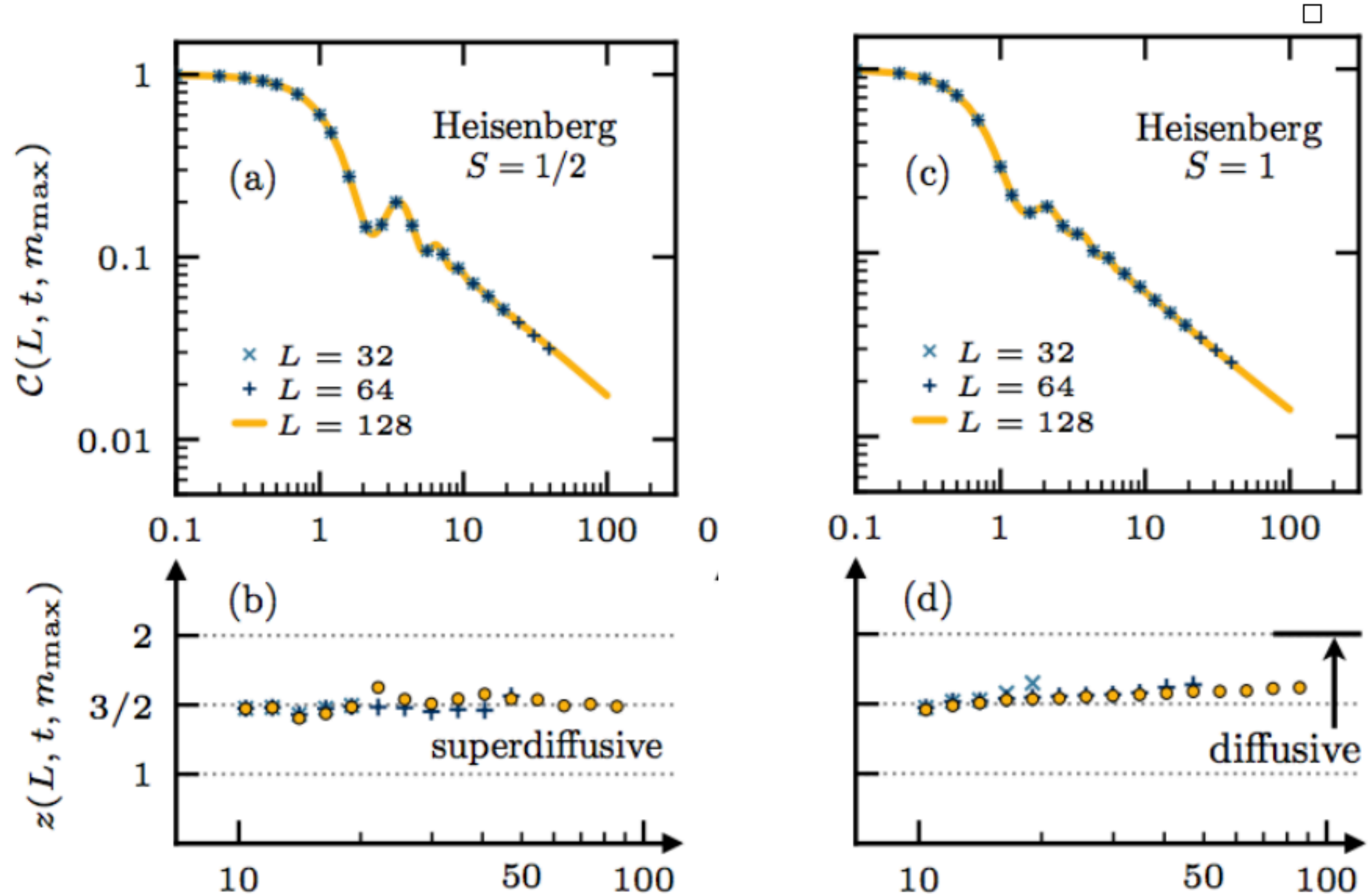
Ljubotina, Znidaric, Prosen, 2018

AND NON-INTEGRABLE CHAINS?

$$H_S = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$$

Quantum SU(2) chains

$$C_{ss}(x,t) = \langle S^z_x(t) S^z_0(0) \rangle_{T=\infty} \qquad \langle S^z_0(t) S^z(0) \rangle \sim t^{-1/z}$$



De Nardis, Medenjak, Karrasch, Ilievski, 2019

Dupont, Moore, 2019

DIFFUSION AND SUPER-DIFFUSION IN CHAINS

Main Statements:

Quantum&Classical chains

1. In **integrable** chains there is usually coexistence of ballistic transport with diffusive spreading

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$$\partial_t \phi + \partial_x (\phi^n + \nu \partial_x \phi + \eta) = 0$$

Bulchandani, 2019

De Nardis, Medenjak, Karrasch, Ilievski, 2020

DIFFUSION AND SUPER-DIFFUSION IN CHAINS

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Bulchandani, 2019

De Nardis, Medenjak, Karrasch, Ilievski, 2020

$$\partial_t \phi + \partial_x (\phi^n + \nu \partial_x \phi + \eta) = 0$$

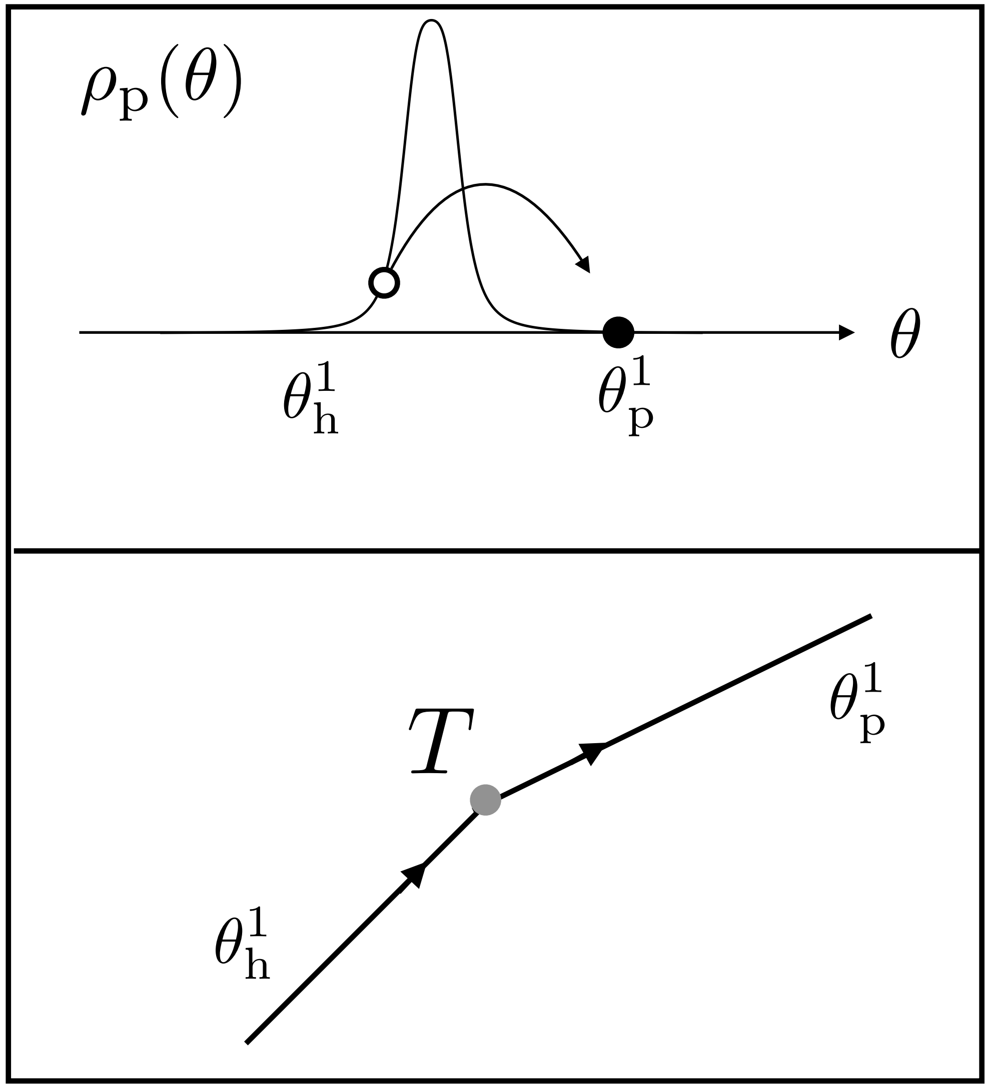
3. In **rotationally invariant non-integrable** chains the local magnetisation is also a Burgers' (KPZ) field but in a time-dependent noisy environment

De Nardis, Medenjak, Karrasch, Ilievski, 2020

$$\partial_t \phi + \partial_x (\phi^n + \nu \partial_x \phi + \gamma(t) \eta) = 0$$

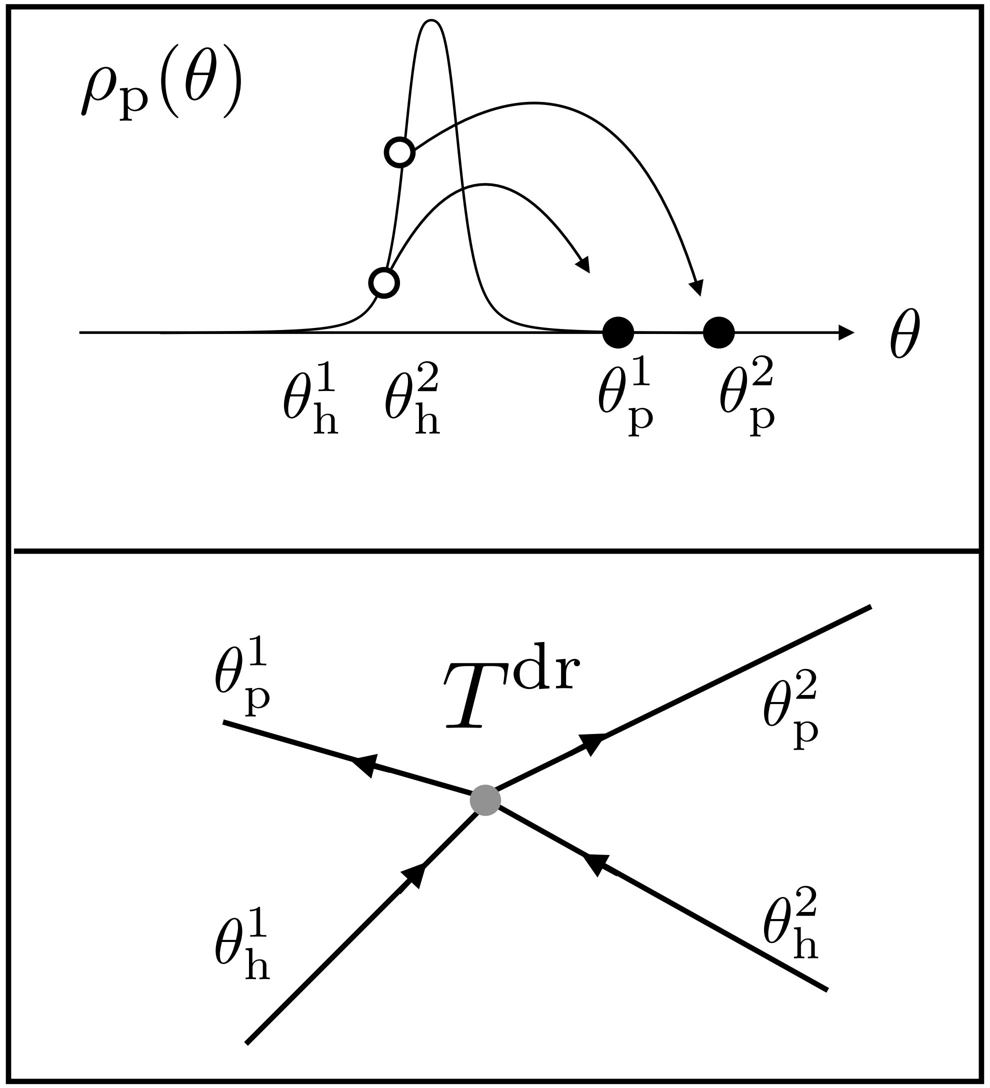
DIFFUSION IN INTEGRABLE SYSTEMS

1 particle-hole excitations



Ballistic motion: effective velocities

2 particle-hole excitations

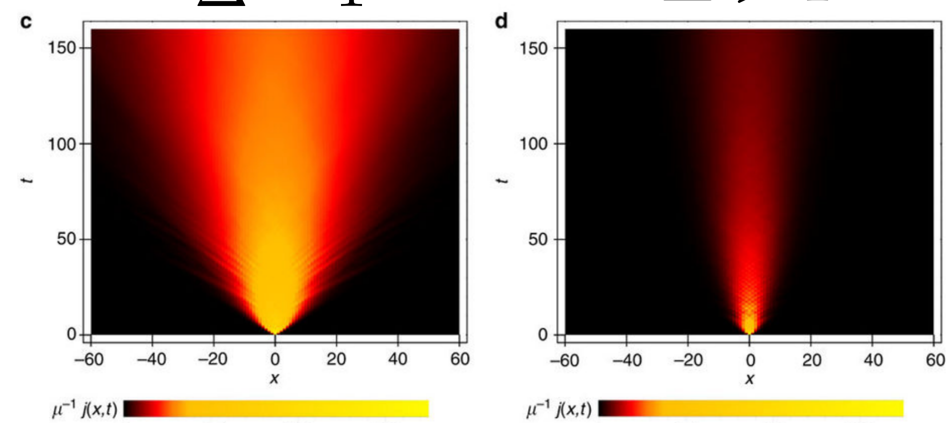


Diffusion spreading: diffusion constants

SPIN SUPER-DIFFUSION IN THE SPIN-1/2 XXX CHAIN

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$$

$\Delta = 1$ $\qquad$ $\Delta > 1$



$\mathfrak{D}_{\text{spin}} \sim t^{1/3}$ $\qquad$ $\mathfrak{D}_{\text{spin}} \to \text{const}$

$$\mathfrak{D}_{\text{spin}} = \sum_{s \geq 1} \overline{v_s^{\text{eff}}}\ \lambda_s$$

$$= \downarrow + \downarrow\downarrow + \downarrow\downarrow\downarrow + \cdots + \overbrace{\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow}^{s}$$

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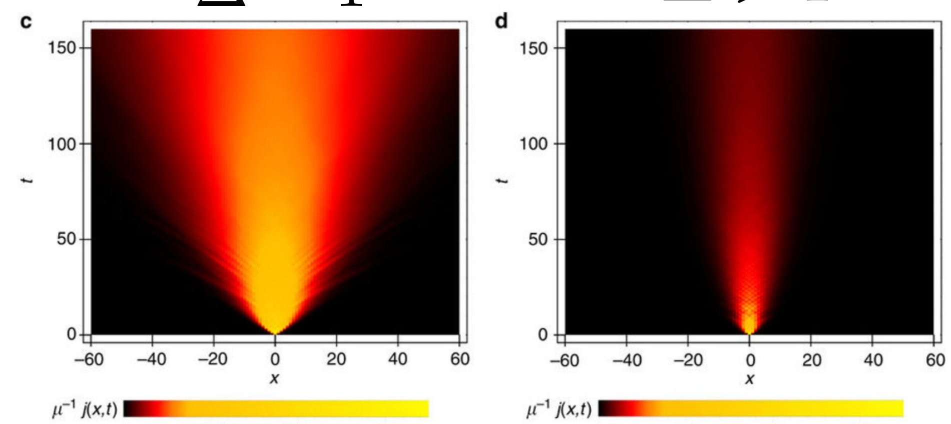

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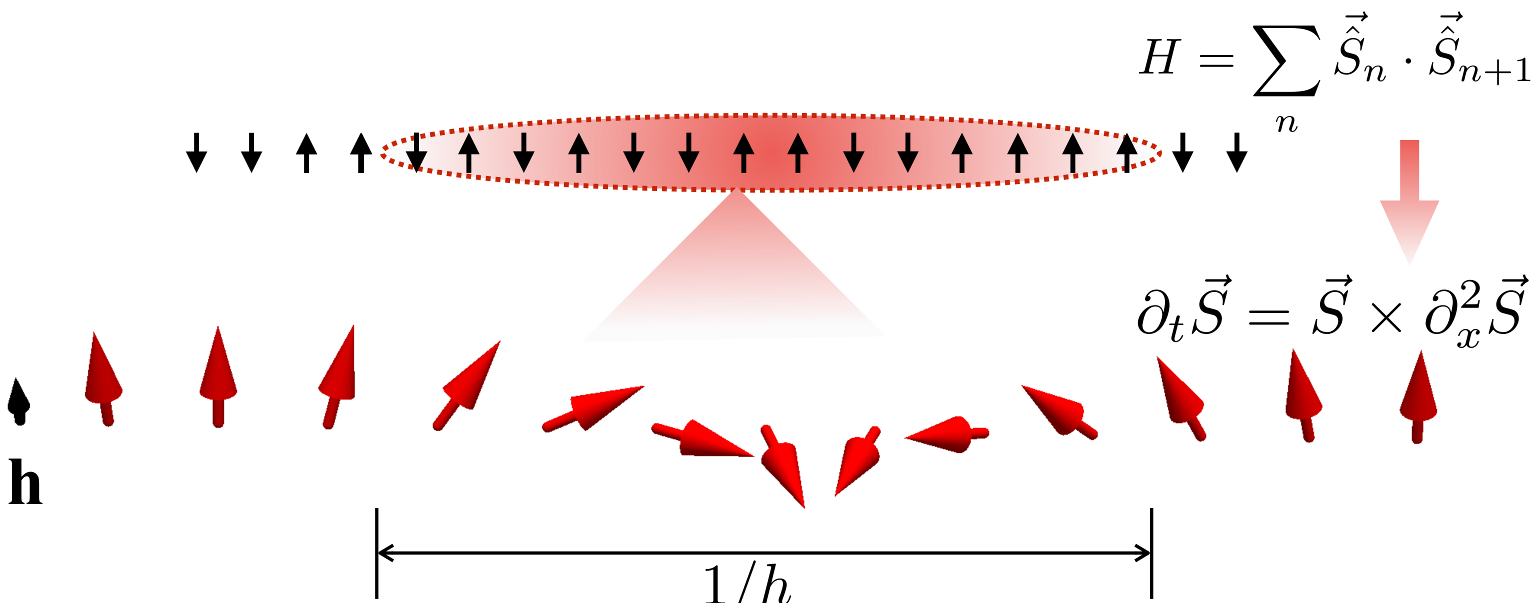
$$\lambda_s \sim s$$

$$\overline{v_s^{\rm eff}} \sim 1/s$$

$$\mathfrak{D}_{\rm spin} \sim s_{\rm max}$$

SPIN SUPER-DIFFUSION IN THE SPIN-1/2 XXX CHAIN



De Nardis, Gopalakrishnan, Ilievski, Vasseur, 2020

$$T_{s,s'}(\theta) \to T^{\text{giant}}_{\xi,\xi'}(u) = \frac{1}{\pi} \log \frac{4u^2 + (\xi + \xi')^2}{4u^2 + (\xi - \xi')^2}$$

$$H_{\text{LL}} = \frac{1}{2} \int dx \, \partial_x \vec{S}(x) \cdot \partial_x \vec{S}(x)$$

SPIN SUPER-DIFFUSION IN THE SPIN-1/2 XXX CHAIN

$$\partial_t s^z + \partial_x \left(\frac{\lambda_{\mathrm{KPZ}}}{2\sqrt{2}} (s^z)^2 + \gamma \partial_x s^z + \sqrt{2\gamma}\ \eta \right) = 0$$

$$\langle \hat{S}_n^z(t) \hat{S}_0^z(0) \rangle \simeq \frac{\chi}{(\lambda_{\mathrm{KPZ}}\, t)^{2/3}} f_{\mathrm{KPZ}} \left(\frac{n}{(\lambda_{\mathrm{KPZ}}\, t)^{2/3}} \right)$$

De Nardis, Gopalakrishnan, Ilievski, Vasseur, 2020

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$$\lambda_{\mathrm{KPZ}}(T) = 4 D_0(T) \sqrt{\chi(T)} / \sigma_{\mathrm{KPZ}}^{3/2}$$

Parameter of the emergent soliton thermal gas

SPIN SUPER-DIFFUSION IN THE SPIN-1/2 XXX CHAIN

$$\langle \hat{S}^z_n(t)\hat{S}^z_0(0)\rangle \simeq \frac{\chi}{(\lambda_{\rm KPZ}\, t)^{2/3}} f_{\rm KPZ}\left(\frac{n}{(\lambda_{\rm KPZ}\, t)^{2/3}}\right)$$

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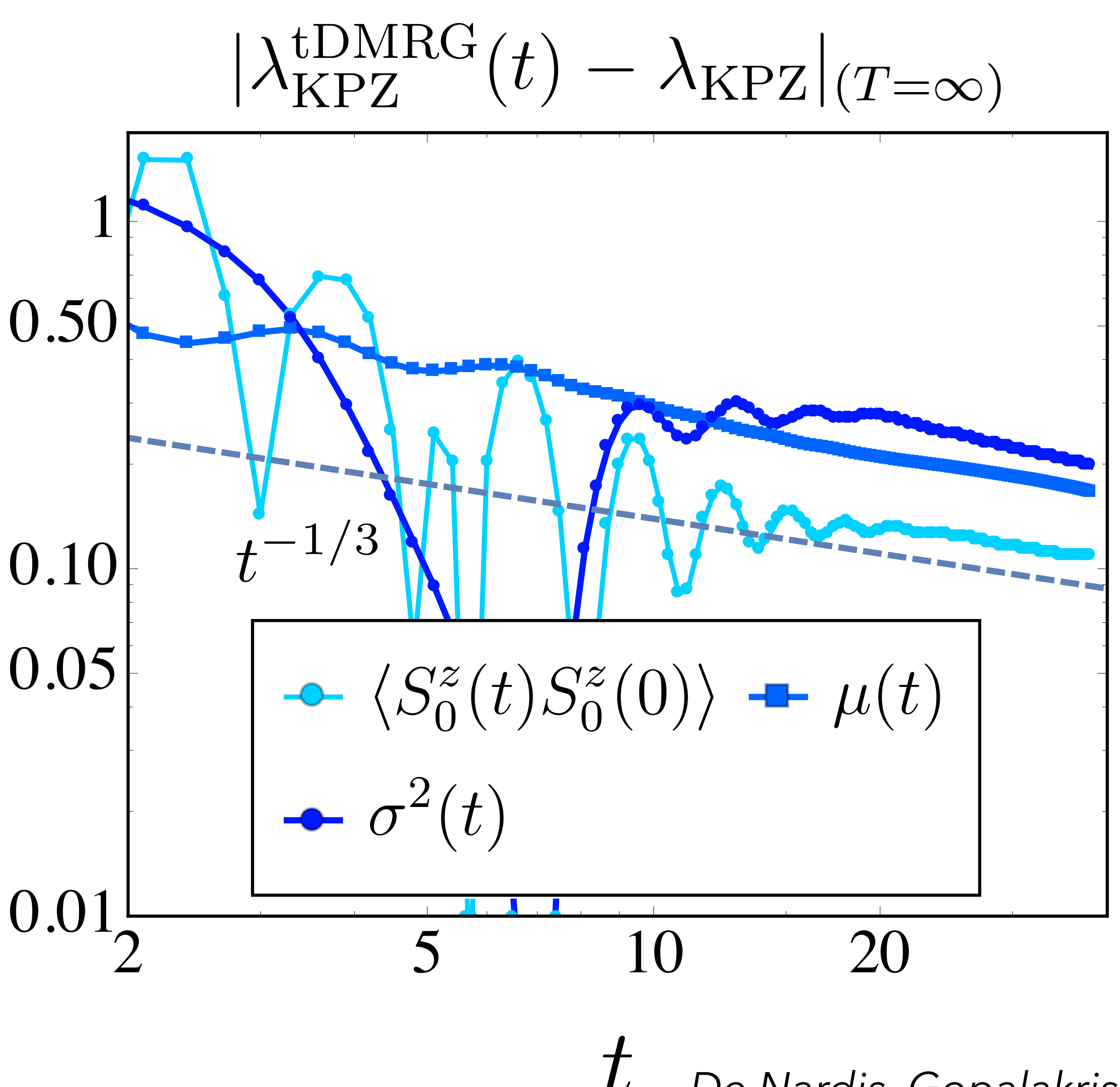
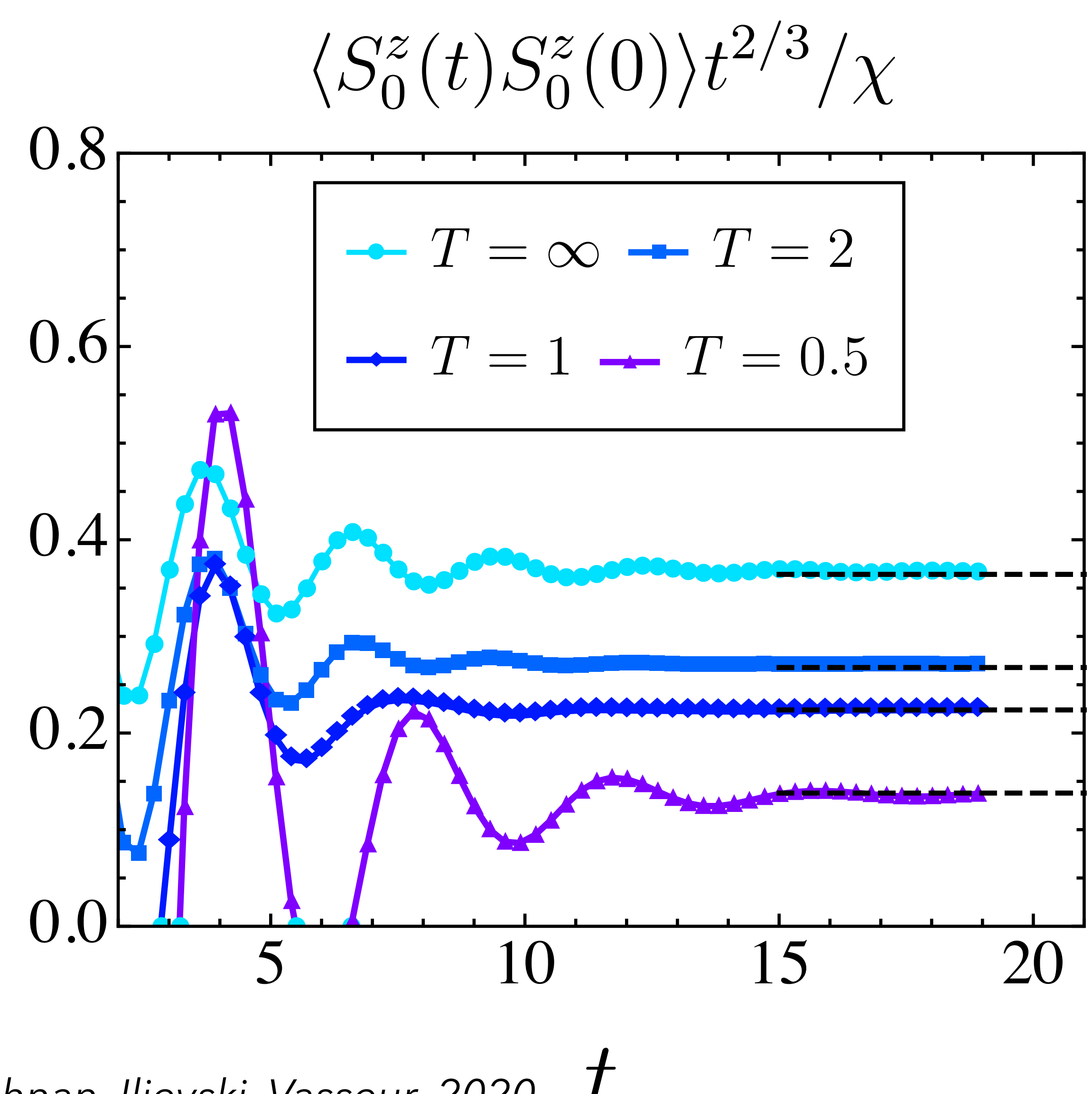




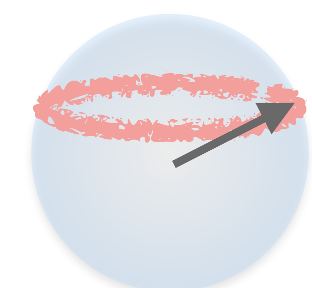
De Nardis, Gopalakrishnan, Ilievski, Vasseur, 2020

SPIN-FIELD THEORY OF ISOTROPIC MAGNETS

H

Semi-classical mean-field limit

$$\partial_t \vec{S} = \vec{S} \times \vec{B}[\vec{S}, \partial_x \vec{S}, \ldots, \partial_x^n \vec{S}] \qquad |\vec{S}| = 1$$



Frenet-Serret reference frame

$$\kappa^2 = \partial_x \vec{S} \cdot \partial_x \vec{S}$$

$$\tau = \frac{1}{\kappa^2} \vec{S} \cdot (\partial_x \vec{S} \times \partial_x^2 \vec{S})$$

$$\langle \delta S^z(x,t) \delta S^z(0,0) \rangle \sim \langle \delta \tau(x,t) \delta \tau(0,0) \rangle$$

$$\partial_t \tau + \partial_x (\lambda \tau^n + \sqrt{\gamma} \eta + D_\gamma \partial_x \tau) = \partial_x (O(\partial_x^{m_1} \tau^{n+2}, \partial_x^{m_2} \kappa^{2n}))$$

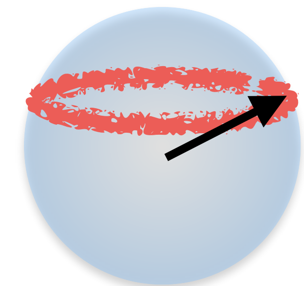
$$\langle \eta(x,t) \eta(0,0) \rangle = \delta(x - x') \delta(t - t')$$

Bulchandani, 2019

De Nardis, Medenjak, Karrasch, Ilievski, 2020

EFFECTIVE BURGERS' EQUATION

$$H[\vec{S}, \partial_x \vec{S}, \ldots, \partial_x^{n-1} \vec{S}]$$



$$\partial_t \tau + \partial_x (\lambda \tau^n + \sqrt{\gamma}\eta + D_\gamma \partial_x \tau) = 0$$

Role of integrability?

Integrable:

other modes are ballistic

$\gamma \sim \gamma_0$

Non-Integrable:

other modes are diffusive

$\gamma \sim \ell^{-1} \sim t^{-1/2}$

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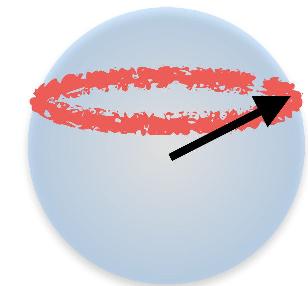

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Stochastic growth in time dependent environments

Guillaume Barraquand,[1] Pierre Le Doussal,[1] and Alberto Rosso[2]

[1] *Laboratoire de Physique de l'École Normale Supérieure, ENS, CNRS, Université PSL, Sorbonne Université, Université de Paris, 24 rue Lhomond, 75231 Paris, France*

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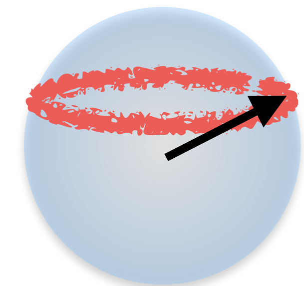
(Dated: October 1, 2019)

TRANSPORT IN SO(3) MAGNETS: CLASSIFICATION

$$H[\vec{S}, \partial_x \vec{S}, \ldots, \partial_x^{n-1} \vec{S}]$$



$$\partial_t \tau + \partial_x(\lambda \tau^n + \sqrt{\gamma}\eta + D_\gamma \partial_x \tau) = 0$$

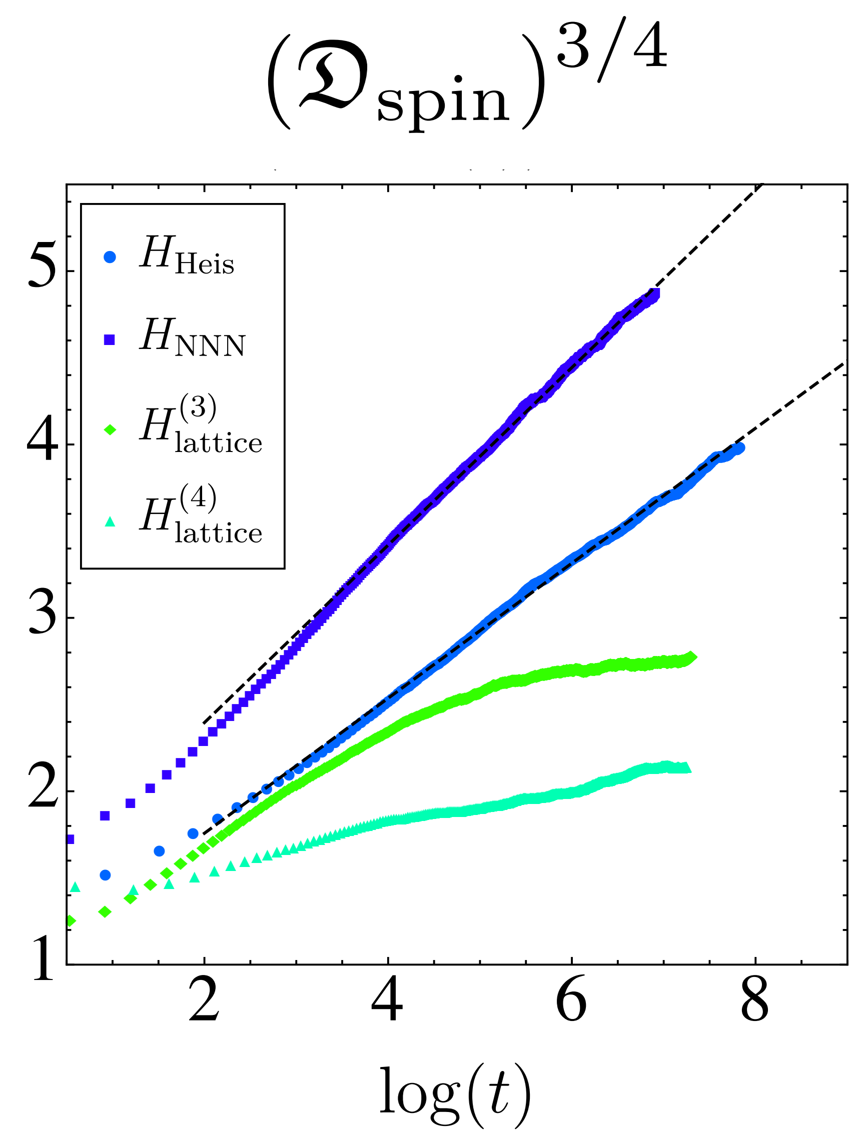
			relevant $n = 2$	marginal $n = 3$	irrelevant $n \geq 4$
integrable	$\gamma(t) \sim t^0$	ballistic $\zeta = 0$	**KPZ** $z = 3/2$, $b = 0$	$\mathbf{D}_{\log}^{(3)}$ $z = 2$, $b = 1/2$	**D** $z = 2$, $b = 0$
non-integrable	$\gamma(t) \sim t^{-1/2}$	diffusive $\zeta = 1/2$	$\mathbf{D}_{\log}^{(2)}$ $z = 2$, $b = 2/3$	**D** $z = 2$, $b = 0$	**D** $z = 2$, $b = 0$

$$\langle \vec{S}(0,t) \cdot \vec{S}(0,0) \rangle \sim t^{-1/z} (\log t)^{-b}$$

De Nardis, Medenjak, Karrasch, Ilievski, 2020

NON-INTEGRABLE SO(3) MAGNETS

		relevant $n=2$	marginal $n=3$	irrelevant $n \geq 4$
ballistic	$\zeta = 0$	**KPZ** $z = 3/2$, $b = 0$	$\mathbf{D}_{\log}^{(3)}$ $z = 2$, $b = 1/2$	**D** $z = 2$, $b = 0$
diffusive	$\zeta = 1/2$	$\mathbf{D}_{\log}^{(2)}$ $z = 2$, $b = 2/3$	**D** $z = 2$, $b = 0$	**D** $z = 2$, $b = 0$



$$C_0(t) = \langle S^z_{L/2}(t) S^z_{L/2}(0) \rangle$$

$$H_{\text{Heis}} = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$$

$$H_{\text{NNN}} = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + \alpha \vec{S}_j \cdot \vec{S}_{j+2}$$

$$C_0(t) \sim t^{-1/2} (\log t)^{-2/3}$$

$$H^{(3)} = \sum_j \vec{S}_j \cdot (\vec{S}_{j+1} \times \vec{S}_{j+2})$$

$$H^{(4)} = \sum_j \vec{S}_j \cdot (\vec{S}_{j+1} \times (\vec{S}_{j+2} \times \vec{S}_{j+4}))$$

$$C_0(t) \sim t^{-1/2}$$

INTEGRABLE SU(2) QUANTUM CHAINS

$$H^{(2)} = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$$

$$H^{(3)} = \sum_j \vec{S}_j \cdot (\vec{S}_{j+1} \times \vec{S}_{j+2})$$

		relevant $n=2$	marginal $n=3$	irrelevant $n \geq 4$
ballistic	$\zeta = 0$	**KPZ** $z = 3/2$ $b = 0$	$\mathbf{D}^{(3)}_{\log}$ $z = 2$ $b = 1/2$	**D** $z = 2$ $b = 0$
diffusive	$\zeta = 1/2$	$\mathbf{D}^{(2)}_{\log}$ $z = 2$ $b = 2/3$	**D** $z = 2$ $b = 0$	**D** $z = 2$ $b = 0$

$$H^{(4)} = \sum_j 2\vec{S}_j \cdot (\vec{S}_{j+1} \times (\vec{S}_{j+2} \times \vec{S}_{j+3})) + 2\vec{S}_j \cdot \vec{S}_{j+2} - 4\vec{S}_j \cdot \vec{S}_{j+1}$$

Here we can use
the exact expression
for the diffusion constant

$$\mathfrak{D}_{\text{spin}} = \sum_{s \geq 1} \int \frac{dk}{2\pi} n_s(k)(1 - n_s(k)) |v_s^{\text{eff}}(k)| (d_s^{\text{dr}})^2$$

$$\partial_t \tau + \partial_x (\lambda \tau^n + \sqrt{\gamma} \eta + D_\gamma \partial_x \tau) = 0$$

INTEGRABLE SU(2) QUANTUM CHAINS

$$H^{(2)} = \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$$

$$H^{(3)} = \sum_j \vec{S}_j \cdot (\vec{S}_{j+1} \times \vec{S}_{j+2})$$

		relevant $n=2$	marginal $n=3$	irrelevant $n \geq 4$
ballistic	$\zeta = 0$	**KPZ** $z = 3/2$ $b = 0$	$\mathbf{D}_{\log}^{(3)}$ $z = 2$ $b = 1/2$	**D** $z = 2$ $b = 0$
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$$\partial_t \tau + \partial_x(\lambda \tau^n + \sqrt{\gamma}\eta + D_\gamma \partial_x \tau) = 0$$

Derrida, Lebowitz, Speer, Spohn, 1991

$$\mathfrak{D}_{\rm spin}^{(2)} \sim t^{1/3} \qquad \mathfrak{D}_{\rm spin}^{(3)} \sim \log t \qquad \mathfrak{D}_{\rm spin}^{(n>3)} \sim {\rm const}$$

NON-INTEGRABLE SU(2) QUANTUM CHAINS

$$H = \sum_j \vec{S}_j \cdot \vec{S}_{j+1} \qquad S = 1$$

$$H_\delta = H_{S=1} + \delta S^z_j S^z_{j+1}$$

		relevant $n = 2$	marginal $n = 3$	irrelevant $n \geq 4$
ballistic	$\zeta = 0$	**KPZ** $z = 3/2$, $b = 0$	$\mathbf{D}^{(3)}_{\log}$ $z = 2$, $b = 1/2$	**D** $z = 2$, $b = 0$
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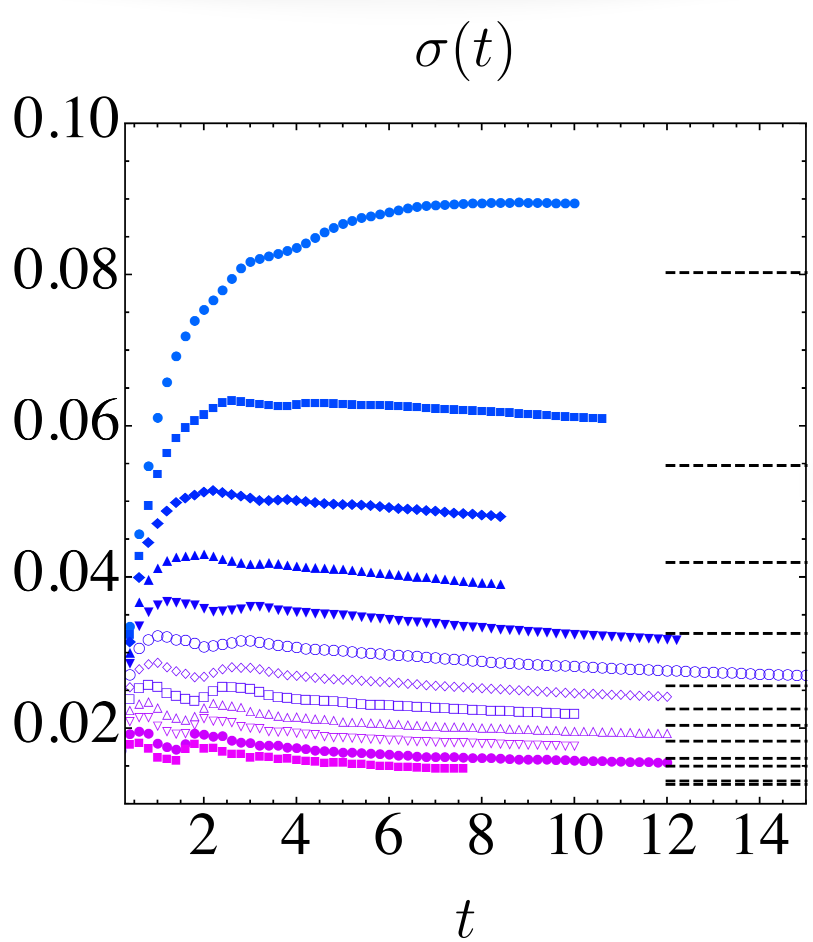

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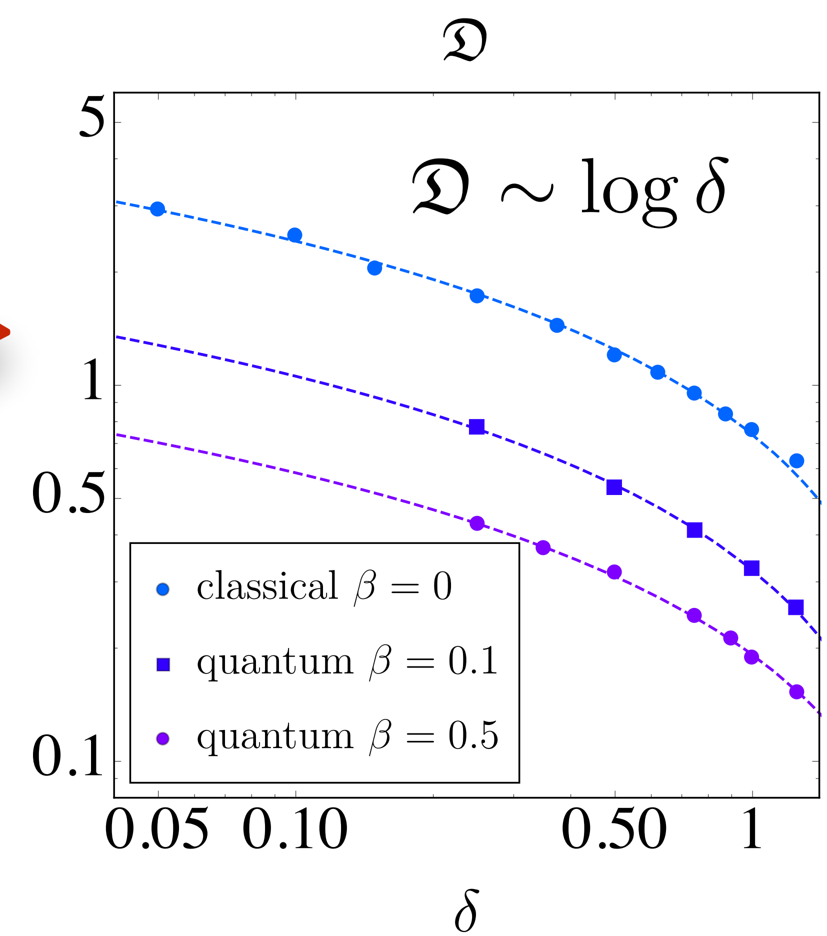
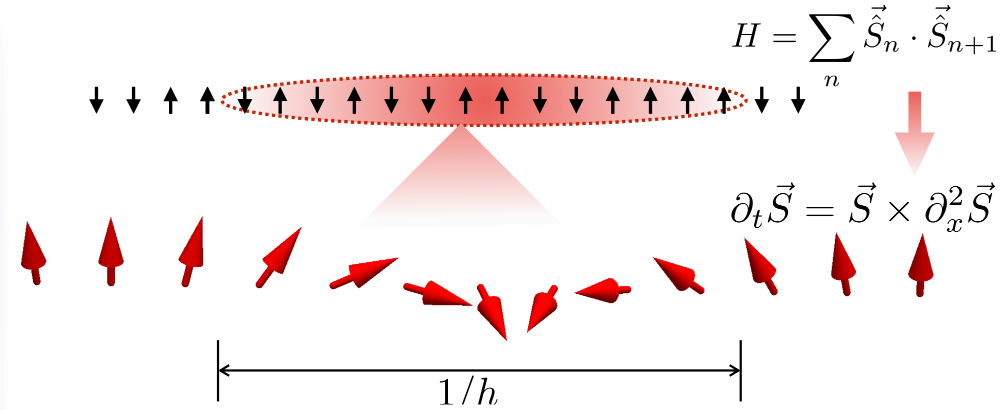


Conclusions

Anomalous KPZ transport in **isotropic quantum chains** is related to the emergent soliton gas (goldstone modes)



A **full classifications** of different types of transport is achieved by examining simply the torsion mode and the presence/absence of ballistic modes

	relevant $n = 2$	marginal $n = 3$	irrelevant $n \geq 4$
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diffusive $\zeta = 1/2$	$\mathbf{D}^{(2)}_{\log}$ $z = 2$, $b = 2/3$	**D** $z = 2$, $b = 0$	**D** $z = 2$, $b = 0$

$$\langle \vec{S}(0,t) \cdot \vec{S}(0,0) \rangle \sim t^{-1/z} (\log t)^{-b}$$

OPEN QUESTIONS

Prove the emergence of KPZ equation from first principle in integrable chains

Prove anomalous log-divergence of diffusion constant in non-integrable chains